Active and Sterile Neutrino Cosmology

Graciela Gelmini - UCLA

DE-SC0009937

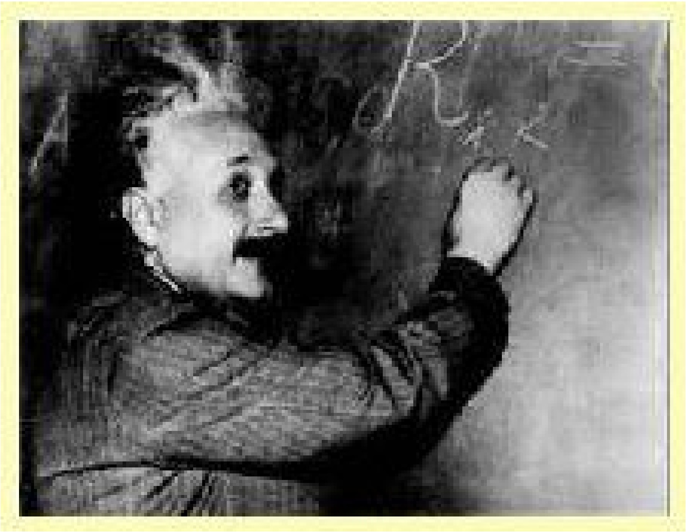
INSS-2026, UCSB, June 29- July 10, 2026

Content:

- Historical development of Standard Model of cosmology
- Basic cosmology of the Early Universe
- Neutrinos and BBN
- Neutrinos and CMB, BAO
- Neutrinos and large scale structure (LSS) in the Universe
- Sterile neutrinos as warm dark matter candidates

The expanding Universe

Some history: Let us start with Albert Einstein 1905



Some history

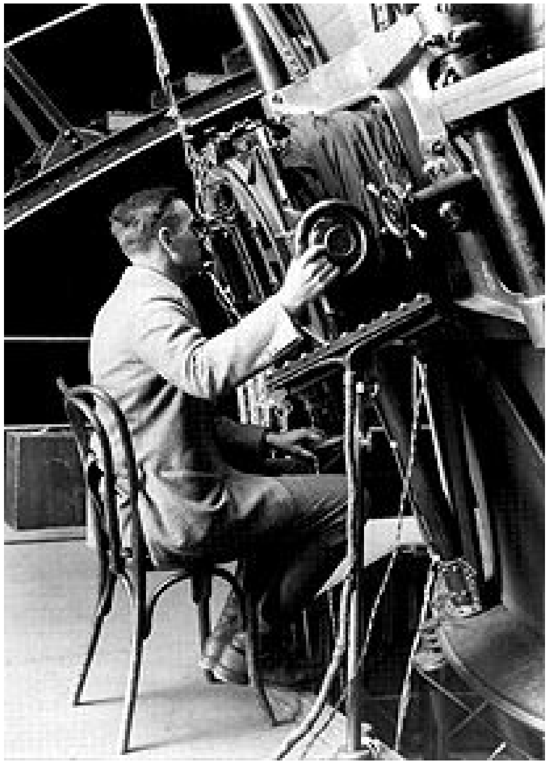
- 1905- Einstein - Special relativity
- 1915- Einstein- General Relativity (the theory of gravity)
- 1917- Einstein believed the Universe was static! He introduced

  the cosmological constant Λ (to equilibrate the gravitational attraction of matter and energy he needed a REPULSIVE gravitational interaction-just in the right amount)

- 1920's - Are "spiral nebulae" part of our galaxy?

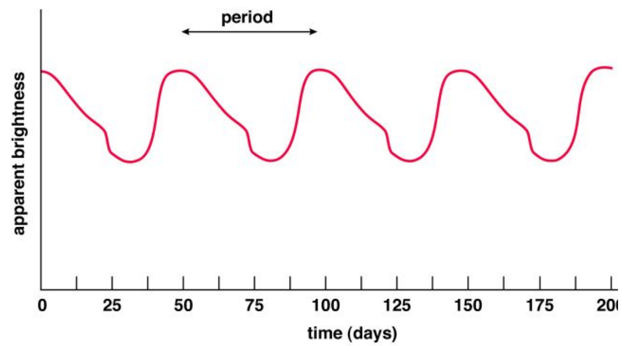
(picture of a "spiral nebula" and stars)

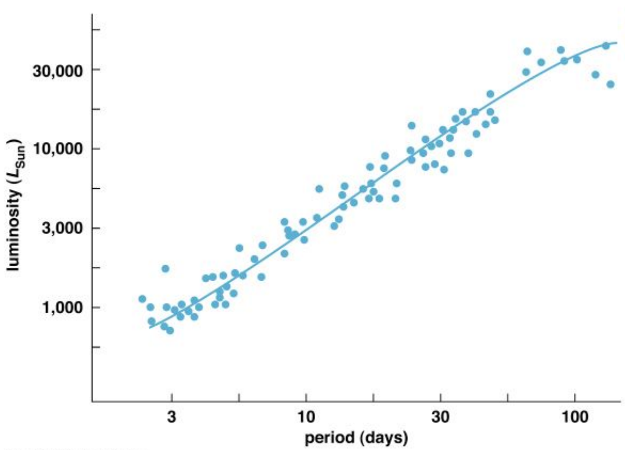
- 1924- Edwin Hubble- measured the distance to a “spiral nebula” the Andromeda Galaxy (used variable stars called Cepheids as “standard candles”) proving it is a separate galaxy larger than ours

