FACULTY OF SCIENCES

DEPARTMENT OF PHYSICS AND ASTRONOMY

Theory of neutrino interactions

Lecture 1 : Nuclear models for neutrino physics

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Neutrinos

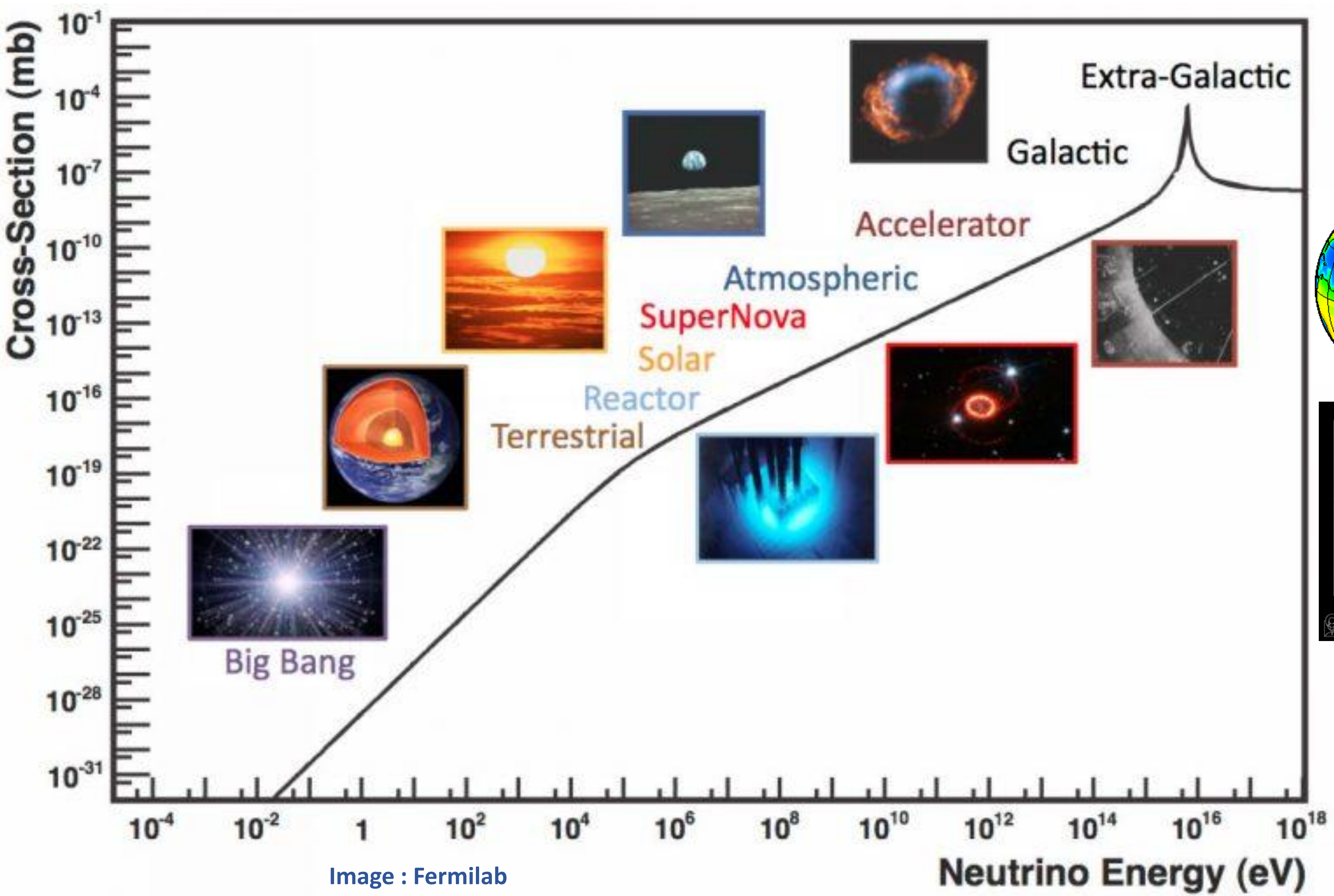
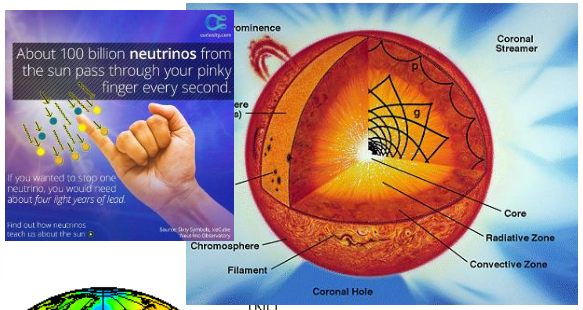
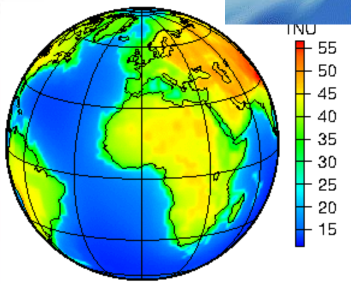
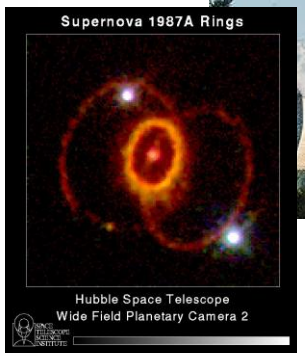
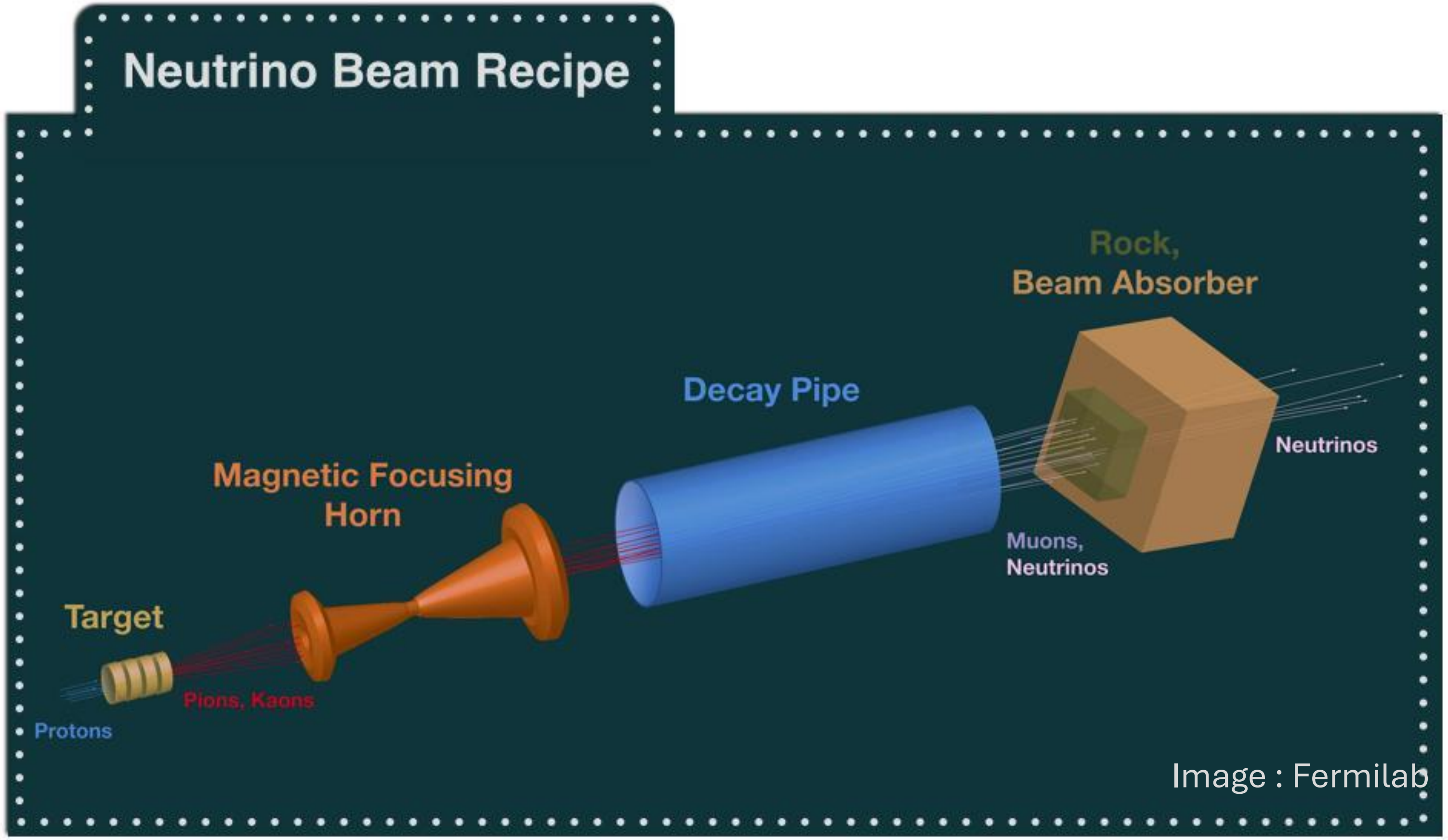


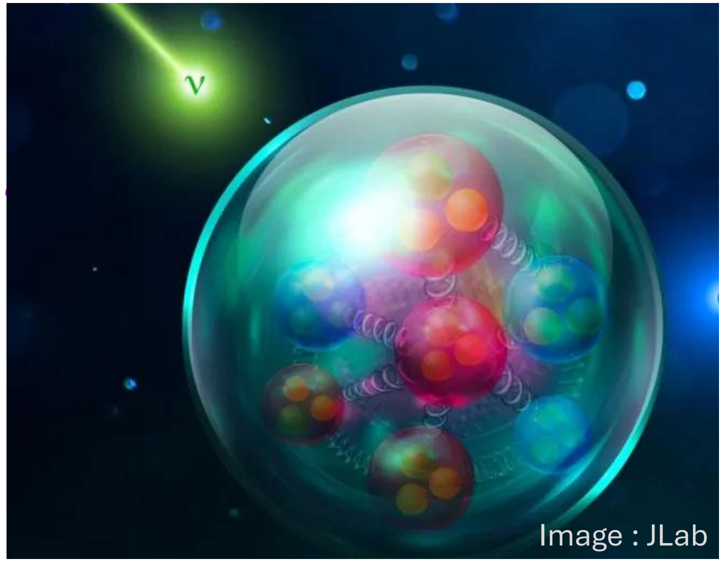
Image : Fermilab











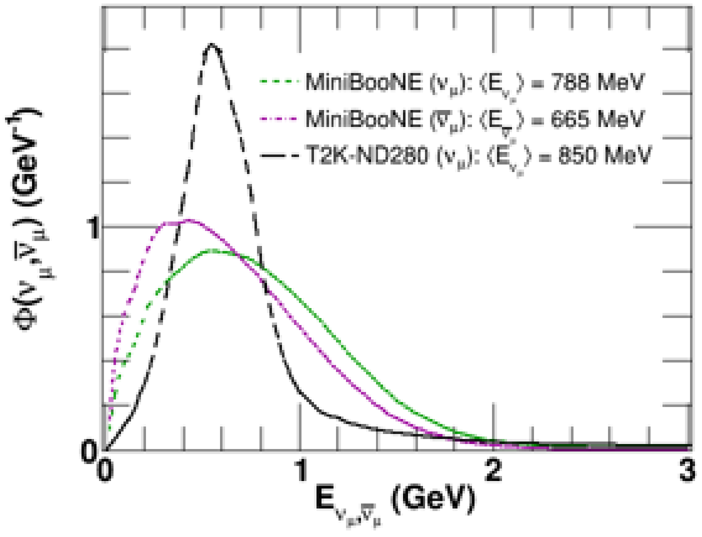
- A detailed understanding of neutrino-nucleus interactions is pivotal for the accuracy of accelerator-based oscillation studies
- Near detector studies of neutrino cross sections provide valuable information about weak interactions and the axial structure of the nucleus

Oscillation analysis in a near/far detector experiment :

$$P_{\nu_i \to \nu_j} = sin^2 2\theta sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

$$\frac{N_{far}(\overline{E}_\nu)}{N_{near}(\overline{E}_\nu)} = \frac{\int \Phi(E_\nu)\sigma(E_\nu)P(\overline{E}_\nu|E_\nu)P_{i\to j}(E_\nu)dE_\nu}{\int \Phi(E_\nu)\sigma(E_\nu)P(\overline{E}_\nu|E_\nu)(E_\nu)dE_\nu}$$

Accelerator neutrinos : broad energy distribution



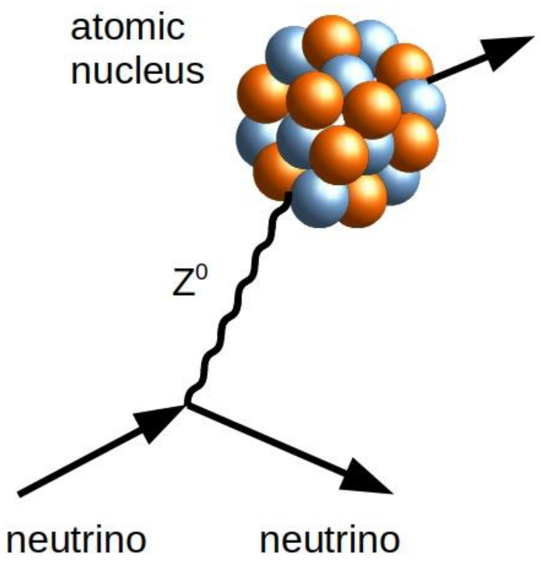
Reconstructed energy

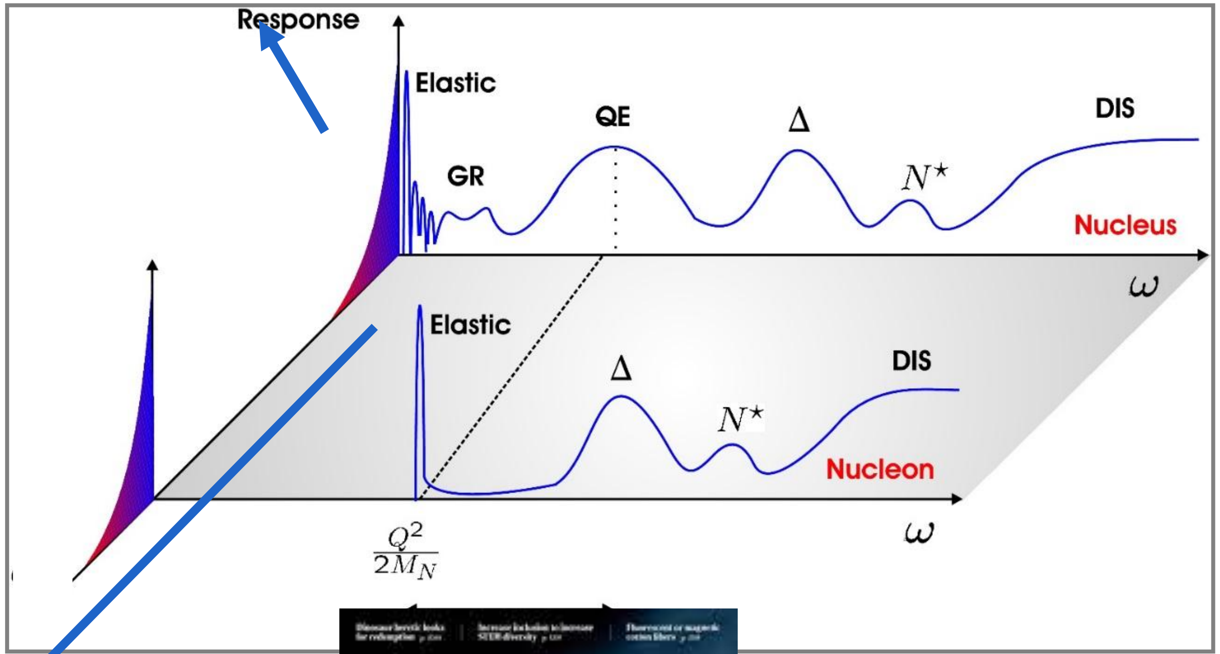
$$\overline{E}_\nu = \frac{2M'_n E_l - (M'^2_n + m_l^2 - M_p^2)}{2(M'_n - E_l + P_l \cos\theta)}$$

- Cross section, flux, detector efficiency, and oscillation probability are all energy dependent
- Energy dependent cross section information is needed
- Tension between event topology and genuine interaction mode
- Performant models need a consistent treatment of all relevant reaction mechanisms

Probing the nucleus

Coherent elastic scattering





- Scattering off the nucleus as a whole
- Small nuclear recoil
- Important for neutral current neutrino scattering

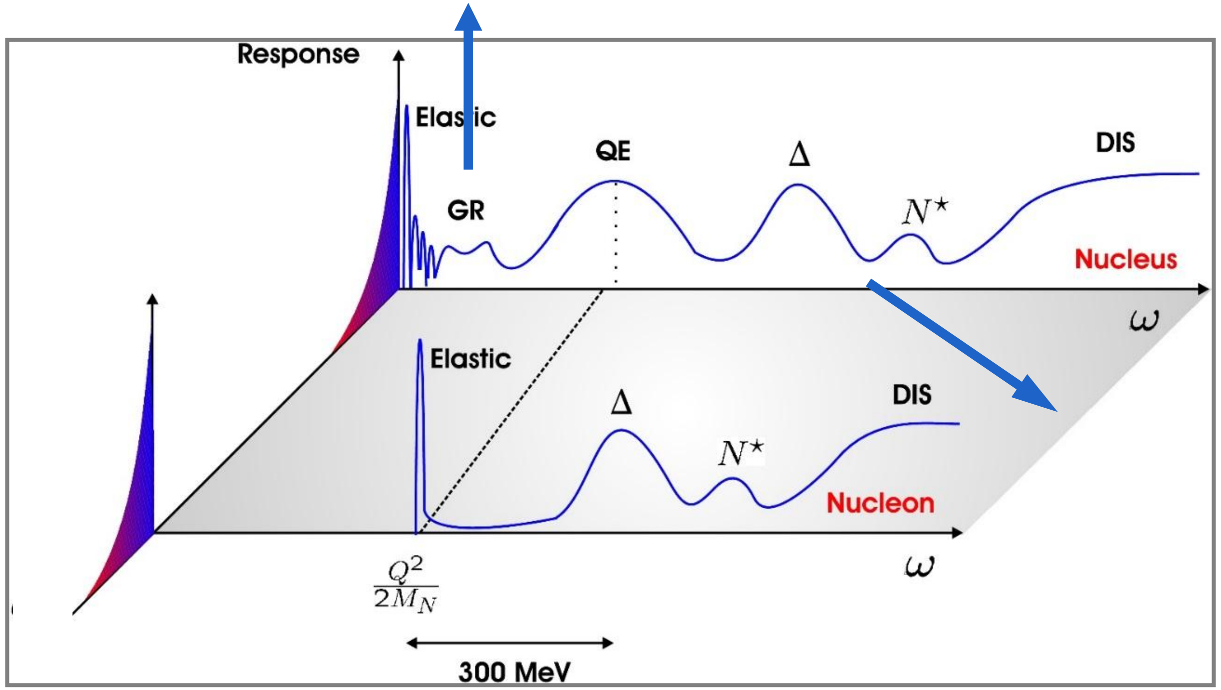
Science, September 2017

Nucleus form factor

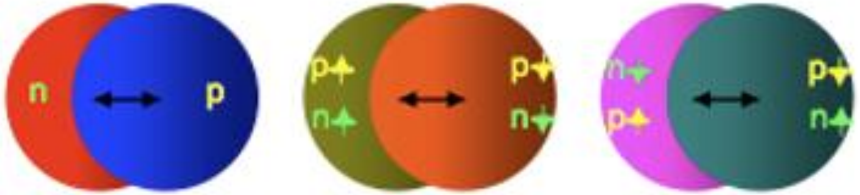
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_f} = \frac{G_F^2}{2\pi} E_i^2 (1+\cos\theta_f) \frac{Q_W^2}{4} F(Q)^2$$

