Precision predictions of partial decay widths in the general Two-Higgs-Doublet Model

Katharsis of Ultimate Theory Standards
Meeting 15 - KIT

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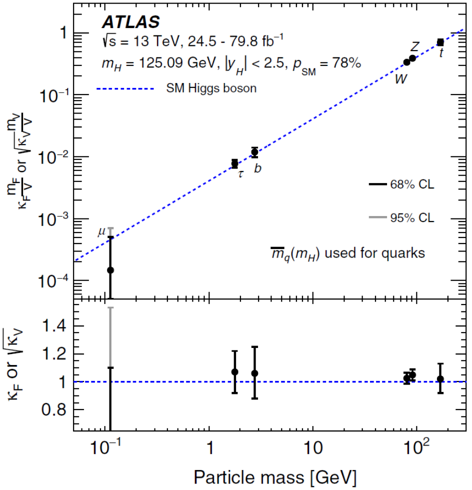
1. Motivation

Searching for new physics through Higgs-boson measurements

- Higgs couplings agree with the Standard Model



arXiv:1909.02845

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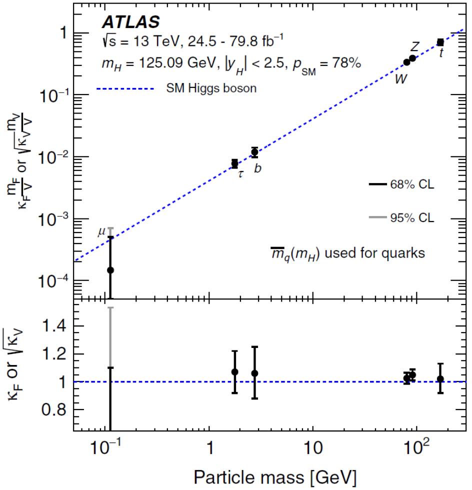

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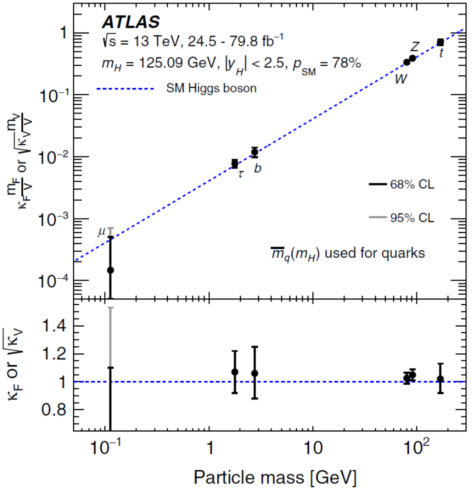

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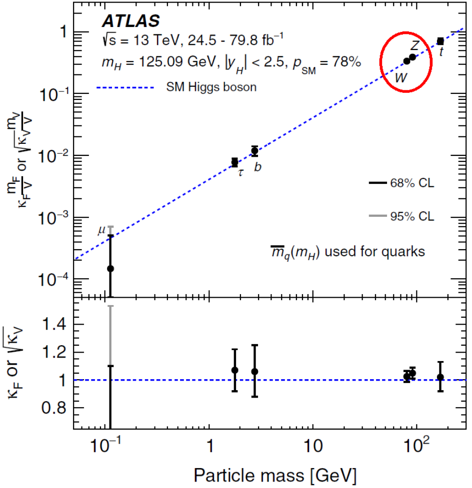

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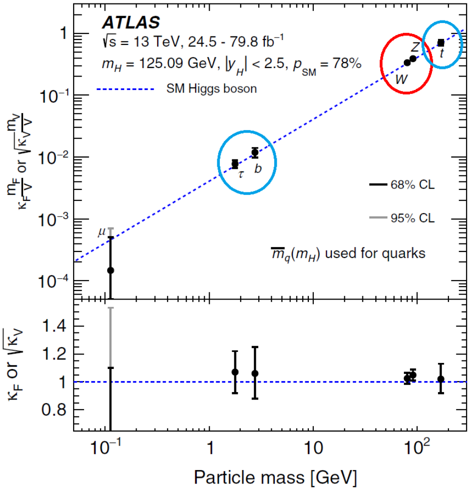

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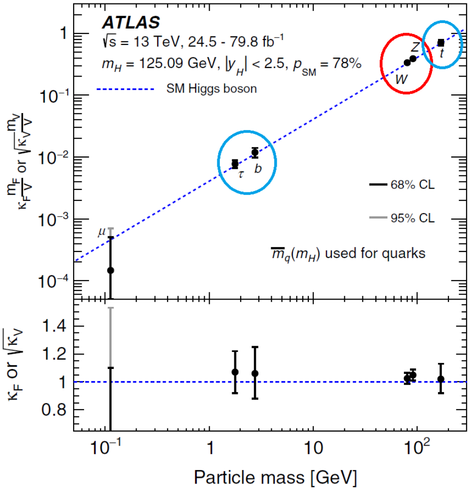

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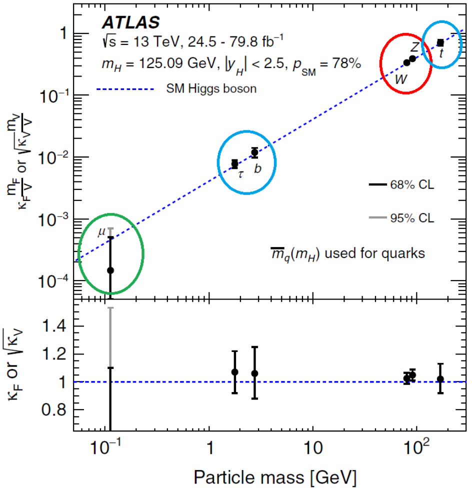

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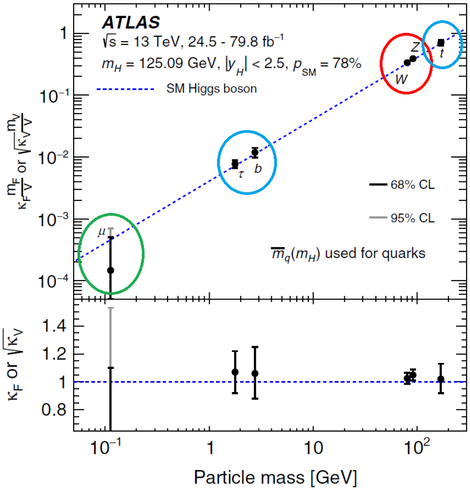

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$$\rho_{\text{exp}} = 1.00031 \pm 0.00019$$

PhysRevD.110.030001

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- THDMs can provide additional sources of CP violation relevant for the baryogenesis.

- The general THDM can reproduce many specific THDM scenarios.

2. The general Two-Higgs-Doublet Model

The scalar potential and Yukawa Lagrangian of the Standard Model

Scalar potential of the Standard Model:

$$V(\Phi) = \frac{\lambda}{4}\left(\Phi^\dagger\Phi\right)^2 - \mu^2\left(\Phi^\dagger\Phi\right),$$

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + h + iG_0\right) \end{pmatrix}, \qquad v = 2\sqrt{\frac{\mu^2}{\lambda}}, \quad \lambda, \mu^2 \geq 0.$$

Tadpole and bilinear terms of the scalar potential:

$$V(\Phi)\Big|_{\text{Tadpole}} = -t_h h, \quad V(\Phi)\Big|_{\text{Bilinear}} = -\frac{1}{v}t_h G^+G^- + \frac{1}{2}\begin{pmatrix} G_0 & h \end{pmatrix}\begin{pmatrix} -\frac{1}{v}t_h & 0 \\ 0 & m_h^2 \end{pmatrix}\begin{pmatrix} G_0 \\ h \end{pmatrix},$$

$$t_h \equiv v\left(\mu^2 - \frac{v^2\lambda}{4}\right), \quad m_h^2 \equiv 2\frac{\lambda}{2}v^2 - \frac{t_h}{v}.$$

The general THDM in the generic basis

Most general renormalizable scalar potential with two complex $SU(2)_\mathsf{L}$ doublets:

$$
\begin{aligned}
V_{\mathsf{THDM}}(\Phi_1,\Phi_2) = {} & m_{11}^2\left(\Phi_1^\dagger\Phi_1\right) + m_{22}^2\left(\Phi_2^\dagger\Phi_2\right) - \left[m_{12}^2\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right] + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 \\
& + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \frac{\lambda_3}{2}\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \frac{\lambda_4}{2}\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\
& + \left[\frac{\lambda_5}{2}\left(\Phi_1^\dagger\Phi_2\right)^2 + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right].
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In the generic basis, the Higgs doublets have a non-vanishing vacuum expectation value (vev):

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}\left(v_1 + \rho_1 + i\eta_1\right) \end{pmatrix}, \qquad \Phi_2 = e^{i\delta} \begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \rho_2 + i\eta_2\right) \end{pmatrix}, \qquad \tan\beta = \frac{v_2}{v_1}.$$

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General Yukawa structure in the general THDM (fermions couple to both doublets):

$$\mathcal{L}_{\text{Yukawa, THDM}} = -\sum_{k=1}^{2}\sum_{i,j}\left(\overline{L}_i'^{\mathsf{L}} G_{ij}^{l,k} l_j'^{\mathsf{R}}\Phi_k + \overline{Q}_i'^{\mathsf{L}} G_{ij}^{u,k} u_j'^{\mathsf{R}}\Phi_k^c + \overline{Q}_i'^{\mathsf{L}} G_{ij}^{d,k} d_j'^{\mathsf{R}}\Phi_k + \text{h.c.}\right).$$

Conversion from the generic basis to the Higgs basis

It is more convenient to work in the Higgs basis (only one doublet has a non-vanishing vev).

