TECHNISCHE UNIVERSITÄT DRESDEN

INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

On-shell Renormalisation of Vector-like Lepton Models

KUTS15 @ KIT

Kilian Möhling

in Collaboration with:

Dominik Stöckinger ▪ Hyejung Stöckinger-Kim ▪ Peter Athron ▪ Gregor Daberstiel

[JHEP 10 (2024) 170], [Prog.Part.Nucl.Phys. 148 (2026)] and [2603.21414] (today ☺)

TU Dresden, Institut für Kern- und Teilchenphysik

Karlsruhe, 24.03.2026

Open questions in the Standard Model

- Baryon asymmetry
- Flavour patterns
- Neutrino Masses
- Dark Matter
- ...

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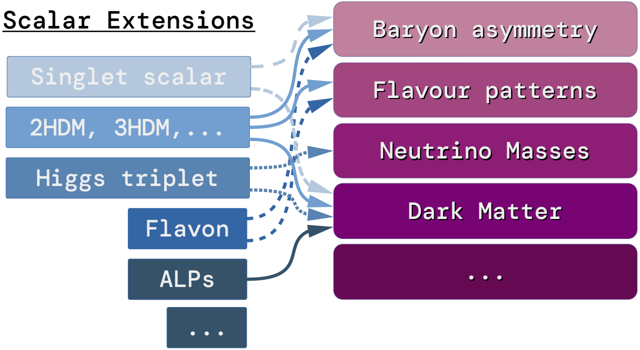

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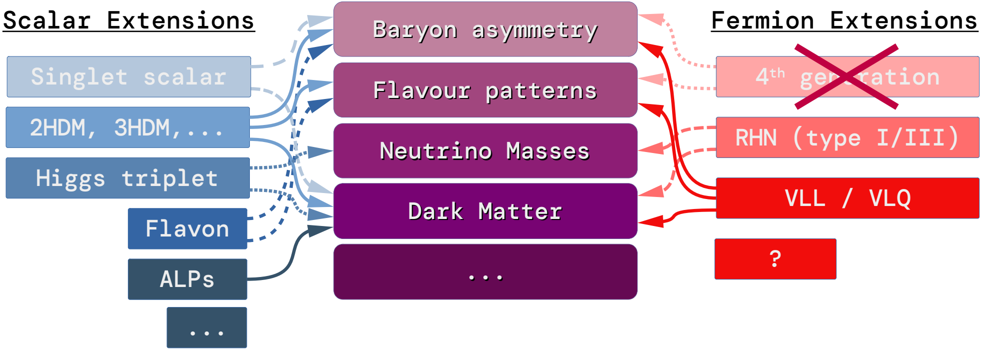

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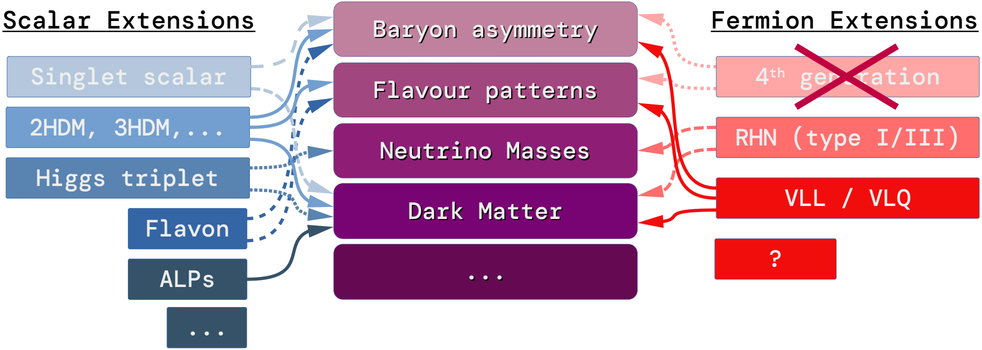

$\Longrightarrow$ **Motivations for VLF**: SM mass hierarchies, CP violation, GUT, extra dimensions, ...

$\hookrightarrow$ offer complementary phenomenology to many scalar extensions

Outline

- Phenomenology at leading order
- Why are higher orders relevant?
- The technical stuff (Renormalization, OS conditions, ...)
- Impact of higher order corrections

Vector-like lepton representations

Special case: tree-level coupling to SM leptons and Higgs $\Longrightarrow$ *vector-like leptons* (VLL)

Name	N	E	L	$L_{\frac{3}{2}}$	N^a	E^a
Rep.	$(\mathbf{1}, 0)$	$(\mathbf{1}, -1)$	$(\mathbf{2}, -\frac{1}{2})$	$(\mathbf{2}, -\frac{3}{2})$	$(\mathbf{3}, 0)$	$(\mathbf{3}, -1)$
$\mathcal{L}$	$\overline{l_L}\tilde{\Phi}N$	$\overline{l_L}\Phi E$	$\overline{L}\Phi e_R$	$\overline{L_{\frac{3}{2}}}\tilde{\Phi}e_R$	$\overline{l_L}\sigma^a\tilde{\Phi}N^a$	$\overline{l_L}\sigma^a\Phi E^a$

vector-like: $F_{L,R} \in$ same representation $\Longrightarrow$ $-M_F\overline{F_L}F_R + h.c.$ gauge-invariant!

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Interaction terms between VLL possible for: $L \oplus E^{(a)}$, $L \oplus N^{(a)}$, $L_{\frac{3}{2}} \oplus E^{(a)}$

e.g. $L \oplus E$: $\mathcal{L} \supset \overline{L}\,[\underbrace{\lambda P_R}_{\text{"SM-like"}} + \underbrace{\bar{\lambda}^* P_L}_{\text{opposite chirality}}]\,E\,\Phi$

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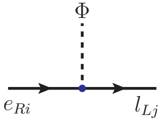
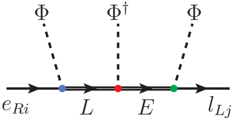
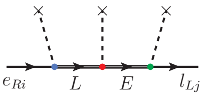
$\hookrightarrow$ independent SM chiral symmetry breaking



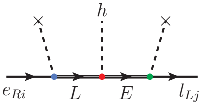
$\sim y_{ji}$



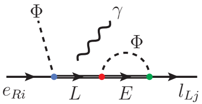
$\sim \dfrac{\lambda_i^L \bar{\lambda} \lambda_j^E}{M_E M_L}$



$\sim \Delta m_i$

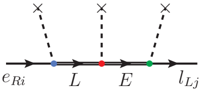


$\sim 3\dfrac{\Delta m_i}{v}$



$\sim -\dfrac{m_i \Delta m_i}{16\pi^2 v^2}$

Phenomenology at leading order: Higgs coupling and $g-2$



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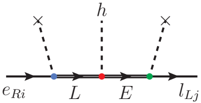

