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Two-Loop Pole-Mass Matching in the CP-Violating NMSSM

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compute v/M_{SUSY}-dependence of
the $\mathcal{O}(\alpha_s\alpha_t)$ threshold corrections to $\lambda_{\text{SM}}^{\text{eff.}}$

15th KUTS, March 2026, KIT, Karlsruhe

What's special about the Higgs boson (mass)?

Assumption: heavy new physics particle with mass m_{heavy}.

Perturbatively calculate vector/fermion/Higgs mass:

- SM Vector bosons: $\delta m_V^2 \propto m_V^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by gauge symmetries, $m_V \to 0$

- SM Fermions: $\delta m_f \propto m_f \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  protected by chiral symmetry, $m_f \to 0$

- Higgs: $\delta m_h^2 \propto m_{\text{heavy}}^2 \log \frac{m_{\text{heavy}}^2}{\mu_{\text{ren.}}^2}$

  which symmetry protects m_h?

  $\to$ SUSY

Imposing SUSY: Higgs boson mass corrections are controlled by the SUSY-breaking scale:
$\delta m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0.$

The SM-like Neutral Higgs Boson Mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \leq \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

- MSSM: $m_h^{\text{tree}} \leq m_Z \ll 125\,\text{GeV}$ ↯

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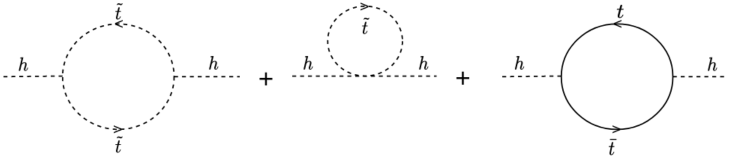

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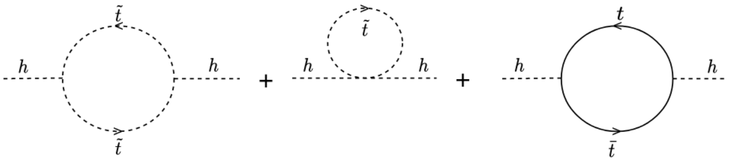

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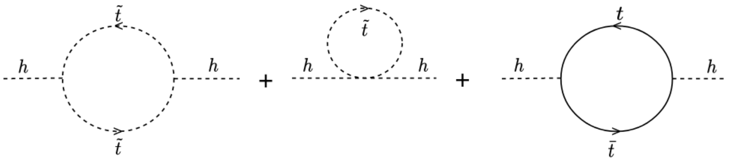

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- $M_{\tilde{t}} \approx m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{M_{\text{SUSY}} \to 0} 0$
- but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$! $\rightarrow$ higher-orders required

Determination of m_h^{SUSY}

$$\det\left[p^2 \mathbb{1} - \boldsymbol{m}_h^{2,\,(tree)} + \hat{\Sigma}_h(p^2)\right] = 0$$

Perturbative series of $\hat{\Sigma}_h$ can be organized in:

- number of loops: $(4\pi)^{-2}$
- powers of coefficients: $\mathcal{O}(\alpha_s \alpha_t)$ etc.
- powers of logs: $\log \frac{m^2_{\text{SUSY}}}{m^2_{\text{SM}}}$
- suppression by heavy scales: $\frac{m^2_{\text{SM}}}{m^2_{\text{SUSY}}}$
- (+ combinations)

If $M_{\text{SUSY}} \gg m_{\text{SM}}$, the $\log$-expansion at fixed-order might not converge well.

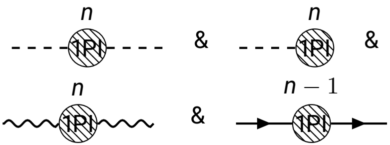
$\rightarrow$ Large-log resummation required for many SUSY scenarios.

m_h^{SUSY}: Fixed-Order Ingredients

A fixed-order n-loop result will incorporate the full logarithmic dependence $(4\pi)^{-2n}\sum_{k=0}^{n}\log^k$ (good and bad ones!) and constant $\frac{m_{\text{SM}}^2}{M_{\text{SUSY}}^2}$-terms:

$\alpha \propto (4\pi)^{-1}$

0	α^0				
1	$\alpha\log$	α			
2	$\alpha^2\log^2$	$\alpha^2\log$	α^2		
	$\vdots$	$\vdots$	$\vdots$		
n	$\alpha^n\log^n$	$\alpha^n\log^{n-1}$	$\alpha^n\log^{n-2}$	...	α^n
	LL	NLL	NNLL	...	N^{n}LL



- diagrammatic: calculate $\Sigma_{ij}^{(n)}(p^2)$ and $T_i^{(n)}$
  - computational expensive
  - mass hierarchies: expand loop-integrals
  - numerical evaluation: no access to logs
- effective potential: $\partial^2_{h_ih_j}V_{eff}^{(n)} \leftrightarrow \Sigma_{h_ih_j}^{(n)}(p^2=0)$
  - V_{eff} known up to 2- and 3-loops for general QFT [Martin, Patel, '18] [Martin, '17]
  - ∂V_{eff} numerically difficult
  - $p^2 = 0 \rightarrow$ massless particles are troublesome
- other ingredients:
  - (OS) v_{SM}: n-loop vector boson masses
  - (OS) s/top sector: $(n-1)$-loop selfenergies
  - n-loop SUSY RGEs from m_Z to M_{SUSY}

m_h^{SUSY}: EFT ingredients

fixed-order ↓

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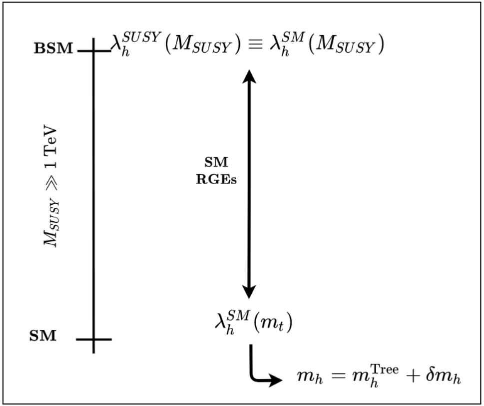
	LL	NLL	NNLL	...	N^nLL

EFT →

- resummation of all leading, next-to-leading logs ...('LL', 'NLL', ...) using one-, two-, ... loop SM RGEs
- inclusion of SUSY threshold corrections at the tree-, one-loop-, ... level (but sometimes 'n is better than $n-1$' [Braathen, Goodsell, Krauss, Opferkuch, Staub '17])

m_h^{SUSY}: EFT Ingredients

Idea: avoid large logs by separation of SUSY/SM contributions.