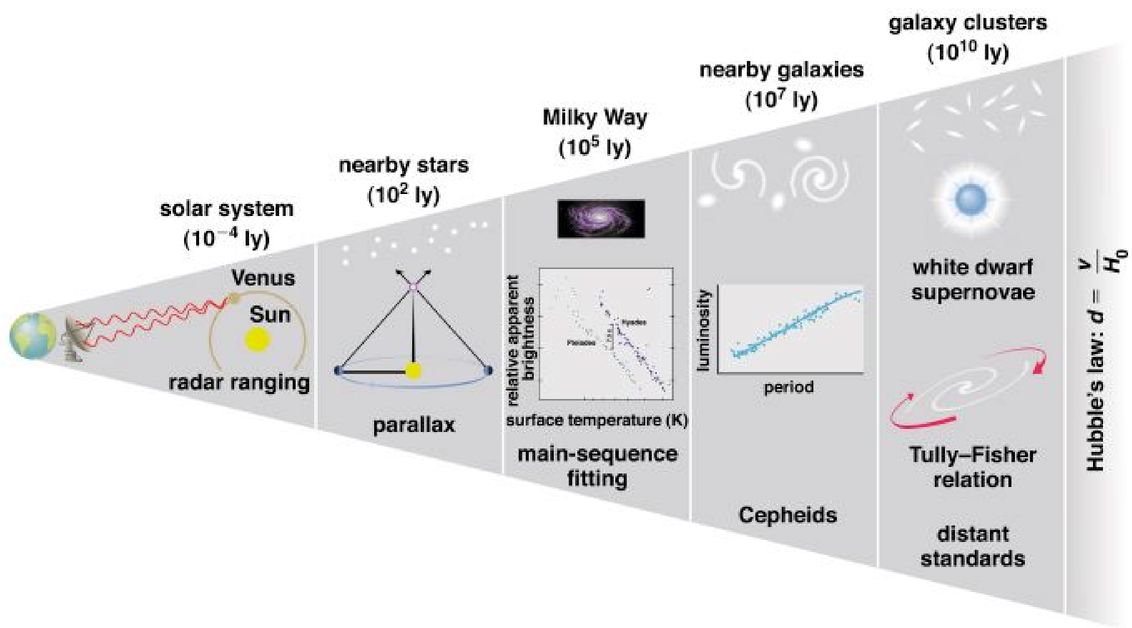


Andromeda Galaxy

- 1924- Hubble used Cepheids variable stars as "standard candles", i.e. sources whose "intrinsic luminosity" L (power) we know thus measuring the "apparent brightness" (flux) $F = L/4\pi