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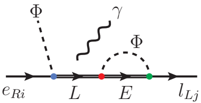

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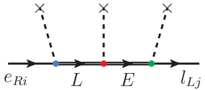
$$\Longrightarrow \quad R_{\mu\mu} = \frac{\Gamma(h \to \mu\mu)}{\Gamma(h \to \mu\mu)_{\text{SM}}} \approx \left(1 - \frac{\Delta a_\mu}{\mathcal{Q}\, 10^{-9}}\right)^2 + \left(\frac{d_\mu\, [e\, \text{cm}]}{2\mathcal{Q} \times 10^{-22}}\right)^2$$



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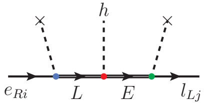

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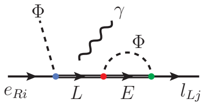

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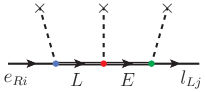

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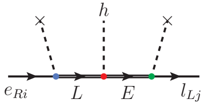

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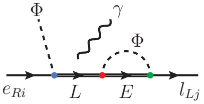

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$R_{\mu\mu} = 1.21 \pm 0.35$

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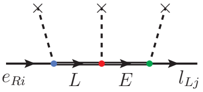

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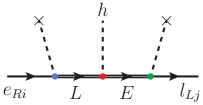

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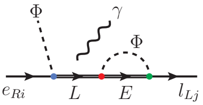

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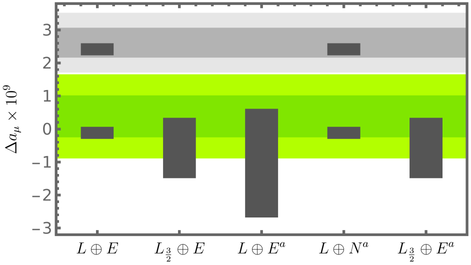

[Athron, KM, Stöckinger, Stöckinger-Kim '25]

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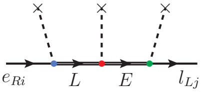

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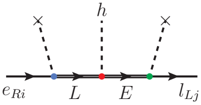

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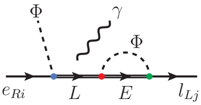

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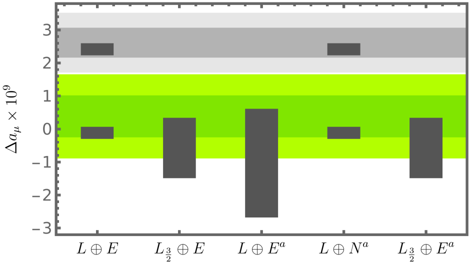

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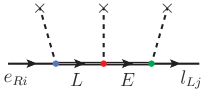

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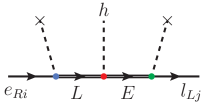

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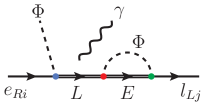

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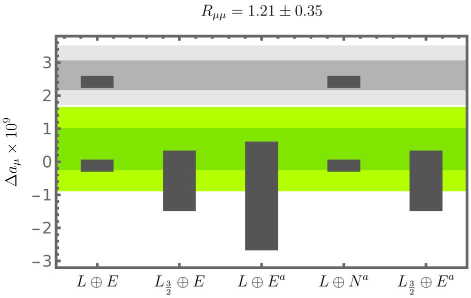

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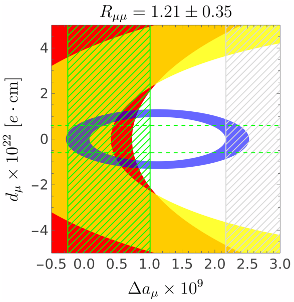
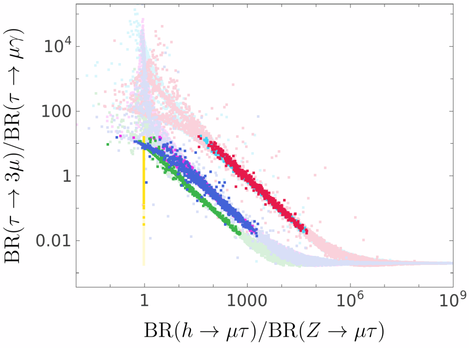
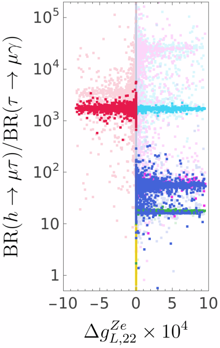


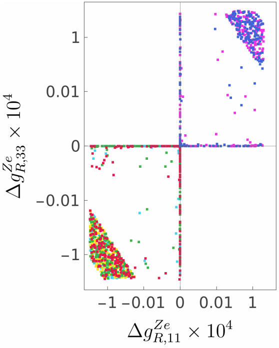
[Daberstiel, KM, Stöckinger, Stöckinger-Kim '26]

Phenomenology at leading order: charged lepton flavour violation

$$\frac{\mathrm{BR}(h \to \mu\tau)}{\mathrm{BR}(\tau \to \mu\gamma)} \approx \frac{1690}{\mathcal{Q}^2}, \qquad \frac{\mathrm{BR}(\mu \to 3e)}{\mathrm{BR}(\mu \to e\gamma)} \propto \frac{1}{|\bar{\lambda}\mathcal{Q}|^2}, \qquad \dots$$







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Never trust a tree-level calculation

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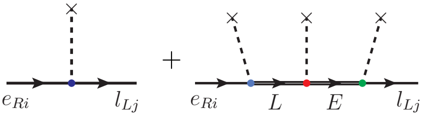
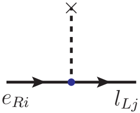
SM Lepton **mass** at tree-level

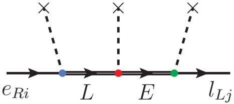


$$+ \quad \dots \quad \sim \quad y_{ij} v + \frac{\lambda_L^i \bar{\lambda} \lambda_E^j}{M_E M_L} v^3 + \mathcal{O}(v^5)$$

SM Lepton **mass** at tree-level vs one-loop



$+$



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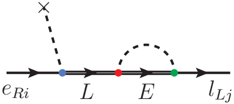
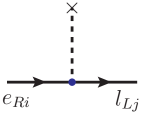


$$+ \quad \dots \quad \sim \quad \frac{\lambda_L^i \bar{\lambda} \lambda_E^j}{16\pi^2} v \; + \; \mathcal{O}(v^3)$$

Never trust a tree-level calculation

SM Lepton **mass** at tree-level vs one-loop



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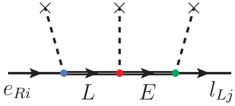

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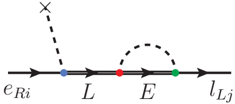

