Infrared divergences and the choice of regularization

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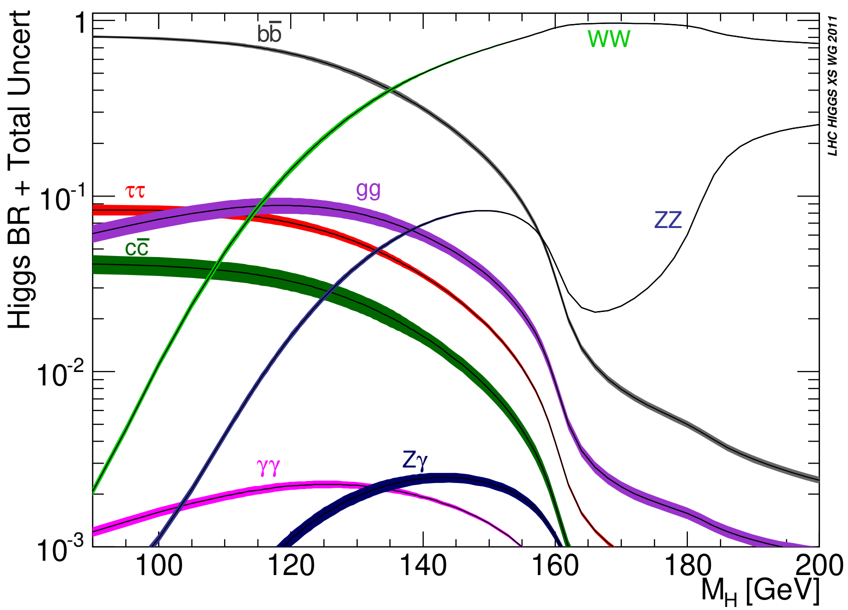
Motivation

- $\overline{\text{MS}}$ and related schemes are important for the automatization of calculation of higher-order corrections in BSM models
- What if we would like to compute decays in the $\overline{\text{MS}}$ scheme and took this scheme to its natural conclusion? What happens with infrared divengencies in the $\overline{\text{MS}}$ schem
- At 1-loop, we compute tree-level, 1-loop virtual and real amplitudes
  - tree – fixed by the scheme as any change in the renormalization scheme is of 1-loop order
  - virtual/real – in principle the scheme does not matter as scheme difference is of 2-loop order. So long as one uses the same masses everywhere (internal lines, external lines, helicity/spinor sums etc) there is no problem
- But what happens if we don't? What happens if you keep mass asignement **exactly** as prescribed by the renormalization scheme?

Test case: 1-loop QCD corrections to $h \to q\bar{q}$

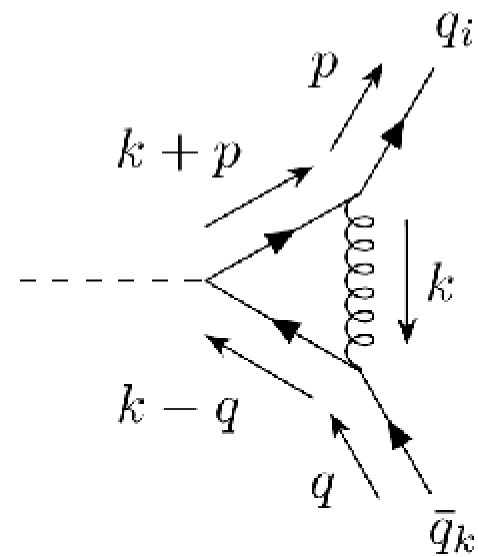
- We consider 1-loop QCD corrections to $h \to q\bar{q}$ (the same arguments will hold for any 2-body $h \to f\bar{f}$ decay).
- The non-abelian nature of QCD does not play a role at this level.
- We use $h \to q\bar{q}$ as an example but $h \to b\bar{b}$ is phenomenologically very relevant. It is the biggest partial width for SM-like Higgs boson so even relatively small changes could be detectable at the level of branching ratios into other final states
- We want to reproduce the know result $\overline{\text{MS}}$ result, which approximately looks like