Generic basis $\xrightarrow{U(\beta,\delta)}$ **Higgs basis**

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}\left(v_1 + \rho_1 + i\eta_1\right) \end{pmatrix} \qquad H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + N_1 + iG_0\right) \end{pmatrix}$$

$$\Phi_2 = e^{i\delta}\begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \rho_2 + i\eta_2\right) \end{pmatrix} \qquad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(N_2 + iN_3\right) \end{pmatrix}$$

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = U(\beta,\delta)\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta e^{-i\delta} \\ -s_\beta & c_\beta e^{-i\delta} \end{pmatrix}\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix},$$

$$v^2 = v_1^2 + v_2^2, \quad v_2 = v\sin\beta, \quad v_1 = v\cos\beta.$$

Bilinear terms of the scalar potential in the Higgs basis:

$$V_{\text{THDM}}\Big|_{\text{Bilinear}} = \begin{pmatrix} G^+ & H^+ \end{pmatrix} M_{\text{charged}} \begin{pmatrix} G^- \\ H^- \end{pmatrix} + \frac{1}{2} \begin{pmatrix} G_0 & N_3 & N_2 & N_1 \end{pmatrix} M_{\text{neutral}} \begin{pmatrix} G_0 \\ N_3 \\ N_2 \\ N_1 \end{pmatrix}.$$

Charged scalar mass matrix:

$$M_{\text{charged}} = \begin{pmatrix} -\frac{t_{N_1}}{v} & -\frac{t_{N_2}+it_{N_3}}{v} \\ -\frac{t_{N_2}-it_{N_3}}{v} & m_{H^+}^2 \end{pmatrix}.$$

Neutral scalar mass matrix:

$$M_{\text{neutral}} = \begin{pmatrix} -\frac{t_{N_1}}{v} & -\frac{t_{N_2}}{v} & \frac{t_{N_3}}{v} & 0 \\ -\frac{t_{N_2}}{v} & m_{H^+}^2 + \frac{1}{2}\left(\tilde{\lambda}_4 - \text{Re}[\tilde{\lambda}_5]\right) v^2 & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5] v^2 & -\text{Im}[\tilde{\lambda}_6] v^2 - \frac{t_{N_3}}{v} \\ \frac{t_{N_3}}{v} & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5] v^2 & m_{H^+}^2 + \frac{1}{2}\left(\tilde{\lambda}_4 + \text{Re}[\tilde{\lambda}_5]\right) v^2 & \text{Re}[\tilde{\lambda}_6] v^2 - \frac{t_{N_2}}{v} \\ 0 & -\text{Im}[\tilde{\lambda}_6] v^2 - \frac{t_{N_3}}{v} & \text{Re}[\tilde{\lambda}_6] v^2 - \frac{t_{N_2}}{v} & \tilde{\lambda}_1 v^2 - \frac{t_{N_1}}{v} \end{pmatrix}.$$

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Mass eigenstates and Yukawa structure in the Higgs basis

Mass eigenstates of the neutral scalar fields

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$$N^{(3\times3)}_{\text{neutral}} = R^T(\theta_1,\theta_2,\theta_3) M^{(3\times3)}_{\text{neutral}} R(\theta_1,\theta_2,\theta_3) = \text{diag}\left(m^2_{A_0}, m^2_h, m^2_H\right).$$

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The Yukawa matrix associated to the Higgs doublet H_1 is diagonalized

$$\widetilde{G}_{ij}^{f,1} = \frac{\sqrt{2} m_{f,i}}{v}\delta_{ij}, \qquad f = l, u, d.$$

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The Yukawa matrix $\widetilde{G}^{f,2}_{ij}$ associated with the doublet H_2 remains more general

Tadpole terms of the scalar potential in the Higgs basis

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$$V_{\mathsf{THDM}}\Big|_{\mathsf{Tadpoles}} = -t_{N_1} N_1 - t_{N_2} N_2 - t_{N_3} N_3,$$

$$t_{N_1} = -\widetilde{m}_{11}^2 v - \tfrac{1}{2}\widetilde{\lambda}_1 v^3, \quad t_{N_2} = \mathrm{Re}\{\widetilde{m}_{12}^2\} v - \tfrac{1}{2}\,\mathrm{Re}\{\widetilde{\lambda}_6\} v^3, \quad t_{N_3} = -\,\mathrm{Im}\{\widetilde{m}_{12}^2\} v + \tfrac{1}{2}\,\mathrm{Im}\{\widetilde{\lambda}_6\} v^3.$$

After diagonalizing the neutral mass matrix, the tadpole constants are rotated

$$V_{\mathsf{THDM}}\Big|_{\mathsf{Tadpoles}} = -t_H H - t_h h - t_{A_0} A_0,$$

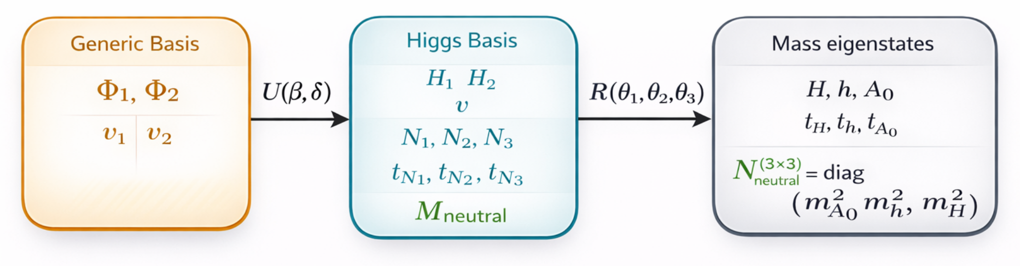
$$\begin{pmatrix} t_H \\ t_h \\ t_{A_0} \end{pmatrix} = R(\theta_1, \theta_2, \theta_3)^{\mathsf{T}} \begin{pmatrix} t_{N_1} \\ t_{N_2} \\ t_{N_3} \end{pmatrix}.$$

From the generic basis to physical parameters

Parameters in the generic basis:

$$p_{\text{generic}} = \{m_{11}^2, m_{12}^2, m_{22}^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, G_{ij}^{f,1}, G_{ij}^{f,2}, v_1, v_2, \delta\}.$$

The following steps rewrite the theory in terms of the physical parameters.



arXiv:9411288

Physical parameters associated with the mass eigenstates:

$$p_{\text{mass}} = \{t_H, t_h, t_{A_0}, m_{H^+}^2, m_{A_0}^2, m_h^2, m_H^2, \theta_1, \theta_2, \theta_3, \widetilde{\lambda}_2, \widetilde{\lambda}_3, \widetilde{\lambda}_7, m_{f,i}, \widetilde{G}_{ij}^{f,2}, v\}.$$

3. Renormalization of the general Two-Higgs-Doublet Model

Bare input parameters and bare scalar fields

The general THDM introduces additional bare parameters in the Higgs and Yukawa sectors:

$$p_0 = \{t_H, t_{A_0}, m_H^2, m_{A_0}^2, m_{H^+}^2, \theta_1, \theta_2, \theta_3, \widetilde{\lambda}_2, \widetilde{\lambda}_3, \widetilde{\lambda}_7, \widetilde{G}_{ij}^{f,2}\}.$$

$$\underbrace{p_0}_{\text{bare parameter}} = \underbrace{p}_{\text{renormalized parameter}} + \underbrace{\delta p}_{\text{renormalization constant}}$$

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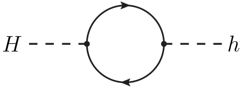
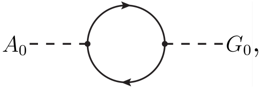
$$\underbrace{p_0}_{\text{bare parameter}} = \underbrace{p}_{\text{renormalized parameter}} + \underbrace{\delta p}_{\text{renormalization constant}}$$

The additional Higgs fields must also be renormalized.





