NLO Calculation of the Process $h \to f\bar{f}$ in the Decoupling Renormalization Scheme

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Introduction

- We can look at α_S with five-and six quark flavors [Bernreuther & Wetzel'81]
- At one-loop Greens-functions expressed through $\overline{\mathrm{MS}}$-parameters contain remnants of the heaviest quark

⇒ No decoupling a la Applequist and Carazzone [Appelquist & Carazzone'74]

- In a momentum subtraction scheme there is always decoupling
- They derived a relation between the two $\overline{\mathrm{MS}}$ couplings

$$\frac{\alpha_{S5}^{\overline{\mathrm{MS}}}}{\pi} = \frac{\alpha_{S6}^{\overline{\mathrm{MS}}}}{\pi} \underbrace{\left(1 + \sum_{k=1}^{\infty} \left(\frac{\alpha_{S6}^{\overline{\mathrm{MS}}}}{\pi}\right)^k C_k \left(\log\left(\frac{m_{\overline{\mathrm{MS}}}^2}{\mu^2}\right)\right)\right)}_{\zeta(\mu, m_{\overline{\mathrm{MS}}})}$$

- Here the function $\zeta(\mu, m_{\overline{\mathrm{MS}}})$ acts as decoupling parameter

⇒ If we now calculate processes in the six flavor theory and express them in terms of $\alpha_{S5}^{\overline{\mathrm{MS}}}$ the non-decoupling contributions of the heavy quark are absorbed

Motivation

- Going away from the QCD sector, we want to apply this to the EW sector of any BSM model
- We want to find renormalization schemes in the full BSM theory that exhibit a decoupling property
- Among the list of properties **Gauge-Independence**, **Numerical Stability** and **Process Independence** for renormalization schemes we can add a fourth

$\Rightarrow$ **Decoupling Property**

- One advantage of this is that it allows us to include higher order SM QED and QCD corrections

How to extend the description?

- An extention to the previous formulation is applied in *FlexibleDecay* [Stöckinger & Kotlarski et.al'21]
- We set up a renormalization scheme in the full theory that leads to decoupling
- We have to make a distinction between the low-energy SM-like and BSM parameters
- Comparing back to Bernreuther & Wetzel we can define a decoupling renormalization scheme as

$$P_{\mathrm{Dec}}(\mu) = P^{\mathrm{SM}}_{\overline{\mathrm{MS}}}(\mu)$$

$\Rightarrow$ **Renormalization condition for a SM-like parameter!**

- This does not fix the complete scheme!
  - How to treatment the BSM parameters?
  - What if there is no one-to one correspondence of a SM-like parameter?

How to setup the analytic calculation?

- For analytic calculations the renormalization condition is not too helpful

$$P_{\text{Dec}} = P^{\text{SM}}_{\overline{\text{MS}}}$$

- We want to extract an expression for the counter term in the decoupling scheme

$$P^{\text{Dec}} + \delta P^{\text{Dec}} \stackrel{!}{=} P^{\text{OS}} + \delta P^{\text{OS}}$$
$$P^{\overline{\text{MS}}}_{\text{SM}} + \delta P^{\overline{\text{MS}}}_{\text{SM}} \stackrel{!}{=} P^{\text{OS}}_{\text{SM}} + \delta P^{\text{OS}}_{\text{SM}}$$

Master Equation

$$\delta P^{\text{Dec}} = \delta P^{\overline{\text{MS}}}_{\text{SM}} + \delta P^{\text{OS}} - \delta P^{\text{OS}}_{\text{SM}}$$

- Generally one can fix the BSM parameters in the $\overline{\text{MS}}$-scheme $\Rightarrow$there are models where this needs to be decided individually

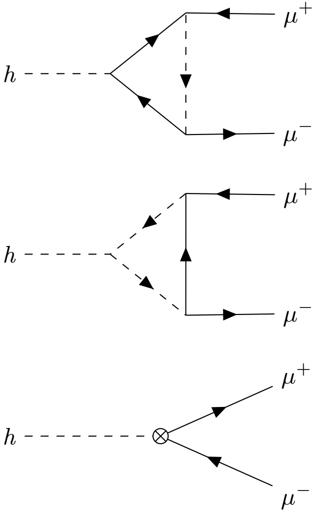
How to apply the decoupling scheme, practically?

- Explore the S_1-Leptoquark model with ϕ being the Leptoquark transforming as $\left(3, 1, -\frac{1}{3}\right)$

$$\mathcal{L}_{Y\phi} = Y_{ij}^{LL}\,(\overline{Q^C}_i^T i\sigma^2 L_j)\phi^\dagger + Y_{ij}^{RR}\,\overline{q_u^C}_i l_j \phi^\dagger + \text{h.c.}$$

$$\mathcal{L}_{H\phi} = -g_{h\phi}(\Phi^\dagger\Phi)\,\mathrm{Tr}\left\{\phi^\dagger\phi\right\}$$

- The parameters that describe the EW-sector are
  - SM-like: e, m_{W}, m_{Z}, m_{h}, m_i^{f}
  - BSM: Y_{ij}^{LL}, Y_{ij}^{RR}, $g_{h\phi}$
- The advantage is the unaltered EW-sector!
- Leading to the same tree-level results. At one-loop, BSM effects only appear in loops!



How to calculate the renormalization constants?

