EWPOs, effective potential and vacuum stability

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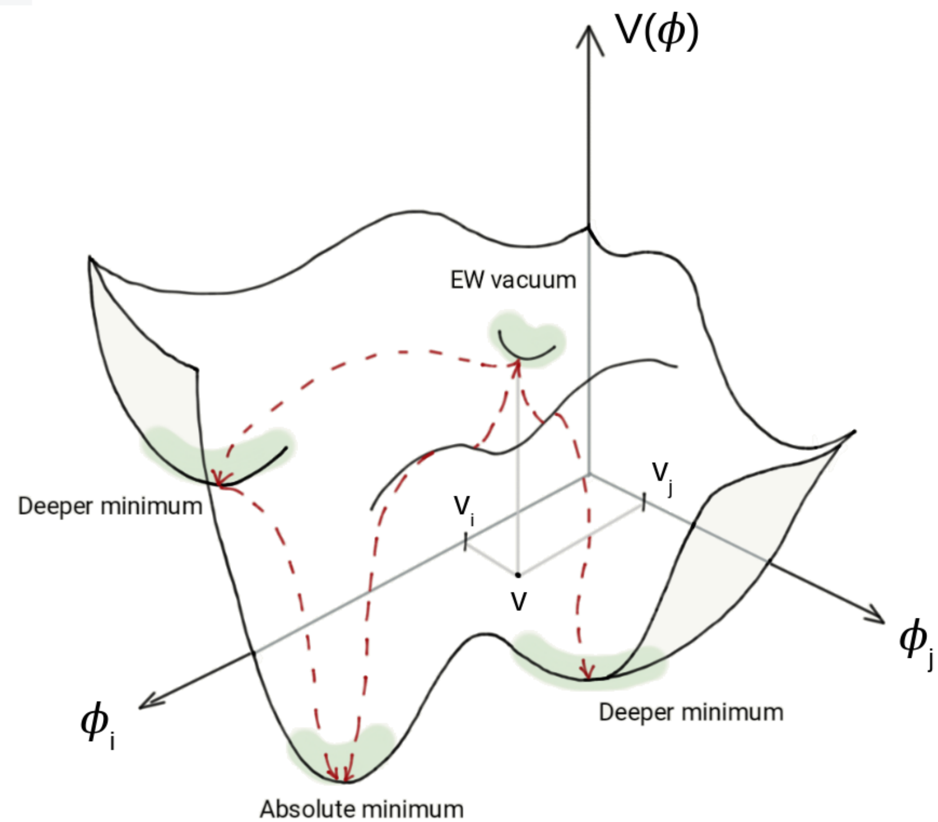
Outline

- Introduction
- BSM predictions for EWPOs
- Some remarks about the renormalisation of BSM models
- Constraints on BSM models from vacuum stability
- Effective Higgs potential with temperature-dependent effects
- Conclusions

Introduction

Electroweak precision observables (EWPOs): test of BSM models via quantum corrections
Focus here as example on predictions for the W mass, M_W

Higgs potential: crucial for EWSB, thermal evolution of the early universe, baryogenesis, vacuum stability, …



[K. Radchenko ’24]

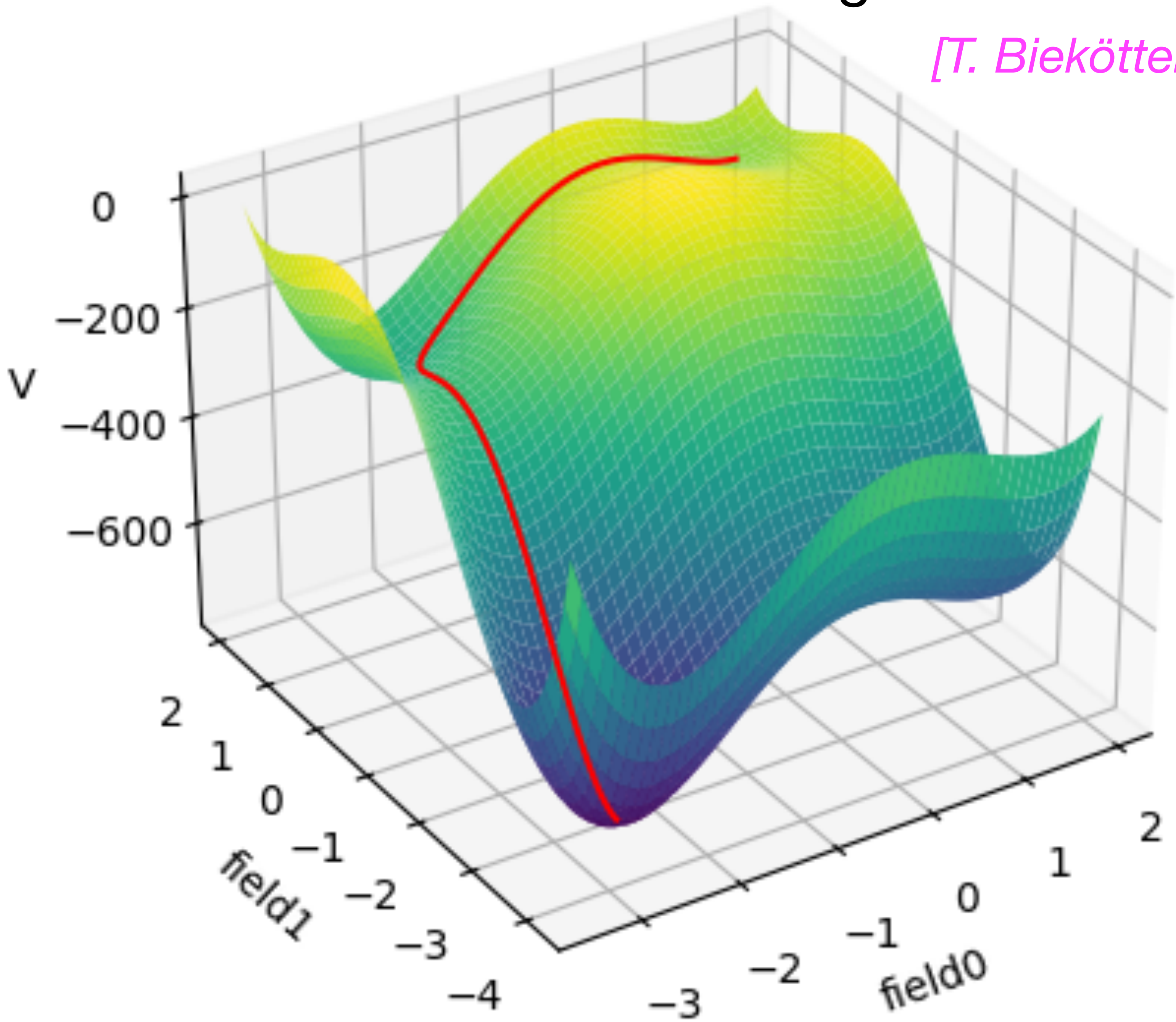
Shape of the Higgs potential realised in nature and underlying physics largely undetermined so far

The Higgs potential and vacuum stability

Simple toy example: two singlet-type Higgs fields

Tunneling from a local minimum into the global minimum:

[T. Biekötter, F. Campello, G. W. '25]



⇒ Proceeds via intermediate local minimum

The Higgs potential and vacuum stability

Extended Higgs sectors in general yield additional minima of the Higgs potential; the electroweak minimum may not be the global minimum

Need to check stability of the electroweak vacuum w.r.t. tunneling into all deeper minima (analysis at T = 0)

Decay rate of ``false" vacuum state: $$\frac{\Gamma}{V_S} = K e^{-B}$$

B: bounce action, related to solution of Euclidean equation of motion

Public code *Evade* *[W.G. Hollik, G. W., J. Wittbrodt '18]*

Original version uses straight path approximation for tunnelling path

Recent improvements: path-deformation algorithm, machine-learning based solution of partial differential equation

[T. Biekötter, F. Campello, G. W. '26] *[T. Biekötter, A. Simon, G. W. '26]*

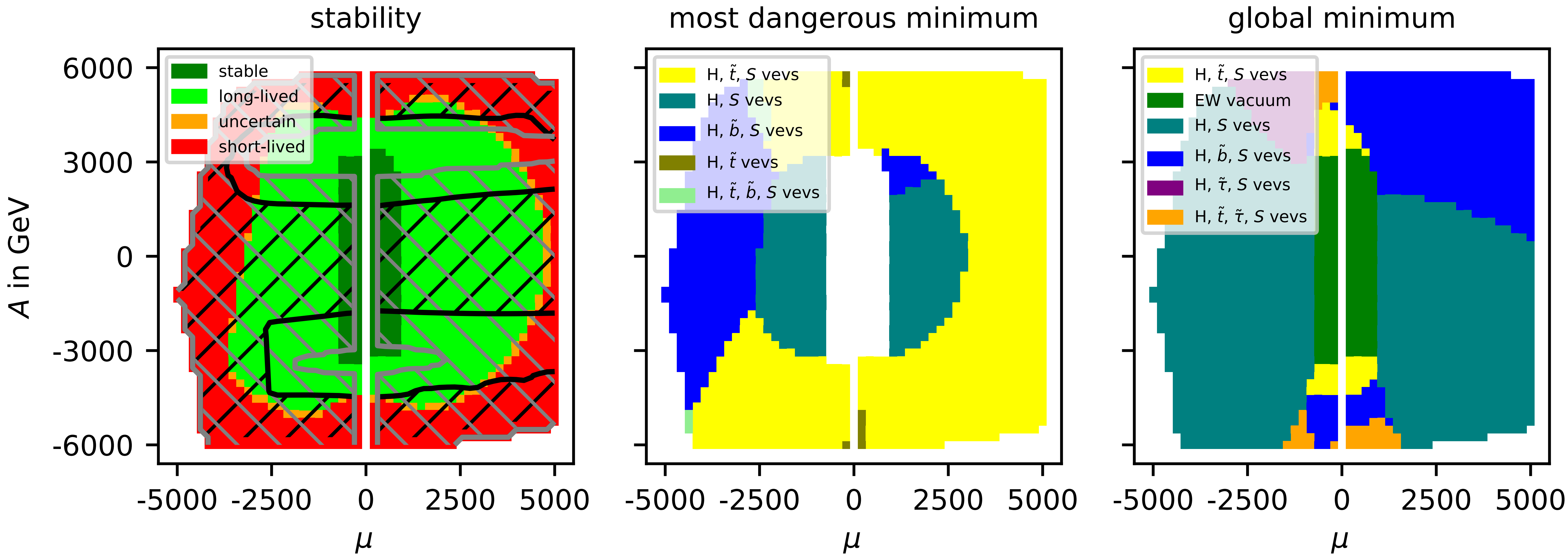
Other existing tools: *CosmoTransitions* *[C.L. Wainwright '11]*

Vevacious *[J.E. Camargo-Molina et al '13, '14]* *BSMPT* *[P. Basler, M. Mühlleitner '18]*

The Higgs potential and vacuum stability

Example: constraints from vacuum stability in the NMSSM on the region allowed by *HiggsBounds* and *HiggsSignals*

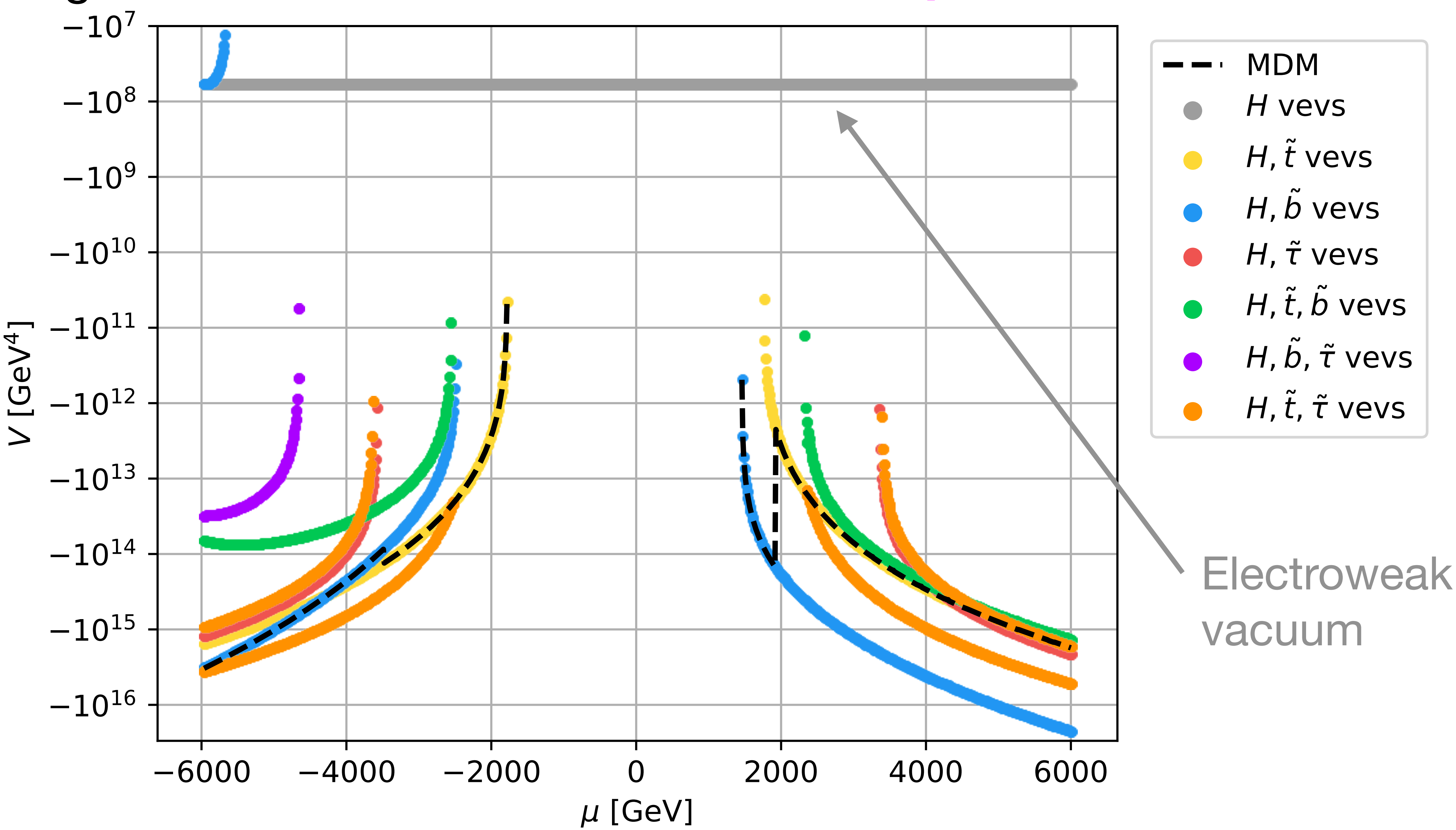
[T. Biekötter, F. Campello, G. W. '26]



⇒ Vacuum stability yields important constraints on parameter space of BSM models

Depth of stationary points of the Higgs potential

Along line with X_t = 2.8 TeV: [W.G. Hollik, J. Wittbrodt, G. W. '18]



⇒ Most dangerous minimum (MDM) often differs from the global minimum and also from the one that is closest in field space

Inclusion of temperature-dependent contributions

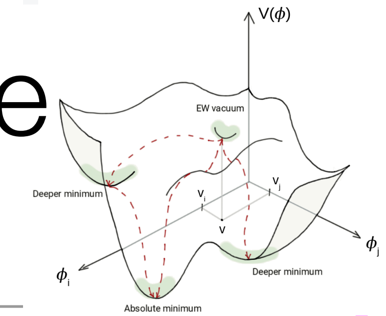
Vacuum stability analysis at T=0 investigates whether in the present Higgs potential predicted by BSM models the electroweak vacuum is sufficiently long-lived

The treatment of the thermal evolution of the early universe and in particular of the electroweak phase transition (EWPT) requires the incorporation of T-dependent contributions into the Higgs potential

In this way in can be analysed whether the thermal evolution was such that the universe actually ended up in the electroweak vacuum of the considered model

Other possibility: ``vacuum trapping”

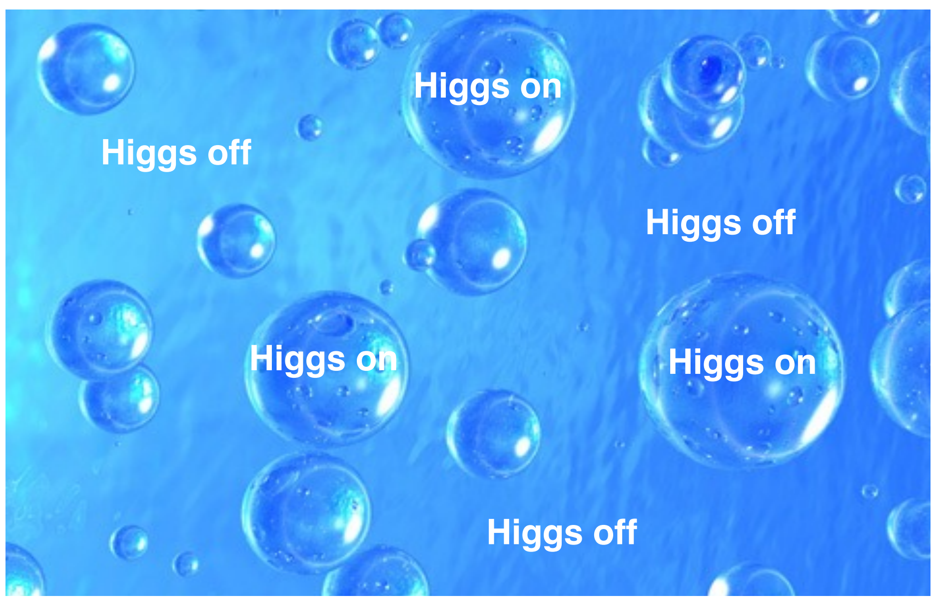
The Higgs potential and the electroweak phase transition (EWPT)

