A Path to Quantum Simulations of Topological Phases: (2+1)D Quantum Electrodynamics with Wilson Fermions

Sriram Bharadwaj

Mani L. Bhaumik Institute for Theoretical Physics,
University of California, Los Angeles (UCLA)

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Preview of key contributions

1. **Lattice QED_3 Hamiltonian realizing IR topological phases at finite lattice spacing**
   - Wilson versus staggered fermions
   - Integer Quantum Hall (IQH) phase of $N_f = 1$ QED_3

2. **Finite-density topological phase diagram**
   - $N_f = 2$ QED_3 IQH/Quantum Spin Hall (QSH)

3. **Path to quantum simulation**
   - Numerical benchmark by exact diagonalization (ED)
   - Gauge-invariant Variational Quantum Eigensolver (VQE) ansatz

Motivation

Why simulate QED_3 on a quantum computer at finite density?

- QCD_4-**like:** propagating gauge fields, confinement, asymptotic freedom, nontrivial topology, sign problem
- **Cond-mat:** quantum spin liquids, topological phases w/ Chern numbers [Fradkin 2013]
- Surge of $(2+1)\mathrm{D}$ gauge-theory quantum simulations on hardware w/ staggered fermions
  1. confinement/deconfinement,
  2. string breaking, real-time dynamics,
  3. topological order of pure $\mathbb{Z}_N$, ...

  [Too many excellent refs. to list – I'm sorry!]
- Sets the stage for analyzing topological phases arising from lattice gauge theory
- A UV Chern-Simons (CS) term is not easy to simulate [Peng et al. 2024; Bulgarelli et al. 2026]
- Here: CS emerges dynamically in the IR from QED_3 w/ Wilson fermions $\Rightarrow$ simulable

Connection to Chern-Simons theory

- $(2+1)$D Dirac ψ of mass m coupled to A_μ at weak coupling $e^2 \ll m$:

$$\underbrace{\int_{\mathbb{R}\times\mathbb{T}^2} d^3x\,\bar{\psi}(i\gamma^\mu D_\mu - m)\psi}_{\text{Dirac } \psi \,+\, A_\mu} + \dots \xrightarrow[\text{RG flow}]{\text{Euclidean space}} S_{\text{EFT}} \supset \mathbf{i}\,\underbrace{\frac{\text{sign}(m)}{4\pi}\int_{\mathbb{R}\times\mathbb{T}^2} AdA}_{\text{Chern}-\text{Simons TQFT}} + \dots$$

- Lattice Lagrangian: $\text{U}(1) + \text{Wilson fermion} \Rightarrow \text{CS term}$! [Golterman et al. 1993]
- **Motivation.**
  1. (Theoretical.) Topological phases in Hamiltonian formulation?
  2. (Simulation.) Quantum simulations of topological phases? **i** sign problem
- Hamiltonian: Topological phases $\leftrightarrow$ Chern number $C(m)$ [Fradkin 2013]
- Compute Chern number two ways: weak-coupling analytic + many-body ED
- Excellent agreement with lattice Lagrangian/continuum expectations

Setup: Staggered Versus Wilson Fermions?

Which Fermion Discretization? Fixed by Time-Reversal

	Staggered	**Wilson**
Simulation cost	$1/\text{site}$	$2/\text{site}$
Chiral symmetry	some	no
Doublers	some	no
Time-reversal	no	yes*

Theory/Fermion	**TR Broken**
Cont. $\mathsf{U}(1)$ + Dirac	✓
Latt. $\mathsf{U}(1)$ + Staggered	✗
Latt. $\mathsf{U}(1)$ + Wilson	✓

Informal Theorem: Time-reversal unbroken $\Longrightarrow$ Chern number zero!

Use Wilson Fermions!

