Revisiting the Higgs-mass calculation in the scale-invariant THDM

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THEORIQUE ET HAUTES ENERGIES

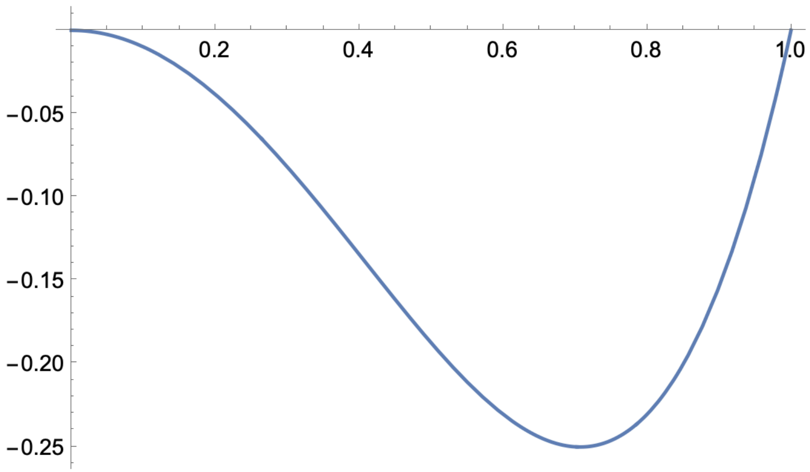
Based on: P.S., 2602.17643

KUTS 15, Karlsruhe, 23-25 March 2026

SSB without mass parameters

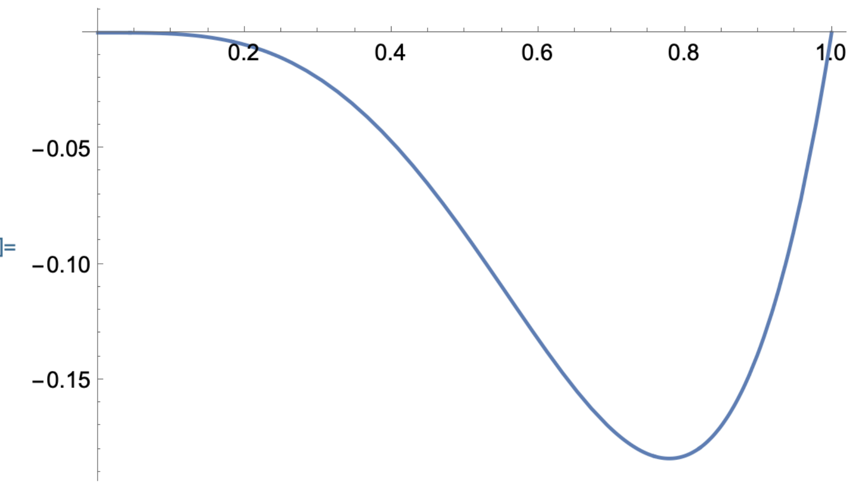
In[1]:= `Plot[-x^2 + x^4, {x, 0, 1}]`

Out[1]=



In[2]:= `Plot[x^4 Log[x^2], {x, 0, 1}]`

Out[2]=



PHYSICAL REVIEW D VOLUME 7, NUMBER 6 15 MARCH 1973

Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

Sidney Coleman
and
Erick Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 8 November 1972)

PHYSICAL REVIEW D VOLUME 13, NUMBER 12 15 JUNE 1976

Symmetry breaking and scalar bosons*

Eldad Gildener and Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 19 January 1976)

The CW / GW mechanism

*We look at the scalar potential along the direction of the field **h** that gets a vev **v***

$$V_0 \; = \; \lambda_h \, (v+h)^4$$

(the tree-level minimum condition implies $v = 0$ or $\lambda_h = 0$)

All particle masses are in the form $m^2_\varphi \; = \; \widetilde{\Lambda} \, v^2$, with $\widetilde{\Lambda} \; = \; \lambda_i, \; g_i^2, \; y_f^2, \; \ldots$

One-loop CW potential: $$V \; = \; V_0(Q) \; + \; (v+h)^4 \left[A + B \, \ln \frac{(v+h)^2}{Q^2} \right]$$

$$A \; = \; \sum_\varphi \frac{n_\varphi \, m^4_\varphi}{64 \pi^2 v^4} \left(\ln \frac{m^2_\varphi}{v^2} - k_\varphi \right) \; , \qquad B \; = \; \sum_\varphi \frac{n_\varphi \, m^4_\varphi}{64 \pi^2 v^4} \; ,$$

(n_φ = ± number of d.o.f., k_φ = scheme-dependent constants)

$$A, B = \mathcal{O} \left(\frac{\widetilde{\Lambda}^2}{16 \pi^2} \right)$$

We can choose a scale $Q_{\rm GW}$ such that $\lambda_h(Q_{\rm GW}) = 0$. The minimum condition implies:

$$\left.\frac{dV}{dh}\right|_{h=0} = 0 \quad \longrightarrow \quad \ln\frac{v^2}{Q^2_{\rm GW}} = -\frac{1}{2} - \frac{A}{B}$$

The vev is connected to the scale via "dimensional transmutation":

$$v = e^{-\left(\frac{1}{4}+\frac{A}{2B}\right)} Q_{\rm GW}$$

The scalar potential becomes:

$$V = B\,(v+h)^4\left(\ln\frac{(v+h)^2}{v^2} - \frac{1}{2}\right)$$

And the loop-induced mass of *h* is:

$$M_h^2 = \left.\frac{d^2V}{dh^2}\right|_{h=0} = 8\,v^2\,B = \sum_\varphi \frac{n_\varphi\, m_\varphi^4}{8\pi^2 v^2}$$

Does it work?