- SUSY: n loop threshold corrections to matching conditions
  - match 4-pt. function in unbroken phase
  - all SM particles massless, scaleless SM-loops
    $\rightarrow$ no large logs, only $\log \frac{M_{\text{SUSY}}}{Q_{\text{match}}}$
  - also: n loop thresholds to SM parameters that entered at $n-1$
- SM: $n+1$ loop RGE running down to m_t
  $\rightarrow$ resums large logs $\log \frac{M_{\text{SUSY}}}{m_t}$
- SM: n loop m_h pole-mass calculation
  $\rightarrow$ no large logs, only $\log \frac{m_{\text{SM}}}{m_t}$

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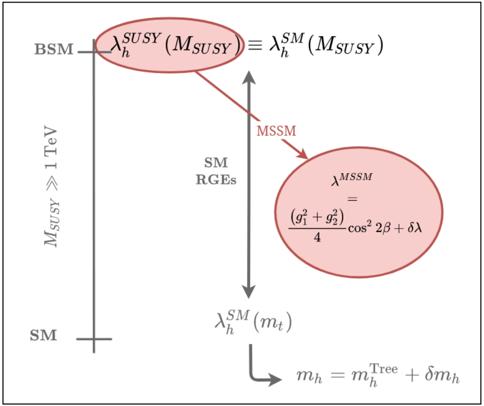

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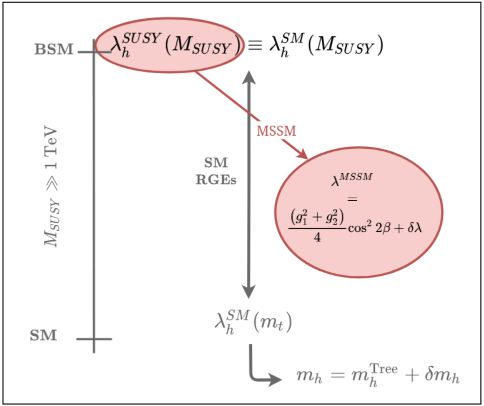

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Attention: no p^2- and no $\frac{v^2}{M^2_{\text{SUSY}}}$-dependence, no mixing contributions

Alternative EFT: Pole-Mass Matching

Idea: "recycling" fixed-order results and incorporate v^2/M^2_{SUSY}-terms [Athron, Park, Steudtner, Stöckinger, Voigt '16]

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- tree-level matching: diagonalize $\boldsymbol{m}_h^{\text{NMSSM}}$ and take SM-like eigenvalue m_h^{NMSSM}

$$(m_h^{\text{SM}})^2 = 2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} \stackrel{!}{=} (m_h^{\text{NMSSM}})^2, \;\; v^{\text{SM}} = v^{\text{NMSSM}} \Rightarrow \qquad \lambda_h^{\text{SM,II}} = \frac{(m_h^{\text{NMSSM}})^2}{2(v^{\text{NMSSM}})^2}$$

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- one-loop: ignore mixing (two-loop effect)

$$2(v^{\text{SM}})^2\,\lambda_h^{\text{SM,II}} - \hat{\Sigma}_h^{\text{SM}}((m_h^{\text{NMSSM}})^2) \overset{!}{=} (m_h^{\text{NMSSM}})^2 - \hat{\Sigma}_h^{\text{NMSSM}}((m_h^{\text{NMSSM}})^2)$$

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  - VEV-matching: $(v^{\text{SM}})^2 = (v^{\text{NMSSM}})^2 + \delta v^2$, $\frac{\delta v^2}{v^2} = \left[\hat{\Sigma}_h^{\text{NMSSM}\prime}(0) - \hat{\Sigma}_h^{\text{SM}\prime}(0)\right] + \mathcal{O}(v^2/M^2_{\text{SUSY}})$
  - (include also thresholds to gauge couplings)
  - expand self-energies: $\hat{\Sigma}_h^X\big((m_h^{\text{NMSSM}})^2\big) = \hat{\Sigma}_h^X(0) + (m_h^{\text{NMSSM}})^2\,\hat{\Sigma}_h^{X\prime}(0) + \mathcal{O}\big((m_h^{\text{NMSSM}})^4\big)$

$$\Rightarrow \Delta\lambda_h^{\text{SM (1) II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h'\right],\ \Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$$

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- two-loop: today (later)

Caveats

$$\Delta\lambda_h^{\text{SM}\,(1)\,\text{II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h'\right], \quad \Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$$

- there are large logs in both self-energies, Σ^{NMSSM} and Σ^{SM}!
- proper cancellation of large logs in ΔX requires expansion of self-energies around $p^2 \approx v^2$ and a parametrisation in terms of input-parameters that **does not** induce higher-orders (avoid momentum iteration + consistent EFT/UV parametrisation)
- two-loops: also large numerical cancellations among (N)MSSM topologies
- possible check: perform $v \to 0$ limit and compare with $\lambda_h^{\text{SM}\,(1)\,\text{IV}}$

Intermediate Summary: The Three Frontiers in Higgs Mass Calculations

(Apart from increasing loops)

Fixed Order (FO)

- Weak/TeV-scale SUSY
- $m_{SUSY} \lesssim 1-2\,\mathrm{TeV}$
- calculate full $\delta m_h^{\mathrm{SUSY}}(m_{\mathrm{SUSY}})$
- full $\frac{m_{\mathrm{SM}}}{M_{\mathrm{SUSY}}}$ dependence

RGE Improved (EFT)

- High/Split-scale SUSY
- $m_{\mathrm{SUSY}} \gtrsim 1-2\,\mathrm{TeV}$
- matching, calculate $\delta\lambda_h^{\mathrm{SUSY}}$
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Ren. Conditions*

- $\overline{\mathrm{DR}}$: minimal subtraction
- OS: Express result through physical quantities

$$m_h(m_t^{\overline{DR}}) \leftrightarrow m_h(m_t^{OS})$$
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Also "hybrid approaches" exist: combine FO and EFT results

- using pole mass matching [Athron, Park, Steudtner, Stöckinger, Voigt, '16] [Porod, Staub, '17]:

$$\Delta\lambda_h^{\text{SM}\,(1)\,\text{II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h'\right], \quad \Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$$

- combine $m^2_{\text{SM}}/m^2_{\text{SUSY}}$ from FO with resummed EFT results [Hahn, Heinemeyer, Hollik, Rzehak, Weiglein, '13] [Bahl, Hollik, '16] [Harlander, Klappert, Voigt, '19]

$$m_h^2 \equiv \left(m_h^{\text{FO}}\right)^2 - \left(m_h^{\text{FO, large-logs}}\right)^2 + \left(m_h^{\text{EFT-resummed}}\right)^2$$

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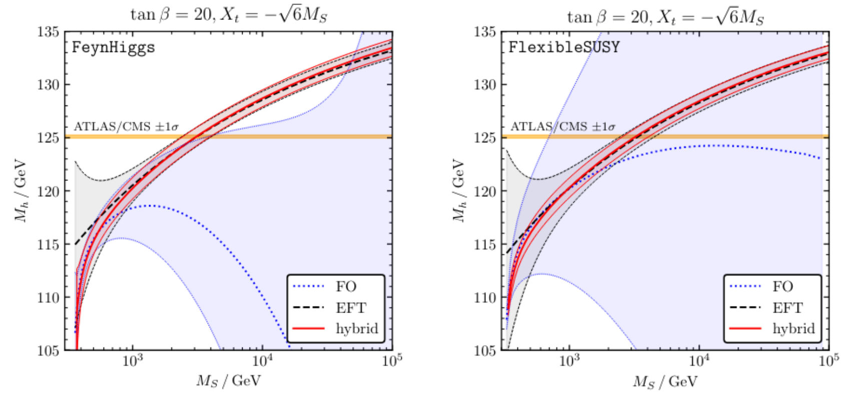

[Slavich et al. '21]

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$\Delta\lambda_h^{\rm SM}$

- combine $m^2_{\rm SM}/m^2_{\rm SUSY}$ fro
[Harlander, Klappert, Voigt, '19]

$$m_h^2 \equiv \left(m_h^{\rm FO}\right)^2 - \left(m_h^{\text{FO, large-logs}}\right)^2 + \left(m_h^{\text{EFT-resummed}}\right)^2$$