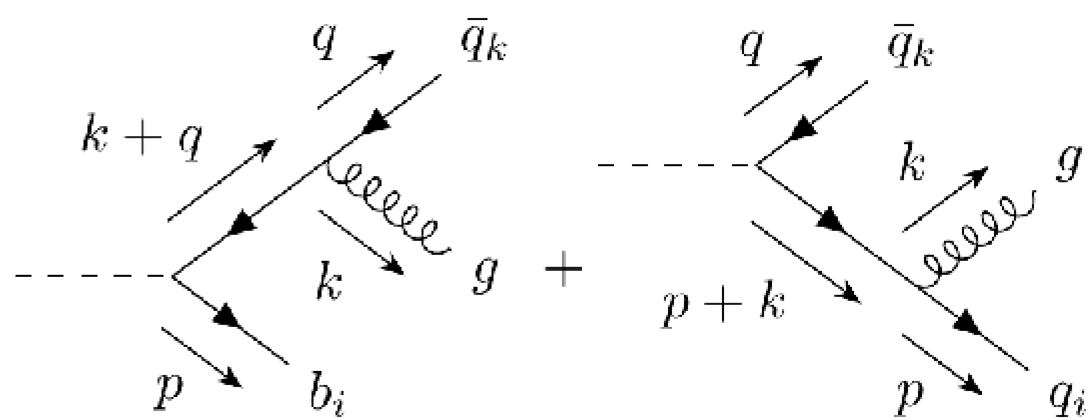
$$\Gamma_{\overline{\text{MS}}}(\mu = m_H) \underset{m_H \gg \bar{m}}{\approx} \Gamma^0 \left(1 + \frac{17\alpha_s C_F}{4\pi}\right)$$



Ingredients

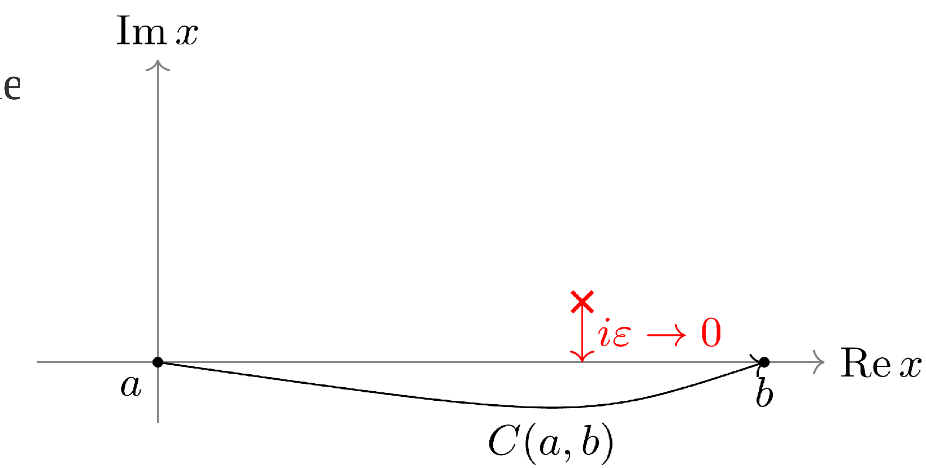
- We renormalize the 1-loop amplitude in the MS scheme: internal quark mass $\overline{m}$ and external mass m, m≠m
- The Higgs is not influenced at order α_s
- For cancelation of IR divergencies the same mass assignement is used for real matrix elements