r^2$ we can know the distance to the source. Beginning of the "distance ladder"







SN Ia as "calibrated" standard candles

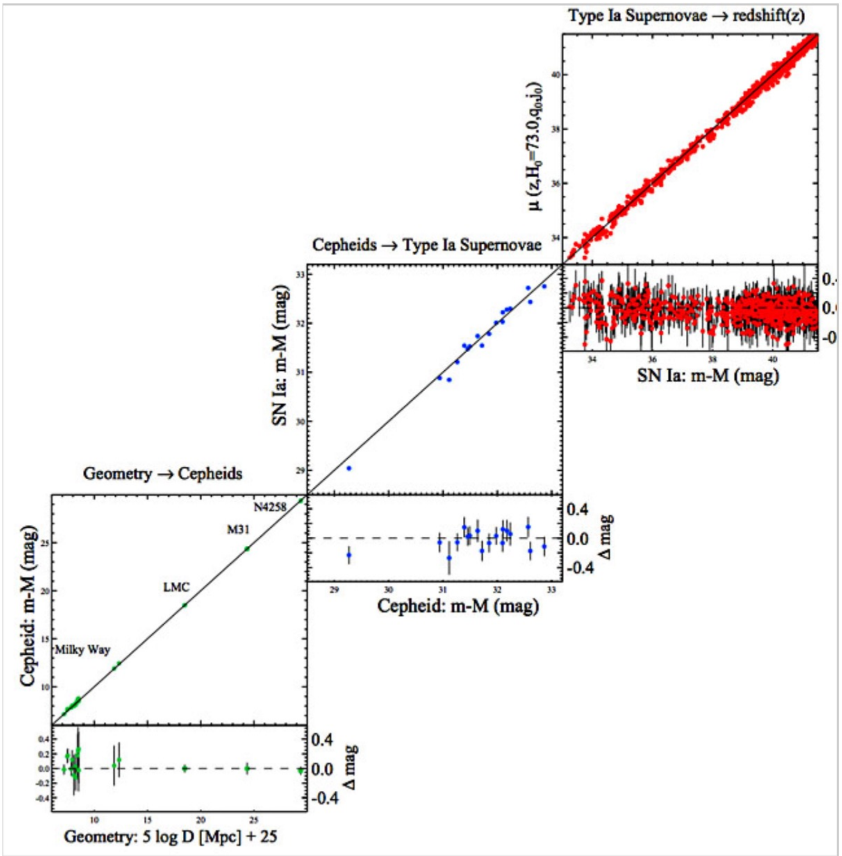
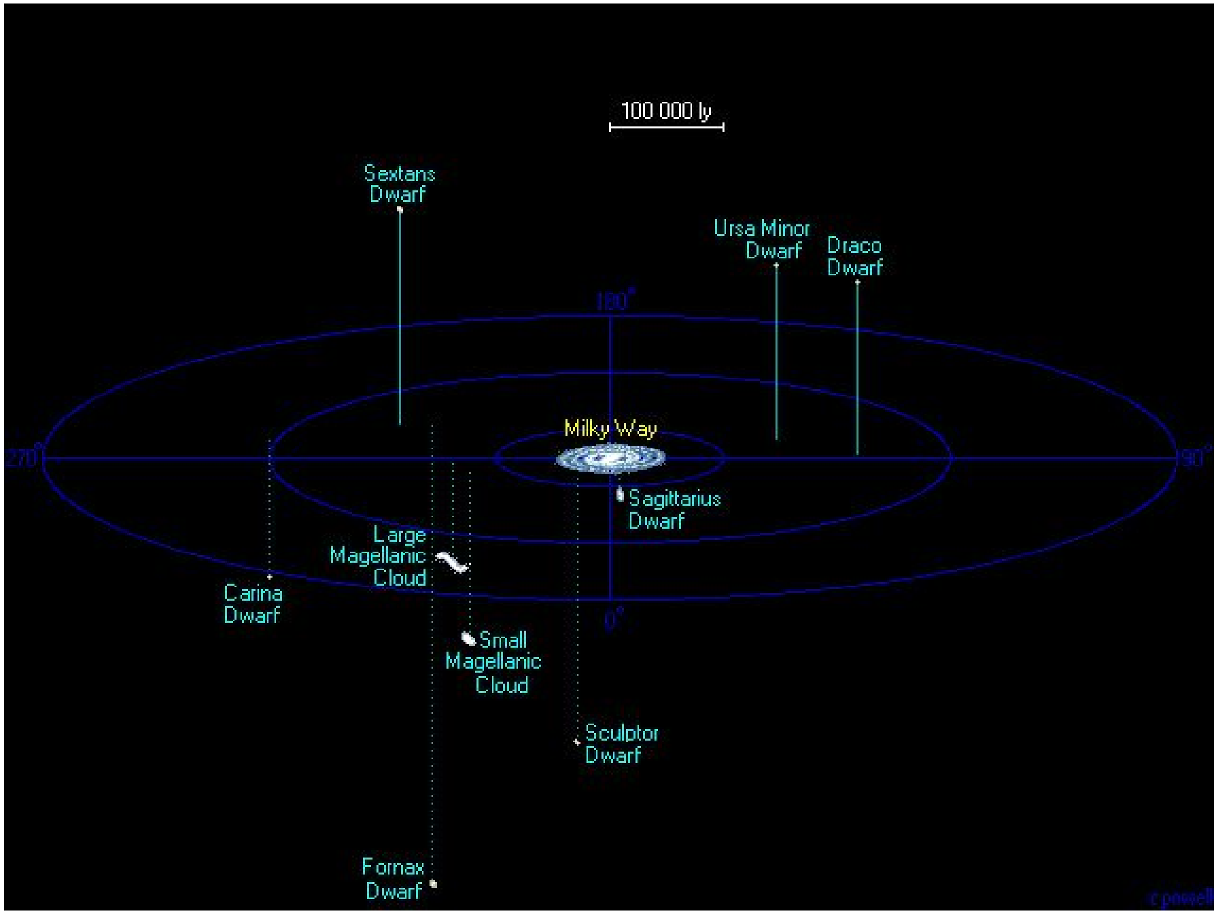
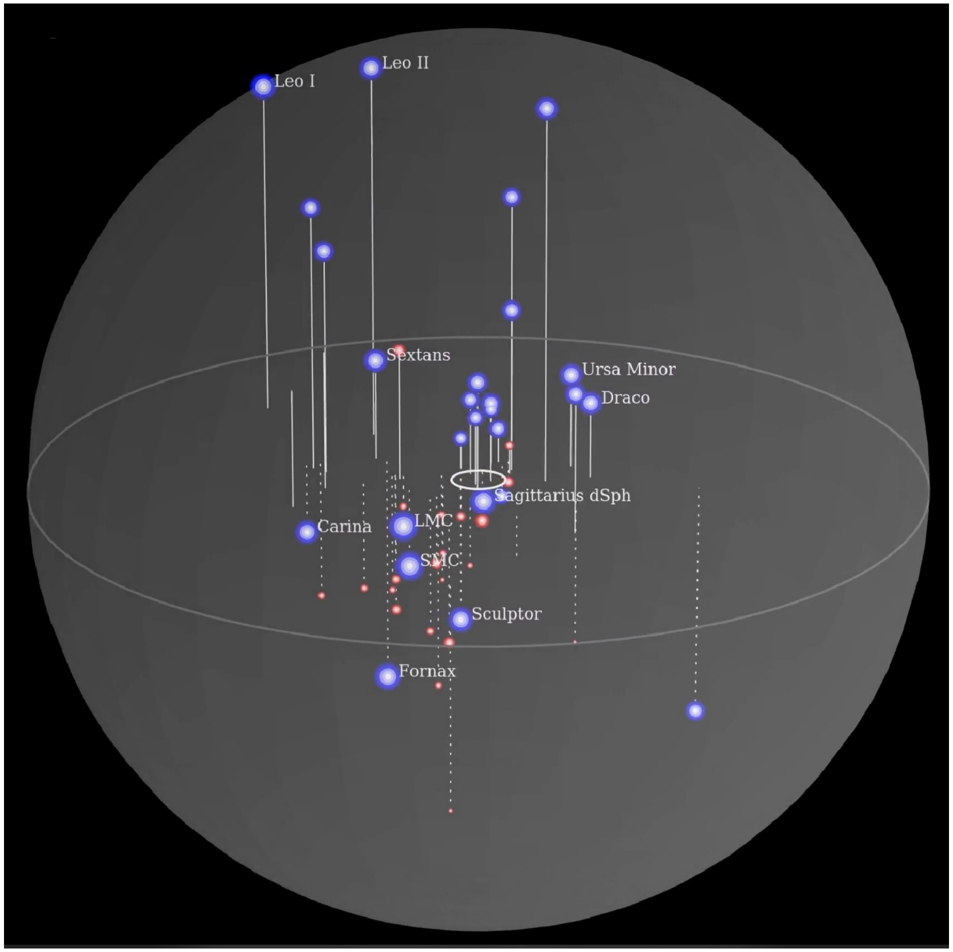