Now: This but in the NMSSM

The Complex Next-to-Minimal Supersymmetric Standard Model

- Singlet extension of the MSSM.
- Theoretically well-motivated (solves μ- and little-hierarchy-problem).
- Rich phenomenology in the Higgs boson sector:

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \quad S = \frac{e^{i\varphi_s}}{\sqrt{2}} (v_S + h_s + i a_s)$$

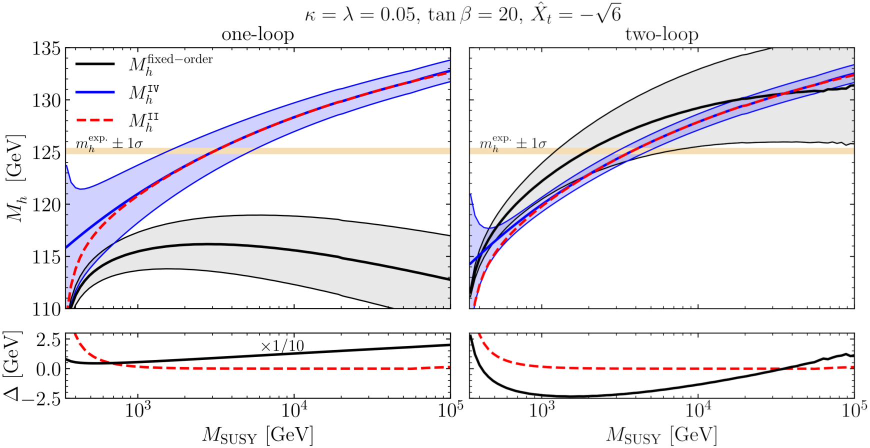
mix to

$$h_1, h_2, h_3, h_4, h_5, G^0 \text{ (mass ordered) and } h^{\pm}, G^{\pm}$$

- LHC measurements: h_1, h_2 **or** h_2 play the role of the Higgs boson h measured at LHC ($h_{1/2/3}$ is "SM-like"). MSSM: no CPV at tree-level and always $h_1 = h$.
- Two important new parameters: λ and κ (singlet-doublet and singlet-singlet couplings)

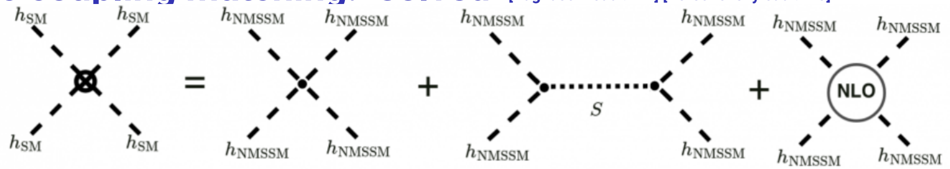
Now: This but with NMSSMCALC

MSSM-limit: agreement with MSSM codes within theory uncertainties:



- **fixed-order**: large uncertainties for large M_{SUSY}
- **EFT**: resummation, but misses $M_{\text{SM}}/M_{\text{SUSY}}$-dependence
- **Hybrid**: best of both worlds

Quartic coupling matching: 'solved' [Bagnaschi et al. '22] [Borschensky et al. '25]



$$\lambda_h^{\rm II,tree} = \underbrace{\frac{1}{8}(g_1^2+g_2^2)\cos^2 2\beta}_{\text{MSSM } D\text{-terms}} + \underbrace{\frac{1}{4}|\lambda|\sin^2 2\beta}_{\text{NMSSM } F\text{-terms}}$$
$$\underbrace{- \frac{1}{48|\kappa|^2 M_s^2(3M_s^2+M_{a_s}^2)}\Big(3|\kappa|^2 M_{H\pm}^2 - 3|\kappa|^2 M_{H\pm}^2\cos 4\beta + (3M_s^2+M_{a_s}^2)\left(|\kappa||\lambda|\cos\varphi_y \sin 2\beta - 2|\lambda|^2\right)\Big)^2}_{s/t/u\text{-channel } s}$$
$$\underbrace{- \frac{3}{16M_{a_s}^2}|\lambda|^2(3M_s^2+M_{a_s}^2)\sin^2 2\beta \sin^2\varphi_y}_{s/t/u\text{-channel } a_s} + \delta^{(1)}\lambda_h^{\rm IV} + \delta^{(2)}\lambda_h^{\rm IV}$$

- M_s, M_{a_s}: singlet masses
- $M_{H\pm}$: mass of the heavy doublet
- φ_y: CP-violating phase
- $\delta^{(1)}\lambda_h^{\rm IV}$: with SARAH*
- $\delta^{(2)}\lambda_h^{\rm IV}$:
  - MSSM-QCD [Vega, Villadoro]
  - MSSM mixed-QCD-EW [Bagnaschi, Degrassi, Paßehr, Slavich '19]
  - and leading-NMSSM-QCD [Bagnaschi, Goodsell, Slavich '22]

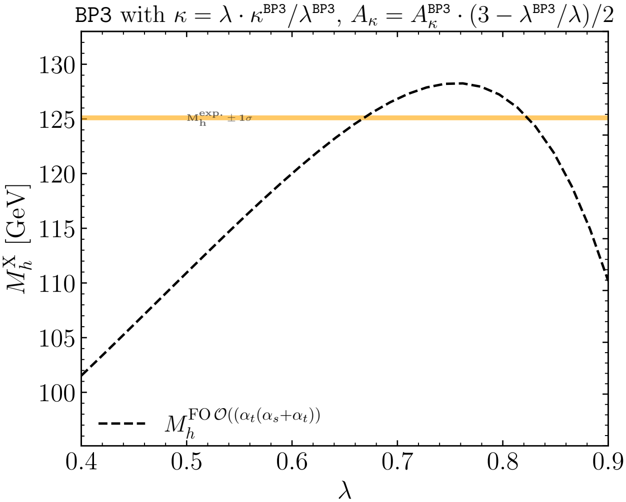
Pole-Mass Matching

- reminder:
  $$\Delta\lambda_h^{\text{SM}\,(1)\,\text{II}} = -\frac{1}{2(v^{\text{NMSSM}})^2}\left[\Delta\hat{\Sigma}_h + 2(m_h^{\text{NMSSM}})^2\Delta\hat{\Sigma}_h'\right], \quad \Delta X \equiv X^{\text{NMSSM}} - X^{\text{SM}}$$
- all self-energies already contained in `NMSSMCALC`!
  - full one-loop
  - two-loop QCD
  - two-loop α_t^2
  - two-loop $(\alpha_\lambda + \alpha_\kappa + \alpha_t)^2$
  - including CP-violation
- $\rightarrow$ need only SM h self-energy (trivial...)
- (+ minor trivialities regarding $\overline{\text{DR}} \rightarrow \overline{\text{MS}}$ shifts)

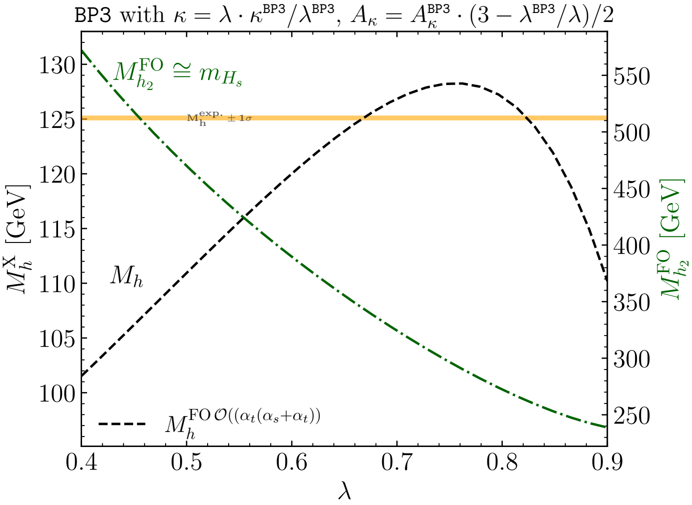
extremely convenient and even get v^2/M_{SUSY}^2-corrections for free!