SM: $$M_h^2 \approx \frac{1}{8\pi^2 v^2}\left(6\,m_W^4 + 3\,m_Z^4 - 12\,m_t^4\right) < 0$$ ✗

BSM: $$M_h^2 \approx \frac{1}{8\pi^2 v^2}\left(\sum_i n_i\, m_{H_i}^4 + 6\,m_W^4 + 3\,m_Z^4 - 12\,m_t^4\right)$$ ✓

Sum rule for the BSM-Higgs masses: $$\sum_i n_i\, m_{H_i}^4 \approx (540\ \text{GeV})^4$$

A simple realization: the scale-invariant THDM

- 1201.4891 Lee, Pilaftsis
  Radiative corrections to scalar masses and mixing in a scale invariant THDM
- 1808.07927 Lane, Shepherd
  Natural stabilization of the Higgs boson's mass and alignment
- 1909.02111 Lane, Pilon
  Phenomenology of the new light Higgs bosons in the GW models
- 2102.07242 Eichten, Lane
  Higgs alignment and the top quark
- 2011.07580 Braathen, Kanemura, Shimoda
  2-loop analysis of classically scale-invariant models with extended Higgs sectors
- 2209.06632 Eichten, Lane
  Gildener-Weinberg THDM at two loops
- 2509.14708 Nhi, Senaha
  Exploring CP violation and vanishing eEDM in a scale-invariant general 2HDM
- 2511.06049 Baouche, Ahriche
  Phenomenology of the minimal scale-invariant THDM

Scale-invariant THDM at the “tree level”

Z_2-symmetric potential (*no mass terms = classical scale invariance*)

$$V_0 = \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\,\Phi_1^\dagger\Phi_1\,\Phi_2^\dagger\Phi_2 + \lambda_4\,\Phi_1^\dagger\Phi_2\,\Phi_2^\dagger\Phi_1 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + \text{h.c.}\right]$$

$$\Phi_k = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\,\phi_k^+ \\ v_k + \phi_k^0 + i\,a_k \end{pmatrix} \qquad (k=1,2)$$

Minimum conditions for $v_1, v_2 \neq 0\,, \;\; \tan\beta \equiv \frac{v_2}{v_1}$:

$$\lambda_1 = -\,\lambda_{345}\,\tan^2\beta\,, \qquad \lambda_2 = -\,\lambda_{345}\,\cot^2\beta$$

Neutral scalar, pseudoscalar and charged sectors all diagonalized by the same rotation:

$$\begin{pmatrix} \Phi_{\rm SM} \\ \Phi_{\rm BSM} \end{pmatrix} = R(\beta)\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}$$

One eigenstate is aligned with the vev $v \equiv v_1^2 + v_2^2$:

$$\Phi_{\rm SM} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + i\,G^0) \end{pmatrix}, \qquad \Phi_{\rm BSM} = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H + i\,A) \end{pmatrix}$$

“Tree-level” masses:

$$m_h^2 = m_{G^0}^2 = m_{G^\pm}^2 = 0\,, \quad m_H^2 = -\,\lambda_{345}\,v^2\,, \quad m_A^2 = -\,\lambda_5\,v^2\,, \quad m_{H^\pm}^2 = -\,\frac{\lambda_{45}}{2}\,v^2$$

Also directly in the "Higgs basis":

$$
\begin{aligned}
V_0 \;=\;& \frac{\Lambda_1}{2}\left(\Phi^\dagger_{\rm SM}\Phi_{\rm SM}\right)^2 + \frac{\Lambda_2}{2}\left(\Phi^\dagger_{\rm BSM}\Phi_{\rm BSM}\right)^2 \\
&+ \Lambda_3\left(\Phi^\dagger_{\rm SM}\Phi_{\rm SM}\right)\left(\Phi^\dagger_{\rm BSM}\Phi_{\rm BSM}\right) + \Lambda_4\left(\Phi^\dagger_{\rm SM}\Phi_{\rm BSM}\right)\left(\Phi^\dagger_{\rm BSM}\Phi_{\rm SM}\right) \\
&+ \left[\frac{\Lambda_5}{2}\left(\Phi^\dagger_{\rm SM}\Phi_{\rm BSM}\right)^2 + \left(\Lambda_6\,\Phi^\dagger_{\rm SM}\Phi_{\rm SM} + \Lambda_7\,\Phi^\dagger_{\rm BSM}\Phi_{\rm BSM}\right)\Phi^\dagger_{\rm SM}\Phi_{\rm BSM} + \text{h.c.}\right]
\end{aligned}
$$

Minimum conditions for $v \neq 0$: $\Lambda_1 = 0\,, \qquad \Lambda_6 = 0$

$$m_h^2 = m_{G^0}^2 = m_{G^\pm}^2 = 0$$

"Tree-level" masses:

$$m_H^2 = \frac{1}{2}\,\Lambda_{345}\,v^2\,, \quad m_A^2 = \frac{1}{2}\left(\Lambda_3 + \Lambda_4 - \Lambda_5\right)v^2\,, \quad m_{H^\pm}^2 = \frac{1}{2}\,\Lambda_3\,v^2$$

One-loop corrections in the literature [Lee-Pilaftsis, Lane *et al.*]