Probing the nucleus

Low energy collective excitations

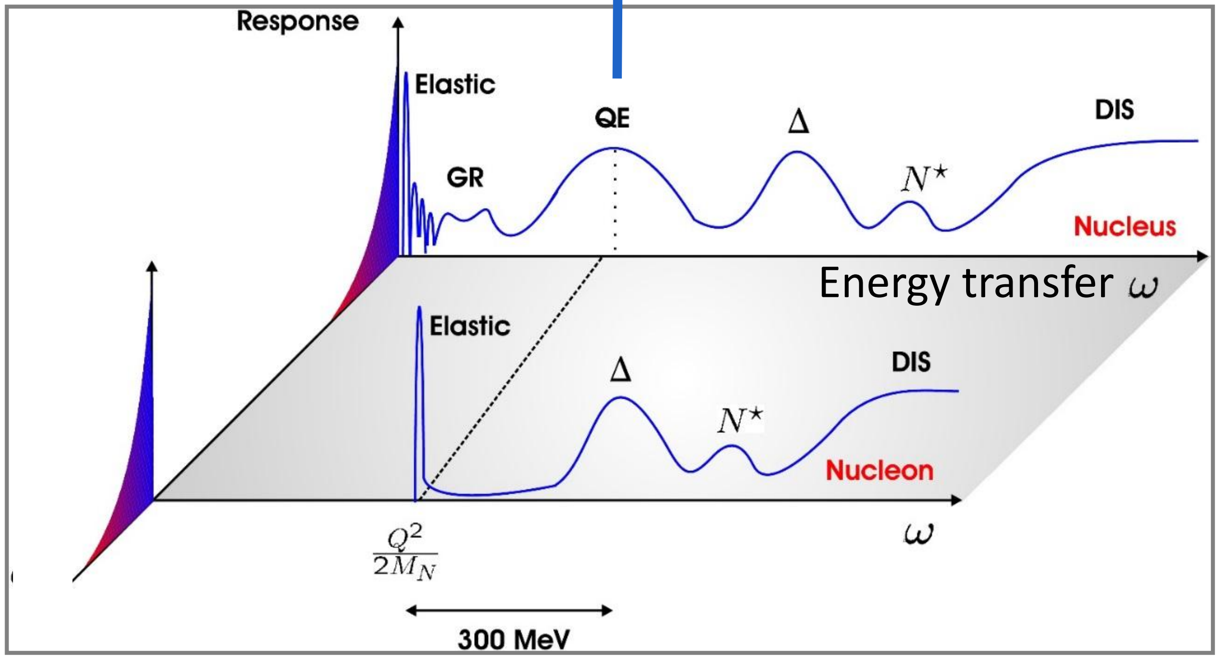


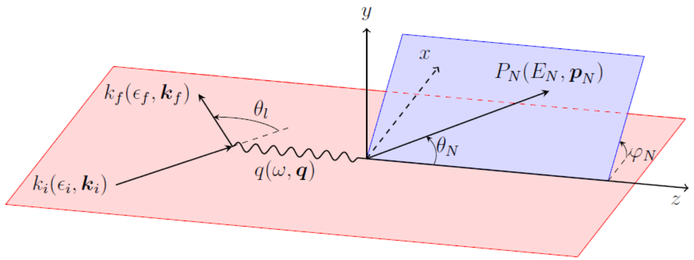
- Long-range correlations are correlations over the whole size of the nucleus
- They can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents.
- They manifest themselves in collective excitations such as giant resonances
- Breaking the impulse approximation

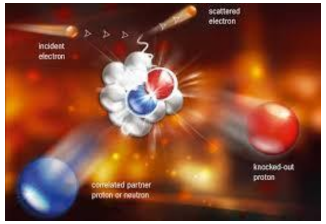


Probing the nucleus

Scattering off a nucleon bound in the nucleus = quasi-elastic scattering





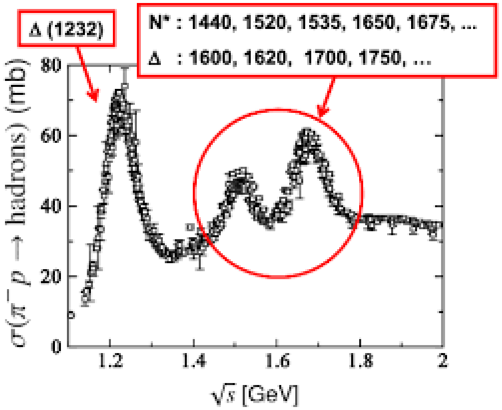


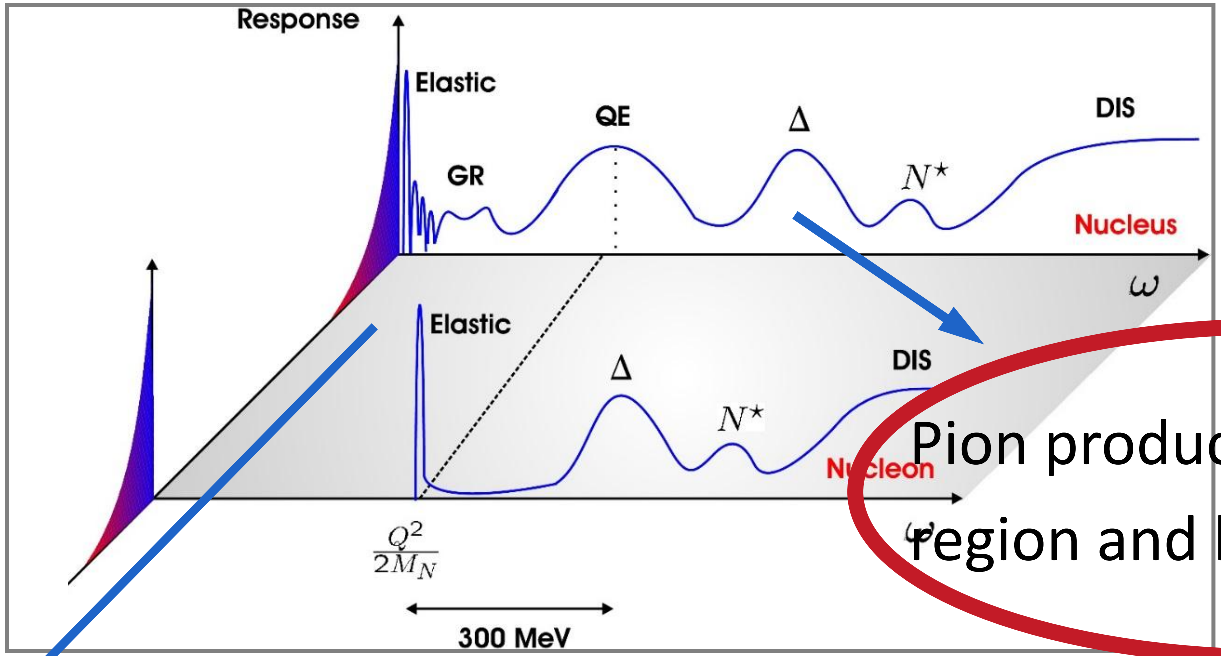
$$
\begin{aligned}
M_N + \varepsilon_i &= E_f + \varepsilon_f \\
(\varepsilon_i - \varepsilon_f) + M_N &= \sqrt{P_f^2 + M_N^2} \\
\omega + M_N &= \sqrt{\vec{q}^2 + M_N^2} \\
\omega^2 + 2\omega M_N + M_N^2 &= \vec{q}^2 + M_N^2 \\
2\omega M_N &= q^2 - \omega^2
\end{aligned}
$$

$$
\omega = \frac{Q^2}{2M_N} \rightarrow \omega_{QE} = \frac{Q^2}{2M_N} \pm E_{Fermi\ motion}
$$

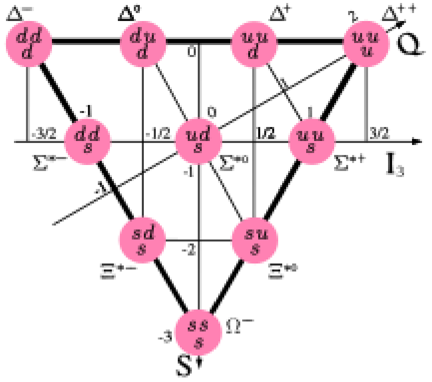
Probing the nucleus

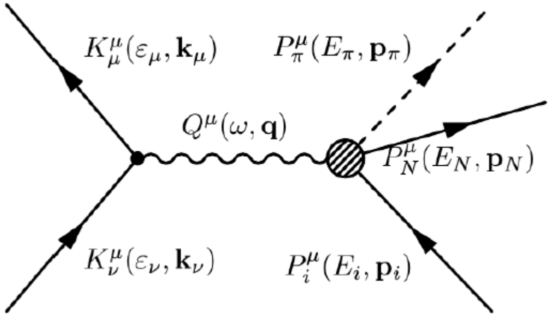
At sufficiently high energy transfers, nucleon excitations become important





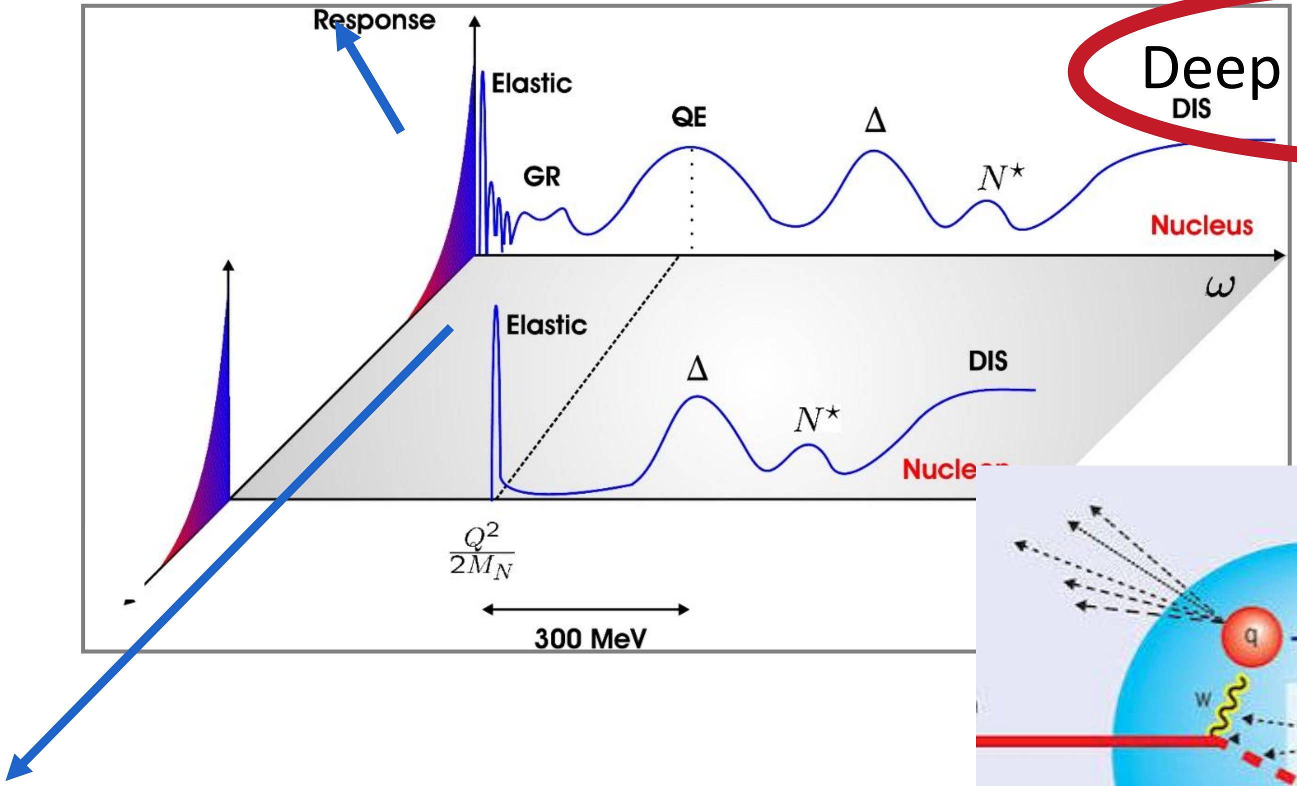
Pion production in the delta region and beyond

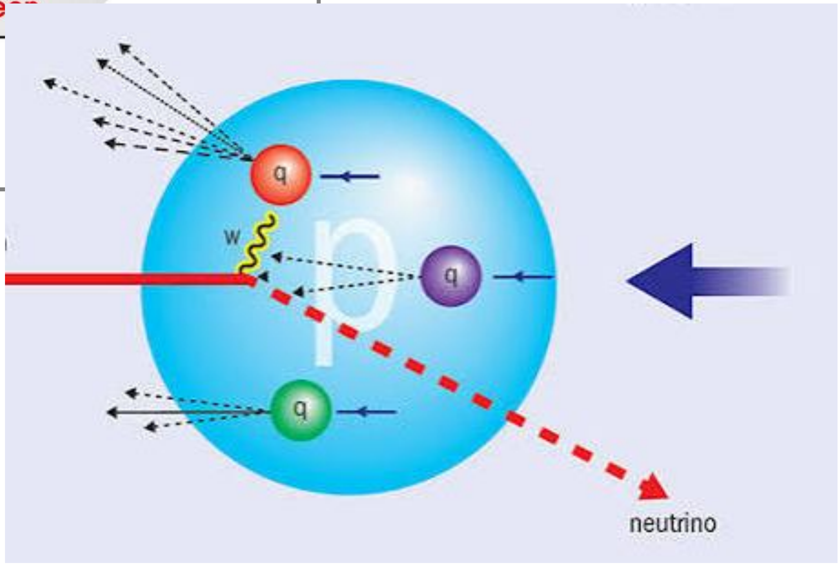


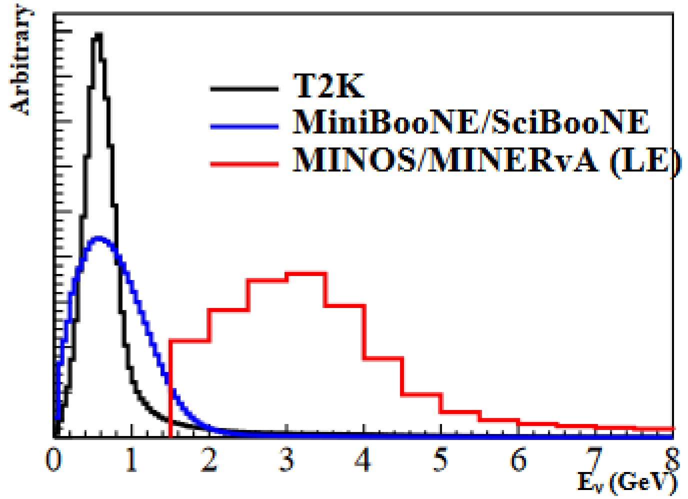


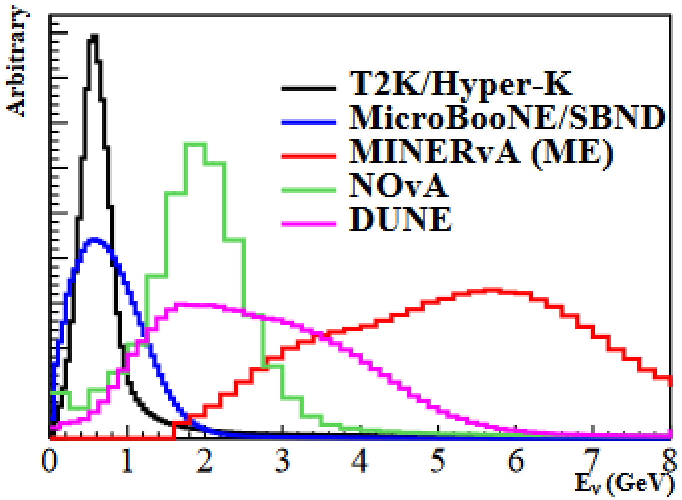
Probing the nucleus

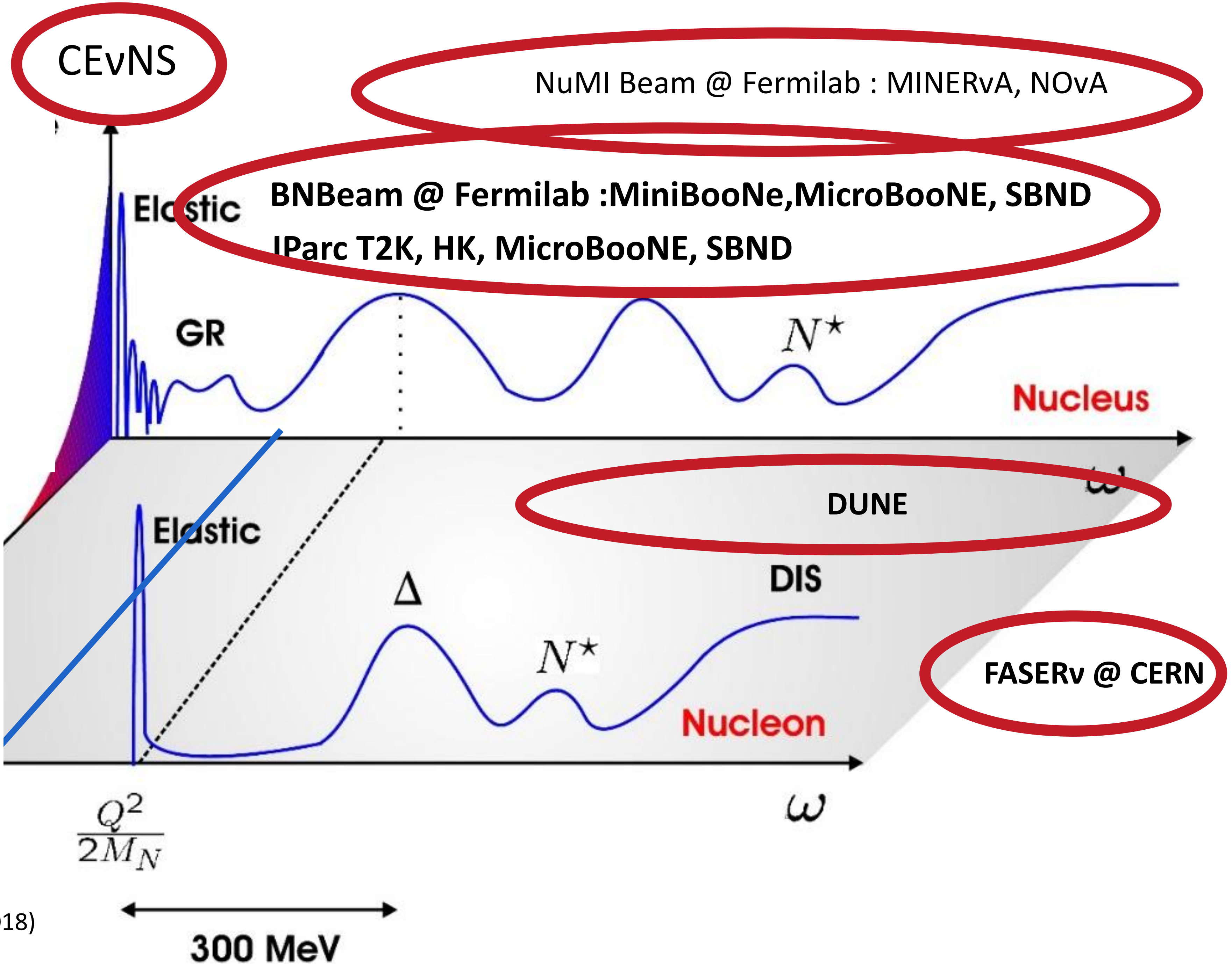
Deep Inelastic Scattering









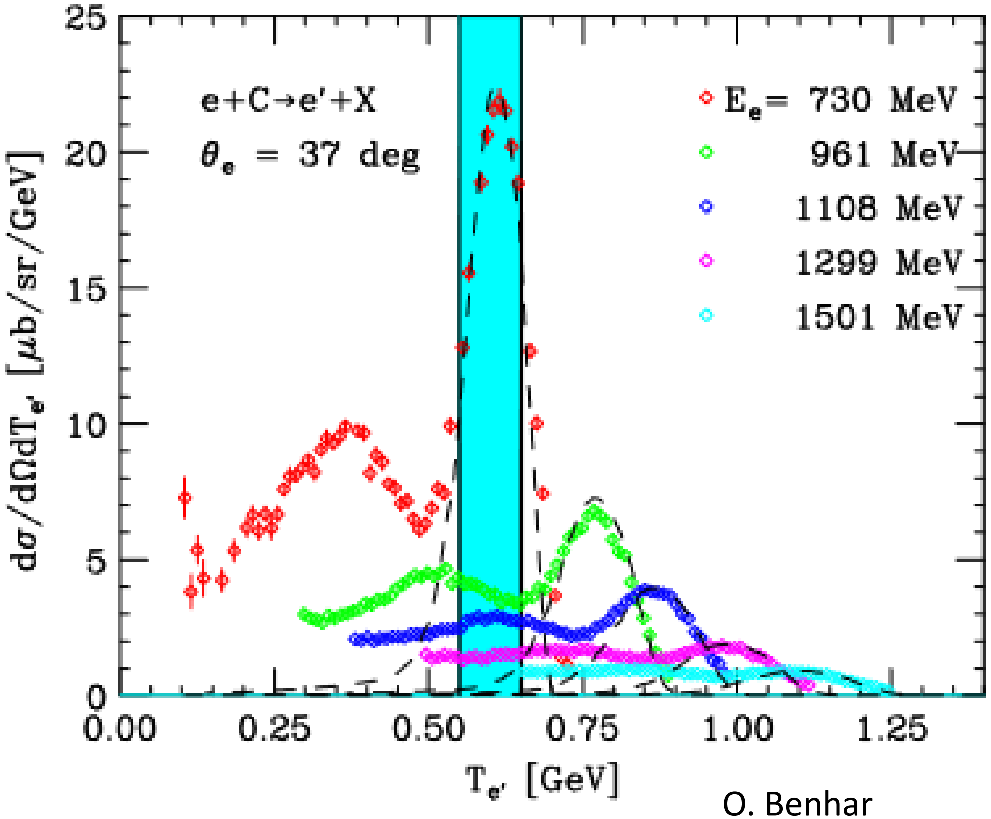


T. Katori & M. Martini, JPhysG 45, 013001 (2018)

(One of) The real challenge(s) : modeling and interpreting a flux-averaged signal

- In neutrino-nucleus interactions the incoming neutrino energy is unknown
- For different incoming energies, different reaction mechanisms contribute to the 'same' final state
- It is difficult to identify the reaction mechanism and reconstruct the energy transfer at the primary vertex
- Reconstructing the incoming neutrino energy is crucial for the oscillation analysis !

e-scattering spectral function calculation :



O. Benhar

→ For neutrinos, we simply have to work a bit harder ... !