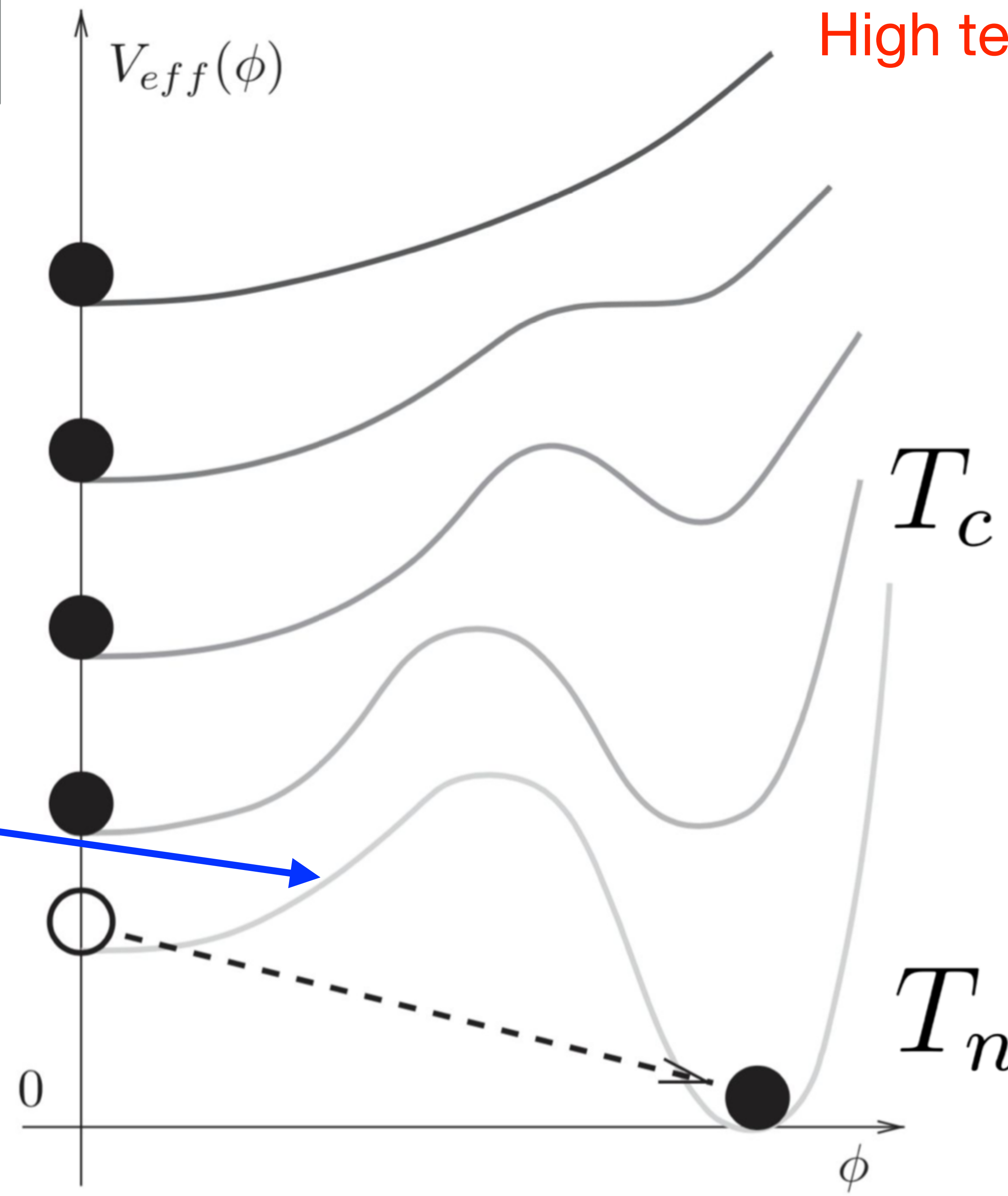


[D. Gorbunov, V. Rubakov]

Temperature evolution of the Higgs potential in the early universe:

$$V(\phi, T) = V_0(\phi) + V^{loop}(\phi, T)$$





High temperature

Critical temperature (degenerate minima)

Transition temperature (strong first-order EWPT)

Potential barrier depends on trilinear Higgs coupling(s)

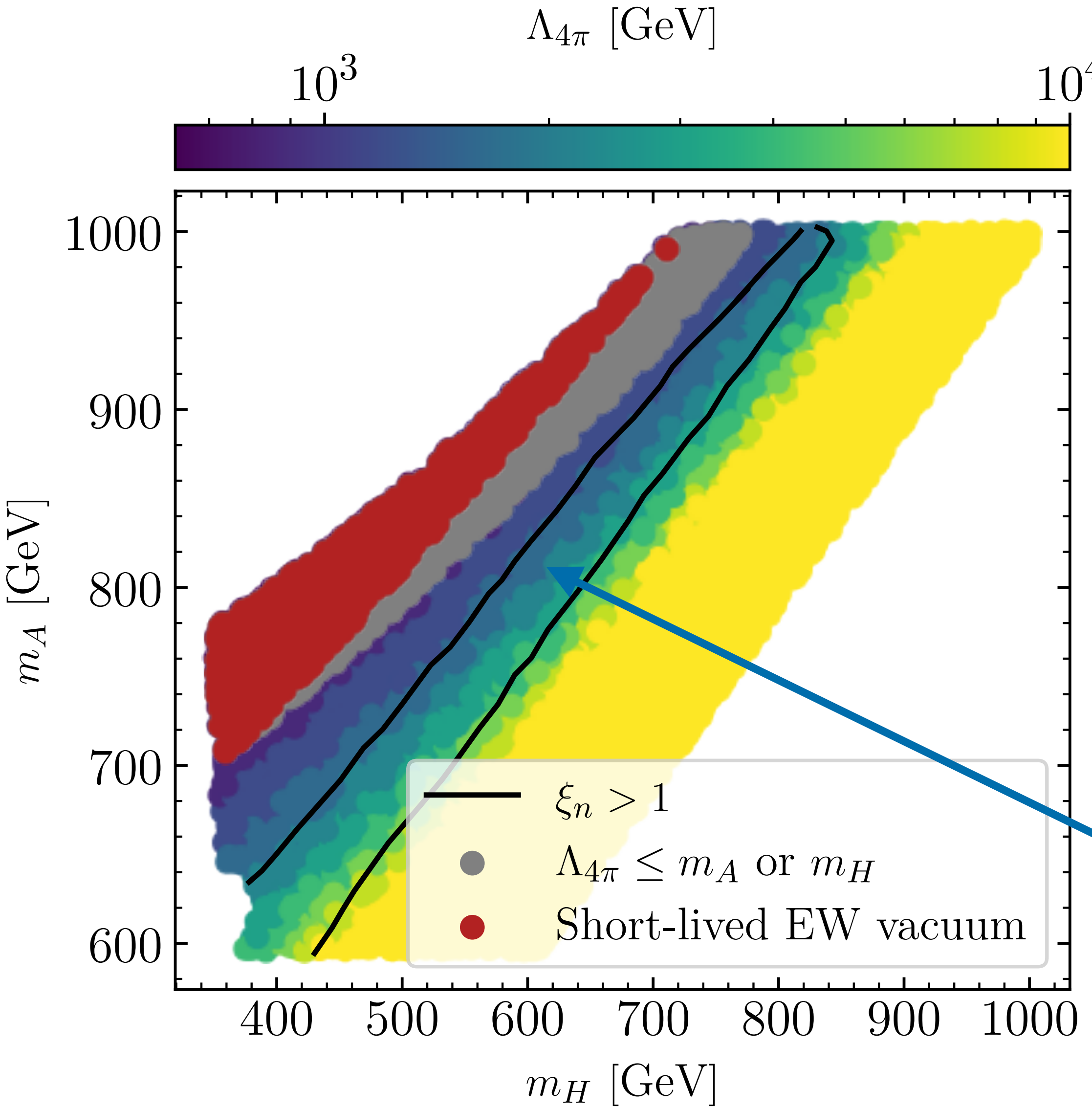
EW baryogenesis: creation of the asymmetry between matter and antimatter in the universe requires strong first-order EWPT

Correlation of strong first-order EWPT with splitting between BSM Higgs masses of extended Higgs sector

[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]

2HDM, N2HDM, … : the parameter region giving rise to a strong first-order EWPT, which may cause a detectable gravitational wave signal, yields an enhancement of the trilinear Higgs self-coupling and "smoking gun" signatures at the LHC

2HDM of type II, alignment limit, tanβ = 3:

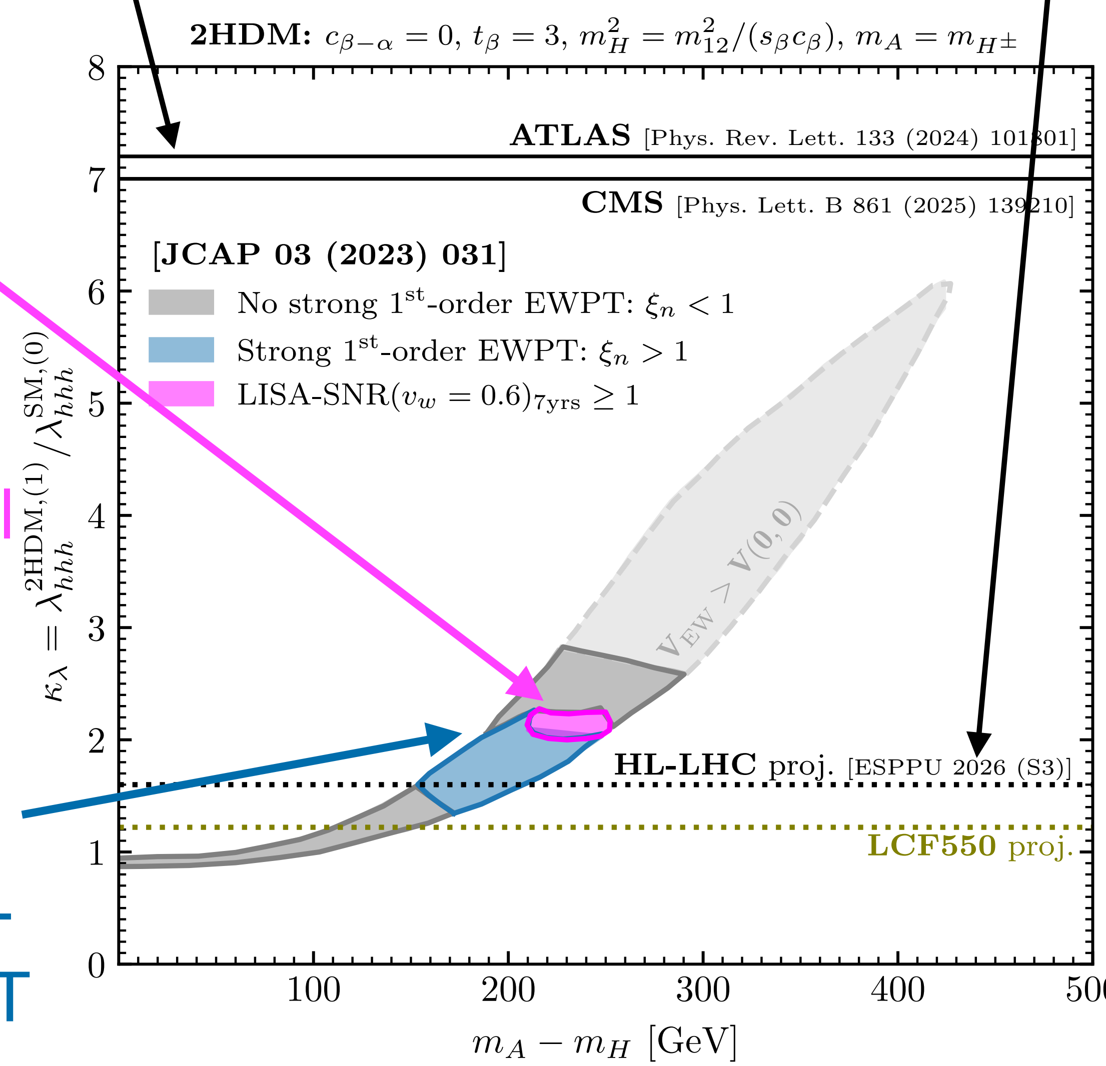


region with potentially observable gravitational wave (GW) signal

region with strong first-order EWPT

current bound

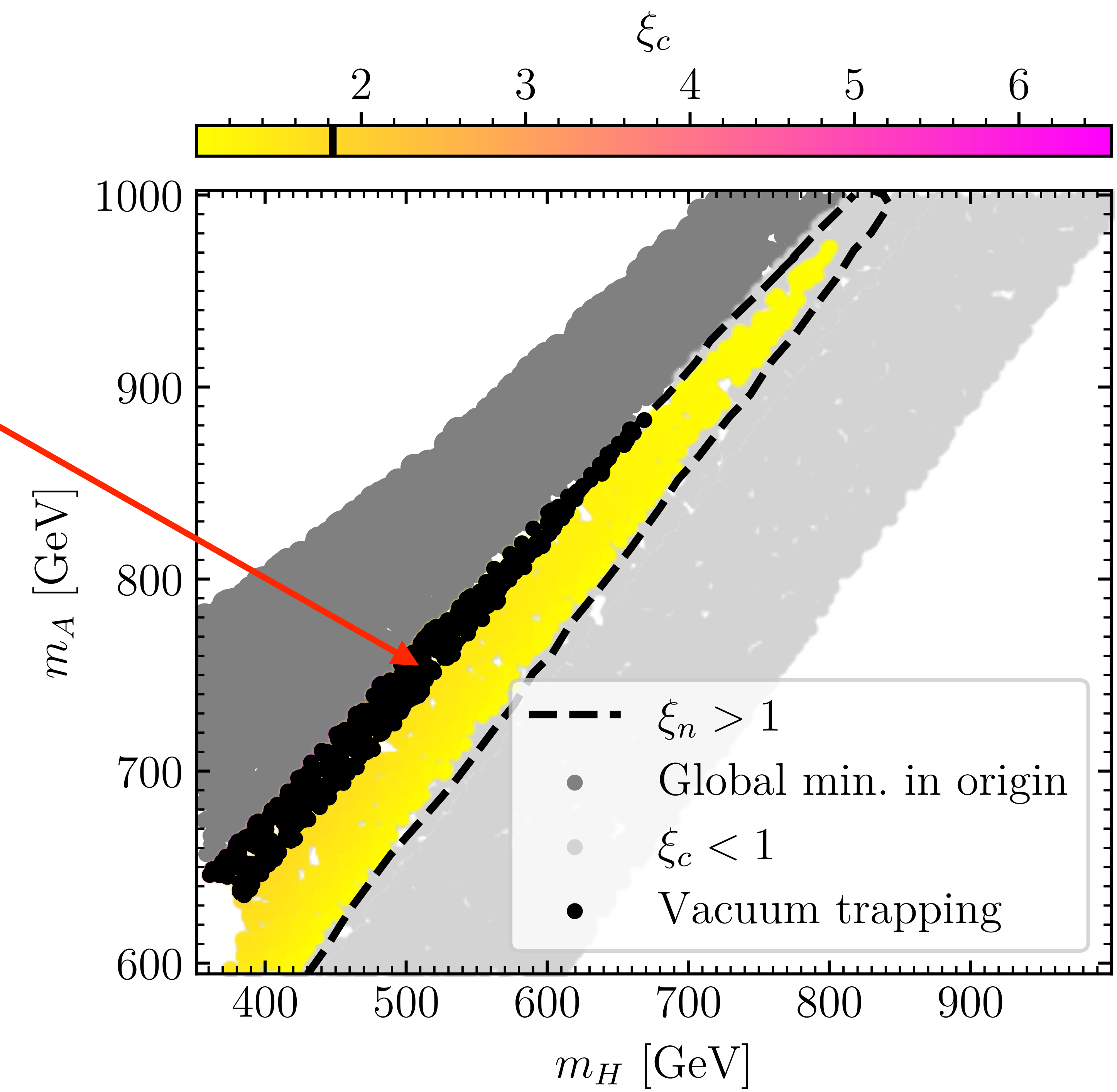
HL-LHC sensitivity



2HDM of type II: region of strong first-order EWPT

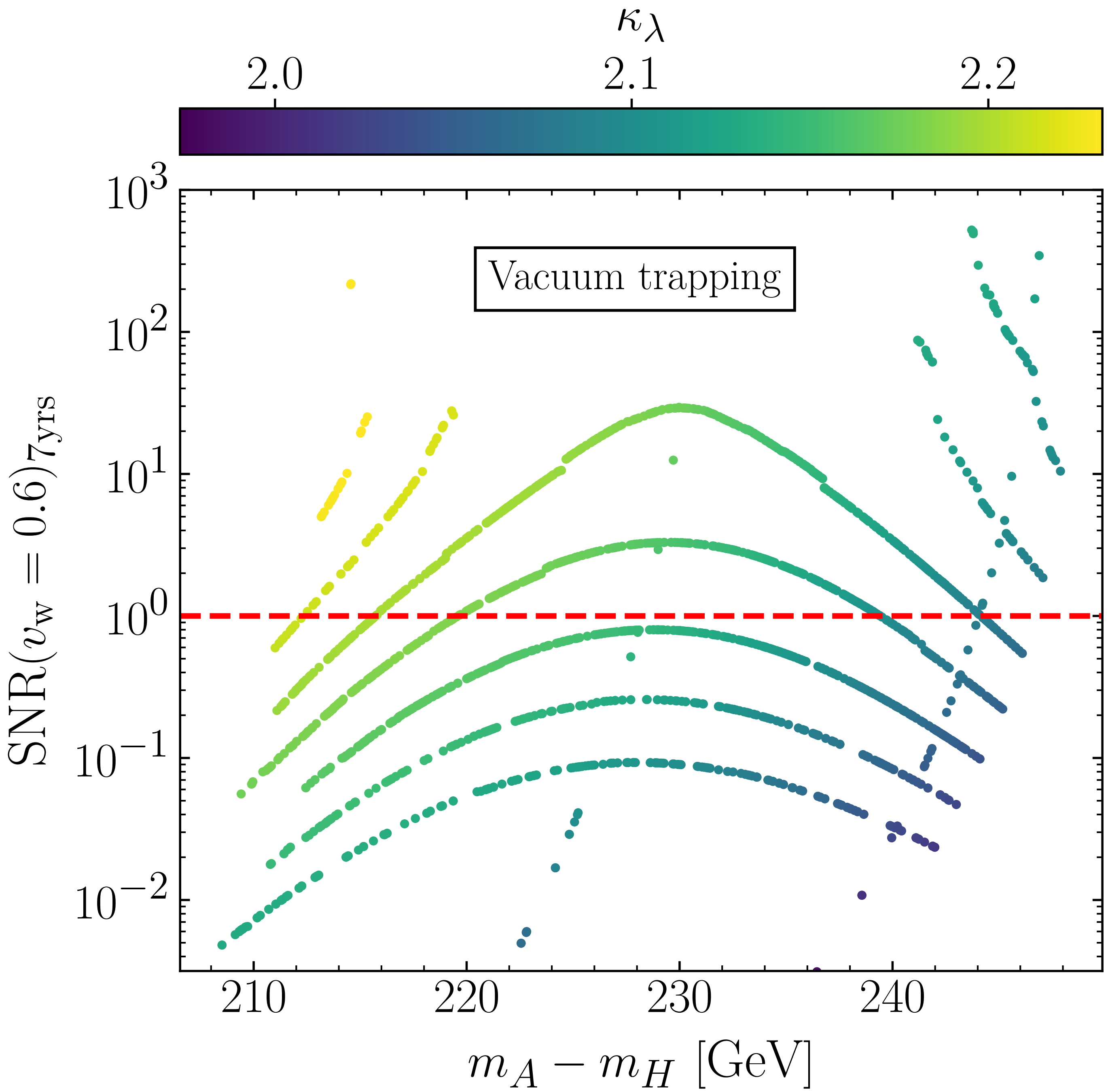
[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]

Constraints from "vacuum trapping": the universe may remain "trapped" in a symmetry-conserving vacuum at the origin, because the conditions for a transition into the deeper EW-breaking minimum are not fulfilled



Correlation of $\varkappa_\lambda$ with the signal-to-noise ratio (SNR) of a gravitational wave signal at LISA

[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, G. W. '22]



⇒Region with potentially detectable gravitational wave signal: significant enhancement of $\varkappa_\lambda$ and non-vanishing mass splitting

BSM predictions for EWPOs

Experimental accuracy for M_W and $\sin^2\theta_{eff}$ provides high sensitivity to loop contributions at the two-loop level and beyond!