Previous work in the renormalization of THDM

Significant progress has been made in the literature in renormalizing THDMs:

- Kanemura et al (see e.g. arXiv:2408.08033)
- Mühlleitner et al (see e.g. arXiv:1605.04853)
- Denner et al (see e.g. arXiv:1607.07352)
- Dittmaier et al (see e.g. arXiv:1704.02645)
- Degrande (see e.g. arXiv:1406.3030)

On-shell renormalization conditions

Masses and **fields** were renormalized by **on-shell** conditions (see e.g. arXiv:1912.06823):

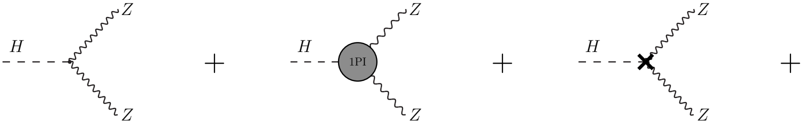
- **Masses** are defined from the location of the real part of the poles of the propagators.
- **Fields** correspond to canonically normalized mass eigenstates.

Modified Minimal Subtraction ($\overline{\text{MS}}$) conditions

- **Higgs mixing angles**, coupling **lambda parameters**, and **Yukawa matrices** were renormalized using $\overline{\textbf{MS}}$ conditions
- The terms proportional to the UV divergence are subtracted (see e.g. arXiv:9212285).

$$\Gamma_{\mathsf{R}}^{HZZ}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\mathsf{UV}} = 0, \quad \Gamma_{\mathsf{R}}^{A_0 ZZ}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\mathsf{UV}} = 0, \quad \Gamma_{\mathsf{R}}^{W^+ H H^+}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\mathsf{UV}} = 0,$$

$i\Gamma_{\mathsf{R}}^{HZZ} =$



$\mathcal{O}(e^4)$.

There are other renormalization schemes that have been applied for mixing angles.

- Kanemura–Okada–Senahan–Yuan scheme (arXiv:0408364).
- Pinched scheme (arXiv:0909.2536).

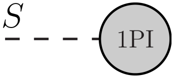
Tadpole renormalization

- Three tadpole contributions arise in the scalar potential due to a non-vanishing vev

$$V_{\mathsf{THDM}}\Big|_{\mathsf{Tadpoles}} = -t_H H - t_h h - t_{A_0} A_0$$

- The vacuum is fixed to vanish the tadpole contributions at tree level
- Higher order tadpole contributions arise from Feynman diagrams containing

$$T^S =$$



$$S = H, h, A_0.$$

- Tadpole renormalization schemes are based on linear/non-linear parametrizations of the Higgs doublets

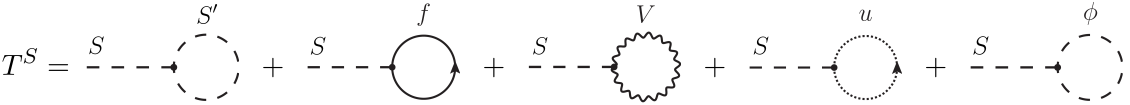
Tadpole renormalization: Linear parametrization of the Higgs doublets

- Most EW corrections are computed using the linear parametrization.

$$\boldsymbol{\Phi}_n \equiv (\Phi_n^c, \Phi_n) = \frac{1}{\sqrt{2}}\left[(v_n + \rho_n)\mathbb{1} + 2i\boldsymbol{\phi}_n\right], \quad \boldsymbol{\phi}_n = \frac{\phi_{nj}\boldsymbol{\sigma}_j}{2}, \quad \omega_n^{\pm} = \frac{(\phi_{n2} \pm i\phi_{n1})}{\sqrt{2}}, \quad \eta_n = -\phi_{n3}.$$

- Higgs fields and tadpole contributions are gauge dependent

- One-loop tadpole contributions:



$$S, S' = H, h, A_0, \quad f = e, \mu, \tau, \mathsf{u}, \mathsf{c}, \mathsf{t}, \mathsf{d}, \mathsf{s}, \mathsf{b}, \quad V = W^{\pm}, Z, \quad u = u^Z, u^{\pm}, \quad \phi = G^{\pm}, G_0$$

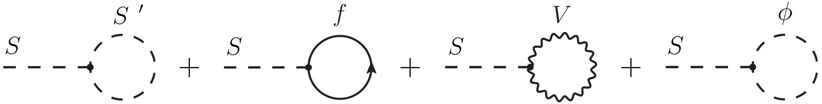
Tadpole renormalization: Non-linear parametrization of the Higgs doublets

- A non-linear parametrization of the Higgs doublets can be used to avoid gauge dependences.

$$\mathbf{\Phi}_n \equiv \exp\left(\frac{2i\boldsymbol{\zeta}}{v}\right)\cdot\frac{1}{\sqrt{2}}\left[(v_n+\rho_n)\mathbb{1}+2ib_n\boldsymbol{\phi}\right], \quad \boldsymbol{\zeta}=\frac{\zeta_j\boldsymbol{\sigma}_j}{2}, \quad \boldsymbol{\phi}=\frac{\phi_j\boldsymbol{\sigma}_j}{2}, \quad b_1=-s_\beta, \quad b_2=c_\beta.$$

- Higgs fields and tadpole contributions are gauge invariant

- One-loop tadpole contributions (no ghost loops):

$T^{S,\mathsf{nl}} =$ 

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Tadpole renormalization: Tadpole schemes

Three different tadpole schemes were used to define the tadpole renormalization constants:

- Parameter Renormalized Tadpole Scheme (PRTS) arXiv:0709.1075
- Fleischer–Jegerlehner Tadpole Scheme (FJTS) PhysRevD:23.2001
- Gauve-Invariant Vacuum expectation value Scheme (GIVS) arXiv:2206.01479

Tadpole schemes	Parametrization	Counterterms per neutral scalar	Counterterm introduction	Gauge invariant	Small corrections
PRTS	Linear	1	Renormalization	No	Yes
FJTS	Linear	1	Shifting fields	Yes	No*
GIVS	Non-linear and linear	2	Both	Yes	Yes

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PRTS

$$t_{H,0} = t_H + \overbrace{(-T^H)}^{\delta t_H}, \qquad t_{h,0} = t_h + \overbrace{(-T^h)}^{\delta t_h}, \qquad t_{A_0,0} = t_{A_0} + \overbrace{(-T^{A_0})}^{\delta t_{A_0}}$$

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FJTS

$$H \to H + \overbrace{\left(T^{H}\right)}^{-\delta t_H} / m_H^2, \qquad h \to h + \overbrace{\left(T^{h}\right)}^{-\delta t_h} / m_h^2, \qquad A_0 \to A_0 + \overbrace{\left(T^{A_0}\right)}^{-\delta t_{A_0}} / m_{A_0}^2$$

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GIVS

$$t_{H,0} = t_H + \overbrace{(-T^{H,\mathrm{nl}})}^{\delta t_{H,1}}, \qquad t_{h,0} = t_h + \overbrace{(-T^{h,\mathrm{nl}})}^{\delta t_{h,1}}, \qquad t_{A_0,0} = t_{A_0} + \overbrace{(-T^{A_0,\mathrm{nl}})}^{\delta t_{A_0,1}}$$

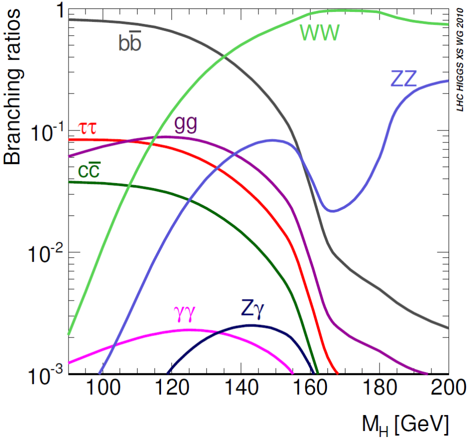
$$H \to H + \overbrace{(T^H - T^{H,\mathrm{nl}})}^{-\delta t_{H,2}}/m_H^2, \quad h \to h + \overbrace{(T^h - T^{h,\mathrm{nl}})}^{-\delta t_{h,2}}/m_h^2, \quad A_0 \to A_0 + \overbrace{(T^{A_0} - T^{A_0,\mathrm{nl}})}^{-\delta t_{A_0,2}}/m_{A_0}^2$$