- one-loop: available in `NMSSMCALC` and `FlexibleEFTHiggs`
- two- and three-loop: in the MSSM-limit [Kwasnitzaa, Stöckinger, Voigt, Wünsche '25]

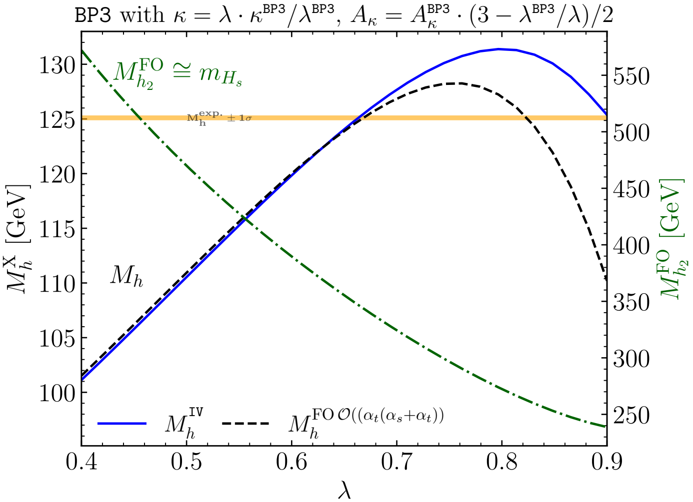
Size of v/M_{SUSY}: One-Loop



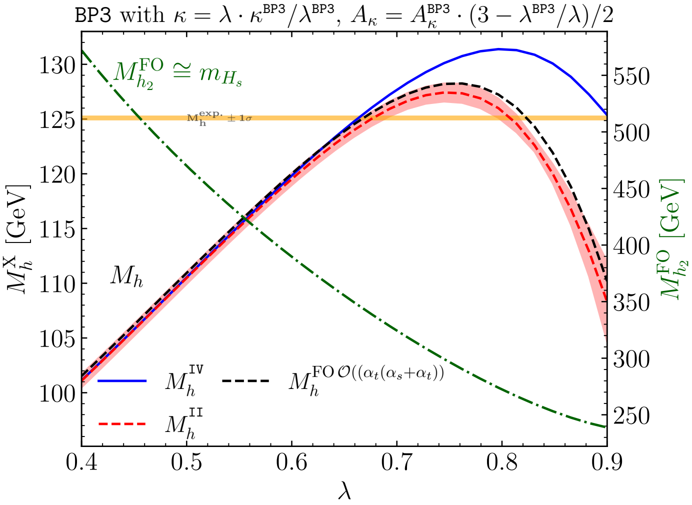
Size of v/M_{SUSY}: One-Loop



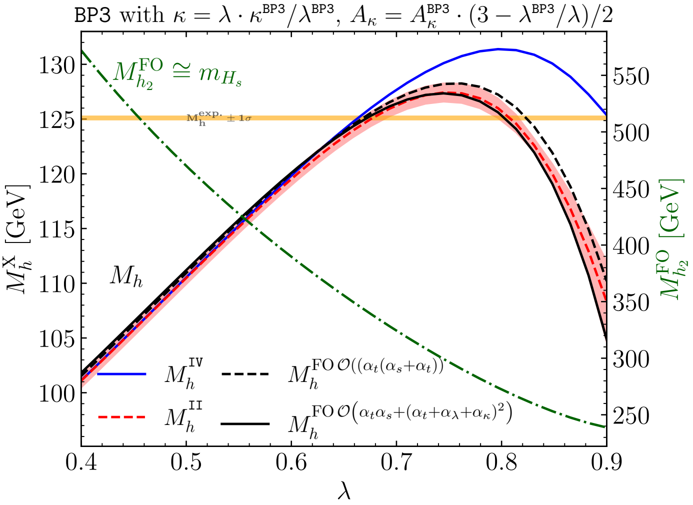
Size of v/M_{SUSY}: One-Loop



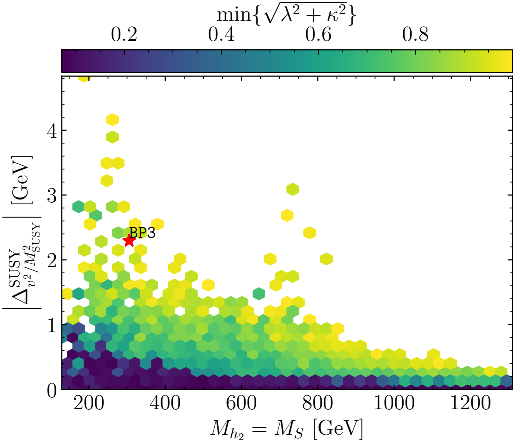
Size of v/M_{SUSY}: One-Loop



Size of v/M_{SUSY}: One-Loop



Size of v/M_{SUSY}: Scatterplot



$\rightarrow$ large effects for large NMSSM-specific couplings and/or light(ish) singlet
$\rightarrow$ behaviour at two-loop?

What is not (yet) known?

- threshold corrections for various non-SM EFT towers (singlet+THDM, ...)
- NMSSM-specific corrections beyond leading-QCD (only known for fixed-order)
- $\mathcal{O}(\frac{v^2}{M^2_{\text{SUSY}}})$-terms beyond one-loop and MSSM-limit $\leftarrow$ **this work**

Pole-Mass Matching: Two-Loops

Difference in the matching condition: one-loop-squared terms from mixing and from momentum iteration:

$$(m_h^{\text{SM}})^2 - \hat{\Sigma}_h^{(1)\,\text{SM}} - \hat{\Sigma}_h^{(2)\,\text{SM}} + \hat{\Sigma}_h^{(1)\,\text{SM}} \partial_{p^2} \hat{\Sigma}_h^{(1)\,\text{SM}}$$

$$\overset{!}{=}$$

$$(m_h^{\text{NMSSM}})^2 - \hat{\Sigma}_{11}^{(1)\,\text{NMSSM}} - \hat{\Sigma}_{11}^{(2)\,\text{NMSSM}} + \hat{\Sigma}_{11}^{(1)\,\text{NMSSM}} \partial_{p^2} \hat{\Sigma}_{11}^{(1)\,\text{NMSSM}} + \sum_{j=2}^{5} \frac{\hat{\Sigma}_{1j}^{(1)\,\text{NMSSM}} \hat{\Sigma}_{j1}^{(1)\,\text{NMSSM}}}{((m_h^{\text{NMSSM}})^2 - (m_j^{\text{NMSSM}})^2)}$$