From Teppei Katori's lectures last week :

1.1 Neutrino cross-section formula

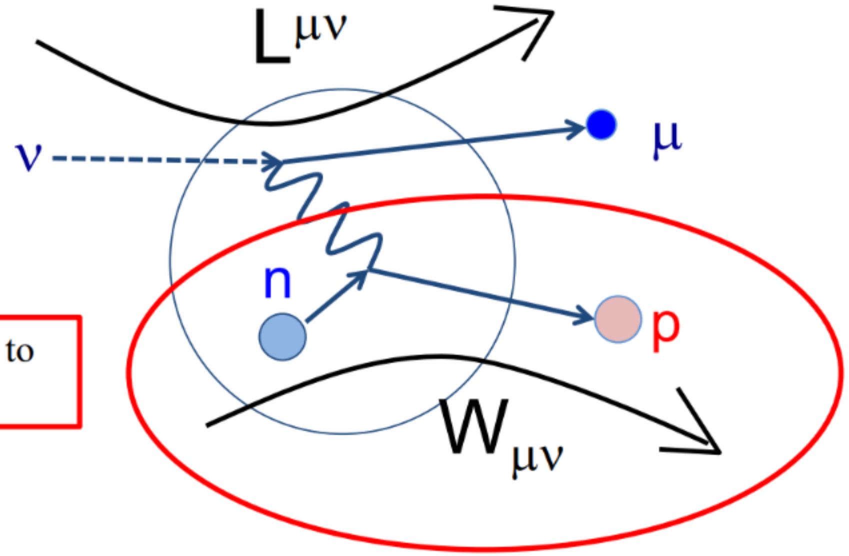
This is what my lectures will focus on

Cross-section
- product of Leptoni… d Hadronic tensor

$$d\sigma \;\ldots\; L^{\mu\nu}W_{\mu\nu}$$

Leptonic tensor
→ the Standard … asy)

Hadronic tensor
→ nuclear physics (hard)



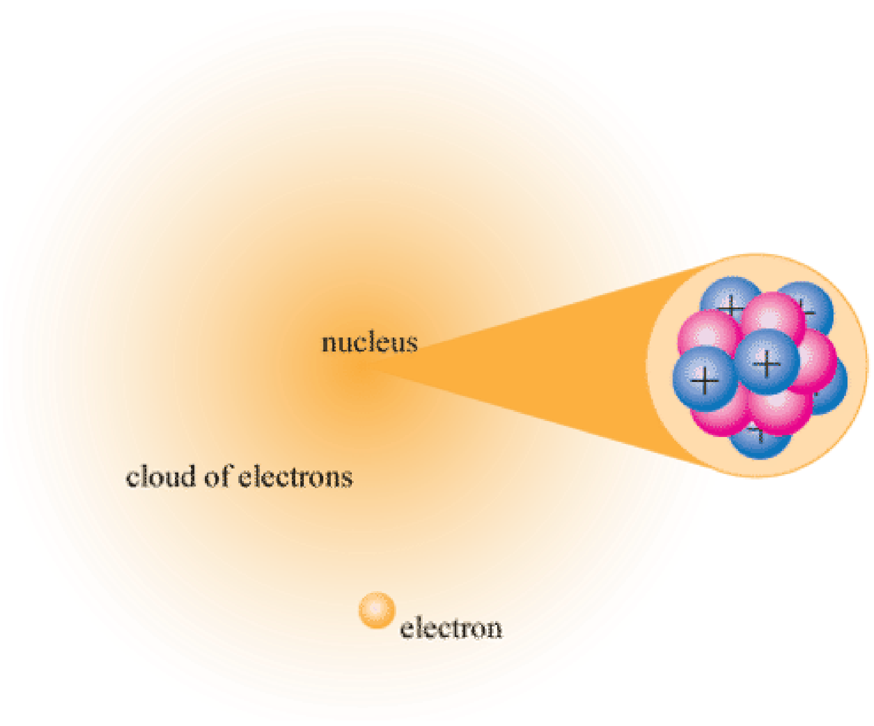
All complication of neutrino cross-section is how to model the hadronic tensor part

I. Understanding the force keeping nuclei together

Nuclei Facts

- A typical grain of sand contains more than 10 million trillion nuclei. That's 100 times more than the number of seconds since the beginning of the Universe.
- The nucleus accounts for more than 99.9994% of the total atomic mass, but occupies less than one ten-trillionth of the atomic volume.
- All nuclei have approximately the same density. If the Moon was smashed to the same density, it would fit inside Yankee Stadium.

https://www.energy.gov/science/doe-explainsnuclei

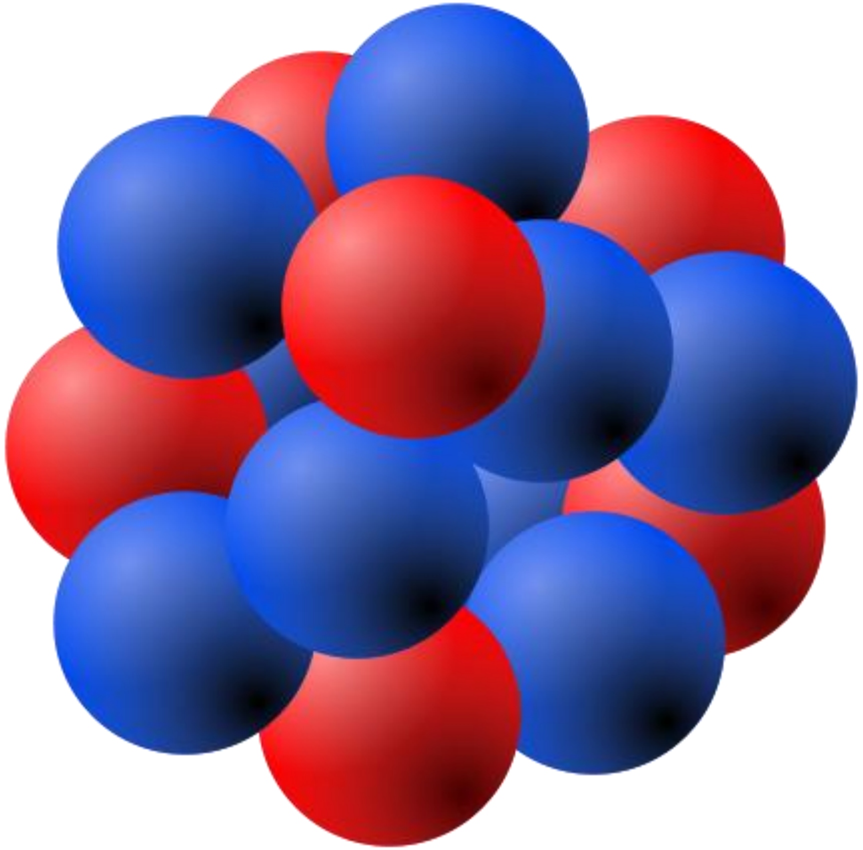


Modeling the nucleus

The nucleus is a mesoscopic system :

- usually too big for few-body techniqes
- usually too small for statistical methods

Nuclear physics is hard work !



Modeling the nucleus

The nucleus is a mesoscopic system :

-usually too big for few-body techniqes
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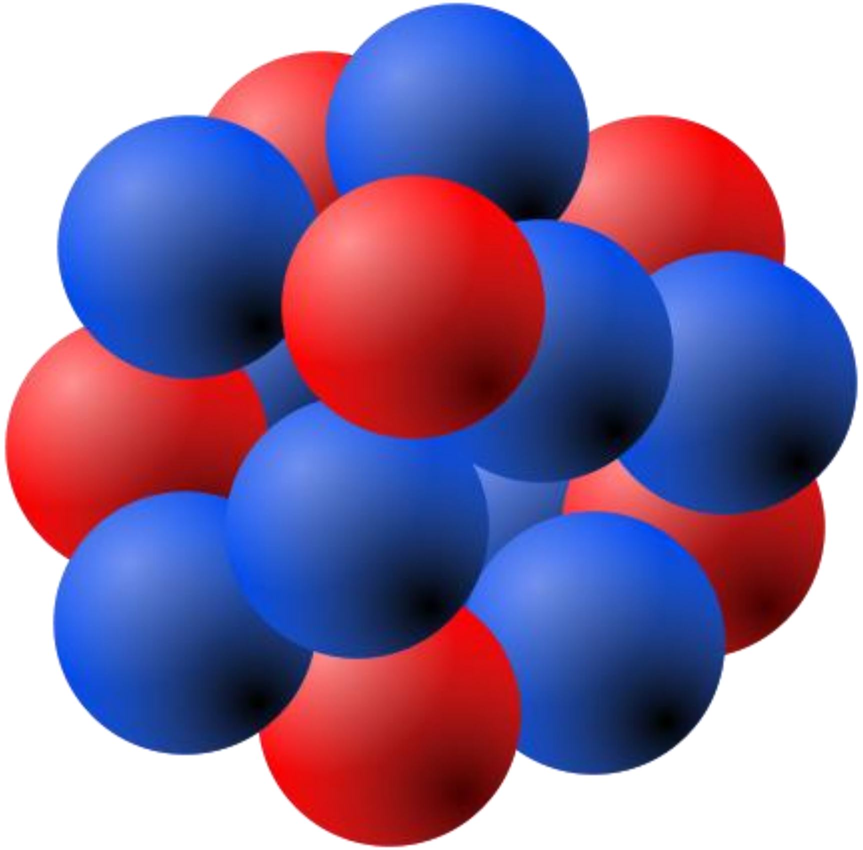
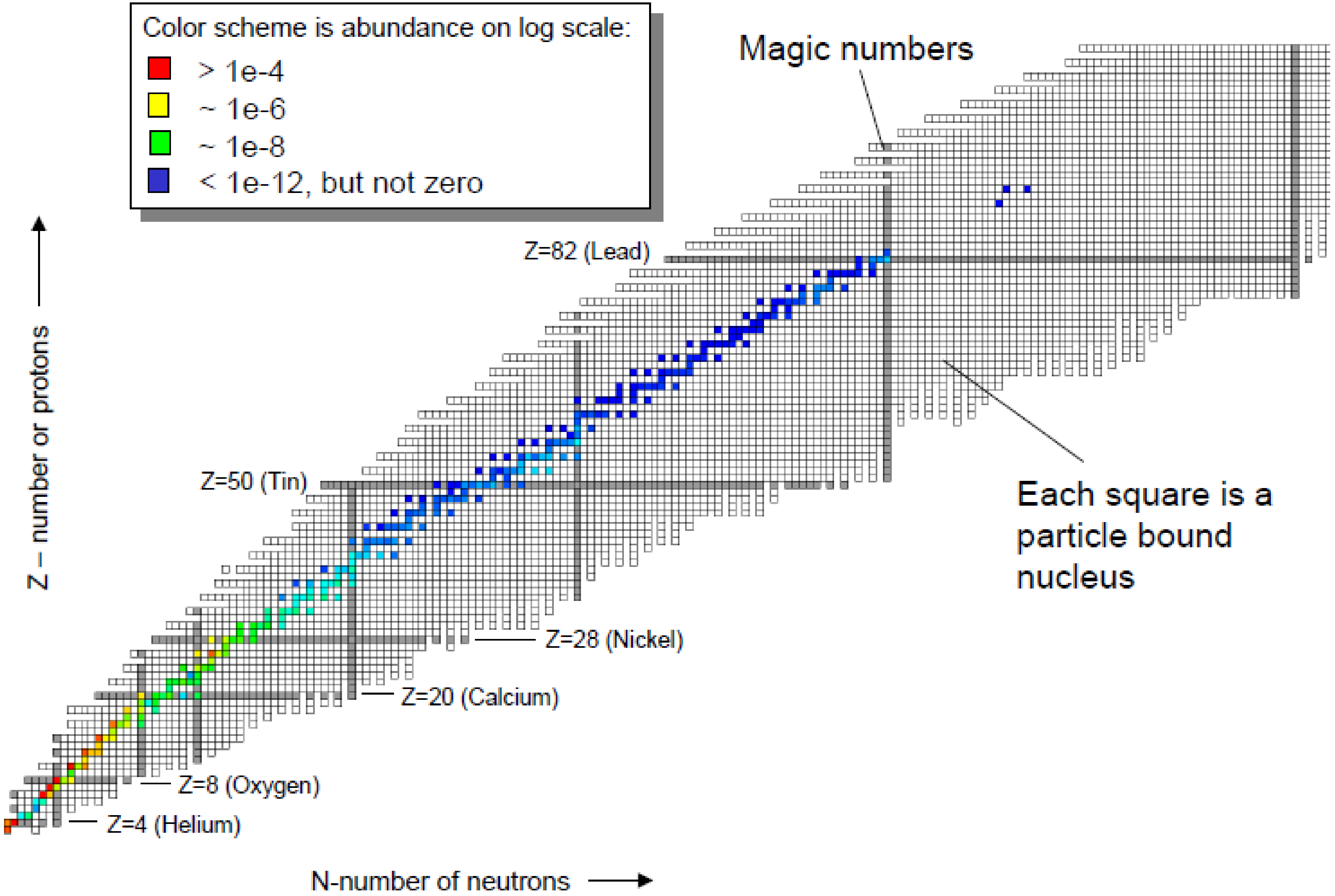
Nuclear physics is ~~hard~~ challenging work !

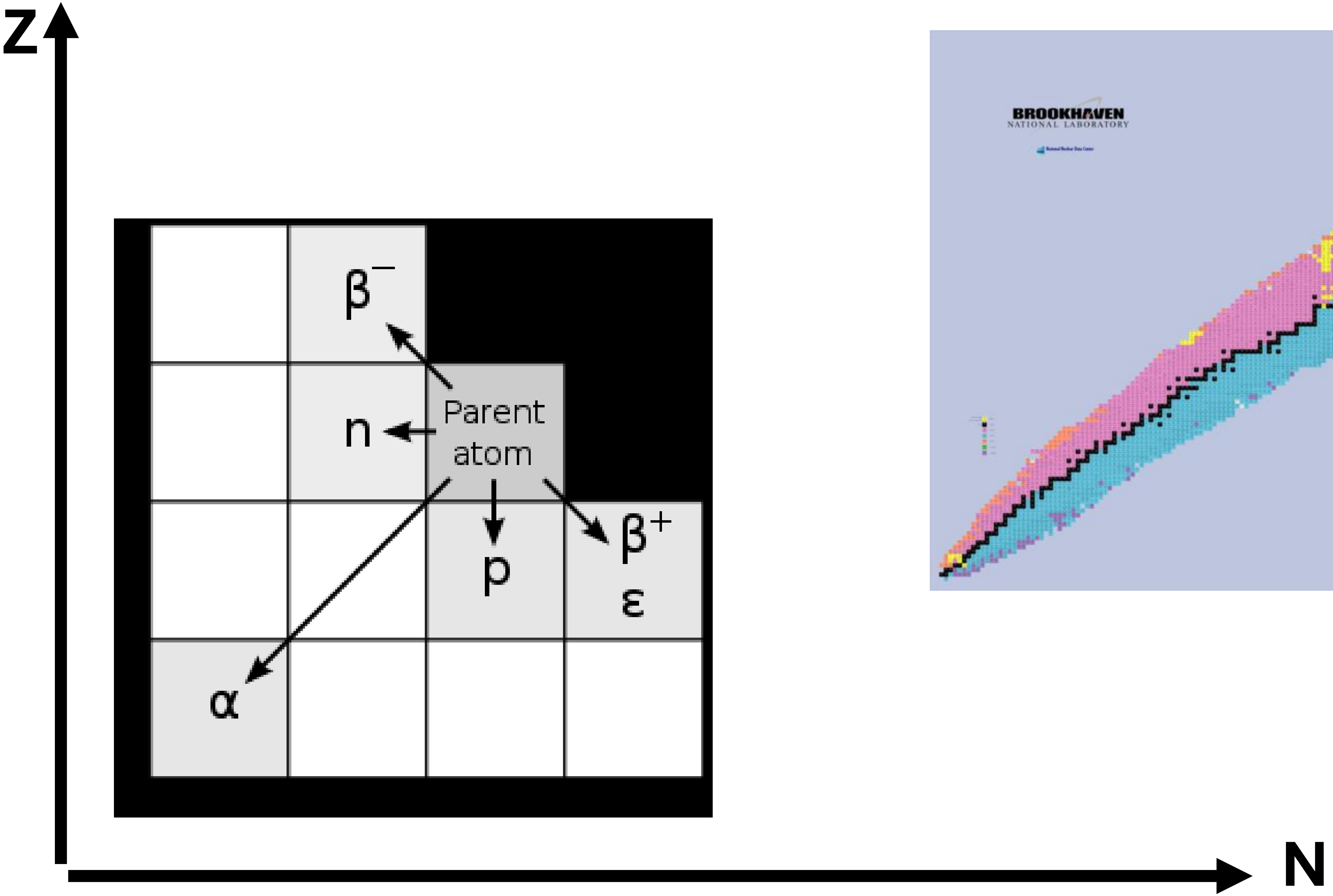


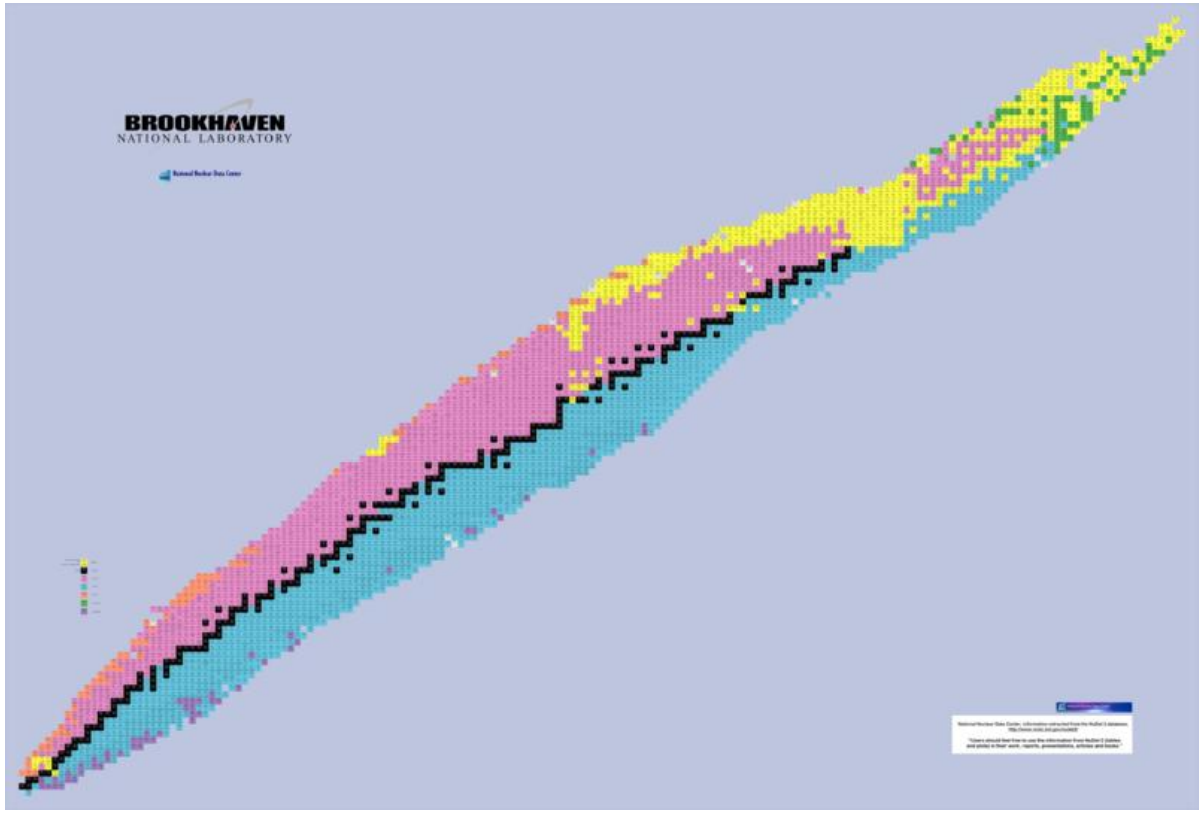
Modeling the nucleus

Abundances of nuclei on the chart of nuclides:

