

Setting the physical scale for lattice simulations using the gluon propagator and the understanding of finite size effects

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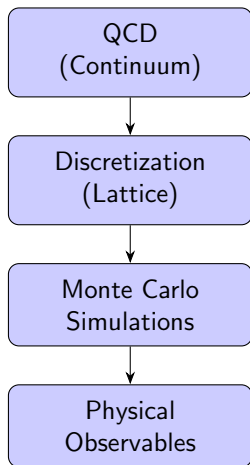
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Outline

- 1 Introduction and Motivation
- 2 Lattice Setup
- 3 Lattice Artefacts & Finite Size Effects
- 4 Summary & Outlook

Lattice QCD and Scale Setting



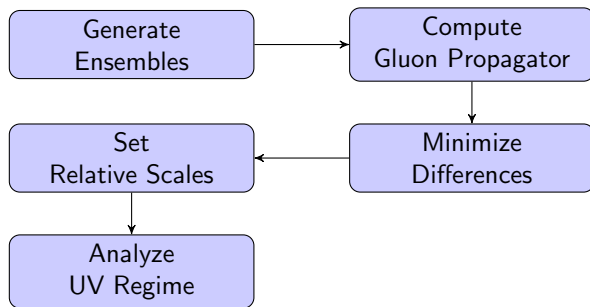
- LQCD is a **first-principles, nonperturbative** approach to QCD
- Simulations yield **dimensionless quantities**
- Comparison with experiment need a **physical scale**, i.e. **lattice spacing** a in physical units
- **Traditional Methods:**
 - Heavy quark potential / Sommer scale (r_0, r_1) .
 - Wilson flow scales (t_0, w_0) .
 - Hadronic observables (m_Ω, f_π) .
 - **The Ambiguity:** Different methods typically show systematic variations at the few-percent level.
- Issues not related:
 - **breaking of rotational symmetry**
The symmetry group is H_4 and not $O(4)$
 - **finite volume effects**

The Landau Gauge Gluon Propagator and the Physical Scale

Advantages

- Relies **solely** on the Landau gauge gluon propagator.
- **Computational Efficiency:** Avoids ghost propagator computation, which is very time-consuming.
- **Accessibility:** Modern LQCD codes include a gluon propagator module (gauge fixing, and $D(p^2)$).
- **Large statistical ensembles** ($\sim 10^4$ configurations per ensemble) produce smooth propagator.
- **Efficient** way of accessing $a(\beta)$

Methodology Overview



- 1 Generate large ensembles of gauge configurations for different β values.
- 2 Compute the Landau gauge gluon propagator for each β value.
- 3 Minimize the difference between propagators from different setups to get the relative scale.
- 4 Study the UV regime to identify and handle finite size effects.

Important: the function $a(\beta)$ depends on discretization (action) used and on the momentum definitions.

Definitions & Tensorial Structure

The gluon propagator in the Landau gauge is the two-point vacuum correlation function:

$$\langle A_\mu^a(p') A_\nu^b(p) \rangle = V \delta(p' + p) D_{\mu\nu}^{ab}(p)$$

In the Landau gauge, the propagator is strictly orthogonal to the gluon momentum

$$D_{\mu\nu}^{ab}(p) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) D(p^2) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}$$

→ $Z(p^2)$ is the gluon dressing function

Lattice Setup

Lattice Setup

- Pure Yang-Mills SU(3) theory with Wilson action
- Two different β values and Two different physical volumes
- 10^4 configurations per ensemble
- Only the Landau gauge will be considered (Gribov noise will be disregarded)

β	Volume	# Configs	$a^{-1}(\text{GeV})$	$L(\text{fm})$
6.0	32^4	10^4	1.943	3.25
6.0	48^4	9034	1.943	4.88
6.2	48^4	10^4	2.718	3.49
6.2	64^4	10^4	2.718	4.64

- Lattice spacing measured from the string tension.
- $a^{-1}(\beta = 6.0)$ will be used to benchmark the relative value of $a^{-1}(\beta = 6.2)$.