Figure: Complete distance ladder. The simultaneous agreement of pairs of geometric and Cepheid-based distances (lower left), Cepheid and SN Ia-based distances (middle panel) and SN and redshift-based distances provides the measurement of the Hubble constant.

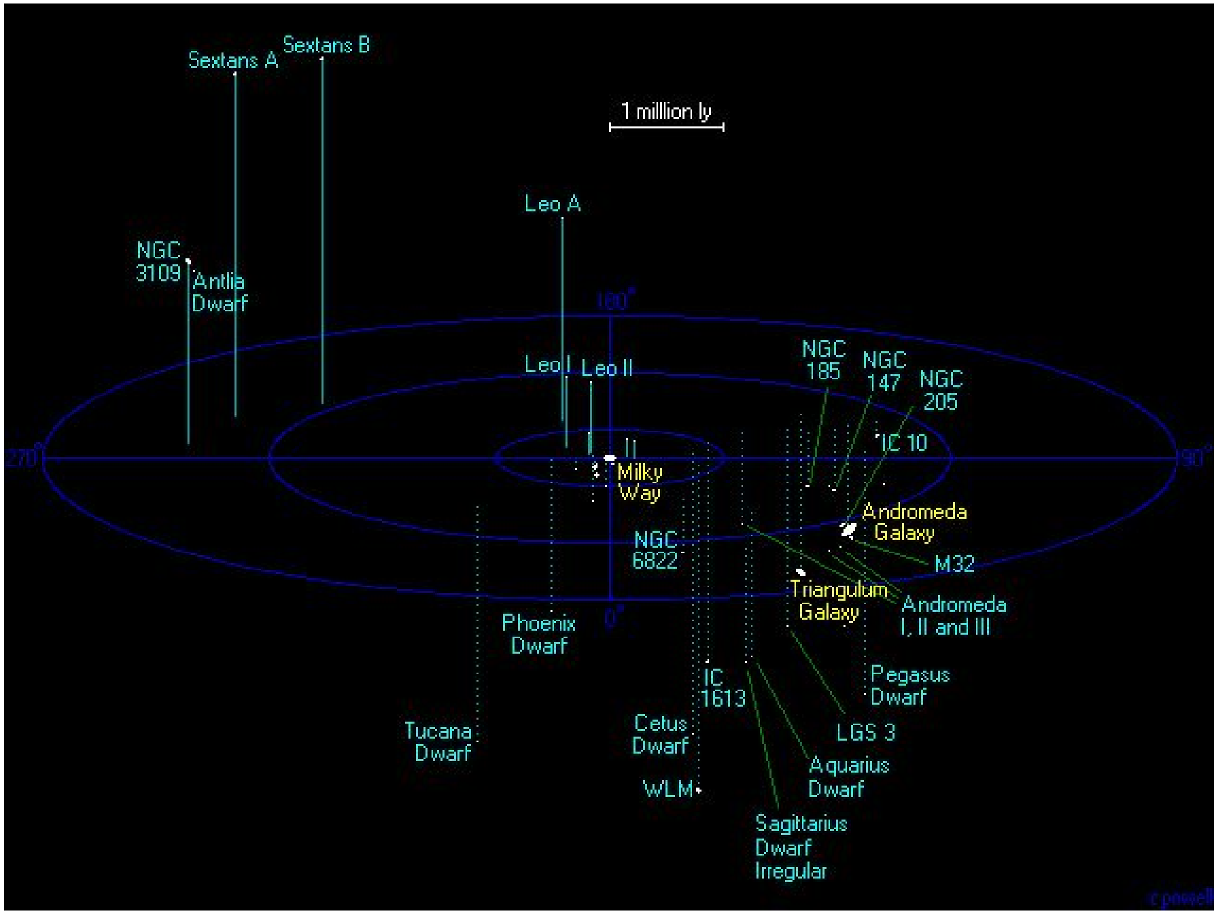
The Milky Way has many small satellite galaxies more than 60 dwarf galaxies have been found so far



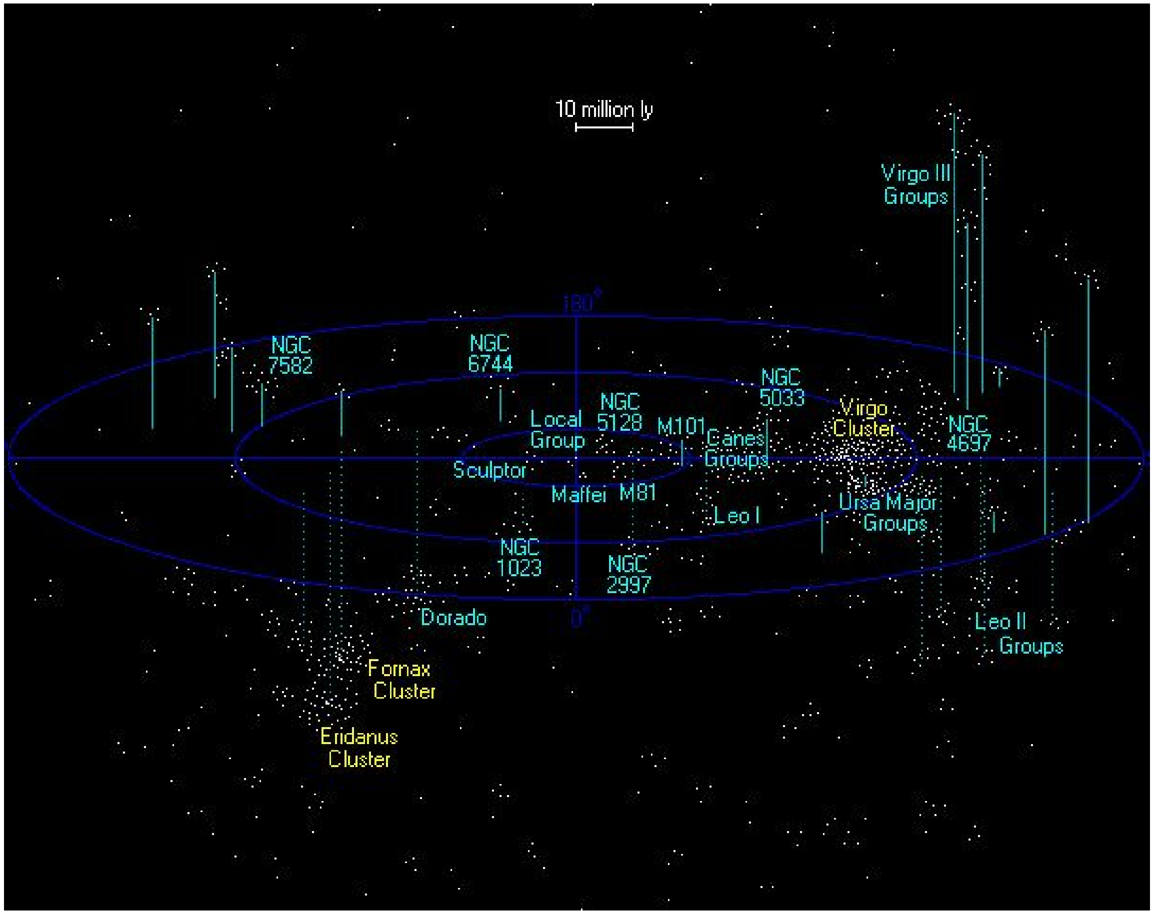
The Milky Way has many small satellite galaxies- dwarfs as of 2016 (in red DES)



