Gradient Flow Renormalization for Composite Fermion Operators

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Based on M. Black, AH, O. Witzel arXiv:2607.00493
see also: M. Dai, next talk
O. Witzel, Friday 5:30 pm
M. Black et al arXiv:2506.16327

Why renormalization?

Quantum Field Theories require

- Regularization :

  Lattice : $\Lambda_{cutoff} \sim 1/a \quad \Longrightarrow \quad O_{\text{bare}}(a)$
- Renormalization:

  Remove regulator dependence: $O_R(\mu_{IR}) = Z_O(\mu_{IR}, a)\, O_{\text{bare}}(a)$
- Continuum connection:

  Match to $\overline{MS}$: $O_R(\mu_{IR}) \longrightarrow O_{\overline{MS}}(\mu_{UV})$

$$O_{\text{bare}}(a) \xrightarrow{\text{nonperturbative renormalization}} O_R(\mu_{IR}) \xrightarrow{\text{continuum matching}} O_{\overline{MS}}(\mu_{UV})$$

Why renormalization?

Ideal lattice renormalization:

- **Gauge invariant**
- **Nonperturbative definition**
  applicable at hadronic scales, easy to calculate
  on the lattice
- **Practical window**
  small truncation, finite volume, cutoff effects
- **Controlled evolution across scales**
  from the lattice regime to high energy;
  preferably nonperturbative
- **Perturbative matching**
  to $\overline{MS}$

Why Gradient Flow renormalization?

Gradient flow renormalization covers it all:

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- **S**hort **F**low **T**ime e**x**pansion in the $\tau \to 0$ limit
- NLO, NNLO available

Gradient Flow Renormalization

Gradient Flow Renormalization

Gradient flow (GF): continuous, invertible smearing transformation (flow)

➡ **Gauge flow**

$$\partial_\tau B_\mu(\tau;x) = D_\nu G_{\nu\mu}(\tau;x), \quad B_\mu(0;x) = A_\mu(x)$$

- Renormalization ✓

$$g^2_{GF}(\tau) = \mathcal{N}\tau^2\langle E\rangle_\tau\ , \quad \mu \sim 1/\sqrt{8\tau}$$

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- ➡ **Fermion flow** $\partial_\tau\chi(\tau;x) = \Delta\,\chi(\tau;x)\,, \quad \partial_\tau\bar\chi(\tau;x) = \bar\chi(\tau;x)\overleftarrow{\Delta}$
  - Renormalization: needs wave function renormalization

$$\chi_R(\tau;x) \;=\; Z_\chi^{1/2}(\tau)\,\chi(\tau;x)$$

$$O_R^{(\mathrm{GF})}(\tau;x) \;=\; Z_\chi^{n/2}(\tau)\,O(\tau;x)$$

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➡ **Connect to $\overline{MS}$**

- short flow time expansion (SFTX):

$$O_{\overline{\mathrm{MS}}}(\mu;x) \;=\; \lim_{\tau\to 0}\zeta_O^{-1}(\mu,\tau)\,O_R^{(\mathrm{GF})}(\tau;x)$$
$$=\; \lim_{\tau\to 0}\zeta_O^{-1}(\mu,\tau)\,Z_\chi(\tau)O(\tau;x)$$

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Define $Z_\chi(\tau)$

↓

Calculate the renormalized operators

↓

Match to $\overline{MS}$

Gradient Flow schemes - $Z_\chi(\tau)$

Different Z_χ definitions lead to different renormalization schemes

Ringed scheme: Makino, Suzuki, *PTEP* 2014 (2014) 063B02

$$\mathring{Z}_\chi(\tau) \propto \tau^2 \left\langle \bar{\chi}(\tau;x)\,\overleftrightarrow{\mathbb{D}}\chi(\tau;x) \right\rangle^{-1/2}$$

Ringed scheme is preferred for perturbative (SFTX) calculations Borgulat et al, *JHEP* 05 (2024) 179

$\mathring{Z}_\chi$ is difficult to evaluate on the lattice; $\langle \bar{\chi}(\tau;x)\,\overleftrightarrow{\mathbb{D}}\chi(\tau;x) \rangle$ requires "backward" flow(?)

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GF Axial / Vector schemes:

Black, AH, Witzel, arXiv:2607.00493

The axial current is partially conserved, in $\overline{MS}$ scheme $A_{\overline{MS}}(\mu) = A_{\text{bare}}$

Reproduce that in GF :

$$\begin{aligned} \langle A_R^{\text{GF}}(\tau) A_{\text{probe}} \rangle &= \langle A_{\text{bare}} A_{\text{probe}} \rangle \\ \langle A_R^{\text{GF}}(\tau) A_{\text{probe}} \rangle &= \widetilde{Z}_\chi^{(A)}(\tau) \langle A(\tau) A_{\text{probe}} \rangle \end{aligned} \quad \Longrightarrow \quad \widetilde{Z}_\chi^{(A)}(\tau) = \frac{\langle A_{\text{bare}} A_{\text{probe}} \rangle}{\langle A(\tau) A_{\text{probe}} \rangle}$$

$\widetilde{Z}_\chi^{(A)}(\tau)$ is straightforward to evaluate on the lattice from forward-flowed 2-point functions

Vector scheme: $A_0 \to V$

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Ratio notation:

The ratio

$$R_O(\tau) = \frac{\langle O(\tau) O_{\text{probe}} \rangle}{\langle O(0) O_{\text{probe}} \rangle}$$

is independent of Euclidean time and the probe if

- consider zero-momentum correlators at large time separation
- O_{probe} couples to O
  - correlator projects to the ground state
  - O_{probe} cancels

With this notation

$\widetilde{Z}_\chi^{(A)}(\tau) = R_A(\tau)$ or $\widetilde{Z}_\chi^{(A)}(\tau) = Z_A R_A(\tau)$ if the lattice current is not conserved

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$\widetilde{Z}^{(A)}_\chi(\tau) = R_A(\tau)$ or $\widetilde{Z}^{(A)}_\chi(\tau) = Z_A R_A(\tau)$ if the lattice current is not conserved