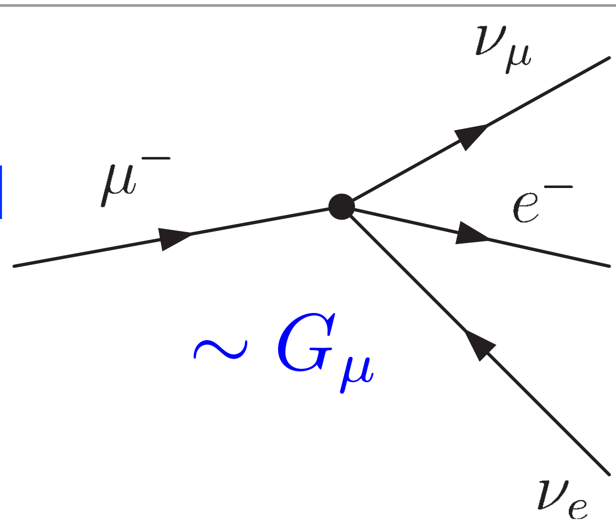
While within the SM the predictions for M_W and $\sin^2\theta_{eff}$ are known at the level of full two-loop and leading higher-order contributions, no full two-loop predictions exist in any BSM model

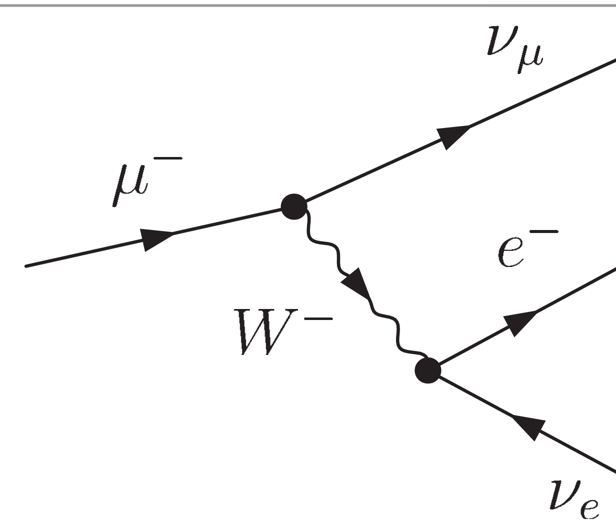
However, restricting the BSM predictions for the EWPOs to the 1-loop level would result in a prediction that is completely off because of the missing SM-like higher-order contributions!

⇒ BSM predictions for (at least) M_W and $\sin^2\theta_{eff}$ need to take into account all known SM-like contributions + the prediction for (BSM – SM) at the level of accuracy for which the BSM prediction is known!

Theoretical prediction for the W-boson mass from muon decay: relation between M_W, M_Z, α, G_μ

Fermi model





SM

$M_{\rm W}$: Comparison of prediction for muon decay with experiment (Fermi constant G_μ); QED corrections in Fermi model incl. in def. of G_μ

$$\Rightarrow \quad M_{\rm W}^2\left(1-\frac{M_{\rm W}^2}{M_{\rm Z}^2}\right)=\frac{\pi\alpha}{\sqrt{2}G_\mu}\left(1+\Delta r\right),$$

$$\Updownarrow$$

$$\Rightarrow M_W^2 = M_Z^2\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\alpha\pi}{\sqrt{2}G_\mu M_Z^2}\left(1+\Delta r\right)}\right)$$

loop corrections

(except QED corrections in the Fermi model)

$\Rightarrow$ Theo. prediction for $M_{\rm W}$ in terms of $M_{\rm Z}$, α, G_μ, $\Delta r(m_{\rm t}, m_{\tilde{\rm t}}, \ldots)$

Tree-level prediction: $M_W^{\rm tree}$ = 80.939 GeV, $M_W^{\rm exp}$ = 80.369 ± 0.013 GeV

⇒ off by many σ (accuracy of 1.6 x 10^{-4})

W-mass prediction in the Standard Model (SM)

One-loop contribution:

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_\mathrm{w}^2}{s_\mathrm{w}^2}\Delta\rho + \Delta r_\mathrm{rem}(M_\mathrm{H}, \ldots)$$

≈ 6% ≈ –3% < 1%

$$\Delta\rho = \frac{\Sigma_T^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_T^{WW}(0)}{M_W^2}$$

contribution from isospin splitting: $\sim (m_t^2 - m_b^2) \approx m_t^2$

custodial symmetry: $\rho = 1$ at lowest order

M_W prediction in the Standard Model

Contributions beyond one-loop order:

$$\Delta r^{\mathrm{SM(h.o.)}} = \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}} + \Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)} + \Delta r^{(G_\mu m_t^2 \alpha_s^3)}$$

Chetyrkin, Kuhn, Steinhauser, Djouadi, Verzegnassi, Awramik, Czakon, Freitas, Weiglein, Faisst, Seidensticker, Veretin, Boughezal, Kniehl, Sirlin, Halzen, Strong, ...

Impact of different contributions to Δr (x 10^4) for fixed M_W = 80.385 GeV and M_H^{SM} = 125.09 GeV:

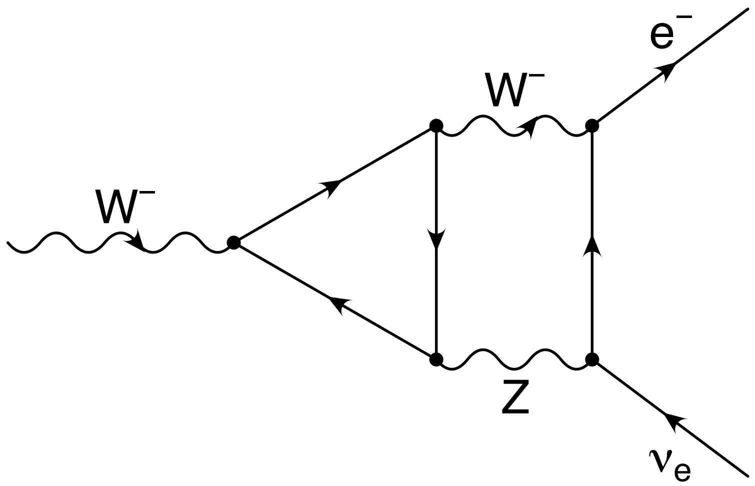
[O. Stål, G. W., L. Zeune '15]

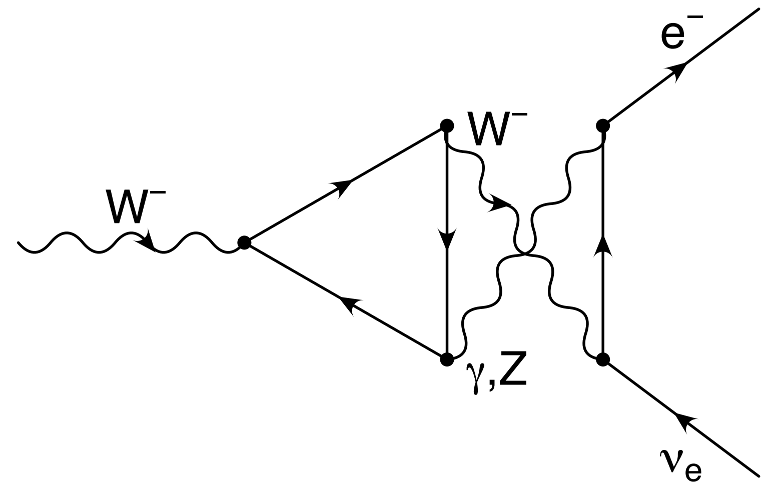
$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha\alpha_s)}$	$\Delta r^{(\alpha\alpha_s^2)}$	$\Delta r^{(\alpha^2)}_{\mathrm{ferm}} + \Delta r^{(\alpha^2)}_{\mathrm{bos}}$	$\Delta r^{(G_\mu^2 \alpha_s m_t^4)} + \Delta r^{(G_\mu^3 m_t^6)}$	$\Delta r^{(G_\mu m_t^2 \alpha_s^3)}$
297.17	36.28	7.03	29.14	-1.60	1.23

Shift of 1 x 10^{-4} in Δr roughly corresponds to –1.5 MeV shift in M_W

Needed for obtaining the full electroweak 2-loop res.

``Anomaly-type” contributions, careful treatment of γ_5 needed

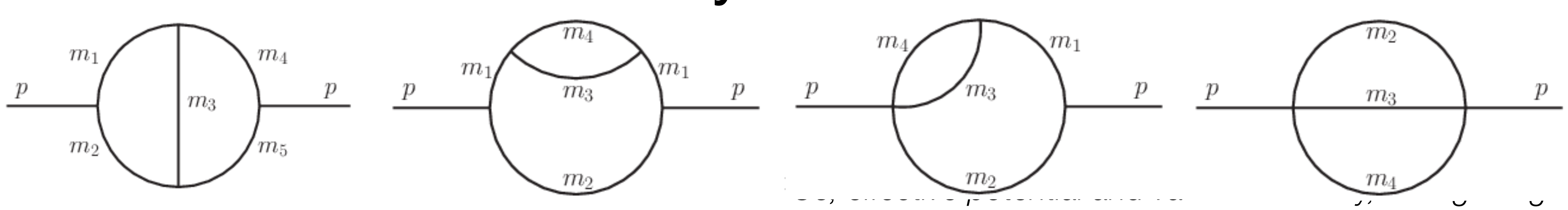




Consistent treatment of W and Z as unstable particles up to the 2-loop level, determination of the physical mass from expansion around the complex pole

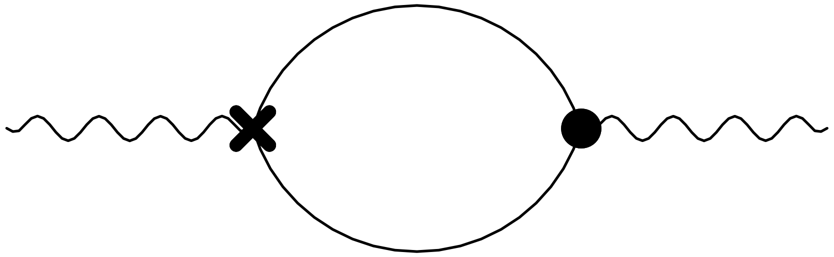
$$M^2 - iM\Gamma - m^2 + \underbrace{\hat{\Sigma}(M^2 - iM\Gamma)}_{\cong \hat{\Sigma}(M^2) - iM\Gamma\hat{\Sigma}'(M^2) + \ldots} = 0$$

2-loop 2-point integrals with non-vanishing external momentum and different internal masses: only numerical results available



Pure fermion-loop contributions at n-loop order

Contribution of n fermion loops at n-loop order:
no irreducible n-loop diagram, but e.g. 1-loop diagram with n–2 counterterm insertions
2-loop example:



Result up to n-loop order (n = 2, 3, 4, …) for running width definition of M_W and M_Z [A. Stremplat '98] [G. W. '98]

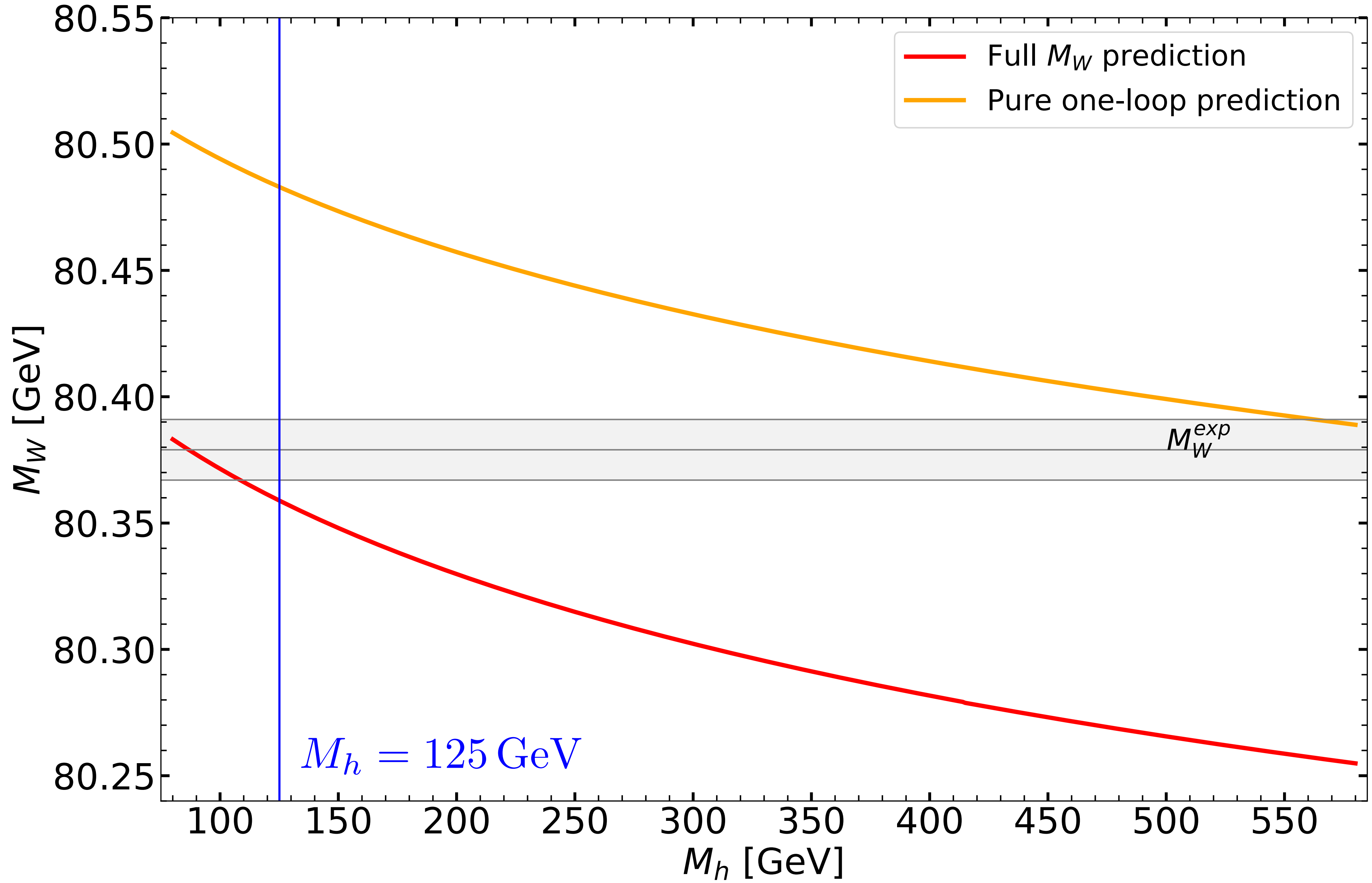
3-loop result for fixed width definition of M_W and M_Z [L. Chen, A. Freitas '20]

3-loop result for 2-loop QCD contribution + closed fermion loop [L. Chen, A. Freitas '20]

W-mass prediction within the SM: one-loop result vs. state-of-the-art prediction

(up to two weeks ago)

[M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]