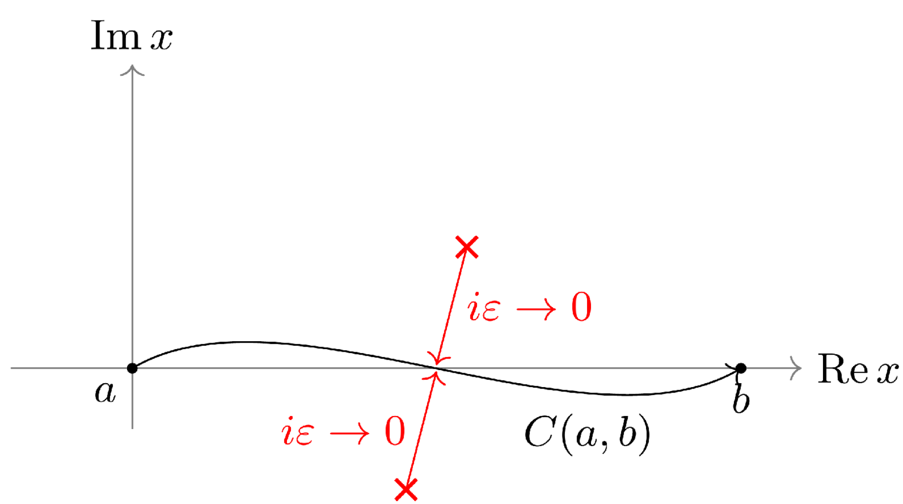


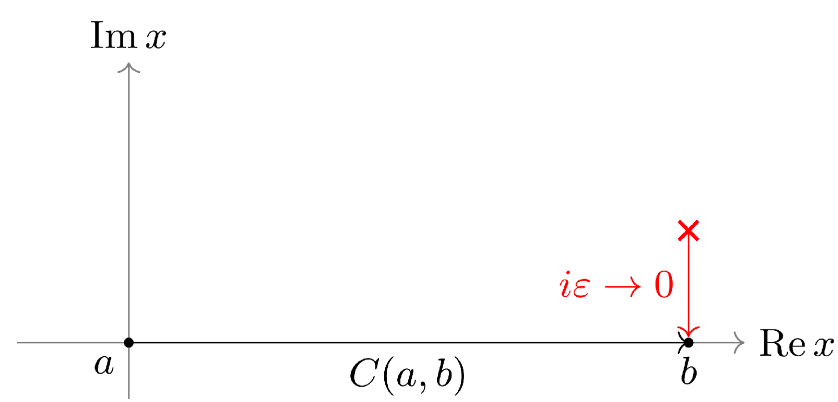


Singularities in 1d

- Due to the $i\varepsilon$ prescription the poles in Feynman parameters are not on the real line but can approach it in the $\varepsilon \to 0$ limit.
- The contour can be deformed unless
  - it gets pinched by 2 poles
  - it lands at the end point

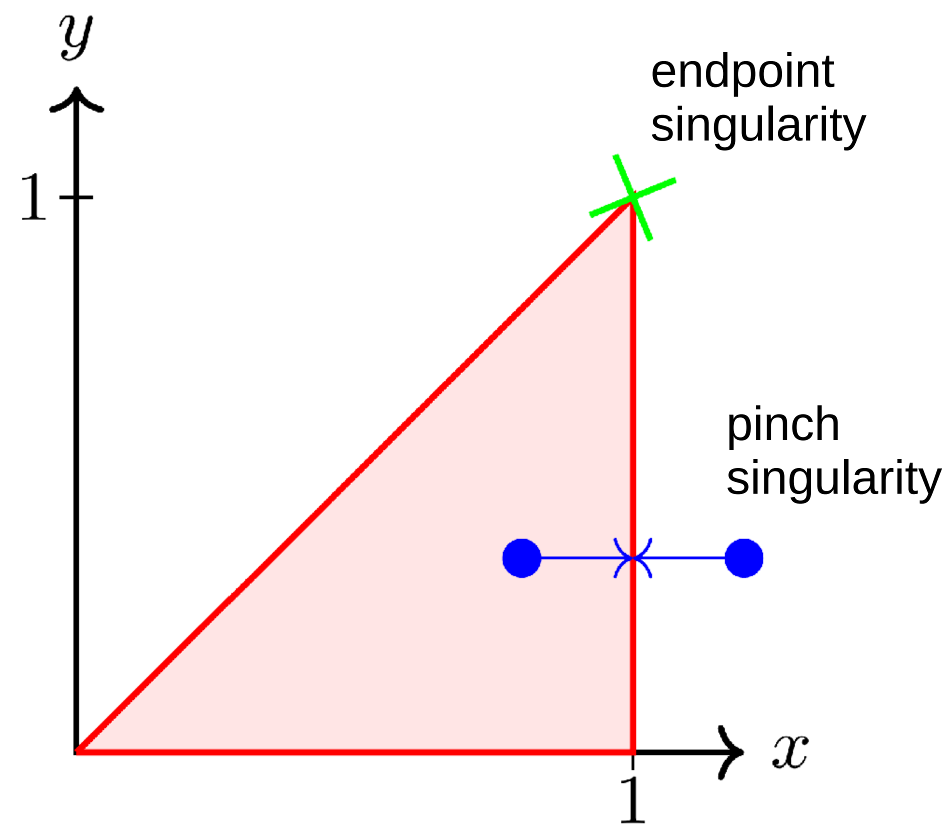






Singularities in 2d and more

- In the case of multiple Feynman parameter integrals pinch singularities can only appear at an edge of the integration region and the endpoint singularity at some corner point of the volume.
- Otherwise there is always the possibility of avoiding the pole.



C_0 function

- We consider 3 regulators:
  - dimensional regularization
  - Gluon mass regulator λ
  - off-shell regulator $\Delta = m^2 - p^2$
- C0 with all 3 of them has the form

$$C_0(p^2, m_H^2, p^2, \lambda, m, m) \sim \int_0^1 dx \int_0^x dy \frac{1}{(m_H^2(x-1)(x-y)+m^2(y-1)^2+\Delta^2 y(1-y)+\lambda^2 y)^{1+\epsilon}}$$

- You can check that in DimReg the limits can be exchanged: you can set Δ and λ to 0 in one-loop integral and get a correct result.
- The endpoint x=y=1 give

$$D\left(x=1, y=1, \Delta^2, \lambda, \epsilon\right) = \left(\lambda^2\right)^{1+\epsilon}$$

The off-shell regularization encounters an endpoint singularity. One cannot guarantee that we can pull the limits of $\lambda \to 0$ and $\varepsilon \to 0$ into the integral.