Pole-Mass Matching: Two-Loops leading QCD

$\Rightarrow$ no one-loop(-squared) contributions:

$$(m_h^{\text{SM}})^2 - \hat{\Sigma}_h^{(1)\,\text{SM}} - \hat{\Sigma}_h^{(2)\,\text{SM}} \stackrel{!}{=} (m_h^{\text{NMSSM}})^2 - \hat{\Sigma}_{11}^{(1)\,\text{NMSSM}} - \hat{\Sigma}_{11}^{(2)\,\text{NMSSM}}$$

same structure as the one-loop case. However,

- shift from replacing the top-quark Yukawa coupling Y_t^{NMSSM} in Σ^{NMSSM} with Y_t^{SM}
- as usual for the NMSSM, even in the gaugeless limit, m_h^{tree} is non-zero (due to λ and κ)
- expansion in small momentum:
  $\Sigma^{(2)\,\text{NMSSM}}(m_h^2) = \Sigma^{(2)\,\text{NMSSM}}(0) + (m_h^{\text{tree}})^2 \partial_{p^2} \Sigma^{(2)\,\text{NMSSM}}(0)$
  $\rightarrow$ need derivatives including the full v-dependence

Derivatives of the Two-Loop Selfenergies

- UV-parts known for long

$$\delta^{(2)} Z_{H_u} = -\frac{g_3^2 Y_t^2}{32\pi^4 \epsilon^2}(1-\epsilon)$$

$$\delta^{(2)} Z_{H_d} = \delta^{(2)} Z_{H_s} = 0$$ [Mühlleitner, Dao , Rzehak, Walz '14]

- finite parts: known only in the MSSM-limit 1908.01670 and also used in the pure-EFT NMSSM result for λ^{IV} in 2206.04618 [Slavich et al.]
- NMSSM (leading QCD diagrams): differ from MSSM by the mixing of Higgses on the external legs
  $\rightarrow$ these are v/M_{SUSY} effects not captured in the pure-EFT calculation
- also full v-dependence of stop-masses in the loops
- in practice: `FeynArts`+`FeynCalc`+`TARCER` + S.P.Martin loop-integral basis [hep-ph/0307101]
- had to work-out some special cases for all derivatives
- full agreement for UV-divergent part; UV-finite part cross-checked with two independent calculations

Current status

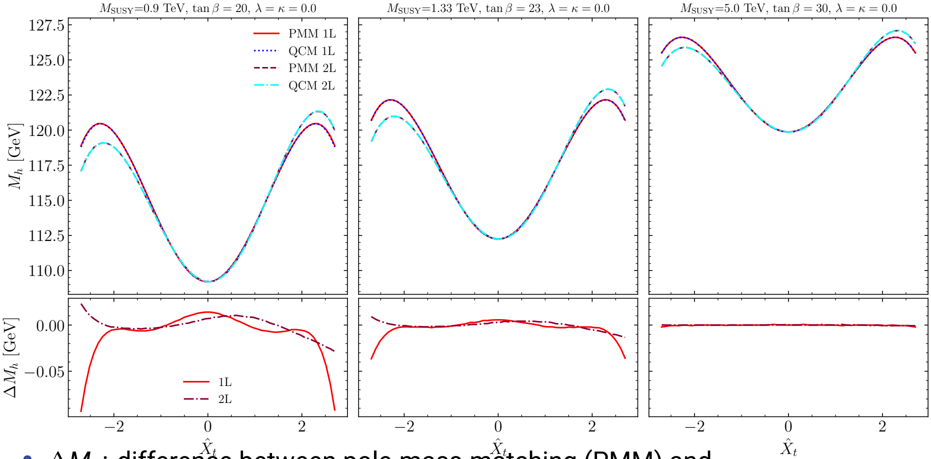
Stuck in comparison mode

Current status

Stuck in comparison mode

- observe large cancellations among (N)MSSM topologies
- re-organize selfenergies or maybe need quadruple precision
  $\rightarrow$ no easy ‘recycling’ of inbuilt selfenergies?
- analytic proof-of-concept calculation in MSSM-limit works (see next slides)

Preliminary results: 'MSSM-limit', $\lambda = \kappa = 0$



- ΔM_h: difference between pole-mass-matching (PMM) and quartic-coupling-matching (QCM) result
- simple check: $\Delta M_h \sim \mathcal{O}(\frac{v^2}{M^2_{\mathrm{SUSY}}}) \xrightarrow{M_{\mathrm{SUSY}} \gg v} 0$

Pole-Mass Matching in the Limit $v \to 0$

- Assuming the case of $M_3 = m_{\tilde{Q}_3} = m_{\tilde{t}_R} = M_{\text{SUSY}}$ and 'MSSM-limit' $(\lambda, \kappa \to 0)$:

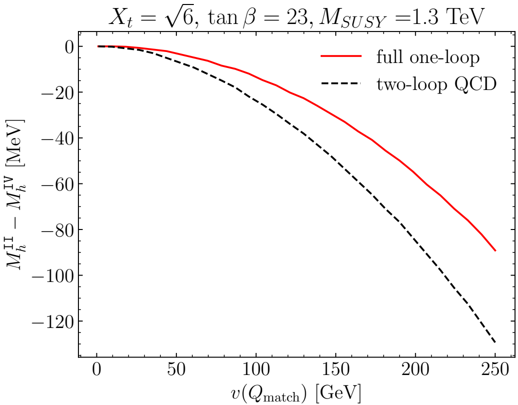
$$\delta^{(2)}\lambda = \frac{\Sigma^{(2)\,\text{NMSSM}\,\overline{\text{DR}}} - \Sigma^{(2)\,\text{SM}\,\overline{\text{MS}}} + \delta^{(1)}\, Y_t^{\overline{\text{MS}}\text{-}\overline{\text{DR}}} \frac{\partial}{\partial Y_t} \Sigma^{(1)\,\text{NMSSM}\,\overline{\text{DR}}}}{v^2}$$

$$\xrightarrow{v \to 0} \frac{g_3^2 Y_t^4}{96\,\pi^4} \left[-12\hat{X}_t - 6\hat{X}_t^2 + 14\hat{X}_t^3 + \frac{1}{2}\hat{X}_t^4 - \hat{X}_t^5\right] + \mathcal{O}\left(\frac{v^2}{M_{\text{SUSY}}^2}\right)$$

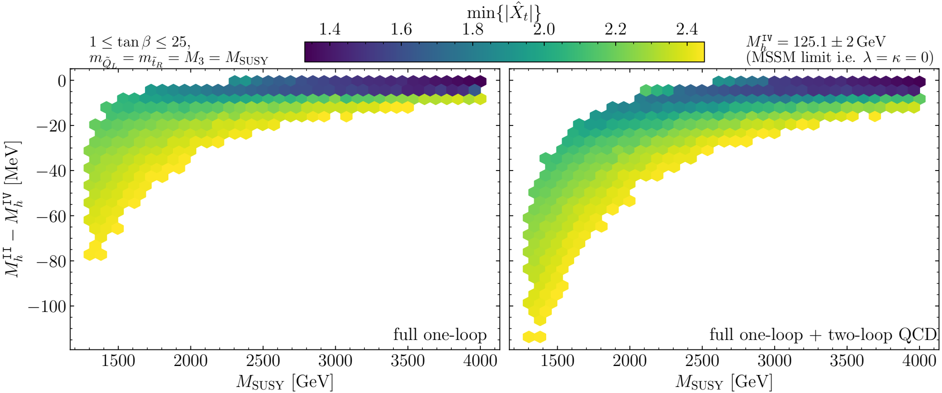
- Perfect agreement with the quartic-coupling-matching result [Bagnaschia, Giudice, Slavich, Strumia '14] once $v \to 0$ limit is taken analytically.
- full dependence on $\mathcal{O}\left(\frac{v^2}{M_{\text{SUSY}}^2}\right)$ terms (no expansion whatsoever)

Pole-Mass Matching in the limit $v \to 0$: numerically

Idea: manually fix $v(Q_{\text{match}})$ to a given value.