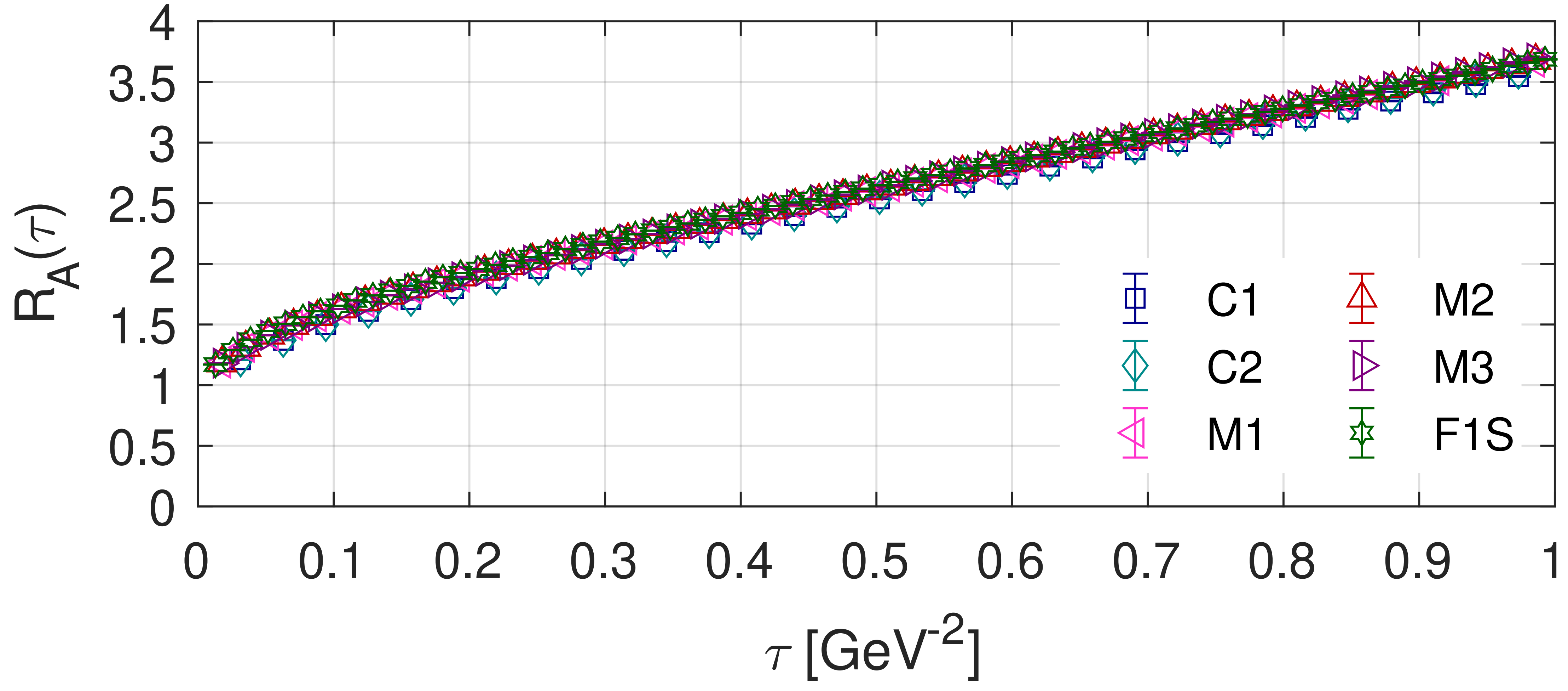
Numerical results : $\widetilde{Z}_\chi^{(A)}(\tau), \quad \widetilde{Z}_\chi^{(V)}(\tau)$

Numerical study:

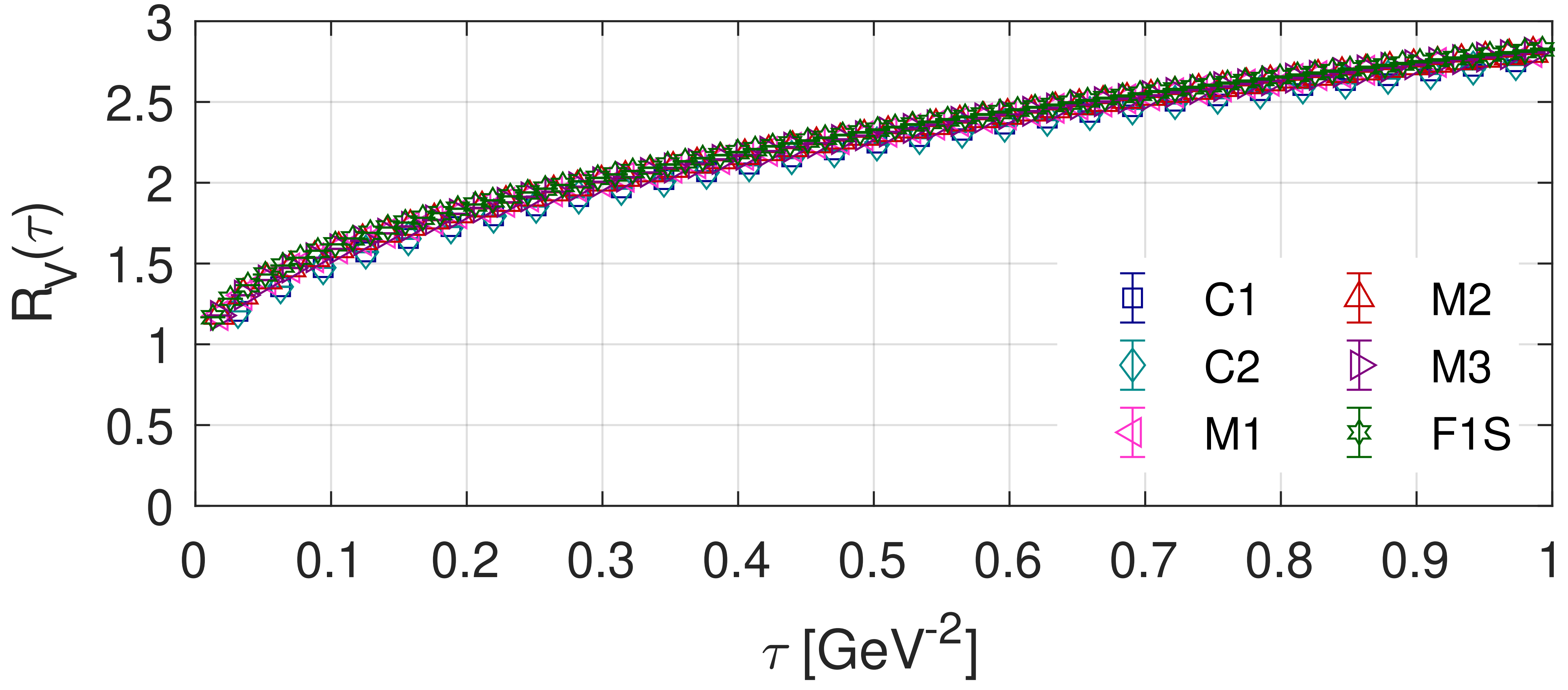
- 2+1 flavor RBC-UKQCD DWF fermion ensembles (from Black et al 2506.16327, 2603.28516, 2603.28517)
- 3 lattice spacings, ($a^{-1} = 1.78 - 2.78$ GeV), 6 ensembles