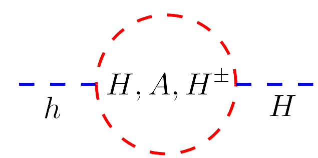
Effective-potential approach:

$$V^{1\ell} \;=\; V_0 \,+\, \Delta V \;, \qquad \left(\mathcal{M}^2_{S,P,C}\right)^{1\ell}_{ij} \;=\; \left.\frac{\partial^2 V^{1\ell}}{\partial\varphi_i\partial\varphi_j}\right|_{\rm min}$$

(ΔV = one-loop CW potential expressed in terms of field-dependent masses)

$$\left(\mathcal{M}^2_S\right)^{1\ell}_{hh} \;\approx\; \frac{1}{8\pi^2 v^2}\left(m_H^4 + m_A^4 + 2\,m_{H^\pm}^4 + 6\,m_W^4 + 3\,m_Z^4 - 12\,m_t^4\right)$$

$$\left(\mathcal{M}^2_S\right)^{1\ell}_{hH} \;\propto\; \frac{m_t^4}{v^2}$$

What about 

???

However…

$$\left(\mathcal{M}^2_{P,C}\right)^{1\ell} = \left(\mathcal{M}^2_{P,C}\right)^{\rm tree}$$

Scale- and gauge-dependent ???

Bad assumption for the field-dependent masses in Lee-Pilaftsis, later borrowed by Lane *et al.* :

where Q is the RG scale and the background field-dependent masses are given by

$$M_H^2 = -2\lambda_{345}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right), \qquad M_A^2 = -2\lambda_5\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right),$$
$$M_{H^\pm}^2 = -\lambda_{45}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right), \qquad M_W^2 = \frac{g^2}{2}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right),$$
$$M_Z^2 = \frac{g^2}{2c_w^2}\left(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2\right), \qquad m_t^2 = |h_I|^2\Phi_I^\dagger\Phi_I\,. \tag{2.18}$$

[1201.4891]

$$\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2 = \frac{1}{2}\left[(v+h)^2+G^{0\,2}+2\,G^+G^-+H^2+A^2+2H^+H^-\right]$$

This works along the *h* direction, but misses mixed terms in the scalar potential:

$$\begin{aligned}V_0|_{\rm min} = & -\frac{1}{2}\lambda_{345}\left[(v+h)H+G^0A+G^+H^-+G^-H^++(H^2+A^2+2H^+H^-)\cot 2\beta\right]^2\\ & -\frac{1}{2}\lambda_{45}\big[((v+h)^2+G^{0\,2})H^+H^-+(H^2+A^2)G^+G^-\\ & \qquad -((v+h)H+G^0A)(G^+H^-+G^-H^+)-i((v+h)A-G^0H)(G^+H^--H^+G^-)\big]\\ & -\frac{1}{2}\lambda_5\left[(v+h)A-G^0H+i(G^+H^--H^+G^-)\right]^2\end{aligned}$$

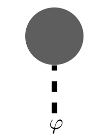
Compute the correct field-dependent masses accounting for all of the mixed terms?

Compute the correct field-dependent masses accounting for all of the mixed terms?

We bypass all complications with a diagrammatic calculation

The effective-potential approx (i.e., $p^2 = 0$) is anyway inadequate for the heavy-boson masses

The minimum conditions at one loop

$T_\varphi = \frac{d\Delta V}{d\varphi}\Big|_{\rm min} =$ 

Z$_2$-symmetric basis:

$$\lambda_1 = -\lambda_{345}\tan^2\beta - \frac{2}{v^3 c_\beta^2}\,(T_h - T_H\tan\beta)$$

$$\lambda_2 = -\lambda_{345}\cot^2\beta - \frac{2}{v^3 s_\beta^2}\,(T_h + T_H\cot\beta)$$

Higgs basis:

$$\Lambda_1 = -\frac{2}{v^3}\,T_h\,, \qquad \Lambda_6 = -\frac{2}{v^3}\,T_H$$

The Gildener-Weinberg scale is defined by $\Lambda_1(Q_{\rm GW}) = 0$ (hence, $T_h = 0$)

[*flat direction of the tree-level potential along h*]

However, the calculation can be performed at any reasonable scale *Q* of $\mathcal{O}(v)$

Some questionable claims in the literature:

Eichten, Lane: tree-level minimum conditions “true at all orders” at Q_{GW}

Nhi, Senaha: “maintain” both $\Lambda_1(Q_{\rm GW}) = 0$ and $\Lambda_6(Q_{\rm GW}) = 0$ (hence, $T_h = T_H = 0$)

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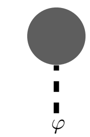
$T_\varphi = \left.\frac{d\Delta V}{d\varphi}\right|_{\rm min} =$ 

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Arbitrary restriction on the parameter space!

One-loop-corrected mass matrices

$$\mathcal{M}_S^2(p^2) \;=\; \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345}\, v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{hh}(p^2) & \Pi_{hH}(p^2) \\ \Pi_{hH}(p^2) & \Pi_{HH}(p^2) \end{pmatrix} - \frac{3}{v} \begin{pmatrix} T_h & T_H \\ T_H & \widetilde{T} \end{pmatrix}$$

$$\mathcal{M}_P^2(p^2) \;=\; \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_5\, v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{G^0G^0}(p^2) & \Pi_{G^0A}(p^2) \\ \Pi_{G^0A}(p^2) & \Pi_{AA}(p^2) \end{pmatrix} - \frac{1}{v} \begin{pmatrix} T_h & T_H \\ T_H & \widetilde{T} \end{pmatrix}$$

$$\mathcal{M}_C^2(p^2) \;=\; \begin{pmatrix} 0 & 0 \\ 0 & -\frac{\lambda_{45}}{2}\, v^2 \end{pmatrix} + \begin{pmatrix} \Pi_{G^\pm G^\mp}(p^2) & \Pi_{G^\pm H^\mp}(p^2) \\ \Pi_{G^\pm H^\mp}(p^2) & \Pi_{H^\pm H^\mp}(p^2) \end{pmatrix} - \frac{1}{v} \begin{pmatrix} T_h & T_H \\ T_H & \widetilde{T} \end{pmatrix}$$

$$\widetilde{T} = T_h + 2\cot 2\beta\, T_H$$

Computed all tadpoles and self-energies with `FeynArts`, in a generic R_ξ gauge

Compared with S.P. Martin's general formulas, found ξ^2 discrepancies in $\Pi_{GG}(p^2)$