Size of v/M_{SUSY}: mass-degenerate and MSSM-limit



(keep in mind: only simple mass-degenerate scenario in MSSM-limit)

To-Do List

- cure numerical instabilities (larger numerical cancellations); internal cross-checks
- cross-checks with results from quartic-coupling matching for non-zero λ, κ
- investigate v/M_{SUSY} with not-too-heavy singlet

[product placement]:

NMSSMScanner

A scanning tool for the NMSSM (see also talk by Felix Egle @ KUTS14)

[Rafael Boto, Nhung Dao, Felix Egle, Karim Elyaouti, Martin Gabelmann, Margarete Mühlleitner, Johann Plotnikov]

Recent NMSSMCALCulations

Recently:

- Higgs mass predictions at low [2106.06990] and high scales [2406.17635].
- W-boson mass prediction [2308.04059]
- Trilinear Higgs couplings: $\lambda_{h_i h_j h_k}$ at two-loops [2210.02104], used in:
  - Higgs-to-Higgs decays (extension of HDECAY [Spira et al.]),
  - Di-Higgs production (extension of HPAIR [Spira et al.]).
- Lepton AMMs ,$(g-2)_l$, and EDMs, d_l [2207.12618].

Even more recently:

- All SUSY two-(three-)body decays at NLO (LO) [Egle et al.] (SDECAY [Djouadi et al.] extension)
- Dark Matter relic density and direct detection limits using RelExt [Capucha et al.]
- In-house code for electroweakino cross-sections for $pp \to \tilde{\chi}_i \tilde{\chi}_j$

Annoying To-Do: combine/link to other codes, setup a scan
$\to$ solved by NMSSMScanner

The Quest for Valid Parameter Points

fundamental input parameters

observables

$$W = \lambda \hat{S}\hat{H}_u\hat{H}_d + Y_u\hat{Q}_u\hat{H}_u\hat{u} + \ldots$$

+ SUSY breaking

prediction

inversion

analytic impossible

masses: $m_{h_1}, \ldots, m_{h_5}, m_{H^\pm}$
$m_{\chi^0_{1,\ldots,5}}, m_{\chi^\pm_{1,2}},$
$m_{\tilde{t}_{1,2}}, \ldots$

decay rates and cross-sections

dark matter

...

Need to perform *some* scan over fundamental input parameters.

The quest for specific benchmarks

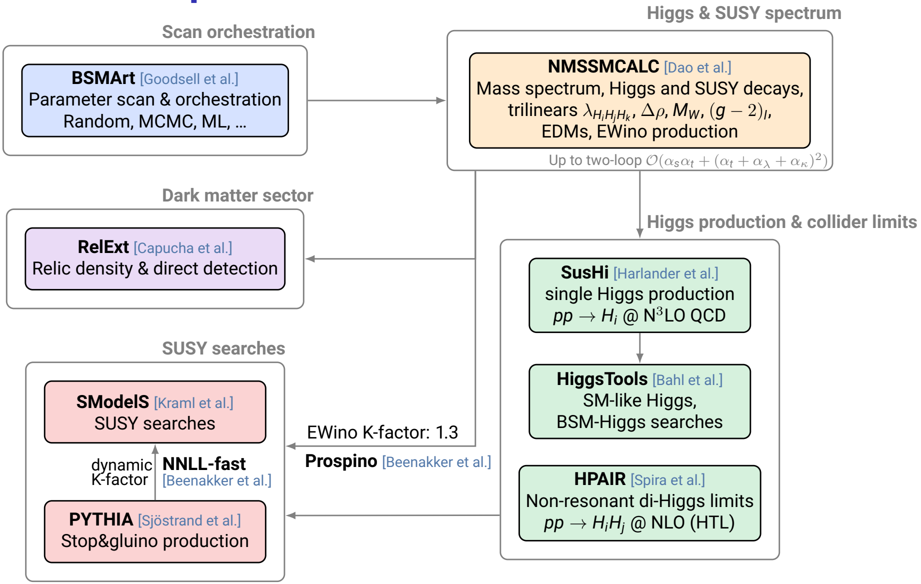
Benchmark searches add boundary conditions in addition to established experimental constraints, e.g. require:

- specific patterns in BSM masses
- specific patterns in $\mathtt{BR}$'s, $\sigma_{\text{prod.}}$'s and/or their combination

This work: develop tool that addresses all three aspects:

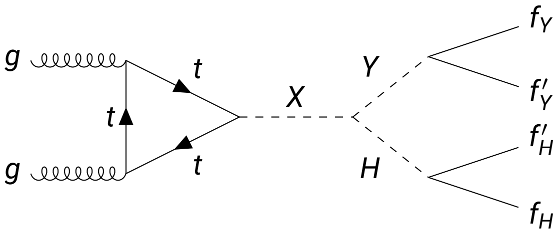
- combination of predictions for the NMSSM within the same theory framework (by means of order in perturbation theory and used renormalisation schemes)
- provide a multitude of different scanning techniques
- possibility to address specific benchmark cases

General scan setup and tool-chain: `NMSSMScanner`



Application: Find Benchmarks for $pp \to X \to YH$

X, Y being scalars or pseudo scalars, H SM-like



For given mass intervals $[m_X^{\text{min}}, m_X^{\text{max}}]/[m_Y^{\text{min}}, m_Y^{\text{max}}]$ and given decay channels for Y and H, what is the maximum cross-section achievable in the NMSSM?

- Need to find valid parameter points with additional boundary condition
- see previous efforts [Ellwanger et al.] to define HH benchmarks
  Strategy:
  - broad random scan
  - discretise sample on pre-defined mass-grid
  - start MCMC scan within each mass-cell, optimize $\sigma_{pp\to X} \times \mathrm{BR}_{X\to YH} \times \mathrm{BR}_{X\to ff'}$

Summary & Outlook

M_h calculation:

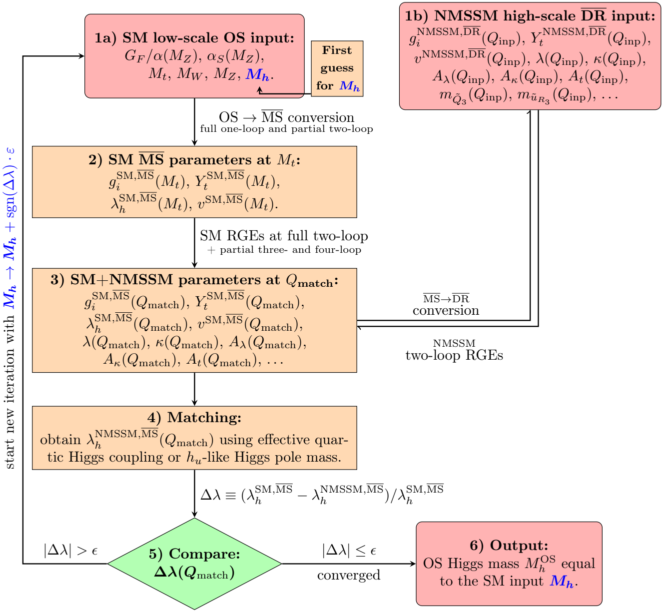
- full one-loop, leading QCD on its way
- inclusion of CPV, v/M_{SUSY} and momentum-dependent effect in hybrid-mode
- also new: top-Yukawa thresholds including v/M_{SUSY}