4. Electroweak NLO corrections to the $h \to b\bar{b}$ and $h \to \tau\bar{\tau}$ decay widths

Dominant fermionic decays of the SM Higgs boson

SM Higgs boson branching ratios with $m_h = 125$ GeV arXiv:1610.07922

$\mathcal{B}(h \to b\bar{b}) =$	0.5824	$\mathcal{B}(h \to WW) =$	0.2137
$\mathcal{B}(h \to gg) =$	0.0819	$\mathcal{B}(h \to \tau^+\tau^-) =$	0.0627
$\mathcal{B}(h \to c\bar{c}) =$	0.0289	$\mathcal{B}(h \to ZZ) =$	0.0262



arXiv:1101.0593

Electroweak NLO corrections in the Type I and Type II THDM

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	$\widetilde{\lambda}_2$	m_{H^+} GeV	m_h GeV	m_H GeV	m_{A_0} GeV
Scenario A	0.6	500	125.25	$200 - 500$	550
Scenario B	0.6	500	125.25	250	$200 - 500$

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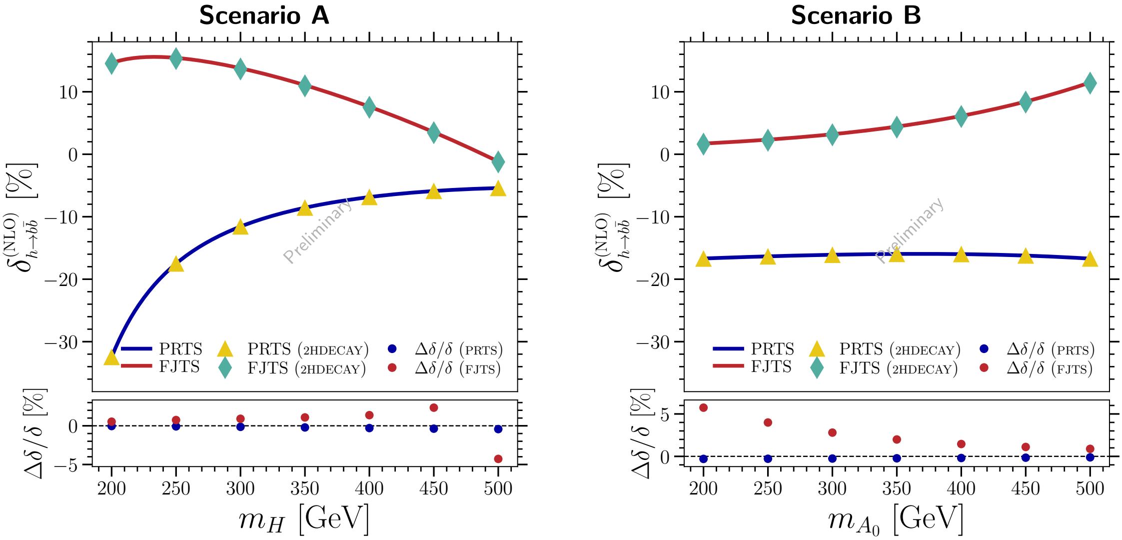
Electroweak NLO corrections in the Type I THDM



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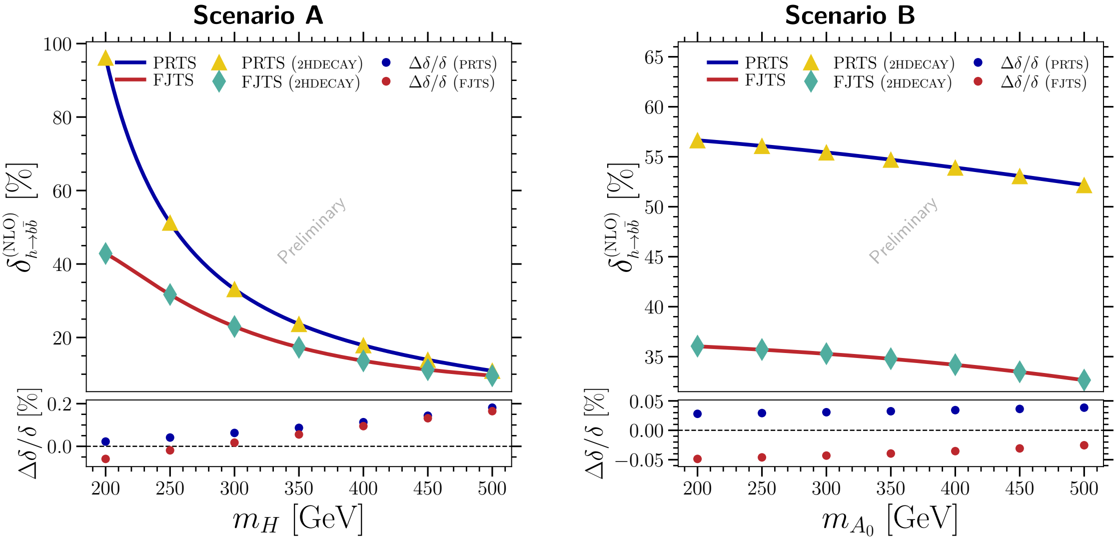
Electroweak NLO corrections in the Type II THDM



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Electroweak NLO corrections in the Type I and II THDM

2HDECAY program implements these couplings instead of the Yukawa matrices:

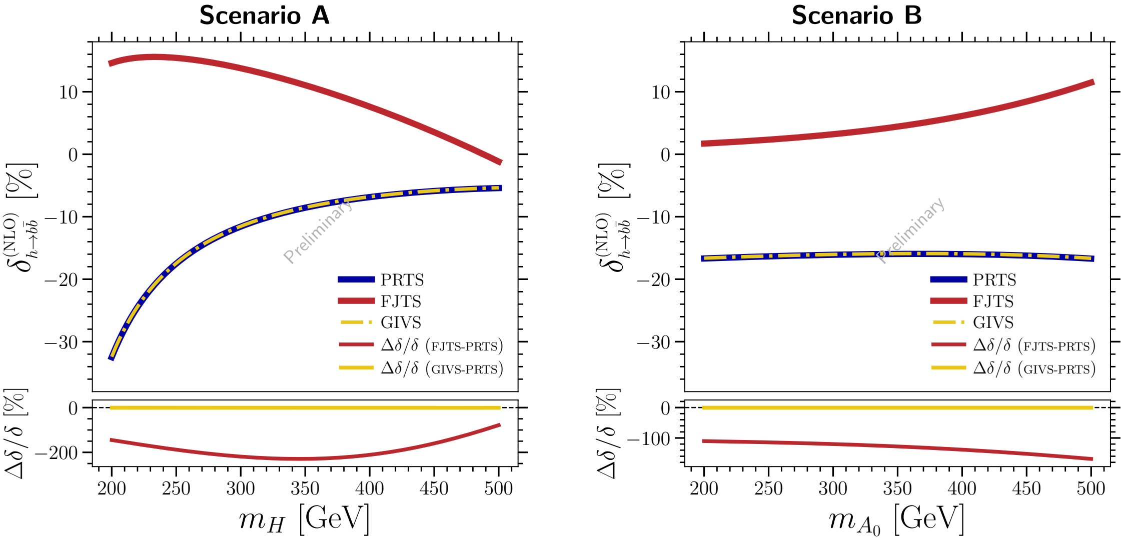
$$\widetilde{G}^{f,2}_{33} = \xi_f(\beta)\frac{\sqrt{2}m_f e}{2 s_{\text{W}} m_{\text{W}}}.$$

	ξ_u	ξ_d	ξ_l
Type I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$

Therefore, the respective counterterms of these couplings are:

$$\delta\widetilde{G}^{f,2}_{33} = \xi_f(\beta)\frac{\sqrt{2}m_f e}{2 s_{\text{W}} m_{\text{W}}}\left(\frac{\delta m_f}{m_f} - \frac{\delta s_{\text{W}}}{s_{\text{W}}} - \frac{\delta m_{\text{W}}^2}{2m_{\text{W}}^2} + \delta Z_e + \frac{\delta\xi_f(\beta)}{\xi_f(\beta)}\right).$$

Comparison of the tadpole schemes in the Type I THDM



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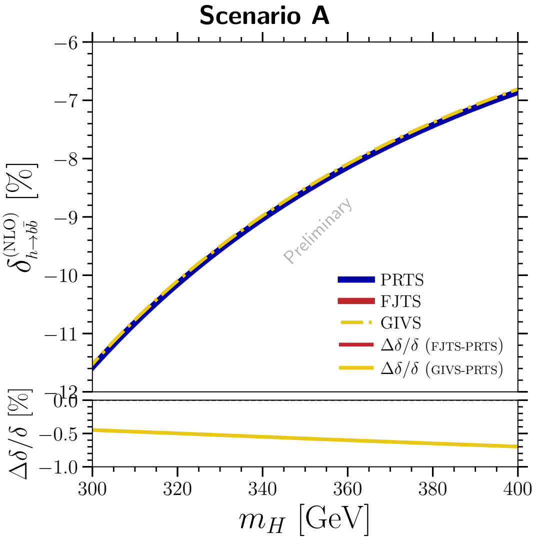