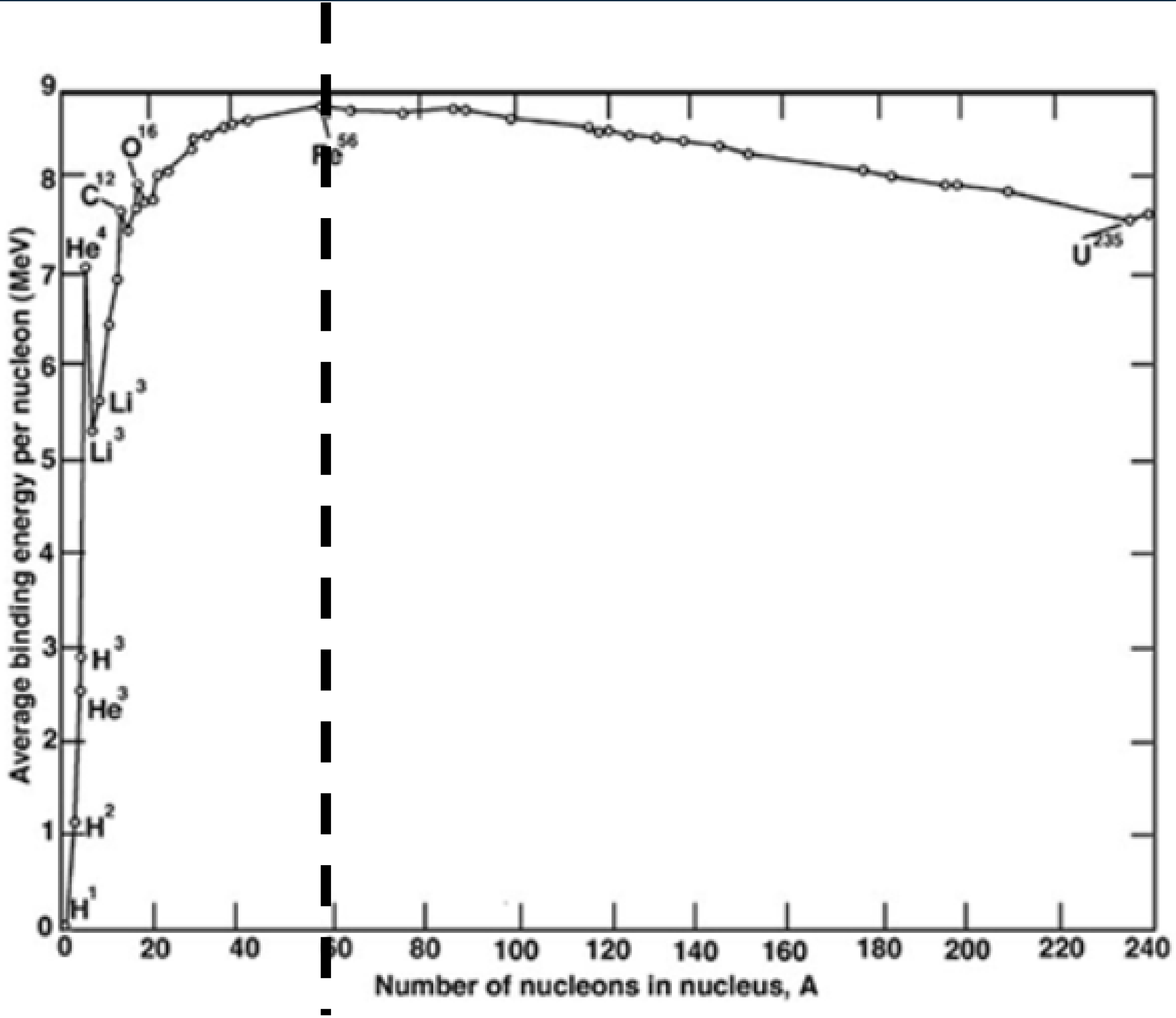


Modeling the nucleus





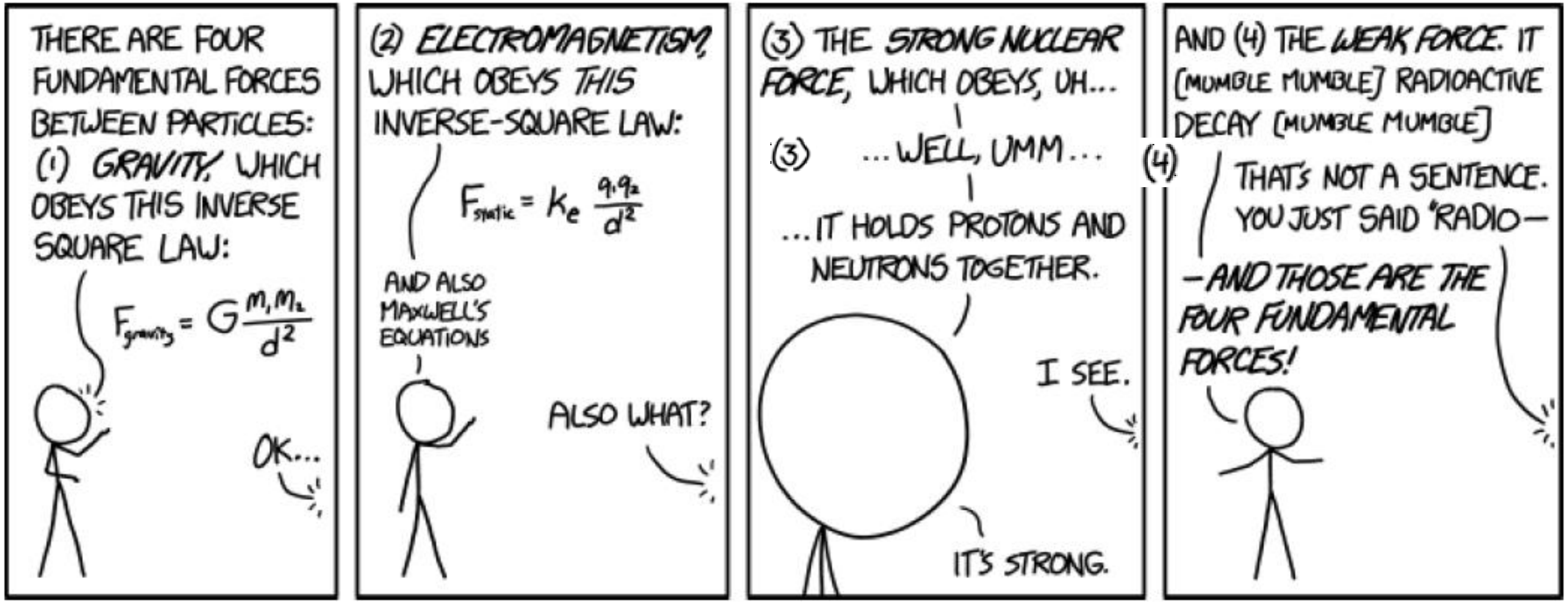
Modeling the nucleus



- B/A~8MeV
- Saturation
- Peaks around A~56

The nucleon-nucleon interaction

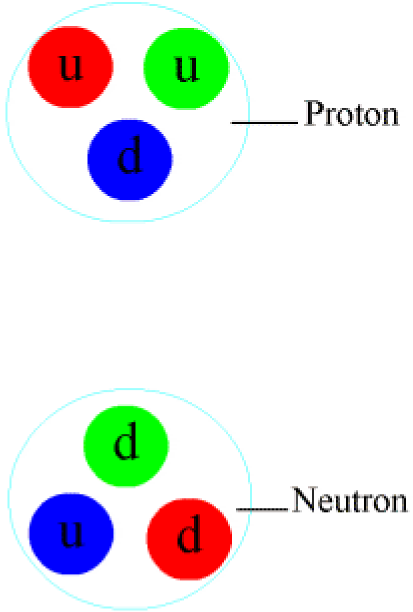
- The nucleons inside the nucleus are bound via the nuclear force. This force is understood as a residual effect of the strong force, which is the force that binds quarks together to form the nucleons.

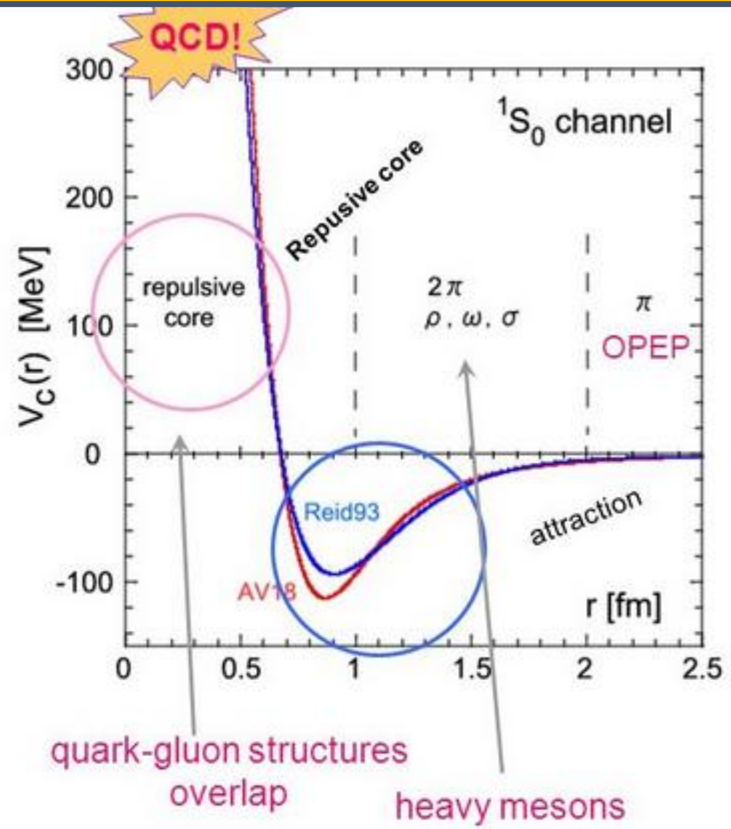


And (quite) a bit by the Coulomb interaction

The nucleon-nucleon interaction

Nuclear interaction = residue of strong force





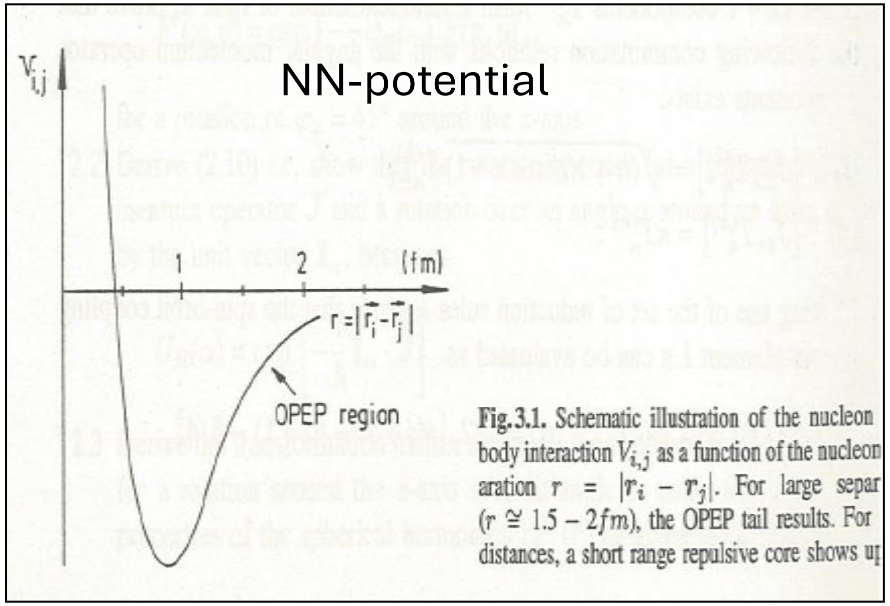
- At relatively long distances, the interaction is dominated by 1-pion exchange resulting in a pseudoscalar field
- At distances of about 1 fm, the exchange of pion pairs introduces a scalar interaction field
- At even shorter distances, heavier vector mesons mediate the interaction between nucleons

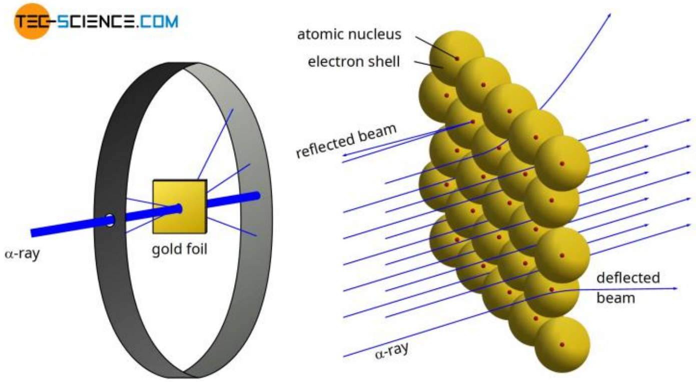
The nucleon-nucleon interaction

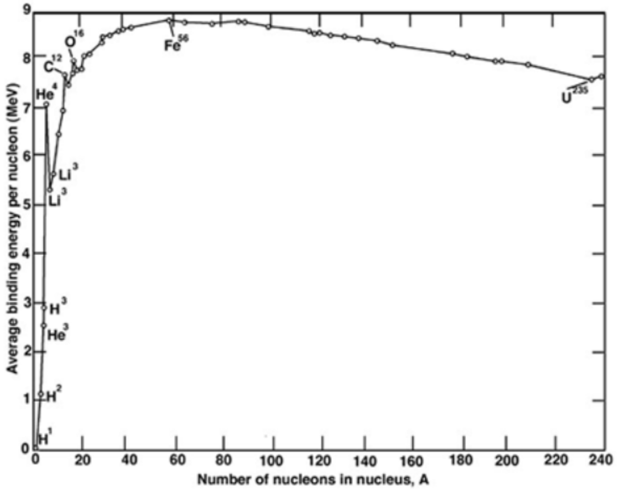
Nuclear Force

- **Short range**

- Rutherford experiment
- Saturation of binding energy curve







The nucleon-nucleon interaction

Nuclear Force

- Short range
- **Repulsive core**

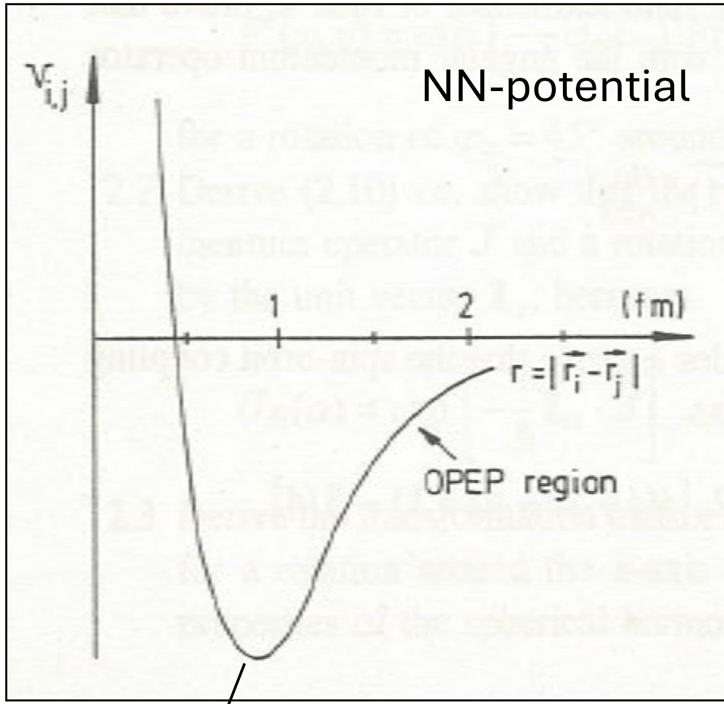


Fig. 3.1. Schematic illustration of the nucleon body interaction $V_{i,j}$ as a function of the nucleon aration $r = |r_i - r_j|$. For large separ ($r \cong 1.5 - 2 fm$), the OPEP tail results. For distances, a short range repulsive core shows up

- Saturation
- ~constant binding energy per nucleon
- constant average distance between nucleons
- ~constant density of nuclear matter

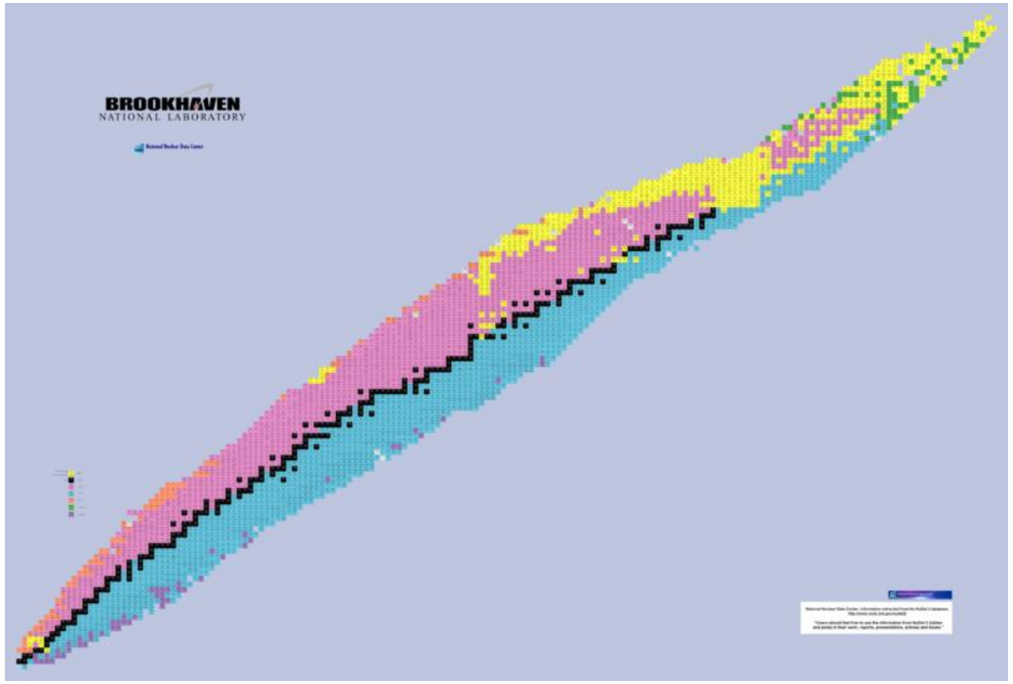
$V \sim A$
$R = 1.2 fm\ A^{1/3}$

The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- **Charge independence**

- n-n,n-p,p-p interactions (almost) identical
- But :
  - Coulomb
  - Pauli exclusion



The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- **Charge independence**
- Spin dependence

- n-n,n-p,p-p interactions almost identical
- But :
  - Coulomb
  - Pauli exclusion

- Isospin symmetry : 'nucleon' N

isospin: $T = 1/2$

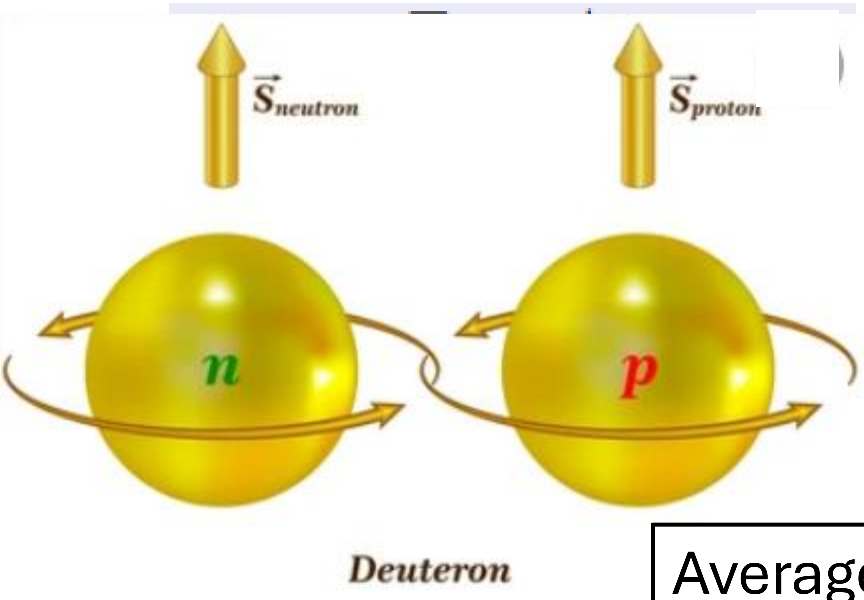
- neutron: $T_z = +1/2$
- proton: $T_z = -1/2$
- nucleus: $T_z = (N - Z)/2$