Hamiltonian lattice QED_3: Wilson fermions coupled to a $\mathrm{U}(1)$ gauge field

- Hamiltonian ($a, b = 1, ..., N_f$ flavor indices, $k = x, y$, $r \in L \times L$ spatial lattice with PBC)

$$H_{\mathrm{Wilson-Dirac}} = \sum_r \left[\frac{1}{2} \psi_a(r)^\dagger \gamma^0 \left(i\gamma^k + R \right) \psi_a(r + \hat{k})\, U_k(r) + h.c. + \psi_a(r)^\dagger M_a \gamma^0 \psi_a(r) \right]$$

$$H_{\mathrm{gauge}} = -\frac{1}{e^2} H_B + e^2 H_E = -\frac{1}{e^2} \sum_r \cos B(r) + \frac{e^2}{2} \sum_{k,r} E_k(r)^2$$

- $M_a = m_a + 2R$: fermion masses m_a, $R = 1$: Wilson coupling

Strategy for Analytical Solution

Two ingredients for an analytical solution at weak coupling

- Goal: find effective low-energy Hamiltonian at weak coupling
- **Observation 0:** $H_E \sim e^2 \sum E^2$ suppressed at weak coupling!
- **Observation 1:** $[H_{\mathrm{Wilson-Dirac}}, H_B] = 0$ – simulataneously diagonalizable in U-basis!

$$\text{Unperturbed: } H = H_{\mathrm{Wilson-Dirac}} - \frac{1}{e^2} H_B \qquad \text{Perturbation: } e^2 H_E = \frac{e^2}{2} \sum_r E_k(r)^2$$

- **Observation 2:** Weak-coupling $\mathsf{U}(1) \times \mathsf{U}(1)$ "flux-symmetry" generated by

$$\mathcal{W}_x = \prod_x U_x(x, 0) \qquad \mathcal{W}_y = \prod_y U_y(0, y) \qquad [\mathcal{W}_k, H_{\mathrm{Wilson-Dirac}} + H_B] = 0$$

  In particular, $[\mathcal{W}_k, H_E] = O(e^2)$ – explicitly broken at strong coupling.
- If $e^2 \ll \mathrm{gap}$: can fix $(\mathcal{W}_x, \mathcal{W}_y) \in \mathsf{U}(1) \times \mathsf{U}(1)$ + solve H sector-by-sector
- Here: fix $(\mathcal{W}_x, \mathcal{W}_y) = (1, 1)$ – captures topological phases
- Observations fix gauge fields in the GS $\Rightarrow$ purely fermionic effective Hamiltonian!

Topological Phases of the $N_f = 1$ theory at zero density

Two-band model and the Chern number

- Observations 0–2 reduce $H_{\mathrm{Wilson-Dirac}}$ to a **Chern insulator**

$$\mathcal{H}(k) = \mathcal{H}_x\sigma^x + \mathcal{H}_y\sigma^y + \mathcal{H}_z\sigma^z, \quad (\mathcal{H}_x, \mathcal{H}_y, \mathcal{H}_z) = (\sin k_x,\ \sin k_y,\ M + \cos k_x + \cos k_y)$$

  with $E_\pm(k) = \pm|\mathcal{H}|$; gapless points $\leftrightarrow$ topological phase transitions

- Two equivalent definitions of the Chern number:

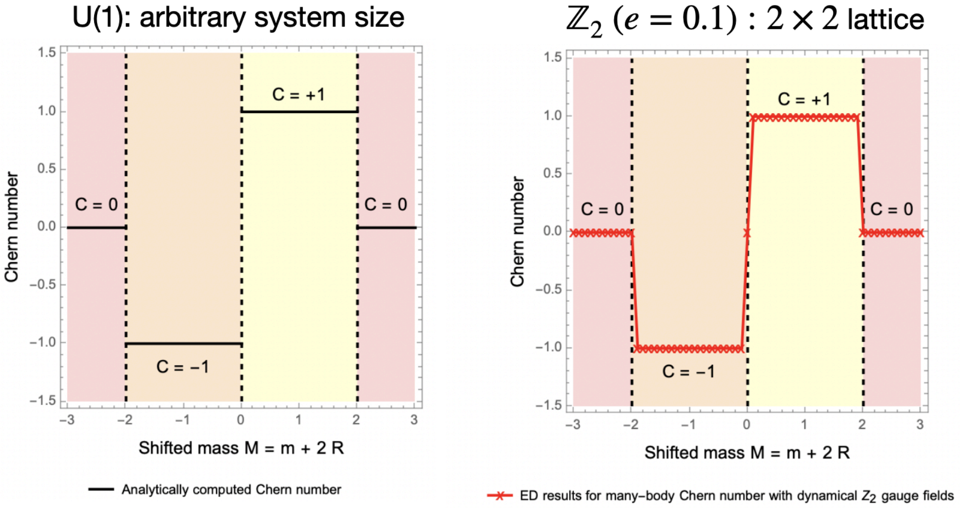
momentum space (analytic): $\quad a_i = -i \langle u_- | \partial_{k_i} | u_- \rangle , \qquad c_1[b] = \int_{\mathcal{B}} \frac{\nabla \times a}{2\pi}$