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mass suppression "loses" to loop-suppression when

$$\frac{16\pi^2 v^2}{M^2} \lesssim 1 \qquad \Leftrightarrow \qquad M \gtrsim \mathcal{O}(1 \text{ TeV})$$

Never trust a tree-level calculation

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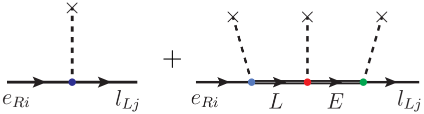
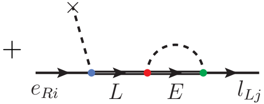


$$+ \quad \dots \quad \sim \quad y_{ij} v + \frac{\lambda_L^i \bar{\lambda} \lambda_E^j}{M_E M_L} v^3 + \mathcal{O}(v^5)$$



$$+ \quad \dots \quad \sim \quad \frac{\lambda_L^i \bar{\lambda} \lambda_E^j}{16\pi^2} v \; + \; \mathcal{O}(v^3)$$

mass suppression "loses" to loop-suppression when

$$\frac{16\pi^2 v^2}{M^2} \lesssim 1 \qquad \Leftrightarrow \qquad M \gtrsim \mathcal{O}(1\ \text{TeV})$$

$\Rightarrow$ what about other observables?

- e.g. $h \to \mu\mu$: $\mathcal{O}(100\%)$ tree-level correction vs $\mathcal{O}(10\%)$ precision at LHC
- NLO corrections to Δa_μ, $Z \to \ell\ell$, CLFV?

Major Issues

- **SVD inversion:** solving for y_i at tree-level is only possible if the *target mass* is known.
  one-loop mass shift Δm_i changes this target to

$$m_i = m_i^{\text{phys}} - \Delta m_i$$

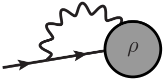
$\implies$ cant get m_i without Δm_i, but cant get Δm_i without y_i (?)

- **UV Divergences:** loop corrections to self-energies, Higgs-couplings, etc. are divergent

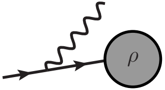
$$\int \frac{d^D k}{[k^2 - \Delta]^2} \sim \frac{1}{\epsilon_{\text{UV}}}$$

$\implies$ cancel after *renormalization*

- **IR Divergences:** external charged particles $\curvearrowright$ divergences at $k^2,\ k \cdot p \sim 0$
  from virtual and real photon spectrum



$$\sim \int \frac{d^D k}{k^2} \frac{\rho_v(k,p)}{k^2 - 2kp}$$



$$\sim \frac{\rho_r(p+k)}{\underbrace{p^2 - m^2}_{=0 \text{ on-shell}} - 2kp}$$

On-shell Renormalization Scheme

SOLUTION: $\Longrightarrow$ on-shell scheme [KM, Stöckinger, Stöckinger-Kim '24]

$\equiv i\Sigma_{ba}(p)$: $\quad \underbrace{\widetilde{\mathrm{Re}}\,\Sigma_{ba}(p)u_a(p) = 0,}_{\text{removes mixing and } \Delta m} \quad \underbrace{\frac{1}{\not{p}-m_a}\widetilde{\mathrm{Re}}\,\Sigma_{aa}u(p) = 0}_{\text{enforces } \mathcal{Z}=\mathbb{1}}$

ren. cond. for $p^2 \to m_a^2$

- **SVD inversion** ✓ induced shift Δm_i and mixing is *absorbed* by renormalization

$$m_i^0 \to m_i^{\text{phys}} + \delta m_{ij}, \qquad e_a^0 \to Z_{ab}^{\frac{1}{2}} e_b$$

- **UV divergences** ✓ remaining $\frac{1}{\epsilon_{\text{UV}}}$ poles after renormalization of the self-energies are *cancelled* by $\overline{\text{MS}}$ constants $\delta\lambda^i_{E/L}$ and $\delta\bar{\lambda}$

- **IR divergences** ✓ $m_i = m_i^{\text{phys}} \curvearrowright$ correctly treated following YFS

Lepton Masses at tree-level

SMEFT $\leftrightarrows$ qualitative understanding, *now*: precise calculation $\curvearrowright$ **full model** (mass basis)

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Example: $L \oplus E$

$$\mathcal{L} \supset -y_i \overline{l_L}_i e_{Ri} \Phi - \lambda_i^E \overline{l_L}_i E_R \Phi - \lambda_j^L \overline{L_L} e_{Rj} \Phi - \overline{L}\big(\lambda \mathsf{P}_\mathsf{R} + \bar{\lambda}^* \mathsf{P}_\mathsf{L}\big) E \Phi - M_E \overline{E_L} E_R - M_L \overline{L_L} L_R$$

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redundancy	$M_L^i \overline{l_L}_i L_R$,	$M_E^i \overline{E_R} e_{Ri}$	$\hookrightarrow$ L_L and E_R same QN as l_{Li} and e_{Ri}

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lepton mass matrix after EWSB

$$\mathcal{M}^- = \begin{array}{c} \\ \overline{e_L}_i \\ \overline{L_L^-} \\ \overline{E_L} \end{array} \begin{array}{c} \begin{matrix} e_{Rj} & E_R & L_R^- \end{matrix} \\ \begin{pmatrix} y_i v & \lambda_i^E v & 0 \\ \lambda_j^L v & \lambda v & M_L \\ 0 & M_E & \bar{\lambda} v \end{pmatrix} \end{array} \quad \Longrightarrow \quad U_L^\dagger \mathcal{M}^- U_R = \mathrm{diag}(m_i)$$

mass basis: $\hat{e}_L = U_L^\dagger (e_{Li}, L_L^-, E_R)^T \qquad \hat{e}_R = U_R^\dagger (e_{Ri}, E_R, L_R^-)^T$

On-shell Scheme: Set-up

Bare couplings: $y_i^0 \to y_i + \delta y, \qquad \lambda_i^0 \to \lambda_i + \delta\lambda_i, \qquad M_i^0 \to M_i + \delta M_i$

Field transformation: $(e_L, L_L^-, E_L)_0^T \to U_L Z_L^{\frac{1}{2}} \hat{e}_L, \qquad (e_R, E_R, L_R^-)_0^T \to U_R Z_R^{\frac{1}{2}} \hat{e}_R$

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$$\delta m = U_L^\dagger \begin{pmatrix} \delta(y_i v) & \delta(\lambda_i^E v) & 0 \\ \delta(\lambda_j^L v) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R$$

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$\hookrightarrow$ *redundancy* δm_{ab} could be absorbed by field-redefinition, but here **unpractical**:

- 5×5 matrices $U_{L,R}$ $\curvearrowright$ explicit parametrization hard to work with
- further obscures relation $\delta m \leftrightarrow \delta y_i, \delta\lambda_i, \delta M_i$