- Approximately conserved in nuclei

The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- **Charge symmetry & independence**
- **Spin dependence**



- n-n,n-p,p-p interactions almost identical
- But :
  - Coulomb
  - Fermions : Pauli exclusion

- nn, pp : T=1
  - ➢ Must have S=0
  - ➢ Marginally unbound
- np : T=0
  - ➢ S=0 (unbound as in nn and pp systems)
  - ➢ S=1 stronger nucleon force : deuteron

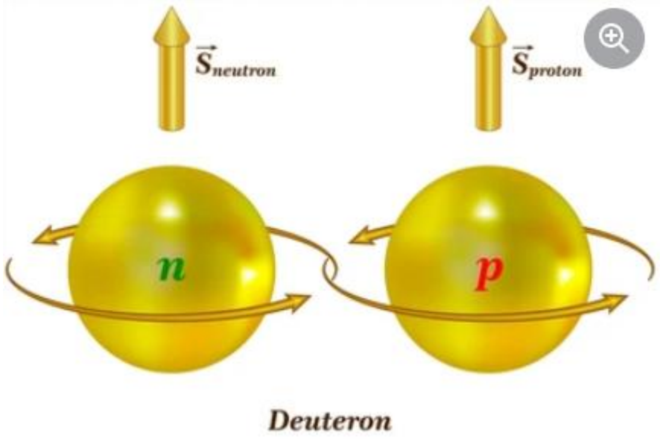
Average force between n-p > force between n-n or p-p in a nucleus

The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- Charge independence
- **Spin dependence**

- n-p force stronger in S=1 state



BE=2.2 MeV

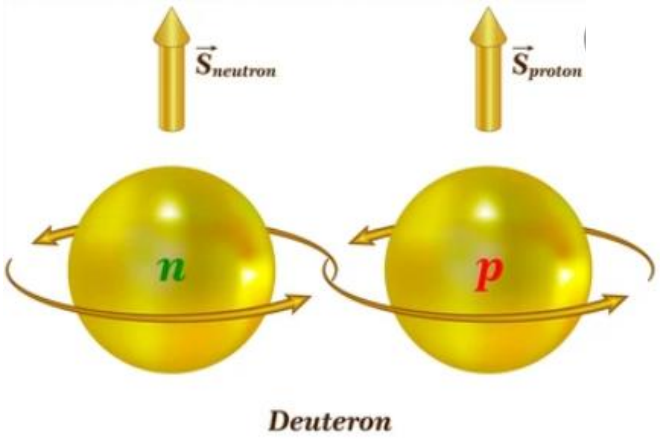
The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- Charge independence
- Spin dependence
- **Spin-orbit contribution**

- n-p force stronger in S=1 state
- Spin-orbit force **L.S**
  - Attractive when **L** and **S** are parallel
  - Repulsive when **L** and **S** are antiparallel

$$
\begin{aligned}
V_{NN}(r) \;=\;& V_0(r) \\
&+V_{ss}(r)\vec{s}_1\cdot\vec{s}_2/\hbar^2 \\
&+V_T(r)(3(\vec{s}_1\cdot\vec{x})(\vec{s}_2\cdot\vec{x})/r^2-\vec{s}_1\cdot\vec{s}_2)/\hbar^2 \\
&+V_{LS}(r)(\vec{s}_1+\vec{s}_2)\cdot\vec{L}/\hbar^2 \\
&+V_{Ls}(r)(\vec{s}_1\cdot\vec{L})(\vec{s}_2\cdot\vec{L})/\hbar^4 \\
&+V_{ps}(r)(\vec{s}_1\cdot\vec{p})(\vec{s}_2\cdot\vec{p})/(\hbar^2 m^2 c^2)
\end{aligned}
$$



BE=2.2 MeV

The nucleon-nucleon interaction

Nuclear Force

- Short range
- Repulsive core
- Charge independence
- Spin dependence
- Spin-orbit contribution
- **Tensor contribution**

$$
\begin{aligned}
V_{NN}(r) \;=\; & V_0(r) \\
& + V_{ss}(r)\vec{s}_1 \cdot \vec{s}_2/\hbar^2 \\
& + V_T(r)(3(\vec{s}_1 \cdot \vec{x})(\vec{s}_2 \cdot \vec{x})/r^2 - \vec{s}_1 \cdot \vec{s}_2)/\hbar^2 \\
& + V_{LS}(r)(\vec{s}_1 + \vec{s}_2) \cdot \vec{L}/\hbar^2 \\
& + V_{Ls}(r)(\vec{s}_1 \cdot \vec{L})(\vec{s}_2 \cdot \vec{L})/\hbar^4 \\
& + V_{ps}(r)(\vec{s}_1 \cdot \vec{p})(\vec{s}_2 \cdot \vec{p})/(\hbar^2 m^2 c^2)
\end{aligned}
$$

Deuteron quadrupole moment

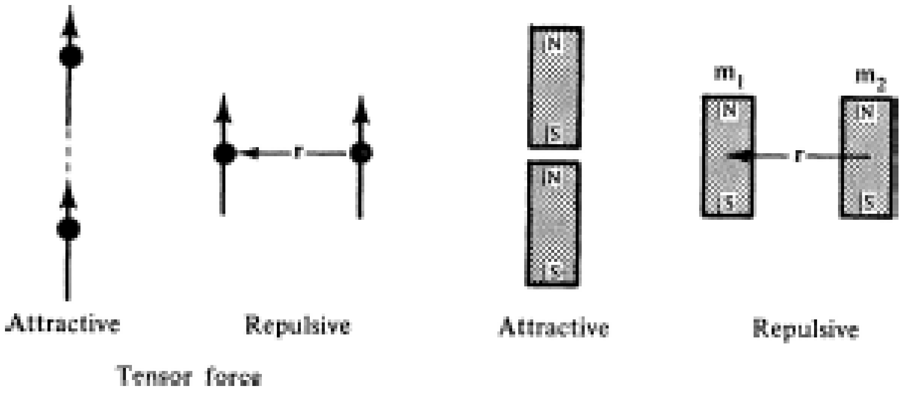


Figure 14.10: The tensor force in the deuteron is attractive in the cigar-shaped configuration and repulsive in the disk-shaped one. Two bar magnets provide a classical example of a tensor force.

Phenomenological Potentials

- Boson field theory is limited in its predictive power – and a lot of approximations have to be made
- Phenomenological potentials provide an efficient way to describe the interaction between 2 nucleons
- Constrained by invariance laws : rotation, reflection, time reversal
- Limiting the description to terms depending only linearly on spin, isospin and momentum :

- 1,
- $\vec{\sigma}_1 \cdot \vec{\sigma}_2$,
- $(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})$
- $(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\mathbf{r} \times \mathbf{p})$

X

- 1, or
- $\vec{\tau}_1 \cdot \vec{\tau}_2$

X

$f(r)$

$f(r) \to 0$ for $r \to \infty$.

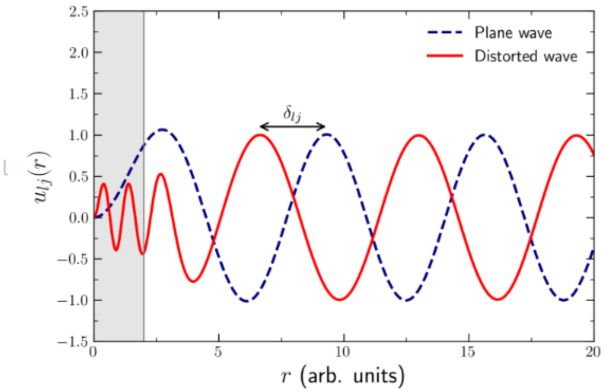
Phenomenological Potentials

$$\begin{aligned} V(\mathbf{r},\mathbf{p},\vec{\sigma},\vec{\tau}) &= [V_c(r) + W_c(r)\,\vec{\tau}_1\cdot\vec{\tau}_2] \\ &+ [V_{sc}(r) + W_{sc}(r)\,\vec{\tau}_1\cdot\vec{\tau}_2]\ \ (\vec{\sigma}_1\cdot\vec{\sigma}_2) \\ &+ [V_T(r) + W_T(r)\,\vec{\tau}_1\cdot\vec{\tau}_2]\,[3(\vec{\sigma}_1\cdot\vec{e}_r)(\vec{\sigma}_2\cdot\vec{e}_r) - \vec{\sigma}_1\cdot\vec{\sigma}_2] \\ &+ [V_{LS}(r) + W_{LS}(r)\,\vec{\tau}_1\cdot\vec{\tau}_2]\,(\vec{\sigma}_1+\vec{\sigma}_2)\cdot(\mathbf{r}\times\mathbf{p}), \end{aligned}$$

- Below the pion production threshold, the 2-nucleon system will either bind or scatter elastically and all r-dependence will be real
- A repulsive core can be added for r<~0.5 fm

Realistic interactions

→ Extensive expressions fit to large datasets os scattering data and phase shifts
e.g. Nijmegen, Reid, Urbana V14, Argonne V18



PHYSICAL REVIEW C VOLUME 51, NUMBER 1 JANUARY 1995

Accurate nucleon-nucleon potential with charge-independence breaking

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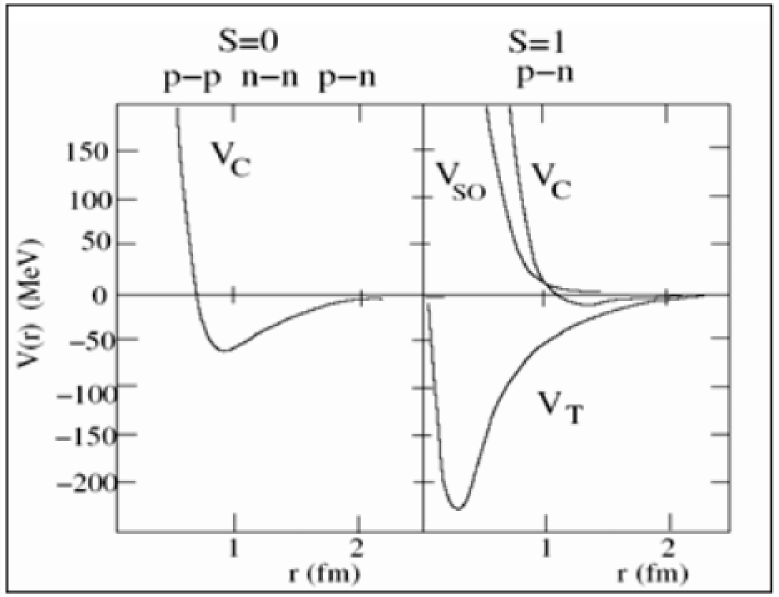
(Received 15 August 1994)

We present a new high-quality nucleon-nucleon potential with explicit charg
charge asymmetry, which we designate Argonne v_{18}. The model has a charge-
with 14 operator components that is an updated version of the Argonne v_{14}
additional charge-dependent and one charge-asymmetric operators are added, alon
electromagnetic interaction. The potential has been fit directly to the Nijmegen *pp*
database, low-energy *nn* scattering parameters, and deuteron binding energy. W
parameters it gives a χ^2 per datum of 1.09 for 4301 *pp* and *np* data in the range

PACS number(s): 13.75.Cs, 12.39.Pn, 21.30



Few-Body Systems Suppl. 99, 1–10 (2018)

Few-Body Systems

The Nijmegen Potentials

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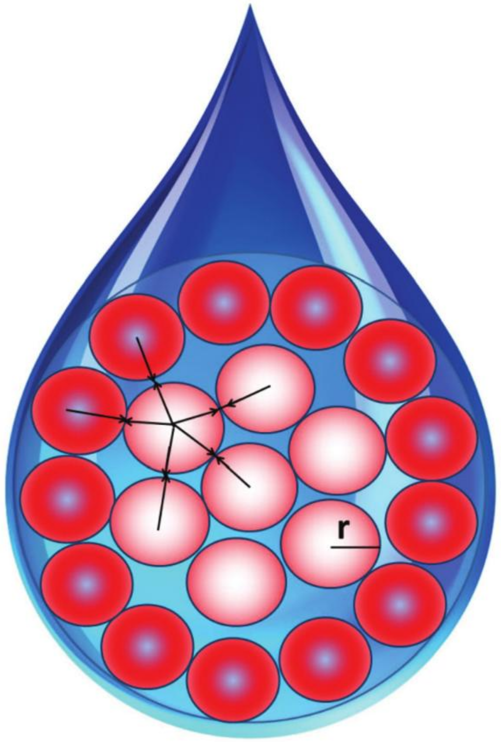
Abstract. A review is given of the various Nijmegen potentials. Special attention is given to some of the newest developments, such as the extended soft-core model, the high-quality potentials, and the Nijmegen optical potentials for NN.

II. From the nucleon-nucleon interaction to understanding the nucleus

Very approximate : (Almost) no correlations

The Liquid Drop Model

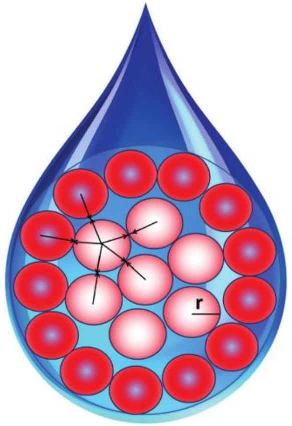


$$M(Z,N) = Z\,M_H + N\,M_n - BE$$

Binding energy

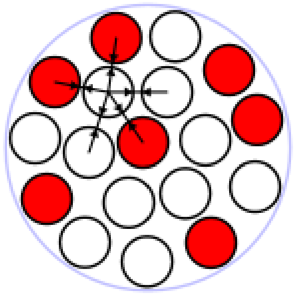
Bulk description of the nucleus
Liquid drop of uniform density

The Liquid Drop Model



Bethe-Weiszäcker Mass Formula

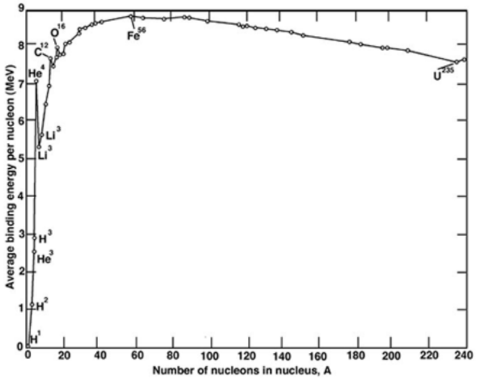
$$B = a_V A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$



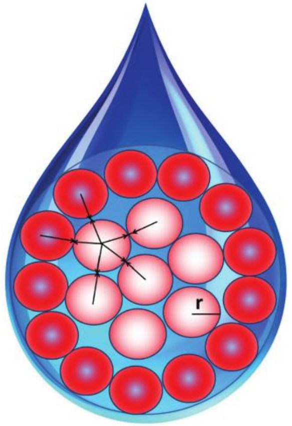
Volume

Volume term

- Binding energy per nucleon is ~constant for stable nuclei over the whole mass table
- The nuclear force is attractive, short-range and saturated
- $$V \propto A$$
  $$R = r_0 A^{1/3} = 1.2\ \text{fm}\ A^{1/3}$$

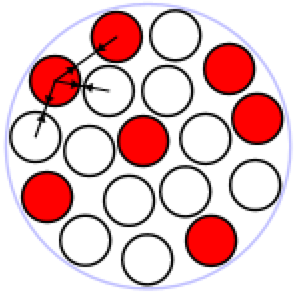


The Liquid Drop Model



$$M(Z, N) = Z\,M_H + N\,M_n - BE$$

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$



Surface term

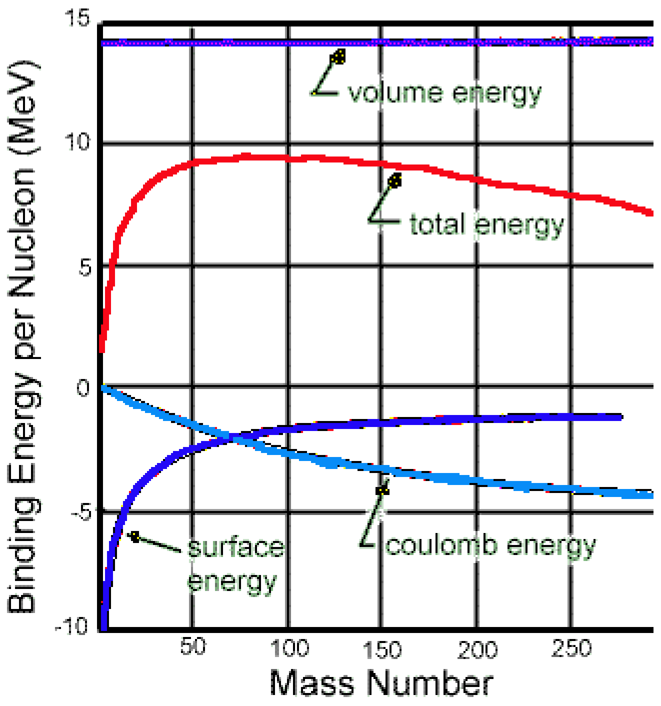
- Nucleons at the surface are not surrounded by other nucleons to attract
- Surface nucleons have reduced binding compared to those in the nuclear interior

The Liquid Drop Model

Bethe-Weiszäcker Mass Formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$

- Volume term
- Surface term
- **Coulomb term** → $\sim Q^2/R$
- Symmetry term
- Pairing

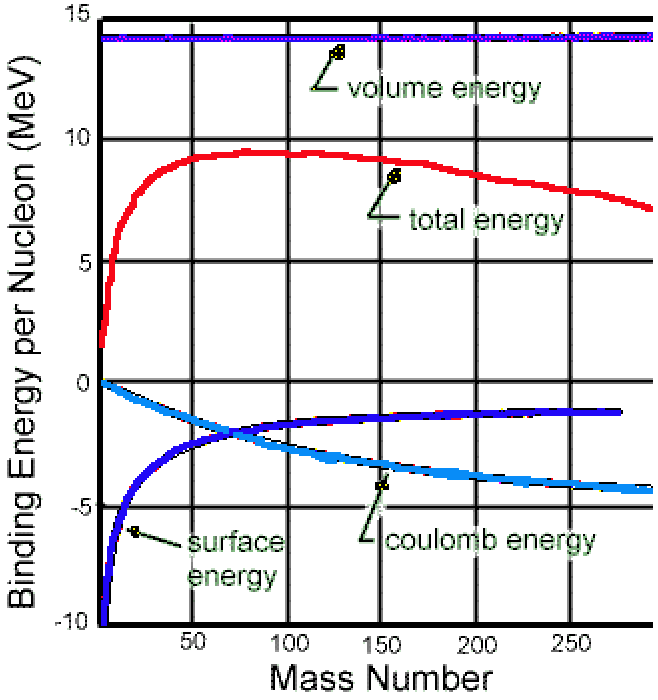


The Liquid Drop Model

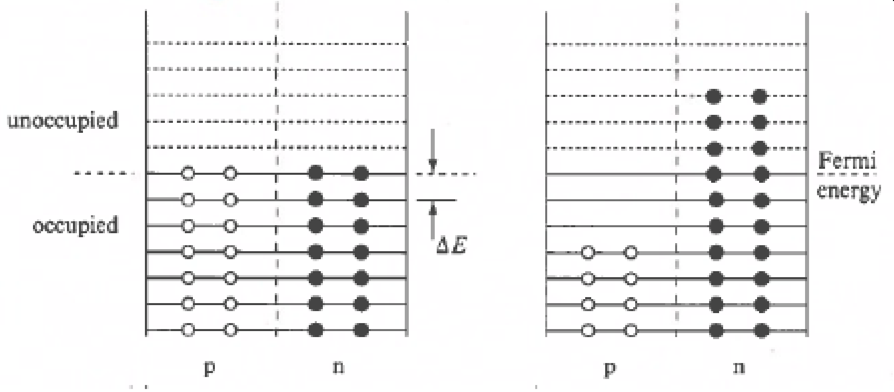
Bethe-Weiszäcker Mass Formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$

