arxiv:2606.16877

Direct calculation of parton distributions in momentum space from lattice QCD

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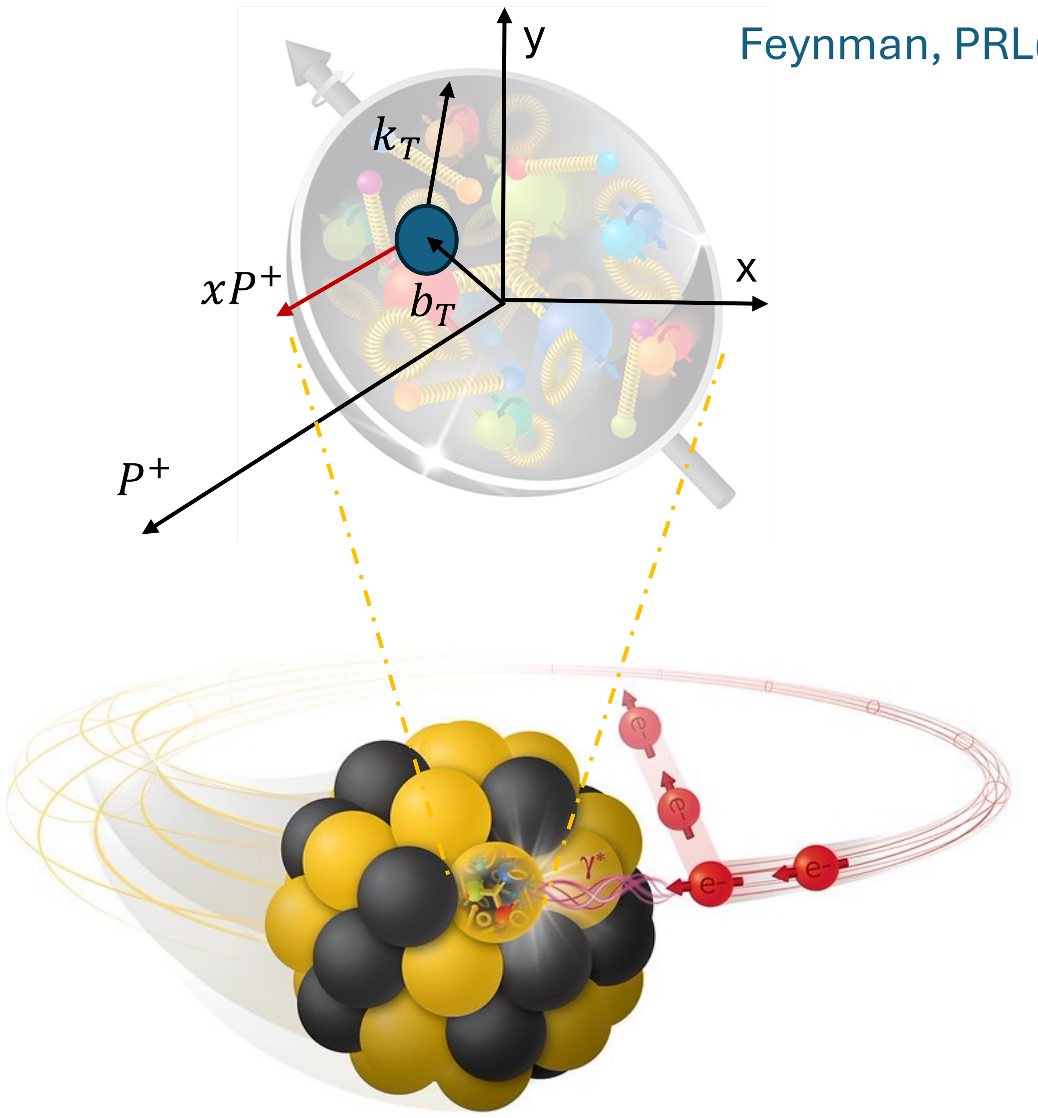
Parton Distributions



Feynman, PRL(1969)

Image credit: BNL

Wigner Distribution/GTMDs $f(x, b_T, k_T)$

- $\int d^2 b_T$ → **TMDs** $f(x, k_T)$
- $\int d^2 k_T$ → **IPDs** $f(x, b_T)$

IPDs $f(x, b_T)$ $\xleftrightarrow{\text{F. T.}}$ **GPDs** $f(x, t)$

- **TMDs** $f(x, k_T)$ $\xrightarrow{\int d^2 k_T}$ **PDFs** $f(x)$
- **IPDs** $f(x, b_T)$ $\xrightarrow{\int d^2 b_T}$ **PDFs** $f(x)$
- **GPDs** $f(x, t)$ $\xrightarrow{\int dx\, x^{n-1}}$ **Form Factors** $A_{n,0}(t) + \cdots$

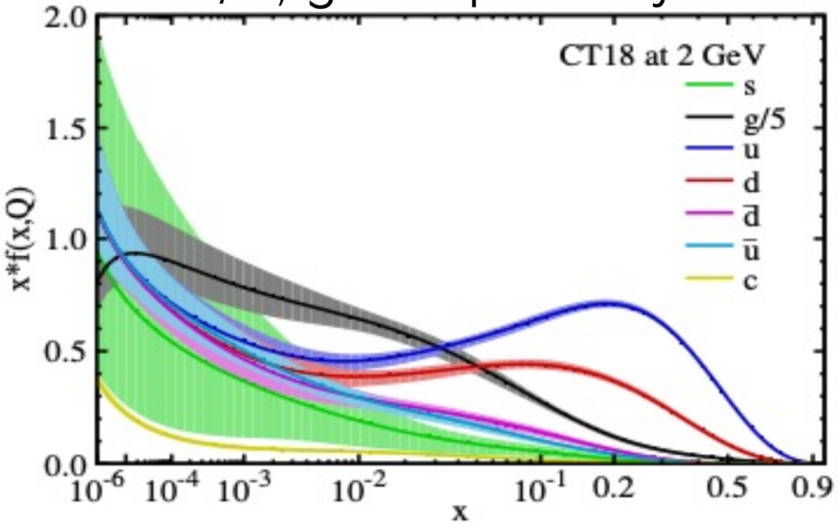
Extracting Parton Physics from Exp Data

Global Fitting is solving an inverse problem (IP) with large amount of data points:

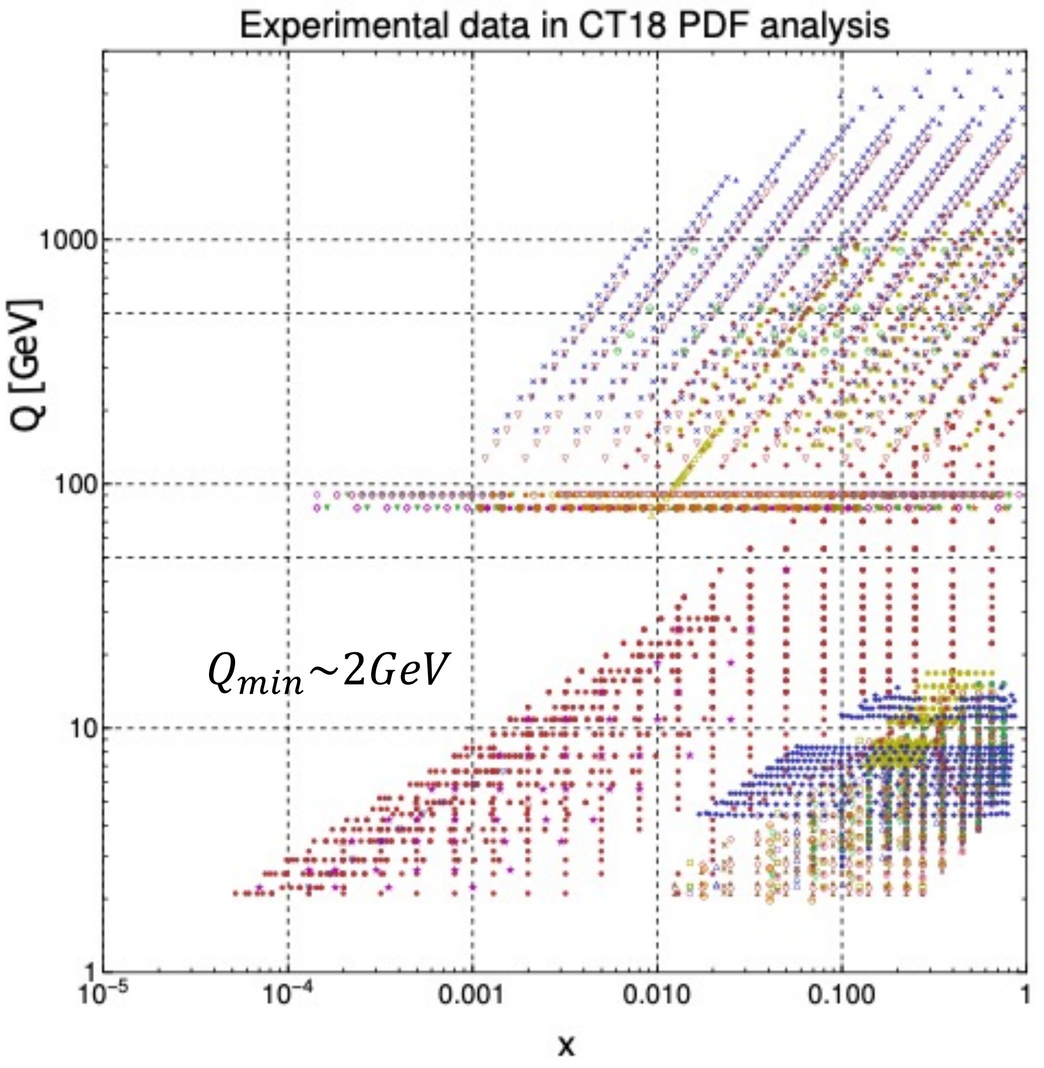
$$\text{Cross Section} = \text{Pert Kernel} \otimes \text{Parton Distributions} + O\left(\frac{\Lambda_{\text{QCD}}^n}{Q^n}\right)$$

More data, less model dependences.