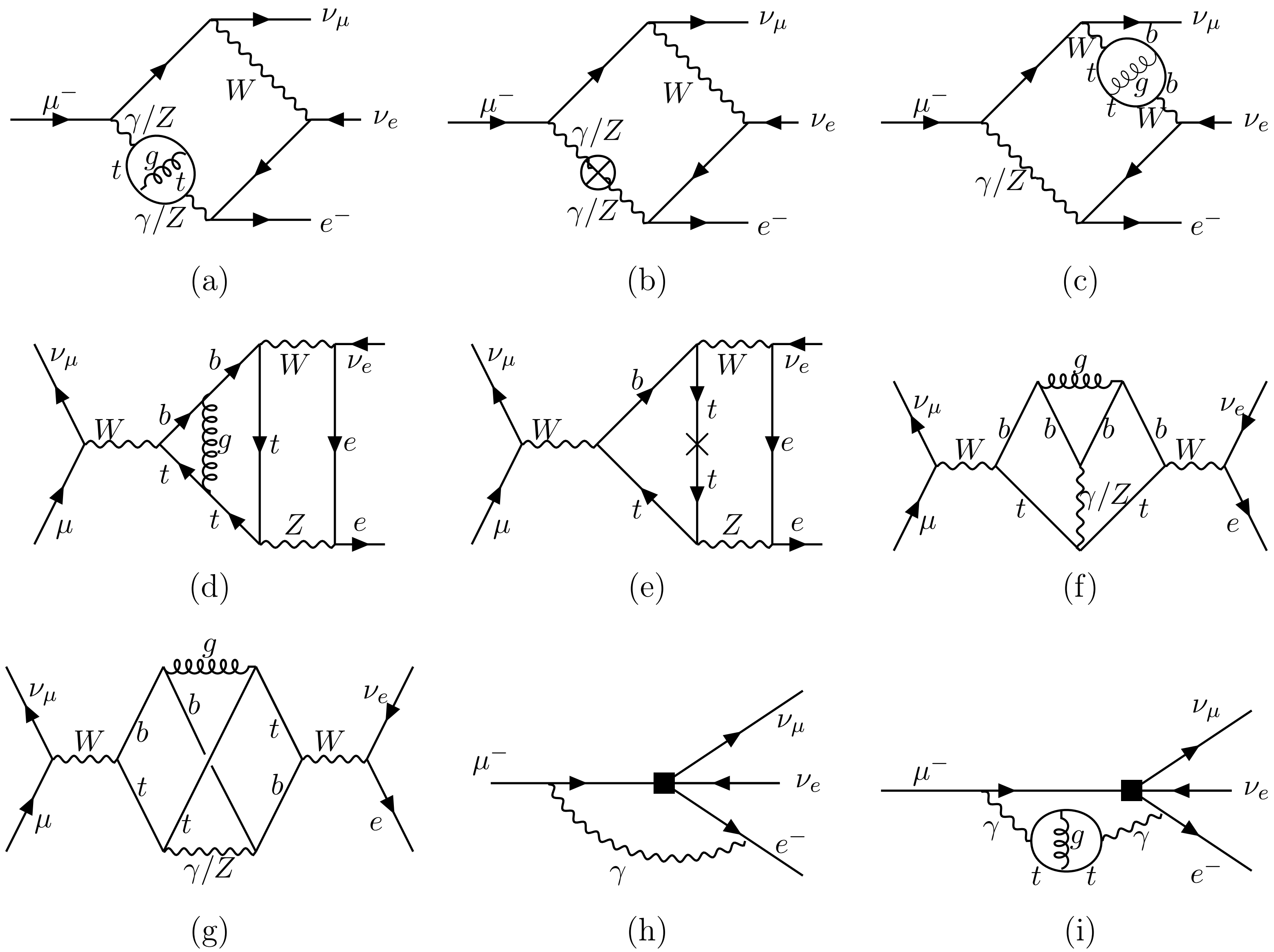


⇒ Pure one-loop result would imply preference for heavy Higgs, M_h > 500 GeV

Corrections beyond one-loop order are crucial for reliable prediction of M_W

New 3-loop result of O($N_f\alpha^2\alpha_s$)

3-loop contributions with one fermion loop, mixed electroweak + QCD



[I. Dubovyk, A. Freitas, J. Gluza, J. Usovitsch ’26]

New contribution to Δr of $-2.0\ 10^{-4}$ ⇔ ≈ +3 MeV in M_W

⇒ Needs to be implemented into BSM predictions

Sources of theoretical uncertainties

- From experimental errors of the input parameters

  $\delta m_t = 0.7$ GeV, $\delta(\Delta\alpha_{\rm had}) = 10^{-4}$, $\delta M_Z = 2.1$ MeV

$$\delta M_W^{\rm para, m_t} = 4 \text{ MeV}, \quad \delta M_W^{\rm para, \Delta\alpha_{had}} = 2 \text{ MeV}, \quad \delta M_W^{\rm para, M_Z} = 2.5 \text{ MeV}$$

- From unknown higher-order corrections (“intrinsic”)

  SM: Complete 2-loop result + leading higher-order corrections known for $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$

  $\Rightarrow$ Remaining uncertainties:

  [*M. Awramik, M. Czakon, A. Freitas, G.W. ’03, ’04*]

  [*M. Awramik, M. Czakon, A. Freitas ’06*]

update after latest result?

$$\Delta M_{\rm W}^{\rm intr} \approx 4 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff}^{\rm intr} \approx 5 \times 10^{-5}$$

BSM prediction for M_W, example: MSSM, NMSSM

Δr in the MSSM and the NMSSM, treatment of higher-order contributions:
full one-loop + higher orders (SM) + higher orders (SUSY)

$$\Delta r^{\mathrm{(N)MSSM}} = \Delta r^{\mathrm{(N)MSSM}(\alpha)} + \Delta r^{\mathrm{(N)MSSM(h.o.)}}$$

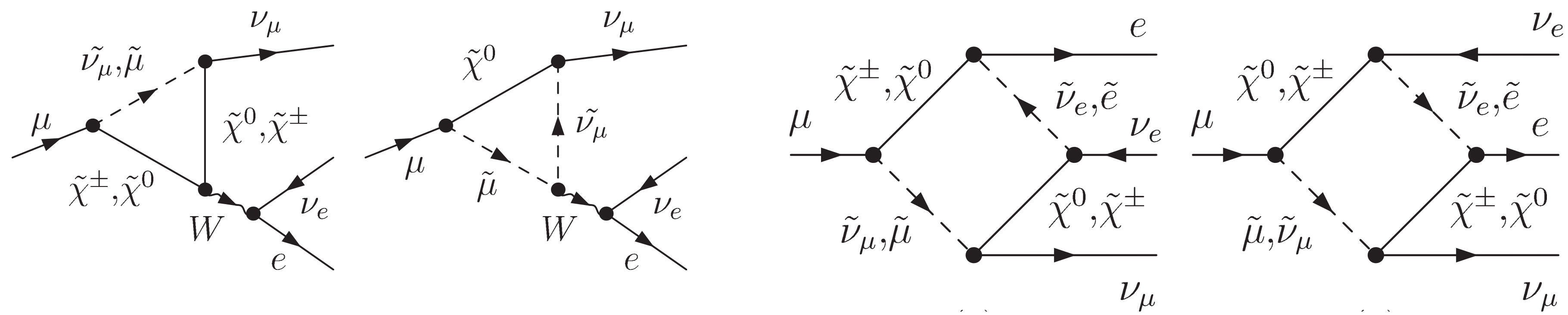
$$\Delta r^{\mathrm{(N)MSSM(h.o.)}} = \Delta r^{\mathrm{SM(h.o.)}} + \Delta r^{\mathrm{SUSY(h.o.)}}$$

⇒ State-of-the art SM prediction recovered in decoupling limit, all available higher-order corrections of SUSY-type included

For relatively light SUSY particles: additional theoretical uncertainty from higher-order SUSY-loop corrections

SUSY higher-order contributions

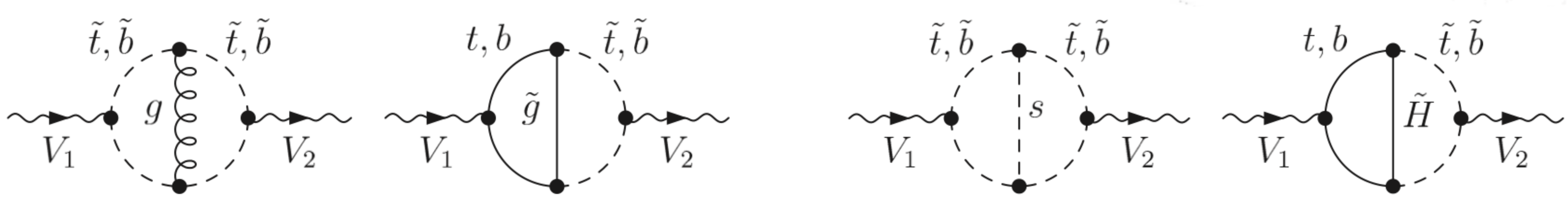
One-loop: complete result known in the MSSM and NMSSM; leading contributions from the scalar superpartners of the top and bottom quarks via Δϱ: additional source of isospin splitting



Two-loop:

leading reducible 2-loop corrections, gluon/gluino 2-loop corrections, higgsino 2-loop corrections

$$\Delta r^{\mathrm{SUSY(h.o.)}} = \Delta r_{\mathrm{red}}^{\mathrm{SUSY}(\alpha^2)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha\alpha_s)} - \frac{c_W^2}{s_W^2}\Delta\rho^{\mathrm{SUSY},(\alpha_t^2,\alpha_t\alpha_b,\alpha_b^2)}$$

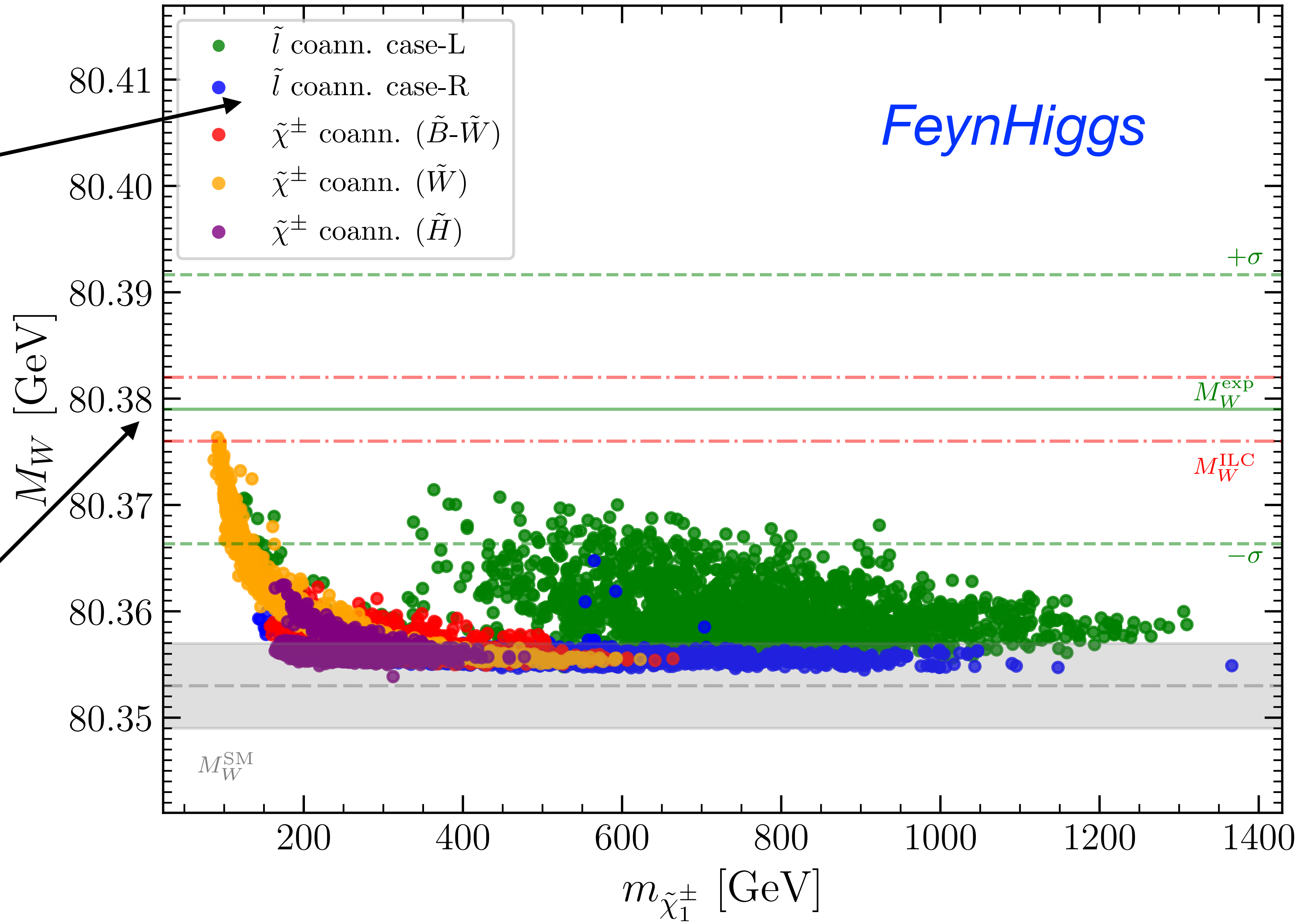


M_W prediction vs. the mass of the lightest chargino

[E. Bagnaschi, M. Chakraborti, S. Heinemeyer, I. Saha, G. W. '22]

Impact of light electroweak SUSY particles (squarks assumed very heavy!):

Different mechanisms for obtaining the right amount of dark matter



2022 world average without the CDF value

$\Rightarrow$ Upward shift w.r.t. SM prediction for light electroweak SUSY particles
Additional shift possible if stops, sbottoms are close to the exp. bounds

MRSSM prediction for M_W

Extended Higgs sectors consisting of doublets and singlets:
custodial symmetry ⇒ $\varrho = 1$ at lowest order

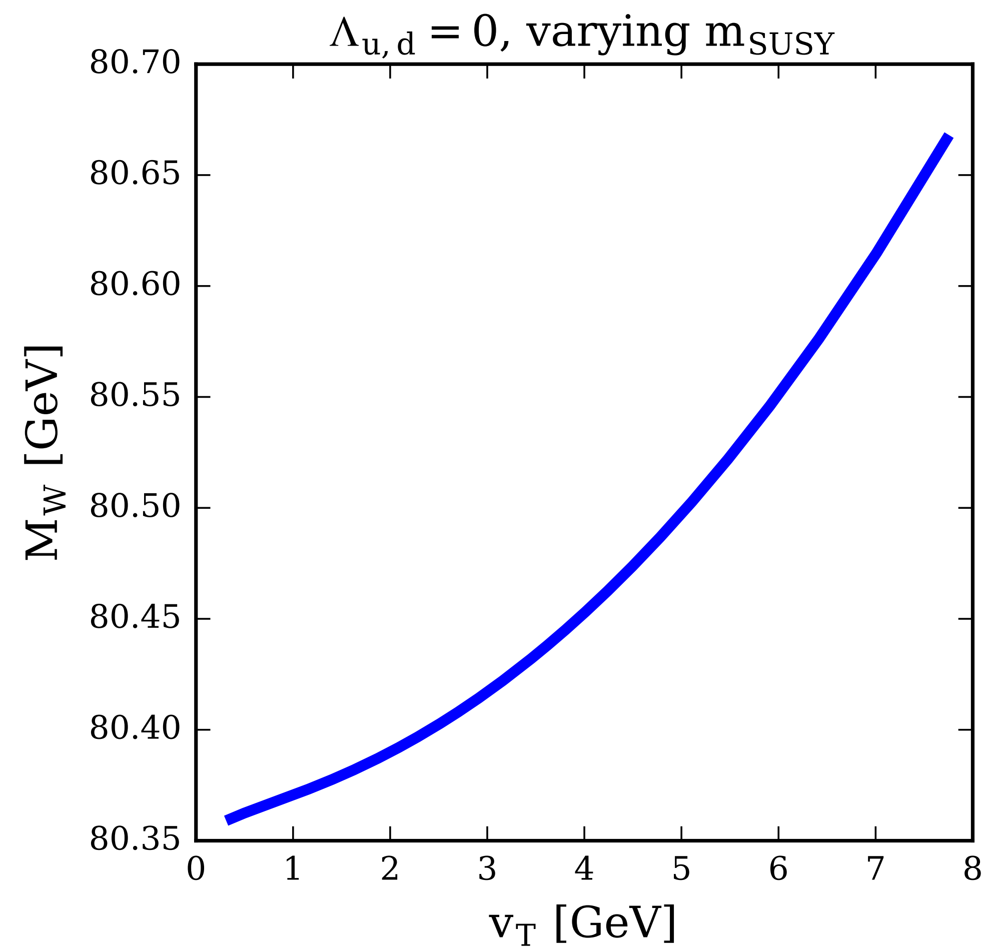
Lowest-order charged Higgs exchange contribution: ~ $(m_\mu m_e)/M_W^2$

⇒Main BSM contributions enter at 1-loop level: $\Delta r(m_i^{SM}, m_j^{BSM}, \ldots)$

Extended Higgs sectors involving triplets:
tree-level contribution from triplet v.e.v. v_T:
$M_W^2 = 1/4\, g_2^2\, v^2 + g_2^2\, v_T^2$

Example: MRSSM *[P. Diessner, G. W. '19]*

⇒Triplet v.e.v. v_T is constrained to be small



What are the prospects for improved M_W predictions?

Treatment of QED corrections: need the difference between the full 3-loop result and the 3-loop QED corrections in the Fermi model

Consistent treatment of W and Z as unstable particles at the 3-loop level?

Full fermionic 3-loop correction (dominant contribution at the 2-loop level)?
Need massive 3-loop 2-point integrals at non-vanishing momentum + consistent γ_5 scheme in D dimensions

Impact of hadronic higher-order uncertainties (see the case of $g_\mu-2$)?