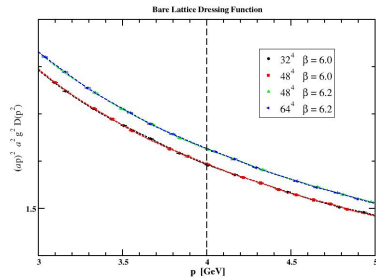
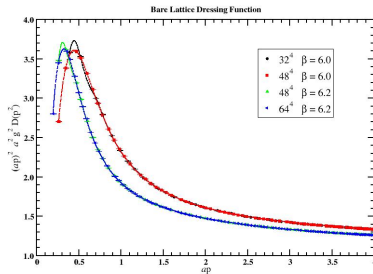
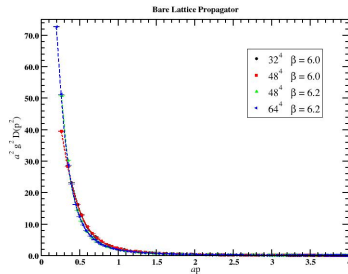
Further definitions

- Use as momentum the improved momentum

$$p_\mu = \frac{2}{a} \sin\left(\frac{\pi}{L} n_\mu\right) \quad \text{with} \quad n_\mu = -L/2, \dots, -1, 0, 1, \dots, L/2 - 1 ;$$

- $D(p^2)$ renormalized in the MOM scheme, i.e. $D(\mu^2) = 1/\mu^2$ at $\mu = 4$ GeV.
- Only data surviving **cylindrical and conical momentum cuts for $p > 0.7$ GeV** will be considered (reducing the effects of breaking of rotational symmetry). For **smaller momenta all data** will be included.
- The large statistical ensembles allows the use of **cubic splines to smoothly interpolate** data. In the interpolation the zero momentum points will be excluded (they introduce a bias for the lower momentum).

Bare Lattice Data



The Relative Scale Setting Minimization

Minimization Target

On each volume, rescale the momentum for $\beta = 6.2$ and minimize the renormalized difference

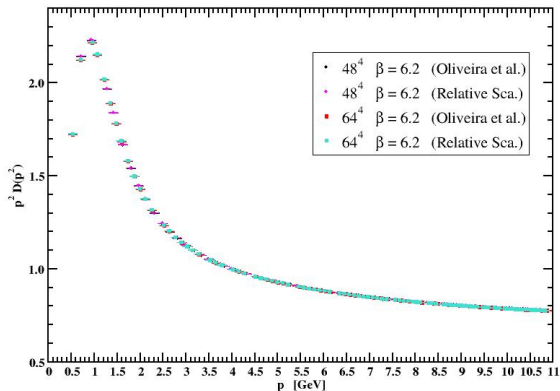
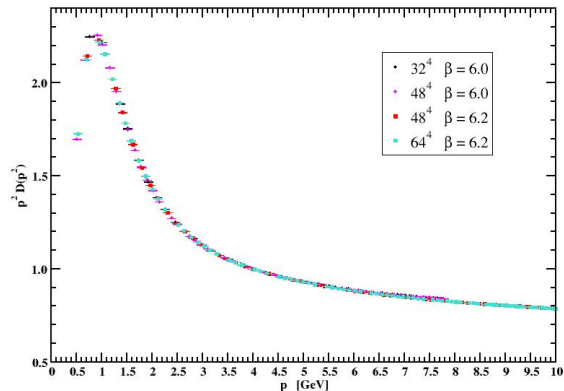
$$\chi_{\text{rel}}^2 = \sum_i \int_{p_{\min}}^{p_{\max}} dp \left[D_{\beta=6.0}(p^2) - D_{\beta=6.2}(p^2) \right]^2$$

- Set $p_{\max} = 7$ GeV (no difference if used instead $p_{\max} = 5$ GeV).
- Set $p_{\min} = 1.5$ GeV.
- By scaling the momentum space values properly, the data seamlessly collapses onto a unique, well-defined smooth curve.
- The minimization gives