- Generally, self-energies have the form

$$\Pi(p^2) = \Pi^{\text{BSM}}(p^2) + \Pi^{\text{SM}}(p^2)$$

- We can define the SM-like renormalization constants as

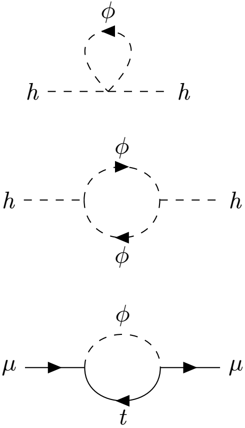
$$\delta Z_{m_\mathrm{W}} = \delta Z^{\overline{\mathrm{MS}}}_{m_\mathrm{W}} + \delta Z^{\mathrm{OS}}_{m_\mathrm{W}}\Big|^{\text{BSM}}_{\text{finite}} \qquad \delta Z_{m_\mathrm{Z}} = \delta Z^{\overline{\mathrm{MS}}}_{m_\mathrm{Z}} + \delta Z^{\mathrm{OS}}_{m_\mathrm{Z}}\Big|^{\text{BSM}}_{\text{finite}}$$

$$\delta Z_{m_\mathrm{h}} = \delta Z^{\overline{\mathrm{MS}}}_{m_\mathrm{h}} + \delta Z^{\mathrm{OS}}_{m_\mathrm{h}}\Big|^{\text{BSM}}_{\text{finite}} \qquad \delta Z_{m_l} = \delta Z^{\overline{\mathrm{MS}}}_{m_l} + \delta Z^{\mathrm{OS}}_{m_l}\Big|^{\text{BSM}}_{\text{finite}}$$

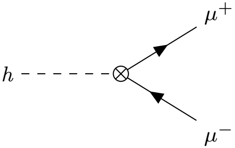
$$\delta Z_e = \delta Z^{\overline{\mathrm{MS}}}_{e} + \delta Z^{\mathrm{OS}}_{e}\Big|^{\text{BSM}}_{\text{finite}}$$

- The remaining renormalization constants for BSM parameters are

$$\delta Z_{\text{LL}} = \delta Z^{\overline{\mathrm{MS}}}_{\text{LL}}, \quad \delta Z_{\text{RR}} = \delta Z^{\overline{\mathrm{MS}}}_{\text{gg}}, \quad \delta Z_{\text{g}} = \delta Z^{\overline{\mathrm{MS}}}_{\text{g}}$$



What are the most important parts?



$$= -\frac{im_\mu}{v}\left\{\frac{1}{2}\left(\delta Z^L_\mu + \delta Z^{l\dagger}_\mu\right) + \delta Z^L_{m_\mu} + \left(\frac{\delta Z_h}{2} - \delta Z_v\right)\right\} P_L$$

$$-\frac{im_\mu}{v}\left\{\frac{1}{2}\left(\delta Z^{L\dagger}_\mu + \delta Z^l_\mu\right) + \delta Z^l_{m_\mu} + \left(\frac{\delta Z_h}{2} - \delta Z_v\right)\right\} P_R$$

- Comparing to the triangle diagrams the most important parts for decoupling are the muon renormalization constants

$$\delta Z^{L,\text{Dec}}_\mu = \delta Z^{L,\overline{\text{MS}}}_\mu + \frac{3}{16\pi^2}\left\{\left(Y^{LL\dagger}\right)_{2k} Y^{LL}_{k2} B_1(m_\mu, m_t, m_\phi) + \dots\right\}$$

$$\delta Z^{l,\text{Dec}}_\mu = \delta Z^{l,\overline{\text{MS}}}_\mu + \frac{3}{16\pi^2}\left\{\left(Y^{RR\dagger}\right)_{2k} Y^{RR}_{k2} B_1(m_\mu, m_t, m_\phi) + \dots\right\}$$

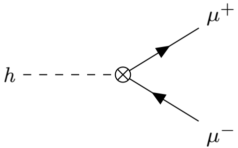
$$\delta Z^{L,\text{Dec}}_{m_\mu} = \delta Z^{L,\overline{\text{MS}}}_{m_\mu} + \frac{3m_t}{16\pi^2}\left(Y^{LL\dagger}\right)_{2k} Y^{RR}_{k2} B_0(m_\mu, m_t, m_\phi) - \frac{3a_{\mu\mu}}{32\pi^2} B_1(m_\mu, m_t, m_\phi)$$

$$\delta Z^{l,\text{Dec}}_{m_\mu} = \delta Z^{l,\overline{\text{MS}}}_{m_\mu} + \frac{3m_t}{16\pi^2}\left(Y^{RR\dagger}\right)_{2k} Y^{LL}_{k2} B_0(m_\mu, m_t, m_\phi) - \frac{3a_{\mu\mu}}{32\pi^2} B_1(m_\mu, m_t, m_\phi)$$

$a_{\mu\mu} = \left(Y^{LL\dagger}_{2k} Y^{LL}_{k2} + Y^{RR\dagger}_{2k} Y^{RR}_{k2}\right)$