Perform expansion in the top-quark mass?
Lesson from the 2-loop case: need to take into account at least the full Higgs-mass dependence

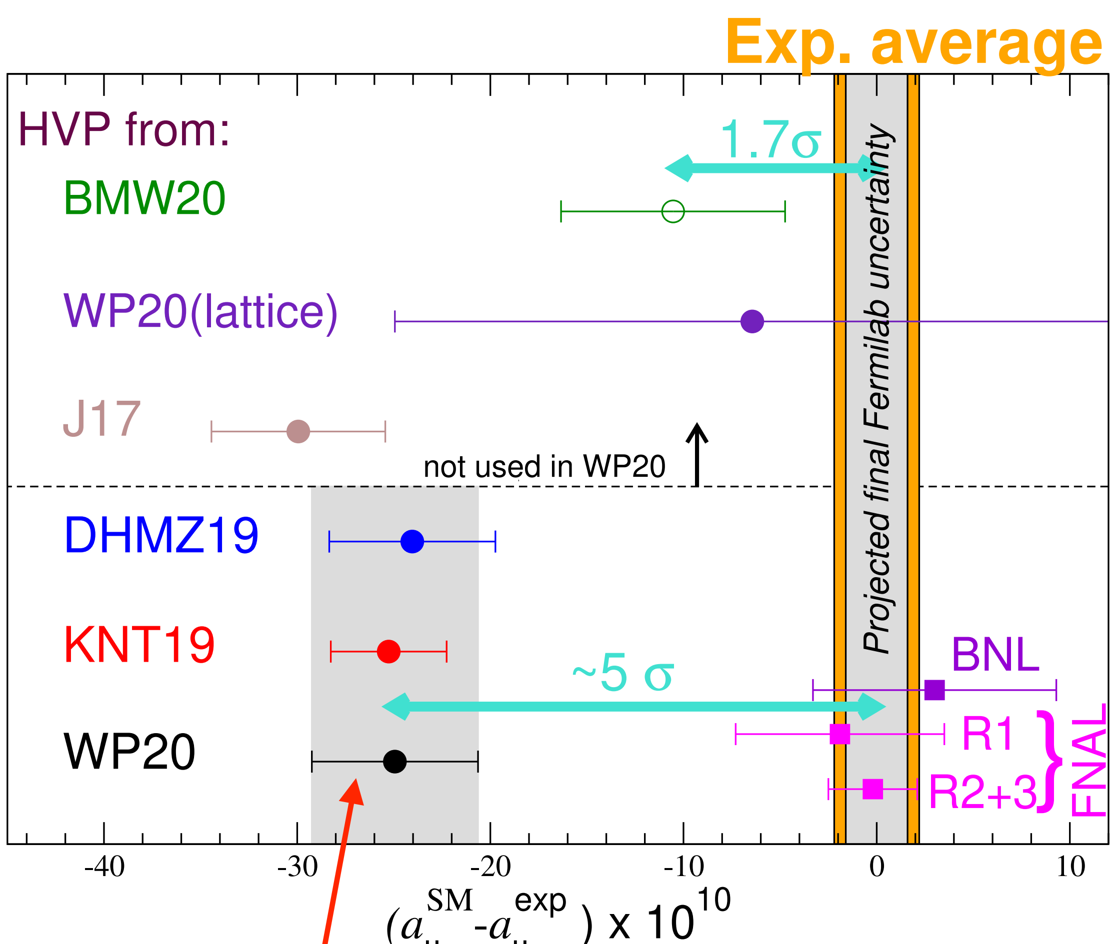
Recent example: g_μ–2, muon anom. mag. moment

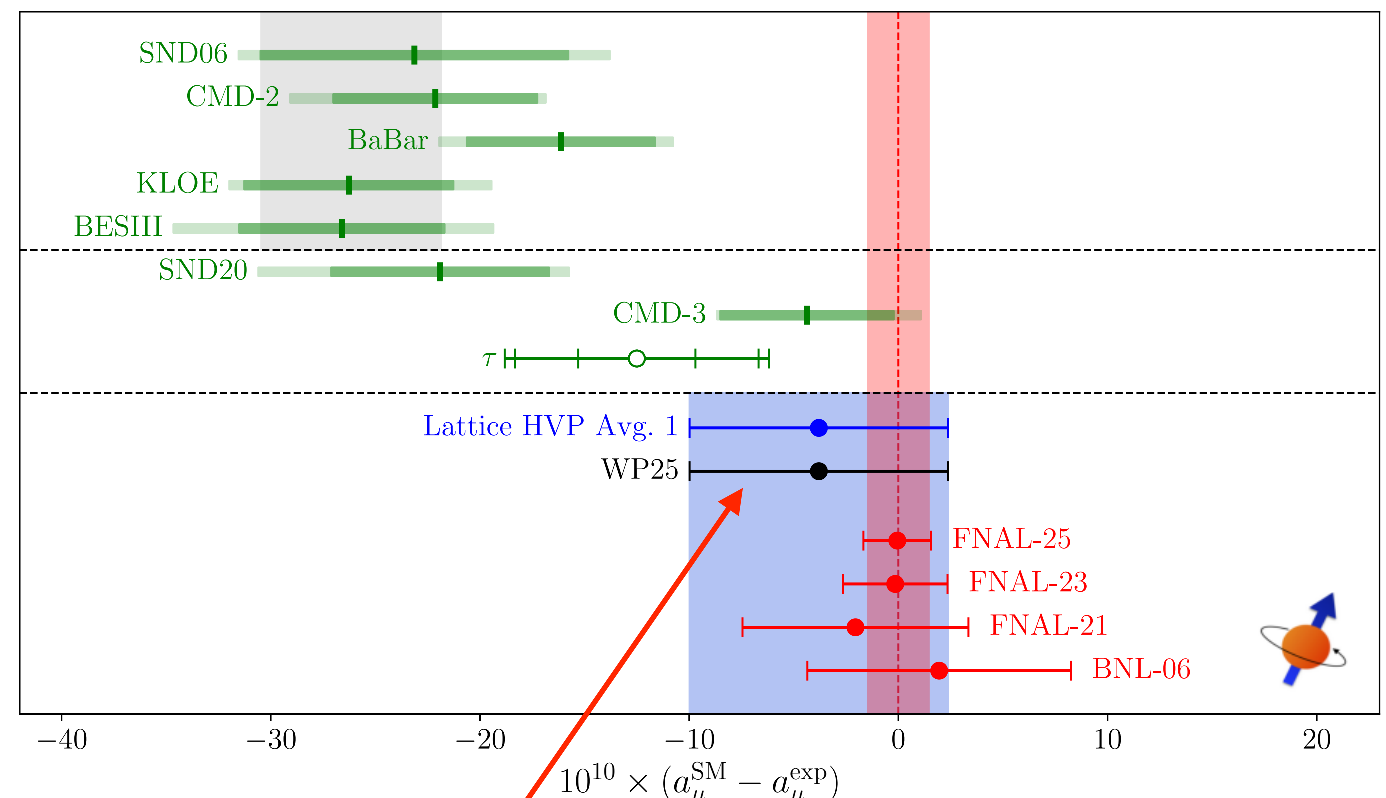
Until six years ago the SM prediction for g_μ–2 seemed very robust and well-established:

[G. Colangelo '25]

After the 2023 Fermilab result

Exp. band=current world average





SM prediction 2020

SM prediction 2025

Future prospects?

The question of how much the theoretical uncertainties of the electroweak precision observables can be reduced in the future was an important topic of the recent discussions of high-precision physics at future e^+e^- colliders:
experimental errors of the input parameters (m_b, m_t, $\alpha(M_Z)$, α_s, …), unknown higher-order corrections, non-perturbative physics, …

Full exploration of Tera-Z and W-physics programme may be limited by conceptual obstacles of full electroweak 3-loop calculations:
massive 3-loop integrals (different scales), renormalisation, treatment of unstable particles, consistent treatment of γ_5, hadronic contributions to the vacuum polarisation, …

Achievable progress even on a time scale of 20 years or more is difficult to predict; from my perspective even the claimed ``conservative” estimate of future theory uncertainties does not look very conservative, not to speak of the ``aggressive” one …

Some remarks about the renormalisation of BSM models

For the renormalisation of BSM models often ``mixed" schemes are proposed where some parameters are renormalised on-shell and others MSbar or DRbar. Schemes of this kind have also been put forward in the ``SPA Convention" *[J.A. Aguilar-Saavedra et al. '06]*

However, it was realised that such schemes can lead to unphysical dependencies on the chosen tadpole scheme

Why unphysical? Because the dependence on the tadpole scheme drops out in relations between physical observables!

Even in relations between physical observables it may still be problematic to use the FJ tadpole scheme because of the numerically large contributions that it induces

How to deal with tadpole scheme dependencies?

Some proposals for curing the unphysical tadpole scheme dependencies just address the unphysical gauge-parameter dependence. The actual benefit of this may be questioned. Not having a visible gauge-parameter dependence does not automatically make a quantity physical!

Ideally one should focus on relations between physical observables, even if this requires predictions for additional process-specific observables, etc.

I suggest to compare, at the very least, with on-shell type renormalisations for mixing angles, $\tan\beta$, etc.

Specific comment on use of the ``pinch technique"

People making statements like ``we have used the pinch technique to make our result gauge-parameter independent" either disregard or have not understood results that have been obtained more than 30 years ago!

It has been shown that the pinch technique corresponds to the special case of the background-field method (BFM) where the quantum gauge parameter is set to $\xi_Q = 1$. One cannot get rid of the dependence on the quantum gauge parameter in this way; other choices of the quantum gauge parameter are equally well motivated as $\xi_Q = 1$!

[A. Denner, S. Dittmaier, G. W. '94]

Extension of mixed schemes to the two-loop level?

tanβ as example: 2-loop MSbar/DRbar tanβ in a mixed scheme will contain one-loop on-shell counterterms, e.g. for the top mass, as subloop

This can be handled by starting with a pure scheme where all parameters are renomalised MSbar/DRbar and then perform a finite reparametrisation from m_t^{MSbar} to m_t^{OS}

If instead one naively applies the mixed scheme, results involving the two-loop MSbar/DRbar tanβ will again have a dependence on an unphysical contribution, in this case the (D–4) part (evanescent) of the top mass counterterm (cannot convert between schemes by a finite reparametrisation) *[see long discussions at previous KUTS meetings]*

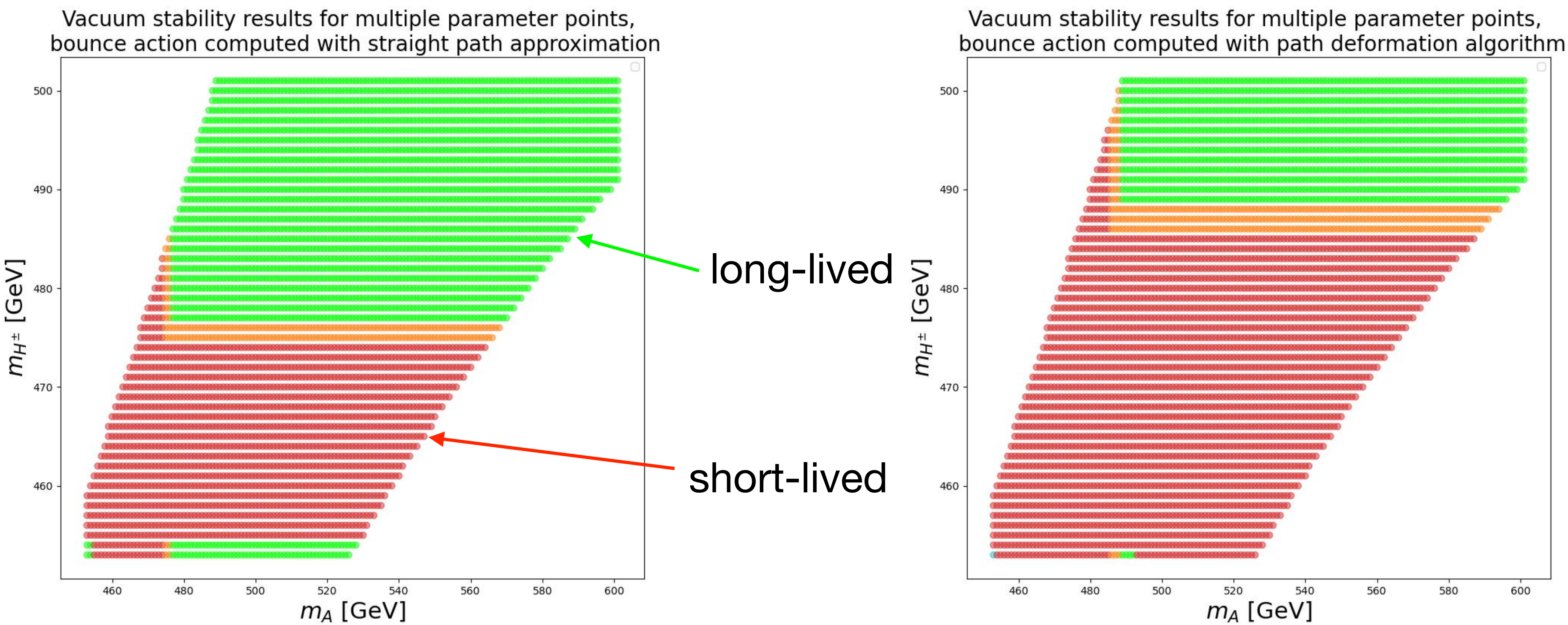
[H. Bahl, D. Meuser, G. W. ’23]

No problems of this kind occur for an OS renormalisation of tanβ at the 2-loop level (using a physical observable, A → ττ decay)

Constraints on BSM models from vacuum stability

N2HDM example: straight path approx. vs. path deformation algorithm

[A. Simon, G. W. ’26]

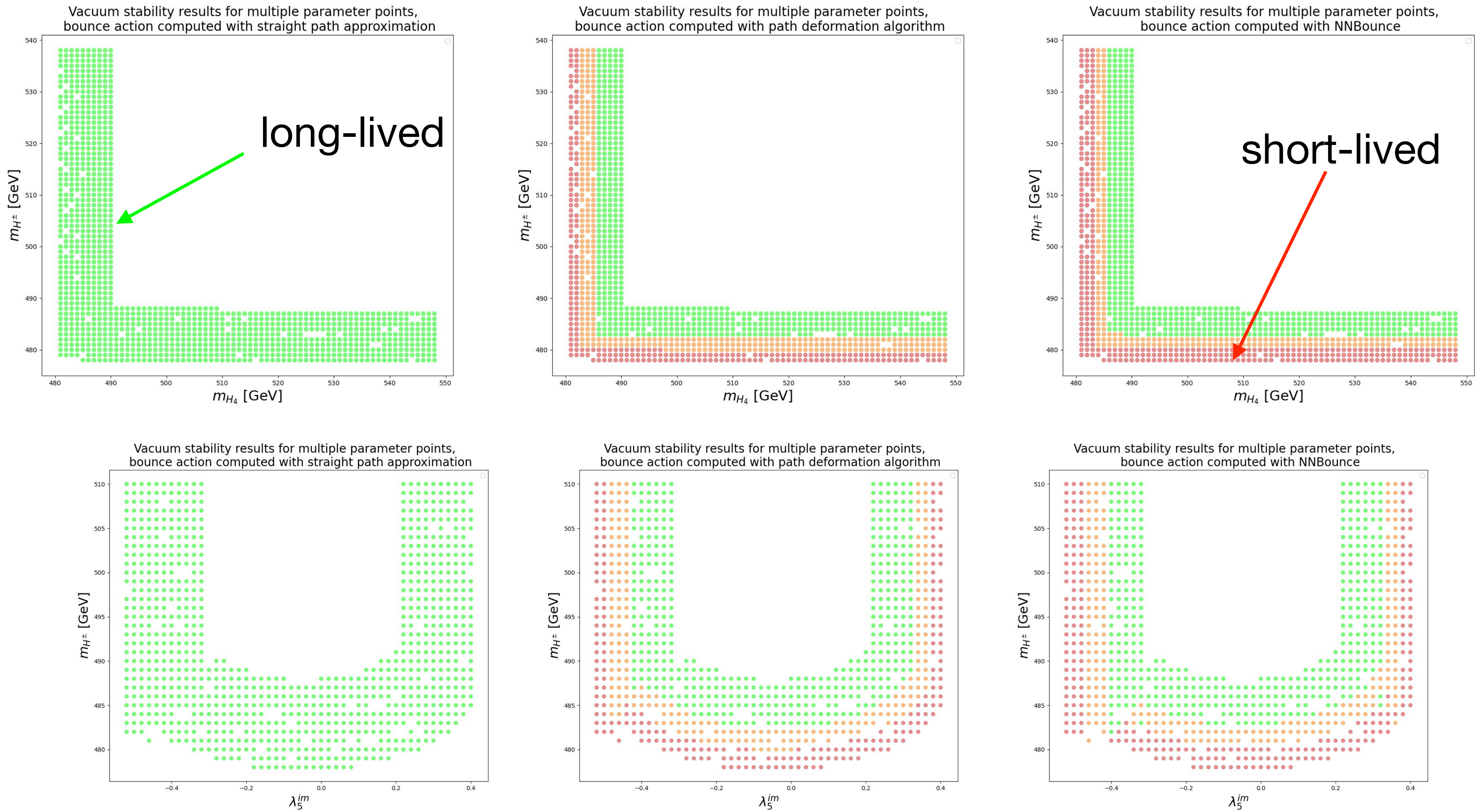


⟹ Path deformation algorithm leads to larger excluded parameter region

cS2HDM examples

[A. Simon, G. W. ’26]

Straight path approx. vs. path deformation algorithm vs. NNBounce



⇒ Path deformation algorithm and machine-learning method yield similar results

Effective potential with T-dependent effects

Important issue: treatment of large T-dependent effects:
``traditional" daisy resummation via Arnold-Espinosa or Parwani,
``partial dressing" … *[P.B. Arnold, O. Espinosa '92] [P.R. Parwani '92] [H. Bahl, M. Carena, A. Ireland, C.E.M. Wagner '24] [P. Bittar, S. Roy, C.E.M. Wagner '25]*

low-energy EFT in 3 dimensions, public tool *DRalgo*
matching between UV theory and EFT *[A. Ekstedt, P. Schicho, T.V.I. Tenkanen '22]*