See also O. Witzel talk, Friday 5:30 pm

$R_A(\tau) = \widetilde{Z}_\chi^{(A)}(\tau)/Z_A,$



$R_V(\tau) = \widetilde{Z}_\chi^{(V)}(\tau)/Z_V$



These are divergent renormalization factors - but the divergence is logarithmic, not observable

Gradient Flow schemes : $Z_O(\tau)$

Renormalization factors for bilinear operators:

$$\begin{aligned} O_R^{(\mathrm{GF})}(\tau;x) &= \widetilde{Z}_\chi(\tau)\, O(\tau;x) \\ O_R^{(\mathrm{GF})}(\tau;x) &= \widetilde{Z}_O(\tau;a) O_{\mathrm{bare}}(a) \end{aligned} \quad \Longrightarrow \quad \widetilde{Z}_O(\tau;a) = \widetilde{Z}_\chi^{(A)}(\tau) \frac{\langle O(\tau) O_{\mathrm{probe}} \rangle}{\langle O(0) O_{\mathrm{probe}} \rangle} \equiv \frac{R_A(\tau)}{R_O(\tau)}$$

Example: renormalized quark mass ($Z_m = 1/Z_P$)

$$\widetilde{m}_{GF}^{(A)}(\tau;a) = \frac{1}{\widetilde{Z}_P^{(A)}(\tau;a)}\, m_{bare}(a) = \frac{R_P(\tau)}{R_A(\tau)}\, m_{bare}(a)$$

Gradient Flow schemes - match to $\overline{MS}$

Ringed scheme:

Ringed scheme is preferred for perturbative (SFTX) calculations

Borgulat et al, *JHEP* 05 (2024) 179

$O_{\overline{\mathrm{MS}}}(\mu) = \lim_{\tau\to 0} \mathring{\zeta}_O^{-1}(\mu,\tau)\,\mathring{Z}_\chi(\tau)\,O(\tau)$, $\quad \mathring{\zeta}_O(\mu,\tau)$ perturbative SFTX matching factors; known to NNLO

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GF Axial scheme:

GF axial scheme has new matching coefficients $\widetilde{\zeta}_O(\mu,\tau)$

$$O_{\overline{\rm MS}}(\mu) = \lim_{\tau\to 0} \widetilde{\zeta}_O^{-1}(\mu,\tau)\, \widetilde{Z}_\chi^{(A)}(\tau)\, O(\tau)$$

In $\overline{MS}$ scheme $A_{\overline{MS}}(\mu) = A_{\rm bare}$, in GF scheme $\langle A_R^{\rm GF}(\tau) A_{\rm probe}\rangle = \langle A_{\rm bare} A_{\rm probe}\rangle$ therefore $\widetilde{\zeta}_A(\mu,\tau) = 1$

and

$$\widetilde{\zeta}_O(\mu,\tau) = \mathring{\zeta}_O(\mu,\tau)\mathring{\zeta}_A^{-1}(\mu,\tau)$$

$$O_{\overline{\rm MS}}(\mu_{UV}) = \lim_{\tau\to 0} \mathring{\zeta}_O^{-1}(\mu_{UV},\tau)\, \mathring{\zeta}_A(\mu_{UV},\tau)\, \widetilde{Z}_\chi^{(A)}(\tau)\, O(\tau)$$

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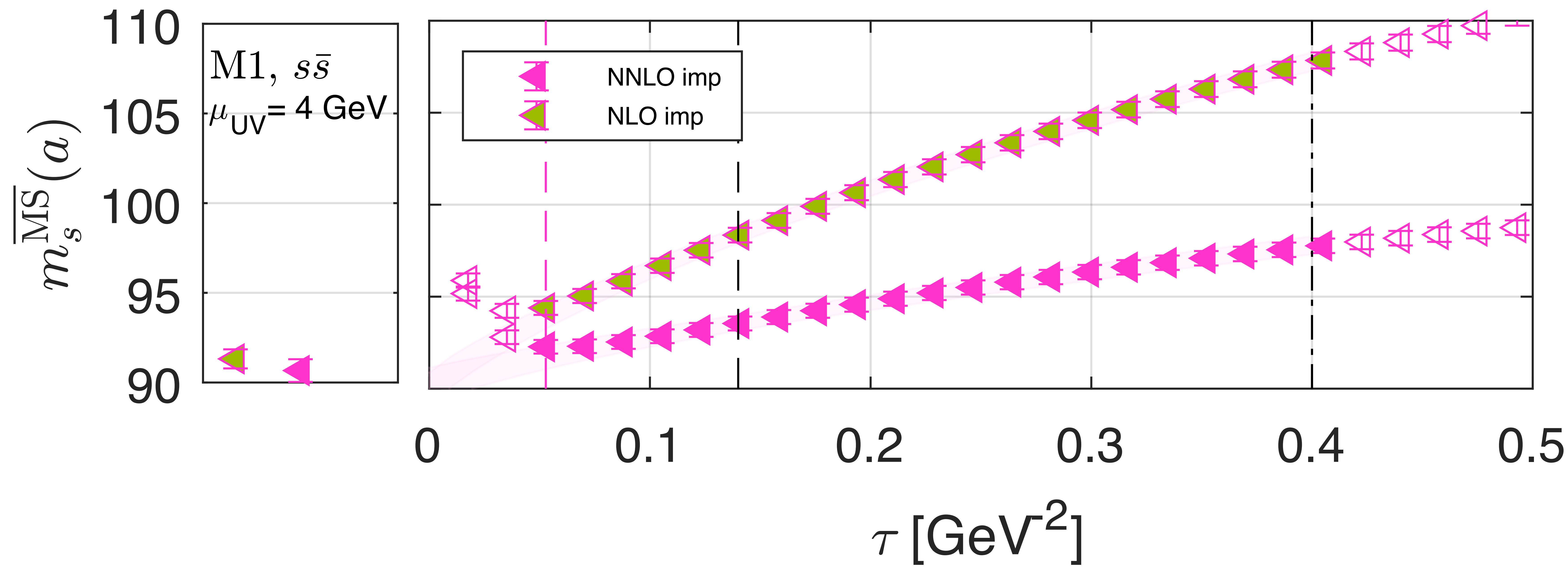
Numerical results - quark masses