- Unpolarized u/d, g PDF precisely determined.



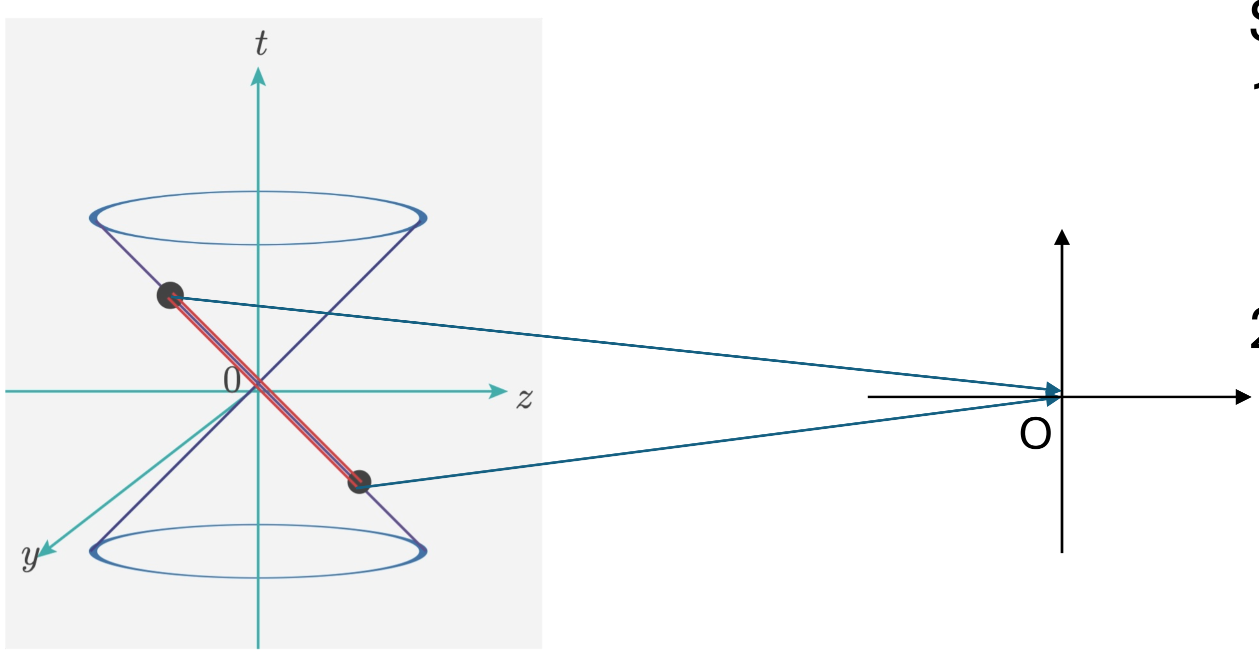
- Many other observables are poorly constrained.
- Lattice QCD calculations are important inputs.



Hou et al. PRD (2019)

Parton physics on lattice ...

Real time dependence. Not directly measurable?



Minkowski Spacetime

Euclidean Lattice

Kronfeld & Photiadis, PRD(1985)
Martinelli & Sachrajda, PLB(1987
Shindler, PRD(2024)

Solutions:

1. Local operators from OPE
    - Measure moments
    - Gradient Flow
2. Indirect probes
    - Hadronic tensor
    - “Lattice cross sections”
    - HOPE
    - Ioffe time distribution
    - Pseudo-PDF
3. Effective theory
    - LaMET

Indirect probe of parton physics

Liu & Dong, PRL(1994)
Detmold & Lin, PRD(2006)
Braun, EPJC(2008)
Radyushkin, PRD(2017)
Ma & Qiu, PRL(2018)
Chambers, et.al, PRL(2017)

Lattice: $h(z,\omega) = \; UV \otimes \; Parton\ Physics + O\left(z^n \Lambda_{\mathrm{QCD}}^n\right)$

($z \to q^{-1}$ in hadronic tensor/ HOPE / Compton amplitude)

Mellin space: $h(z,\omega) = \sum \omega^n \, C(z,\mu)\langle x^n \rangle + O\left(z^n \Lambda_{\mathrm{QCD}}^n\right)$

Require hard scale: $Q \sim z^{-1} \gg \Lambda_{\mathrm{QCD}}$

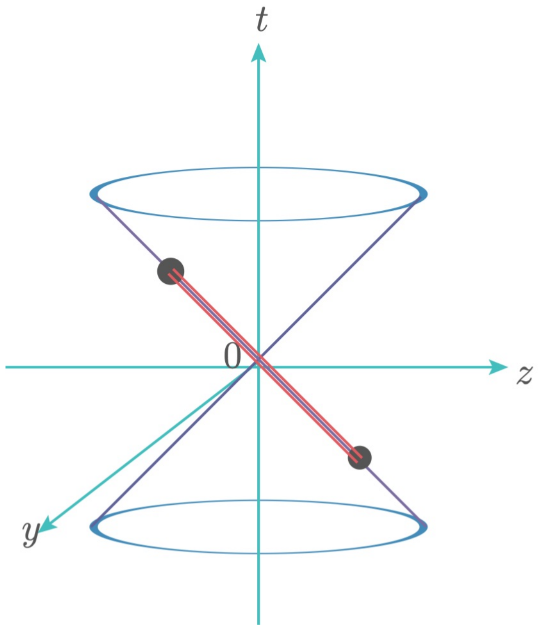
1. Extracting Lowest few Mellin Moments, or
2. Solve the IP to obtain x-dependence

Need assumptions on the x-space functional form.

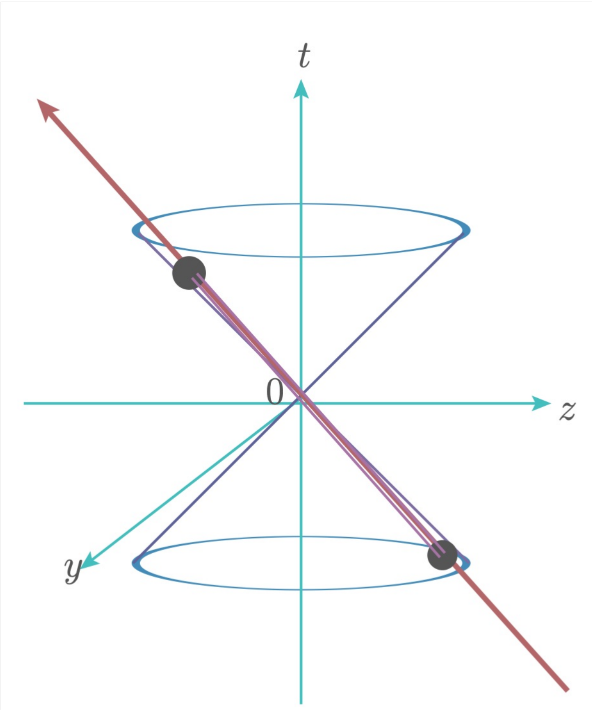
Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)

Forward Matching



Large P_z
Expansion



$$F(x,\mu) = C\,(x,y,\mu,P_z) \otimes \tilde{F}(y,P_z)$$

Lightcone Distribution = Quasi-Distribution

Infinite Power Corrections

$$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$$

$$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2},\frac{\Lambda_{QCD}^2}{\bar{x}^2 P_z^2}\right)$$

LaMET universality class (example for PDFs)

- $\tilde{F}(y, P_z)$ can be anything within a universality class
  - Quasi-PDF: $\int e^{ixzP_z} \langle P|\bar{\psi}(z)W(z,0)\Gamma\psi(0)|P\rangle$
  - Coulomb gauge quasi-PDF: $\int e^{ixzP_z} \langle P|\bar{\psi}_C(z)\Gamma\psi_C(0)|P\rangle$
  - Current-Current distribution: $\int e^{ixzP_z}\, z^3 \langle P|\bar{\psi}(z)\Gamma^a\psi(z)\bar{\psi}(0)\Gamma^b\psi(0)|P\rangle$
  - …