Renormalized Lepton Self-energies

Renormalization of kinetic term:

$$\overline{\hat{e}_a}(i\not{\partial} - m_a)\hat{e}_a \to \overline{\hat{e}_a}\Big[i\not{\partial}\Big(Z_R^{\frac{1}{2}\dagger} Z_R^{\frac{1}{2}} \mathsf{P_R} + Z_L^{\frac{1}{2}\dagger} Z_L^{\frac{1}{2}} \mathsf{P_L}\Big) - \Big(Z_L^{\frac{1}{2}\dagger}[m + \delta m] Z_R^{\frac{1}{2}} \mathsf{P_R} + Z_R^{\frac{1}{2}\dagger}[m + \delta m]^\dagger Z_L^{\frac{1}{2}} \mathsf{P_L}\Big)\Big]\hat{e}_a$$

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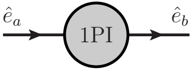
covariant decomposition:



$$\equiv i\Sigma_{ba}(p) = i\Big(\Sigma_{ba}^R(p^2)\, p\!\!\!/ \mathsf{P_R} + \Sigma_{ba}^L(p^2)\, p\!\!\!/ \mathsf{P_L} + \Sigma_{ba}^{SR}(p^2)\, \mathsf{P_R} + \Sigma_{ba}^{SL}(p^2)\, \mathsf{P_L}\Big)$$

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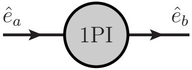

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One-loop expansion: $Z_i^{\frac{1}{2}} = \mathbb{1} + \frac{1}{2}\delta Z_i + \ldots$

$$\overline{\Sigma}^{L/R}(p^2) = \Sigma^{L/R}(p^2) + \frac{1}{2}\Big(\delta Z_{L/R} + \delta Z_{L/R}^\dagger\Big)$$

$$\overline{\Sigma}^{SR}(p^2) = \Sigma^{SR}(p^2) - \frac{1}{2}\Big(m\delta Z_R + \delta Z_L^\dagger m\Big) - \delta m$$

$$\overline{\Sigma}^{SL}(p^2) = \Sigma^{SL}(p^2) - \frac{1}{2}\Big(m\delta Z_L + \delta Z_R^\dagger m\Big) - \delta m^\dagger$$

On-shell Renormalization Constants: Lepton Sector

on-shell conditions:

$$\underbrace{\widetilde{\mathrm{Re}}\,\overline{\Sigma}_{ba}(p)u_a(p) = 0,}_{\text{removes mixing and } \Delta m} \qquad \underbrace{\frac{1}{\not{p}-m_a}\widetilde{\mathrm{Re}}\,\overline{\Sigma}_{aa}u(p) = 0}_{\text{enforces } \mathcal{Z}=\mathbb{1}}$$

$$\text{for } a=b \begin{cases} (\delta m)_{aa} = \frac{1}{2}\,\widetilde{\mathrm{Re}}\left[\Sigma^{SR}_{aa} + \Sigma^{SL*}_{aa} + m_a\Sigma^{R*}_{aa} + m_a\Sigma^{L}_{aa}\right]_{p^2=m_a^2} \\ (\delta Z_{L/R})_{aa} = -\widetilde{\mathrm{Re}}\left[\Sigma^{L/R}_{aa} + m_a\left(\Sigma^{SR\prime}_{aa} + \Sigma^{SL\prime}_{aa}\right) + m_a^2\left(\Sigma^{R\prime}_{aa} + \Sigma^{L\prime}_{aa}\right)\right]_{p^2=m_a^2} \end{cases}$$

$$\text{for } a\neq b \begin{cases} (\delta Z_R)_{ab} = \frac{2}{m_a^2-m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^R_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^L_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} \\ \qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} - m_a(\delta m)_{ab} - m_b(\delta m^\dagger)_{ab}\Big]_{p^2=m_b^2} \\ (\delta Z_L)_{ab} = \frac{2}{m_a^2-m_b^2}\Big[m_b^2\,\widetilde{\mathrm{Re}}\,\Sigma^L_{ab} + m_a m_b\,\widetilde{\mathrm{Re}}\,\Sigma^R_{ab} + m_a\,\widetilde{\mathrm{Re}}\,\Sigma^{SL}_{ab} \\ \qquad + m_b\,\widetilde{\mathrm{Re}}\,\Sigma^{SR}_{ab} - m_a(\delta m^\dagger)_{ab} - m_b(\delta m)_{ab}\Big]_{p^2=m_b^2} \end{cases} .$$

Mass matrix: tree-level vs counterterm

tree-level:

$$\begin{pmatrix} m_i & 0 & 0 \\ 0 & m_4 & 0 \\ 0 & 0 & m_5 \end{pmatrix} = U_L^\dagger \begin{pmatrix} y_i v & \lambda_i^E v & 0 \\ \lambda_j^L v & \lambda v & M_L \\ 0 & M_E & \bar{\lambda} v \end{pmatrix} U_R$$

---

ren. constant:

$$\begin{pmatrix} \delta m_i & \delta m_{i4} & \delta m_{i5} \\ \delta m_{4j} & \delta m_4 & \delta m_{45} \\ \delta m_{5j} & \delta m_{54} & \delta m_5 \end{pmatrix} = U_L^\dagger \begin{pmatrix} \delta(y_i v) & \delta(\lambda_i^E v) & 0 \\ \delta(\lambda_i^L v) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R$$

Counterterm Inversion

$$\delta m_{aa} = \left[U_L^\dagger \begin{pmatrix} \delta(y_i v) & \delta(\lambda_i^E v) & 0 \\ \delta(\lambda_i^L v) & \delta(\lambda v) & \delta M_L \\ 0 & \delta M_E & \delta(\bar{\lambda} v) \end{pmatrix} U_R \right]_{aa}$$

Counterterm Inversion

fixed by OS

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e.g. $\overline{\text{MS}}$

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$$\Longrightarrow \quad \boxed{\big(\delta(y_i v), \delta M_L, \delta M_E\big) = \boldsymbol{\kappa}^{-1}(\boldsymbol{\delta m} - \boldsymbol{\delta c})}$$

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fixed at tree-level ($U_L^\dagger$, U_R); free ren. consts. ($\delta(y_i v)$, δM_L, δM_E); e.g. $\overline{\text{MS}}$ ($\delta(\lambda_i^E v)$, $\delta(\lambda_i^L v)$, $\delta(\lambda v)$, $\delta(\bar{\lambda} v)$)

$$\Longrightarrow \qquad \boxed{\big(\delta(y_i v), \delta M_L, \delta M_E\big) = \boldsymbol{\kappa}^{-1}(\boldsymbol{\delta m} - \boldsymbol{\delta c})}$$