many-body (ED): $\quad \mathsf{A}_k(\theta) = i \langle \psi_0(\theta) | \partial_{\theta_k} | \psi_0(\theta) \rangle , \qquad C = \frac{1}{2\pi} \int_{T^2_\theta} \mathsf{F}_{xy} \in \mathbb{Z}$

  with twisted BC $\psi_{r+L\hat{k}} = e^{i\theta_k}\psi_r$ on the spatial $\mathbb{T}^2$

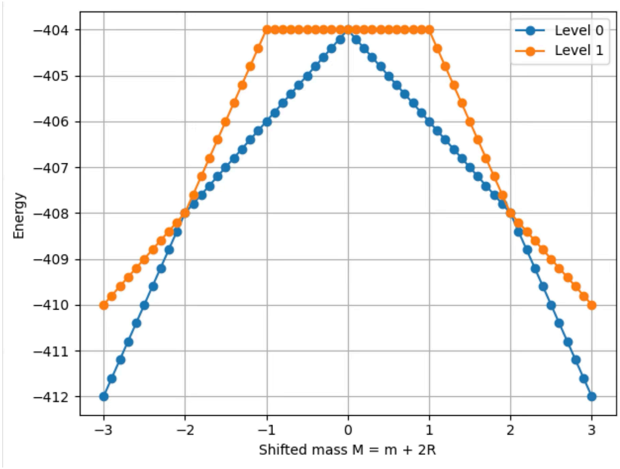
- Both computed independently; agree across the full M phase diagram

Candidate topological phase diagram: robust to truncation finite-size effects



Lattice Lagrangian CS level [Sen 2020] $\xrightarrow{a_0 \to 0}$ Chern no. $C(M)$ [Bharadwaj et al. 2026]

Imprints of topological phase transitions on the spectrum

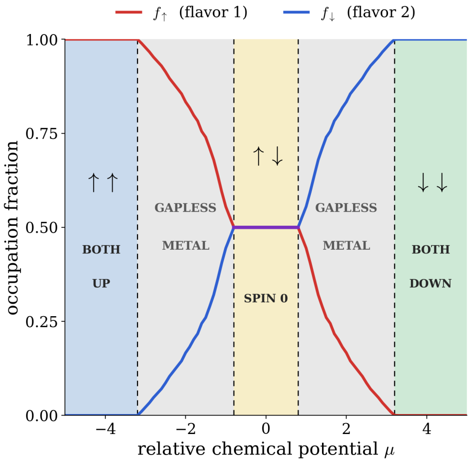


- Gapless points $\longleftrightarrow$ topological phase transition points

Topological Phases of the $N_f = 2$ theory at finite density

Masses & chemical potentials: three gapped phases at finite density

- Chemical potential $H_\mu = \mu \sum_r (\psi_1^\dagger \psi_1 - \psi_2^\dagger \psi_2)$; the two flavors act as spin $|\uparrow\rangle, |\downarrow\rangle$
- Equal masses $M_1 = M_2 = M$: $E = \pm\mu \pm |\mathcal{H}|$, so tuning μ drives **three gapped regimes**