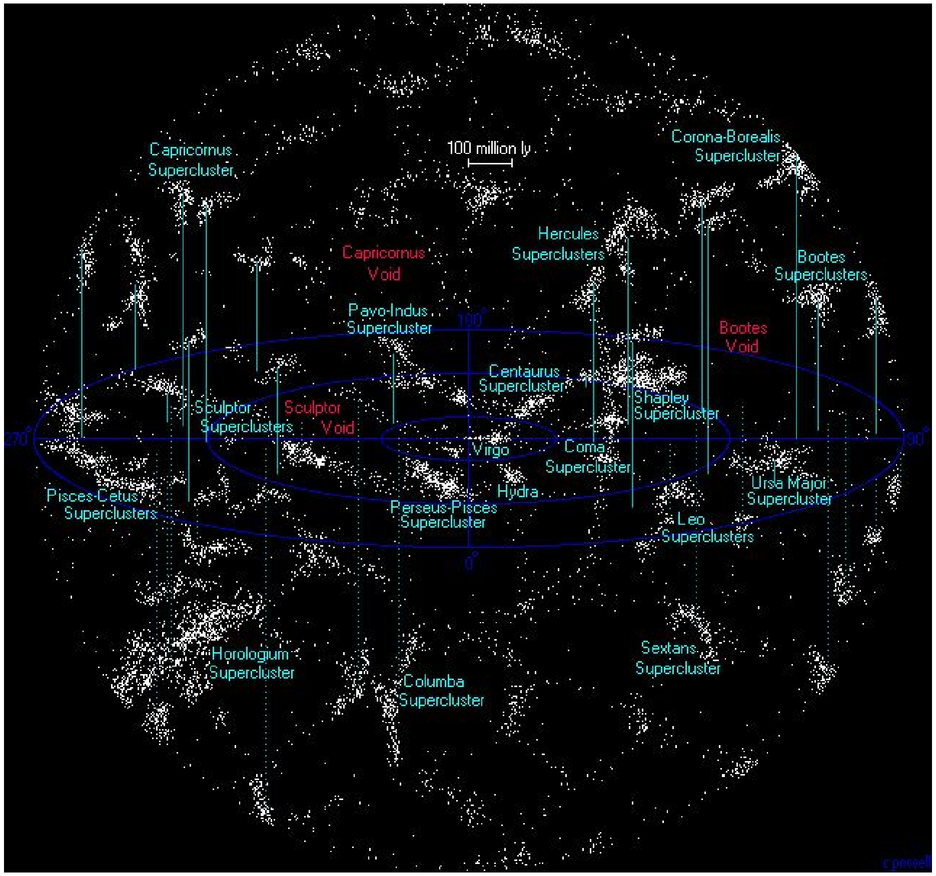
Galaxies come in groups, clusters, superclusters......Our Local Group of galaxies



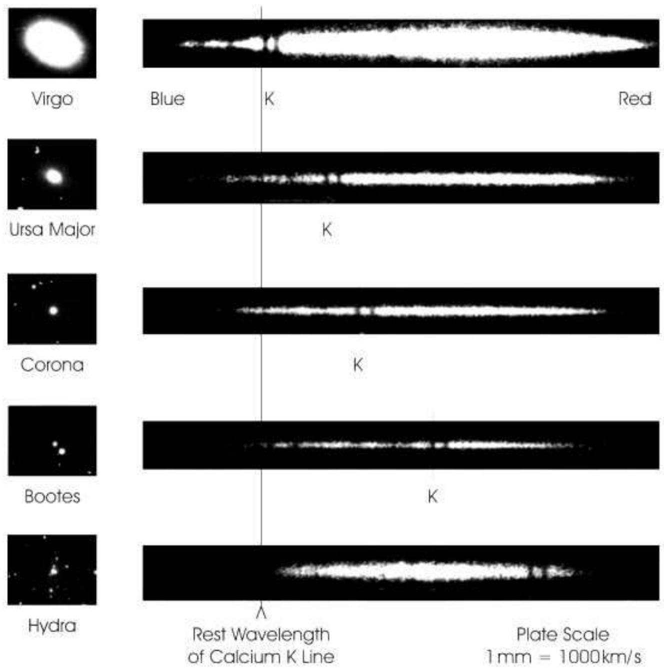
Galaxies come in groups, clusters, superclusters...... Our Local Group of galaxies is in the outskirts of the Virgo Cluster

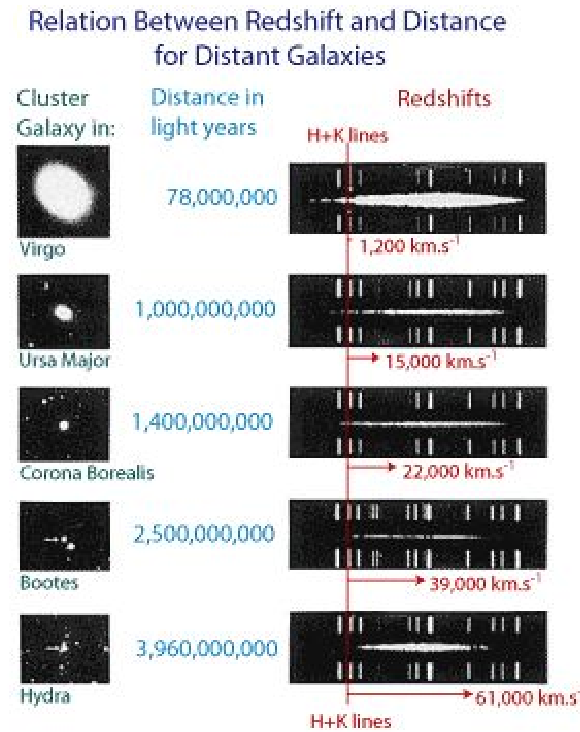


Galaxies are the building block of the Universe: they come in groups, clusters (which form "filaments, walls and voids")



- 1929- Hubble- far away galaxies recede from us $v = Hd$
  "Hubble Law" H=Hubble constant





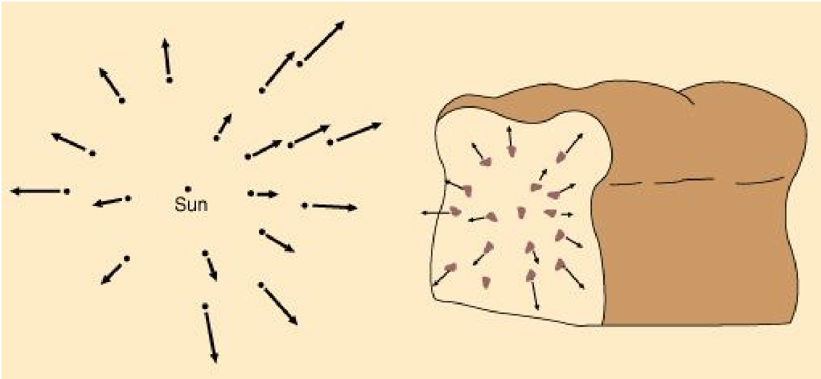
- 1929- Hubble- far away galaxies recede from us
  "Hubble Law" $v = Hd$
  H=Hubble constant, now $H_0 \simeq$70 km/Mpc s (1pc= 3.2ℓy)
  + **Cosmological Principle** ("we are not special", i.e. space is homogeneous and isotropic)=**Universe is expanding**

