

The Collins-Soper kernel from a vacuum soft function

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with

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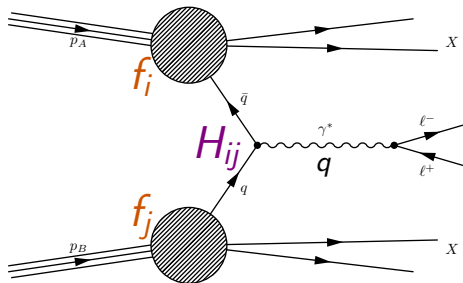
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TMD factorization in Drell-Yan scattering



- $\Lambda_{\text{QCD}} \lesssim |\vec{q}_\perp| \ll Q = \sqrt{q^2}$
- Rapidity divergences in TMDPDFs need to be regulated
- Collins-Soper scale, $\zeta_{a,b}$, dependence

$$\frac{d\sigma}{dQ dY d^2\vec{q}_\perp} = \sum_{i,j} H_{ij}(Q, \mu) \int d^2\vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{q}_\perp} f_i(x_a, \vec{b}_\perp, \mu, \zeta_a) f_j(x_b, \vec{b}_\perp, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_\perp^2}{Q^2}, \frac{\Lambda_{\text{QCD}}^2}{Q^2}\right) \right] \quad \zeta_{a,b} = 2(x_{a,b} P^+ e^{-y_n})^2$$

$$f_i(x_a, \vec{b}_\perp, \mu, \zeta_a) = \lim_{\substack{\epsilon \rightarrow 0 \\ y_B \rightarrow -\infty}} Z_{\text{UV}}(\mu, \zeta_a, \epsilon) \frac{f_i^{\text{bare}}(x_a, \vec{b}_\perp, \epsilon, y_B, x_a P^+)}{\sqrt{S_i(b_\perp, \epsilon, 2y_n - 2y_B)}}$$

Evolution kernel:

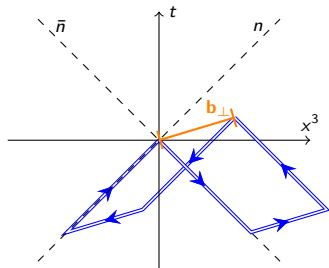
$$\gamma_\mu^q(\mu, \zeta) = \frac{d \log f_q(x, \vec{b}_\perp, \mu, \zeta)}{d \log \mu}$$

Collins-Soper (CS) kernel:

$$\gamma_q(b_\perp, \mu) = \frac{d \log f_q(x, \vec{b}_\perp, \mu, \zeta)}{d \log \sqrt{\zeta}}$$

$$S_q(b_\perp, y_A, y_B, \mu) = S_l(b_\perp, \mu) e^{\gamma_q(b_\perp, \mu)(y_A - y_B)} \left(1 + \mathcal{O}\left(e^{-2(y_A - y_B)}\right)\right)$$

Regulating the soft function



Soft function defined on the lightcone:

$$n^2 = \bar{n}^2 = 0$$

$$n = (1, 0, 0, 1),$$

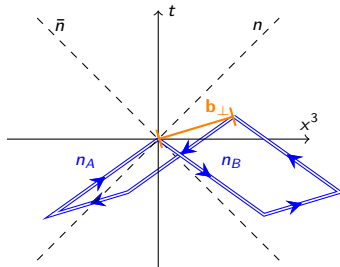
$$\bar{n} = (1, 0, 0, -1)$$

Regulate rapidity divergence with spacelike Wilson lines:

$$y_A, -y_B \rightarrow \infty,$$

$$n_A \equiv n - e^{-y_A} \bar{n},$$

$$n_B \equiv \bar{n} - e^{+y_B} n$$



In the LaMET framework:

$$\begin{aligned} & \sqrt{S_I(b_\perp, \mu)} \tilde{f}_T(x, b_\perp, P^z, \mu) \\ &= f(x, b_\perp, \mu, \zeta) H_f(x, P^z, \mu) \exp \left[\frac{1}{2} \log \frac{(2xP^z)^2}{\zeta} \gamma_q(b_\perp, \mu) \right] \\ &+ \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}}{xP^z}, \frac{1}{b_\perp(xP^z)} \right) \end{aligned}$$