- m and δm of leptons with different charge are not independent
  $\hookrightarrow$ OS cond. on all leptons fixes more parameters (e.g. also λ or $\bar{\lambda}$) [KM, Stöckinger, Stöckinger-Kim '24]
- alternatively: leave masses of some leptons (e.g. doubly charged or heavy neutrinos) off-shell and compute one-loop mass shift (preferable e.g. in the triplet models)

$\overline{\text{MS}}$ Renormalization constants

starting point: β-functions from `RGBeta` package [Thomsen '21]

$$\beta(\Theta_i) = -\xi_i \delta\Theta_i^{[1]} + \sum_k \frac{\partial \delta\Theta_i^{[1]}}{\partial \Theta_k} \xi_j \Theta_k \quad \rightsquigarrow \quad 2\delta\Theta_i^{[1]} \quad \text{(@ one-loop)}$$

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tree-level structure also for renormalization constant $\curvearrowright$ field-redefinition (gauge-basis), e.g.

$$(l_{Li}, L_L) \to (\mathbb{1} + \delta V_L)(l_{Li}, L_L) \qquad \Longrightarrow \qquad \delta M^i_L \to \delta M^i_L + (\delta V_L)_{i4} M_L \stackrel{!}{=} 0$$

$\longrightarrow$ removes off-diagonal terms + RGE running preserves tree-level structure...

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$\longrightarrow$ removes off-diagonal terms + RGE running preserves tree-level structure...

...but *modifies* RGE, e.g.

$$\beta(y_i) = y_i \Big(\delta\Upsilon + 3y_i^2 + 2|\lambda_i^L|^2 + |\lambda_i^E|^2\Big) \quad - \quad 4\lambda_i^E \lambda_i^L \bar{\lambda} \frac{M_E^2 + 2M_L^2}{M_E M_L}$$

Counterterm Matrices

Yukawa couplings:
$$\delta\mathcal{Y}^- = \begin{matrix} & e_{Rj} & E_R & L_R^- \\ \bar{e}_{Li} & \delta y_i & \delta\lambda_E^i & 0 \\ \bar{L}_L^- & \delta\lambda_L^j & \delta\lambda & 0 \\ \bar{E}_L & 0 & 0 & \delta\bar{\lambda} \end{matrix} \quad \Longrightarrow \quad \delta Y^h = {U_L^-}^\dagger \delta\mathcal{Y}^- U_R^-$$

Yukawa vertex counterterm

Counterterm Matrices

Yukawa couplings:
$$\delta\mathcal{Y}^- = \begin{array}{c} \\ \bar{e}_{Li} \\ \bar{L}_L^- \\ \bar{E}_L \end{array} \begin{array}{c} \begin{array}{ccc} e_{Rj} & E_R & L_R^- \end{array} \\ \begin{pmatrix} \delta y_i & \delta\lambda_E^i & 0 \\ \delta\lambda_L^j & \delta\lambda & 0 \\ 0 & 0 & \delta\bar{\lambda} \end{pmatrix} \end{array} \quad \Longrightarrow \quad \delta Y^h = {U_L^-}^\dagger \delta\mathcal{Y}^- U_R^-$$

Vertex Renormalization

$$\mathcal{L} \supset -\overline{\hat{e}_L} Y^h \hat{e}_R \frac{h}{\sqrt{2}} \longrightarrow -\overline{\hat{e}_L} Z_L^{\frac{1}{2}\dagger}\left(Y^h + \delta Y^h\right) Z_R^{\frac{1}{2}} \hat{e}_R \frac{Z_h^{\frac{1}{2}} h}{\sqrt{2}}$$

Yukawa vertex counterterm

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one-loop expansion: $Z_i^{\frac{1}{2}} = 1 + \frac{1}{2}\delta Z_i + \ldots$

$$\Longrightarrow \boxed{\delta\Gamma_{ab}^{ct} = \delta Y_{ab}^h + \frac{1}{2}\left(\delta Z_L^\dagger Y^h + Y^h \delta Z_R\right)_{ab} + \frac{1}{2} Y_{ab}^h \delta Z_h}$$

Muon–Higgs coupling at one-loop

$$Y_{\mu\mu}^{h,\text{eff}} = Y_{\mu\mu}^{h}\big(1 - \alpha B\big) + \Gamma_{\mu\mu}^{1\ell}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma_{\mu\mu}^{ct}$$

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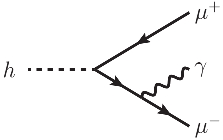
real radiation
(IR div.)



real radiation

Muon–Higgs coupling at one-loop

genuine 1ℓ diagrams
(UV + IR div.)

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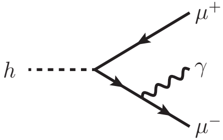

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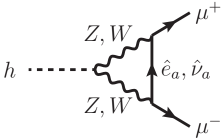

genuine one-loop

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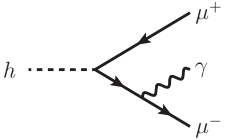
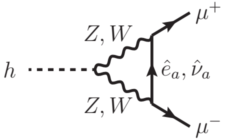
real radiation
(IR div.)

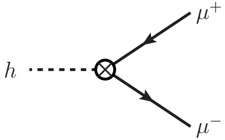
counterterm contribution
(UV + IR div.)



real radiation



genuine one-loop



counterterm

Muon–Higgs coupling at one-loop

genuine 1ℓ diagrams (UV + IR div.)

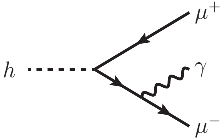
$$Y^{h,\text{eff}}_{\mu\mu} = Y^{h}_{\mu\mu}(1-\alpha B) + \Gamma^{1\ell}_{\mu\mu}(p_h^2, p_{\mu^+}^2, p_{\mu^-}^2) + \delta\Gamma^{ct}_{\mu\mu}$$

real radiation (IR div.)

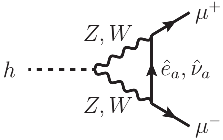
counterterm contribution (UV + IR div.)