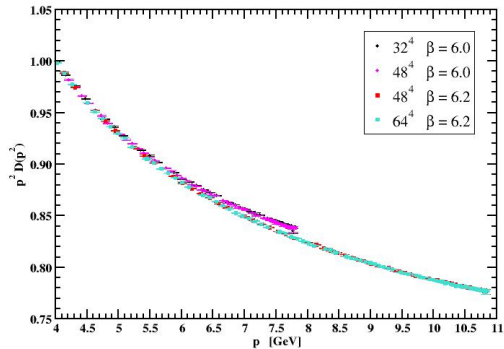
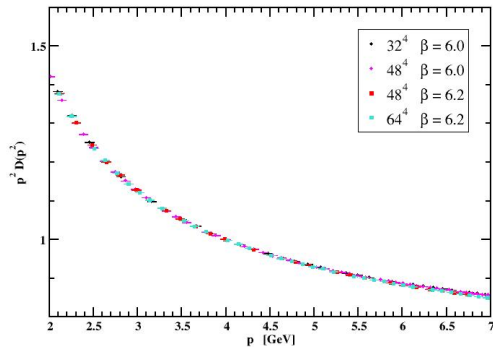
$$a^{-1}(\beta = 6.0) = 1.943 \text{ GeV} \quad \text{and} \quad a^{-1}(\beta = 6.2) = 2.7052 \text{ GeV}.$$

The latter should be compared with the value on the first table $a^{-1}(\beta = 6.2) = 2.718$ GeV.

Lattice Data I



Lattice Data II



Comments on the UV renormalized data

- **Observation I:** good agreement for each lattice spacing and both volumes (small volume effects?)
- **Observation II:** scaling violations above 5 GeV (lattice spacing effects?)
- **Observation III:** all data sets compatible with the functional form for $D(p^2)$ predicted by RG-improved perturbation theory.
- **Origin:** interplay of finite lattice spacing and finite volume effects
- **Our approach:** Assume that the UV differences are due to lattice spacing effects and rely on perturbation theory to try to understand them.

An Effective Description of Finite Size Effects

Working hypothesis

$$D_L(p^2) = D_C(p^2) \left(1 + \sum_j c_j(a, V) \alpha_s^j(\mu) \ln \frac{p^2}{\zeta^2} \right)^j$$

where μ is the **renormalization scale**, ζ is a mass scale and, at large momentum,

$$D_C(p^2) = \frac{Z(\mu)}{p^2} \left(1 + \frac{33 \alpha_s(\mu)}{12 \pi} \ln \frac{p^2}{\zeta^2} \right)^{-\gamma} \quad (\text{one-loop RG improved})$$

with $\gamma = 13/22$ being the gluon anomalous dimension.

Lattice Log-Log Plots

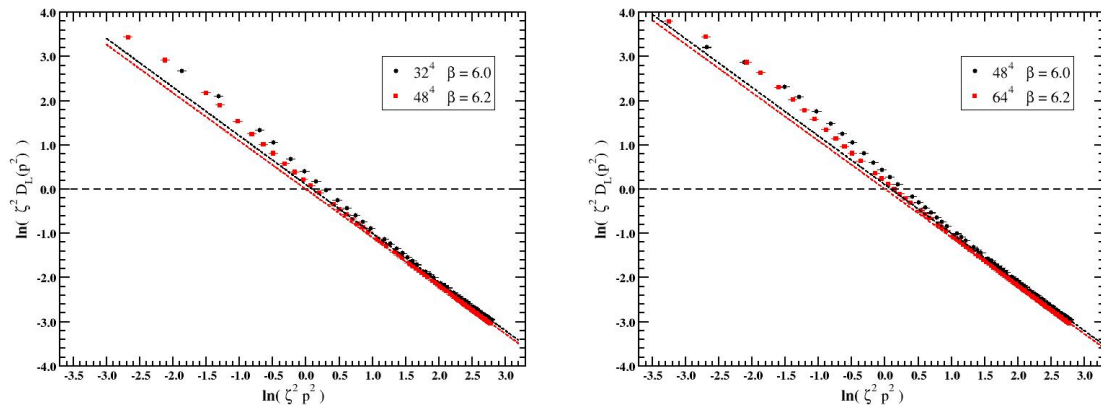


Figure 1: Left: Log-Log plot for the smallest volume. Right: Log-Log plot for the smallest volume. Dashed Lines: outcome of linear fit