The $h \to \mu^+\mu^-$ counter term in the decoupling scheme

- The explicit counter term responsible for decoupling is

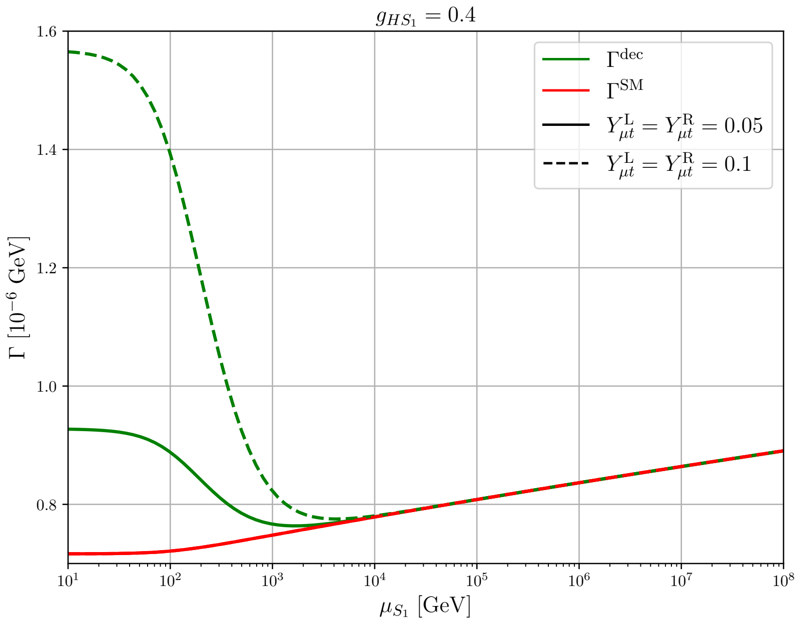


$$\supset -\frac{im_\mu}{v}\left(\frac{1}{2}\left(\delta Z_\mu^{L,\overline{\mathrm{MS}}} + \delta Z_\mu^{l,\overline{\mathrm{MS}}\dagger}\right) + \delta Z_{m_\mu}^{L,\overline{\mathrm{MS}}} + \frac{3m_t}{16\pi^2} Y_{2k}^{LL\dagger} Y_{k2}^{RR} B_0(m_\mu, m_t, m_\phi)\right) P_L$$
$$-\frac{im_\mu}{v}\left(\frac{1}{2}\left(\delta Z_\mu^{L,\overline{\mathrm{MS}}\dagger} + \delta Z_\mu^{l,\overline{\mathrm{MS}}}\right) + \delta Z_{m_\mu}^{L,\overline{\mathrm{MS}}} + \frac{3m_t}{16\pi^2} Y_{2k}^{RR\dagger} Y_{k2}^{LL} B_0(m_\mu, m_t, m_\phi)\right) P_R$$

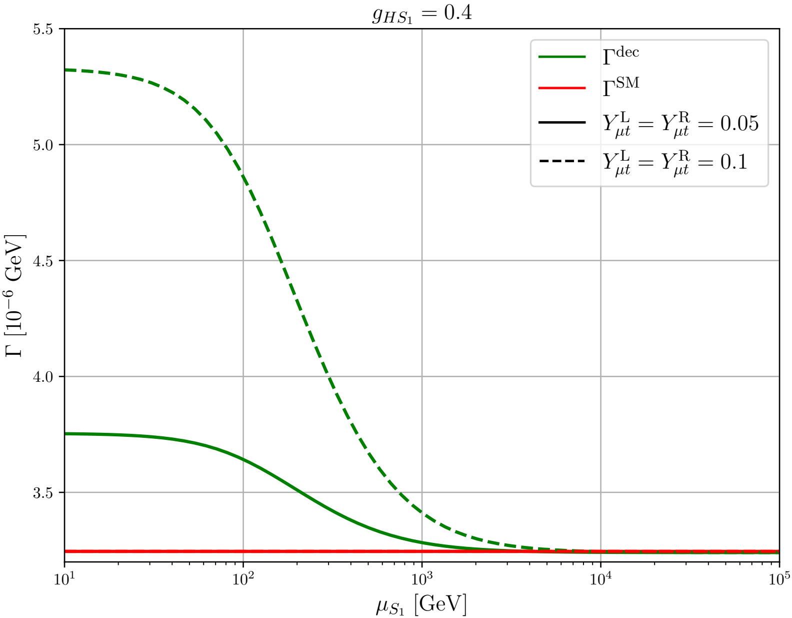
- From the one-loop contributions, including the LQ, we get

$$i\mathcal{A}_{h\to\mu\mu} = \frac{3m_t}{16\pi^2}\left(B_0(m_\mu, m_t, m_\phi) + \text{C-function}\right) Y_{2k}^{LL\dagger} Y_{k2}^{RR} P_L$$
$$+ \frac{3m_t}{16\pi^2}\left(B_0(m_\mu, m_t, m_\phi) + \text{C-function}\right) Y_{2k}^{RR\dagger} Y_{k2}^{LL} P_R$$

- Adding everything results in an amplitude that is suppressed by the LQ mass



$g_{HS_1} = 0.4$
Γ^{dec}
Γ^{SM}
$Y^{\text{L}}_{\mu t} = Y^{\text{R}}_{\mu t} = 0.05$
$Y^{\text{L}}_{\mu t} = Y^{\text{R}}_{\mu t} = 0.1$
$\Gamma\ [10^{-6}\ \text{GeV}]$
1.6
1.4
1.2
1.0
0.8
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
μ_{S_1} [GeV]


$g_{HS_1} = 0.4$
Γ^{dec}
Γ^{SM}
$Y^{\text{L}}_{\mu t} = Y^{\text{R}}_{\mu t} = 0.05$
$Y^{\text{L}}_{\mu t} = Y^{\text{R}}_{\mu t} = 0.1$
$\Gamma\ [10^{-6}\ \text{GeV}]$
5.5
5.0
4.5
4.0
3.5
10^1
10^2
10^3
10^4
10^5
μ_{S_1} [GeV]

Decoupling in a simple model with mixing

- In the HSESM, the SM gains a singlet S

$$\mathcal{L}_{\text{Scalar}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) + \frac{1}{2}\partial_\mu S \partial^\mu S + \mu_2^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2 + \mu_1^2 S^2 - \lambda_s S^4 - \kappa \left(\Phi^\dagger \Phi\right) S^2$$

- The scalar parametrization are

$$S = \frac{1}{\sqrt{2}}(v_s + s), \quad \Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v_h + h + i\phi_Z) \end{pmatrix}$$

- and the physical states become

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} s \\ h \end{pmatrix}$$

What is the decoupling limit of the HSESM?

- We expect that the doublet and the singlet decouple
- At tree level there is only v_s that can lead to decoupling
- The equation fixing the mixing angle is