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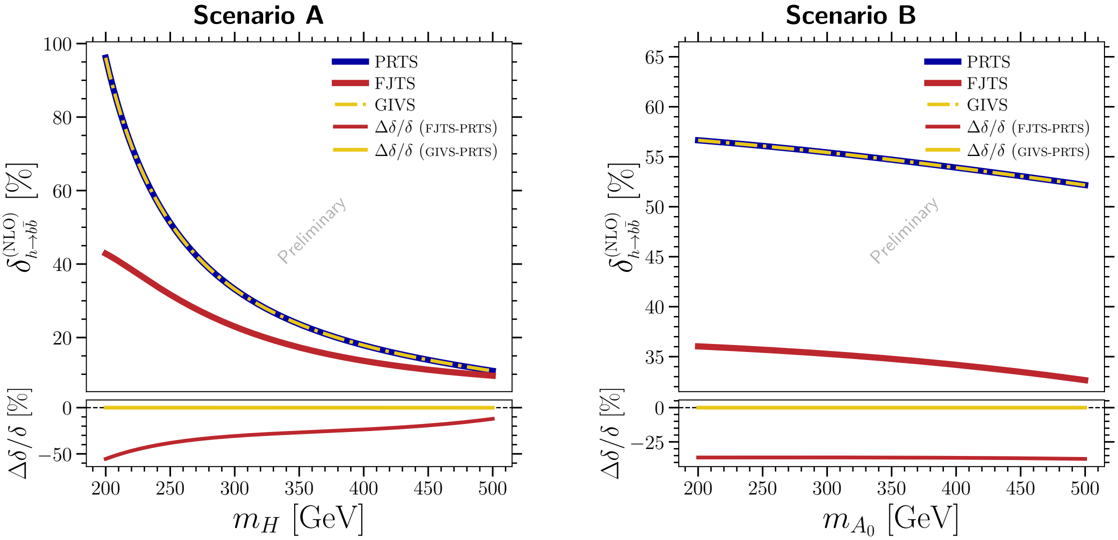

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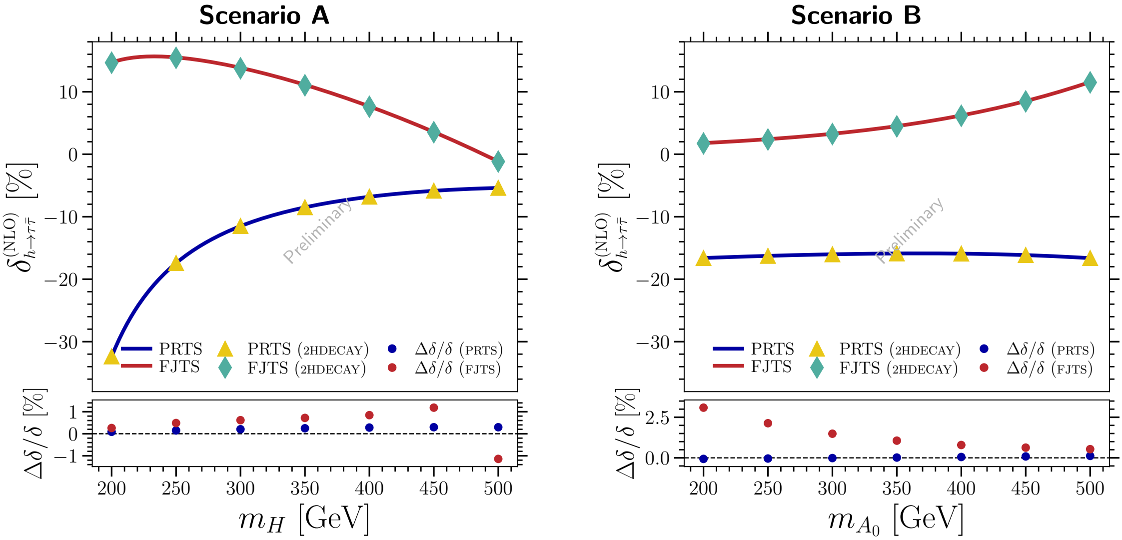

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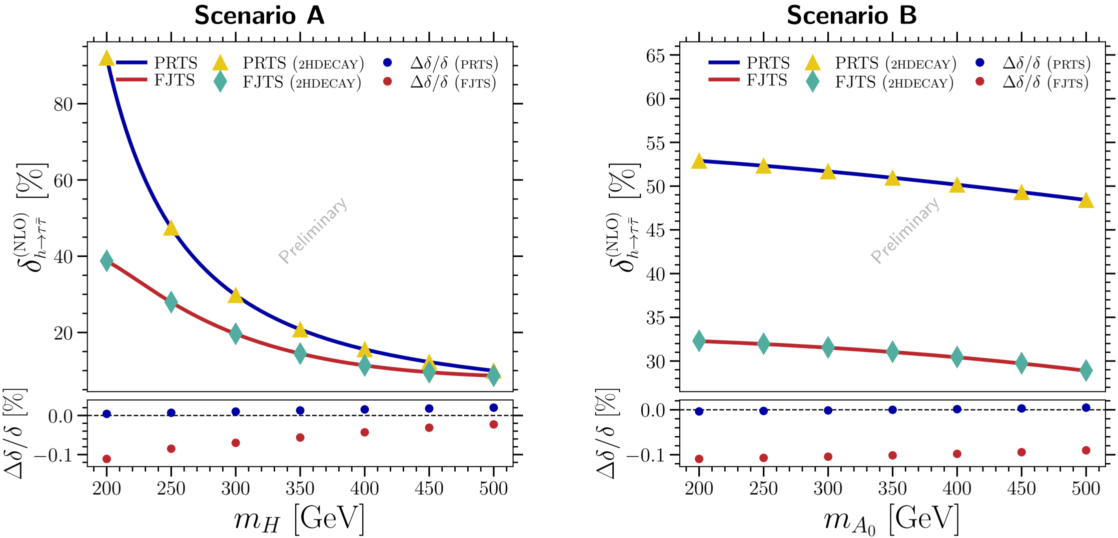

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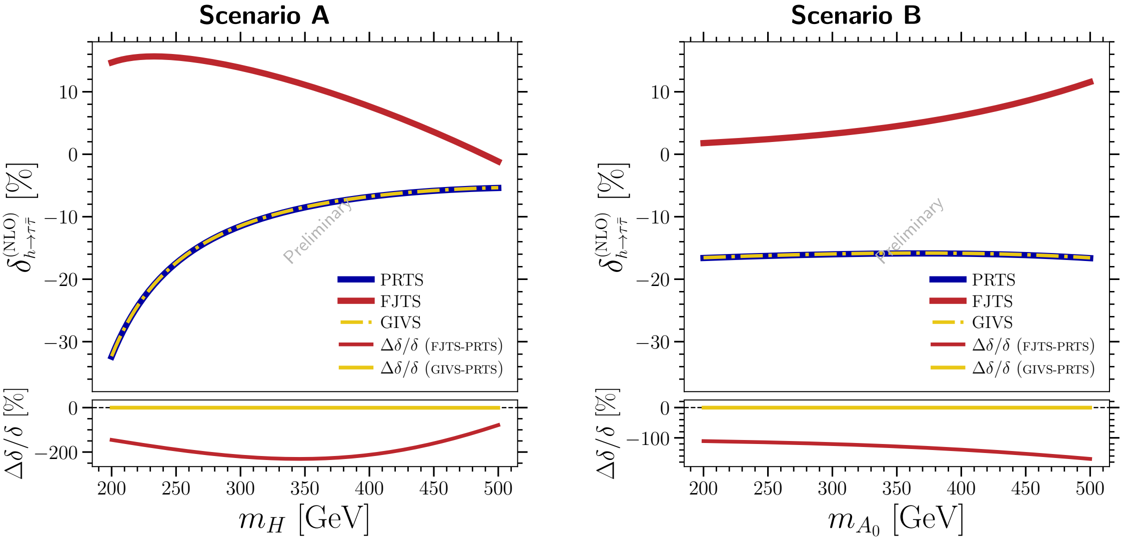

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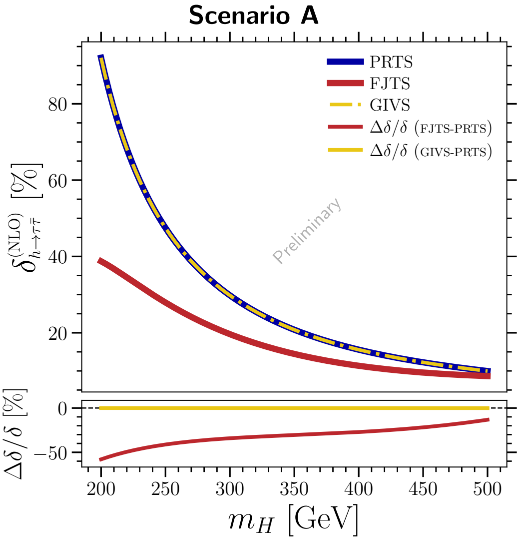

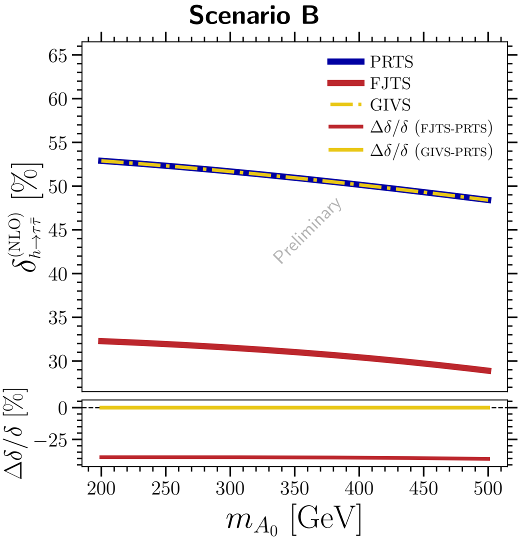

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5. Summary and outlook

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- Extend the analysis to the Type X and Type Y variants of the THDM and to **the MSSM Higgs sector in the low-energy limit**.

