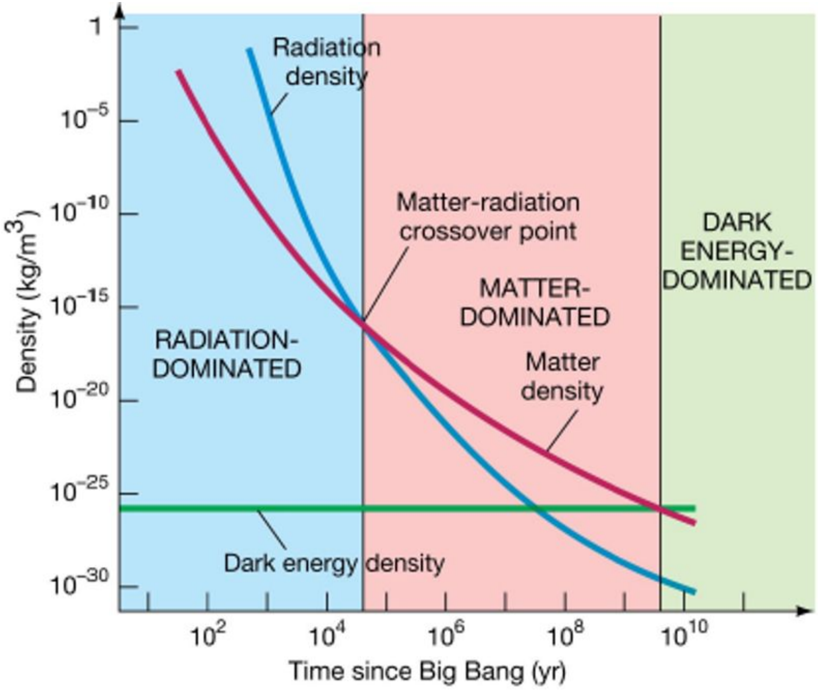
Neutrino Decoupling in the Early Universe

A quick calculation

Neutrino Decoupling in the Early Universe

- Neutrinos decouple from the cosmic soup when the reaction rate $\Gamma(T)$ falls behind the Hubble Rate $H(T)$
  - Same as saying Neutrinos fall out of thermal equilibrium
- This process happened closely after the universe started to expand, about 1 s after the big bang
  - At that time the universe and its expansion were dominated by radiation



Terms to Calculate

Important Equations:

Friedmann Equation in spatially flat radiation dominated universe:

$$H = \sqrt{\frac{8\pi G \rho(T)}{3}}$$

ϱ(T) is fermion energy density, G is the Gravitational Constant (= 1 / M_{Pl}^2)

Weak Reaction Rate:

$$\Gamma(T) = n(T)\,\sigma\, v \simeq n(T)\, G_F^2\, T^2$$

n(T) is fermion number density, G_F is Fermi Constant

n(T) and ϱ(T) can be calculated from Stat Mech Integrals:

$$n(T) = \int \frac{d^3p}{(2\pi)^3}\, f(p,T)$$

$$\rho(T) = \int \frac{d^3p}{(2\pi)^3}\, \sqrt{p^2+m^2}\, f(p,T)$$

Where f(p,T) is the Fermi-Dirac Distribution function

$$f(p,T) = g_i \left(\exp\left(\frac{\sqrt{p^2+m^2}}{T} \right) + 1 \right)^{-1}$$

Solving for densities

$$n(T) = \int \frac{d^3p}{(2\pi)^3} f(p,T)$$

Which evaluates to:

$$n(T) = \frac{3\,\zeta(3)}{4\pi^2} T^3 \sum_i g_i$$

With g_i being the fermion degrees of freedom. For 5 active fermions (3ν, e^+,e^-) this sums to 10

$$n(T) \simeq \frac{30\,\zeta(3)}{4\pi^2} T^3$$

$$\rho(T) = \int \frac{d^3p}{(2\pi)^3} \sqrt{p^2+m^2}\, f(p,T)$$

Which evaluates to:

$$\rho(T) = \frac{7\pi^2}{240} T^4 \sum_i g_i$$

Once More g_i terms sum to 10

$$\rho(T) \simeq \frac{7\pi^2}{24} T^4$$

Solving for Decoupling Temperature

To Find decoupling temperature we set **Γ(T) = H(T)**:

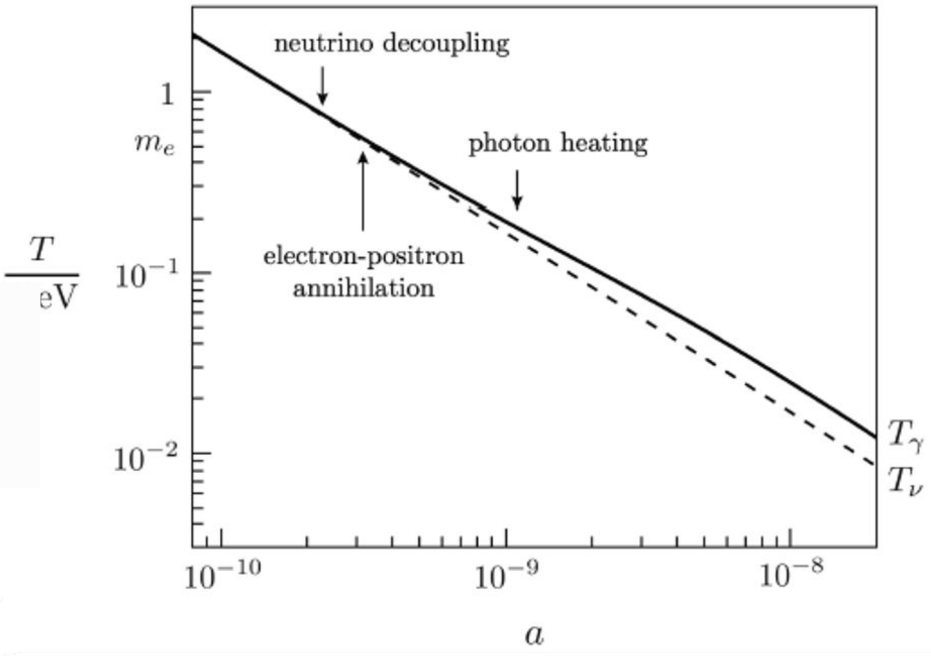
$$30\,\zeta(3)\,T^5\,G_F^2\,/\,4\pi^2 = \frac{\sqrt{7\pi^3/9}\,T^2}{M_{\rm Pl}}$$

Solving for T:

$$T_D = \left[\frac{\sqrt{7\pi^3/9}\;4\pi^2}{30\,\zeta(3)\,G_F^2\,M_{\rm Pl}}\right]^{1/3}$$