Important checks

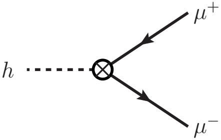
- cancellation of UV and IR divergences ✓
- cancellation of residual renormalization scale dependence ✓
- decoupling behaviour in physical observables ✓



real radiation



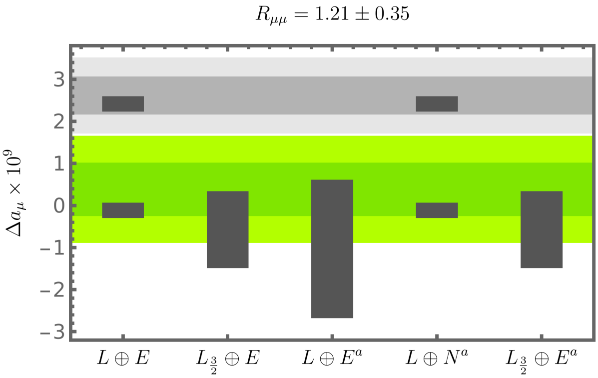
genuine one-loop



counterterm

Effect of NLO correction to $R_{\mu\mu}$

tree-level $R_{\mu\mu}$



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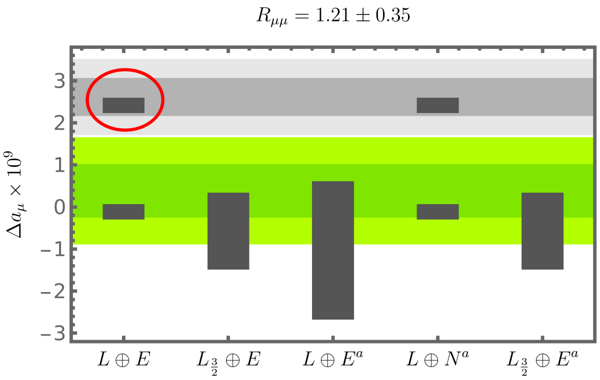

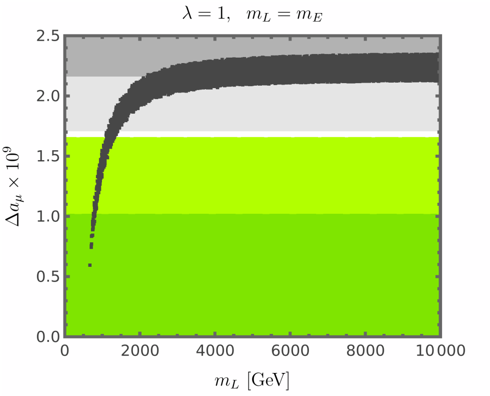

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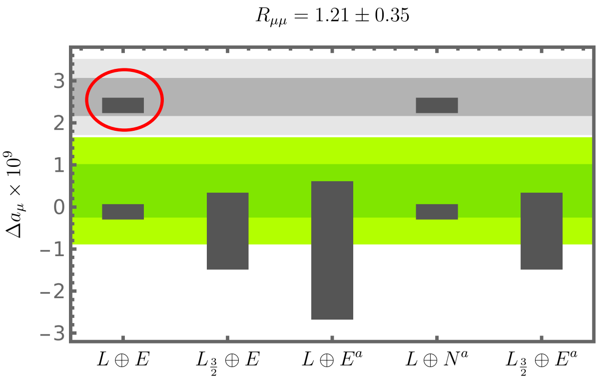
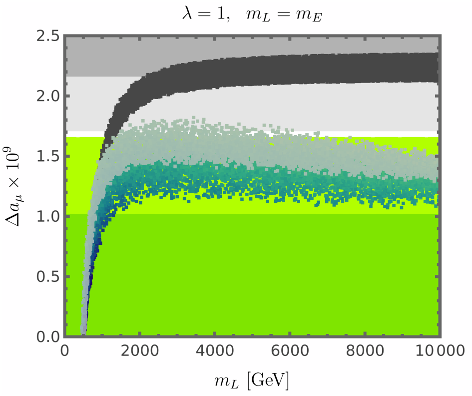
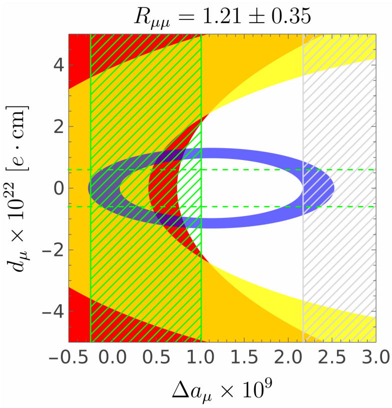


one-loop $R_{\mu\mu}$

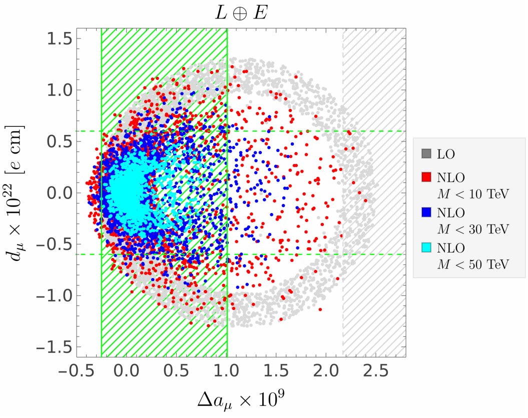


NLO correction to dipole ellipse

tree-level $R_{\mu\mu}$



one-loop $R_{\mu\mu}$



Renormalization group running

mixed OS and $\overline{\text{MS}}$ conditions $\implies$ *residual* renormalization scale dependence

$$\mu\frac{d\Theta_\alpha}{d\mu} \equiv \beta^{\text{OS}}(\Theta_\alpha) \approx \beta^{\overline{\text{MS}}}(\Theta_\alpha) - \mu\frac{d\delta^{[0]}\Theta_\alpha}{d\mu} + \mathcal{O}(2\ell)$$

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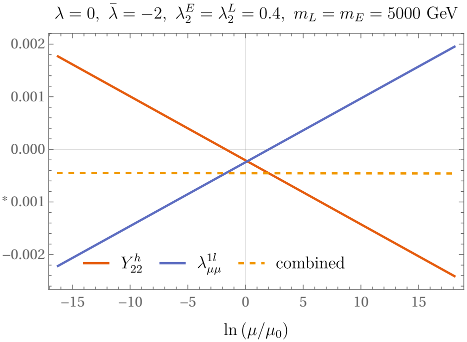
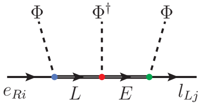
linearized running from the perturbative expression: $Y^h_{ii} \approx \frac{m_i}{v} + \frac{2v^2}{m_4 m_5}\lambda^E_i\bar{\lambda}\lambda^L_i + \mathcal{O}(v^4)$

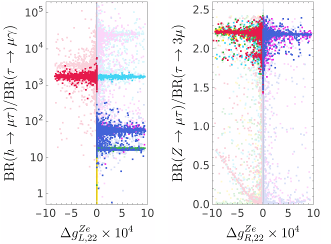
$$Y^h_{ii}(\mu) \approx Y^h_{ii}(\mu_0) + \frac{2v^2}{m_4 m_5}\left[\lambda^i_E\lambda^j_L\beta^{\text{OS}}(\bar{\lambda}) + \lambda^i_E\beta^{\text{OS}}(\lambda^j_L)\bar{\lambda} + \beta^{\text{OS}}(\lambda^i_E)\lambda^j_L\bar{\lambda}\right]\Bigg|_{\mu=\mu_0} \ln\left(\frac{\mu}{\mu_0}\right)$$