$$m_{\overline{MS}}(\mu_{UV}, a) = \lim_{\tau \to 0} \mathring{\zeta}_P^{-1}(\mu_{UV}, \tau)\, \mathring{\zeta}_A(\mu_{UV}, \tau)\, \widetilde{m}_{GF}^{(A)}(\tau; a) \;, \qquad \widetilde{m}_{GF}^{(A)}(\tau; a) = \frac{1}{\widetilde{Z}_P^{(A)}(\tau; a)}\, m_{bare}(a)$$



NLO, NNLO show significant τ dependence, but the τ → 0 limits are consistent
The results are very similar and in τ → 0 limits are consistent with PCAC approach

M. Black et al arXiv:2506.16327

Anomalous dimension $\gamma_O(\tau)$

Operator anomalous dimensions are defined

$$\widetilde{\gamma}_O^{(A)}(\tau) = -2\tau \frac{d}{d\tau} \log \widetilde{Z}_O^{(A)}(\tau) = -2\tau \frac{d}{d\tau} \log \frac{R_A(\tau)}{R_O(\tau)}$$

If $\widetilde{\gamma}_O^{(A)}(\tau)$ is known in the *continuum limit*, we can calculate the flow time evolution

$$C_m(\tau^*, \tau) = \exp\left[-\frac{1}{2}\int_\tau^{\tau^*} \frac{d\tau'}{\tau'}\,\widetilde{\gamma}_O^{(A)}(\tau')\right] = \frac{\widetilde{Z}_O(\tau^*)}{\widetilde{Z}_O(\tau)}$$

and extend lattice τ to larger energies τ^* ($\tau^* < \tau$)

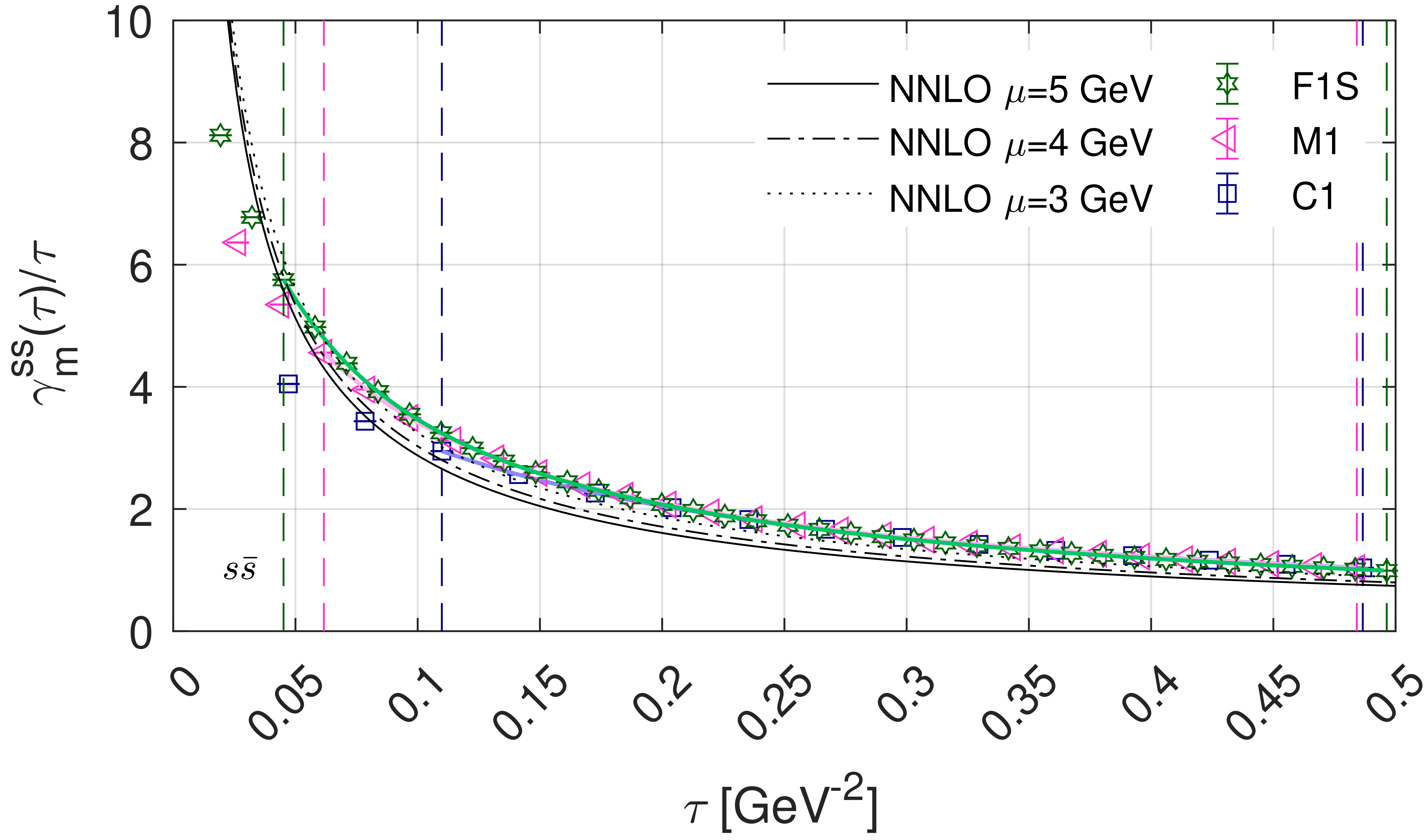
$$\langle \widetilde{O}_{GF}^{(A)}(\tau^*)\, O(0)\rangle = C_m(\tau^*, \tau)\langle \widetilde{O}_{GF}^{(A)}(\tau)\, O(0)\rangle,$$

improving the $\tau \to 0$ SFTX matching

Numerical results : $\widetilde{\gamma}_m^{(A)} \equiv -\widetilde{\gamma}_P^{(A)}$

$$\widetilde{\gamma}_O^{(A)}(\tau) = -2\tau\frac{d}{d\tau}\log\widetilde{Z}_O^{(A)}(\tau) = -2\tau\frac{d}{d\tau}\log\frac{R_A(\tau)}{R_O(\tau)}$$