[*Problem with Goldstone-ghost-ghost couplings, already noticed by Goodsell & Paßehr*]

Pole masses for the physical scalars

Strict one-loop expansion:

$$M_h^2 \;=\; \Pi_{hh}(0) \,-\, 3\,\frac{T_h}{v}$$

$$M_H^2 \;=\; -\frac{\lambda_{345}}{\sqrt{2}G_F} \,-\, m_H^2\,\frac{\delta v^2}{v^2} \,+\, \mathrm{Re}\,\Pi_{HH}(m_H^2) \,-\, 3\,\frac{\widetilde{T}}{v}$$

$$M_A^2 \;=\; -\frac{\lambda_5}{\sqrt{2}G_F} \,-\, m_A^2\,\frac{\delta v^2}{v^2} \,+\, \mathrm{Re}\,\Pi_{AA}(m_A^2) \,-\, \frac{\widetilde{T}}{v}$$

$$M_{H^\pm}^2 \;=\; -\frac{\lambda_{45}}{2\sqrt{2}G_F} \,-\, m_{H^\pm}^2\,\frac{\delta v^2}{v^2} \,+\, \mathrm{Re}\,\Pi_{H^\pm H^\mp}(m_{H^\pm}^2) \,-\, \frac{\widetilde{T}}{v}$$

$$(\sqrt{2}G_F)^{-1} = v^2 + \delta v^2\;, \qquad \delta v^2 \;=\; v^2\left[\frac{\Pi_{WW}(0)}{m_W^2} - \delta_{\rm VB}\right]$$

All pole masses are gauge- and scale-independent at the one-loop order ✓

Mixing is a two-loop effect, but we can compute $M_{h'}^2$ and $M_{H'}^2$ as eigenvalues of the full mass matrix $\mathcal{M}_S^2(p^2)$ computed at $p^2 = 0$ and $p^2 = m_H^2$, respectively.

Notable results:

$$M_h^2 = \frac{1}{8\pi^2 v^2}\left(m_H^4 + m_A^4 + 2\,m_{H^\pm}^4 + 6\,m_W^4 + 3\,m_Z^4 - 4\,N_c\,m_t^4\right)$$

$$\left[\mathcal{M}_S^2(0)\right]_{hH} = \frac{1}{8\pi^2 v^2}\left[m_H^2\,\left(3\,m_H^2 + m_A^2 + 2\,m_{H^\pm}^2\right)\,\cot 2\beta - 4\,N_c\,m_t^4\,\cot\beta\right]$$

When devising benchmark scenarios, M_h = 125 GeV fixes one of the BSM Higgs masses

We can fix another mass by requiring $\left[\mathcal{M}_S^2(0)\right]_{hH} = 0$ (*one-loop alignment*)

[Note: works only for $\tan\beta < 1$]

But what masses are we actually fixing?