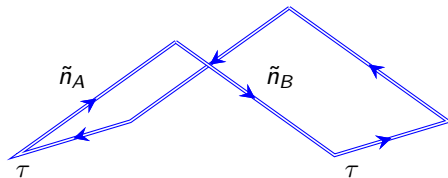
[Ebert, et. al., 2019], [Ji, Liu, Liu, 2019], [Ebert, et. al, 2022]

- Intrinsic soft function: S_I
[Ji, Liu, Liu, 2020]
- quasi-TMD beam function: $\tilde{f}_T$
- TMDPDF: f
- Perturbative hard kernel: H_f
- CS kernel: γ_q

Motivated by a proposal by Liu in [Ji, Liu, Liu, 2019]

Euclidean space directional vectors with purely imaginary time components

$$\tilde{n}_A = (in_A^0, 0, 0, n_A^3), \quad \tilde{n}_B = (in_B^0, 0, 0, n_B^3)$$



$$r_a = \frac{n_A^3}{n_A^0} = \frac{1 + e^{-2y_A}}{1 - e^{-2y_A}}, \quad r_b = \frac{n_B^3}{n_B^0} = \frac{1 + e^{2y_B}}{1 - e^{2y_B}}$$

Auxiliary field definition of the Wilson line

Write Wilson line in terms of one dimensional ‘fermions’ that live along the path:

$$\begin{aligned} & P \exp \left\{ -ig \int_{s_i}^{s_f} ds n^\mu A_\mu(y(s)) \right\} \\ &= Z_\psi^{-1} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \psi \bar{\psi} \exp \left\{ ig \int_{s_i}^{s_f} ds \bar{\psi} i \partial_s \psi - \bar{\psi} n \cdot A \psi \right\} \end{aligned}$$

[Gervais, Neveu 1980], [Aref'eva 1980]

Auxiliary field propagator:

$$in \cdot DH_n(y) = \delta(y) \xrightarrow{\text{Euclidean space}} -i\tilde{n} \cdot D_E H_{\tilde{n}}(y) = \delta(y), \quad \tilde{n} = (in_0, \vec{n})$$

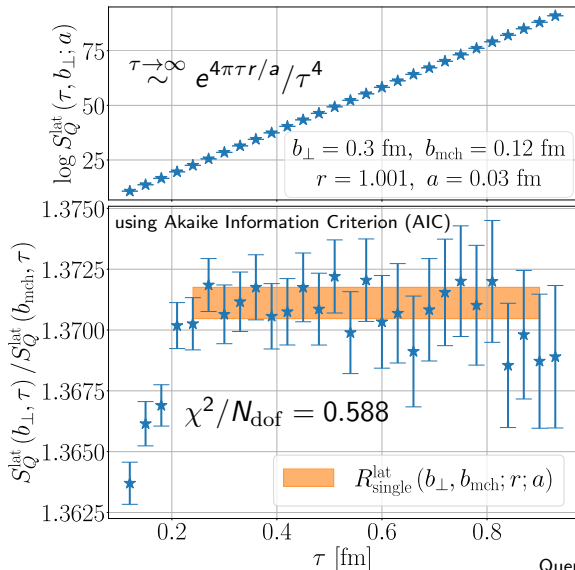
Meaningful solution only obtained with a UV cutoff

[Aglietti, *et. al.* 1992], [Aglietti, 1994]

Perturbative computation of soft factor with auxiliary field propagators requires further investigation

[Liu, 2022]

Single ratio, time dependence



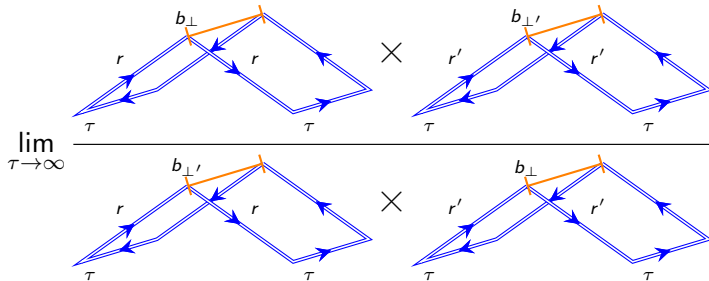
$$R_{\text{single}}^{\text{lat}}(b_{\perp}, b_{\text{mch}}, r, a) = \lim_{\tau \rightarrow \infty} \frac{S_Q^{\text{lat}}(b_{\perp}, \tau; r, r; a)}{S_Q^{\text{lat}}(b_{\text{mch}}, \tau; r, r; a)}$$

$$\begin{aligned} b_{\perp} &= 0.3 \text{ fm} \\ b_{\text{match}} &= 0.12 \text{ fm} \\ r &= 1.001 \end{aligned}$$