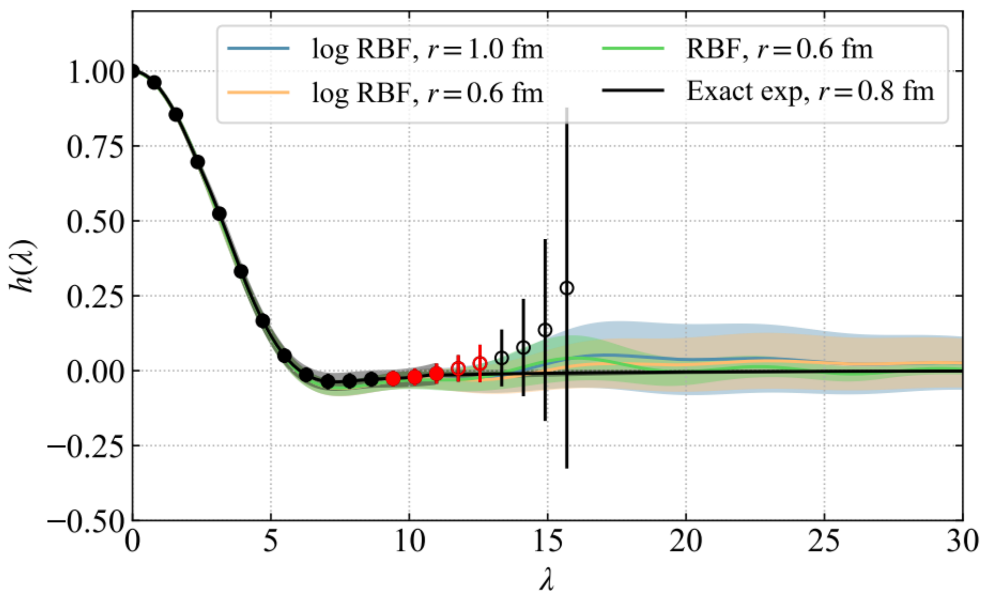
Zhao, PRL (2024)
Gao, Liu & Zhao, PRD(2023)
Jialu Zhang, Ji, Schäfer, **RZ**, Zimmerman, arxiv:2605.03234

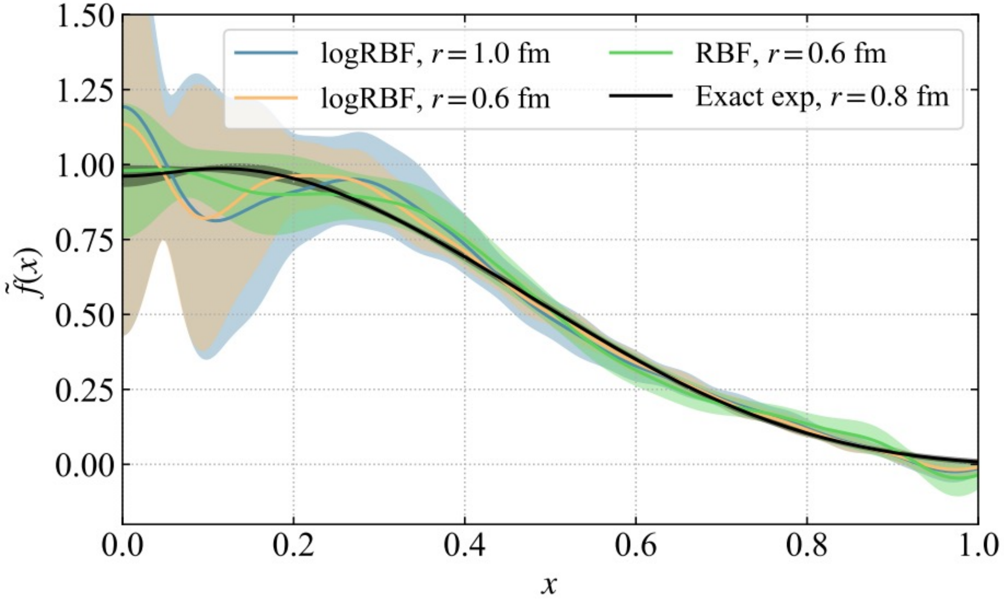
- Same IR physics as lightcone parton distributions
  - Lightcone parton distributions / physical cross sections can be factorized into quasi distributions and perturbative matching, if $\tilde{F}(y, P_z)$ is fully known.

Ji, NPB (2024)

Systematics in x-space LaMET

Chen, et.al., PRD (2026)
Ji, Liu & Su, arxiv:2601.12189





FT from $z(\lambda)$ space to x space:

$$\tilde{F}(x, P_z) = \int_{\infty}^{\infty} \frac{P_z dz}{2\pi} \tilde{h}(z, P_z) e^{ixzP_z}$$

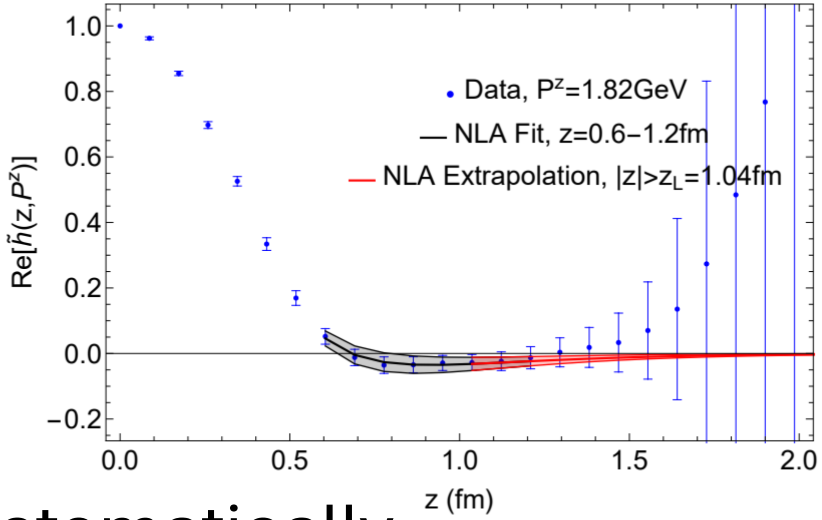
- Noisy $\tilde{h}(z, P_z)$ at large z, need truncation.
- Formally an IP if no constraint.

Solution: Asymptotic Analysis

Ji, Liu & Su, arxiv:2601.12189

Physical Constraint: Hadronic correlations decay exponentially.

- Explicit forms derived by Yushan Su, et.al..
- The long tails are largely constrained.



- When to apply?
  - Data are precise enough near $z\sim0.8(0.5)$ fm

- Why solved? Large z contribution is below a systematically improvable bound.

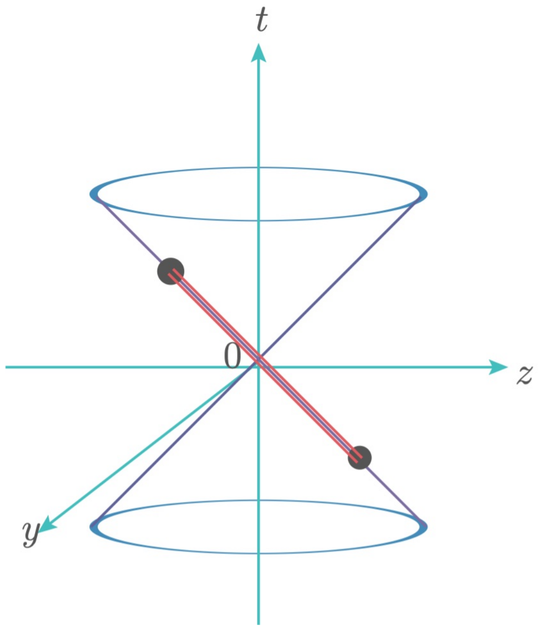
Gao, et.al., PRL (2022)
Chen, et.al., PRD (2026)

$$\delta f(x,P,\lambda_L) < \frac{4\mathcal{O}(1)\left|\tilde{h}(z,P;\lambda_L)\right|_{\max}}{\pi x}$$

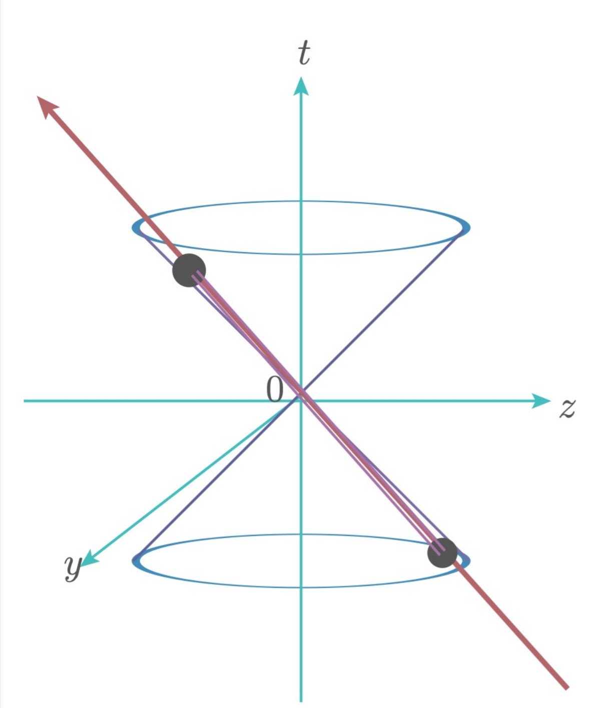
($x \to 0$ is less constrained for PDF. $x \to \{0,1\}$ for DA, $x \to \{\pm\xi, \pm1\}$ for GPD.)

All we need is $\tilde{F}(y, P_z)$

Can we measure $\tilde{F}(y, P_z)$ directly from lattice?