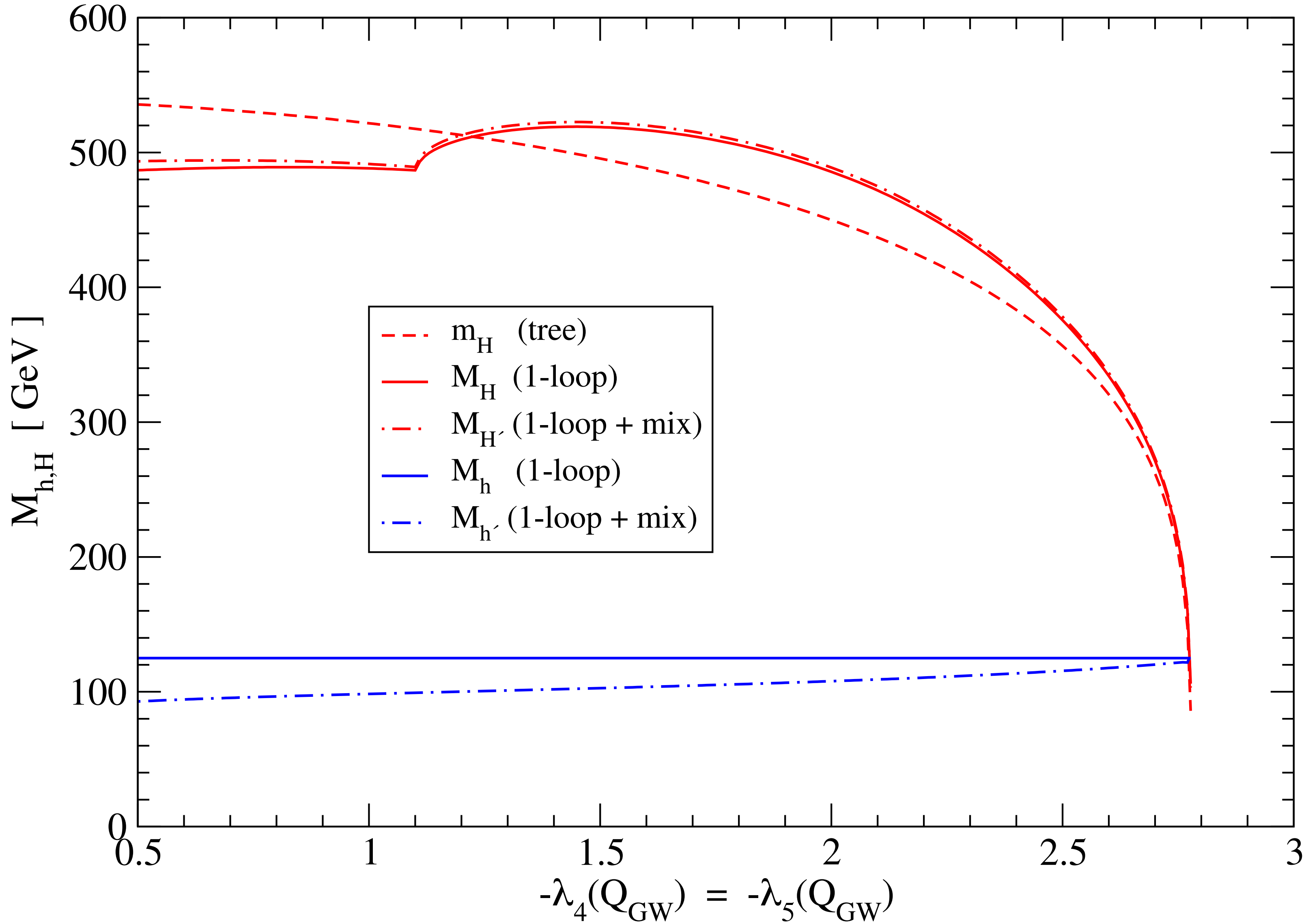
We will identify the tree-level masses with: $m_H^2 = -\dfrac{\lambda_{345}(Q)}{\sqrt{2}G_F}\,,\quad m_A^2 = -\dfrac{\lambda_5(Q)}{\sqrt{2}G_F}\,,\quad m_{H^\pm}^2 = -\dfrac{\lambda_{45}(Q)}{\sqrt{2}G_F}$

Different scale choices = different scenarios; and we could even choose to fix the pole masses!

A two-loop calculation of the mass corrections would resolve the ambiguities

Note: no ambiguities if we fix directly $\tan\beta\,,\ \lambda_3(Q)\,,\ \lambda_4(Q)\,,\ \lambda_5(Q)$ *at some scale Q !*