- Volume term
- Surface term
- Coulomb term
- **Symmetry term**
- Pairing



Charge symmetry of nucleon-nucleon force + Pauli principle

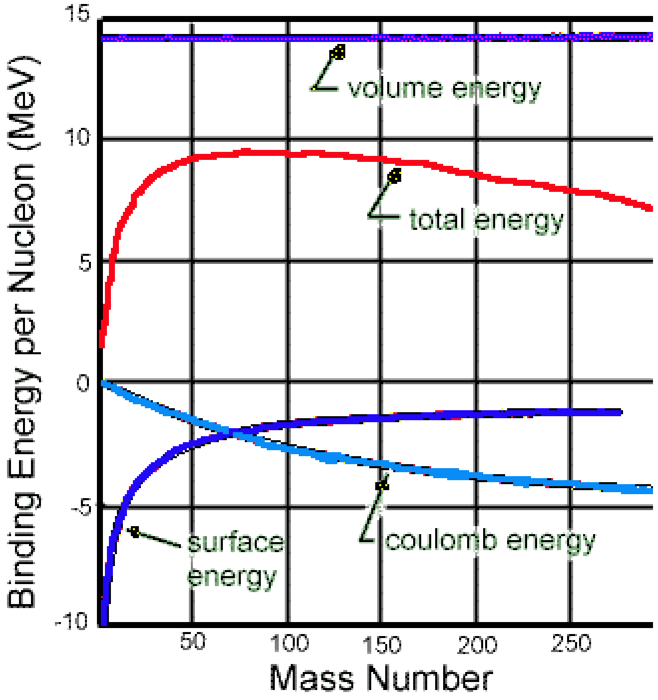


Effective force between n-p > force between n-n or p-p

The Liquid Drop Model

Bethe-Weiszäcker Mass Formula

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$



- Volume term
- Surface term
- Coulomb term
- Symmetry term
- **Pairing**

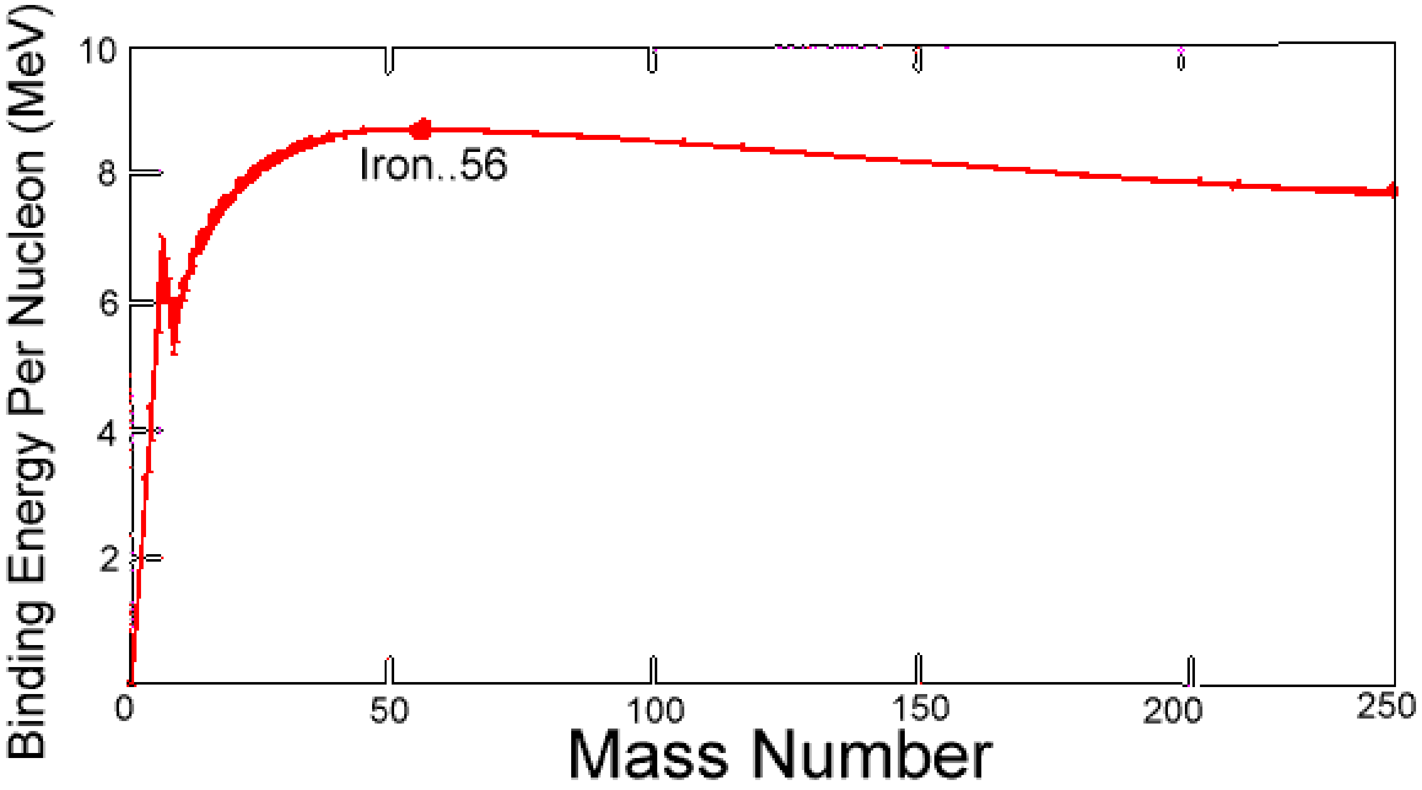
- Short-range interaction
- Nucleons prefer to couple to spatial spin zero pairs to maximize overlap in spatial wave functions

N, Z even: $+\Delta$
A odd: 0
N, Z odd: $-\Delta$

The Liquid Drop Model

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} \pm \Delta$$

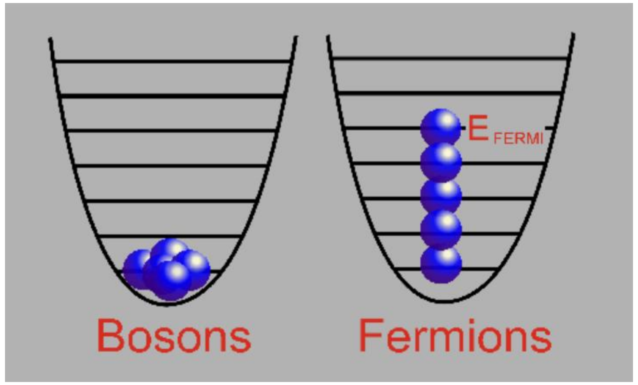
- $a_V = 15.6$ MeV
- $a_S = 17.23$ MeV
- $a_a = 23.28$ MeV
- $a_C = 0.7$ MeV
- $\Delta = \frac{12}{A^{1/2}}$ MeV

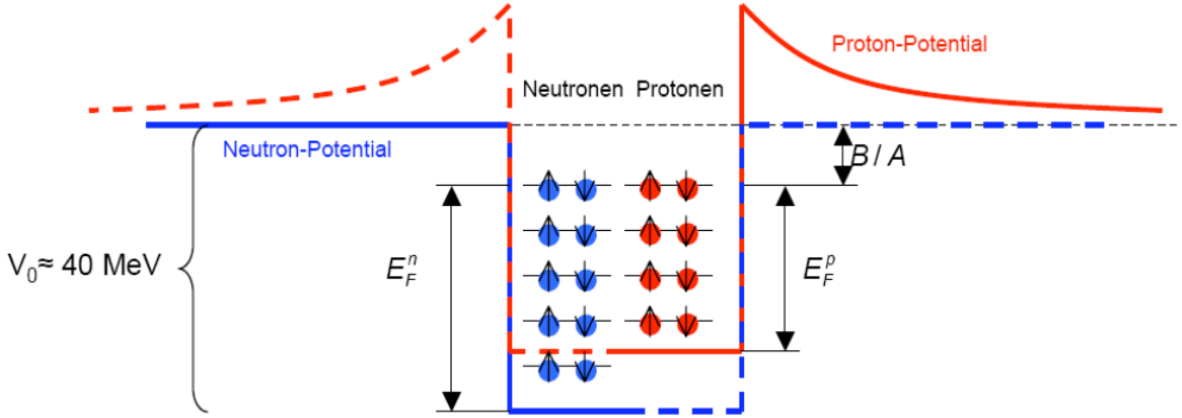


Slightly less approximate : Still (Almost) no correlations

The Fermi Gas model

Most basic microscopic independent particle model





$$n = 2\frac{V}{(2\pi\hbar)^3}\int_0^{p_F} d^3p = 2\frac{V}{(2\pi\hbar)^3}\left(\frac{4}{3}\pi p_F^3\right)$$

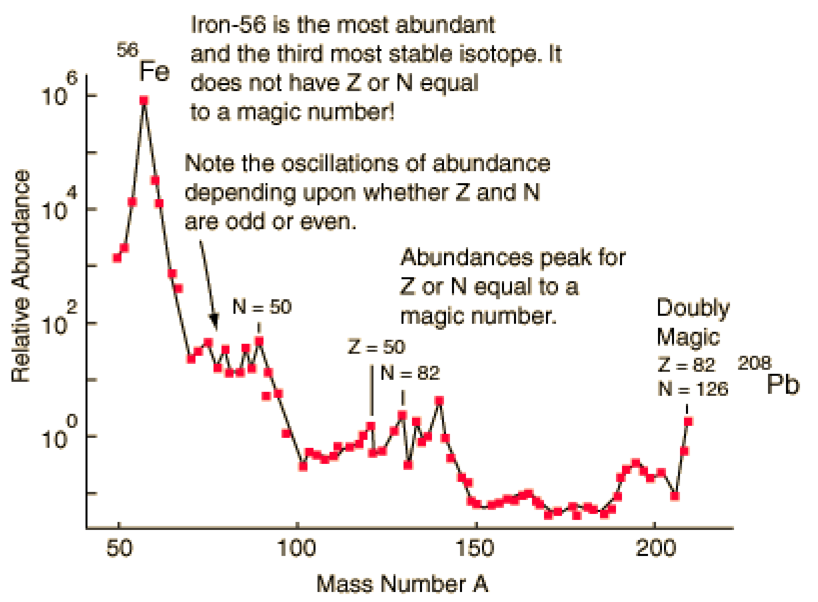
$$\Rightarrow \quad p_F = \hbar\left(\frac{3n\pi}{V}\right)^{1/3} \sim 247\,\mathrm{MeV/c}$$

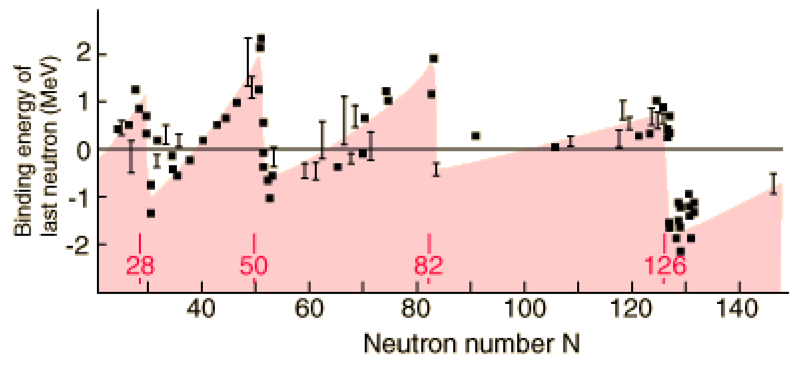
$$E_F = \frac{p_F^2}{2m} \sim 33\,\mathrm{MeV}$$

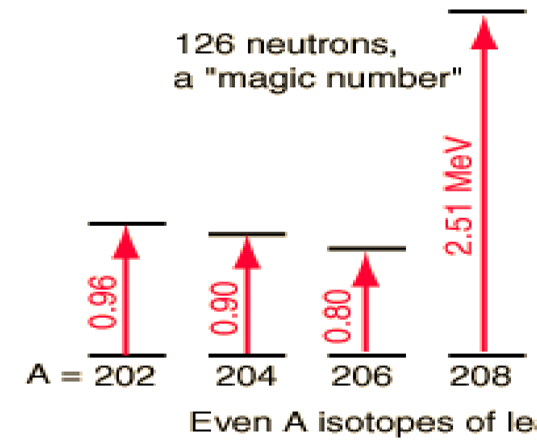
$$\langle E\rangle = \frac{\int_0^{E_F} E_{kin} d^3p}{\int_0^{p_F} d^3p} = \frac{1}{2m}\frac{\int_0^{E_F} p^2 d^3p}{\int_0^{p_F} d^3p} = \frac{3}{5}E_F \sim 20\mathrm{MeV}$$

The Nuclear Shell Model

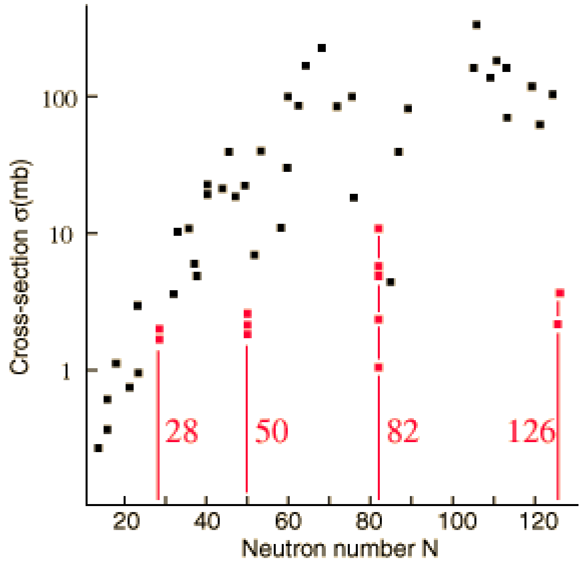
Evidence for Shell Structure







Excitation energy first excited state

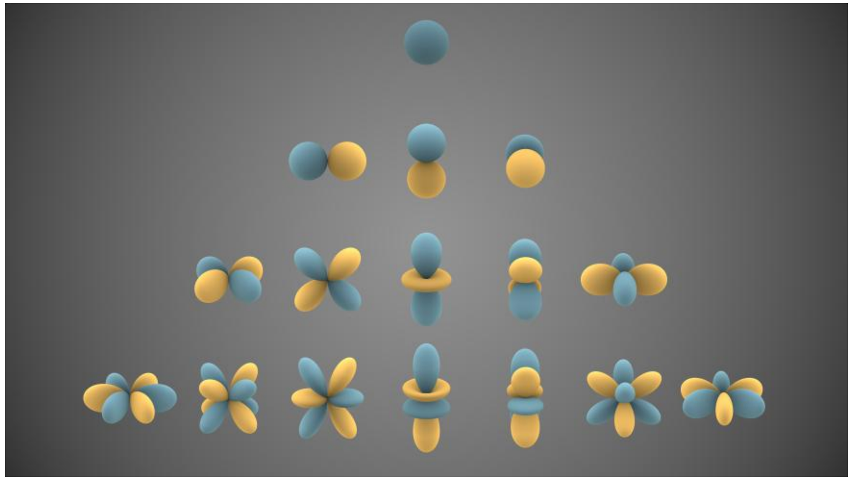


Nuclei with magic numbers of neutrons have neutron absorption cross-sections up to two orders of magnitude less than other nuclei with similar masses

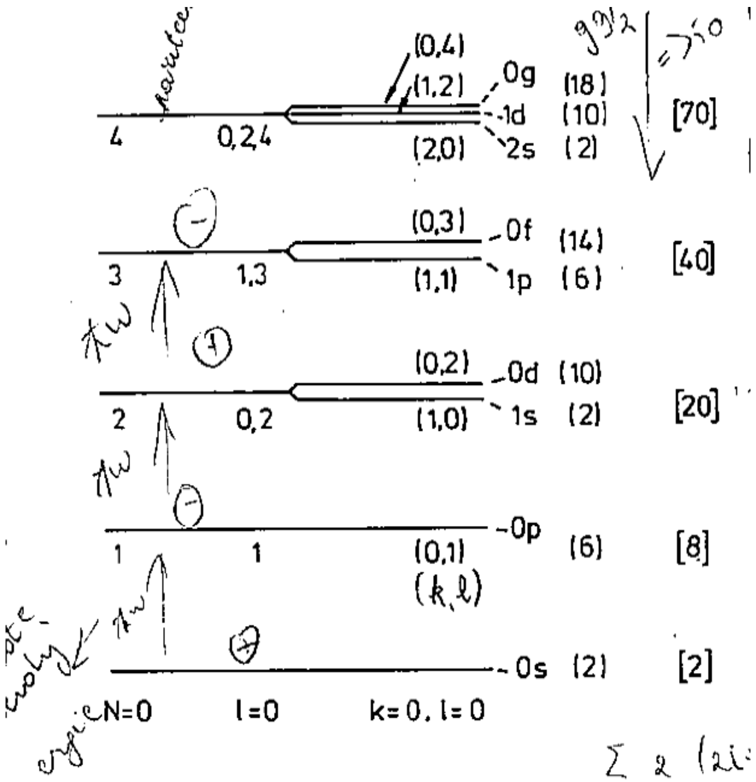
The Nuclear Shell Model

→ solve Schrödinger equation for a nucleon in the nuclear potential

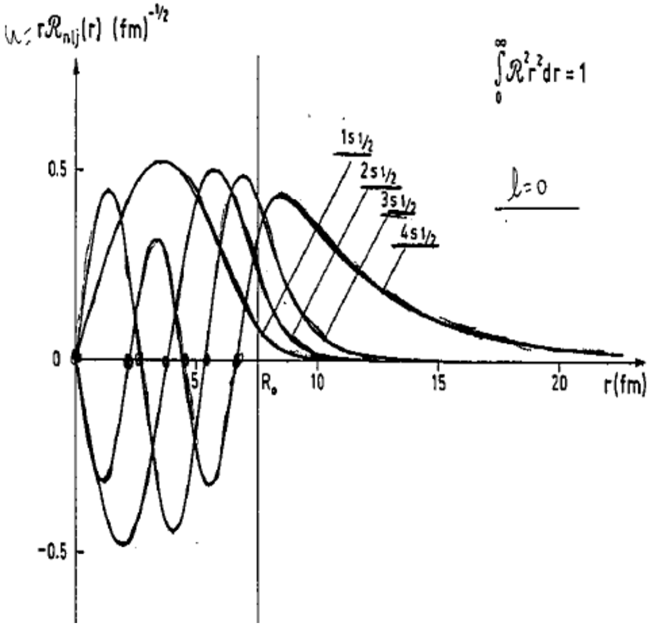
$$\varphi(\vec{r}) = u(r) Y_m^l(\Omega)$$



?