Plugging in Numbers Results in decoupling Temp

T_D ≅ 1.47 MeV



EXTRA/CURIOSITY

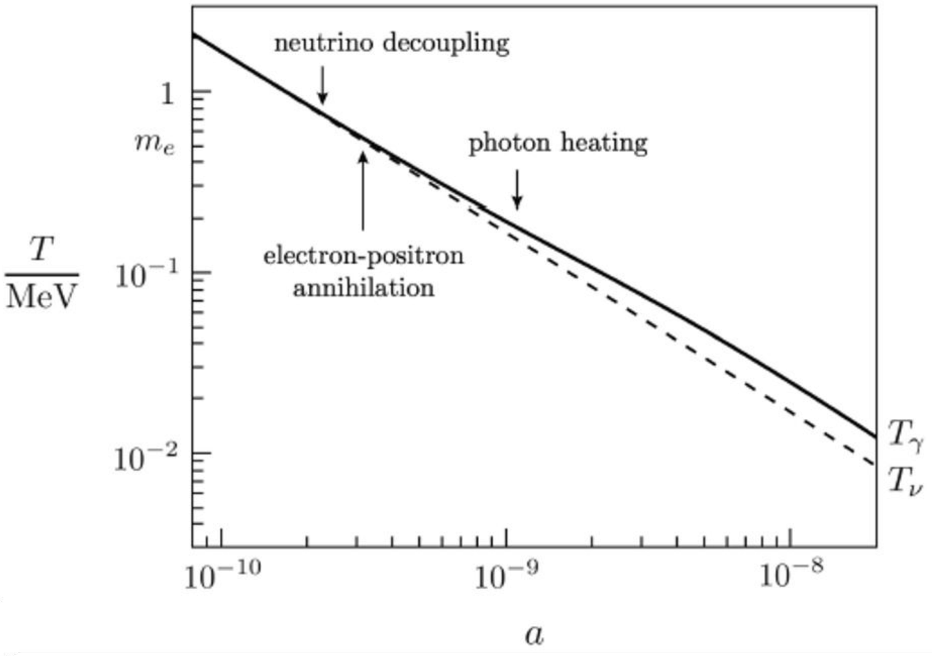
The decoupling of the neutrinos was not instant but gradually

so it was not completely over when at around T~ 2me the electron/positron annihilation started to heat up photons (and also Neutrino)

$$\rho_{\mathrm{R}} = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right)$$

This led to obtain an effective number of neutrino species Neff = 3.044 a little bit higher with respect to the 3 expected by the Standard Model

This value has been computed but not measured. A significant different value of Neff could lead to new physics beyond standard model



Astrophysical Neutrinos

Astrophysical Neutrino Problem #2:

The neutrino fluxes emerge from sources with neutrino flavor ratios that change as the fluxes propagate and oscillate over astrophysical distances to arrive at Earth.

Using current central values of PMNS elements, estimate the neutrino flavor ratios at Earth for

(a) pion dominated fluxes (pions and muons decay promptly)

(b) muon damped (neglect neutrinos from muon decays)

(c) neutron decay dominated (only neutrinos from neutron decay)

*not distinguishing between particle and antiparticle

Astrophysical Neutrino Oscillations:

For neutrino sources located at astrophysical distances (L), the oscillation length, $\lambda_{jk} << L$, with

$$\lambda_{jk} = \frac{4\pi E}{\Delta m^2_{jk}}$$

So, even for extremely high energies the terms involving Δm^2 in the oscillation probabilities are effectively averaged out; oscillation probabilities are reduced to:

$$P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Source flavor composition: $\Phi^0_e : \Phi^0_\mu : \Phi^0_\tau \xrightarrow{\text{Oscillation}}$ Flux at Earth:

$$\Phi_\alpha = \sum_\beta P_{\alpha\beta} \Phi^0_\beta$$

Astrophysical Neutrino Oscillations:

central values of PMNS matrix

$$U_{\mathrm{TBM}} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & \sqrt{2} & 0 \\ -1 & \sqrt{2} & -\sqrt{3} \\ -1 & \sqrt{2} & \sqrt{3} \end{pmatrix} \longrightarrow P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Probability of oscillation for $\lambda_{jk} << L$ sources

$$P_{\mathrm{TBM}} = \frac{1}{18} \begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}$$

Astrophysical neutrino sources:

1. Pion dominated sources: $\pi^+ \to \mu^+ \nu_\mu$, $\mu^+ \to e^+ \nu_e \overline{\nu}_\mu$

$$\boxed{\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0 \equiv 1 : 2 : 0}$$