Numerical examples

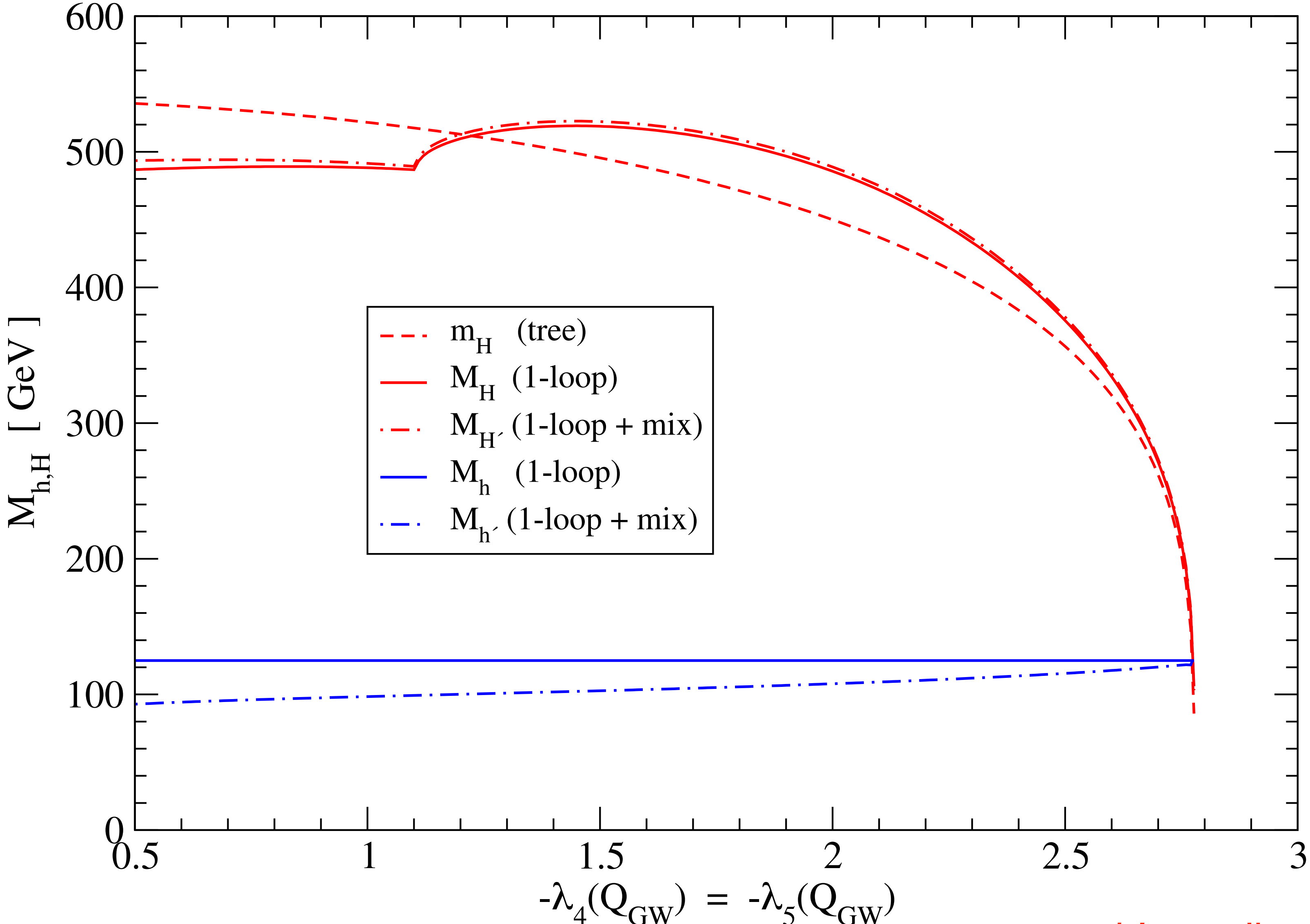


$\tan\beta = 2$

Benchmark scenario A:

$\lambda_4 = \lambda_5 \qquad (\, m_A = m_{H^\pm} \,, \quad \Delta\rho = 0 \,)$

λ_3 fixed by M_h = 125 GeV , i.e., $m_H^4 + m_A^4 + 2\, m_{H^\pm}^4 \;\approx\; (540 \text{ GeV})^4$



h' couplings:

$\kappa_V \approx 0.99$, $\kappa_t \approx 1.06$

$\kappa_{b,\tau} \approx 1.06$ (I) , $\lesssim 0.74$ (II)

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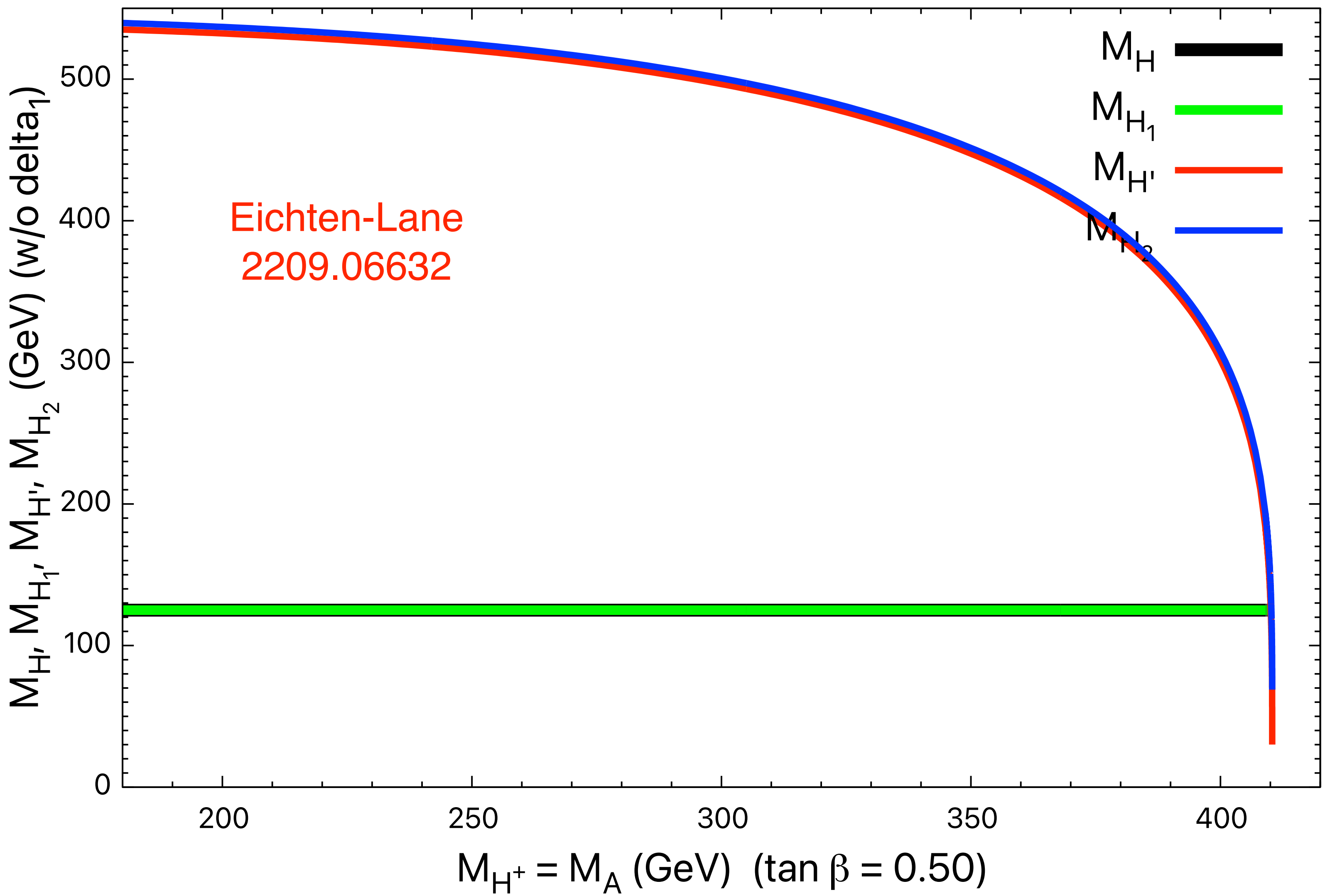