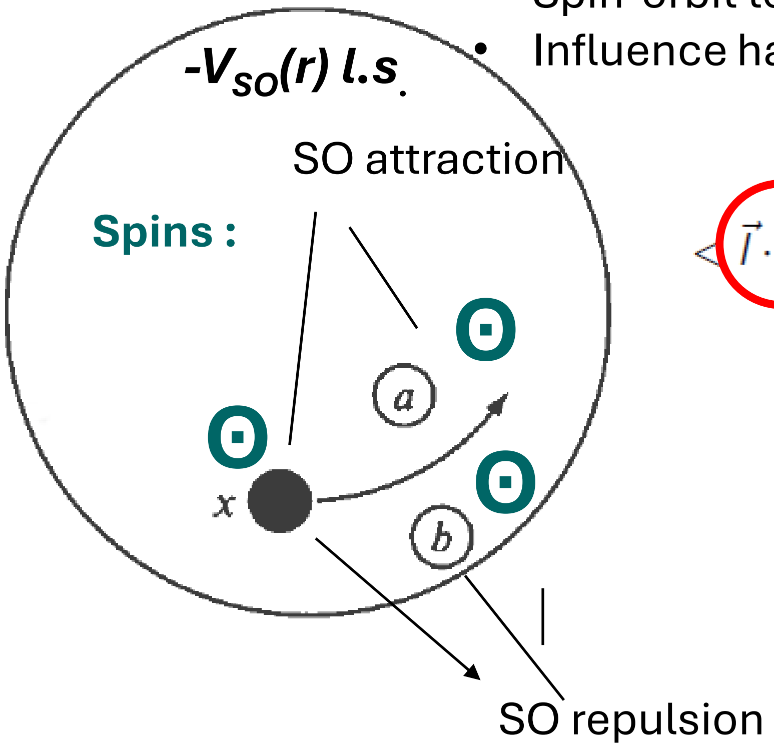


Harmonic oscillator potential



The Nuclear Shell Model

Solution : Spin-orbit coupling (Mayer & Jensen 1949)



SO repulsion

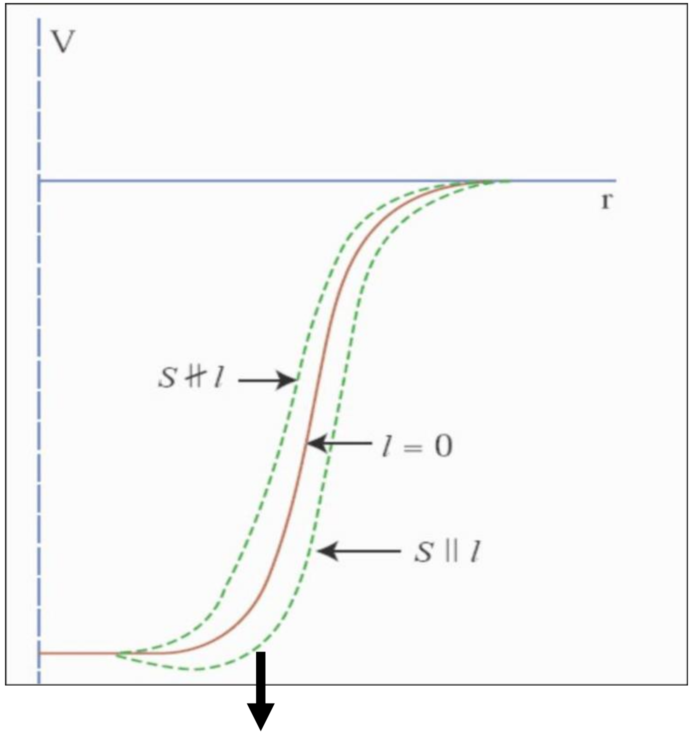
- Spin-orbit term in nucleon-nucleon force
- Influence has to be added on top of the mean field

$$\vec{j}^2 = \vec{l}^2 + \vec{s}^2 + 2\vec{l}\cdot\vec{s}$$

$$<\vec{l}\cdot\vec{s}> = \frac{1}{2}[j(j+1) - l(l+1) - s(s+1)]$$

$$= \frac{1}{2}l\hbar^2 \qquad \text{for } j = l + \frac{1}{2}$$

$$= -\frac{1}{2}(l+1)\hbar^2 \qquad \text{for } j = l - \frac{1}{2}$$



- Little influence on depth of potential
- Affects width of the potential

x : spin // orbital angular momentum
Relative orbital momentum x-a and x-b are in opposite direction : $SO_{x\text{-}a}$ and $SO_{x\text{-}b}$ attractive vs repulsive
⇒ in nuclear interior SO effects will cancel

⇒ at the surface there will be less nucleons at larger radii (more a than b from the viewpoint of x), resulting in a net SO force, attractive for spins parallel to l, repulsive for pairs with spin opposite to l

The Nuclear Shell Model

Nuclear shell model [edit]

Main article: Nuclear shell model

During her time at Chicago and Argonne in the late 1940s, Goeppert Mayer developed a mathematical model for the structure of nuclear shells, which she published in 1950.[36][37] Her model explained why certain numbers of nucleons in an atomic nucleus result in particularly stable configurations. These numbers are what Eugene Wigner called *magic numbers*: 2, 8, 20, 28, 50, 82, and 126. In an account relayed by Joe Mayer, Maria Goppert Mayer attained a critical insight while speaking with Enrico Fermi.

> Fermi and Maria were talking in her office when Enrico was called out of the office to answer the telephone on a long distance call. At the door he turned and asked his question about spin-orbit coupling. He returned less than ten minutes later and Maria started to 'snow' him with the detailed explanation. You may remember that Maria, when excited, had a rapid fire oral delivery, whereas Enrico always wanted a slow detailed and methodical explanation. Enrico smiled and left: 'Tomorrow, when you are less excited, you can explain it to me.'[38]

Maria Goeppert Mayer walking into the Nobel ceremony with King Gustaf VI Adolf of Sweden in 1963

She had realised that the nucleus is a series of closed shells and pairs of neutrons and protons tend to couple together.[39][40] She described the idea as follows:

> Think of a room full of waltzers. Suppose they go round the room in circles, each circle enclosed within another. Then imagine that in each circle, you can fit twice as many dancers by having one pair go clockwise and another pair go counterclockwise. Then add one more variation; all the dancers are spinning twirling round and round like tops as they circle the room, each pair both twirling and circling. But only some of those that go counterclockwise are twirling counterclockwise. The others are twirling clockwise while circling counterclockwise. The same is true of those that are dancing around clockwise: some twirl clockwise, others twirl counterclockwise.[41]

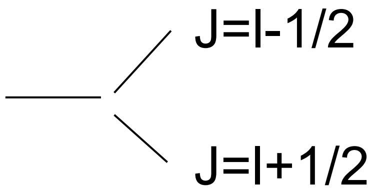
Three German scientists, Otto Haxel, J. Hans D. Jensen, and Hans Suess, were also working on solving the same problem, and arrived at the same conclusion independently. While their results were announced in an issue of the Physical Review before Goeppert Mayer in June 1949, Goeppert Mayer's work was received for review in February 1949, while the work of the German authors was received later in April 1949.[42][43] Afterwards, she collaborated with them. Hans Jensen co-authored a book with Goeppert Mayer in 1950 titled *Elementary Theory of Nuclear Shell Structure*.[44] In 1963, Goeppert Mayer, Jensen, and Wigner shared the Nobel Prize for Physics "for their discoveries concerning nuclear shell structure."[45] She was the second female Nobel laureate in physics, after Marie Curie,[46] and would be the last for over half a century, until Donna Strickland was awarded the prize in 2018.[24]

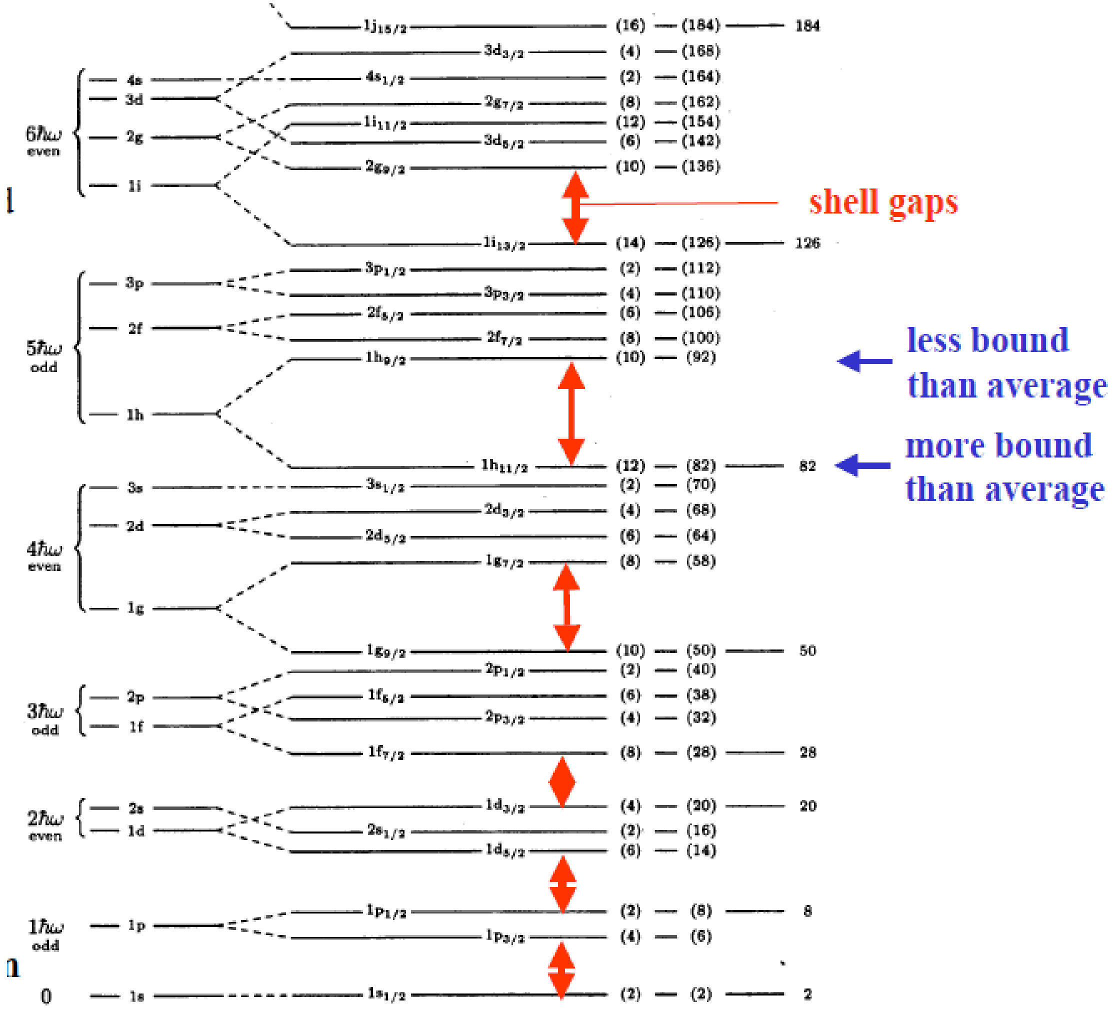
The Nuclear Shell Model

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \frac{2\eta k}{r} + k^2 - \frac{2m}{\hbar^2}(\hat{V}_C(r) + \hat{V}_{SO}(r))\right] u_{lj}(r) = 0$$

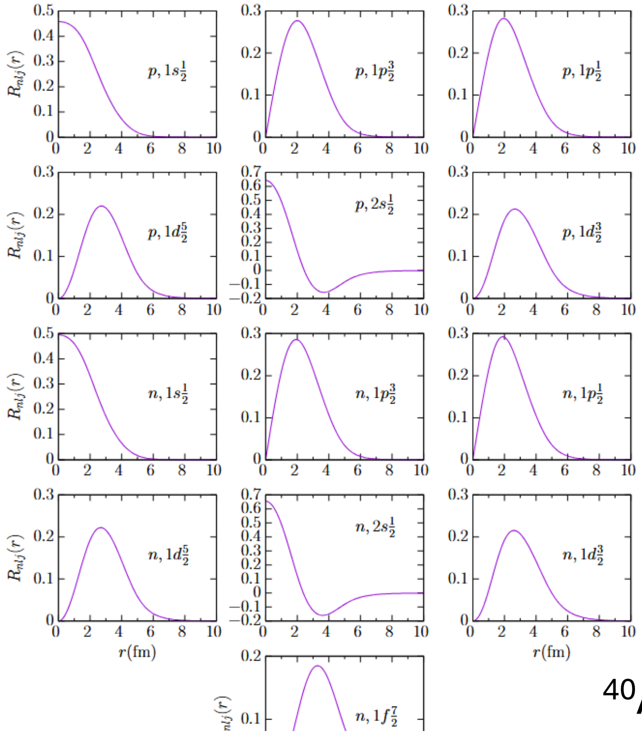
→Work harder : add spin-orbit term

$$\widehat{H} \rightarrow \widehat{H} + \xi(r)\bar{l} \cdot \bar{s}$$





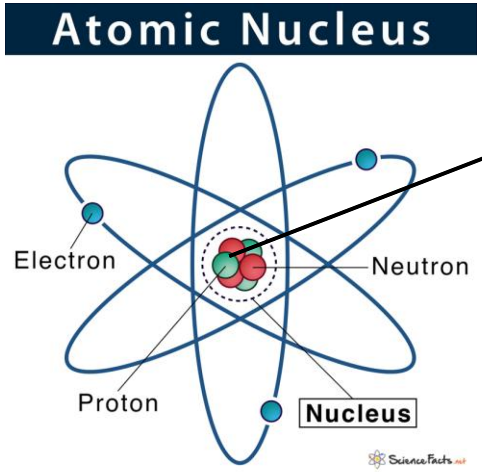
Magic numbers He, Oxygen, Calcium, Lead, ...

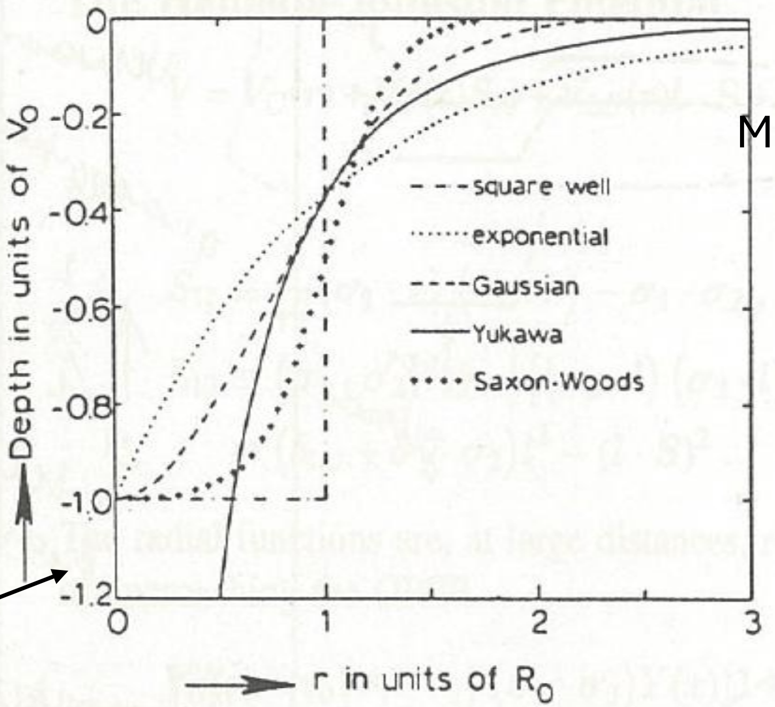


^{40}Ar single-particle Wave functions

The Nuclear Shell Model

- First approximation : nucleons are moving (quasi)independently in a mean field potential
- The correlations are encoded in the mean field potential i.e. the averaged potential generated by all individual nucleon-nucleon interactions
- Pauli exclusion makes this description as independent particles possible





Mean-field potential

Fig. 3.27. Plots of the potentials, given below. In addition, the Woods-Saxon potential with a diffuseness parameter $a = 0.7$ is also shown [taken from (Brussaard, Glaudemans 1977)]

Square well $\quad V(r) = -V_0 \quad r \le R_0$,
$\qquad\qquad\qquad\quad = 0 \quad r > R_0$,

Exponential potential $\quad V(r) = -V_0 \exp(-r/R_0)$,

Gaussian potential $\quad V(r) = -V_0 \exp(-r^2/R_0^2)$,

Yukawa potential $\quad V(r) = -V_0 \exp(-r/R_0)/(r/R_0)$.

The Nuclear Shell Model : Woods-Saxon potential

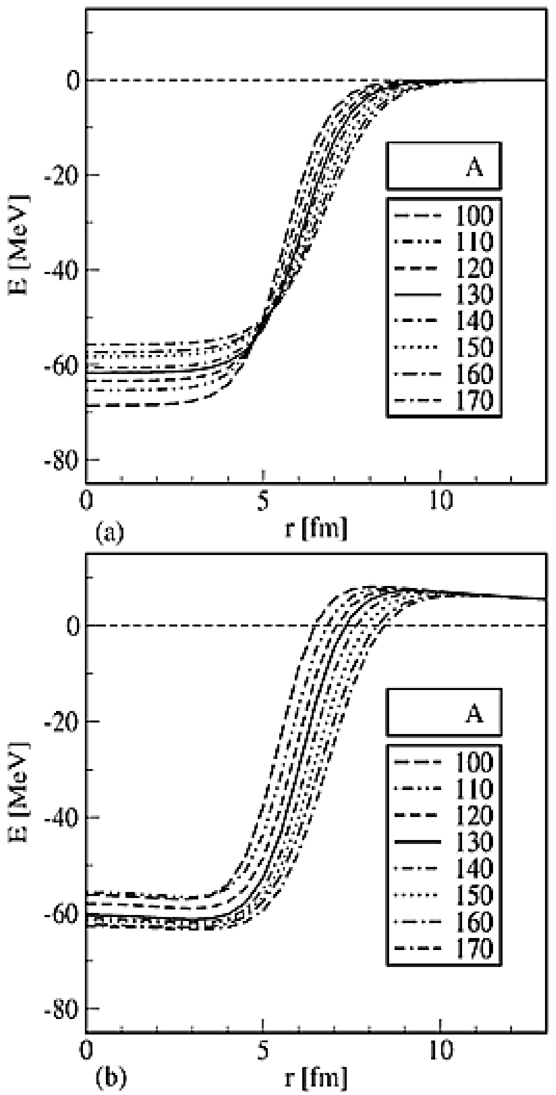


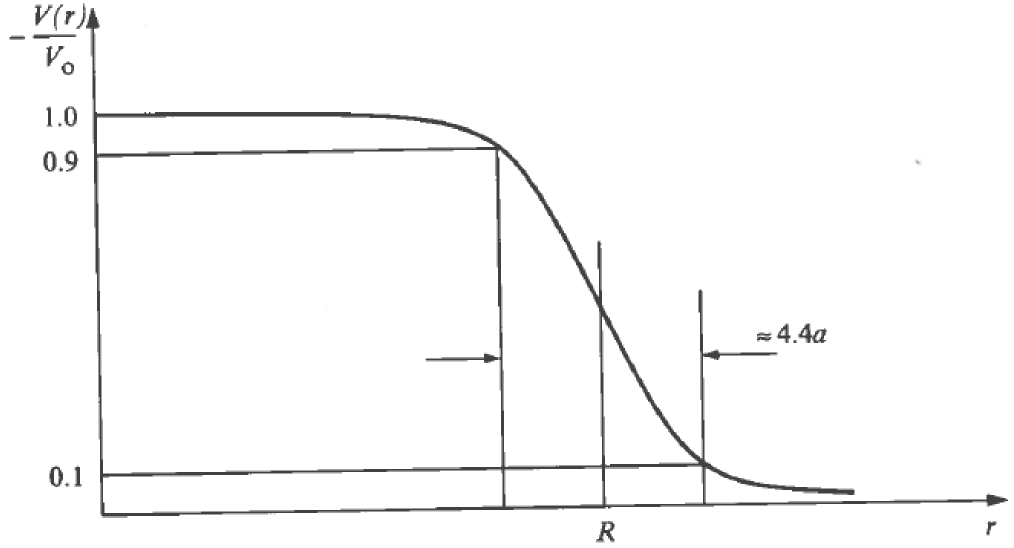
Figure 1. Woods-Saxon potential for neutrons (a) and protons (b) along the Sn isotopic chain.

$$V(r) = \frac{-V_0}{1 + e^{\left(\frac{r-R}{a}\right)}}$$

Depth of the well ~50 MeV

R=1.2 $A^{1/3}$ fm

Diffuseness ~0.6 fm



The Nuclear Shell Model

- Single-particle shells progressively filled by nucleons
- Full shells result in 0^+ nuclear states
- Even-even nuclei have 0^+ ground state
- Extra nucleons (on top of the closed shells) couple in 0^+ pairs
- Uncoupled nucleons determine ground state quantum numbers

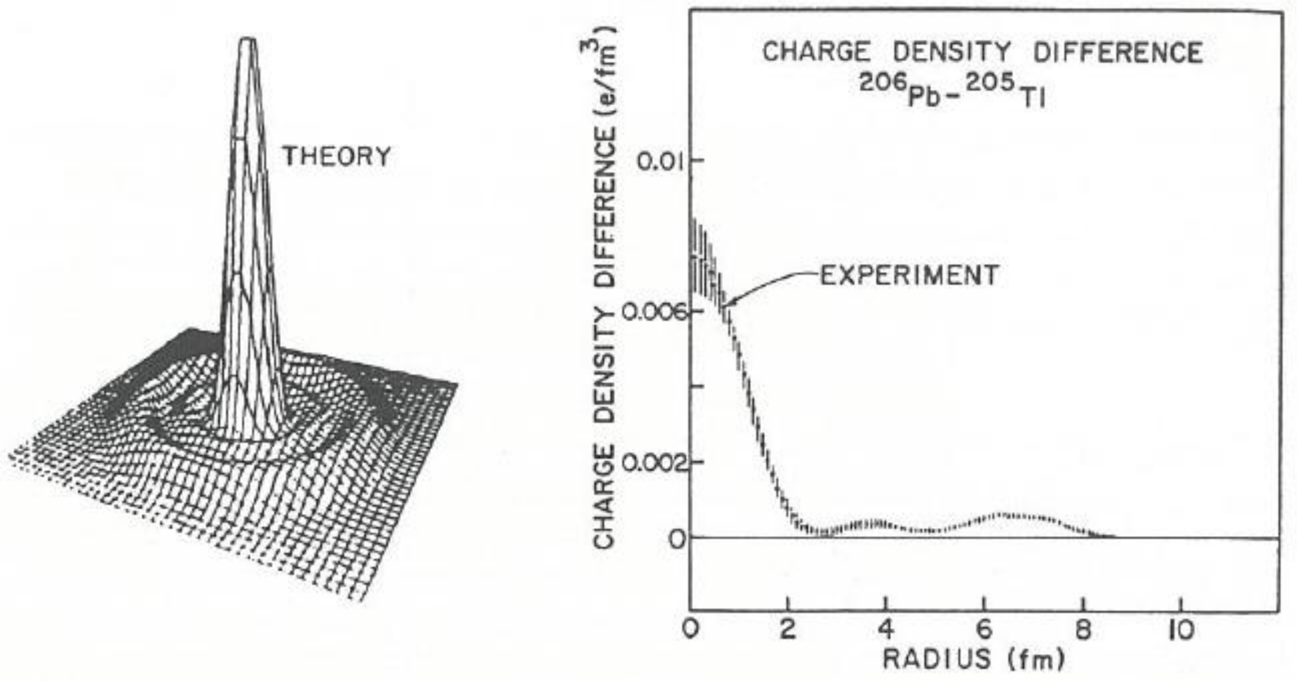
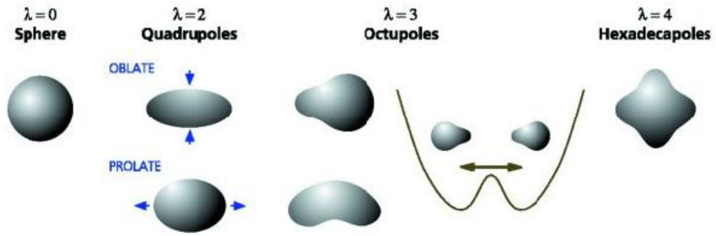
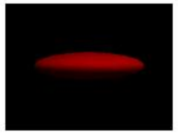