Large P_z
Expansion



Infinite Power Corrections

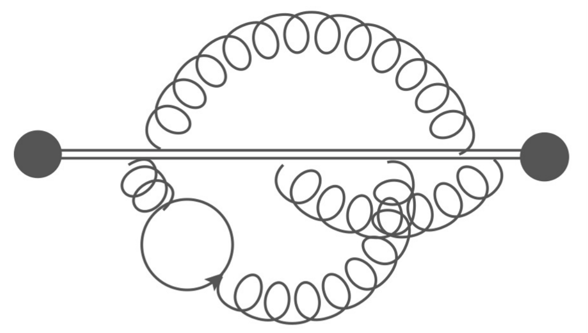
$$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$$

$$F(x, \mu) = C\,(x, y, \mu, P_z) \otimes \tilde{F}(y, P_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mathrm{x}^2 P_z^2}, \frac{\Lambda_{QCD}^2}{\bar{\mathrm{x}}^2 P_z^2}\right)$$

Lightcone Distribution

Quasi-Distribution

Why start from z-space in earlier works?



- Renormalization of operator $O(z) = \bar{\psi}(z)W(z,0)\psi(0)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a) \cdot |z|}$
  - Renormalization constant $Z(a,z,\mu) \sim e^{-\delta m(a) \cdot |z|}$ is z-dependent

$$\tilde{q}^R(x, P_z, \mu) = \int \frac{dzP_z}{2\pi} e^{ixzP_z} \frac{\tilde{h}^B(z, P_z, a)}{Z(z, a, \mu)}.$$

- Renormalization is implemented before F.T.
- Renormalized large z data have to be truncated/modeled due to exponentially growing noise → asymptotic analysis in LaMET

Alternative: Quasi-PDF without Wilson line

Gao, et.al., PRD (2023)
Zhao, PRL (2024)

- Coulomb gauge $O_C(z) = \bar{\psi}_C(z)\Gamma\psi_C(0), \quad \psi_C(x) = e^{-ig\frac{\nabla\cdot A}{\nabla^2}}\psi(x)$
- In the $P_z \to \infty$ limit, $\nabla \cdot A = 0 \to A^+ = 0$
- Renormalization of $O_C(z)$ is independent of z

$$O_C^R(z,\mu) = O_C^B(z,a)/Z_C(a,\mu).$$

- Directly define x-space momentum density operator:

$$\tilde{O}_C(k_z) \equiv \sum_{z=-L/2}^{L/2-a} e^{izk_z}\bar{\psi}(0)\gamma^t\psi(z) \qquad \tilde{q}_C^R(x,P_z,\mu) = \frac{\tilde{q}_C^B(x,P_z,a)}{Z_C(a,\mu)} = P_z\langle P|O_C^R(xP,\mu)|P\rangle$$

- Generalization to 3D TMDPDF:

$$\tilde{O}_C(\vec{k}) \equiv \sum_{\vec{b}\in V} e^{i\vec{k}\cdot\vec{b}}\bar{\psi}(0)\gamma^t\psi(\vec{b})$$

Where does the systematics go?

$z-$ space method

Uncertainty control in FT from $z-$space

Bounded, improvable model dependence of asymptotic analysis

$x-$ space method

Extra finite volume effects for off-grid momenta $xP_z \neq n\times\frac{2\pi}{L}$

$\frac{\Delta\tilde{q}(\mathrm{x})}{\tilde{q}(\mathrm{x})} \sim e^{-\Lambda L}|\sin(xLP_z/2)|, \quad \Lambda \geq m_\pi$

A type of common lattice artifacts. Automatically removed during infinite volume extrapolation.

Lattice test for pion

- The ensemble is generated by MILC collaboration

Lattice Spacing	m_π	Lattice Volume	$m_\pi L$	Fermion Action (sea)	Fermion Action (Valence)
0.06 fm	310 MeV	$48^3 \times 144$	3.7	2+1+1f HISQ	Clover-Wilson

Momentum Smearing	P_z	Gauge Smear	Samples	Sources/cfg
$k = 1.7$ GeV	{2.15,2.6}GeV	1-HYP	100	16

Bali, et.al., PRD (2016)

RZ, Grebe, Hackett, Wagman, Zhao, PRD (2025)

Kinematically enhanced interpolators to improve precision (~15X)

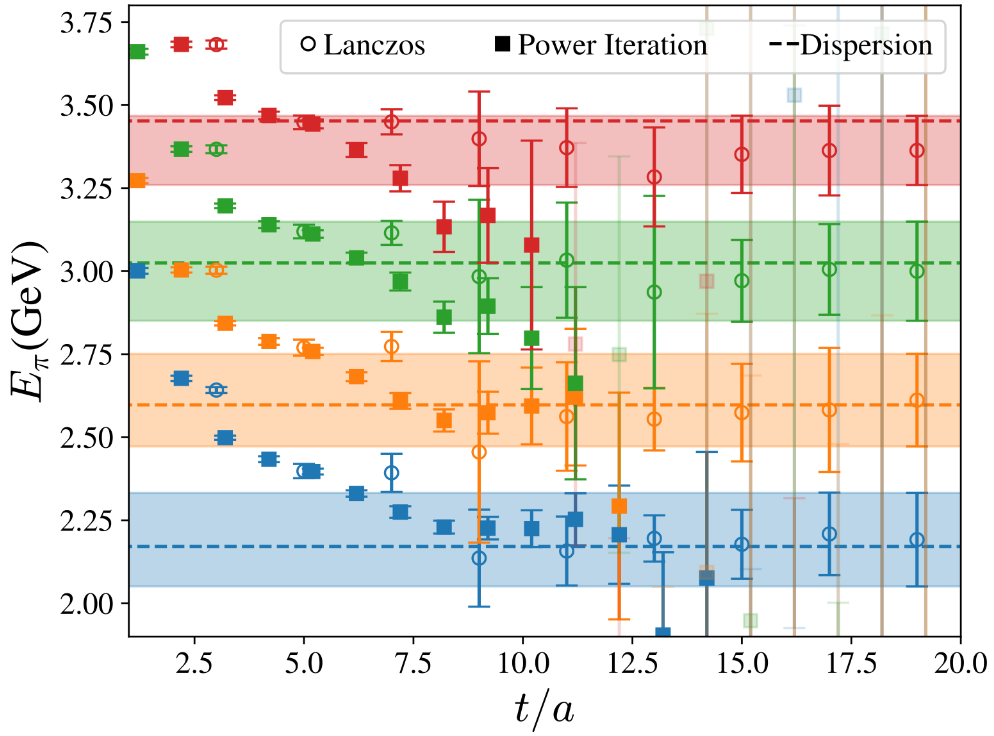
Anisotropic momentum smearing width $\vec{\sigma} = (8a, 8a, 3a)$ in preparation

Extraction of ground-state matrix elements

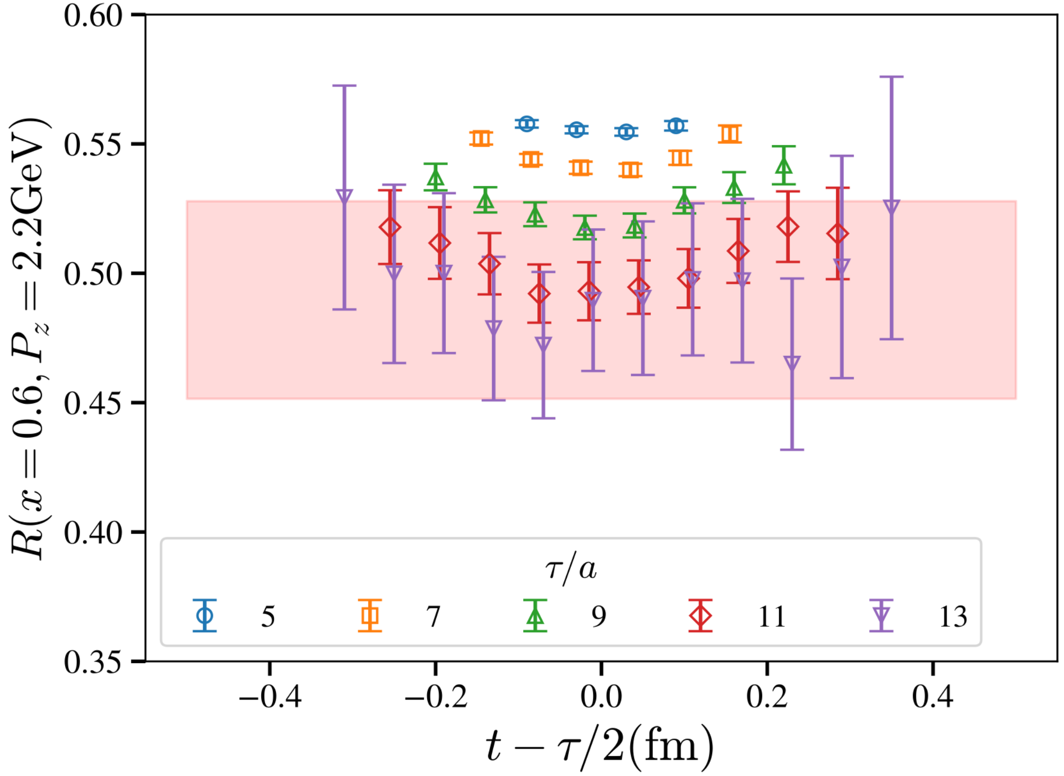
Wagman, PRL (2025)
Hackett & Wagman., PRD(2025)