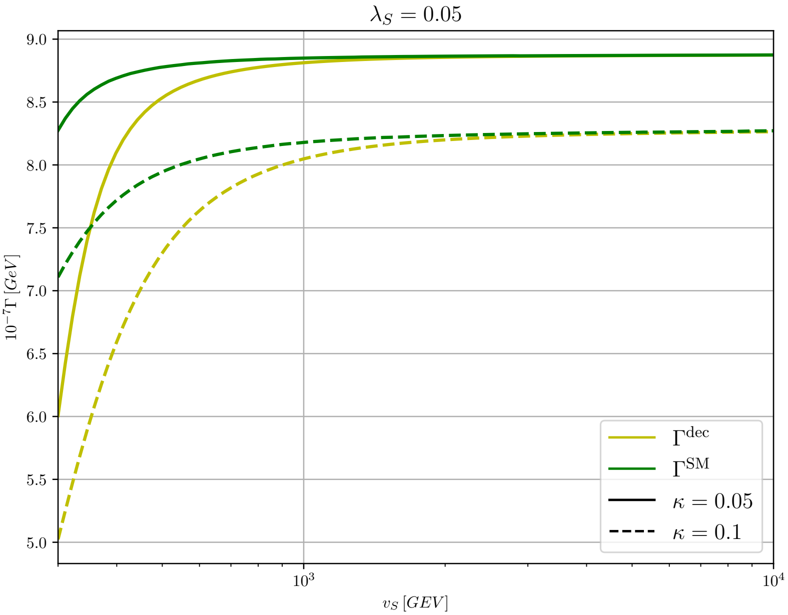
$$0 = -(\lambda_s v_s - \lambda v_h)\sin(2\alpha) + \kappa v_s v_h \cos(2\alpha)$$

- In the decoupling limit, the mixing angles behave as

$$\sin(2\alpha) \sim \frac{\kappa v_h}{\lambda_s v_s} \to 0$$
$$\cos(2\alpha) \sim 1 - \frac{\kappa v_h^2}{2\lambda_s^2 v_s^2} \to 1$$

- The only possible decoupling limit is therefore $v_s \to \infty$ such that $\alpha \to 0$ and we have a Higgs field h from the doublet and a completely decoupled singlet s

$\Rightarrow$ Also at one-loop we have a simple decoupling trajectory


$\lambda_S = 0.05$
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
$10^{-7}\Gamma\,[GeV]$
10^3
10^4
$v_S\,[GEV]$
Γ^{dec}
Γ^{SM}
$\kappa = 0.05$
$\kappa = 0.1$

Intermediate Conclusion

- We can set up a decoupling renormalization scheme, which handles large logarithms from BSM contributions
- At heart the decoupling scheme is a partial momentum subtraction scheme
- This allows for consideration of higher order $\mathrm{SM}-\overline{\mathrm{MS}}$ corrections
- An easy recipe is that SM renormalization constants contain a finite part from the BSM contributions

$\sim$

Type-II 2HDM

- motivation for considering the 2HDM:
  - step forward in complexity $\rightarrow$ impact of mixing angle treatment and parametrisation
  - large variety of existing schemes
  - similar structures of Higgs sector in many BSM models

Higgs Potential

We assume a softly broken $\mathbb{Z}_2$-symmetry and CP-conservation:

$$
\begin{aligned}
V_H = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\
& + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\
& + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right]
\end{aligned}
$$

with $m_{12}^2, \lambda_5 \in \mathbb{R}$. Φ_1 and Φ_2 aquire VEVs:

$$
\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1^0 + i\chi_1^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2^0 + i\chi_2^0) \end{pmatrix}, \quad \tan\beta = \frac{v_2}{v_1}
$$

Type-II 2HDM

- rotation from gauge into mass basis:

$$\begin{pmatrix} H \\ h \end{pmatrix} = R(\alpha) \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G_0 \\ A \end{pmatrix} = R(\beta) \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}$$

Type-II Yukawa Coupling Structure

Down-type fermions couple to Φ_1, while up-type fermions couple to Φ_2:

$$\mathcal{L}_{\text{Yuk}} = -y_u \overline{Q} \Phi_2^c u_R - y_d \overline{Q} \Phi_1 d_R - y_e \overline{L} \Phi_1 e_R + \text{h.c.}$$

Type-II 2HDM: Parametrisation Issues

- **Higgs potential parametrisation**: $P_{\text{pot}} = \{e, g_w, g_s, y_{f_i}, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, m_{12}^2, \beta, v\}$
- use tree-level relations to replace $\lambda_{1,\ldots,4}$ and m_{12}^2 by Higgs mass parameters and α:

$$\lambda_1 = \frac{m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha - (m_A^2 + \lambda_5 v^2)\sin^2\beta}{v^2\cos^2\beta}$$

$$\lambda_2 = \frac{m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha - (m_A^2 + \lambda_5 v^2)\cos^2\beta}{v^2\sin^2\beta}$$

$$\lambda_3 = \frac{(m_H^2 - m_h^2)\sin 2\alpha + (2m_{H^\pm}^2 - m_A^2)\sin 2\beta}{v^2\sin 2\beta} - \lambda_5$$

$$\lambda_4 = \frac{2(m_A^2 - m_{H^\pm}^2)}{v^2} + \lambda_5$$

$$m_{12}^2 = (m_A^2 + \lambda_5 v^2)\sin\beta\cos\beta$$

$\Rightarrow$ **physical parametrisation**:
$P_{\text{phys}}^{\{\alpha,\lambda_5\}} = \{e, g_s, m_W^2, m_Z^2, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, \lambda_5, \alpha, \beta\}$

- variations of physical parametrisation:

$$P_{\text{phys}}^{\{\lambda_3,\lambda_5\}} = \{e, g_s, m_W^2, m_Z^2, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, \lambda_3, \lambda_5, \beta\}$$

$$P_{\text{phys}}^{\{\alpha,m_{12}^2\}} = \{e, g_s, m_W^2, m_Z^2, m_h^2, m_H^2, m_A^2, m_{H^\pm}^2, m_{12}^2, \alpha, \beta\}$$

Type-II 2HDM: Decoupling Trajectories at Tree-Level

- appearance of decoupling behaviour of low-energy observables depends on details of chosen trajectory in parameter space
- reasonable decoupling trajectory should involve:
  - light CP-even Higgs h becoming SM-like (alignment)
  - BSM Higgs bosons becoming heavy
  - λ_i remaining perturbative
- from perspective of P_{pot}, decoupling at tree-level achieved via:

$$m_{12}^2 \to \infty \quad \text{while} \quad \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \beta, v \quad \text{fixed}$$