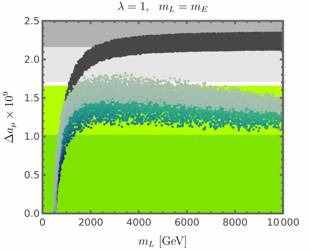


Conclusions

- VLL strongly affect **lepton–Higgs** physics
  $\hookrightarrow$ unique correlations testable in upcoming experiments
- LO study for all six models completed ✓
- precision era requires accurate (higher-order) predictions
  $\hookrightarrow$ **OS scheme** provides consistent framework
- one-loop corrections implemented ✓







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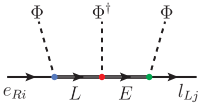
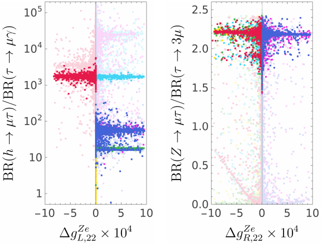
<u>Outlook</u>

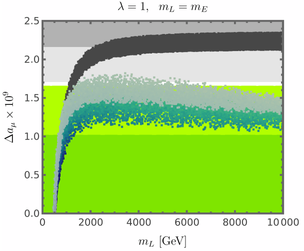
▷ NLO for many more interesting observables:
  $Z \to \ell\ell$, CLFV, EDM, trilinear Higgs (?), ...

▷ RGE improved results ⇝ SMEFT analyses

▷ theory studies: vacuum stability, different RS







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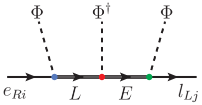

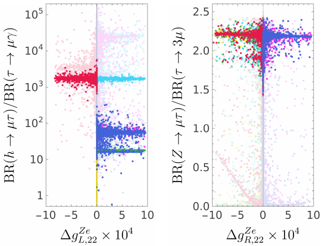

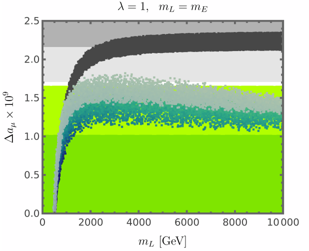

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- ▷ RGE improved results $\rightsquigarrow$ SMEFT analyses
- ▷ theory studies: vacuum stability, different RS

$\Longrightarrow$ still some work left to do!

Backup

Mass Basis: SVD Problem

PROBLEM: masses $m_{1,2,3} \equiv m_{e,\mu,\tau}$ known, but *fundamental* Yukawa couplings are not!

GOAL: For given $\{\lambda_i, M_i\}$, find y_i such that SVD yields the correct SM masses

1 Perturbative Solution expansion in small parameter: $\epsilon \sim v/M_{L/E} \ll 1$

$$\mathcal{M}^- = \mathcal{M}_0^- + \epsilon\,\mathcal{M}_1^- \quad \Longrightarrow \quad U^- = U_0^-\left(\mathbb{1} + \epsilon\,\Delta_1^- + \epsilon^2\Delta_2^- + \ldots\right)$$

↪ **approximate** results in terms of *fundamental* parameters (equivalent to SMEFT tree-level expansion)

$$m_i \approx y_i v + \frac{\lambda_E^i \lambda_L^i \bar{\lambda} v^3}{M_E M_L} + \mathcal{O}(\epsilon^2), \qquad m_4 \approx M_E + \mathcal{O}(\epsilon), \qquad m_5 \approx M_L + \mathcal{O}(\epsilon)$$

2 Numeric Solution transform to eigenvalue problem ...

$$U_L^{-\dagger}\mathcal{M}^- U_R^- = \text{diag}(m_i) \quad \text{(SVP)} \qquad \Longrightarrow \qquad U_R^{-\dagger}\mathcal{M}^{-\dagger}\mathcal{M}^- U_R^- = \text{diag}(m_i^2) \quad \text{(EVP)}$$

... and solve for roots of characteristic polynomial

$$\chi_i(y_j) = \det\left(\mathcal{M}^{-\dagger}\mathcal{M}^- - \mathbb{1} m_i^2\right) \qquad \Longrightarrow \qquad \texttt{FindRoot}[\{\chi_1, \chi_2, \chi_3\}, \{\texttt{y}_1, \texttt{y}_2, \texttt{y}_3\}]$$

↪ much **more precise** (in principle arbitrary precision), but "black box".

Mass Basis: Effective Couplings

Mass Matrix:
$$\mathscr{M}^- = \begin{array}{c} \\ \bar{e}_{Li} \\ \bar{L}_L^- \\ \bar{E}_L \end{array} \begin{array}{c} \begin{array}{ccc} e_{Rj} & E_R & L_R^- \end{array} \\ \begin{pmatrix} y^e_{ij} v & \lambda_E^i v & 0 \\ \lambda_L^j v & \lambda v & M_L \\ 0 & M_E & \bar{\lambda} v \end{pmatrix} \end{array} \implies {U_L^-}^\dagger \mathscr{M}^- U_R^- = \text{diag}(m_i)$$

Yukawa couplings:
$$\mathcal{Y}^- = \begin{array}{c} \\ \bar{e}_{Li} \\ \bar{L}_L^- \\ \bar{E}_L \end{array} \begin{array}{c} \begin{array}{ccc} e_{Rj} & E_R & L_R^- \end{array} \\ \begin{pmatrix} y^e_{ij} & \lambda_E^i & 0 \\ \lambda_L^j & \lambda & 0 \\ 0 & 0 & \bar{\lambda} \end{pmatrix} \end{array} \implies {U_L^-}^\dagger \mathcal{Y}^- U_R^- \neq \text{diag}\left(\frac{m_i}{v}\right)$$

Z couplings:
$$\mathcal{G}_R^Z = \begin{array}{c} \\ \bar{e}_{Ri} \\ \bar{E}_R \\ \bar{L}_R^- \end{array} \begin{array}{c} \begin{array}{ccc} e_{Rj} & E_R & L_R^- \end{array} \\ \begin{pmatrix} \delta_{ij} s_W^2 & 0 & 0 \\ 0 & s_W^2 & 0 \\ 0 & 0 & s_W^2 - \frac{1}{2} \end{pmatrix} \end{array} \implies {U_R^-}^\dagger \mathcal{G}_R^Z U_R^- \not\propto \mathbb{1} s_W^2$$

On-shell Renormalization Constants: Gauge Sector

$$\widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2) = 0, \qquad \lim_{p^2\to M_h^2} \frac{1}{p^2-M_h^2}\widetilde{\mathrm{Re}}\,\Sigma'_h(M_h^2) = 0, \qquad \text{and}$$

$$\widetilde{\mathrm{Re}}\,\Sigma^{\mu\nu}_{VV'}(q)\epsilon_\nu(q)\Big|_{q^2=M_{V'}^2} = 0, \qquad \lim_{q^2\to M_V^2} \frac{1}{q^2-M_V^2}\Sigma^{\mu\nu}_{VV}(q)\epsilon_\nu(q) = 0$$