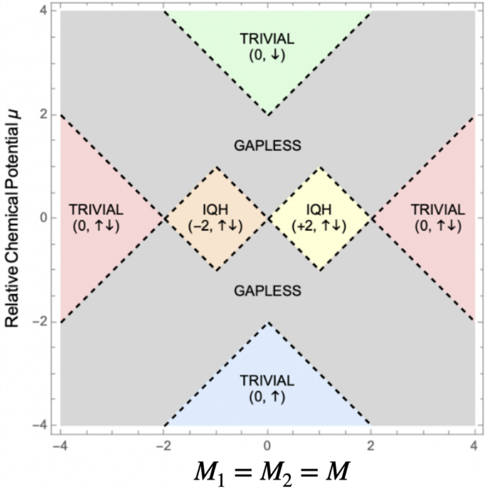
Occupation fraction with $a = \uparrow, \downarrow$:

$$f_a(\mu, M) = \frac{1}{2L^2} \sum_{k \in \mathcal{B}} \left\langle \psi_a(k)^\dagger \psi_a(k) \right\rangle_{\mu, M}$$

Mass $M = 1.2$

- Opposite masses $M_1 = -M_2 = M$: same logic

Proposed Phase Diagram in the (M, μ)-plane



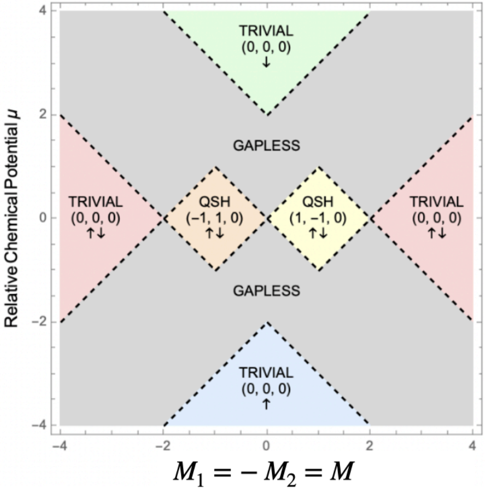


Figure: Left: $(c_1[b], \langle s\rangle)$. Right: $(c_\uparrow, c_\downarrow, c_{\text{tot}})$. $\langle s\rangle = \uparrow, \downarrow, \uparrow\downarrow$, $\uparrow\downarrow$ is zero spin

Outlook and summary

Topological transitions in the $N_f = 1$ theory on a 2×2 lattice w/ $\mathbb{Z}_2$ truncation

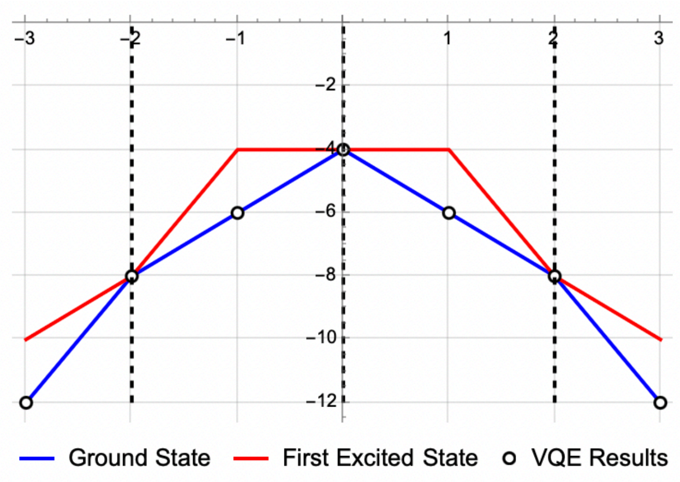


Figure: Level crossings as M varies. VQE (dots) converges to ED (solid) with one layer!

- Dramatic truncation for classical simulation: (2×2) w/ $\mathbb{Z}_2$ gauge fields!
- Classical hardness: $\dim \mathcal{H} = 2^{16} \approx 6.5 \times 10^4$!
- Need quantum hardware!