We use Lanczos analysis to extract ground state

Grounds-state energy for different momenta

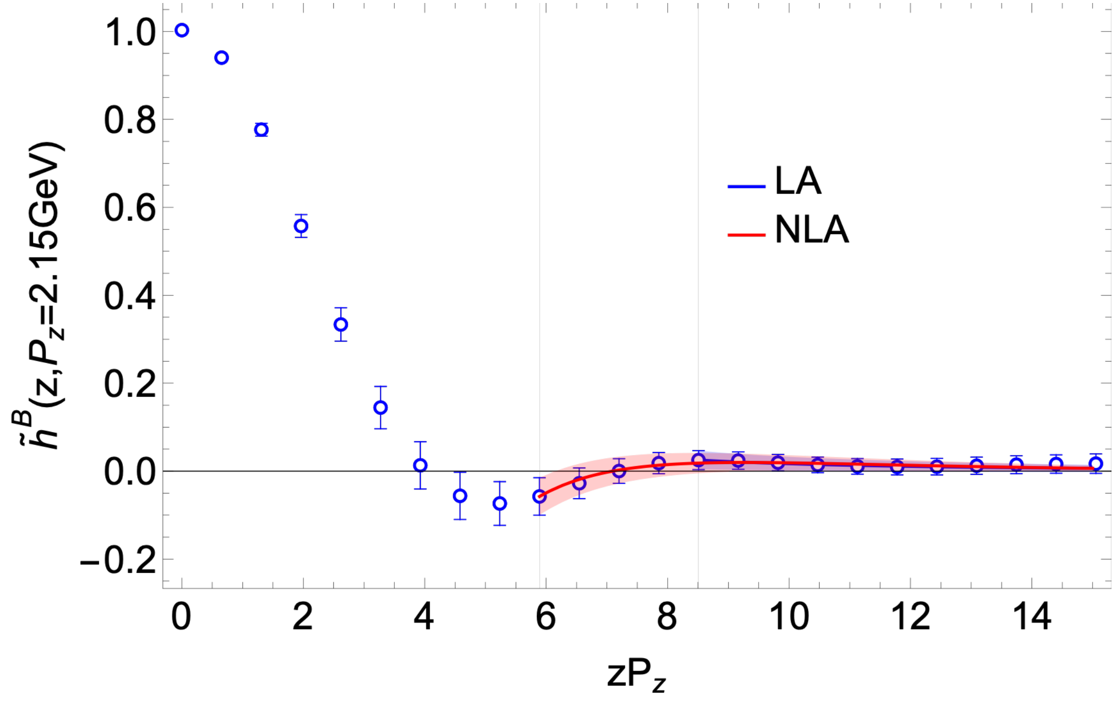


Ground-state matrix elements in x-space

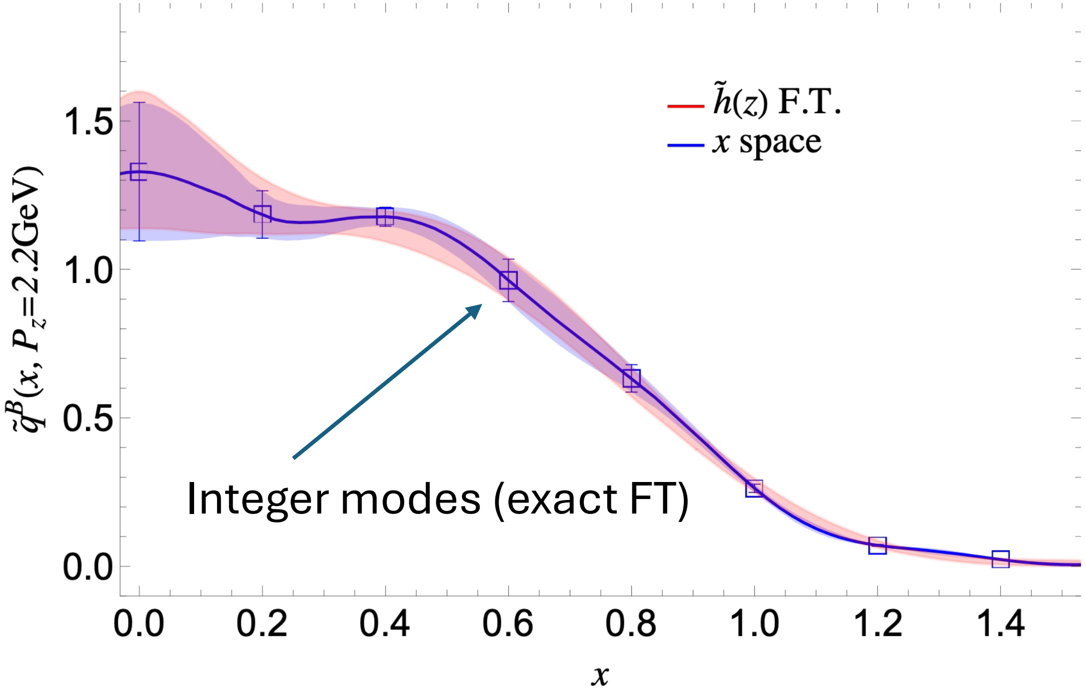


Quasi-PDF: Compare with z-space analysis

Asymptotic analysis in z-space



Compare two approaches

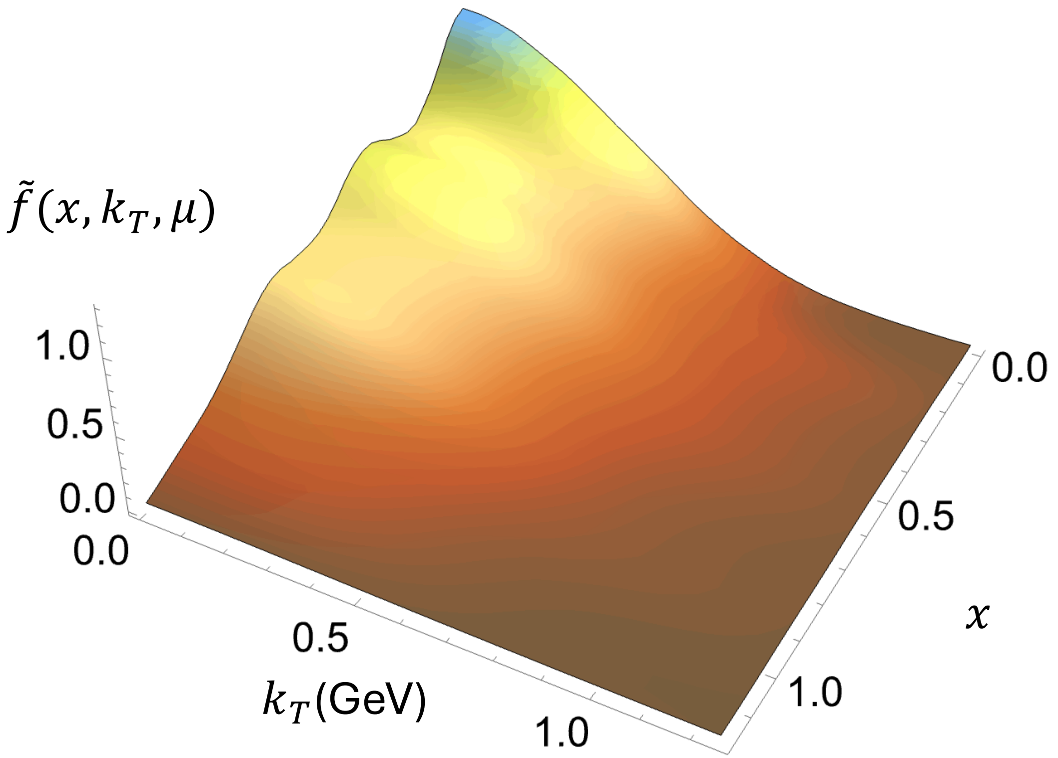


With fast enough decay in the first period, the two approaches are consistent.