$\widetilde{\gamma}_m(\tau)/\tau$ at the strange quark mass



The agreement between different ensembles for $\tau/a^2 \gtrsim 0.3$ suggests mild continuum limit extrapolation

Agreement with SFTX at small $\tau \lesssim 0.05\mathrm{GeV}^{-2}$ suggests smooth connection to PT

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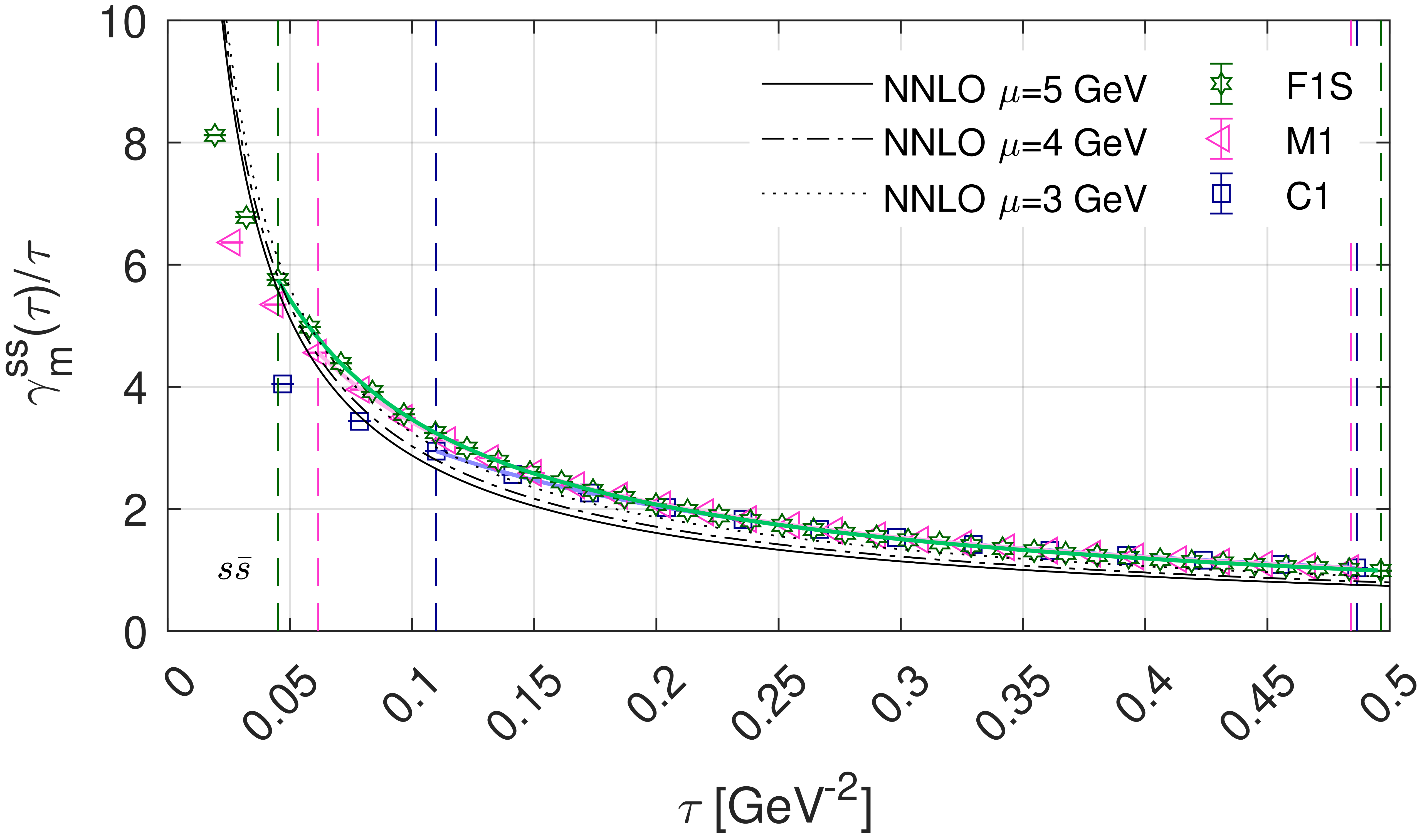

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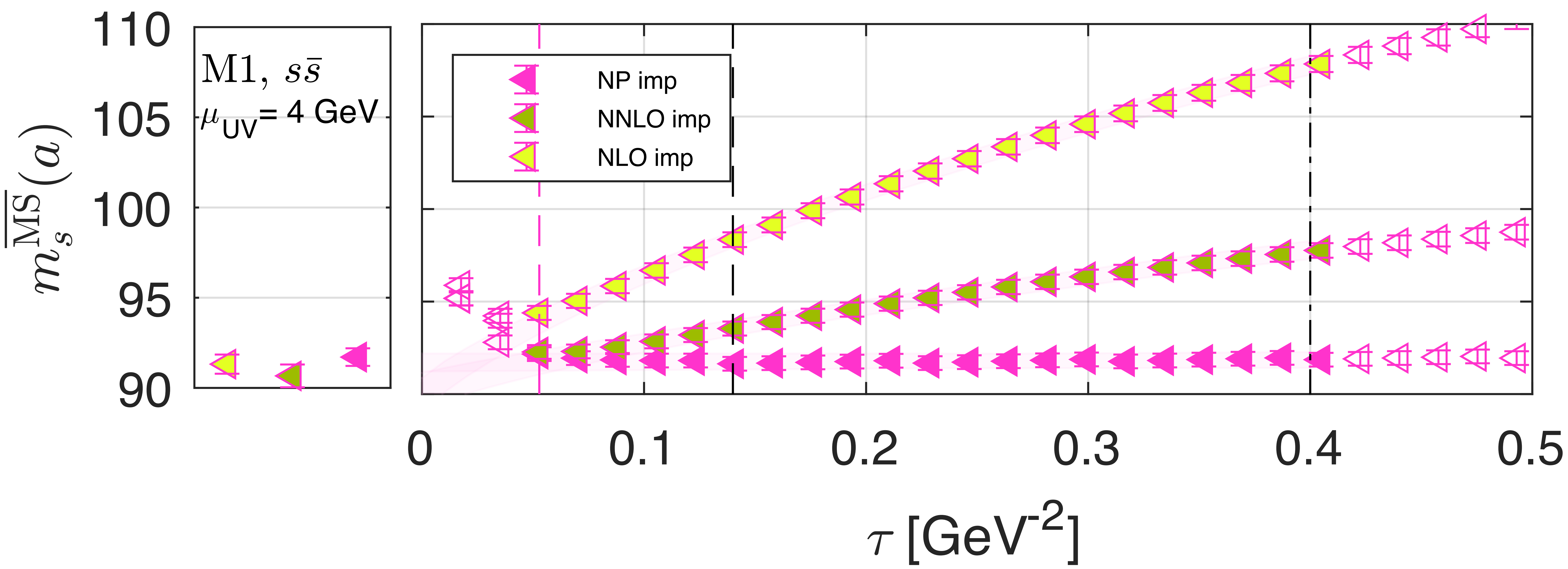
Proxy for the continuum limit $\widetilde{\gamma}_m(\tau)/\tau$