Summary

1. **Hamiltonian realizing topological phases at finite lattice spacing**
   - Hamiltonian lattice QED_3 with **Wilson** fermions realizes Chern-insulator phases
   - Chern number from weak-coupling analytics and many-body ED agree
   - matches the continuum/lattice Lagrangian $N_f = 1$: IQH
2. **Finite-density phase diagram**
   - $N_f = 2$ QED_3 masses and chemical potential drive IQH/QSH phases in (M, μ)-plane
   - Occupation fraction $f_a(\mu, M)$ diagnoses each gapped vacuum
3. **Path to quantum simulation**
   - gauge-invariant VQE ansatz
   - reproduces the topological transitions with 2×2, $\mathbb{Z}_2$(!)
   - concrete target for quantum hardware

Questions?

Vacuum solution from gauge-fixing with symmetry constraints

- For simplicity: focus on $\mathbb{Z}_N$ truncated gauge group on a $L \times L$ lattice
- Then at $e^2 \to 0$ with

$$\mathcal{H}_{\text{phys}} \xrightarrow[e^2 \to 0]{} \bigoplus_{(w_x, w_y) \in \mathbb{Z}_N^2} \mathcal{H}_{w_x, w_y} \qquad \mathcal{H}_{\text{phys}} = \frac{\mathcal{H}_{\text{fermion}} \otimes \mathcal{H}_{\text{gauge}}}{\{|\phi\rangle : [\nabla \cdot E - \psi^\dagger \psi] |\phi\rangle = 0\}}$$

- Consider $(\mathcal{W}_x, \mathcal{W}_y) = (1, 1)$ – "**Trivial-Flux Sector**"
- Pick a gauge with $\mathcal{W}_x = \mathcal{W}_y = 1$, e.g. $U_k \equiv 1 \, \forall \, r \iff U_k |\mathfrak{g}\rangle = |\mathfrak{g}\rangle \in \mathcal{H}_{\text{gauge}}$
- Fermionic GS $|\mathfrak{f}_\mathfrak{g}\rangle$ subject to the gauge configuration $\mathfrak{g}$ is:

$$H_{\text{Wilson-Dirac}}[\mathfrak{g}] \, |\mathfrak{f}_\mathfrak{g}\rangle = E_0[\mathfrak{g}] \, |\mathfrak{f}_\mathfrak{g}\rangle$$

- A many-body ground state:

$$|\mathfrak{f}_{\mathfrak{g}_*}, \mathfrak{g}_*\rangle = |\mathfrak{f}_{\mathfrak{g}_*}\rangle \otimes |\mathfrak{g}_*\rangle \qquad |\mathfrak{g}_*\rangle = \arg\min_{|\mathfrak{g}\rangle} \langle \mathfrak{f}_{\mathfrak{g}}| \, H_{\text{Wilson}-\text{Dirac}}[\mathfrak{g}] \, |\mathfrak{f}_{\mathfrak{g}}\rangle$$

- To create a gauge invariant state, use projector:

$$\mathbb{P} = \frac{1}{\sqrt{|G_N|}} \sum_{g \in G_N} g \qquad G_N = \langle e^{\frac{2\pi i}{N}[\nabla \cdot E - \psi^\dagger \psi]} \rangle \cong_{\text{group}} (\mathbb{Z}_N)^{L^2-1}$$

- Gauge-invariant many-body GS: $|0\rangle = \mathbb{P}\,|\mathfrak{f}_{\mathfrak{g}_*}, \mathfrak{g}_*\rangle$
- $(\mathcal{W}_x, \mathcal{W}_y) = (1, 1)$ represents the true weak-coupling vacuum in the thermodynamic limit!
- Shown to match continuum phase diagram with $e^2 \ll m$
- Note: $U_k \equiv 1$ does **not** trivialize gauge dynamics as $[E, U] \sim U \neq 0$
- $N \to \infty$ limit reproduces $\mathsf{U}(1)$ results