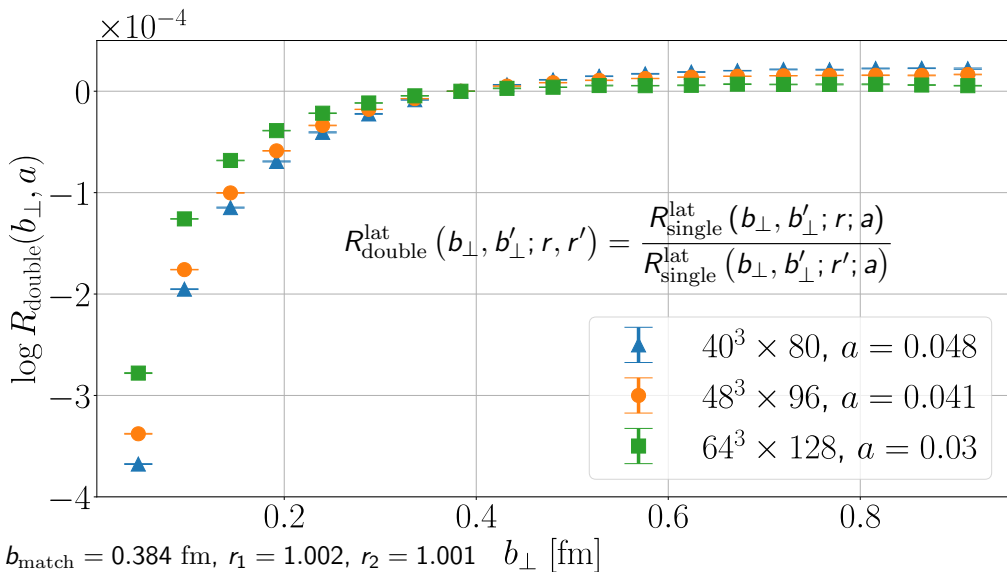
Quenched configurations from [Detmold, Endres, 2018]

$$R_{\text{double}}^{\text{lat}}(b_{\perp}, b'_{\perp}; r, r') = \frac{R_{\text{single}}^{\text{lat}}(b_{\perp}, b'_{\perp}; r; a)}{R_{\text{single}}^{\text{lat}}(b_{\perp}, b'_{\perp}; r'; a)} = e^{\Delta\gamma_q(b_{\perp}, b'_{\perp})\Delta Y(r, r')}$$

$$\Delta\gamma_q(b_{\perp}, b'_{\perp}) = \gamma_q(b_{\perp}, a) - \gamma_q(b'_{\perp}, a), \quad \Delta Y(r, r') = 2 \log \left(\frac{r-1}{r+1} / \frac{r'-1}{r'+1} \right)$$



Interpolated double ratio, $b_{\perp}$ dependence



$$\gamma_q(b_{\perp,1}, \mu) = \gamma_q(b_{\perp,2}, \mu) + \frac{\frac{1}{2} \log(R_{\text{double}}(b_{\perp,1}, b_{\perp,2}, \mu, r_1, r_2))}{\log\left(\frac{r_1+1}{r_1-1} / \frac{r_2+1}{r_2-1}\right)}$$

- Implies that we need at least one perturbative value of γ_q .
- Another perturbative value of γ_q is needed, because $r_{1,2}$ is renormalized due to $O(4)$ symmetry breaking. [Aglietti, et. al. 1992], [Aglietti, 1994]

- Model the double ratio as a piecewise function:

$$\log R_{\text{double}}^{\text{lat}}(b_{\perp}, b_{\text{mch}}; r, r') = \Delta Y_i^{\text{ren}} \left(\left\{ \begin{array}{l} \gamma_q^{\overline{\text{MS}}}(b_{\perp}, \mu) - \gamma_q^{\text{mch}} \\ \{d_{\ell}\} \end{array} \right. , \begin{array}{l} b_{\text{cut}} \leq b_{\perp} \leq b_{\text{th}} \\ b_{\ell} = b_{\perp} > b_{\text{th}} \end{array} \right\} + c_1 \left(\frac{a^2}{b_{\perp}^2} - \frac{a^2}{b_{\text{mch}}^2} \right) \right),$$