2. Muon damped (secondary muons do not decay):

$$\pi^+ \to \mu^+ \nu_\mu$$

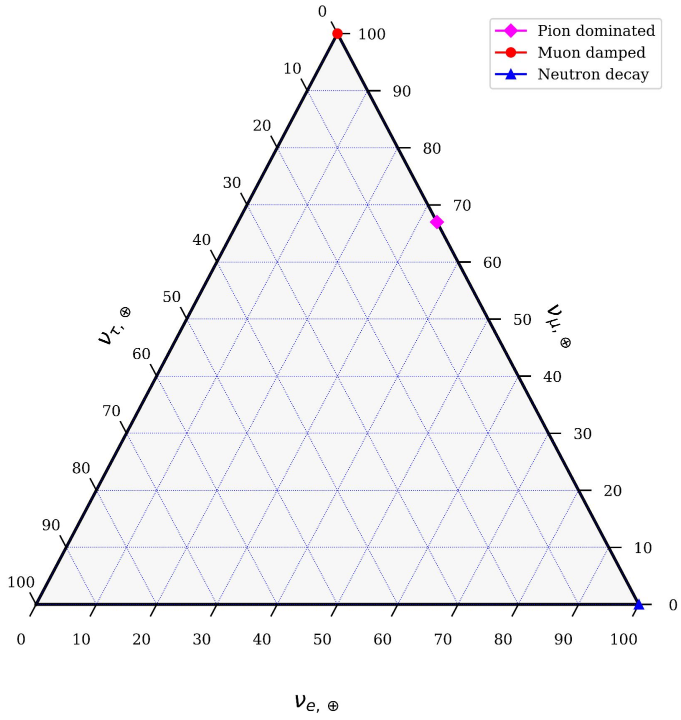
$$\boxed{\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0 \equiv 0 : 1 : 0}$$

3. Neutron decay sources:

$$n \to p\, e^- \,\overline{\nu}_e$$

$$\boxed{\Phi_e^0 : \Phi_\mu^0 : \Phi_\tau^0 \equiv 1 : 0 : 0}$$

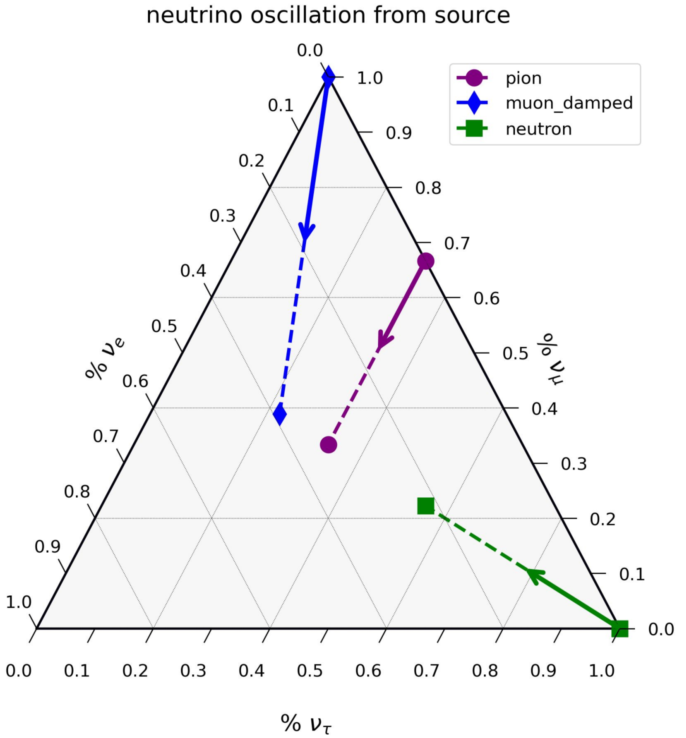
Flavor composition ternary plot at source:



PMNS matrix central values and flavor ratio on Earth:

		Normal Ordering ($\Delta\chi^2 = 0.6$)	
		bfp $\pm 1\sigma$	3σ range
IC19 without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \to 0.596$
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \to 50.5$
	$\sin^2\theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \to 0.02376$
	$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \to 8.87$
	$\delta_{\mathrm{CP}}/^\circ$	177^{+19}_{-20}	$96 \to 422$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \to +2.606$