Agreement with the lattice Lagrangian formulation

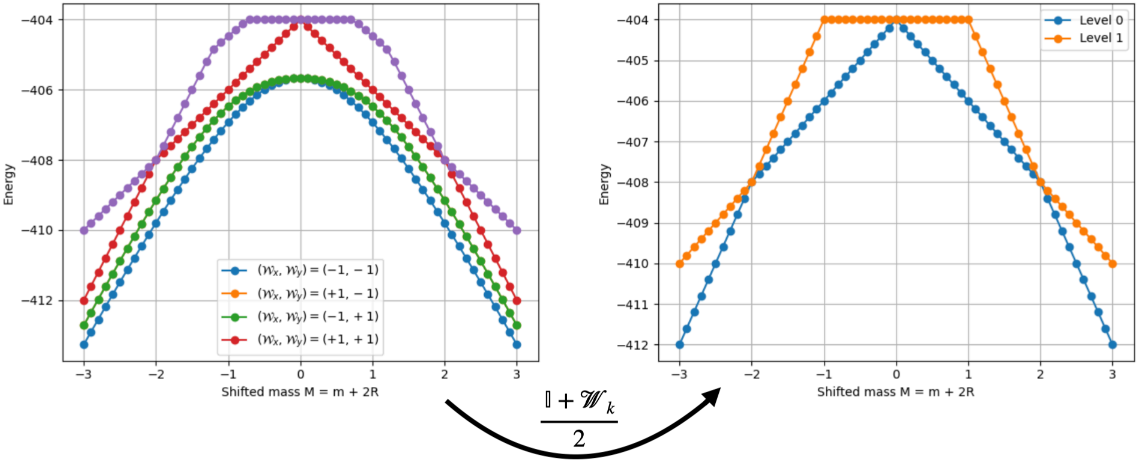
- CS level from lattice Lagrangian (time lattice spacing a_0, spatial lattice spacing $a = 1$) [Sen 2008.01743]

$$C(m, a, a_0) = \frac{1}{2}\left[(-1)^0 \frac{m}{|m|} + (-1)^1 \left(2\frac{m+2}{|m+2|} + \frac{m+\frac{2}{a_0^2}}{\left|m+\frac{2}{a_0^2}\right|}\right)\right.$$
$$\left. + (-1)^2 \left(\frac{m+4}{|m+4|} + 2\frac{m+2+\frac{2}{a_0^2}}{\left|m+2+\frac{2}{a_0^2}\right|}\right) + (-1)^3 \left(\frac{m+2+\frac{4}{a_0^2}}{\left|m+2+\frac{4}{a_0^2}\right|}\right)\right]$$

- Agreement with Hamiltonian calculation as we take $a_0 \to 0$ with $M = m + 2$

$$C = -\lim_{a_0 \to 0} c(M, a, a_0) = -\frac{1}{2}\left[\frac{M-2}{|M-2|} - 2\frac{M}{|M|} + \frac{M+2}{|M+2|}\right] = \begin{cases} 0, & M < -2 \\ -1, & M \in (-2, 0) \\ +1, & M \in (0, 2) \\ 0, & M > 2 \end{cases}$$

Other sectors for $\mathbb{Z}_2$?