`NMSSMCScanner`:

- `NMSSMCALC`: *consistently* combine many *precision* predictions for the NMSSM
- input for collider, dark matter, flavour, ..., processes
- `BSMArt`: non-trivial scan setups, easy to extend

Backup

The algorithm



Threholds to gauge-couplings in pole-mass matching

In order to include the gauge shifts consistently at the one-loop order, we expand the tree-level mass in $\delta g_i = \delta g_i^{\text{reg}} + \delta g_i^{\text{thr}}$ $(i = 1, 2)$ to first order

$$\begin{aligned}\left(m_h^{\text{NMSSM}}\right)^2 &\equiv \left(m_h^{\text{NMSSM}}(g_i^{\text{NMSSM},\overline{\text{DR}}} \to g_i^{\text{SM},\overline{\text{MS}}} + \delta g_i)\right)^2 \\ &= \left(m_h^{\text{NMSSM,tree}}(g_i^{\text{SM},\overline{\text{MS}}})\right)^2 + \delta^{\text{gauge}} m_h^2 + \mathcal{O}\left((\delta g_i)^2\right)\end{aligned} \tag{3.33}$$

The pole-mass matching involves a rotation into the mass basis,

$$\delta^{\text{gauge}} m_h^2 = \left(\mathcal{R}^H(v)\,\delta^{\text{gauge}}\boldsymbol{M}_H \mathcal{R}^{H^T}(v)\right)_{11}\,, \tag{3.34}$$

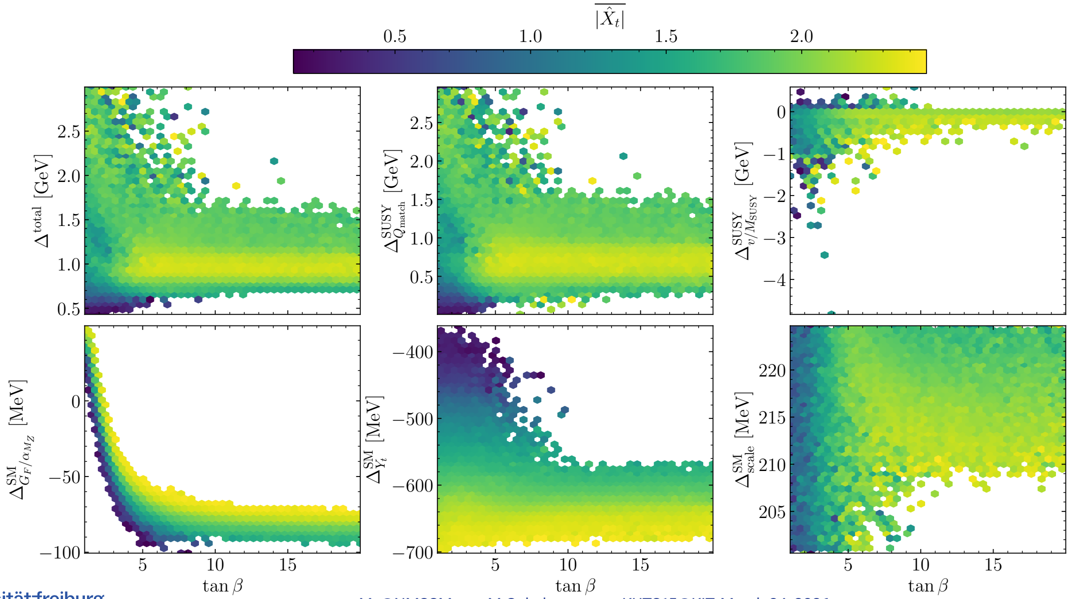
where $\mathcal{R}^H(v)$ are the rotation matrices that diagonalise the squared neutral Higgs mass matrix, $\mathcal{R}^H(v)\boldsymbol{M}_H\mathcal{R}^{H^T}(v) = (m_{h_i}^{\text{NMSSM}})^2\delta_{ij}$, in the broken phase (*i.e.* not as in Eq. (2.10) but for the case of non-zero v) and

$$\delta^{\text{gauge}}\boldsymbol{M}_H = \sum_{i=1,2}\left(\delta g_i^{\text{thr}} + \delta g_i^{\text{reg}}\right)\frac{\partial}{\partial g_i^{\text{NMSSM},\overline{\text{DR}}}}\boldsymbol{M}_H\Bigg|_{g_i^{\text{NMSSM},\overline{\text{DR}}}\to g_i^{\text{SM},\overline{\text{MS}}}}\,. \tag{3.35}$$