Thank you!

Backup

The ρ parameter

The ρ parameter is an important quantity in electroweak physics that measures the ratio between neutral and charged current interaction strengths in the low-energy limit [?]:

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_{k=1}^{n}\left[I_k(I_k+1) - \frac{1}{4}Y_k^2\right] v_k^2}{\sum_{k=1}^{n} \frac{1}{2} Y_k^2 v_k^2}, \quad n = \#\text{ of scalar multiplets.} \tag{7}$$

Conversion from the generic basis to the Higgs basis

In the generic basis, the Higgs doublets are linearly expanded according to

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}\left(v_1 + \rho_1 + i\eta_1\right) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}\left(v_2 + \rho_2 + i\eta_2\right) \end{pmatrix}. \tag{8}$$

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It is more convenient to rotate the Higgs doublets to the Higgs basis [?], where only one of the Higgs doublets (H_1) has a non-vanishing vacuum expectation value v in the neutral component,

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + N_1 + iG_0\right) \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(N_2 + iN_3\right) \end{pmatrix},$$

$$v^2 = v_1^2 + v_2^2, \qquad \tan\beta = \frac{v_2}{v_1}. \tag{9}$$

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Here, the scalar fields in the generic basis and the Higgs basis are related through the following field rotations

The Higgs and Yukawa Lagrangians of the THDM in the Higgs basis

The parameters of the potential in both basis are related according to the following equations

$$
\begin{aligned}
\widetilde{m}_{11}^2 &= m_{11}^2 c_\beta^2 + m_{22}^2 s_\beta^2 - \mathrm{Re}[m_{12}^2] s_{2\beta}, \quad \widetilde{m}_{22}^2 = m_{11}^2 s_\beta^2 + m_{22}^2 c_\beta^2 + \mathrm{Re}[m_{12}^2] s_{2\beta},\\
\widetilde{m}_{12}^2 &= \frac{1}{2}\left(m_{11}^2 - m_{22}^2\right) s_{2\beta} + \mathrm{Re}[m_{12}^2] c_{2\beta} + i\,\mathrm{Im}[m_{12}^2],\\
\widetilde{\lambda}_1 &= \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 + 2 s_{2\beta}\left(c_\beta^2\,\mathrm{Re}[\lambda_6] + s_\beta^2\,\mathrm{Re}[\lambda_7]\right),\\
\widetilde{\lambda}_2 &= \lambda_1 s_\beta^4 + \lambda_2 c_\beta^4 + \frac{1}{2}\lambda_{345} s_{2\beta}^2 - 2 s_{2\beta}\left(s_\beta^2\,\mathrm{Re}[\lambda_6] + c_\beta^2\,\mathrm{Re}[\lambda_7]\right),\\
\widetilde{\lambda}_3 &= \lambda_3 + \frac{1}{4} s_{2\beta}^2\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right) - s_{2\beta} c_{2\beta}\,\mathrm{Re}[\lambda_6 - \lambda_7],\\
\widetilde{\lambda}_4 &= \lambda_4 + \frac{1}{4} s_{2\beta}^2\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right) - s_{2\beta} c_{2\beta}\,\mathrm{Re}[\lambda_6 - \lambda_7],\\
\widetilde{\lambda}_5 &= \frac{1}{4} s_{2\beta}^2\left(\lambda_1 + \lambda_2 - 2\lambda_{345}\right) + \mathrm{Re}[\lambda_5] + i c_{2\beta}\,\mathrm{Im}[\lambda_5] - s_{2\beta} c_{2\beta}\,\mathrm{Re}[\lambda_6 - \lambda_7] - i s_{2\beta}\,\mathrm{Im}[\lambda_6 - \lambda_7],\\
\widetilde{\lambda}_6 &= -\frac{1}{2} s_{2\beta}\left(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_{2\beta} - i\,\mathrm{Im}[\lambda_5]\right) + c_\beta c_{3\beta}\,\mathrm{Re}[\lambda_6] + s_\beta s_{3\beta}\,\mathrm{Re}[\lambda_7] + i c_\beta^2\,\mathrm{Im}[\lambda_6] + i s_\beta^2\,\mathrm{Im}[\lambda_7],\\
\widetilde{\lambda}_7 &= -\frac{1}{2} s_{2\beta}\left(\lambda_1 s_\beta^2 - \lambda_2 c_\beta^2 + \lambda_{345} c_{2\beta} + i\,\mathrm{Im}[\lambda_5]\right) + s_\beta s_{3\beta}\,\mathrm{Re}[\lambda_6] + c_\beta c_{3\beta}\,\mathrm{Re}[\lambda_7] + i s_\beta^2\,\mathrm{Im}[\lambda_6] + i c_\beta^2\,\mathrm{Im}[\lambda_7].
\end{aligned}
\tag{11}
$$

Physical Higgs bosons in the THDM

$$M_{\text{neutral}} = \begin{pmatrix} -\frac{t_{N_1}}{v} & -\frac{t_{N_2}}{v} & \frac{t_{N_3}}{v} & 0 \\ -\frac{t_{N_2}}{v} & m_{H^+}^2 + \frac{1}{2}\left(\tilde{\lambda}_4 - \text{Re}[\tilde{\lambda}_5]\right) v^2 & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5]v^2 & -\text{Im}[\tilde{\lambda}_6]v^2 - \frac{t_{N_3}}{v} \\ \frac{t_{N_3}}{v} & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5]v^2 & m_{H^+}^2 + \frac{1}{2}\left(\tilde{\lambda}_4 + \text{Re}[\tilde{\lambda}_5]\right) v^2 & \text{Re}[\tilde{\lambda}_6]v^2 - \frac{t_{N_2}}{v} \\ 0 & -\text{Im}[\tilde{\lambda}_6]v^2 - \frac{t_{N_3}}{v} & \text{Re}[\tilde{\lambda}_6]v^2 - \frac{t_{N_2}}{v} & \tilde{\lambda}_1 v^2 - \frac{t_{N_1}}{v} \end{pmatrix}. \quad (16)$$

Now we diagonalize $M_{\text{neutral}}^{(3\times 3)}$ (i.e. the components of M_{neutral} associated with the mixing terms of the fields N_1, N_2, and N_3)

$$\begin{pmatrix} G_0 \\ N_3 \\ N_2 \\ N_1 \end{pmatrix} = R_{\theta_1,\theta_2,\theta_3} \begin{pmatrix} G_0 \\ A_0 \\ h \\ H \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 c_3 & s_1 s_2 c_3 - c_1 s_3 & c_1 s_2 c_3 + s_1 s_3 \\ 0 & c_2 s_3 & s_1 s_2 s_3 + c_1 c_3 & c_1 s_2 s_3 - s_1 c_3 \\ 0 & -s_2 & s_1 c_2 & c_1 c_2 \end{pmatrix} \begin{pmatrix} G_0 \\ A_0 \\ h \\ H \end{pmatrix},$$

$$= \begin{pmatrix} T_{G_0 G_0} & T_{G_0 A_0} & T_{G_0 h} & T_{G_0 H} \\ T_{A_0 G_0} & m_{A_0}^2 & 0 & 0 \\ \ldots \end{pmatrix}$$