- F1S values for $\tau \gtrsim 0.045\,\mathrm{GeV}^{-2}$
- and SFTX for $\tau \lesssim 0.045\,\mathrm{GeV}^{-2}$

Numerical results - quark masses

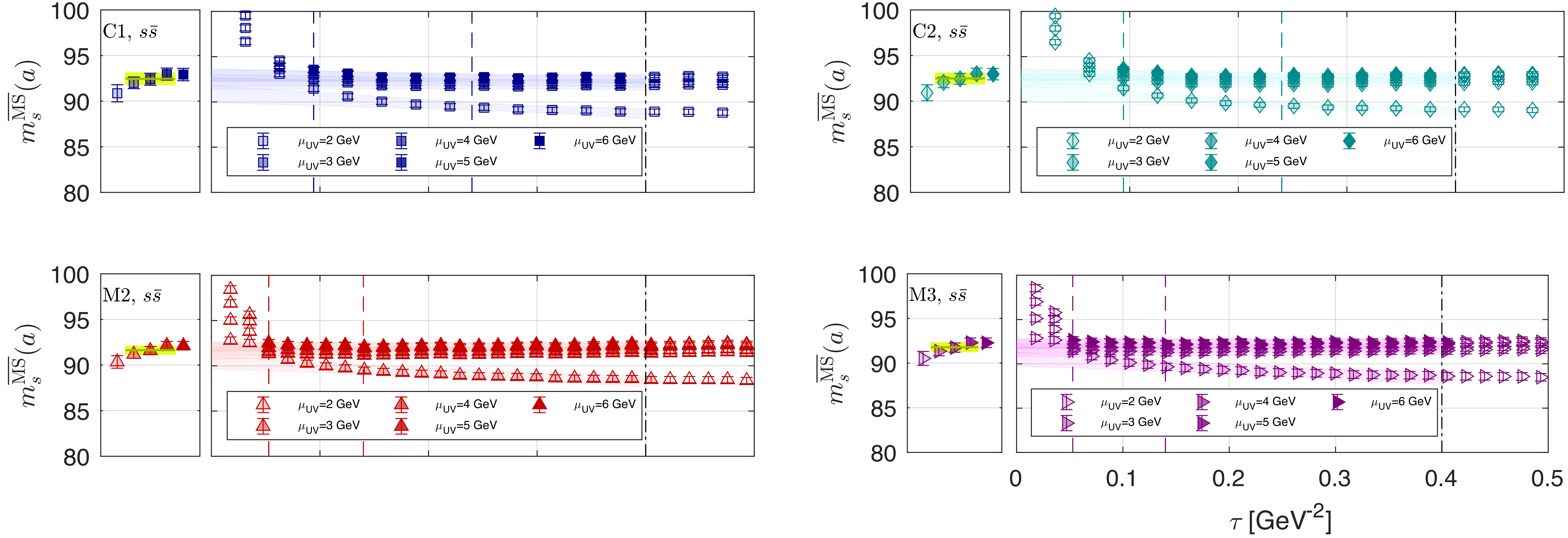
NP - improved : $m_{\overline{MS}}(\mu_{UV}, a) = \lim_{\tau \to 0} \mathring{\zeta}_P^{-1} \mathring{\zeta}_A(\mu_{UV}, \tau^*)\, C_m(\tau^*, \tau)\, \widetilde{m}_{GF}^{(A)}(\tau; a)$, $\tau^* = 0.045\mathrm{GeV}^{-2}$



The nonperturbative $\gamma(\tau)$ removes practically all τ dependence, making SFTX more reliable

Numerical results - NP improved quark masses

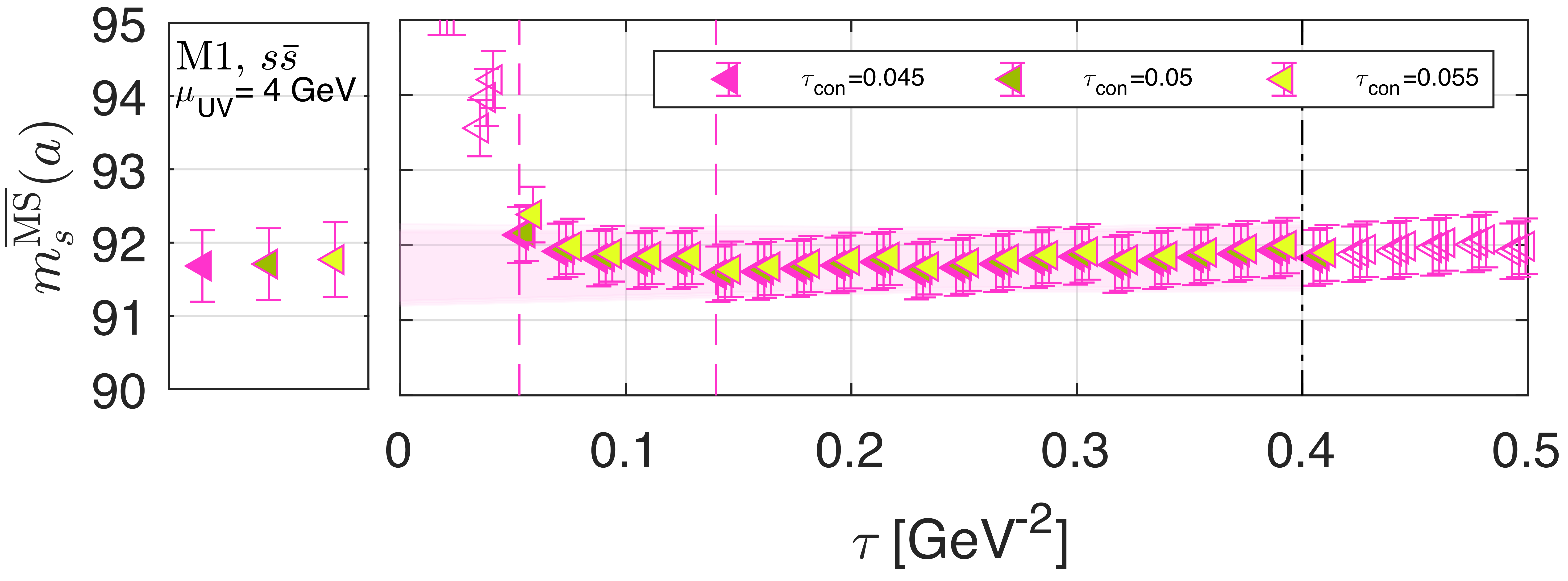
Results are similar for other ensembles - most τ dependence is removed