- The Universe is not static: Einstein said that introducing a cosmological constant Λ was "the biggest blunder of my life"

- The expansion has no center:
  all inter-distances grow
  in the same way

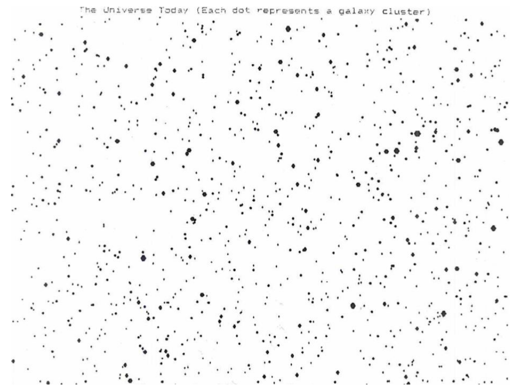
  $d(t) = a(t)d_0 \Rightarrow v = \dot{d} = \frac{\dot{a}}{a}d$

  $H(t) \equiv \dot{a}/a$ (const. in space-not in t)
  $a(t)$: scale factor of the Universe

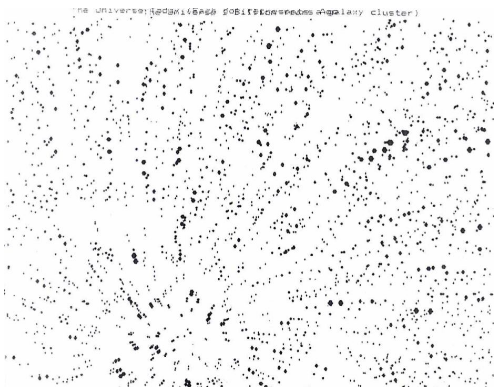


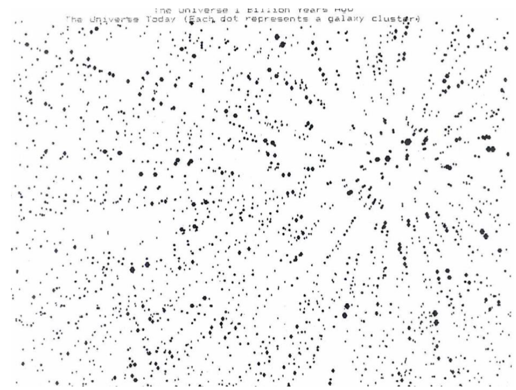
Expansion of the Universe

Sometime in the past



Now





The galaxy from which we observe, seems always the center of expansion

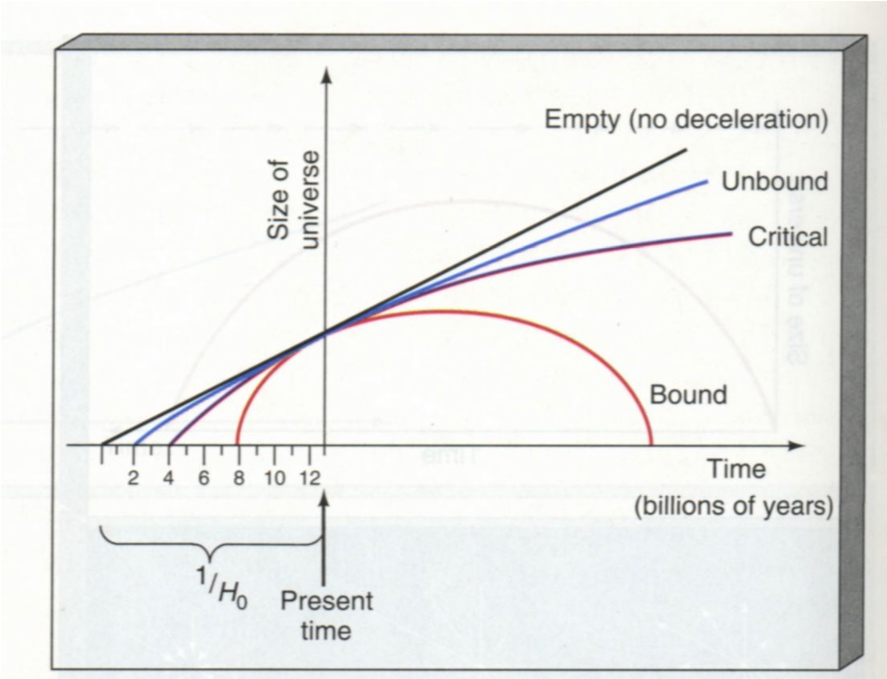
Lifetime of the Universe: counted from $a = 0$ forwards

$$t_U \sim 1/H$$

With $a_0 = 1$ (sub 0 means "now")
$H(t) \equiv \dot{a}/a \Rightarrow \dot{a}_0 = H_0$
Thus if $\dot{a} =$ constant it is $\dot{a} = H_0$
Thus taking $t = 0$ when $a = 0$ we have
$a_0 = H_0 t_0$, i.e. $t_0 = 1/H_0$

Radiation and matter attractive gravitational interactions slow down the initial expansion, so $\ddot{a} > 0$, $t_0 = 1/2$ or $2/3 \times 1/H_0$

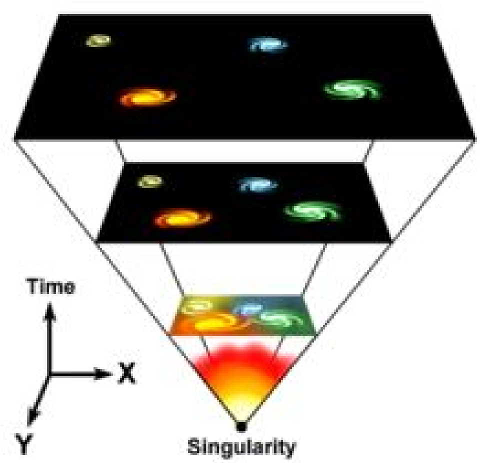
But Hubble measured $H_0 \simeq 500$ km/Mpc s.
this gave $t_U \simeq 4.5 \times 10^9$y
shorter than Earth's known then!