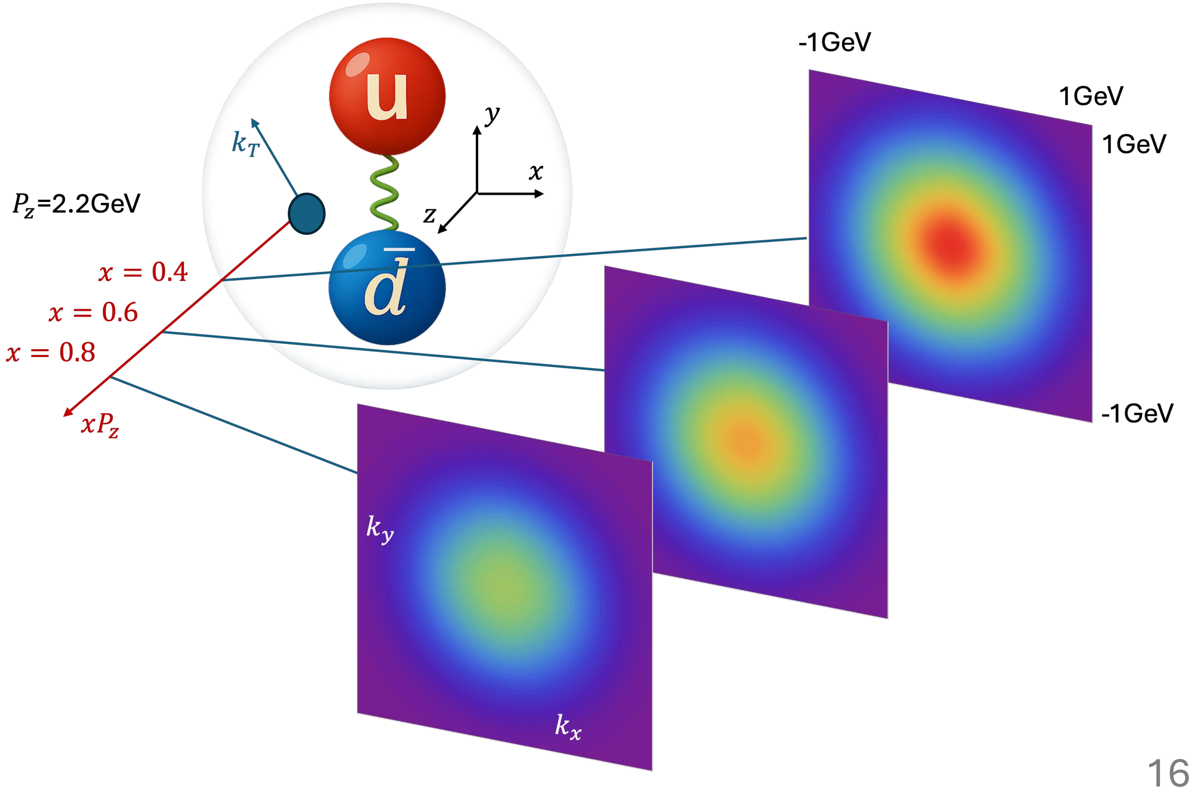
Numerical Results: TMDPDF

First model-independent 3D imaging of a moving pion from LQCD

- 3D Momentum Distribution



- k_T distribution at different x



Summary

- We propose an approach based on Coulomb gauge operators to directly calculate parton distributions in momentum space.
- The FT uncertainty is encoded in finite volume effects thus can be automatically removed during the infinite-volume extrapolation.
- Consistent with traditional LaMET approach, testify both approaches on the well control of FT uncertainty. Can be used to cross-check with traditional approach with almost no extra cost.
- The method is straightforwardly generalized to multi-dimensional parton distributions. A first direct lattice 3D imaging of a boosted pion is presented.

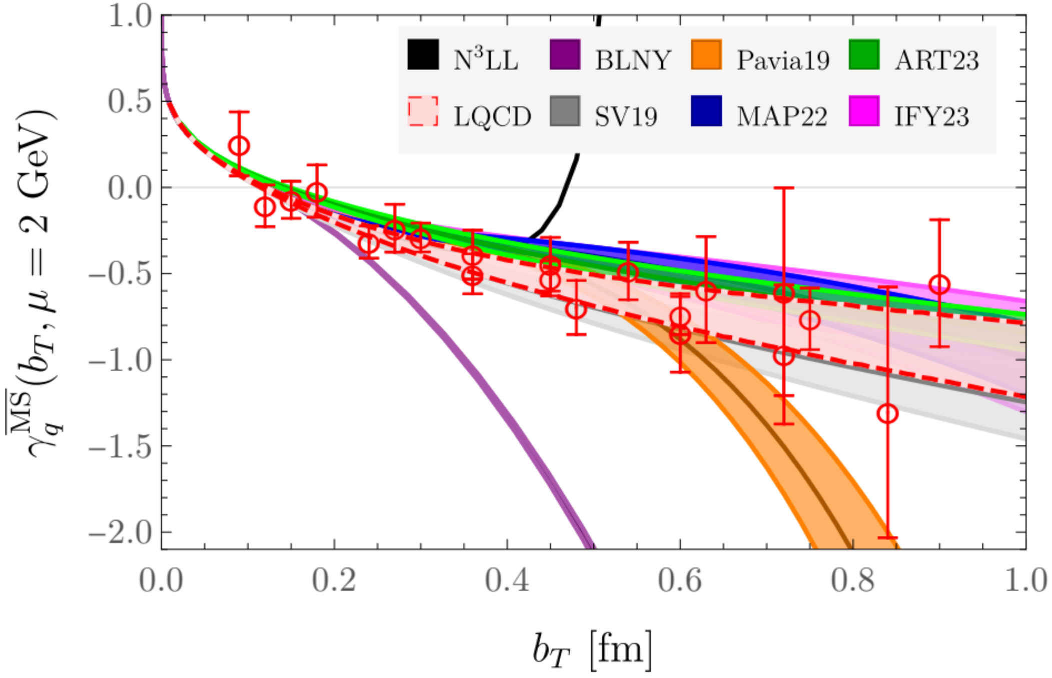
Thank you for listening!

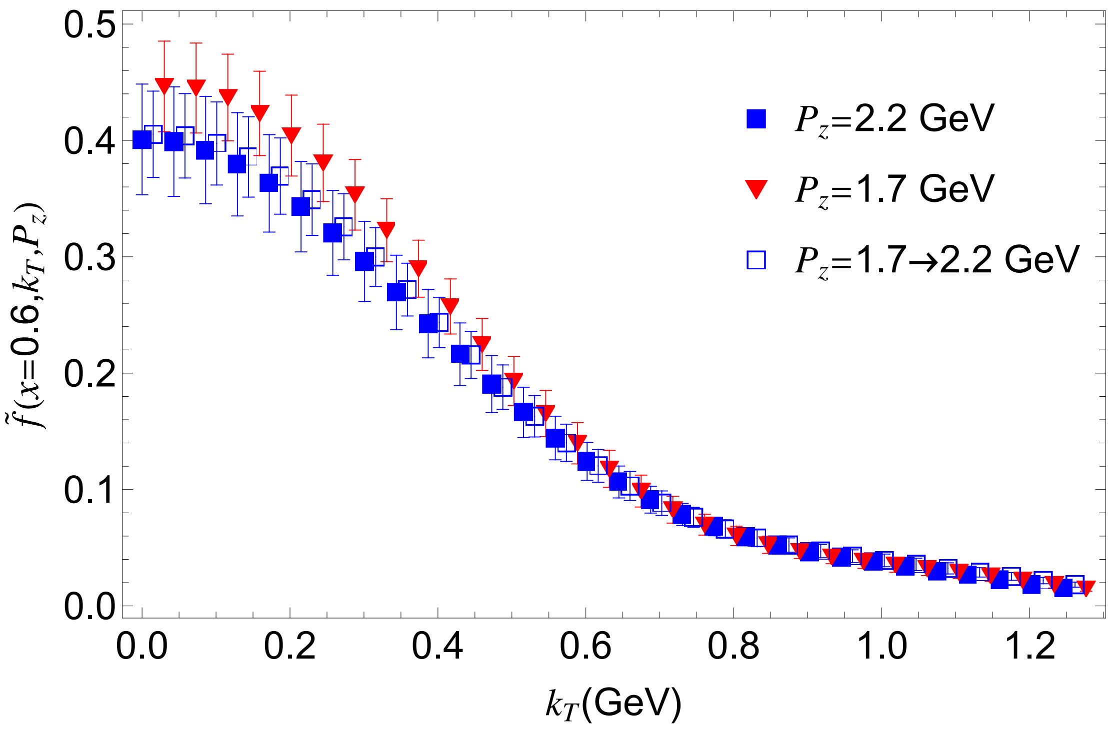
Backup Slides

Evolution of TMDPDF

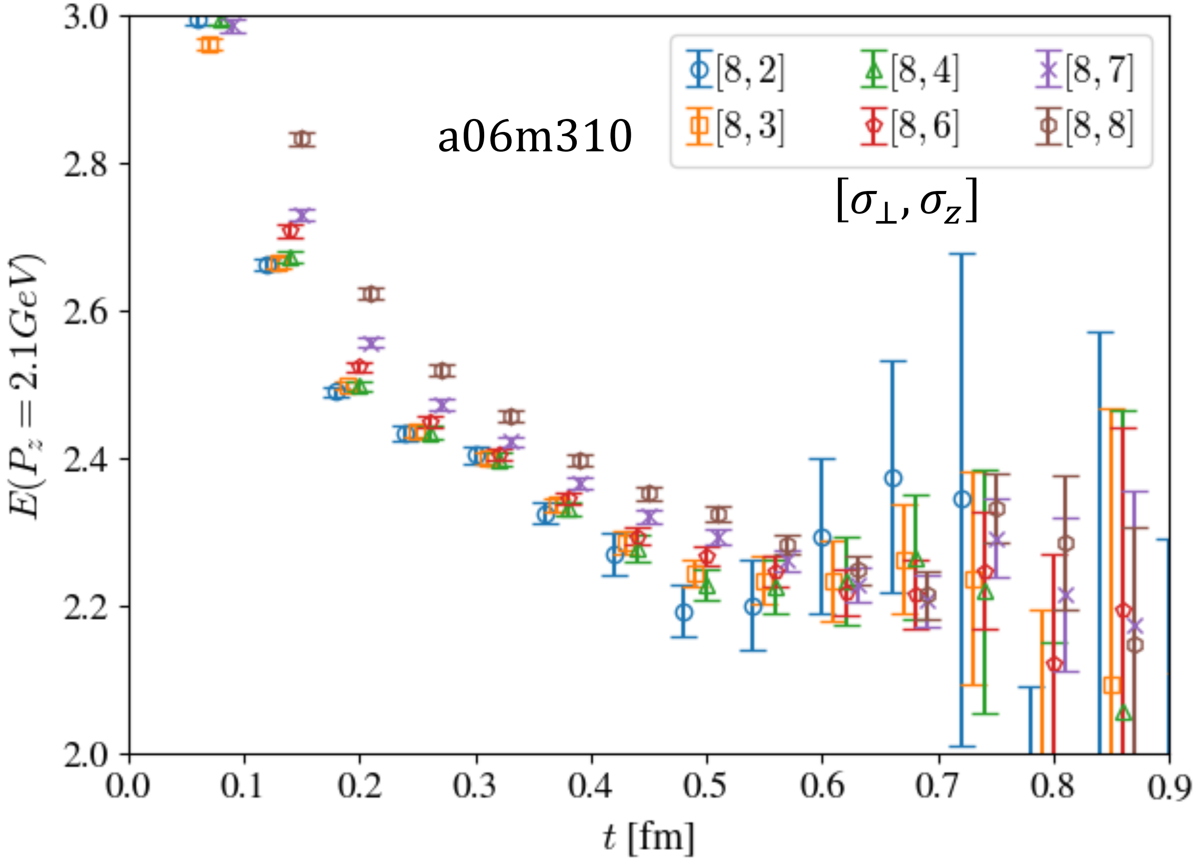
$$\frac{d}{d\ln P_z}\tilde{f}(x, b_T, \mu, P_z) = \gamma_q(b_T, \mu) + \delta\gamma_q(\mu, xP_z)$$