Explicit for of C_0

- Explicit results for real part of

$$\mathrm{Re}\{C_0(m^2, m_H^2, m^2, \lambda, m, m)\} = \frac{1}{\beta m_H^2}\left(\log\left(\frac{1-\beta}{1+\beta}\right)\log\left(\frac{\lambda^2}{m^2}\right) + \frac{1}{2}\log^2\left(\frac{1-\beta}{1+\beta}\right) + 2\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \pi^2\right)$$

$$\mathrm{Re}\{C_0(m^2, m_H^2, m^2, 0, m, m)\} = \frac{1}{\beta m_H^2}\left(\log\left(\frac{1-\beta}{1+\beta}\right)\left(\frac{1}{\bar\epsilon} + \log\left(\frac{\mu^2}{m^2}\right)\right) + \frac{1}{2}\log^2\left(\frac{1-\beta}{1+\beta}\right) + 2\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right) - \pi^2\right)$$

$$\mathrm{Re}\{C_0(p^2, m_H^2, p^2, 0, m, m)\} = \frac{1}{\beta m_H^2}\left(\log\left(\frac{1-\beta}{1+\beta}\right)\log\left(\frac{\Delta^4}{m^4}\right) - 2\pi^2\theta\left(\Delta^2\right) + \log^2\left(\frac{1-\beta}{1+\beta}\right) + 4\mathrm{Li}_2\left(\frac{2\beta}{1+\beta}\right)\right)$$

- gluon mass and MS regulated results related by

$$\log\lambda^2 \to \frac{1}{\bar\epsilon} + \log\mu^2$$

Optical theorem

- Alternatively, once can use optical theorem to calculate the squared amplitude

$$2\,\mathrm{Im}\{\mathcal{M}(i \to i)\} = \sum_n \prod_{f=1}^{n} \int \frac{\mathrm{d}^3 q_f}{(2\pi)^3 2 q_f^0} \times$$
$$(2\pi)^4 \delta^{(4)}(\Sigma_i p_i - \Sigma_f q_f)|\mathcal{M}(i \to f)|^2$$

- And the partial width

$$\Gamma_H = \frac{1}{2m_H} 2\,\mathrm{Im}\{\mathcal{M}(H \to H)\}$$

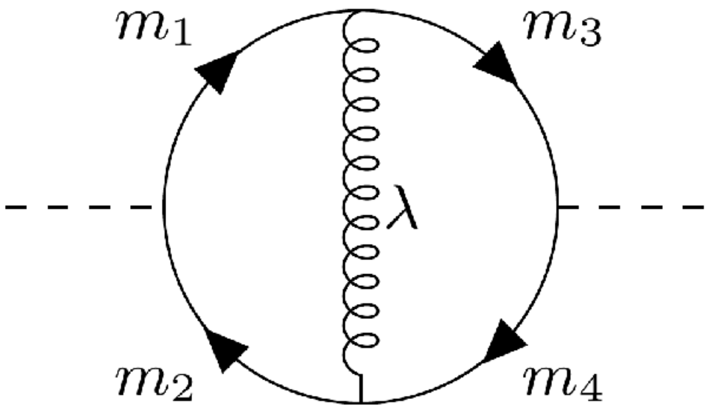
- The imaginary part can be obtained from the discontinuity of the amplitude

$$\mathrm{Disc}(i\mathcal{M}) = -2\,\mathrm{Im}\,\mathcal{M}$$

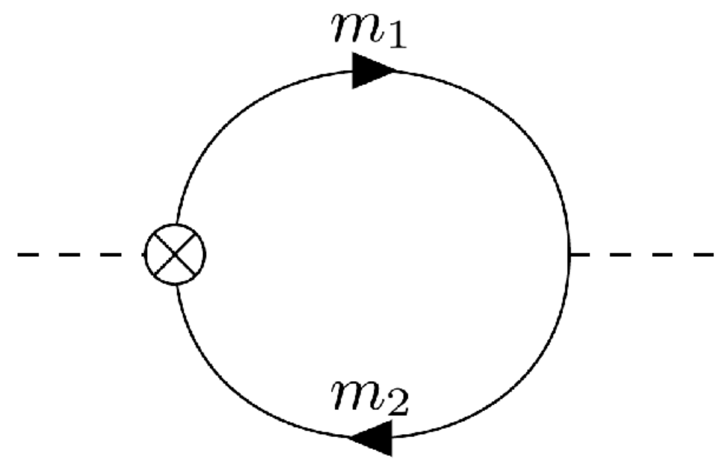
with the help of cutting rules

$$\frac{1}{(p^2 - m^2 + i\epsilon)^n} \longrightarrow -2\pi i \frac{(-1)^{n-1}}{(n-1)!} \delta^{(n-1)}(p^2 - m^2)$$