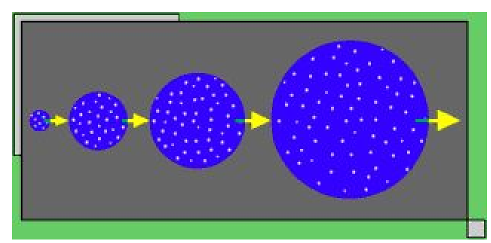
- 1929- Hubble- far away galaxies recede from us with $v = Hd$
  $+$ Cosmological Principle "we are not special" $=$ **Universe is expanding**

- "Big-Bang" (George Gamow, 1946)
  Hot beginning, cooling through adiabatic expansion $T \sim 1/a$. Predicted ^{4}He Nucleo-synthesis (BBN) when $T \simeq E_{\text{nuclear binding}}$

  and a $\sim 5^o$K Cosmic Microwave Background Radiation (CMB) released when atoms became stable $T \simeq E_{\text{atomic binding}}$



- or "Stationary Universe" (Fred Hoyle, 1948)
  no background radiation and all nuclei created in stars



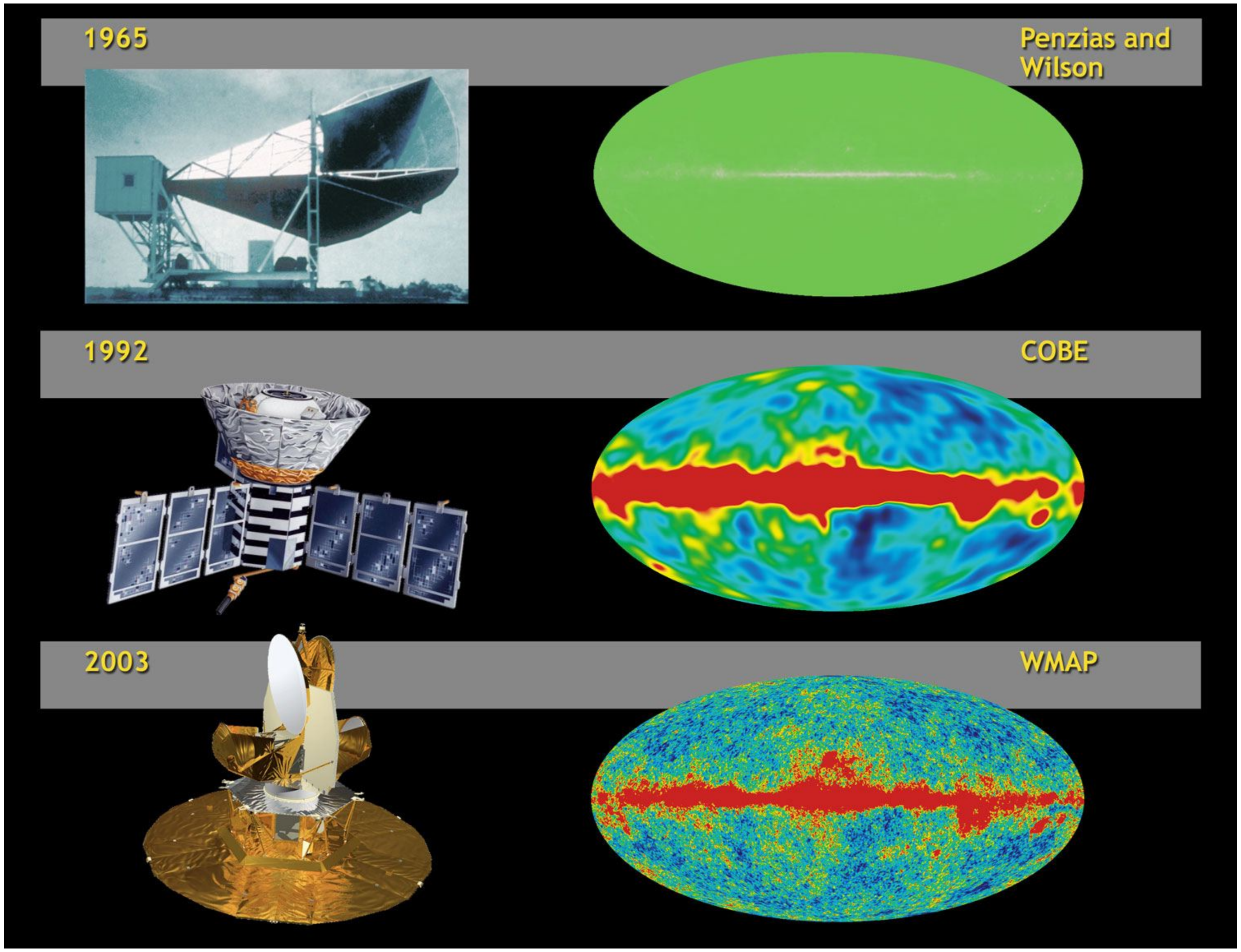
- 1946- Big-Bang (BB): Gamow et al. predict ^{4}He BBN and a $\sim 5^o$K CMB.

- 1940- Missed opportunity for the BB: Andrew Mc Kellar (1940 Proc. Ast. Soc. Pac. 52 187) had measured absorption lines of CN interstellar molecules (in the spectrum of stars) excited in high rotational levels- consistent with 3^{o}K background radiation.