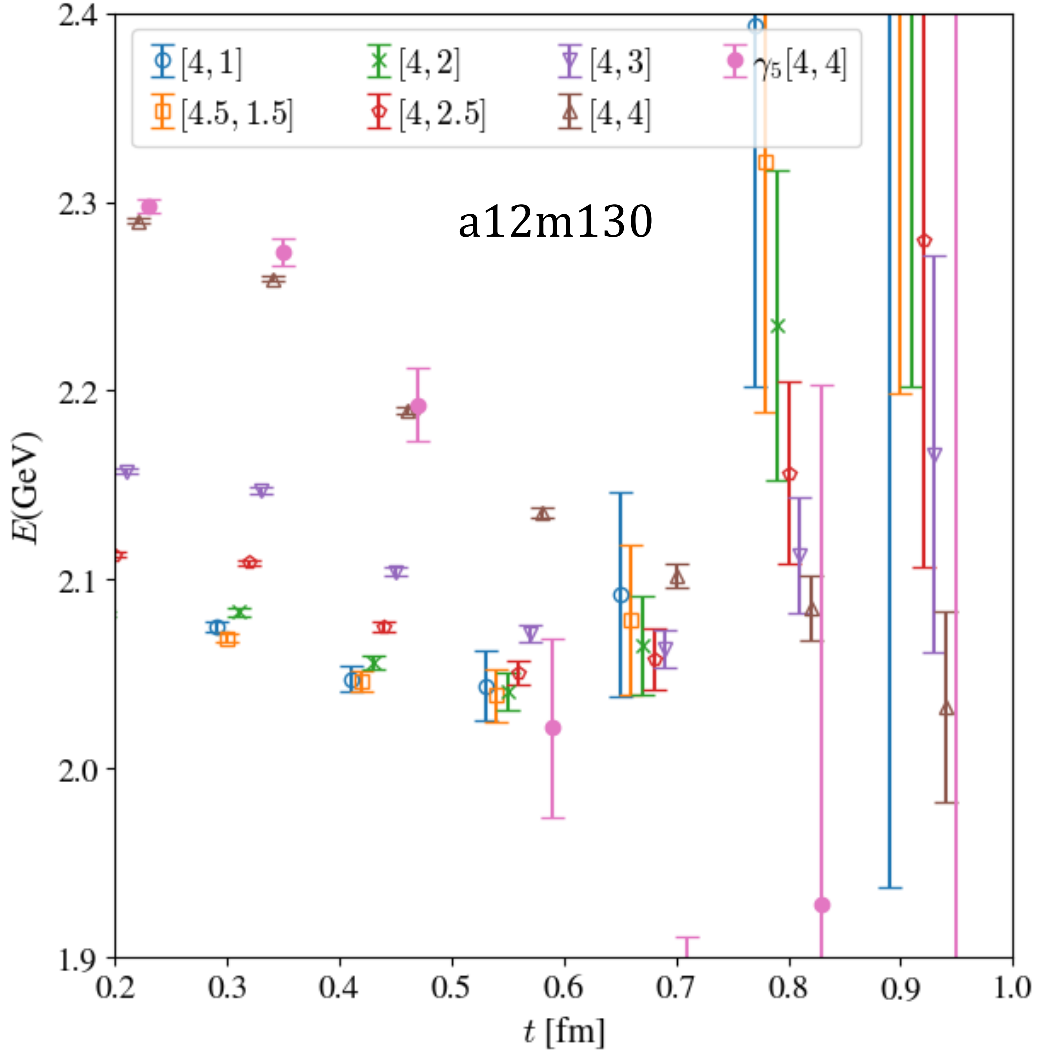
Avkhadiev, Shanahan, Wagman & Zhao, PRL (2024)