The vector two-point function has the general covariant decomposition

$$-i\Sigma^{\mu\nu}_{VV'}(q) = -i\Sigma^T_{VV'}(q^2)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - i\Sigma^L_{VV'}(q^2)\frac{q^\mu q^\nu}{q^2}$$

Inserting this into the on-shell conditions gives the following renormalization constants at one-loop order

$$\begin{aligned}
\delta Z_h &= -\widetilde{\mathrm{Re}}\,\Sigma'_h(M_h^2) & \delta M_h^2 &= \widetilde{\mathrm{Re}}\,\Sigma_h(M_h^2)\\
\delta Z_{VV} &= -\widetilde{\mathrm{Re}}\,\Sigma^{T\prime}_{VV}(M_V^2), & \delta M_V^2 &= \widetilde{\mathrm{Re}}\,\Sigma^T_{VV}(M_V^2)\\
\delta Z_{AZ} &= -\frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(M_Z^2) & \delta Z_{ZA} &= \frac{2}{M_Z^2}\widetilde{\mathrm{Re}}\,\Sigma^T_{AZ}(0)
\end{aligned}$$

The charge and vev renormalization constants are given by

$$\frac{\delta e}{e} = -\frac{1}{2}\left(\delta Z_{AA} + \frac{s_W}{c_W}\delta Z_{ZA}\right), \qquad \frac{\delta v}{v} = \frac{\delta M_W^2}{2M_W^2} + \frac{c_W^2}{s_W^2}\left(\frac{\delta M_Z^2}{2M_Z^2} - \frac{\delta M_W^2}{2M_W^2}\right) - \frac{\delta e}{e}$$

Muon–Higgs coupling at one-loop

$\implies$ remaining large loop effects e.g. from corrections to δv

$$\frac{\delta v}{v}\bigg|_{\text{VLL}} \sim \frac{1}{16\pi^2}\sum_i |\lambda_i|^2 \ln\left(\frac{M}{m_h}\right)$$

$+$ enhancement from mixing

$$\lambda_{\mu\mu}^{\text{eff}}\Big|_{\delta v} \simeq C_{e\Phi}^{\mu\mu} v^2 \times 6\frac{\delta v}{v}$$

Leading correction $(L \oplus E)$

$$\frac{\lambda_{\mu\mu}^{\text{eff}}}{\lambda_{\mu\mu}^{\text{SM}}}\bigg|_{1\ell} \simeq \frac{\lambda_E^{\mu}\bar{\lambda}\lambda_L^{\mu} v^3}{16\pi^2 M^2 m_\mu}\left[\sum_i \lambda_i^2 - 12\bar{\lambda}\lambda\right] \ln\left(\frac{M}{m_h}\right)$$

Full NLO correlation?

the one-loop corrections to $\lambda_{\mu\mu}$ result in

$$\frac{\lambda_{\mu\mu}}{\lambda_{\mu\mu}^{\mathsf{SM}}}\bigg|_{@1\ell} \approx 1 - \frac{2\Delta a_\mu}{2.3\times 10^{-9}}\Big(1 + \mathcal{O}(1\ell)\Big) \qquad \ldots$$

... but since the only relevant chiral symmetry breaking comes from

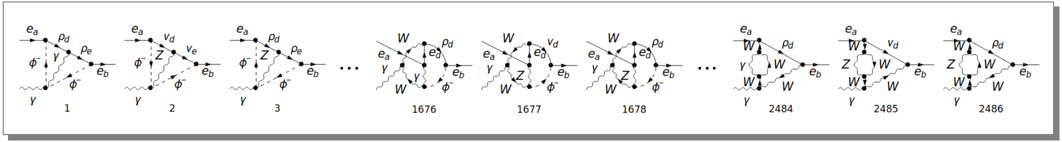
$$\frac{\lambda_E^i \lambda_L^i \bar{\lambda}}{M_E M_L}$$

so do the two-loop corrections to Δa_μ

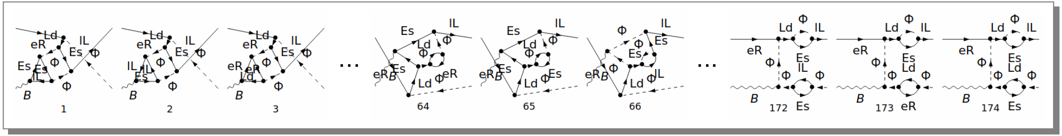
$$\Delta a_\mu\big|_{@2\ell} \sim \Delta a_\mu^{1\ell}\Big(1 + \mathcal{O}(1\ell)\Big)$$

Δa_μ at Two-loop

MASS BASIS: in principle yields *trustworthy* result from *established* calculation methods, 2486 genuine (1PI) 2-loop diagrams + $\mathcal{O}(100)$ counterterm diagrams



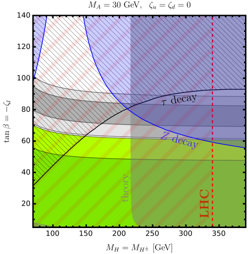
GAUGE BASIS: In analogy to SMEFT one-loop matching: calculate $l_L^i + B/W^a \to e_R^j \Phi$
only 262 genuine (1PR) 2-loop diagrams + 280 counterterm diagrams



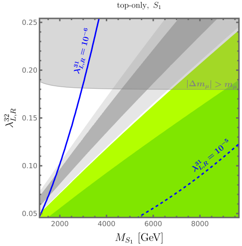
$\rightsquigarrow$ mass-insertion method, should give the leading (chirally enhanced) two-loop contribution

Consequences for new Physics?

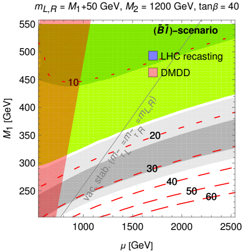
2HDM



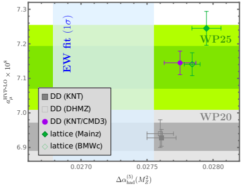
LQs



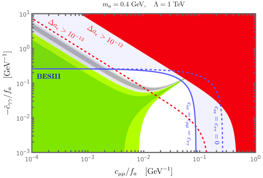
SUSY



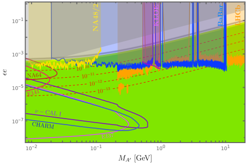
EW fit



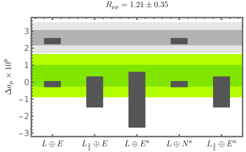
ALPs



Dark Photons



VLL



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