We use the NuFit 2024 parameters of theta_12, theta_23, theta_13 and delta_cp for the central values of the PMNS matrix and calculate the final flavor ratio on Earth.



backup slides

COSMOLOGY - PROBLEM 10

Assuming that radiation now contains three relativistic neutrino species and the CMB photons, and that the current matter density is $\rho_{m_0} = \Omega_m \rho_c = 0.3\rho_c$, **estimate the temperature of radiation/matter equality,** T_{eq}, defined as $\rho_{rad}(T_{eq}) = \rho_m(T_{eq})$

COSMOLOGY - PROBLEM 10 ASSUMPTIONS

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We assume that the radiation is made up of photons (as a blackbody with $T = 2.7260$ K, Fixsen 2009) and neutrinos with $\rho_\nu = N_{\rm eff}(7/8)(4/11)^{4/3}\rho_\gamma$ and[9] $N_{\rm eff} = 3.046$ (Mangano et al. 2002). The neutrinos are assumed to have very low masses, which we approximate as a single eigenstate with $m_\nu = 0.06$ eV.

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ASIDE 1: ρ vs. Ω

$$\rho_{m_0} = \Omega_m \rho_c = 0.3\rho_c$$

- $\rho_{(m)}$ = mass-energy density (of matter) in the universe
- $\Omega_{(m)}$ = mass-energy density (of matter) in the universe *with respect to* ρ_c

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$$\frac{\rho_{m_0} = \Omega_m \cancel{\rho_c} = 0.3\cancel{\rho_c}}{\cancel{\rho_c}} \qquad \therefore \Omega_i = \frac{\rho_i}{\rho_c}, \Omega_m = 0.3$$

COSMOLOGY - PROBLEM 10 *IN SIMPLE TERMS*

Assuming that radiation now contains three relativistic neutrino species and the CMB photons, and that the current matter density is $\rho_{m_0} = \Omega_m \rho_c = 0.3\rho_c$, **estimate the temperature of radiation/matter equality,** T_{eq}, defined as $\rho_{rad}(T_{eq}) = \rho_m(T_{eq})$

What was the temperature when $\Omega_{\mathrm{matter}} = \Omega_{\mathrm{radiation}}$?

ASIDE 2: Redshift (z) as a proxy

To look back in time in cosmology, we need a quantitative way to relate our current conditions with the conditions during the time of interest. The Hubble parameter (H), the scale factor (a), and the redshift (z) are ~~basically the same thing~~ closely related in terms of how quantities evolve over time:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \qquad a(t) = \frac{1}{1+z}$$

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$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

rate of universe's expansion

$$a(t) = \frac{1}{1 + z}$$

(relative) size of expanding universe

redshift!

COSMOLOGY - PROBLEM 10 *IN SIMPLER TERMS*

Assuming that radiation now contains three relativistic neutrino species and the CMB photons, and that the current matter density is $\rho_{m_0} = \Omega_m \rho_c = 0.3\rho_c$, **estimate the temperature of radiation/matter equality,** T_{eq}, defined as $\rho_{rad}(T_{eq}) = \rho_m(T_{eq})$

What was the t redshift e when $\Omega_{\mathrm{matter}} = \Omega_{\mathrm{radiation}}$?

Density parameter as a function of time

We can relate the current values of Ω_i with the values at any given time through their proportional relationship to the scale factor a(t):

$$\Omega_{\mathrm{matter}} \propto a(t)^{-3} = (1+z)^3$$

$$\Omega_{\mathrm{radiation}} \propto a(t)^{-4} = (1+z)^4$$

$$\therefore \Omega_{\mathrm{matter}} = \Omega_{\mathrm{radiation}} \Rightarrow \boxed{\Omega_{\mathrm{matter}_0}(1+z_{\mathrm{eq}})^3 = \Omega_{\mathrm{radiation}_0}(1+z_{\mathrm{eq}})^4}$$