gapless points $\longleftrightarrow$ topological phase transition points

Other sectors for higher N? Expectations from the unperturbed problem

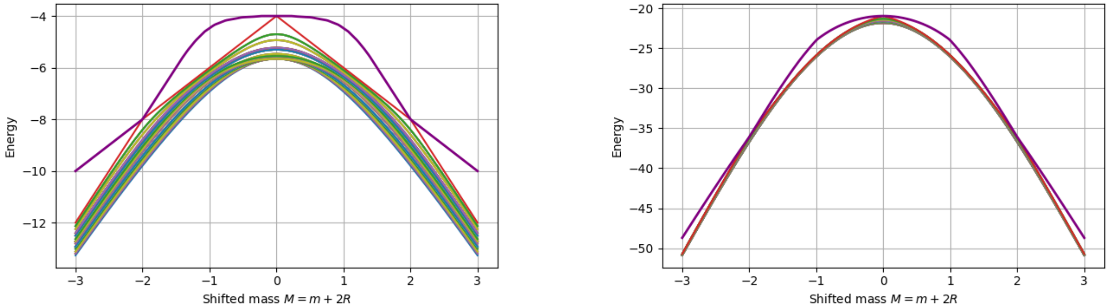
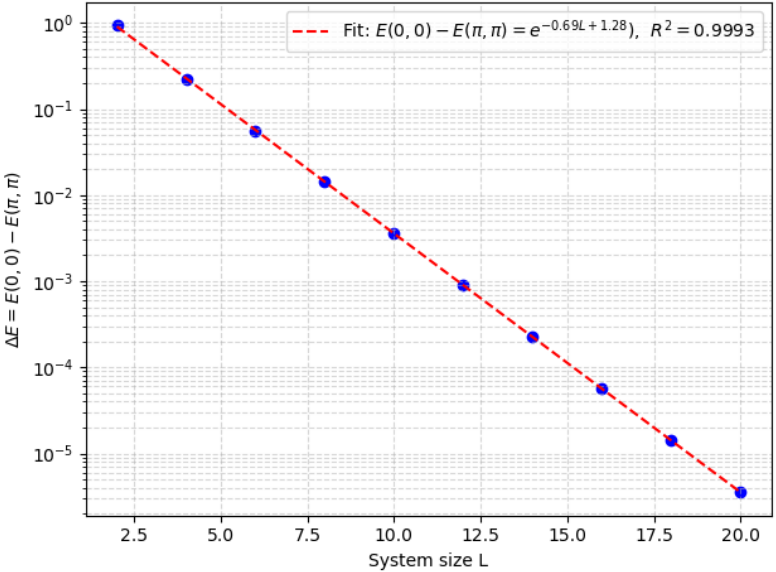


Figure: Lowest $N^2 + 1$ energy levels with $N = 8$ for 2×2 (left) and 4×4 (right) lattices

- Lowest energy level always has $(\varphi_x, \varphi_y) = (\pi, \pi)$ with $\mathcal{W}_k = e^{i\varphi_k} = -1$
- Splitting between $(\mathcal{W}_x, \mathcal{W}_y)$-sectors is a finite-size effect, vanishes as $L \to \infty$

Sectors with distinct $(\mathcal{W}_x, \mathcal{W}_y) = (e^{i\varphi_x}, e^{i\varphi_y})$ degenerate exponentially as $L \to \infty$



Gap persists as $L \to \infty$ provided $M \neq 0, \pm 2$

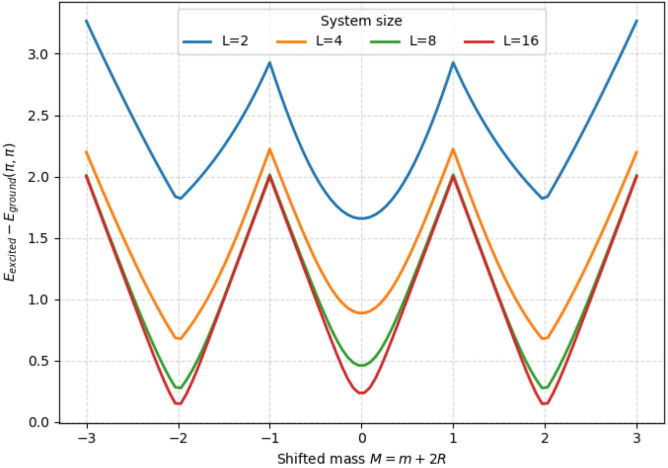


Figure: Gap between true ground state w/ $(\mathcal{W}_x, \mathcal{W}_y) = (-1, -1)$ and would-be first excited state in the infinite-volume limit for various system sizes L.