Mass eigenstates and Yukawa structure in the Higgs basis

Mass eigenstates of the neutral scalar fields

$$V_{\text{THDM}}\Big|_{\text{Bilinear}} = \begin{pmatrix} G^+ & H^+ \end{pmatrix} M_{\text{charged}} \begin{pmatrix} G^- \\ H^- \end{pmatrix} + \frac{1}{2} \begin{pmatrix} G_0 & A_0 & h & H \end{pmatrix} N_{\text{neutral}} \begin{pmatrix} G_0 \\ A_0 \\ h \\ H \end{pmatrix},$$

$$N^{(3\times3)}_{\text{neutral}} = R^T(\theta_1, \theta_2, \theta_3) M^{(3\times3)}_{\text{neutral}} R(\theta_1, \theta_2, \theta_3) = \text{diag}\left(m^2_{A_0}, m^2_h, m^2_H\right).$$

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The Yukawa matrix associated to the Higgs doublet H_1 is diagonalized as in the SM

$$\widetilde{G}^{f,1}_{ij} = \frac{\sqrt{2} m_{f,i}}{v}\delta_{ij} = \sum_{k,m} V^{f,\mathsf{L}}_{ik} G^{f,\mu}_{km} U^\dagger_{\mu,1} V^{f,\mathsf{R}\dagger}_{mi}\delta_{ij}, \qquad f_i^{\mathsf{L/R}} = \sum_k V^{f,\mathsf{L/R}}_{ik} f'^{\mathsf{L/R}}_k, \qquad f = l,u,d.$$

In contrast, the Yukawa matrix associated with the doublet H_2 remains more general

$$\widetilde{G}^{f,2} \ldots \sum V^{f,\mathsf{L}} G^{f,\mu} U^\dagger \ldots V^{f,\mathsf{R}\dagger}$$

Relations between quartic couplings, Higgs masses and mixing angles

$$\widetilde{\lambda}_1 = \frac{1}{v^2}\left(m_H^2 c_1^2 c_2^2 + m_h^2 s_1^2 c_2^2 + m_{A_0}^2 s_2^2\right) + \frac{t_{N_1}}{v^3},$$
$$\widetilde{\lambda}_4 = \frac{1}{4v^2}\left(3m_H^2 + 3m_h^2 + 2m_{A_0}^2 - 8m_{H^\pm}^2 + 2(m_h^2 - m_H^2)c_{21}c_2^2 - (m_H^2 + m_h^2 - 2m_{A_0}^2)c_{22}\right)$$
$$\mathrm{Re}[\widetilde{\lambda}_5] = \frac{1}{4v^2}((2(m_H^2 + m_h^2 - 2m_{A_0}^2)c_2^2 + (m_H^2 - m_h^2)c_{21}(c_{22} - 3))c_{23} + 4(m_h^2 - m_H^2)s_{21}s_2 s_{23}), \tag{18}$$

$$\mathrm{Im}[\widetilde{\lambda}_5] = \frac{1}{4v^2}((2(m_H^2 + m_h^2 - 2m_{A_0}^2)c_2^2 + (m_H^2 - m_h^2)c_{21}(c_{22} - 3))s_{23} - 4(m_h^2 - m_H^2)s_{21}s_2 c_{23}),$$
$$\mathrm{Re}[\lambda_6] = \frac{1}{v^2}((m_h^2 - m_H^2)s_1 c_1 c_3 + (m_H^2 c_1^2 + m_h^2 s_1^2 - m_{A_0})s_2 s_3)c_2 + \frac{t_{N_2}}{v^3},$$
$$\mathrm{Im}[\lambda_6] = \frac{1}{v^2}((m_h^2 - m_H^2)s_1 c_1 s_3 + (m_H^2 c_1^2 - m_h^2 s_1^2 - m_{A_0})s_2 c_3)c_2 - \frac{t_{N_3}}{v^3}. \tag{19}$$

Modified Minimal Subtraction conditions

The renormalization constants fixed by modified minimal subtraction ($\overline{\text{MS}}$) conditions only subtract the terms proportional to the UV divergence (i.e. $\Delta_{\text{UV}} = 1/\epsilon - \gamma_{\text{E}} + \ln(4\pi)$).

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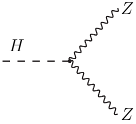
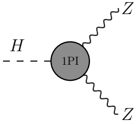
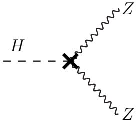
Higgs mixing angles:

$$\Gamma_{\text{R}}^{HZZ}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\text{UV}} = 0, \quad \Gamma_{\text{R}}^{A_0 ZZ}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\text{UV}} = 0, \quad \Gamma_{\text{R}}^{W^+ H H^+}(\delta\theta_1, \delta\theta_2, \delta\theta_3)\Big|_{\text{UV}} = 0,$$

$i\Gamma_{\text{R}}^{HZZ} =$ 

$+$ 

$+$ 

$+ \quad \mathcal{O}(e^4).$ (20)

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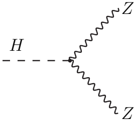

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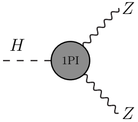

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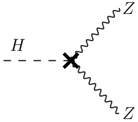

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$$\Gamma_{\text{R}}^{HH^+H^-}(\delta\widetilde{\lambda}_3, \delta\widetilde{\lambda}_7)\Big|_{\text{UV}} = 0, \quad \Gamma_{\text{R}}^{hH^+H^-}(\delta\widetilde{\lambda}_3, \delta\widetilde{\lambda}_7)\Big|_{\text{UV}} = 0, \quad \Gamma_{\text{R}}^{A_0 H^+H^-}(\delta\widetilde{\lambda}_3, \delta\widetilde{\lambda}_7)\Big|_{\text{UV}} = 0, \tag{21}$$

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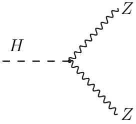
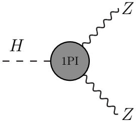
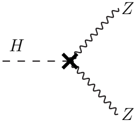
$i\Gamma_{\mathsf{R}}^{HZZ} =$ 

$+$ 

$+$ 

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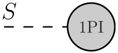
Tadpole renormalization (TR): Linear parametrization

At higher orders, tadpole contributions are given by Feynman diagrams containing subdiagrams of the form

$$\Gamma^S = T^S = \quad (23)$$



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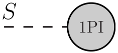

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Most of the EW corrections are computed by parametrizing the Higgs doublets in the linear representation [**?**]:

$$\mathbf{\Phi}_n \equiv (\Phi_n^c, \Phi_n) = \frac{1}{\sqrt{2}}\left[(v_n + \rho_n)\mathbb{1} + 2i\boldsymbol{\phi}_n\right], \quad \boldsymbol{\phi}_n = \frac{\phi_{nj}\boldsymbol{\sigma}_j}{2}, \quad \omega_n^{\pm} = \frac{(\phi_{n2} \pm i\phi_{n1})}{\sqrt{2}}, \quad \eta_n = -\phi_{n3} \tag{24}$$

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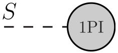
$$\Gamma^S = T^S = \quad$$ 

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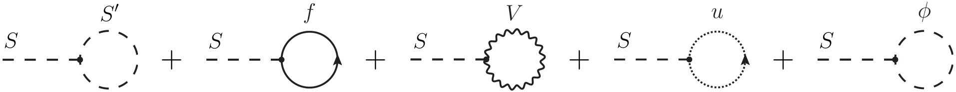
In this parametrization, the Higgs fields H, h, A_0 and the tadpole contributions T^{H,h,A_0} are gauge dependent

$$\Gamma^S = T^S = $$ 

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Tadpole renormalization (TR): Non-linear parametrization

Additionally, a non-linear parametrization can be used in the calculation of EW corrections to avoid gauge dependences [?]

$$\mathbf{\Phi} \equiv \exp\left(\frac{2i\boldsymbol{\zeta}}{v}\right) \cdot \frac{1}{\sqrt{2}}\left[(v_n + \rho_n)\mathbb{1} + 2ib_n\boldsymbol{\phi}\right], \quad \boldsymbol{\zeta} = \frac{\zeta_j\boldsymbol{\sigma}_j}{2}, \quad \boldsymbol{\phi} = \frac{\phi_j\boldsymbol{\sigma}_j}{2}, \quad b_1 = -s_\beta, \quad b_2 = c_\beta,$$

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}, \quad \begin{pmatrix} G_0 \\ N_3 \end{pmatrix} = -\begin{pmatrix} \zeta_3 \\ \phi_3 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \zeta_2 \pm i\zeta_1 \\ \phi_2 \pm i\phi_1 \end{pmatrix}. \quad (26)$$

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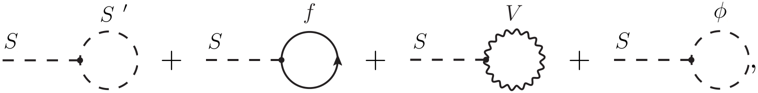
In this representation, the Higgs fields H, h, A_0 and the tadpole contributions $T_{\mathrm{nl}}^{H,h,A_0}$ are gauge-invariant

$$\Gamma_{\mathrm{nl}}^S = T_{\mathrm{nl}}^S = \quad (27)$$



$S_{\mathrm{nl}} = H, h, A_0, \quad {S_{\mathrm{nl}}}' = H, h, A_0, H^\pm, \quad f = e, \mu, \tau, \mathrm{u, c, t, d, s, b}, \quad V = W^\pm, Z, \quad u = u^Z, u^\pm, \quad \phi$