$\mu_{UV} = 2\mathrm{GeV}$ is slightly off, $\mu_{UV} \geq 3\mathrm{GeV}$ are consistent

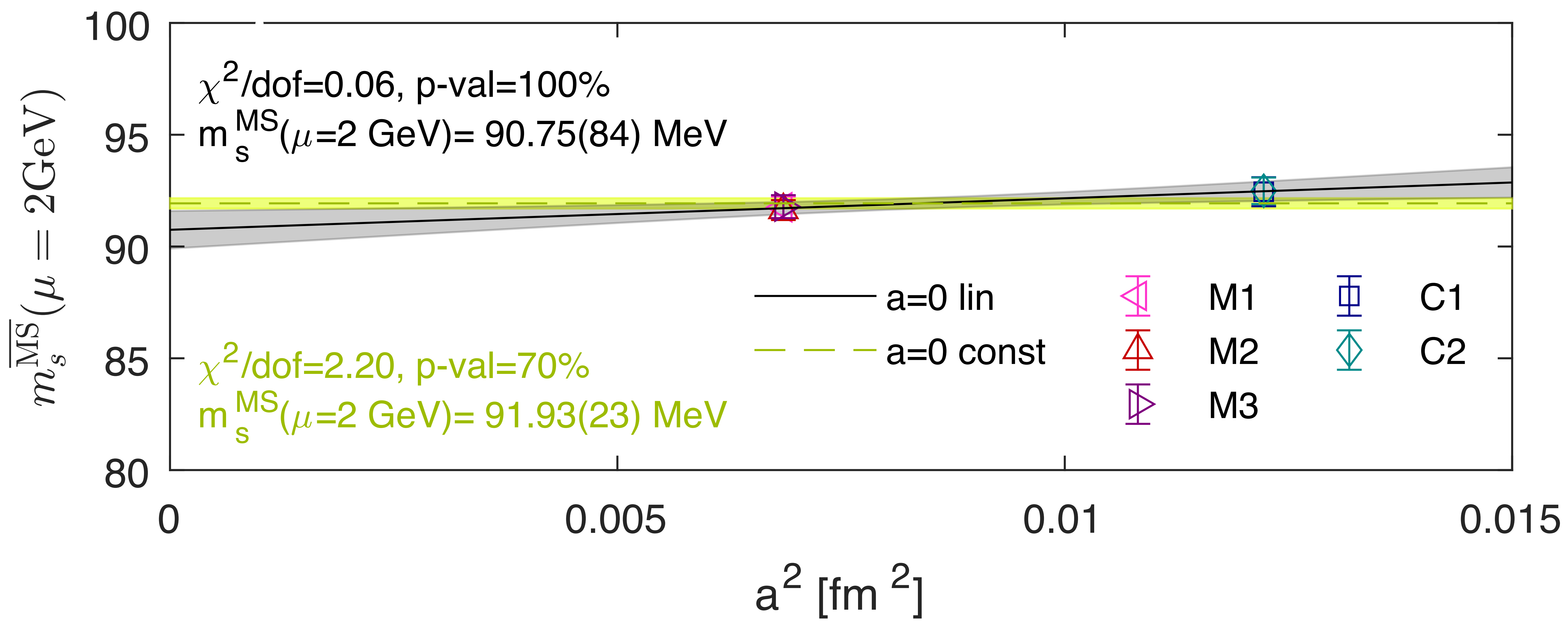
Numerical results - NP improved quark masses

Connecting NP and SFTX at $\tau^* = 0.45\,, 0.50\,, 0.55\,\mathrm{GeV}^{-2}$ has very small effect



Numerical results - NP improved quark masses

Ensembles show only mild cutoff effetcs; $a^2 \to 0$ limit is consistent with expectations



Summary

Gradient flow renormalization for composite fermions is attractive option

- Only wave function renormalization Z_χ is needed
- Connecting to $\overline{MS}$ is done via SFTX

Axial and vector schemes define Z_χ via conserved currents

- Z_χ, Z_O are determined via ratios of 2-point function
- Numerically simple observables

Renormalization factors for arbitrary operators Z_O are expressed via 2-point fn's

Nonperturbative γ_O predicted as the logarithmic derivative of Z_O.
Integrating the *continuum limit* $\gamma_O(\tau)/\tau$ removes most τ dependence, making the $\tau \to 0$ limit of SFTX well controlled

Acknowledgements

- A.H. acknowledges support from DOE grant DE-SC001005.
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- O.W. received support from the Deutsche Forschungsgemeinschaft (through grant 396021762 — TRR 257 and under Germany's Excellence Strategy -- EXC 3107 -- Project 533766364.

- We thank the RBC/UKQCD Collaboration for generating and making theirgauge field ensembles publicly available.

- The data analyzed here were generated as part of the project to calculate heavy meson lifetimes (Black et al)
- These computations used resources provided by the OMNI cluster at the University of Siegen, the HAWK cluster at the High-Performance Computing Center Stuttgart, and LUMI-G at the CSC data center Finland (DeiC National HPC g.a.~DEIC-SDU-L5-
  and DEIC-SDU-N5-2024053).