- translated to $P_{\text{phys}}^{\{\alpha,\lambda_5\}} = \{m_h^2, m_H^2, m_A^2, m_{H\pm}^2, \lambda_5, \alpha, \beta, v\}$:

$$\zeta^2 := m_A^2 = m_H^2 + \Delta_H = m_{H\pm}^2 + \Delta_{H\pm} \to \infty \quad \text{and} \quad \cos(\alpha-\beta) \propto \zeta^{-2} \to 0$$
$$\text{while} \quad m_h^2, \Delta_H, \Delta_{H\pm}, \lambda_5, \beta, v \quad \text{fixed}$$

- **beyond tree-level**: loop corrections complicate statements on decoupling significantly

Type-II 2HDM: Renormalisation Schemes

- different sets of input parameters
- different tadpole treatments:
  - Parameter Renormalized Tadpole Scheme (PRTS)
  - Fleischer–Jegerlehner Tadpole Scheme (FJTS) [Denner et al.'16, Mühlleitner et al.'16, Altenkamp/Dittmaier/Rzehak'17, Kanemura et al.'24]
  - Gauge-Invariant VEV Scheme (GIVS) [Dittmaier/Rzehak'22]
- different mixing angle renormalisations:
  - $\overline{\mathrm{MS}}$ scheme [Altenkamp/Dittmaier/Rzehak'17, Denner/Dittmaier/Lang'19]
  - $\overline{\mathrm{MS}}(\lambda_3)$ scheme [Altenkamp/Dittmaier/Rzehak'17]
  - Kanemura scheme [Kanemura et al.'04, Mühlleitner et al.'16]
  - process-dependent schemes [Mühlleitner et al.'16, Denner/Dittmaier/Lang'19]
  - pinched schemes [Mühlleitner et al.'16]

Type-II 2HDM: Constructing Decoupling Schemes

- Why should we construct decoupling schemes for the 2HDM?
  - transparent separation of SM and BSM effects
  - simple inclusion of multiloop SM results in $\overline{\text{MS}}$ scheme possible
- choice of parametrization: $P_{\text{phys}}^{\{\alpha,\lambda_5\}}$ $\rightarrow$ incorporates m_h^2 as parameter
- divide parameter set into SM-like and BSM-like:

$$P_{\text{phys}}^{\{\alpha,\lambda_5\}} = \underbrace{\{e, g_s, m_W^2, m_Z^2, m_{f_i}, m_h^2\}}_{\text{SM-like}} \cup \underbrace{\{m_H^2, m_A^2, m_{H\pm}^2, \lambda_5, \alpha, \beta\}}_{\text{BSM-like}}$$

- tadpole treatment: PRTS
- field renormalisation: on-shell
- SM-like parameters: renormalisation fixed by decoupling prescription ("master equation")
- $m_H^2, m_A^2, m_{H\pm}^2, \lambda_5$: $\overline{\text{MS}}$ scheme
- different renormalisations schemes applied to the mixing angle

Type-II 2HDM: Mixing Angle Renormalization

- Higgs field renormalization in gauge and mass basis:

$$\begin{pmatrix} H_0 \\ h_0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{1}{2}\delta Z_{HH} & \frac{1}{2}\delta Z_{Hh} \\ \frac{1}{2}\delta Z_{hH} & 1+\frac{1}{2}\delta Z_{hh} \end{pmatrix}}_{=:\sqrt{Z_H}} \begin{pmatrix} H \\ h \end{pmatrix} \qquad \begin{pmatrix} \phi^0_{1,0} \\ \phi^0_{2,0} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{1}{2}\delta Z_{11} & \frac{1}{2}\delta Z_{12} \\ \frac{1}{2}\delta Z_{21} & 1+\frac{1}{2}\delta Z_{22} \end{pmatrix}}_{=:\sqrt{Z_\phi}} \begin{pmatrix} \phi^0_1 \\ \phi^0_2 \end{pmatrix}$$

- rotation with (bare) α connects (bare) Higgs fields in gauge and basis basis:

$$\begin{pmatrix} H_0 \\ h_0 \end{pmatrix} = R(\alpha_0) \begin{pmatrix} \phi^0_{1,0} \\ \phi^0_{2,0} \end{pmatrix} \quad \Rightarrow \quad \sqrt{Z_H} = R(\delta\alpha) R(\alpha) \sqrt{Z_{\phi^0}}\, R(\alpha)^\dagger$$

$$\begin{aligned} \Rightarrow \quad \delta Z_{HH} &= c_\alpha^2 \delta Z_{11} + s_\alpha c_\alpha \left(\delta Z_{12} + \delta Z_{21}\right) + s_\alpha^2 \delta Z_{22} \\ \delta Z_{Hh} &= s_\alpha c_\alpha \left(-\delta Z_{11} + \delta Z_{22}\right) + c_\alpha^2 \delta Z_{12} - s_\alpha^2 \delta Z_{21} + 2\delta\alpha \\ \delta Z_{hH} &= s_\alpha c_\alpha \left(-\delta Z_{11} + \delta Z_{22}\right) - s_\alpha^2 \delta Z_{12} + c_\alpha^2 \delta Z_{21} - 2\delta\alpha \\ \delta Z_{hh} &= s_\alpha^2 \delta Z_{11} - s_\alpha c_\alpha \left(\delta Z_{12} + \delta Z_{21}\right) + c_\alpha^2 \delta Z_{22} \end{aligned}$$

$\Rightarrow$ relation for $\delta\alpha$:

$$\delta\alpha = \frac{1}{4}\left(\delta Z_{Hh} - \delta Z_{hH}\right) + \frac{1}{4}\left(\delta Z_{21} - \delta Z_{12}\right)$$

Type-II 2HDM: Mixing Angle Renormalization

$\mathrm{Kan}(\alpha)$ Scheme [Kanemura et al.'04]

Use symmetric $\sqrt{Z_\phi}$:

$$\sqrt{Z_\phi} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{11} & \frac{1}{2}\delta Z_{12} \\ \frac{1}{2}\delta Z_{12} & 1 + \frac{1}{2}\delta Z_{22} \end{pmatrix} \quad \Rightarrow \quad \delta\alpha^{\mathrm{Kan}} = \frac{1}{4}\left(\delta Z_{Hh}^{\mathrm{OS}} - \delta Z_{hH}^{\mathrm{OS}}\right)$$