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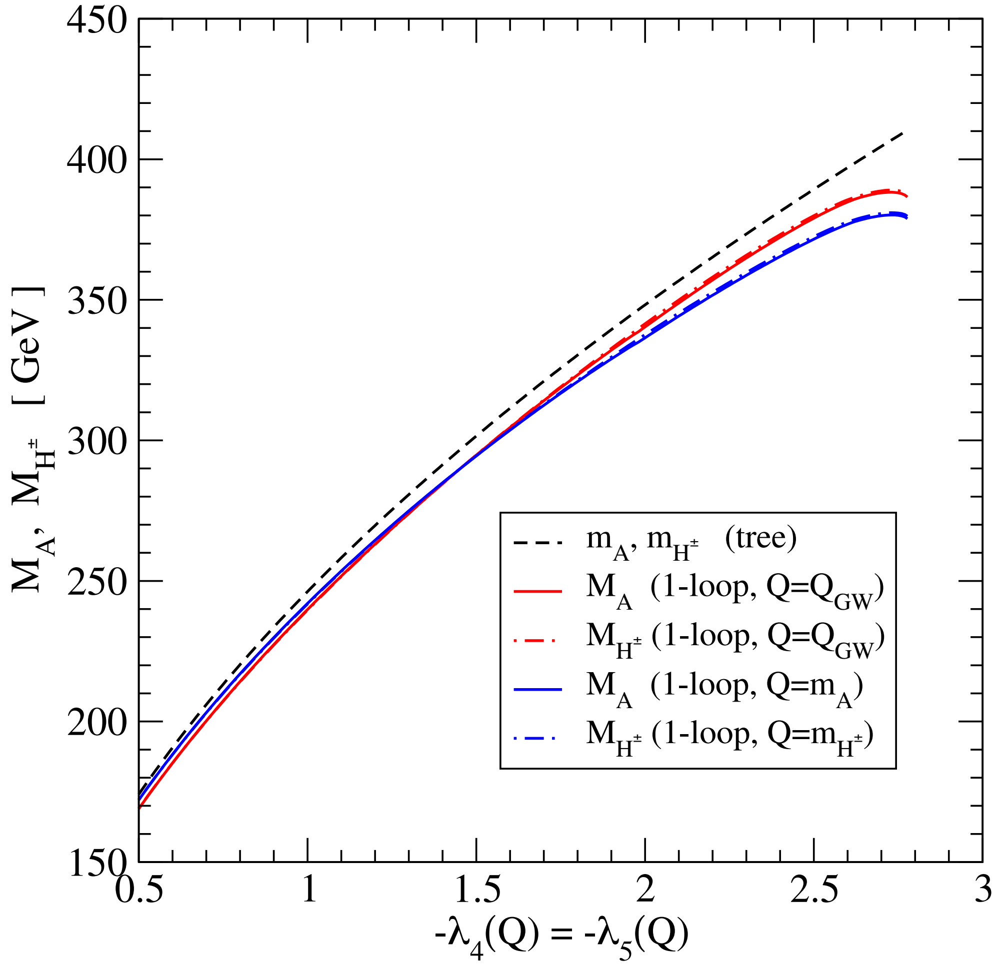

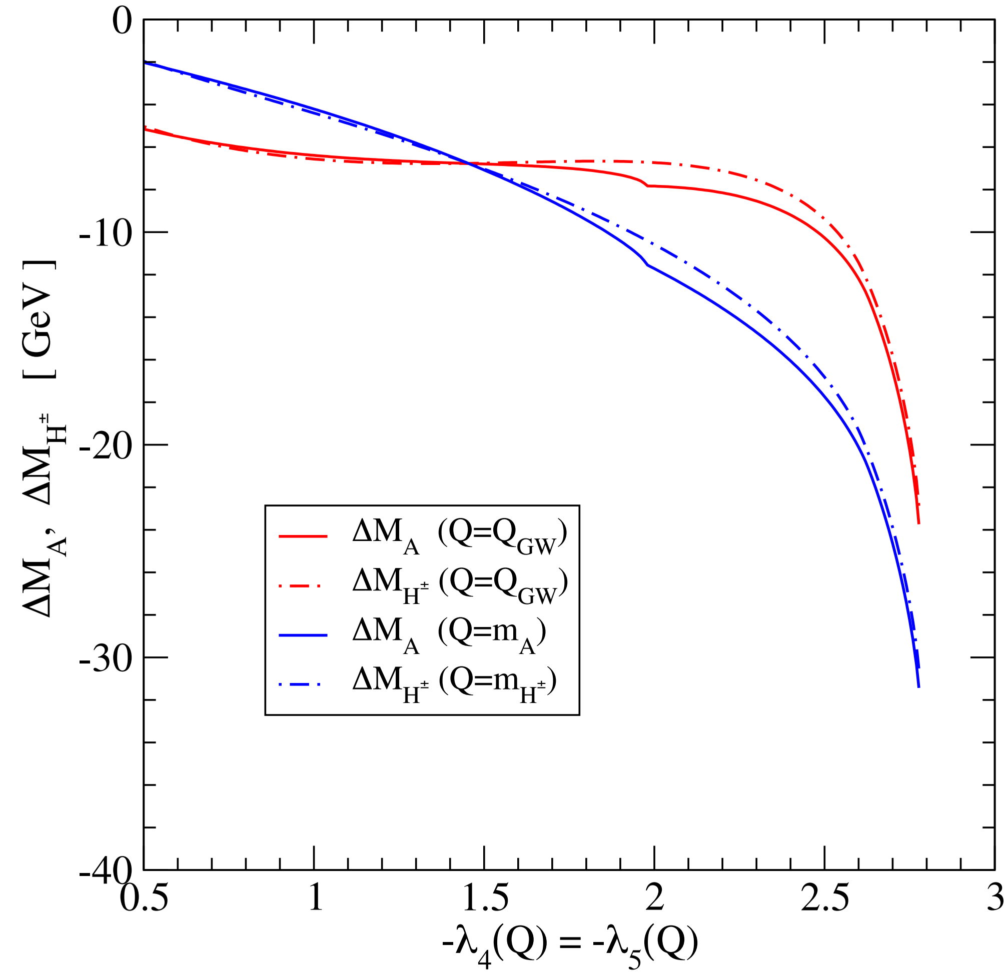

Benchmark scenario A:

$\tan\beta = 2$

$\lambda_4 = \lambda_5$ $\quad (m_A = m_{H^\pm} \, , \quad \Delta\rho = 0 \,)$ SO(3) symmetry!