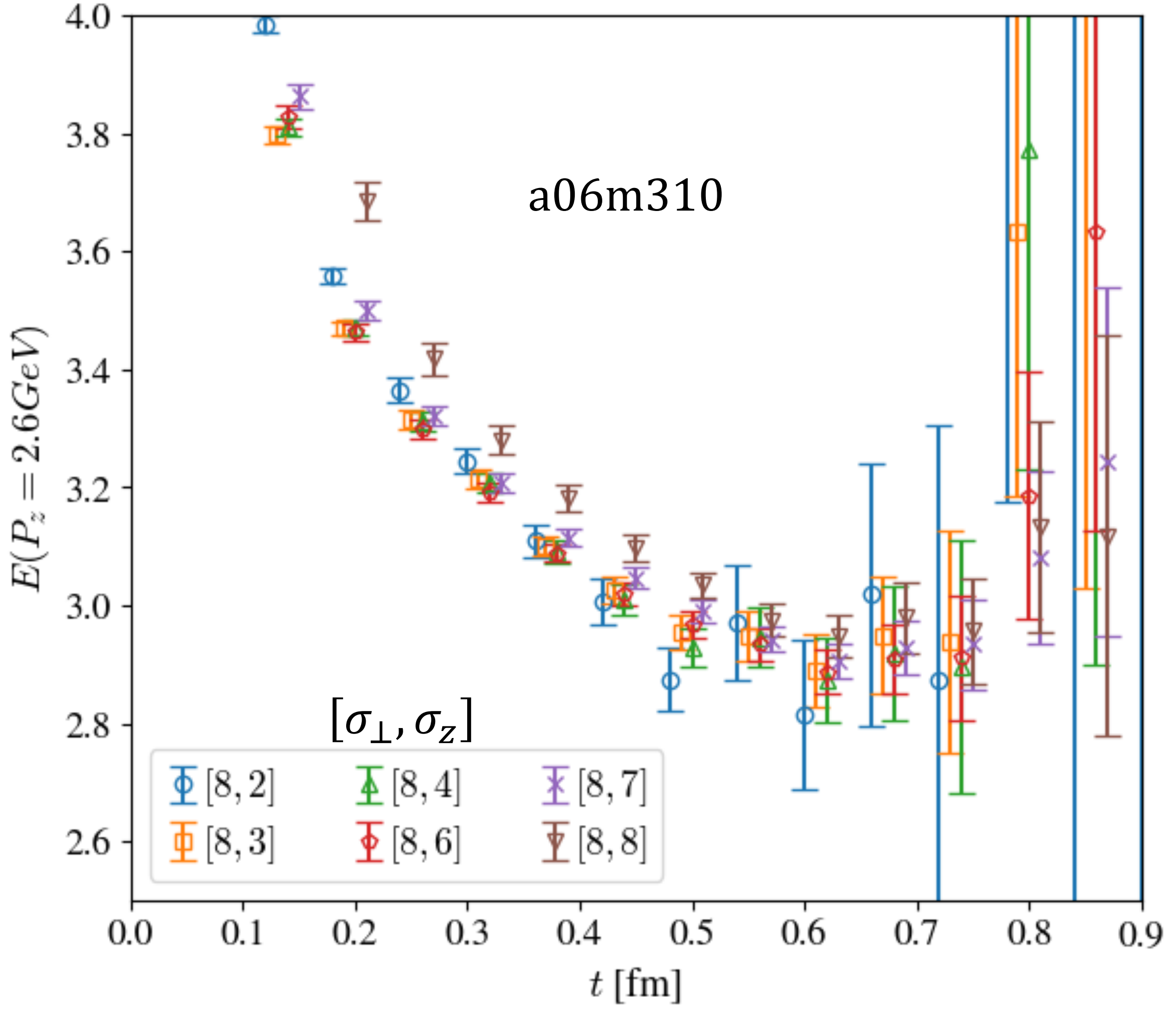


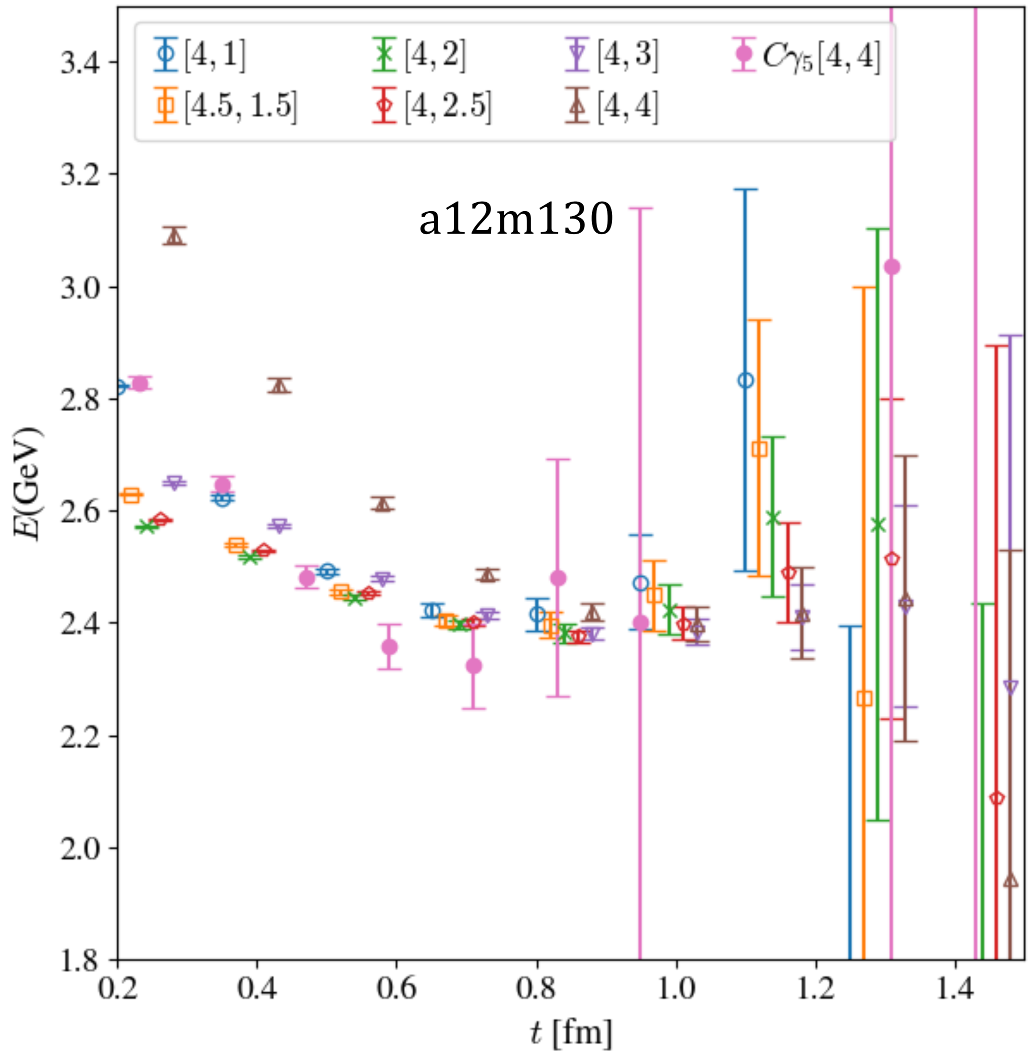
Anisotropic Smearing-pion



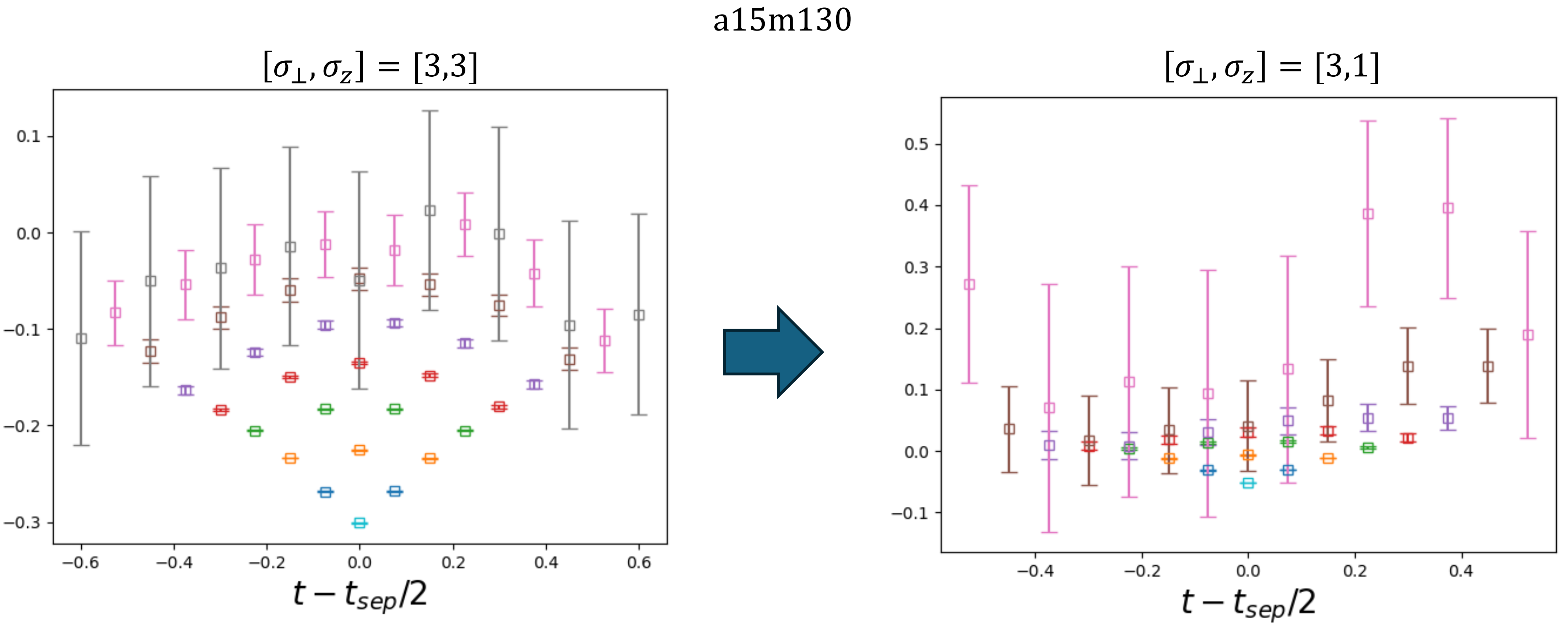


Anisotropic Smearing - nucleon





Anisotropic Smearing – 3pt

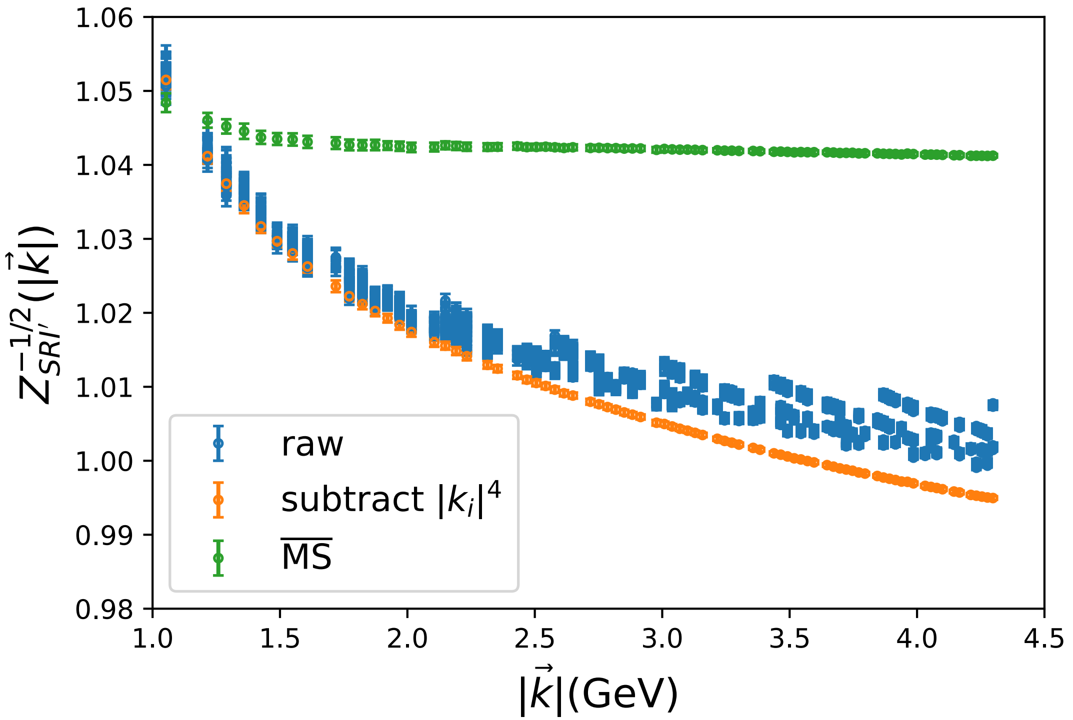


Renormalization for CG operator

Gao, et.al., arxiv:2602.11283

$$Z_{\mathrm{SRI'}}(|\vec{k}|) = \frac{\mathrm{Tr}\left[\sum_{i=1}^{3} \gamma_i (\hat{\vec{k}}_1)_i \; S_0(\vec{k})\right]}{\mathrm{Tr}\left[\sum_{i=1}^{3} \gamma_i (\hat{\vec{k}}_1)_i \; S^{\mathrm{lat.}}_{\mathrm{tree}}(\vec{k})\right]}$$

$$\sqrt{\frac{Z^{\mathrm{RI'}}}{Z^{\overline{\mathrm{MS}}}}} = 1 - \frac{\alpha_S C_F}{2 \cdot (4\pi)} \left[\left(13 - \frac{4\pi^2}{3} - 2\log 2 \right) + \log \frac{\mu^2}{\vec{k}^2} \right]$$



Cross-validation with perturbative matching

- The z-space results are from hybrid scheme with asymptotic analysis.
- The x-space method is from the direct calculation of x-space quasi-PDF with $\overline{\mathrm{MS}}$ matching.
- Two results are consistent.
- Little P_z dependence (small power correction).