Finite-size scaling of spectral gap at topological phase transitions

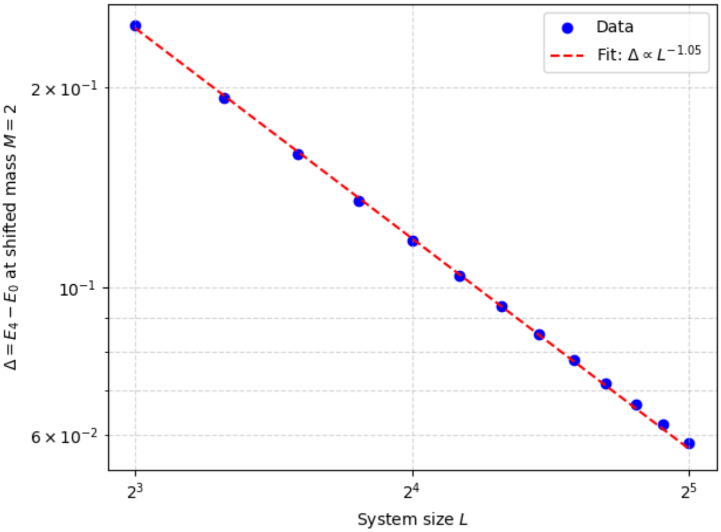
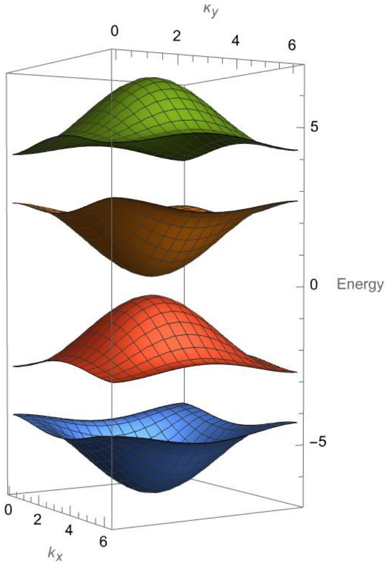
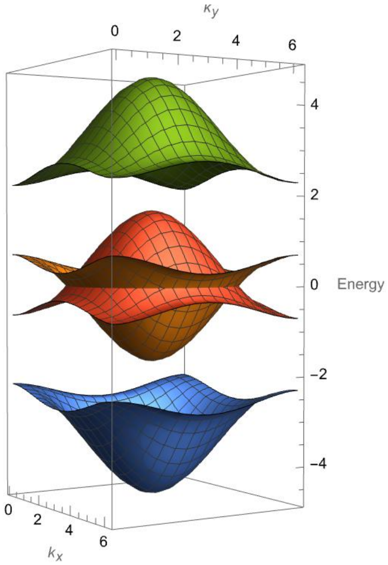


Figure: Power law vanishing of the gap as $L \to \infty$ at phase transition

The vacuum structure I – fixed mass $M \neq 0, \pm 2$

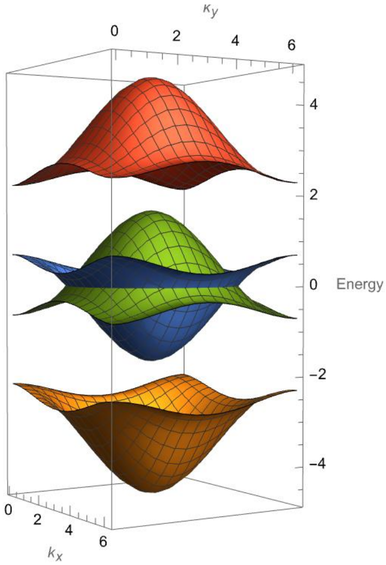


(a) $\mu = -3.5$

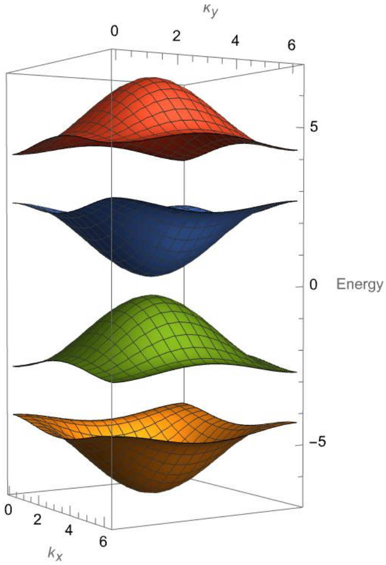


(b) $\mu = -1.5$

The vacuum structure II – fixed mass $M \neq 0, \pm 2$



(a) $\mu = 1.5$



(b) $\mu = 3.5$