Working with z_{eq}

$$\Omega_{\text{matter}_0}(1+z_{\text{eq}})^3 = \Omega_{\text{radiation}_0}(1+z_{\text{eq}})^4$$

Rearranging to get z_{eq} on one side,

Working with z_{eq}

$$\Omega_{\text{matter}_0}(1+z_{\text{eq}})^3 = \Omega_{\text{radiation}_0}(1+z_{\text{eq}})^4$$

Rearranging to get z_{eq} on one side,

$$\frac{\Omega_{\text{matter}_0}}{\Omega_{\text{radiation}_0}} = (1+z_{\text{eq}})$$

٩(´ᗜ`*)و how simple! ♡

Planck 2018 Data

Currently, Ω_{matter} and $\Omega_{\text{radiation}}$ are ***not*** equal:

Table 2. Derived Cosmological Parameters

Parameter	Mean (68% confidence range)
Amplitude of Galaxy Fluctuations	$\sigma_8 = 0.9 \pm 0.1$
Characteristic Amplitude of Velocity Fluctuations	$\sigma_8 \Omega_m^{0.6} = 0.44 \pm 0.10$
Baryon Density/Critical Density	$\Omega_b = 0.047 \pm 0.006$
Matter Density/Critical Density	$\Omega_m = 0.29 \pm 0.07$
Age of the Universe	$t_0 = 13.4 \pm 0.3$ Gyr
Redshift of Reionization[b]	$z_r = 17 \pm 5$
Redshift at Decoupling	$z_{dec} = 1088^{+1}_{-2}$
Age of the Universe at Decoupling	$t_{dec} = 372 \pm 14$ kyr
Thickness of Surface of Last Scatter	$\Delta z_{dec} = 194 \pm 2$
Thickness of Surface of Last Scatter	$\Delta t_{dec} = 115 \pm 5$ kyr
Redshift at Matter/Radiation Equality	$z_{eq} =$ 👀
Sound Horizon at Decoupling	$r_s = 144 \pm 4$ Mpc
Angular Diameter Distance to the Decoupling Surface	$d_A = 13.7 \pm 0.5$ Gpc
Acoustic Angular Scale[c]	$\ell_A = 299 \pm 2$
Current Density of Baryons	$n_b = (2.7 \pm 0.1) \times 10^{-7}$ cm^{-3}
Baryon/Photon Ratio	$\eta = (6.5^{+0.4}_{-0.3}) \times 10^{-10}$

where is $\Omega_{\text{radiation}}$? (°▭°``) !!

→ let's calculate it!

Calculating $\Omega_{\mathrm{radiation}}$

We know the *total* Ω for a flat universe must be 1., and the three contributions are Ω_{matter} , $\Omega_{\mathrm{radiation}}$, and Ω_{Λ} (dark energy):

$$\Omega_{\mathrm{matter}} + \Omega_{\mathrm{radiation}} + \Omega_{\Lambda} = 1$$

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	Ω_{Λ}	0.6847 ± 0.0073	0.6889 ± 0.0056
+	Ω_{m}	0.3153 ± 0.0073	0.3111 ± 0.0056

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	Σ(°□°)!!!	1.0000 ± 0.0146	1.0000 ± 0.0112

Calculating $\Omega_{\text{radiation}}$ differently?

We *also* know that, according to the problem, radiation now contains three relativistic neutrino species and the CMB photons. In other words,

$$\rho_{\text{radiation}} = \rho_\nu + \rho_\gamma$$

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$$\rho_{\text{radiation}} = \rho_\nu + \rho_\gamma$$

$$\rho_\nu = N_{\text{eff}}(7/8)(4/11)^{4/3}\rho_\gamma$$

$$\therefore \rho_{\text{radiation}} = \left(N_{\text{eff}} \left(\frac{7}{8}\right) \left(\frac{4}{11}\right)^{4/3} + 1 \right) \rho_\gamma$$

Plugging in numbers

Neff = effective

Crunching the numbers

Plugging in our values,

$$\frac{\Omega_{\mathrm{matter}_0}}{\Omega_{\mathrm{radiation}_0}} = \frac{0.3}{2.47 \times 10^{-5}}$$

How'd we do?