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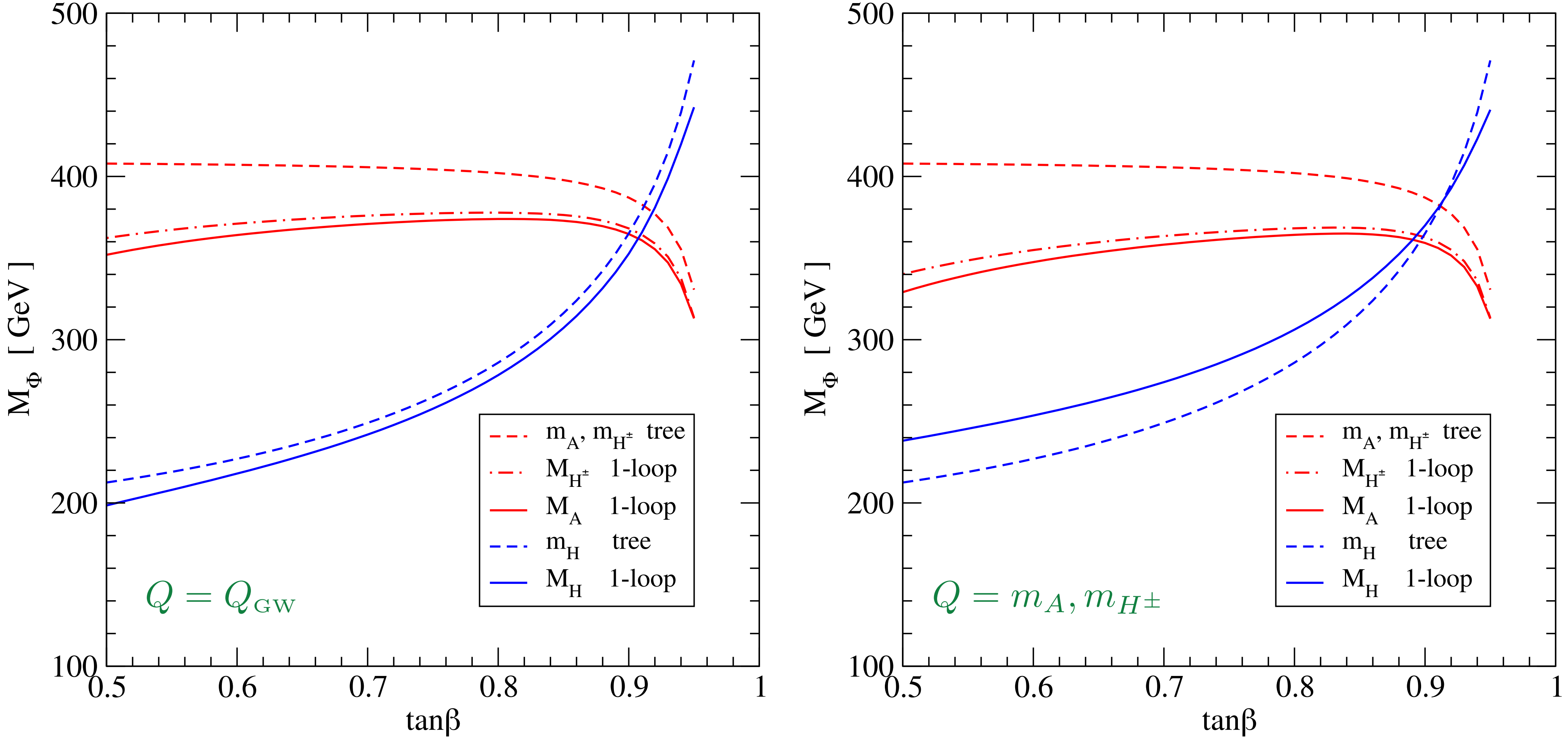

Benchmark scenario B:

$\lambda_4 = \lambda_5 \qquad (m_A = m_{H\pm} \,, \quad \Delta\rho = 0)$

λ_3 fixed by *M$_h$* = 125 GeV , i.e., $m_H^4 + m_A^4 + 2\, m_{H\pm}^4 \;\approx\; (540 \text{ GeV})^4$

λ_4, λ_5 fixed by $\left[\mathcal{M}_S^2(0)\right]_{hH} = 0$ (*one-loop alignment*)

Summary:

- We revised the one-loop calculation of the Higgs masses in the SI-THDM
- The corrections to the BSM-Higgs masses can be of $\mathcal{O}(10\%)$
- Mixing affects the M_h prediction only at two loops, but its effect is not negligible (*must* be taken into account in the two-loop calculation!)
- A *"maximally SM-like"* scenario with one-loop alignment can be devised

To do:

- Extend the two-loop calculation (also to remove ambiguities in the scenarios)
- Study the constraints from Higgs measurements and BSM searches at the LHC

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Any volunteers?

Thank you!!!