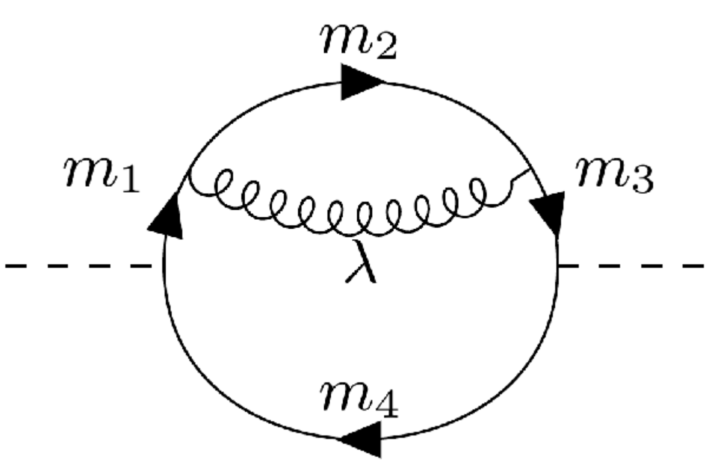
Classes of 2-loop diagrams

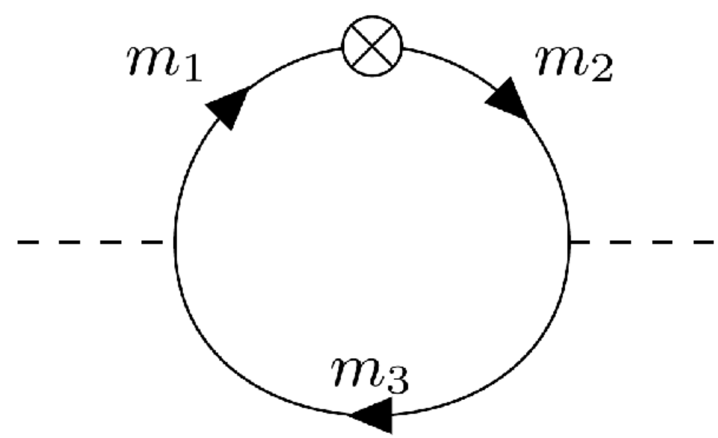


(a) $iS_1(m_1, m_2, m_3, m_4, \lambda)$



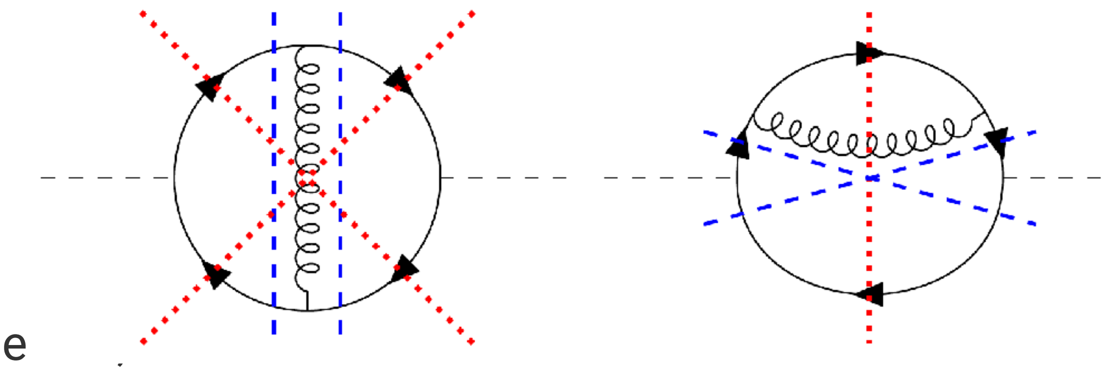
(b) $iS_1^{\mathrm{ct1}}(m_1, m_2)$





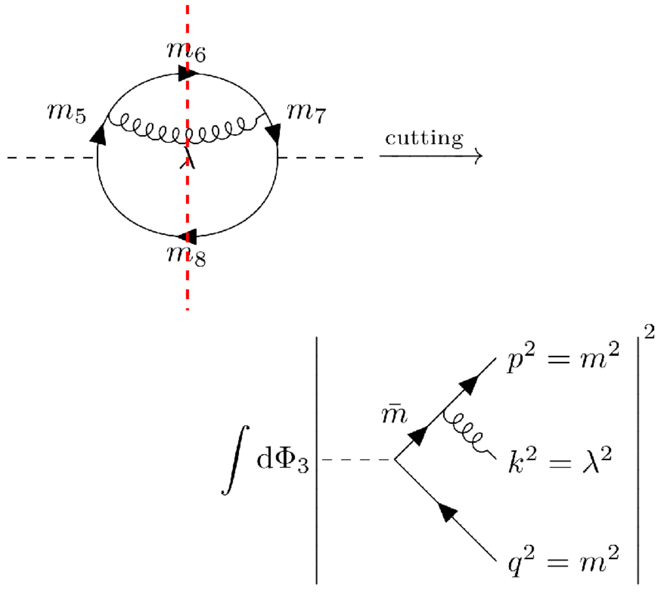
Cuts

- Types:
  - Red: correspond to real correction
  - Blue: correspond to virtual correction
- The blue dashed cuts of the diagram correspond to only one since we cut a squared propagator
- Left diagram generates vertex corrections and interference part of real emissions
- Right diagram generates squared real emission and wave function correction