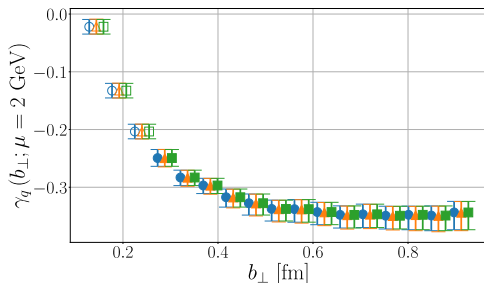
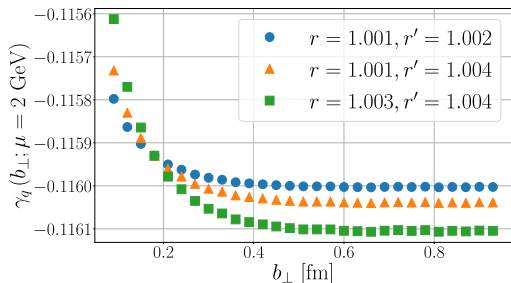
- Combined fit on three ensembles
- Model imposes a constraint on the large- $b_{\perp}$ region

$$b_{\text{cut}} = 0.144 \text{ fm}, \quad b_{\text{th}} = 0.24 \text{ fm}, \quad b_{\text{mch}} = 0.384 \text{ fm} > b_{\text{th}}$$

$$d_{\ell} = \gamma_q(b_{\ell}, \mu) - \gamma_q(b_{\text{mch}}, \mu), \quad \gamma_q^{\text{ext}}(b_{\ell}, \mu) = \gamma_q^{\text{mch}} + d_{\ell}$$

- Model does not adequately estimate the lattice spacing dependence

Bare vs renormalized rapidity factor

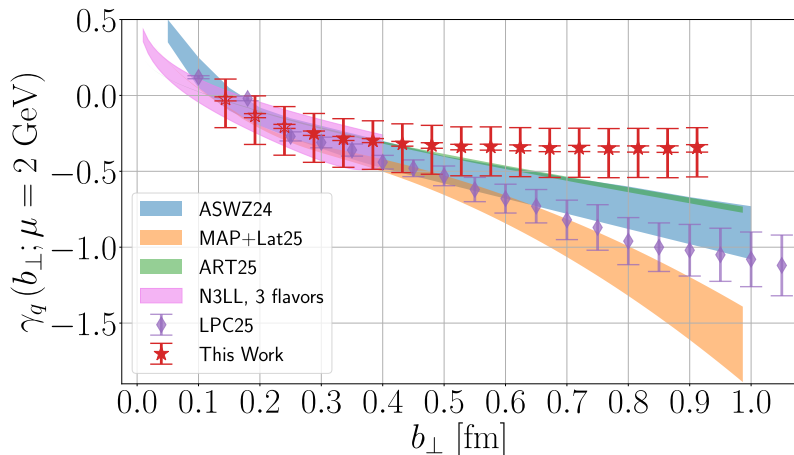


The CS kernel determined on the $a = 0.03$ fm ensemble with three sets of bare rapidities.

- Left plot uses bare rapidity:

$$2(y - y') = \log \left(\frac{r+1}{r-1} \middle/ \frac{r'+1}{r'-1} \right)$$

- Right plot from model fit with renormalized rapidity
- Included statistical, truncation and discretization error.



- Our result is pure gauge. Comparison plots have $n_f = 3, 4$.
- Inner error bars include statistical, truncation and discretization errors.
- Outer error bars include UV scale variation uncertainty, $\sqrt{2}\mu$.

- Euclidean space soft function with complex directional vectors maps directly to Minkowski space result
- Lattice data at high statistical precision, low computational cost
- Double ratio method gives: $\gamma_q(b_\perp, \mu) - \gamma_q(b'_\perp, \mu)$
- Error primarily from UV scale uncertainty
- Our results are competitive with existing results

Open questions and future direction:

- Ongoing exploration of perturbative results for auxiliary field computation
- Improve systematic uncertainties by simulating at finer lattices or with an alternative scheme for the rapidity renormalization

Thank you!