TR: Fleischer–Jegerlehner Tadpole Scheme (FJTS)

In this scheme [?], the bare tadpole constants $t_{H,0}^{\mathsf{FJTS}}, t_{h,0}^{\mathsf{FJTS}}, t_{A_0,0}^{\mathsf{FJTS}}$ are set to zero and the tadpole counterterms are introduced by shifting the bare Higgs fields according to

$$H_B \to H_B + \Delta v_H^{\mathsf{FJTS}}, \quad h_B \to h_B + \Delta v_h^{\mathsf{FJTS}}, \quad A_{0,B} \to A_{0,B} + \Delta v_{A_0}^{\mathsf{FJTS}}. \tag{28}$$

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This shifting yields the terms

$$\mathcal{L}_{\mathsf{Higgs}} \supset -m^2_{H,0}\Delta v^{\mathsf{FJTS}}_H \cdot H_B - m^2_{h,0}\Delta v^{\mathsf{FJTS}}_h \cdot h_B - m^2_{A_0,0}\Delta v^{\mathsf{FJTS}}_{A_0} \cdot A_{0,B} \tag{29}$$

where the constants $\Delta v^{\mathsf{FJTS}}_H, \Delta v^{\mathsf{FJTS}}_h, \Delta v^{\mathsf{FJTS}}_{A_0}$ are determined by

$$\begin{aligned} \Delta v^{\mathsf{FJTS}}_H &= -\,\delta t^{\mathsf{FJTS}}_H / m^2_H = T^H / m^2_H, \\ \Delta v^{\mathsf{FJTS}}_h &= -\,\delta t^{\mathsf{FJTS}}_h / m^2_h = T^h / m^2_h, \\ \Delta v^{\mathsf{FJTS}}_{A_0} &= -\,\delta t^{\mathsf{FJTS}}_{A_0} / m^2_{A_0} = T^{A_0} / m^2_{A_0}. \end{aligned} \tag{30}$$

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- Might lead to large corrections to the mass renormalization constants in the $\overline{\text{MS}}$ scheme.
- Predictions for observables in the OS and $\overline{\text{MS}}$ schemes are gauge independent.

TR: Parameter Renormalized Tadpole Scheme (PRTS)

In this scheme [?], the bare tadpole constants $t_{H,0}^{\text{PRTS}}, t_{h,0}^{\text{PRTS}}, t_{A_0,0}^{\text{PRTS}}$ are renormalized according to

$$\begin{aligned} t_{H,0}^{\text{PRTS}} &= t_H^{\text{PRTS}} + \delta t_H^{\text{PRTS}}, & \delta t_H^{\text{PRTS}} &= -T^H, \\ t_{h,0}^{\text{PRTS}} &= t_h^{\text{PRTS}} + \delta t_h^{\text{PRTS}}, & \delta t_h^{\text{PRTS}} &= -T^h, \\ t_{A_0,0}^{\text{PRTS}} &= t_{A_0}^{\text{PRTS}} + \delta t_{A_0}^{\text{PRTS}}, & \delta t_{A_0}^{\text{PRTS}} &= -T^{A_0}. \end{aligned} \tag{31}$$

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- Leads to small contributions to the mass renormalization constants in the $\overline{\text{MS}}$ scheme.
- Introduces a gauge dependence in predictions for observables in the $\overline{\text{MS}}$ scheme.

TR: Gauge-Invariant Vacuum expectation value Scheme (GIVS)

This tadpole scheme constructs the tadpole counterterms δt_S^{GIVS} $(S = H, h, A_0)$ from two kind of tadpole counterterms $\delta t_{S,1}^{\text{GIVS}}$ and $\delta t_{S,2}^{\text{GIVS}}$ [?]. The first kind of counterterms $\delta t_{S,1}^{\text{GIVS}}$ are introduced through the application of the PRTS in the non-linear Higgs expansion to take advantage of the gauge independence of T_{nl}^S,

$$t_{S,0}^{\text{GIVS}} = t_S^{\text{GIVS}} + \delta t_{S,1}^{\text{GIVS}}, \qquad t_{S,0}^{\text{GIVS}} \equiv t_{S_{\text{nl}},0}^{\text{PRTS}}, \qquad t_S^{\text{GIVS}} \equiv t_{S_{\text{nl}}}^{\text{PRTS}} = 0, \qquad \delta t_{S,1}^{\text{GIVS}} \equiv \delta t_{S_{\text{nl}}}^{\text{PRTS}} = -T \tag{32}$$

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$\delta t_{H,1}^{\text{GIVS}}$ are not sufficient to fully cancel the tadpole contributions T^S (i.e., $T^S \neq T_{\text{nl}}^S$). The complete cancellation is achieved by the second kind of counterterms $\delta t_{S,2}^{\text{GIVS}}$, which are introduced by shifting the bare Higgs fields as in the FJTS,

$$S_B \to S_B + \Delta v_S^{\text{GIVS}}, \qquad \Delta v_S^{\text{GIVS}} = -\delta t_{S,2}^{\text{GIVS}} / m_S^2, \tag{33}$$

such that

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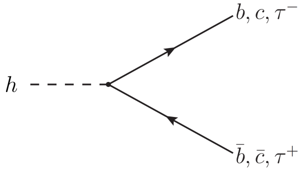
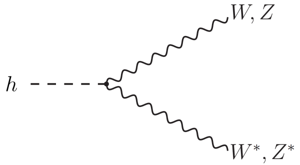
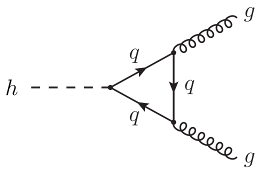
Dominant decay channel of the SM Higgs boson

SM Higgs boson branching ratios with $m_h = 125$ GeV [?].

$\mathcal{B}(h \to b\bar{b}) =$	0.5824	$\mathcal{B}(h \to WW) =$	0.2137
$\mathcal{B}(h \to gg) =$	0.0819	$\mathcal{B}(h \to \tau^+\tau^-) =$	0.0627
$\mathcal{B}(h \to c\bar{c}) =$	0.0289	$\mathcal{B}(h \to ZZ) =$	0.0262







Electroweak NLO correction in the CP-conserving Type I THDM with softly broken $\mathbb{Z}_2$ symmetry

To compare our calculation for a specific point of the space input parameter with the results given by the 2HDECAY program developed by Krause, Mühlleitner, and Spira [?], we diagonalize the neutral mass matrix $M^{(3\times3)}_{\text{neutral}}$ of the bilinear terms excluding the tadpole contributions:

$$M_{\text{neutral}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m^2_{H^+} + \frac{1}{2}\left(\tilde{\lambda}_4 - \text{Re}[\tilde{\lambda}_5]\right)v^2 & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5]v^2 & -\text{Im}[\tilde{\lambda}_6]v^2 \\ 0 & -\frac{1}{2}\,\text{Im}[\tilde{\lambda}_5]v^2 & m^2_{H^+} + \frac{1}{2}\left(\tilde{\lambda}_4 + \text{Re}[\tilde{\lambda}_5]\right)v^2 & \text{Re}[\tilde{\lambda}_6]v^2 \\ 0 & -\text{Im}[\tilde{\lambda}_6]v^2 & \text{Re}[\tilde{\lambda}_6]v^2 & \tilde{\lambda}_1 v^2 \end{pmatrix} + \frac{1}{v}\begin{pmatrix} -t_{N_1} & -t_{N_2} & t_{N_3} & 0 \\ -t_{N_2} & 0 & 0 & -t_{N_3} \\ t_{N_3} & 0 & 0 & -t_{N_2} \\ 0 & -t_{N_3} & -t_{N_2} & -t_{N_1} \end{pmatrix},$$

$$N_{\text{neutral}} = R^T_{\theta_1,\theta_2,\theta_3} M_{\text{neutral}} R_{\theta_1,\theta_2,\theta_3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m^2_{A_0} & 0 & 0 \\ 0 & 0 & m^2_h & 0 \\ 0 & 0 & 0 & m^2_H \end{pmatrix} + \begin{pmatrix} T_{G_0G_0} & T_{G_0A_0} & T_{G_0h_0} & T_{G_0H} \\ T_{A_0G_0} & T_{A_0A_0} & T_{A_0h} & T_{A_0H} \\ T_{h_0G_0} & T_{hA_0} & T_{hh} & T_{hH} \\ T_{HG_0} & T_{HA_0} & T_{Hh} & T_{HH} \end{pmatrix}. \quad (35)$$