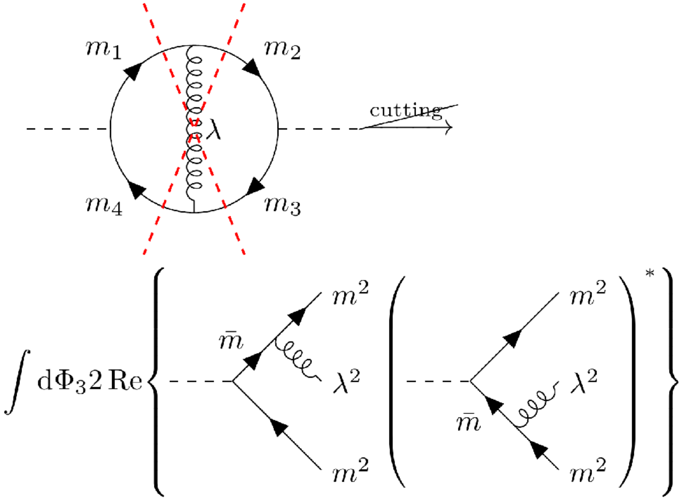


When it works and doesn't

- $m_5 = m_7 = \bar{m}$ and $m_6 = m_8 = m$



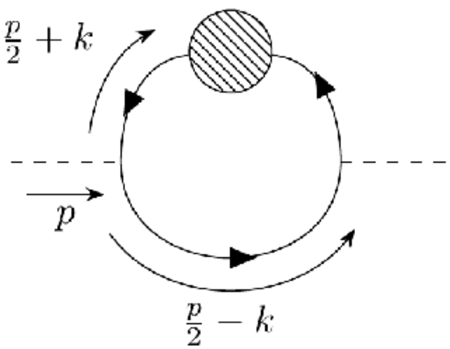
- lines connected to the scalar line once must have the mass $\bar{m}$ and once then the mass m at the same time



Correct $\overline{\text{MS}}$ prescription

- Calculate everything in $\overline{\text{MS}}$ using the optical theorem
- One new class of diagram

$i\delta\mathcal{M}(H \to H) =$



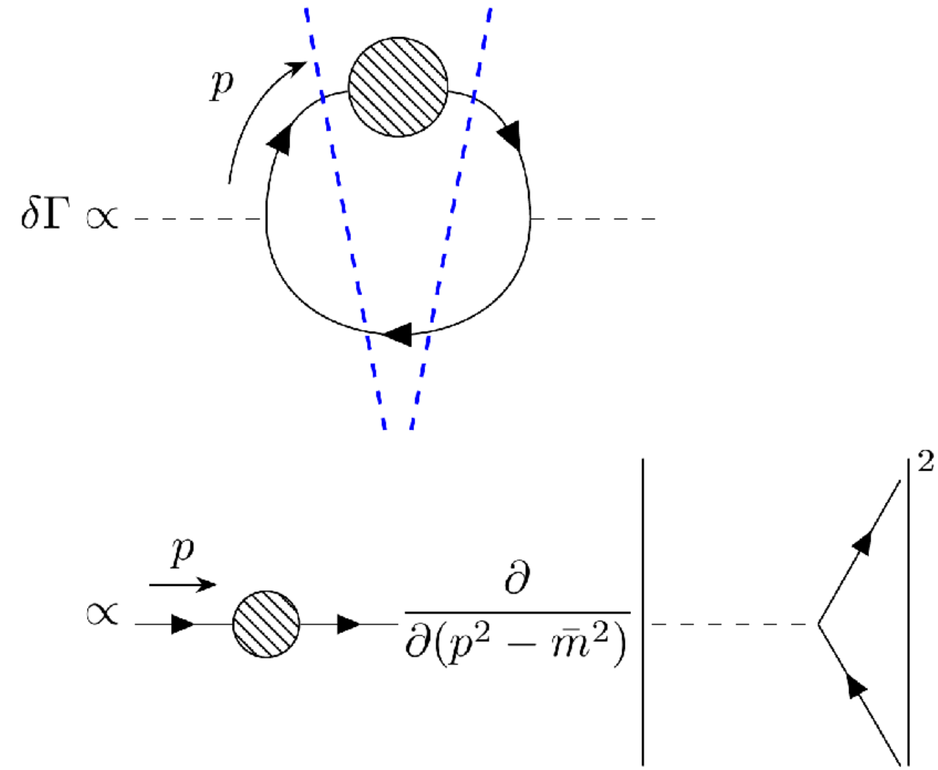
$$= y^2 \mu^{4-D} \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{\mathrm{Tr}\left[\left(-\frac{\not{p}}{2} - \not{k} + \bar{m}\right)\left(\frac{\not{p}}{2} - \not{k} + \bar{m}\right)\left(-\frac{\not{p}}{2} - \not{k} + \bar{m}\right)\hat{\Sigma}\left(-\frac{\not{p}}{2} - \not{k}\right)\right]}{\left[\left(\frac{p}{2} - k\right)^2 - \bar{m}^2\right]\left[\left(\frac{p}{2} + k\right)^2 - \bar{m}^2\right]^2}$$