$\overline{\mathrm{MS}}(\alpha)$ Scheme

$$\delta\alpha^{\overline{\mathrm{MS}}} = \delta\alpha^{\mathrm{Kan}}\Big|_{\mathrm{div}} = \frac{1}{4}\left(\delta Z_{Hh}^{\mathrm{OS}} - \delta Z_{hH}^{\mathrm{OS}}\right)\Big|_{\mathrm{div}}$$

$\rightarrow$ analogous definitions for β in CP-odd sector

Type-II 2HDM: Mixing Angle Renormalization

$\overline{\mathrm{MS}}(\lambda_3)$ Scheme [Altenkamp/Dittmaier/Rzehak'17]

Renormalise α such that CT for λ_3 is purely UV-divergent:

$$\delta\lambda_3\Big|_{\text{fin}} = \frac{1}{v^2}\Bigg[2\left(m_A^2 - 2m_{H^\pm}^2 - (m_H^2 - m_h^2)\frac{\sin 2\alpha}{\sin 2\beta}\right)\delta Z_v - \frac{\sin 2\alpha}{\sin 2\beta}\delta m_h^2 + 2(m_H^2 - m_h^2)\left(\frac{\cos 2\alpha}{\sin 2\beta}\delta\alpha - \frac{\sin 2\alpha \cos 2\beta}{\sin^2 2\beta}\delta\beta\right)\Bigg]\Bigg|_{\text{fin}} \stackrel{!}{=} 0$$

$$\Rightarrow \quad \delta\alpha^{\overline{\mathrm{MS}}(\lambda_3)}\Big|_{\text{fin}} = \frac{\tan 2\alpha}{m_H^2 - m_h^2}\Bigg[\frac{\delta m_h^2}{2} - \left(m_h^2 - m_H^2 + (m_A^2 - 2m_{H^\pm}^2)\frac{\sin 2\beta}{\sin 2\alpha}\right)\delta Z_v - \cot 2\beta(m_H^2 - m_h^2)\delta\beta\Bigg]\Bigg|_{\text{fin}}$$

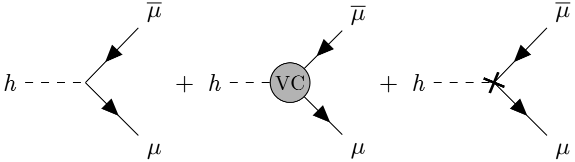
Type-II 2HDM: Constructing Decoupling Schemes

- three schemes constructed with different mixing angle renormalisation:
  - $\mathrm{Dec} + \overline{\mathrm{MS}}(\alpha, \beta)$: α and β both $\overline{\mathrm{MS}}$-renormalised
  - $\mathrm{Dec} + \mathrm{Kan}(\alpha, \beta)$: α and β both renormalised in the Kanemura scheme
  - $\mathrm{Dec} + \overline{\mathrm{MS}}(\lambda_3, \beta)$: β $\overline{\mathrm{MS}}$-renormalised, α renormalised such that λ_3 is $\overline{\mathrm{MS}}$-renormalised
- for pedagogical purposes: $\overline{\mathrm{MS}} + \overline{\mathrm{MS}}(\alpha, \beta)$ constrcuted with $\overline{\mathrm{MS}}$-renormalised SM-like parameters and mixing angles $\rightarrow$ reference scheme with non-decoupling behaviour

Type-II 2HDM: The Process $h \to \mu\mu$

- renormalised NLO amplitude:

$$\mathcal{A}^{\text{NLO}}_{h\to\mu\mu} =$$



$$= \mathcal{A}^{\text{tree}}_{h\to\mu\mu} + \mathcal{A}^{1\ell,\text{VC}}_{h\to\mu\mu} + \mathcal{A}^{1\ell,\text{CT}}_{h\to\mu\mu}$$

$$= \frac{m_\mu \sin\alpha}{v\cos\beta} + \text{genuine 1-loop diagrams}$$

$$+ \frac{m_\mu \sin\alpha}{v\cos\beta}\left[\frac{\delta m_\mu}{m_\mu} - \delta Z_v + \cot\alpha\ \delta\alpha + \tan\beta\ \delta\beta + \frac{1}{2}\left(\delta Z_{hh} - \cot\alpha\ \delta Z_{Hh} + \delta Z^L_\mu + \delta Z^R_\mu\right)\right]$$

Type-II 2HDM: Analytical Considerations

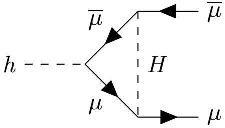
- for demonstration purposes: consider specific parameter trajectory defined by

$$\zeta = m_H^2 = m_A^2 = m_{H^\pm}^2 \text{ and } \delta := \beta - \pi/2 - \alpha = 0$$

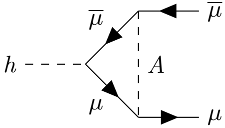
  and extract non-decoupling parts of all quantities