Understanding the data in the UV

Stochastic Lattice Perturbation Theory (extrapolated to the continuum)

$$D_{sPt}(p^2) = Z \frac{2.29368 - 0.24697 \ln(a^2 p^2)}{p^2}$$

Effective Parameterization

$$D_L(p^2) = D_{sPt}(p^2) + \sum_{j=0} c_j p^{2j}$$

Understanding the data in the UV

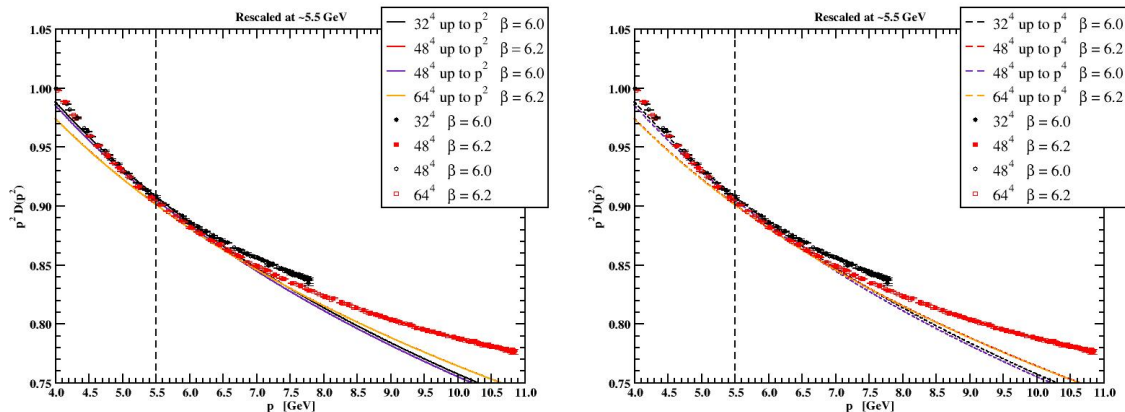
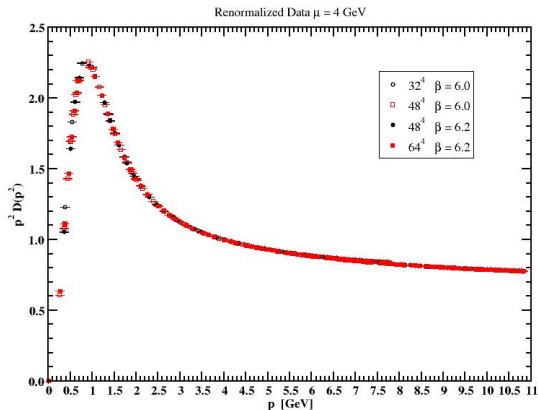
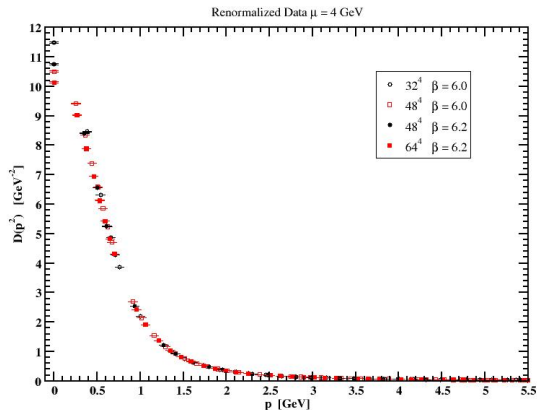


Figure 2: UV gluon dressing function corrected for the lattice artefacts. The lines in the figures refer to the outcome of the fits when only terms up to p^2 are included to model the lattice artefacts. The dashed lines in the figures were obtained including terms up to p^4 to model the lattice artefacts.

All Lattice Data Corrected in UV



Summary and Conclusions

- **Accomplishment:** large statistical ensembles + cuts produce smooth gluon propagator data
 - **Simple & Robust Method:** The Landau gauge gluon propagator offers a reliable scale-setting technique
 - **UV Control:** Effective description and correction of finite size lattice artefacts that dominate the UV regime (> 5 GeV).
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- **Extension to other N -point Green functions:** The minimization and correction procedure discussed can be adapted to other Green functions but need large statistical ensembles
 - **Further developments:** help with the sampling and noise reduction techniques will be welcome

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Thank you!

Questions & Comments?