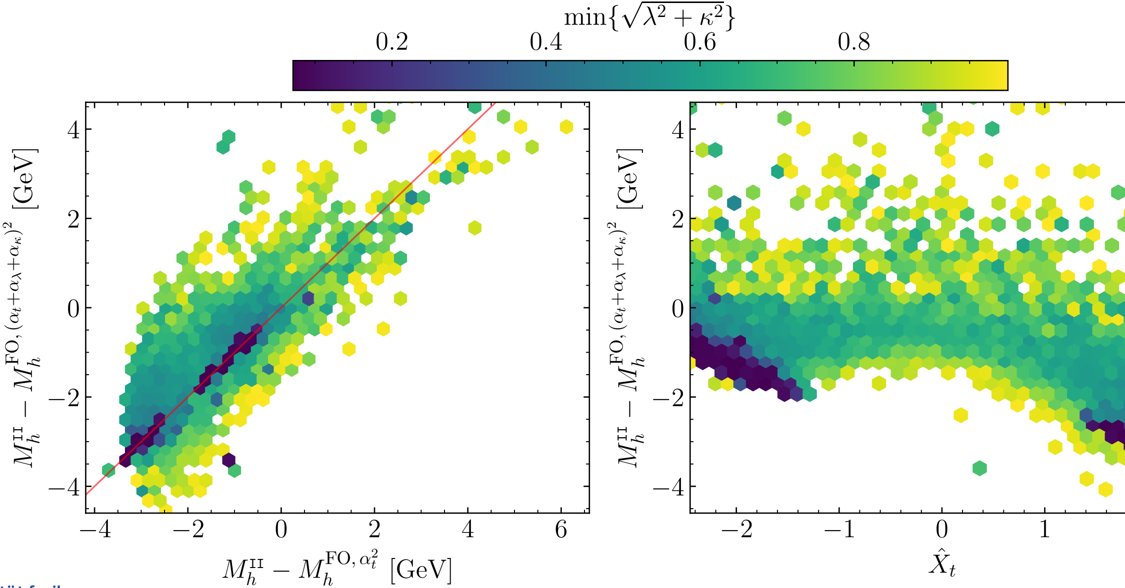
Uncertainty estimate

- SM uncertainty:
  - scale uncertainty: $0 \overset{!}{=} p^2 - 2\lambda^2(Q_{EW})v^2(Q_{EW}) - \mathrm{Re}\Sigma_h^{SM,\overline{MS}}(p^2, Q_{EW})|_{\mathrm{UV-fin}}$
    $\rightarrow \Delta^{SM}_{Q_{\mathrm{EW}}} = \max\{|M_h^{\mathrm{OS}} - M_h^{\overline{\mathrm{MS}},\mathrm{pole}}(2M_t)|, |M_h^{\mathrm{OS}} - M_h^{\overline{\mathrm{MS}},\mathrm{pole}}(M_t/2)|\}$
  - missing gauge corr. $\Delta^{SM}_{G_F/\alpha_{M_Z}} = |M_h^{G_F} - M_h^{\alpha_{M_Z}}|$
  - missing top corr. $\Delta^{SM}_{Y_t} = M_h(Y_t^{\mathcal{O}(\alpha_s^2)}) - M_h(Y_t^{\mathcal{O}(\alpha_s^3)})$
- SUSY uncertainty:
  - scale-uncertainty: $\Delta^{\mathrm{SUSY}}_{Q_{\mathrm{match}}} = \max\{|M_h^{M_{\mathrm{SUSY}}/2} - M_h^{M_{\mathrm{SUSY}}}|, |M_h^{2M_{\mathrm{SUSY}}} - M_h^{M_{\mathrm{SUSY}}}|\}$
  - for the quartic-coupling matching: $\Delta M_h^{\mathrm{IV}} = \left[(\Delta M_h^{\mathrm{II}})^2 + \underbrace{(M_h^{\mathrm{II}} - M_h^{\mathrm{IV}})}_{\Delta^{\mathrm{SUSY}}_{v/M_{\mathrm{SUSY}}}}{}^2 \right]^{\frac{1}{2}}$
- total uncertainty: $\Delta M_h^{\mathrm{II}} = \left[\left(\Delta^{SM}_{G_F/\alpha_{M_Z}}\right)^2 + \left(\Delta^{SM}_{Q_{\mathrm{EW}}}\right)^2 + \left(\Delta^{SM}_{Y_t}\right)^2 + \left(\Delta^{\mathrm{SUSY}}_{Q_{\mathrm{match}}}\right)^2 \right]^{\frac{1}{2}}$

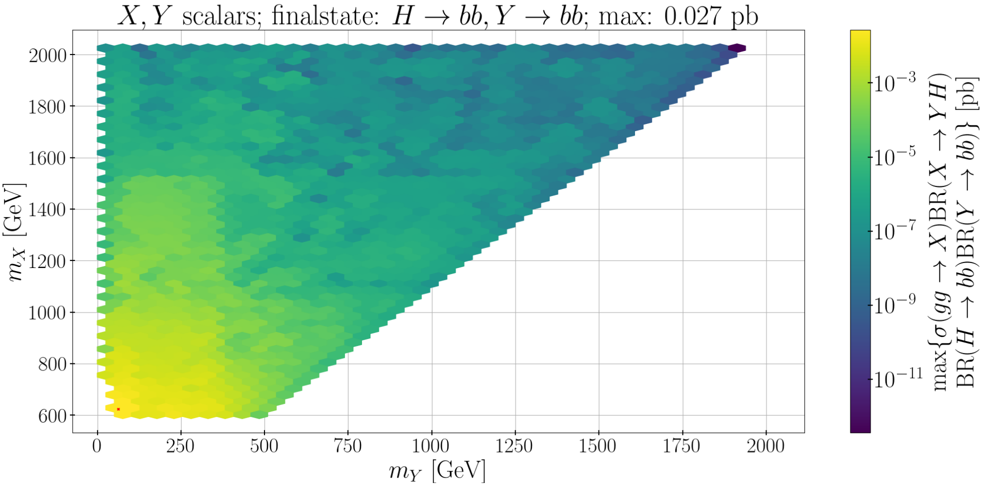
Uncertainty Budget



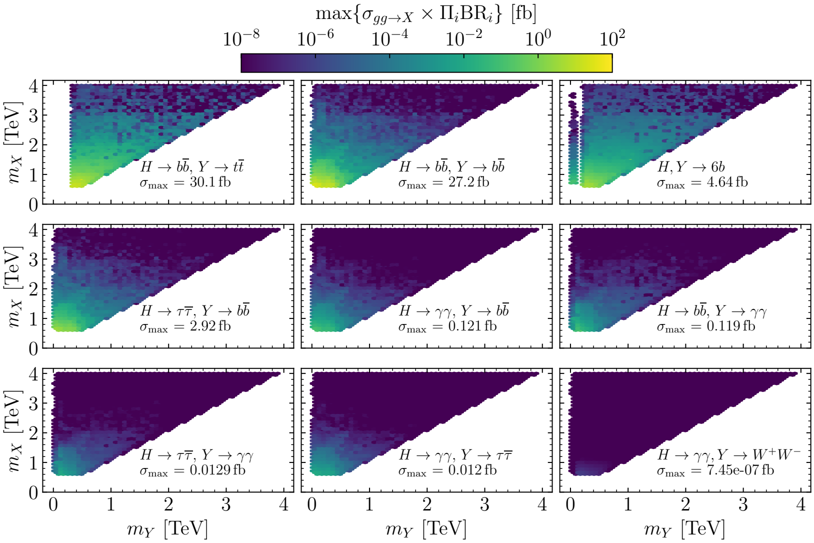
Fixed-order / EFT comparison



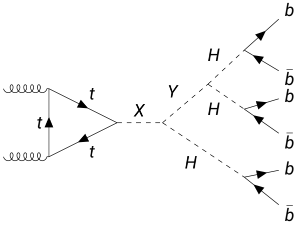
Example: find benchmarks for $pp \to H_3 \to H_2 H_1, H_2 \to b\bar{b}, H_1 \to b\bar{b}$



Example: find benchmarks for $pp \to H_3 \to H_2 H_1$: all considered channels

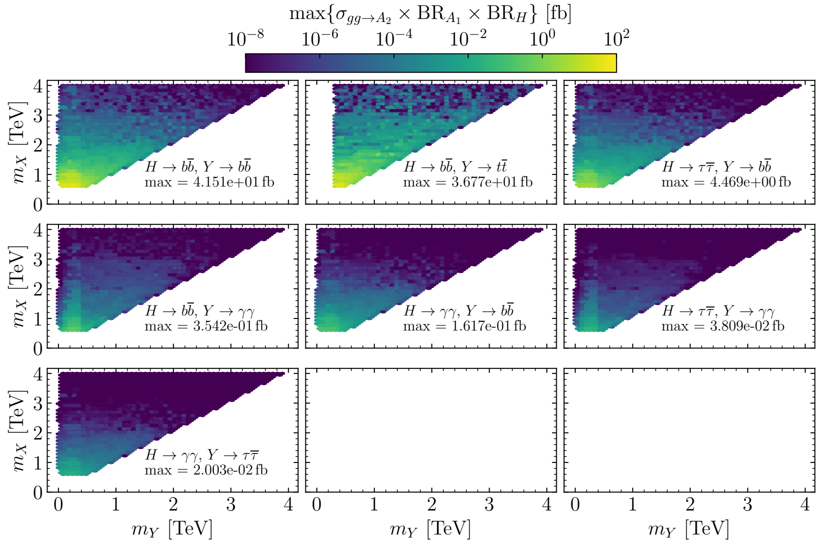


Cascade decay leading to $6b$ final state:



(channels ordered by the maximum cross-section)

Example: find benchmarks for $pp \to A_2 \to A_1 H_1$: all considered channels



Example: find benchmarks for $pp \to H_3 \to H_2 H_1$: channel with largest xs?