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Our answer: $z_{eq} = 3276$

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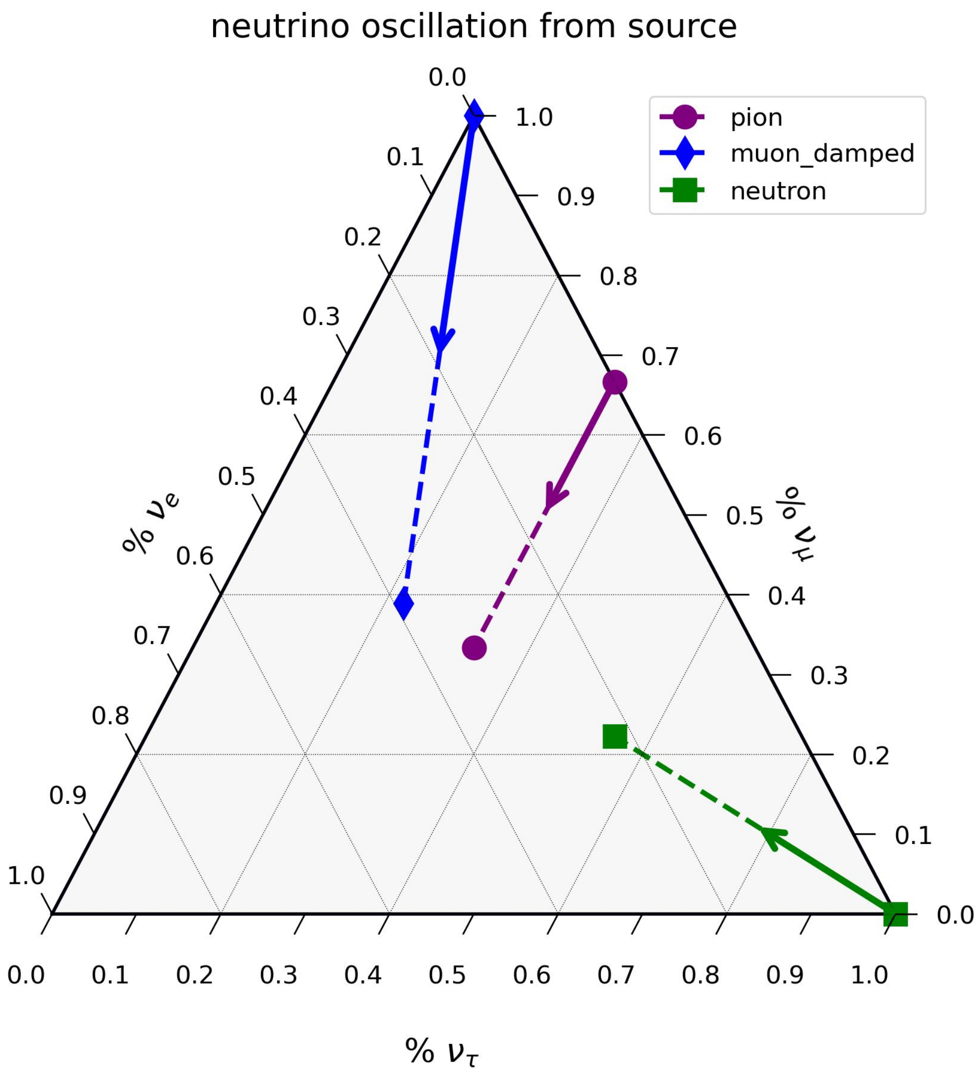
＼(^▽^)／ not bad!



neutrino oscillation from source
pion
muon_damped
neutron
% ν_e
% ν_μ
% ν_τ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0