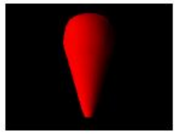
Fig. 3.18. The nuclear density distribution for the least bound proton in ^{206}Pb. The shell-model predicts the last ($3s_{1/2}$) proton in ^{206}Pb to have a sharp maximum at the centre, as shown at the left-hand side. On the right-hand side the nuclear charge density difference $\varrho_c(^{206}\mathrm{Pb}) - \varrho_c(^{205}\mathrm{Tl}) = \varphi^2_{3s_{1/2}}(r)$ is given [taken from (Frois 1983) and Doe 1983)]

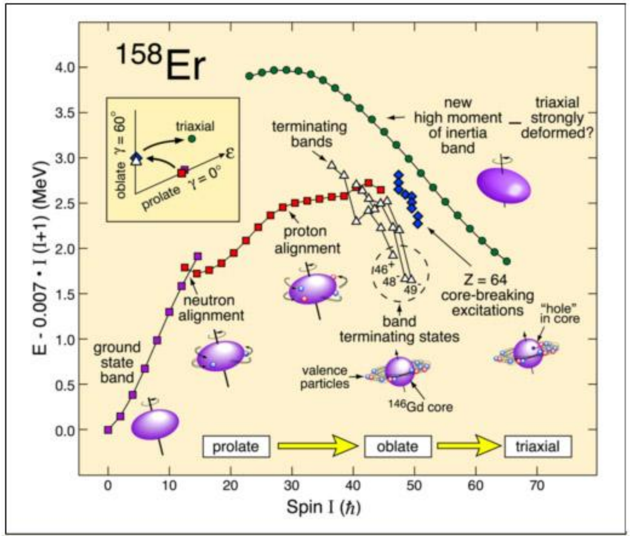
The Nuclear Shell Model

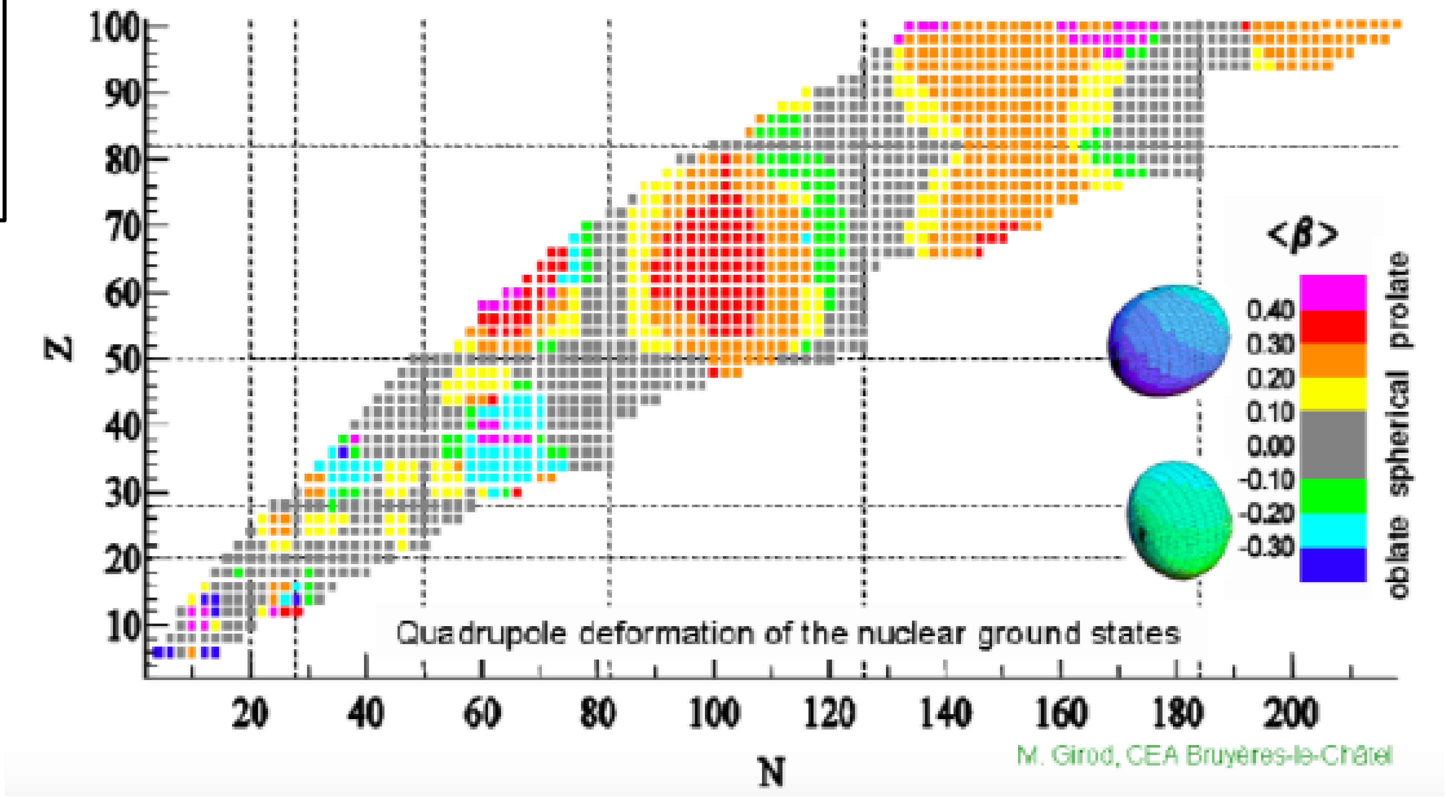
- Between closed shells the valence nucleons give rise to deformation
- Deformed nuclei can exhibit vibration and rotation







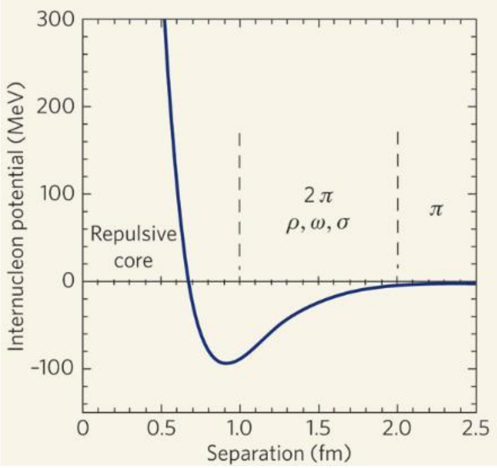


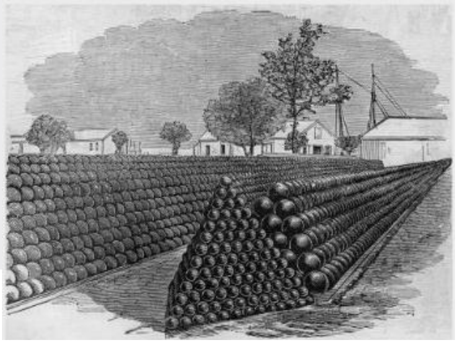


III. Beyond mean-field correlations

Short range correlations

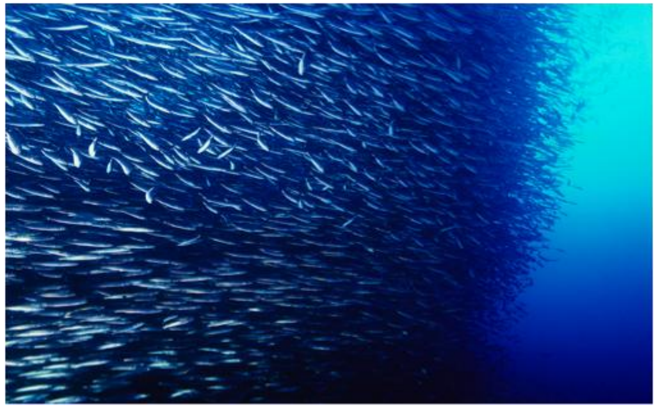
- Nuclear radius $\approx 1.2A^{\frac{1}{3}}$ fm
- Nucleon is a diffuse system
  - Hard core (repulsion) ≈ 0.5 fm
  - RMS charge radius from (e,e′) $= 0.897(18)$ fm
- $0.07 \lesssim \text{NPF} \lesssim 0.42$
  - closest packing fraction of spheres ≈ 0.74
  - packing fraction of Argon liquid ≈ 0.032
  - packing fraction of Argon gas $\approx 3.75 \cdot 10^{-5}$
- The nuclear medium is a rather **dense quantum liquid**





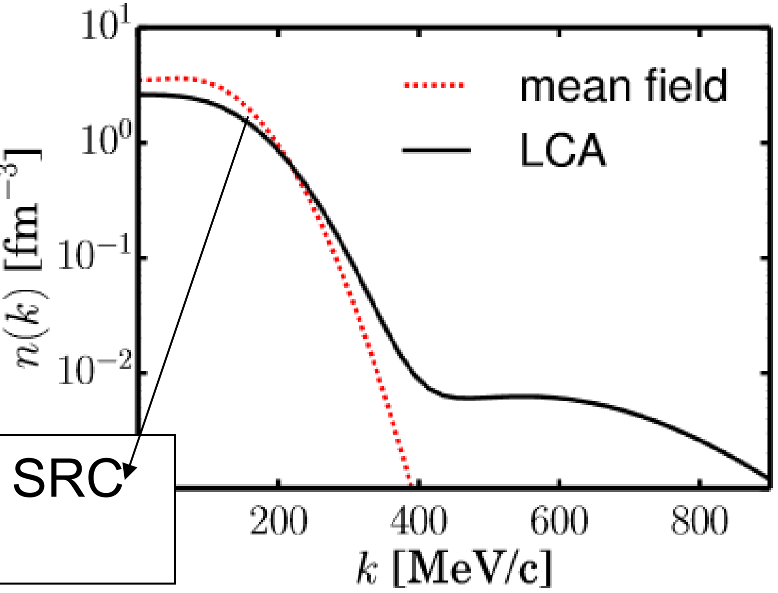
C. Colle, PhD, UGent 2017

Short range correlations



- The short-range repulsive character of this force, which correlates with the Pauli exclusion principle, results in a large mean free path of the nucleons with respect to the size of the nucleus
- In an independent particle model nucleons move independently from each other in a mean field
- This approach fails to capture short-range features of nucleon-nucleon correlations

- SRC : short-range repulsive, tensor component of the nuclear force
- Individual nucleons receive large momenta compared to the Fermi momentum



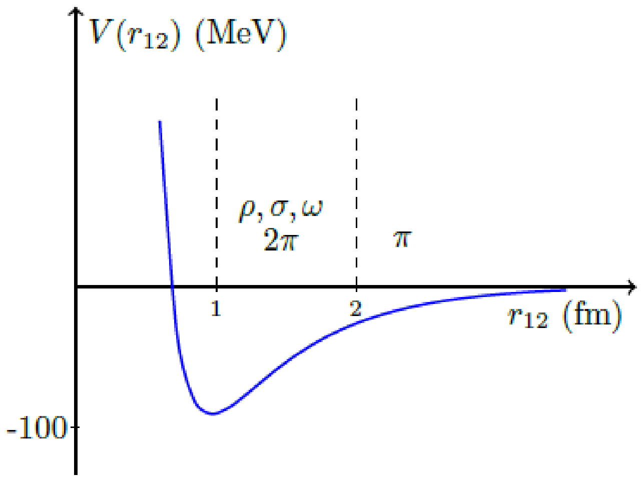
IPM single-particle orbitals are depleted by SRC and higher energy levels are populated

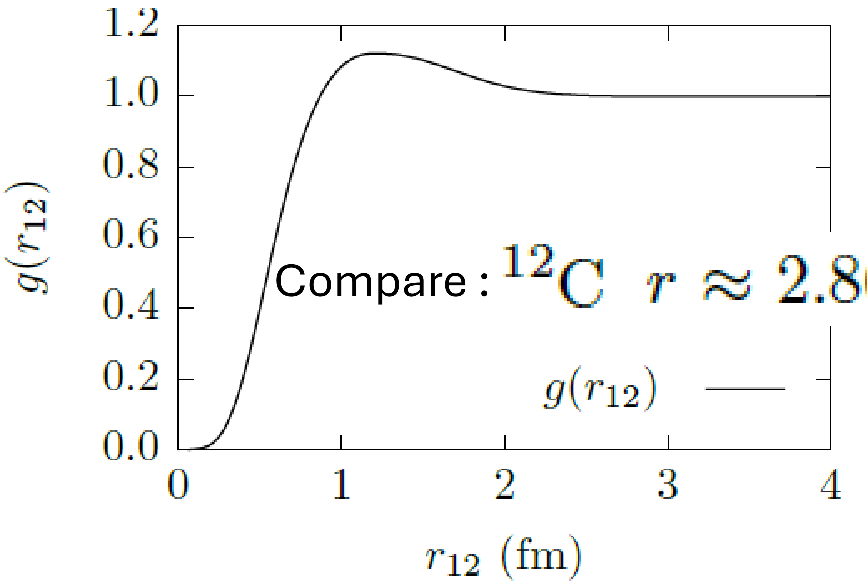
Short range correlations

Two-body density : $\rho^{[2]}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \underbrace{\rho^{[1]}(\boldsymbol{r}_1)\rho^{[1]}(\boldsymbol{r}_2)}g(r_{12})$

Independent particle model

Correlation function

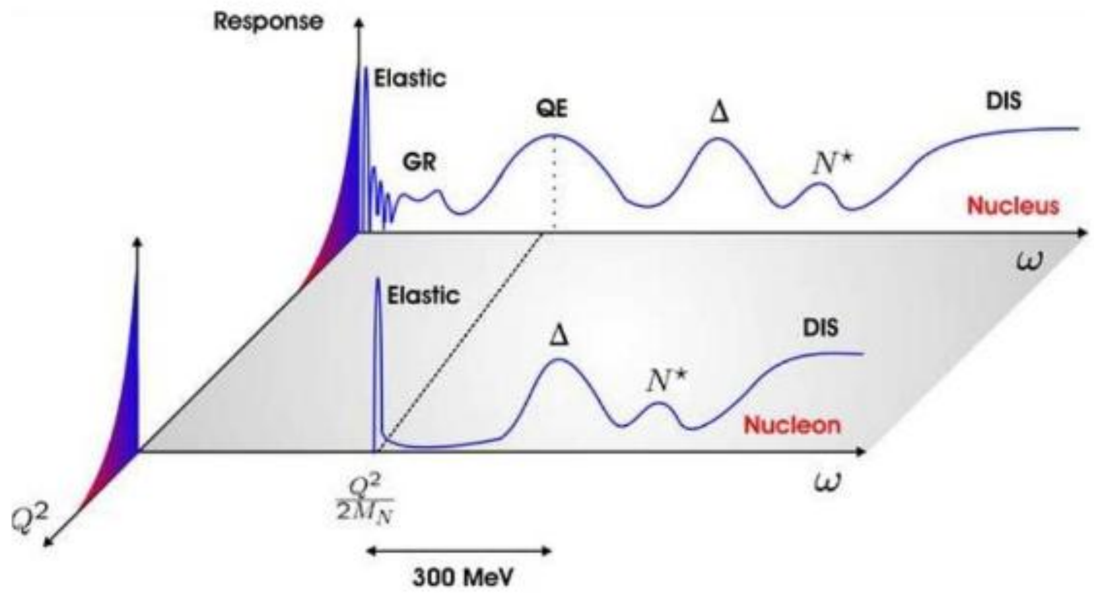




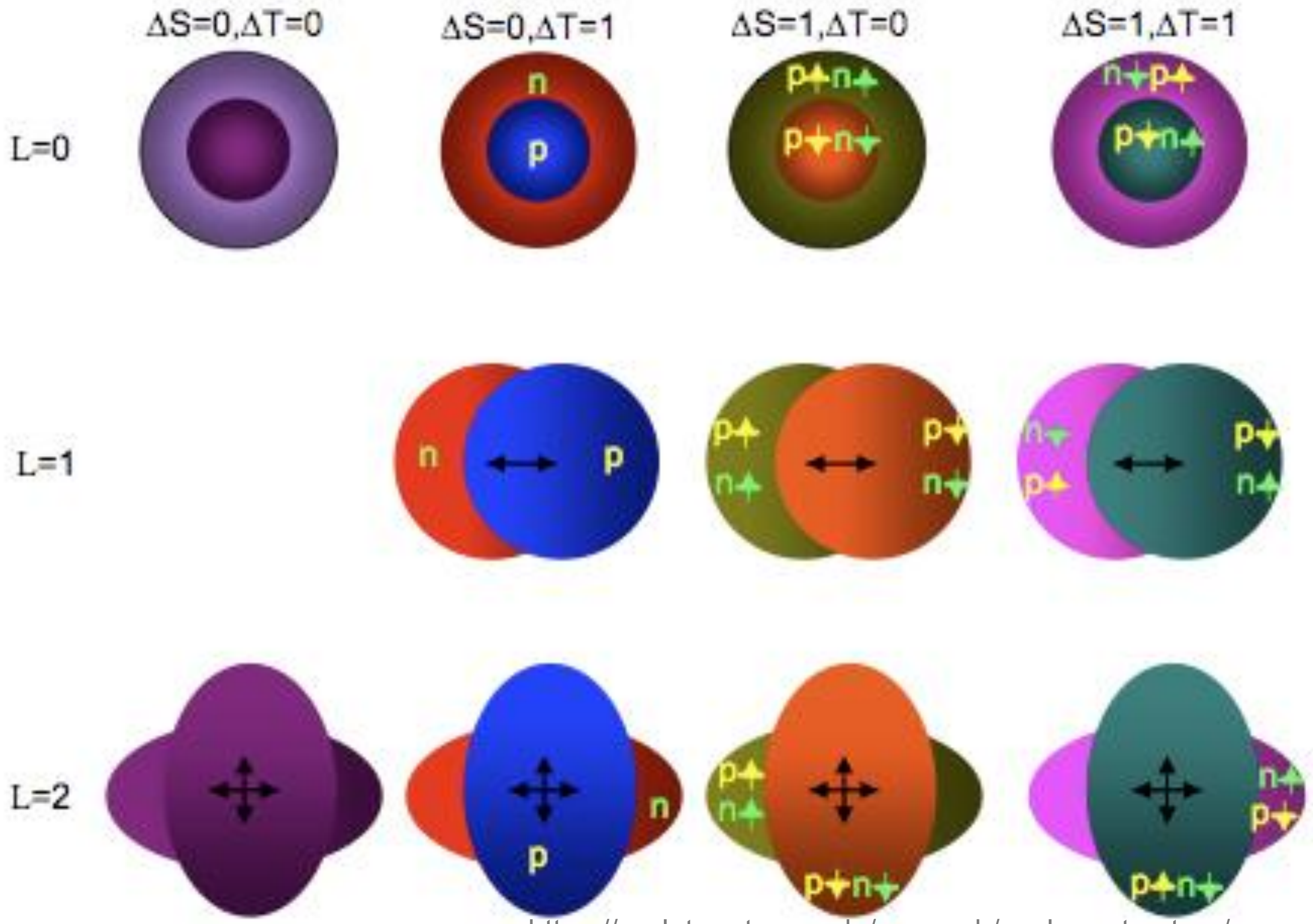
Compare : $^{12}\mathrm{C} \quad r \approx 2.86$ fm

Long range correlations : probing collective effects

- Long-range correlations are correlations over the whole size of the nucleus
- In an interaction,they can redistribute the incoming energy transfer to the nucleus over all the nuclear constituents in a coherent way
- They manifest themselves in collective excitations such as giant resonances
- Especially important for the description of processes in the tens of MeVs range
- Included in Continuum Random Phase Approximation calculations



Long range correlations : probing collective effects



https://cyclotron.tamu.edu/research/nuclear-structure/