Extra slides

GF A-scheme

Correlator ratio should be taken at large time separation (projects to lowest state)

$$\widetilde{Z}_\chi^{(A)}(\tau) = \frac{\langle A_{0,\mathrm{bare}} A_{0,\mathrm{probe}} \rangle}{\langle A_0(\tau) A_{0,\mathrm{probe}} \rangle} = \lim_{t\to\infty} \frac{\langle A_0(\tau=0;t) A_0(\tau=0;t=0)\rangle}{\langle A_0(\tau;t) A_0(\tau=0;t=0)\rangle}$$

Renormalized operator:

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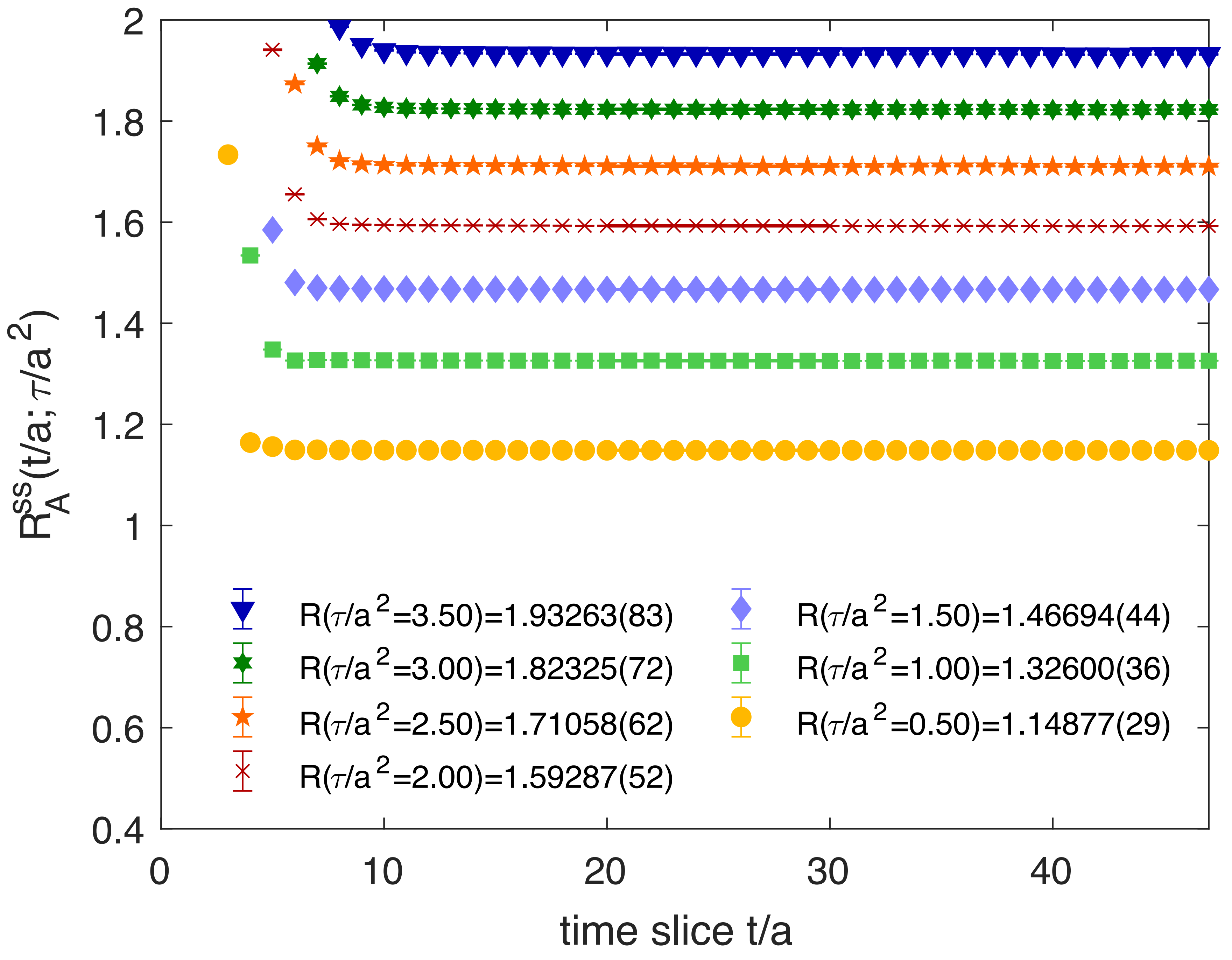

$R_A(\tau)$ is independent of Euclidean separation if $t \to \infty$ since both numerator and denominator couples to the same ground state

Numerical study: 2+1 flavor DWF fermions as in Black et al 2506.16327, 2603.28516, 2603.28517
See O. Witzel talk

Renormalization factors - example

The renormalized quark mass is

$$\widetilde{m}^{(A)}_{GF}(\tau) = \frac{1}{\widetilde{Z}^{(A)}_{P}(\tau)} m_{\mathrm{bare}} = \frac{1}{Z^{(A)}_{\chi}} \frac{\langle P(0)A_0(0)\rangle}{\langle P(\tau)A_0(0)\rangle} m_{\mathrm{bare}}$$

probe: A_0 couples to P

$$= \frac{1}{Z_A} \frac{\langle A_0(\tau)A_0(0)\rangle}{\langle A_0(0)A_0(0)\rangle} \frac{\langle P(0)A_0(0)\rangle}{\langle P(\tau)A_0(0)\rangle} m_{\mathrm{bare}}$$

$$= \frac{1}{Z_A} \frac{\langle A_0(\tau)A_0(0)\rangle}{\langle P_0(\tau)A_0(0)\rangle} \frac{\langle P(0)A_0(0)\rangle}{\langle A_0(0)A_0(0)\rangle} m_{\mathrm{bare}}$$

notice bare and renormalized PCAC forms

$$= \frac{2\widetilde{m}^{(A)}_{GF}(\tau)}{2M_{PS}} \frac{2M_{PS}}{2m_{\mathrm{bare}}} m_{\mathrm{bare}} = \widetilde{m}^{(A)}_{GF}(\tau)$$

In the V-scheme we get different lattice form but identical result in the $\tau \to 0$ limit