- cancellation of non-decoupling effects among genuine 1-loop diagrams:

$$\mathcal{A}^{1\ell,\text{VC}}_{h\to\ell\ell}\Big|_{\text{non-dec.}} = \Bigg($$



$$+$$



$$\Bigg)_{\text{non-dec.}}$$

$$= \frac{m_\mu^3 \tan^2\beta}{16\pi^2 v^3}\left(\log\frac{\mu^2}{\zeta^2} + 1\right) - \frac{m_\mu^3 \tan^2\beta}{16\pi^2 v^3}\left(\log\frac{\mu^2}{\zeta^2} + 1\right)$$

$$= 0$$

Type-II 2HDM: Analytical Considerations

- cancellation of non-decoupling effects among muon-related counterterms:

$$\delta m_\mu^{\overline{\mathrm{MS}}}\Big|_{\mathrm{non-dec.}} = 0$$

$$\delta m_\mu^{\mathrm{Dec}}\Big|_{\mathrm{non-dec.}} = \delta m_\mu^{\mathrm{OS}}\Big|_{\mathrm{non-dec.}} = \frac{3 m_\mu^3 \tan^2\beta}{32\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + \frac{5}{6}\right)$$

$$\delta Z_{\mu\mu}^{L,\mathrm{OS}}\Big|_{\mathrm{non-dec.}} = -\frac{m_\mu^2 \tan^2\beta}{16\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + \frac{3}{2}\right)$$

$$\delta Z_{\mu\mu}^{R,\mathrm{OS}}\Big|_{\mathrm{non-dec.}} = -\frac{m_\mu^2 \tan^2\beta}{16\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + \frac{1}{2}\right)$$

$$\Rightarrow \quad \left(\frac{\delta m_\mu^{\overline{\mathrm{MS}}}}{m_\mu^2} + \frac{1}{2}\left(\delta Z_{\mu\mu}^{L,\mathrm{OS}} + \delta Z_{\mu\mu}^{R,\mathrm{OS}}\right)\right)\Big|_{\mathrm{non-dec.}} = -\frac{m_\mu^2 \tan^2\beta}{16\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + 1\right)$$

$$\left(\frac{\delta m_\mu^{\mathrm{Dec}}}{m_\mu^2} + \frac{1}{2}\left(\delta Z_{\mu\mu}^{L,\mathrm{OS}} + \delta Z_{\mu\mu}^{R,\mathrm{OS}}\right)\right)\Big|_{\mathrm{non-dec.}} = 0$$

Type-II 2HDM: Analytical Considerations

- non-decoupling effects in scalar sector:

$$\delta Z^{\rm OS}_{hh}\Big|_{\rm non-dec.} = \partial_{p^2}\Sigma_{hh}(p^2)\Big|_{p^2=m_h^2} = 0$$

$$\delta Z^{\rm OS}_{Hh}\Big|_{\rm non-dec.} = \frac{2}{\zeta^2 - m_h^2}\mathrm{Re}\Big\{\Sigma_{hH}(m_h^2) - \delta t_{hH}\Big\}\Big|_{\rm non-dec.} = 0$$

$$\delta Z^{\rm OS}_{hH}\Big|_{\rm non-dec.} = \frac{2}{\zeta^2 - m_h^2}\mathrm{Re}\Big\{\Sigma_{hH}(\zeta^2) - \delta t_{hH}\Big\}\Big|_{\rm non-dec.} \approx \frac{3m_t^2\cot\beta}{4\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + 2\right)$$

$$\delta\alpha^{\overline{\rm MS}}\Big|_{\rm non-dec.} = \delta\beta^{\overline{\rm MS}}\Big|_{\rm non-dec.} = 0$$

$$\delta\alpha^{\rm Kan}\Big|_{\rm non-dec.} = \frac{1}{4}\left(\delta Z^{\rm OS}_{Hh} - \delta Z^{\rm OS}_{hH}\right)\Big|_{\rm non-dec.} \approx -\frac{3m_t^2\cot\beta}{16\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + 2\right)$$

$$\delta\beta^{\rm Kan}\Big|_{\rm non-dec.} = \frac{1}{4}\left(\delta Z^{\rm OS}_{G^0 A} - \delta Z^{\rm OS}_{A G^0}\right)\Big|_{\rm non-dec.} \approx -\frac{3m_t^2\cot\beta}{16\pi^2 v^2}\left(\log\frac{\mu^2}{\zeta^2} + 2\right)$$

$$\delta\alpha^{\overline{\rm MS}(\lambda_3,\beta)}\Big|_{\rm non-dec.} = \frac{\tan 2\beta}{2\zeta^2}\delta m_h^{2,\rm Dec}\Big|_{\rm non-dec.} = -\frac{3\lambda_5\cos 2\beta}{16\pi^2}\left(\log\frac{\mu^2}{\zeta^2} + 1\right)$$

Type-II 2HDM: Analytical Considerations

- non-decoupling effects in scalar sector:

$$\left(\cot\alpha\,\delta\alpha + \tan\beta\,\delta\beta + \frac{1}{2}\left(\delta Z_{hh}^{\text{OS}} - \cot\alpha\,\delta Z_{Hh}^{\text{OS}}\right)\right)\Big|_{\text{non-dec.}} = \tan\beta(\delta\beta - \delta\alpha)\big|_{\text{non-dec.}}$$

$\Rightarrow$ for different mixing angle renormalisations:

$$\left(\delta\beta^{\overline{\text{MS}}} - \delta\alpha^{\overline{\text{MS}}}\right)\Big|_{\text{non-dec.}} = 0$$

$$\left(\delta\beta^{\text{Kan}} - \delta\alpha^{\text{Kan}}\right)\Big|_{\text{non-dec.}} = 0$$

$$\left(\delta\beta^{\overline{\text{MS}}} - \delta\alpha^{\overline{\text{MS}}(\lambda_3)}\right)\Big|_{\text{non-dec.}} = \frac{3\lambda_5\cos 2\beta}{16\pi^2}\left(\log\frac{\mu^2}{\zeta^2} + 1\right)$$

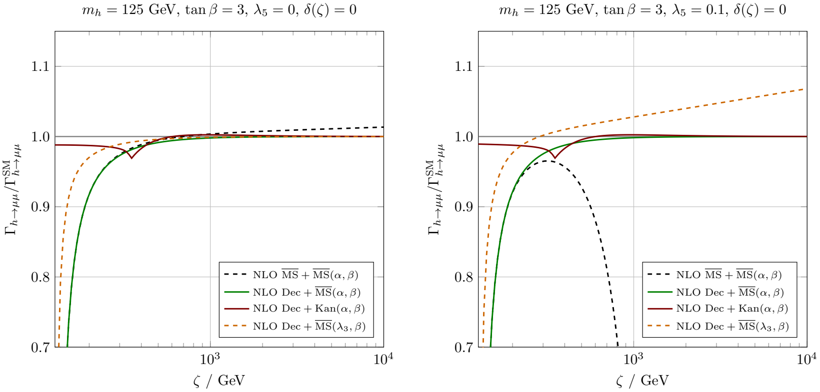
- VEV free of non-decoupling effects: $\delta Z_v^{\text{Dec}}\Big|_{\text{non-dec.}} = \delta Z_v^{\text{OS}}\Big|_{\text{non-dec.}} = 0$