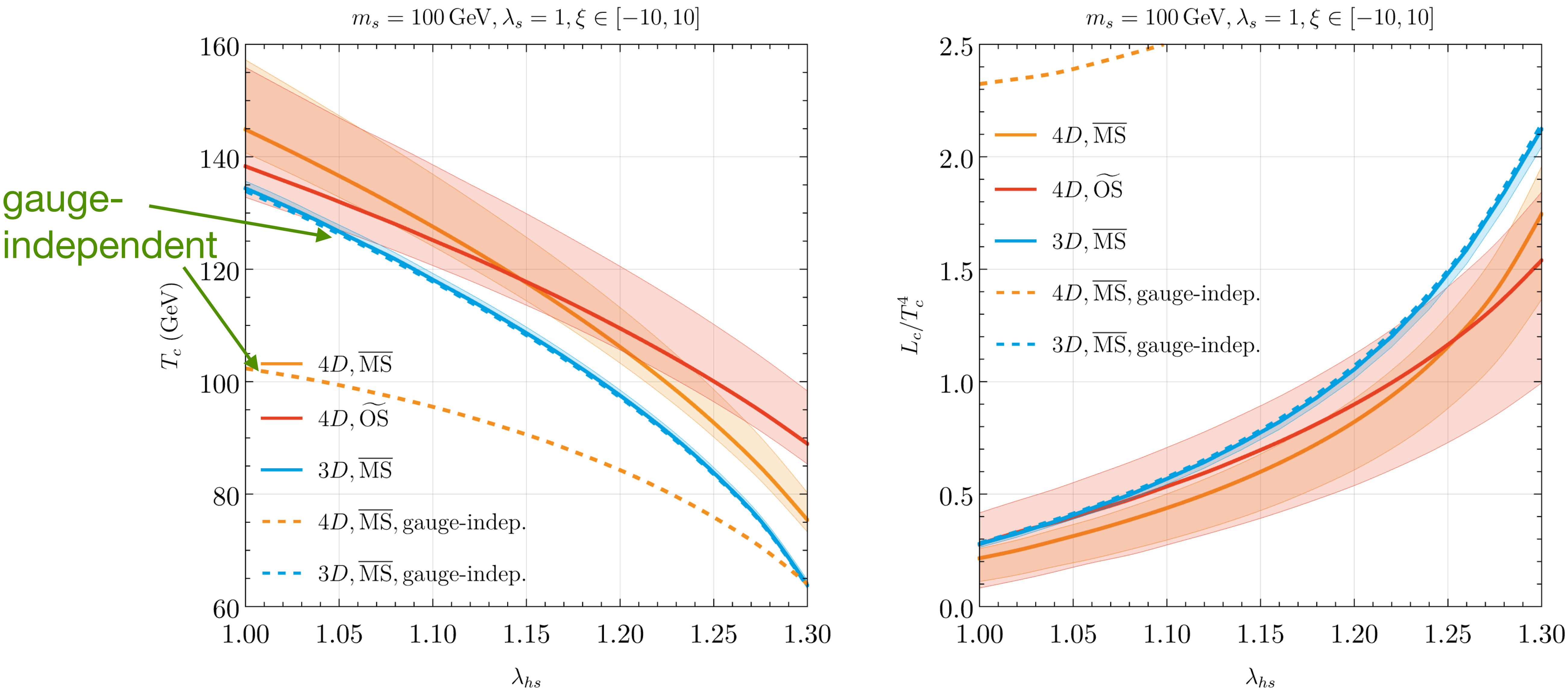
Theoretical uncertainties of predictions derived from the effective potential: gauge-dependence, renormalisation- and matching-scale dependence, effect of higher-dimensional operators …
gauge-independent results can be obtained via $\hbar$ expansion

Investigation of effective potential at two-loop order in R_ξ gauge for extension of the SM by a complex singlet

Predictions for critical temperature and latent heat

[T. Biekötter, A. Dashko, M. Löschner, G. W. '25]

Extension of the SM by a complex singlet: effects of gauge parameter variation on critical temperature (left) and latent heat (right) for 4-dimensional treatment with resummation and for 3-dimensional EFT

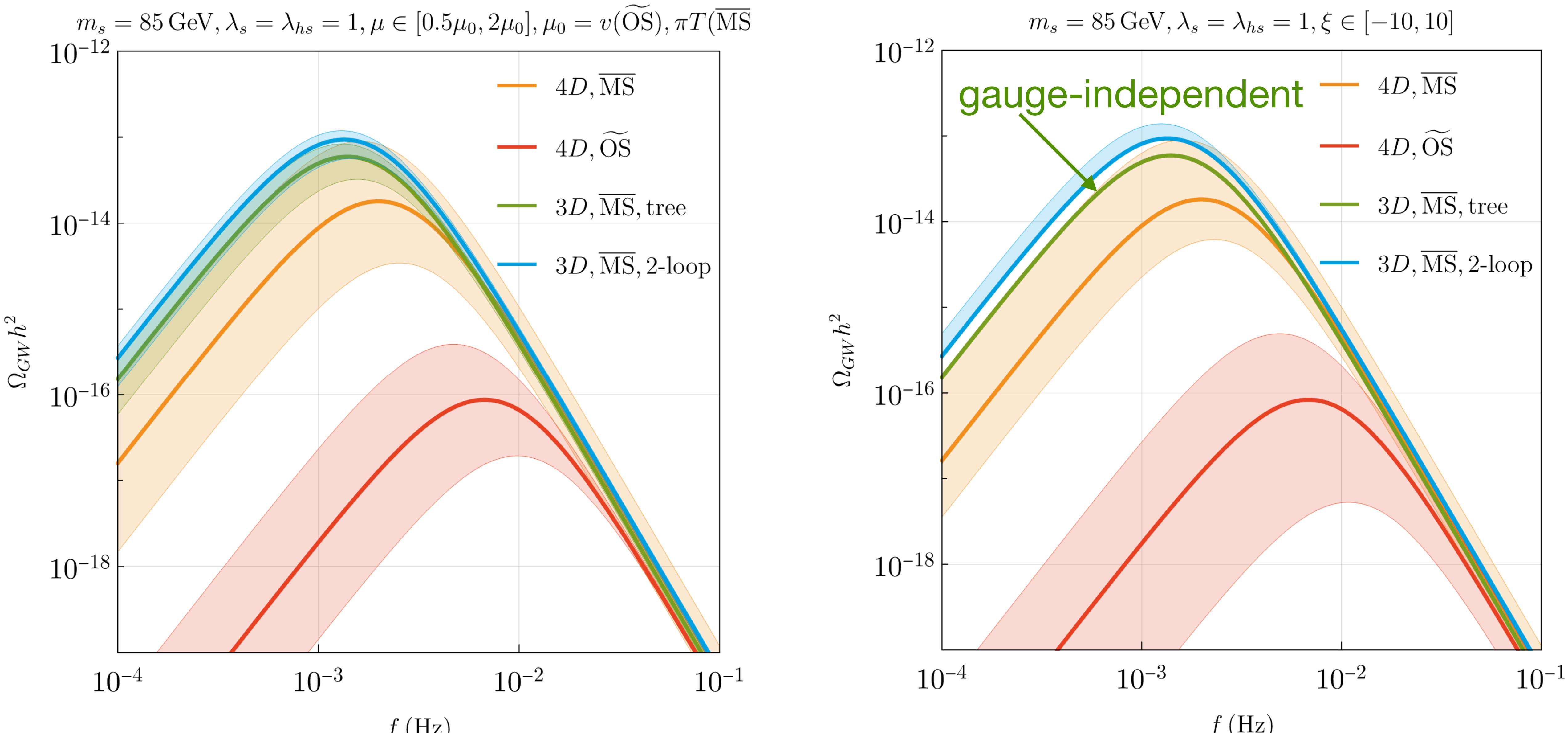


⇒ Smaller gauge-parameter dependence in 3-dimensional EFT approach, good agreement of direct potential minimisation with gauge-indep. res.

Theoretical uncertainties of GW predictions

[T. Biekötter, A. Dashko, M. Löschner, G. W. '25]

Extension of the SM by a complex singlet: effects of renormalisation scale (left) and gauge parameter variation (right) for 4-dimensional treatment with resummation and for 3-dimensional EFT

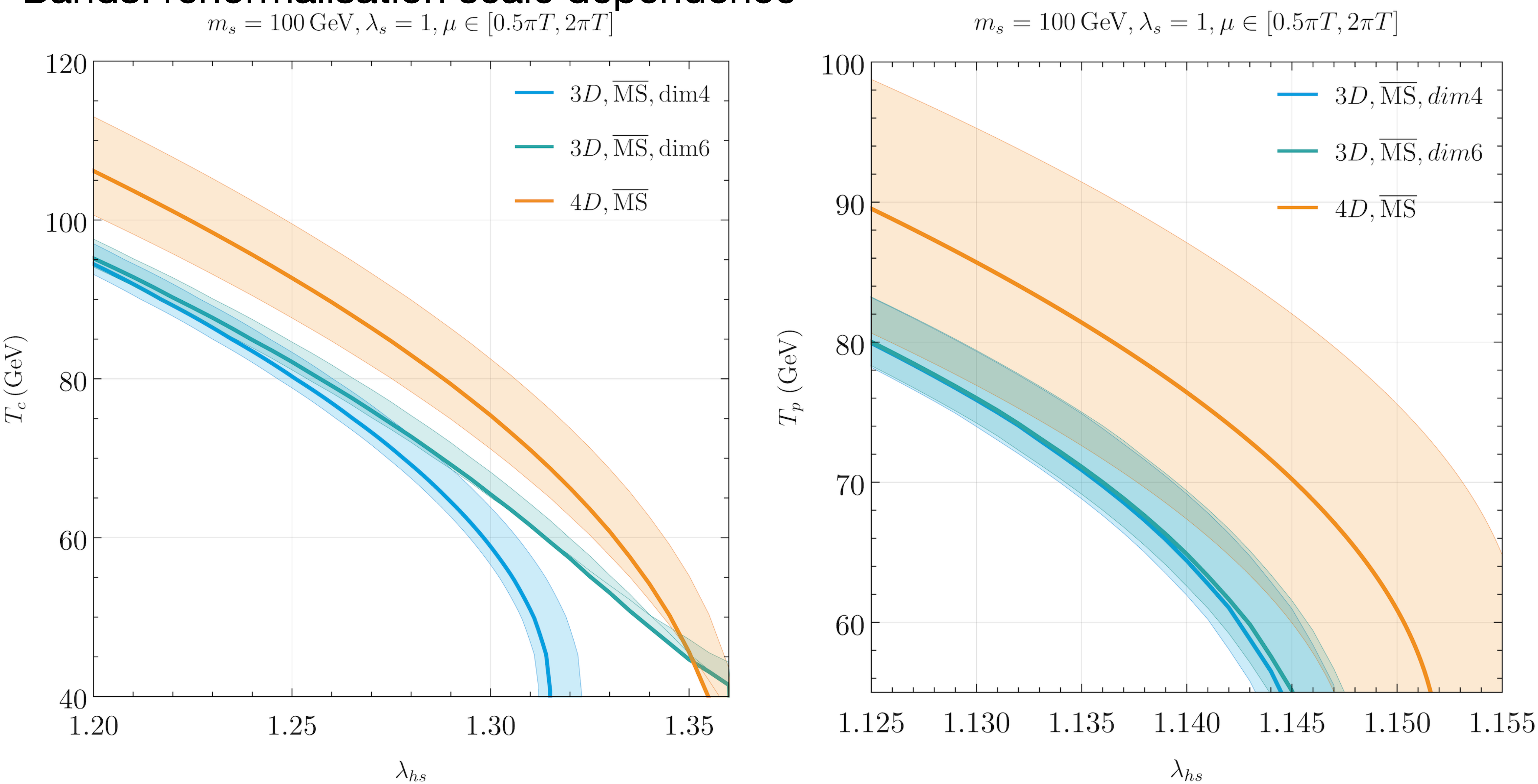


⇒ Significantly smaller scale and gauge-parameter dependence in 3-dimensional EFT approach

Impact of higher-dimensional operators in the 3-dimensional EFT approach on T_c and T_p

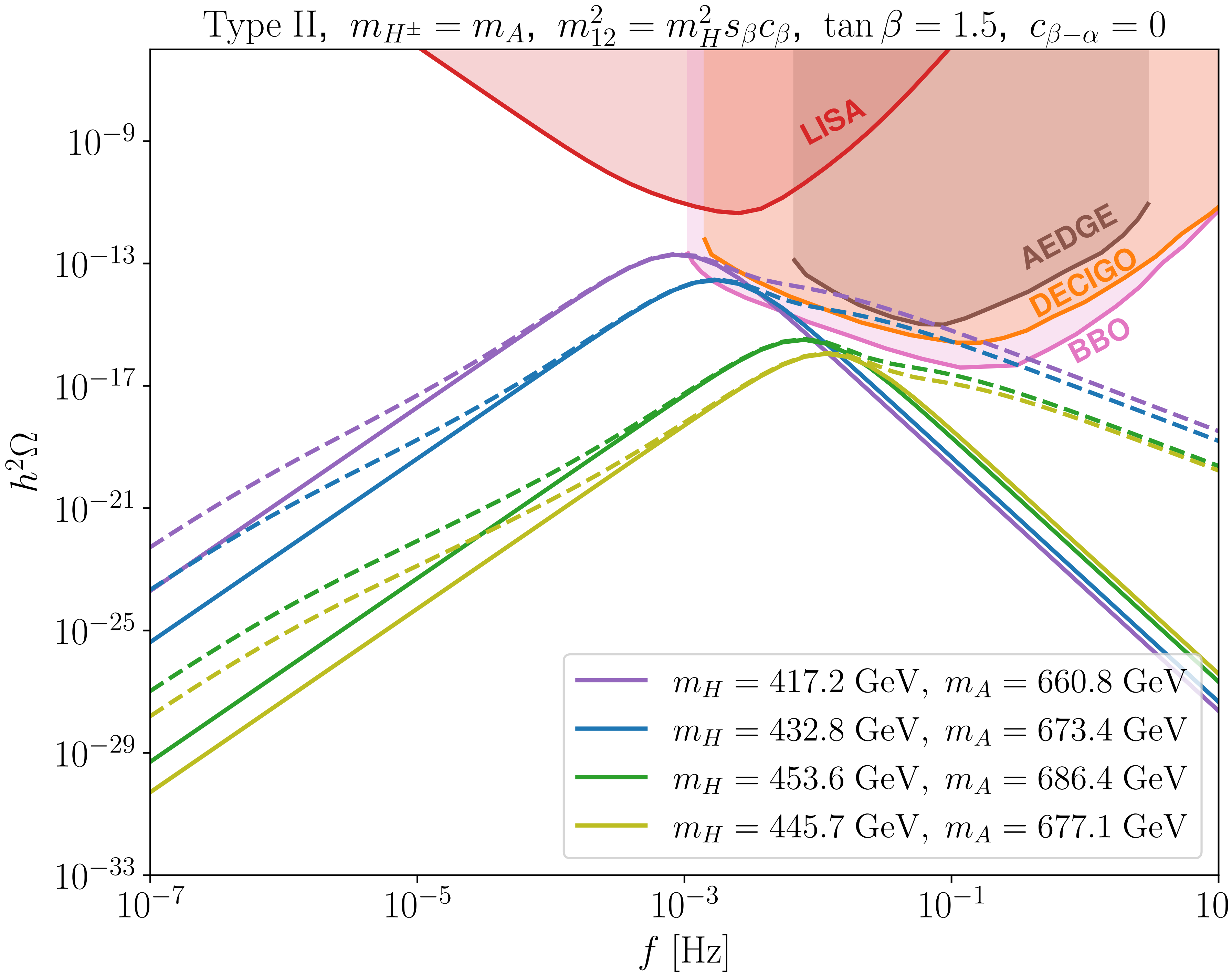
[T. Biekötter, A. Dashko, M. Löschner, G. W. '25]

Bands: renormalisation scale dependence



⇒ Higher-dimensional operators in EFT approach have significant impact for lowest values of critical temperature T_c, i.e. for strongest GW signals

GW spectra vs. future experimental sensitivities



[T. Biekötter, S. Heinemeyer, J. M. No, M. O. Olea, K. Radchenko, G. W. '23]

⇒ Prospects for GW detection depend very sensitively on the precise details of the mass spectrum of the additional Higgs bosons

Conclusions

Different kinds of BSM precision calculations:

- EWPOs, M_W, $\sin^2\theta_{\mathrm{eff}}$, …: very high sensitivity for testing loop effects of BSM physics; crucial to incorporate state-of-the-art SM result into the BSM predictions!
- Constraint from vacuum stability at $T = 0$: different approaches for evaluating the bounce action
- Effective potential with T-dependent effects, electroweak phase transition, possible gravitational wave signals: 4-dim approach with resummation affected by sizeable gauge-parameter and renormalisation scale dependence; 3-dim EFT affected by potentially large contributions of higher-dimensional operators

Backup

Simplified BSM predictions for the W-boson mass

S, T, U parameters: only BSM contributions taken into account that enter via gauge-boson self-energies (only one-loop contributions), external momentum neglected

$$M_W^2 = M_W^2\big|_{\mathrm{SM}} \left(1 + \frac{s_w^2}{c_w^2 - s_w^2}\Delta r'\right)$$

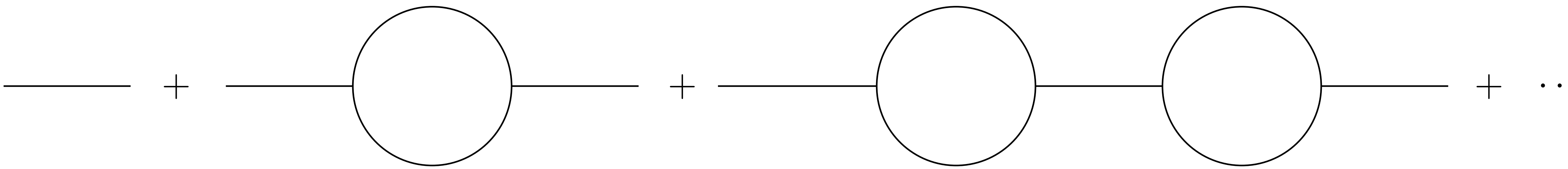
$$\Delta r' = \frac{\alpha}{s_w^2}\left(-\frac{1}{2}S + c_w^2 T + \frac{c_w^2 - s_w^2}{4 s_w^2}U\right)$$

SM prediction for the experimental values of M_H, m_t, …

Global fits to electroweak precision observables:
SM, SM + S, T, U parameters: *GFitter*, …
BSM models (SUSY, …): *MasterCode*, *Gambit*, …
EFT fits

What is meant by the mass of an unstable particle?