- In the on-shell scheme it's renormalized to 0
- You can extract its discontinuity from explicit calculation

$$\mathrm{Disc}(i\delta M) = \frac{3y^2 N_c C_F g_s^2 \bar{\beta} \bar{m}^2}{32\pi^3}\left(3\log\left(\frac{\mu^2}{\bar{m}^2}\right) + 4\right)$$

Diagramatic interpretation

- Diagramatically this corresponds to the cut of a squared propagator
- The correction term in the MS scheme is therefore a derivative of the tree-level result times the renormalized self energy
- This term looks like the correction term for the fact that we used the MS mass as pole mass in the tree-level contribution



Correct $\overline{\text{MS}}$ result

- $\overline{\text{MS}}$ result

$$\Gamma_{\overline{\text{MS}}} = \Gamma^0(1+\Delta_{\overline{\text{MS}}})$$

where

$$\Gamma^0 = \frac{\bar{y}^2 N_c m_H \bar{\beta}^3}{8\pi}$$

and

$$\Delta_{\overline{\text{MS}}} = \frac{\alpha_s C_F}{\pi}\left(\frac{(1+\bar{\beta}^2)}{\bar{\beta}}\left(2\log\left(\frac{1-\bar{\beta}}{1+\bar{\beta}}\right)\log(\bar{\beta}) - 3\log\left(\frac{1-\bar{\beta}}{1+\bar{\beta}}\right)\log\left(\frac{1+\bar{\beta}}{2}\right) + 4\text{Li}_2\left(\frac{1-\bar{\beta}}{1+\bar{\beta}}\right) + 2\text{Li}_2\left(\frac{\bar{\beta}-1}{1+\bar{\beta}}\right) - 4\log(\bar{\beta})\right.\right.$$

$$\left. - 3\log\left(\frac{4}{1-\bar{\beta}^2}\right)\right) + \frac{3}{8\bar{\beta}^2}(-1+7\bar{\beta}^2) - \frac{(3+34\bar{\beta}^2-13\bar{\beta}^4)}{16\bar{\beta}^3}\log\left(\frac{1-\bar{\beta}}{1+\bar{\beta}}\right)$$

$$\left. + \frac{(5\bar{\beta}^2-3)}{4\bar{\beta}^2}\left(3\log\left(\frac{\mu^2}{\bar{m}^2}\right)+4\right)\right)$$

Comparison with on-shell

- On-shell result

$$\Gamma_{\mathrm{OS}} = \Gamma^0(1 + \Delta_{\mathrm{OS}})$$

where

$$\Gamma^0 = \frac{y^2 N_c m_H \beta^3}{8\pi}$$

and

$$\begin{aligned}
&\Delta_{\mathrm{OS}} \\
&= \frac{\alpha_s C_F}{\pi}\left(\frac{(1+\beta^2)}{\beta}\left(2\log\left(\frac{1-\beta}{1+\beta}\right)\log(\beta) - 3\log\left(\frac{1-\beta}{1+\beta}\right)\log\left(\frac{1+\beta}{2}\right) + 4\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right.\right. \\
&\qquad \left.+2\mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right)\right) \\
&\qquad \left.-4\log(\beta) - 3\log\left(\frac{4}{1-\beta^2}\right) + \frac{3}{8\beta^2}\left(-1+7\beta^2\right) - \frac{(3+34\beta^2-13\beta^4)}{16\beta^3}\log\left(\frac{1-\beta}{1+\beta}\right)\right).
\end{aligned}$$