$\Rightarrow$ we indentify two schemes as decoupling schemes:

  - $\text{Dec} + \overline{\text{MS}}(\alpha, \beta)$
  - $\text{Dec} + \text{Kan}(\alpha, \beta)$

Type-II 2HDM: Numerical Results

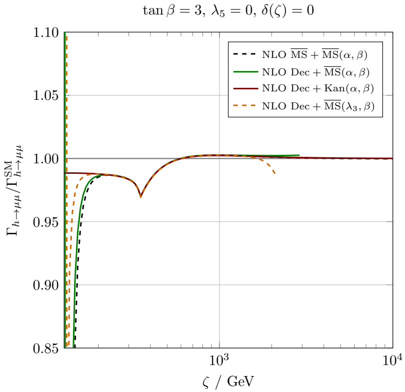
- along same numerical decoupling trajectory in different scheme (different physical trajectories!):

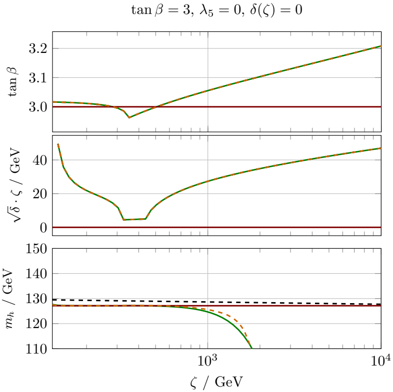


$\Rightarrow$ $\overline{\mathrm{MS}} + \overline{\mathrm{MS}}(\alpha,\beta)$ and Dec $+ \overline{\mathrm{MS}}(\lambda_3,\beta)$ schemes do not decouple although tree-level decoupling limit is approached!

Type-II 2HDM: Numerical Results

- along same physical decoupling trajectory (defined in $\mathrm{Dec} + \mathrm{Kan}(\alpha, \beta)$ scheme):



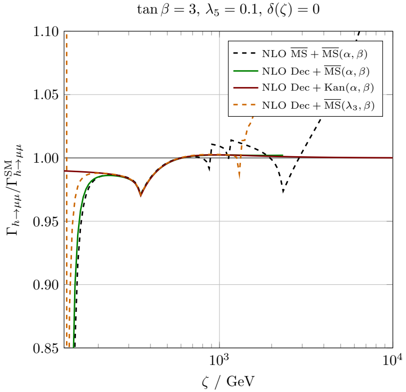


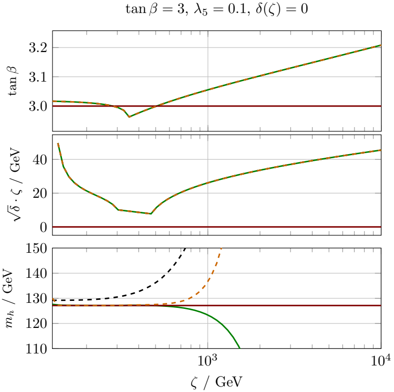
$\Rightarrow$ ratio $\Gamma_{h\to\mu\mu}/\Gamma^{\mathrm{SM}}_{h\to\mu\mu}$ has decoupling behaviour in all schemes (up to 2-loop effects)

$\Rightarrow$ proper scheme translation reveals **non-decoupling differences** in trajectories

Type-II 2HDM: Numerical Results

- along same physical decoupling trajectory (defined in $\mathrm{Dec} + \mathrm{Kan}(\alpha, \beta)$ scheme):





$\Rightarrow$ ratio $\Gamma_{h\to\mu\mu}/\Gamma^{\mathrm{SM}}_{h\to\mu\mu}$ has decoupling behaviour in all schemes (up to 2-loop effects)

$\Rightarrow$ proper scheme translation reveals **non-decoupling differences** in trajectories

Type-II 2HDM: Scheme Comparison

- properties of constructed schemes:

	$\overline{\text{MS}} + \overline{\text{MS}}(\alpha, \beta)$	$\text{Dec} + \overline{\text{MS}}(\alpha, \beta)$	$\text{Dec} + \text{Kan}(\alpha, \beta)$	$\text{Dec} + \overline{\text{MS}}(\lambda_3, \beta)$
m_{f_i}	$\overline{\text{MS}}$	Dec	Dec	Dec
v	$\overline{\text{MS}}$	Dec	Dec	Dec
m_h^2	$\overline{\text{MS}}$	Dec	Dec	Dec
$m_H^2, m_A^2, m_{H^\pm}^2$	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$
λ_3	–	–	–	$\overline{\text{MS}}$
λ_5	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$	$\overline{\text{MS}}$
α	$\overline{\text{MS}}$	$\overline{\text{MS}}$	Kan	–
β	$\overline{\text{MS}}$	$\overline{\text{MS}}$	Kan	$\overline{\text{MS}}$
decoupling		✓	✓	

$\Rightarrow$ Applying decoupling prescription to all SM-like parameters is not sufficient to guarantee decoupling (but it may work)!

Conclusion

- We propose decoupling property as fourth criterion for renormalisation schemes next to gauge independence, numerical stability and process independence.
- We constructed decoupling schemes for the S_1-extension of the SM, the HSESM and the Type-II 2HDM.
- The application to the 2HDM reveals: applying the decoupling prescription to all SM-like parameters is not sufficient, the choice of the mixing angle renormalisation plays a crucial role.
- Further investigations are necessary for the desired general applicability and an implementation in `FlexibleSUSY`/`FlexibleDecay`.

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