Mass of a physical particle: pole of the propagator



$\frac{i}{p^2-m^2}$ $\quad\quad$ $\frac{i}{p^2-m^2} \; i\Sigma(p^2) \; \frac{i}{p^2-m^2}$ $\quad\quad$ $\frac{i}{p^2-m^2} \; i\Sigma(p^2) \; \frac{i}{p^2-m^2} \; i\Sigma(p^2) \; \frac{i}{p^2-m^2}$

$$= \frac{i}{p^2 - m^2 + \Sigma(p^2)}$$

$\Rightarrow$ Pole of the propagator: $\mathcal{M}^2$

Renormalised self-energy (denoted below by $\hat{\Sigma}$)

$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0$$

For a stable particle: $\Sigma(\mathcal{M}^2)$ is real

If $\Sigma(\mathcal{M}^2) \neq 0 \Rightarrow$ Pole shifted by higher-order contributions

Mass of an unstable (elementary) particle

For an unstable particle:

$\Sigma(\mathcal{M}^2)$ is complex $\Rightarrow$ Pole in the complex plane

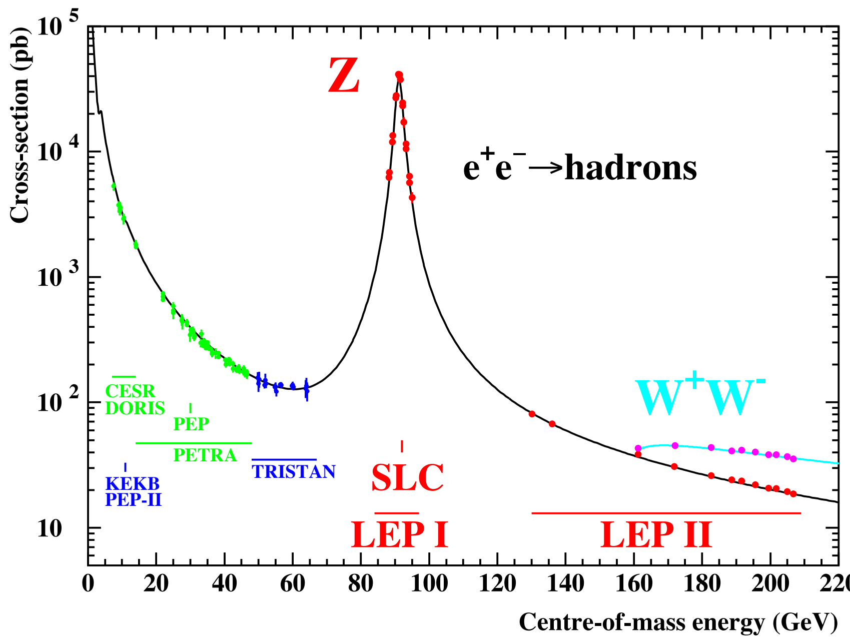
$$\mathcal{M}^2 - m^2 + \Sigma(\mathcal{M}^2) = 0, \quad \mathcal{M}^2 = M^2 - iM\Gamma$$

M: physical mass, Γ: decay width of the unstable particle

$\Rightarrow$ The mass of an unstable (elementary) particle is defined according to the real part of the complex pole

Example:

resonant production of the Z boson and its decay



Which parameter is actually measured?

On the experimental side masses of unstable particles are not directly physical observables (can only measure cross sections, branching ratios, kinematical distributions, …): masses are ``pseudo-observables" whose determination involves a deconvolution procedure (unfolding)

Different parameterisations of the resonance: Breit-Wigner shape with running or constant width (difference: 27 MeV!)

The experimental mass parameter is obtained from the comparison data — Monte Carlo prediction

⇒ The experimental mass parameters M_W, M_Z, m_t, … are not strictly model-independent (example: exp. value for M_Z depends on M_H!)
Note: relation between Monte Carlo mass and well-defined Lagrangian par. is most difficult for m_t (coloured state, renormalon ambiguities, …), but much higher accuracy needed for M_W!

Physical mass of unstable particles: real part of complex pole

⇒ Only the complex pole is gauge-invariant

Expansion around the complex pole leads to a Breit–Wigner shape with constant width

For historical reasons, the experimental values of M_Z, M_W are defined according to a Breit–Wigner shape with running width

⇒ Need to correct for the difference in definition when comparing theory with experiment

M_W: measurements and theoretical predictions

Measurements at LEP, the Tevatron and the LHC:

- LEP: $e^+e^- \to W^+W^-$ in the continuum and at threshold (small amount of data); impact of fully hadronic final state suffered from uncertainties due to BE correlations, colour reconnections
- Tevatron, LHC: transverse mass distribution

In the SM: M_W is a free input parameter

From muon decay (yields Fermi constant G_μ): relation between the four extremely precisely measured quantities G_μ, α, M_W, M_Z

⇒ Use as prediction for M_W in different models, compare with experimental value

Important: precise theoretical predictions are needed both for its predictions in the SM and beyond and for the extraction of M_W from the data (source of systematic uncertainty)!

Comp.: when prediction for $\sin^2\theta_{eff}$ reached the full 2-loop level additional contributions were required for its extraction from the data at LEP

Form factors implemented in *ZFITTER*:

[M. Awramik, M. Czakon, A. Freitas ’06]

$$
\begin{aligned}
\mathcal{A}[e^+e^- \to f\bar{f}] = 4\pi i\,\alpha\,\frac{Q_\mathrm{e}Q_\mathrm{f}}{s}\,\gamma_\mu \otimes \gamma^\mu \\
+ i\frac{\sqrt{2}G_\mu M_\mathrm{Z}^2}{1+i\Gamma_\mathrm{Z}/M_\mathrm{Z}}\, I_\mathrm{e}^{(3)}\, I_\mathrm{f}^{(3)}\, \frac{1}{s-\overline{M}_\mathrm{Z}^2 + i\overline{M}_\mathrm{Z}\overline{\Gamma}_\mathrm{Z}} \\
\times\, \rho_\mathrm{ef}\Big[\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu(1+\gamma_5) \\
- 4|Q_\mathrm{e}|s_\mathrm{W}^2\,\kappa_\mathrm{e}\,\gamma_\mu\otimes\gamma^\mu(1+\gamma_5) \\
- 4|Q_\mathrm{f}|s_\mathrm{W}^2\,\kappa_\mathrm{f}\,\gamma_\mu(1+\gamma_5)\otimes\gamma^\mu \\
+ 16|Q_\mathrm{e}Q_\mathrm{f}|s_\mathrm{W}^4\,\kappa_\mathrm{ef}\,\gamma_\mu\otimes\gamma^\mu\Big]
\end{aligned}
$$

$$
\begin{aligned}
\kappa_\mathrm{ef}(s) &= \kappa_\mathrm{e}(s)\kappa_\mathrm{f}(s) - \frac{M_\mathrm{Z}^2 - s}{s}\,\frac{1}{(a_\mathrm{e}^{(0)} - v_\mathrm{e}^{(0)})(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})} \\
&\quad\times\left[q_\mathrm{e}^{(1)}q_\mathrm{f}^{(0)} + q_\mathrm{f}^{(1)}q_\mathrm{e}^{(0)} - p_\mathrm{f}^{(1)}q_\mathrm{e}^{(0)}\frac{v_\mathrm{f}^{(0)}}{a_\mathrm{f}^{(0)}} - p_\mathrm{e}^{(1)}q_\mathrm{f}^{(0)}\frac{v_\mathrm{e}^{(0)}}{a_\mathrm{e}^{(0)}} - q_\mathrm{e}^{(0)}q_\mathrm{f}^{(0)}\frac{\Sigma_{\gamma\gamma}^{(1)}}{s} + \text{boxes}\right], \\
\kappa_\mathrm{e,f}(s) &= \kappa_\mathrm{Z}^\mathrm{e,f}(s) + \frac{M_\mathrm{Z}^2 - s}{s}\left[\frac{q_\mathrm{e,f}^{(0)}}{a_\mathrm{e,f}^{(0)} - v_\mathrm{e,f}^{(0)}}\,\frac{p_\mathrm{f,e}^{(1)}}{a_\mathrm{f,e}^{(0)}} + \text{boxes}\right], \\
\kappa_\mathrm{Z}^\mathrm{f}(s) &= \kappa_\mathrm{Z}^\mathrm{f}(M_\mathrm{Z}^2) + (s - M_\mathrm{Z}^2)\,\frac{\hat{a}_\mathrm{f}^{(1)\prime}(M_\mathrm{Z}^2)\,v_\mathrm{f}^{(0)} - \hat{v}_\mathrm{f}^{(1)\prime}(M_\mathrm{Z}^2)\,a_\mathrm{f}^{(0)}}{a_\mathrm{f}^{(0)}(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})}.
\end{aligned}
$$

Relation between $\sin^2\theta_{eff}^f$ determined from expansion around the complex pole and the one defined in *ZFITTER*:

$$
\sin^2\theta^f_{\mathrm{eff,pole}} = \overline{s}_\mathrm{W}^2\,\mathrm{Re}\left\{\overline{\kappa}_\mathrm{Z}^\mathrm{f}(M_\mathrm{Z}^2)\right\} = \sin^2\theta^f_{\mathrm{eff,ZFITTER}} - \frac{\Gamma_\mathrm{Z}}{M_\mathrm{Z}}\,\frac{q_\mathrm{f}^{(0)}}{a_\mathrm{e}^{(0)}(a_\mathrm{f}^{(0)} - v_\mathrm{f}^{(0)})}\,\mathrm{Im}\{p_\mathrm{e}^{(1)}\}
$$

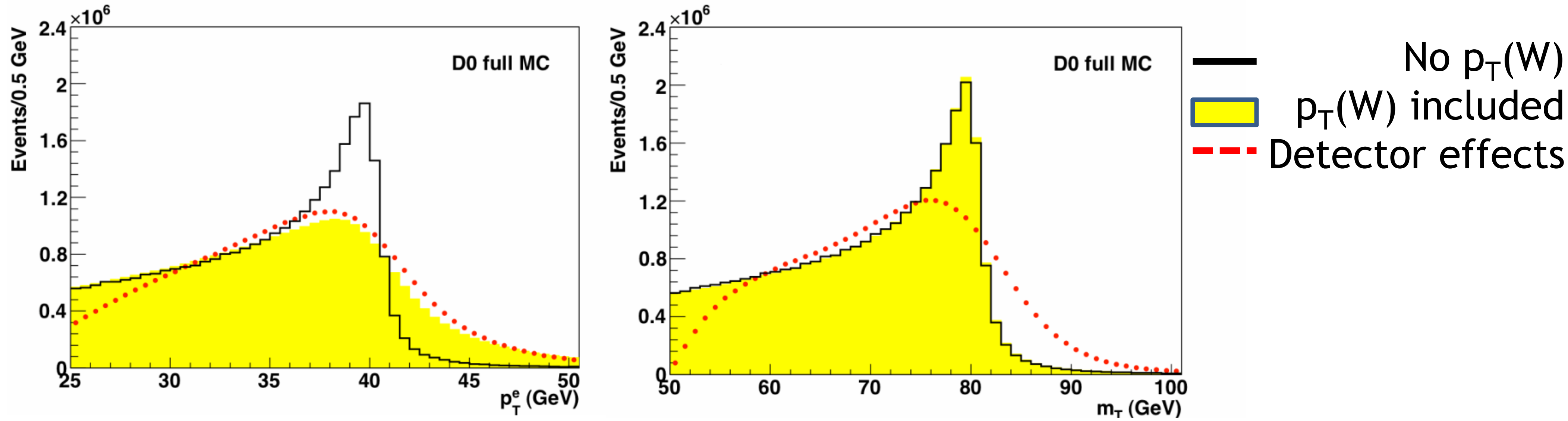
$$
\overline{s}_\mathrm{W}^2 = \left(1 - \frac{\overline{M}_\mathrm{W}^2}{\overline{M}_\mathrm{Z}^2}\right) = s_\mathrm{W}^2\left[1 + \frac{c_\mathrm{W}^2}{s_\mathrm{W}^2}\left(\frac{\Gamma_\mathrm{W}^2}{M_\mathrm{W}^2} - \frac{\Gamma_\mathrm{Z}^2}{M_\mathrm{Z}^2}\right)\right]^{-1}.
$$

numerically small, but required at this order

Extraction of M_W from the data at hadron colliders

problems are due to
- the smearing of the distributions due to difficult neutrino reconstruction
- strong sensitivity to the modelling of initial state QCD effects

[A. Vicini '22]



Templates for different M_W values:

The templates are perturbative predictions.
Their residual theoretical uncertainties will propagate as theoretical systematic errors on the determination of (G_μ, m_W, m_Z)

Given the very high precision goal $\delta m_W / m_W \sim 1 \cdot 10^{-4}$, $\delta \sin^2\theta_{eff} / \sin^2\theta_{eff} \sim 1 \cdot 10^{-3}$
control on the shape of the distributions at the sub-percent level is needed, at a hadron collider…

- very large impact of initial-state QCD radiation on the ptlep distribution
- large radiative corrections due to QED final state radiation at the jacobian peak
- very large interplay of QCD and QED corrections redefining the precise shape of the jacobian peak

W mass measurements: current situation

Combined value: M_W^{exp} = 80.369 ± 0.013 GeV (accuracy of 1.6 x 10^{-4})

[LHC-TeV MW Working Group '23]

The consistency of Drell-Yan cross-section measurements, as well as the m_W combination, is highest for the CT18 PDF set due to its large uncertainties. With this PDF set the combination of LEP, LHC, and Tevatron Run 2 measurements gives a value $m_W = 80394.6 \pm 11.5$ MeV. This value has a χ^2 probability of 0.5% and is therefore disfavoured. Other PDF sets give probabilities of consistency between 2×10^{-5} and 3×10^{-3}.

Good consistency is observed when all experiments other than CDF are combined, with a resulting W-boson mass of 80369.2 ± 13.3 MeV and a 91% probability of consistency for the CT18 PDF set. When using this set and uncertainty for the CDF measurement and for the combination of the others, the values differ by 3.6 standard deviations. Further measurements or studies of procedures and uncertainties are required to improve the understanding and consistency of a world-average value of the W boson mass.

Does not include the CDF measurement: *[CDF Collaboration '22]*

M_W^{exp} = 80.433 ± 0.009 GeV

Expansion around the complex pole for a single resonance

$$p^2 - m^2 + \hat{\Sigma}(p^2) = \underbrace{(p^2 - \mathcal{M}^2)}\underbrace{\left\{1 + \frac{d\,\hat{\Sigma}}{d\,p^2}\right\}\Bigg|_{p^2=\mathcal{M}^2}} + \ldots$$

→ Breit-Wigner factor with fixed width

→ Field renormalisation and wave function normalisation factor of unstable particle

Note:

Wave-function normalisation factor needs to be evaluated at the complex pole

One-loop field renormalisation: $\delta Z^{(1)} = -\frac{\partial\Sigma(p^2)}{\partial p^2}\Bigg|_{p^2=m^2}$

Complex quantity, no restriction to Re(…)

Expansion around the complex pole (example: M_Z)

Expansion of amplitude around complex pole:

$$\mathcal{A}(e^+e^- \to f\bar{f}) = \frac{R}{s - \mathcal{M}_Z^2} + S + (s - \mathcal{M}_Z^2)\, S' + \cdots$$

$$\mathcal{M}_Z^2 = \overline{M}_Z^2 - i\overline{M}_Z\,\overline{\Gamma}_Z$$

Expanding up to $\mathcal{O}(\alpha^2)$ using $\mathcal{O}(\overline{\Gamma}_Z/\overline{M}_Z) = \mathcal{O}(\alpha)$

From 2-loop order on:

real part of complex pole, $\overline{M}_Z \neq$ pole of real part, $\widetilde{M}_Z^2$

$$\delta\overline{M}_{(2)}^2 = \delta\widetilde{M}_{(2)}^2 + \underbrace{\mathrm{Im}\left\{\Sigma'_{\mathrm{T},(1)}(M^2)\right\}\ \mathrm{Im}\left\{\Sigma_{\mathrm{T},(1)}(M^2)\right\}}$$

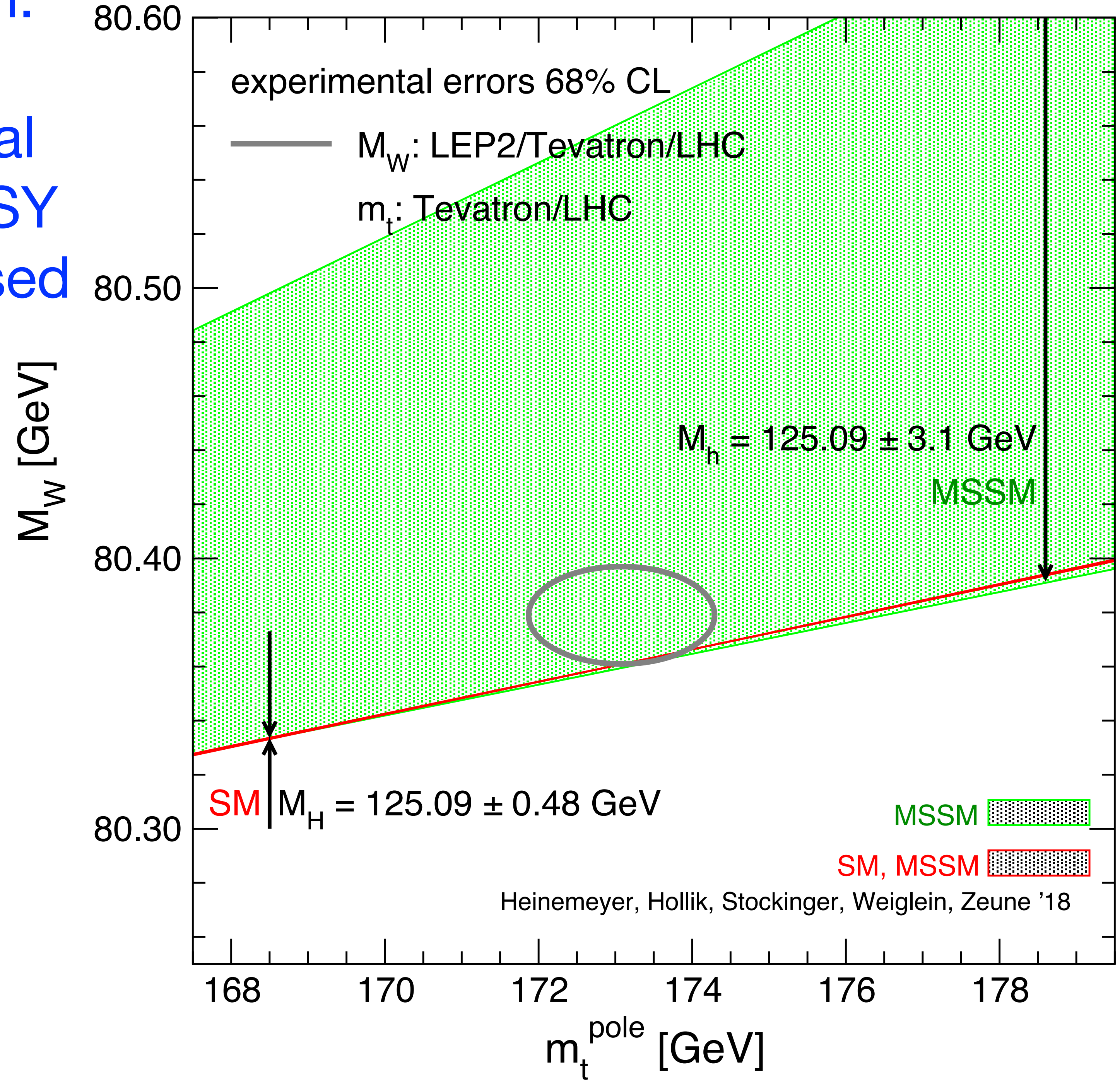
gauge-parameter dependent!