Advantages of the $\overline{\text{MS}}$ scheme

- Some models are not (fully) on-shell renormalizable, e.g. SUSY
- $\overline{\text{MS}}$ calculations are easier in general and are easier to automatise in BSM models. In special schemes, higher-order SM corrections in the $\overline{\text{MS}}$ scheme can be directly applied in the BMS model (see talk by Jonas and Johannes)
- No explicit, unresummed large logs from separate scales
  - on-shell

$$\Gamma_{\text{OS}}(m_H \gg m) = \Gamma^0_{\text{OS}} \left(1 + \frac{3\alpha_s C_F}{2\pi}\left[\frac{3}{2} - \log\left(\frac{m_H^2}{m^2}\right)\right]\right)$$

  - $\overline{\text{MS}}$

$$\Gamma_{\overline{\text{MS}}}(m_H \gg \bar{m}) = \Gamma^0 \left(1 + \frac{3\alpha_s C_F}{2\pi}\left[\frac{17}{6} - \log\left(\frac{m_H^2}{\mu^2}\right)\right]\right)$$

  setting $\mu = m_H$ removes all explicit logs by resumming them in running masses and couplings. Phenomenologically very important. Influences $h \to b\bar{b}$ width, which is the largest contributor to the total SM Higgs width, which influences all bnraching ratios.

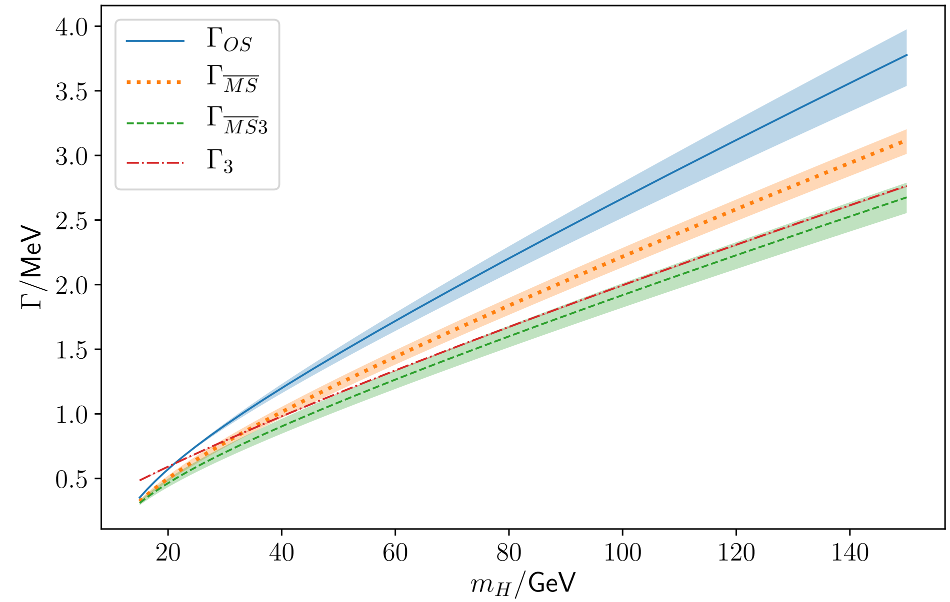
How SARAH does it?

arxiv:1703.09237

- Calculate the virtual corrections using tree-level masses and extract the infrared divergences
- These result are used to obtain $\text{IR}(\overline{\text{M}}^{v,w})$
- We use loop-corrected masses m throughout all infrared divergent diagrams for the external legs and the particles in the loop. We take again the infrared divergent parts of these amplitudes to obtain $\text{IR}(\text{M}^{v,w})$.
- The calculation of the kinematics as well as of the helicity and polarisations sums for both, the virtual and real corrections, is done with loop-corrected masses.

Numerical analysis

- Γ_{OS} is the on-shell result. The sca dependence come only from α_s
- $\Gamma_{\overline{\mathrm{MS}}}$ is $\overline{\mathrm{MS}}$ result at 1-loop
- $\Gamma_{\overline{\mathrm{MS}}3}$ is $\overline{\mathrm{MS}}$ result with 3-loop running bu 1-loop mass dependence
- Γ_3 is 3-loop result in massless qua approximation from Chetyrkin
- Things to note:
  - phase space effect for light Higgse massless result doesn't vanish threshold
  - 3-loop running brings the result closer to the 3-loop calculation
- Justification for the procedure in HDECAY where you interpolate between on-shell and $\overline{\mathrm{MS}}$ result



Conclusions and outlook

- One would expect that an $\mathcal{O}(\alpha_s)$ $m \to \overline{m}$ shift in $\mathcal{O}(\alpha_s)$ amplitude would result in a 2-loop change, but this is not the case! Incorrect regularization leads to the difference already at the 1-loop order.
- There are few ways to obtain an $\overline{\text{MS}}$, all leading to the same, correct, $\mathcal{O}(\alpha_s)$ result
- We plan to use what we learned here to improve predictions for Higgs decays in FlexibleSUSY