Prediction for M_W in the SM and the MSSM vs. experimental results for M_W and m_t

[S. Heinemeyer, W. Hollik, G. W., L. Zeune ’18]

Parameter scan:

No experimental bounds on SUSY particles imposed



FeynHiggs

MSSM region

SM ``line”

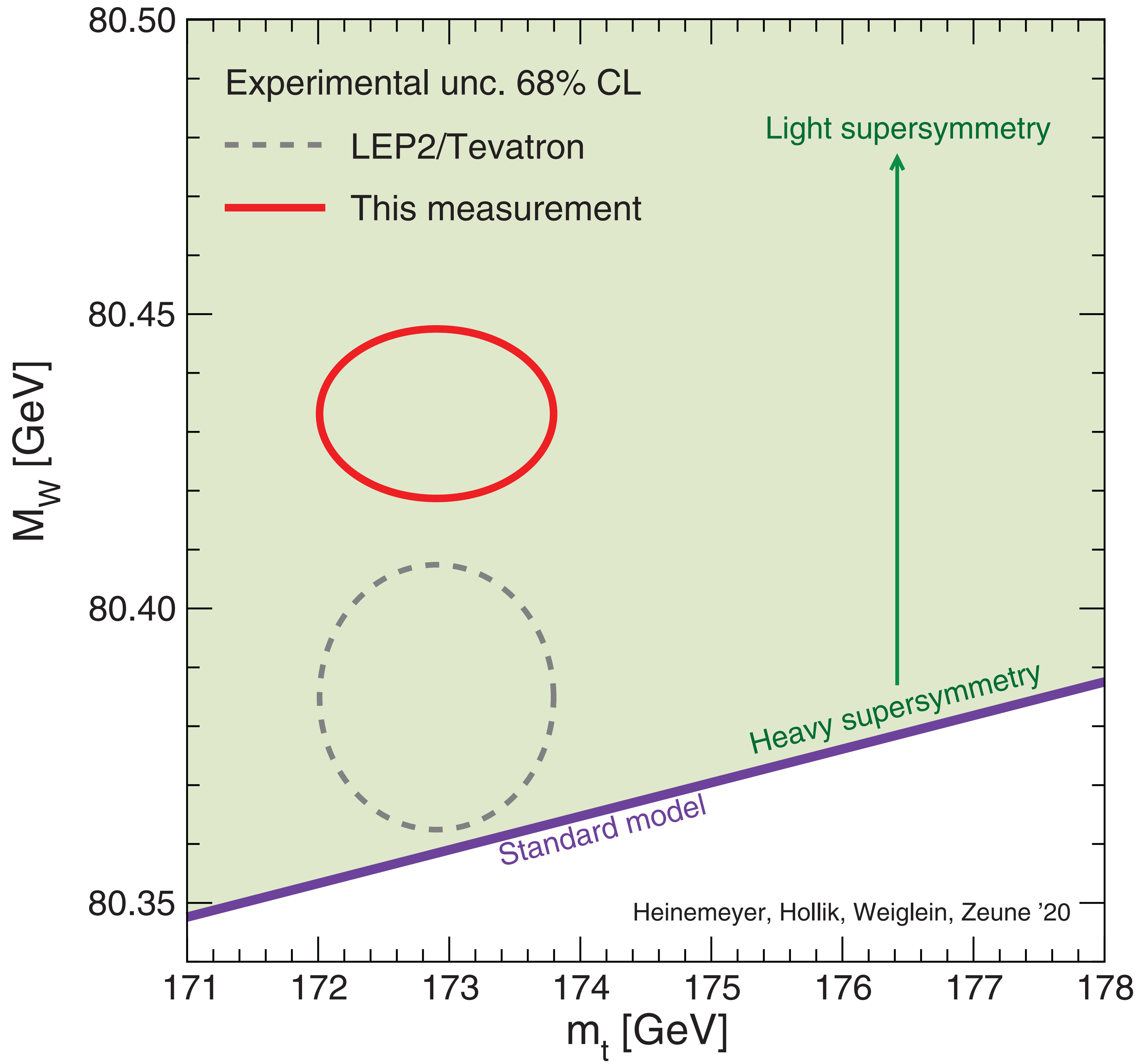
⇒ Large upward shift in M_W possible, large sensitivity to BSM effects

From the CDF paper on their M_W measurement

[CDF Collaboration '22] [S. Heinemeyer, W. Hollik, G. W., L. Zeune '20]

SUSY: in principle large upward shifts in M_W are possible (main source: $\Delta\rho$)

But: no experimental bounds on SUSY particles imposed here!

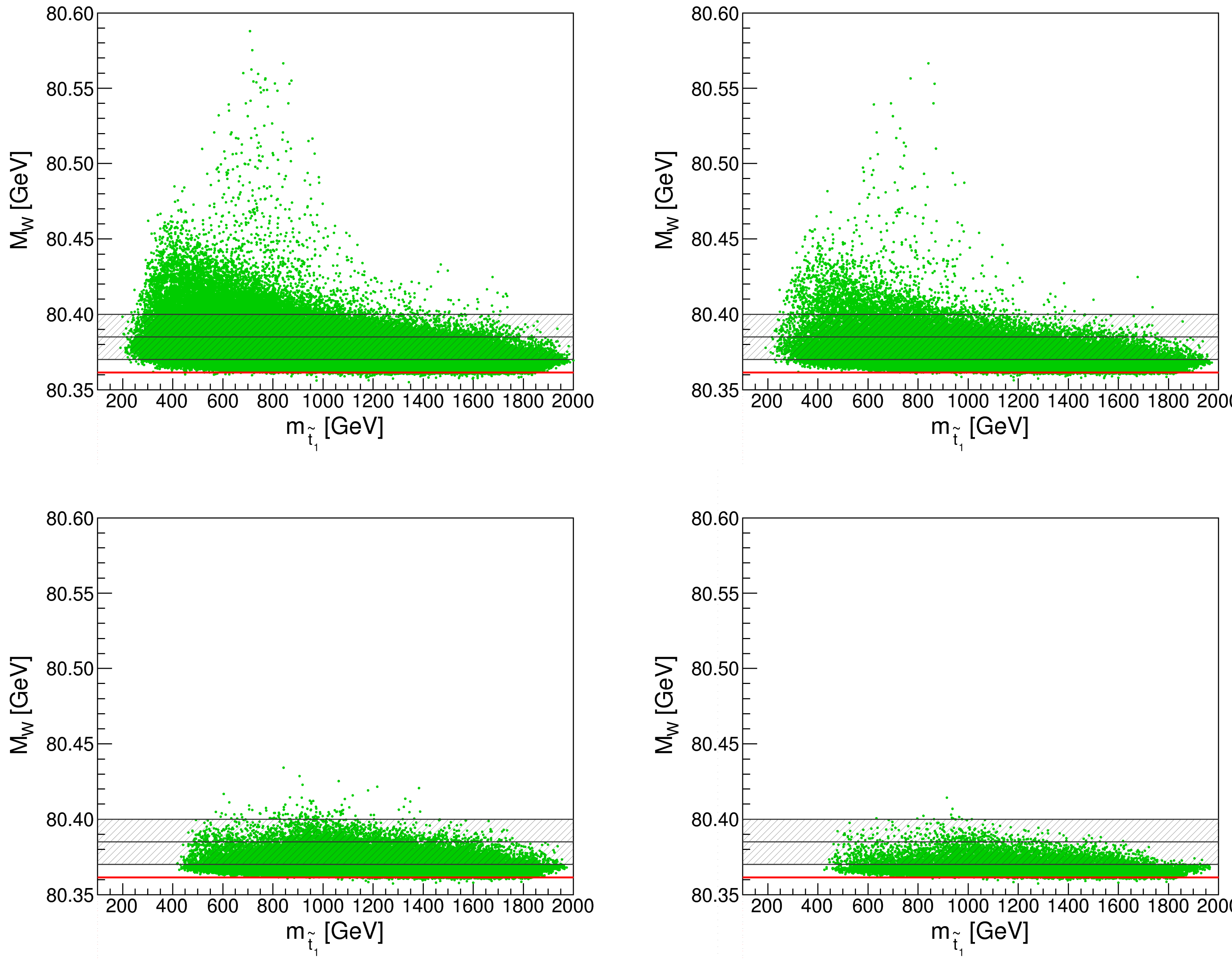


Prediction for M_W in the MSSM depending on the lighter stop mass (parameter scan) *FeynHiggs*

[S. Heinemeyer, W. Hollik, G. W., L. Zeune ’13]

All particles allowed to be light

Heavy gluino, heavy first and second generation squarks



+ heavy sbottoms

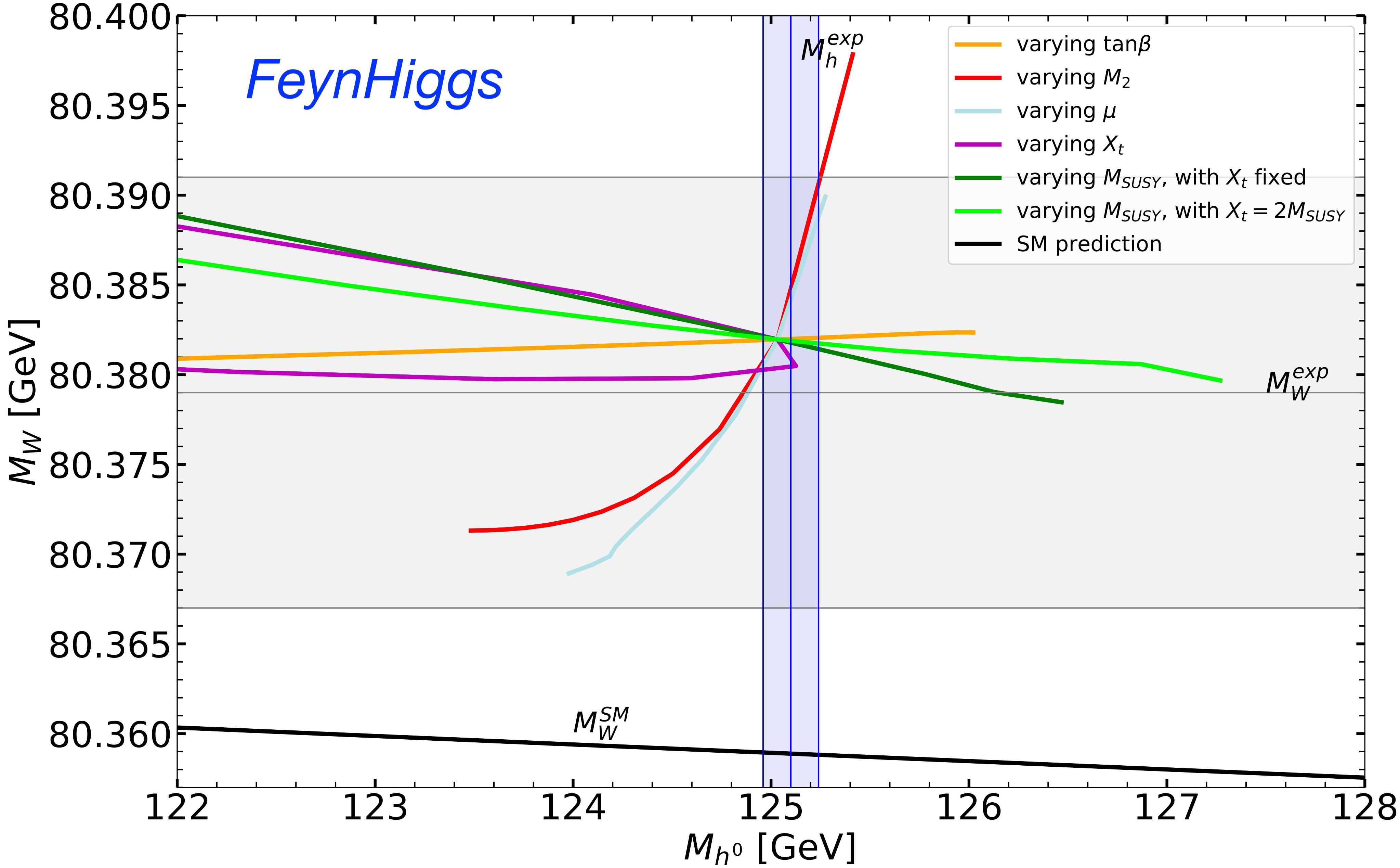
+ heavy sleptons and charginos

⇒Sizeable enhancements possible even for relatively heavy SUSY

Important further constraint: prediction for M_h has to agree with exp. value

Experimental result for M_W vs. prediction in the SM and the MSSM (different parameters varied)

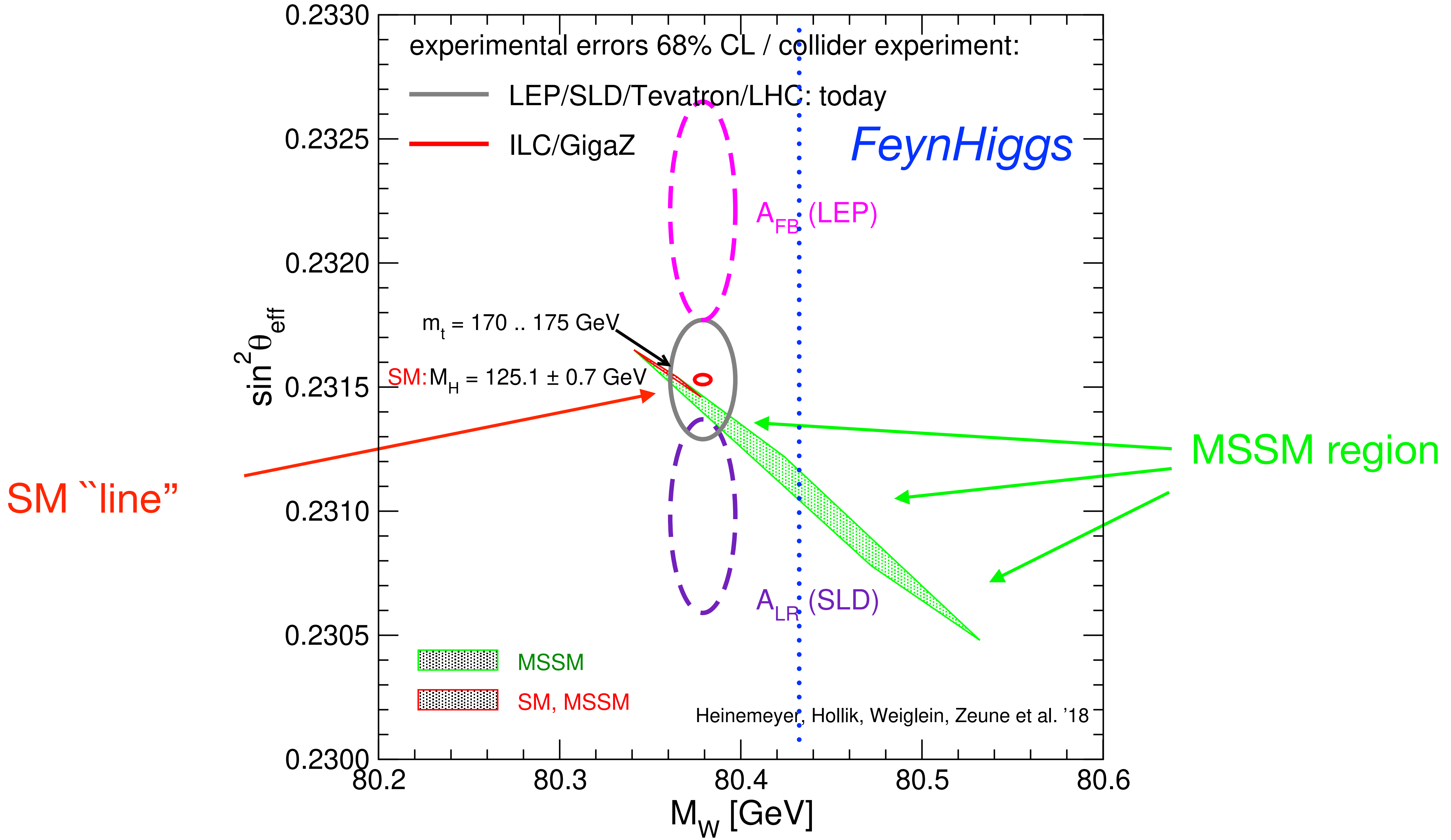
[M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]



⇒ Good agreement in the MSSM, comparison can be used to obtain indirect constraints on the SUSY parameters

Prediction for M_W and $\sin^2\theta_{eff}$ in the SM and MSSM vs. experimental accuracies

[S. Heinemeyer, W. Hollik, G. W., L. Zeune '18]



⇒ M_W and $\sin^2\theta_{eff}$ have high sensitivity for model discrimination