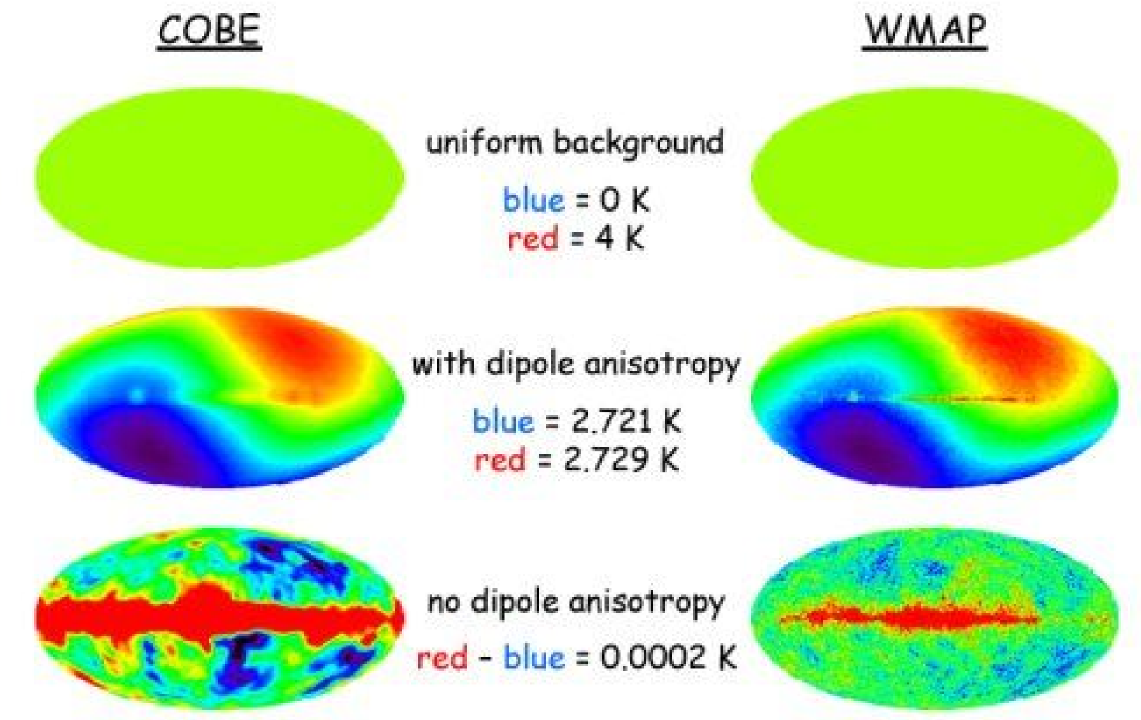
- 1964- Penzias and Wilson (Bell Labs) detect the 2.7^{o}K CMB (1965 Ap.J. 142 419): the Big-Bang is confirmed! Chase for anisotropies starts.
  By 1980 $\Delta T/T < 10^{-3}$ so baryonic density fluctuations did not have time to evolve freely into the nonlinear structures visible today because they can only grow after atoms become stable.Theorists invoked a gravitationally dominant **dark matter** component, not coupled to photons, thus inhomogeneities can start growing earlier.

- 1992- COBE satellite discovers anisotropies in the CMB $\Delta T/T \simeq$ few 10^{-5} (Smoot et al 1990 Astrophys. J. 360 685) confirmed that structure in the Universe is due to gravitational collapse of small inhomogeneities in dark matter

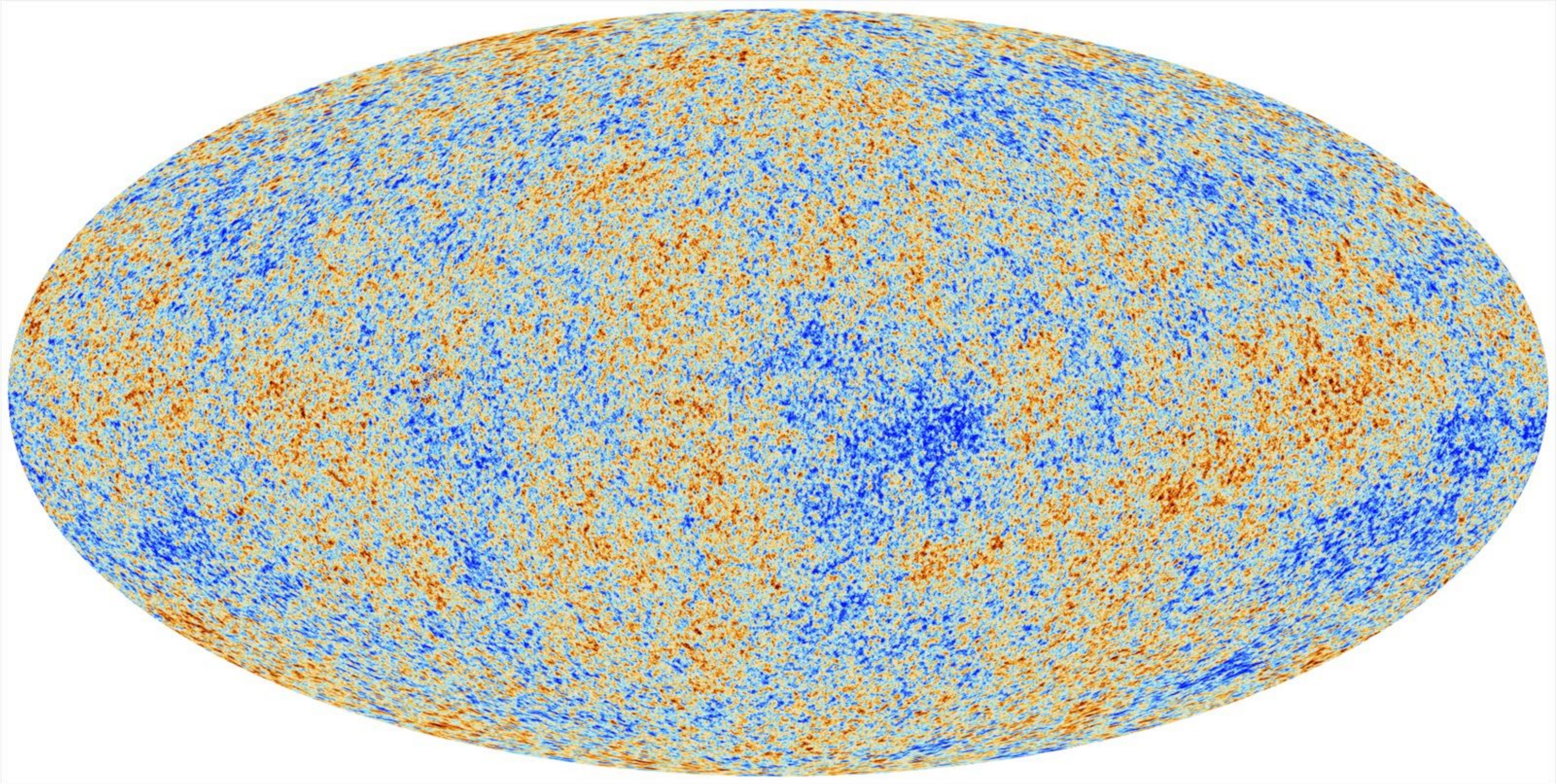
Cosmic Microwave Background radiation (CMB)



Cosmic Microwave Background radiation (CMB)



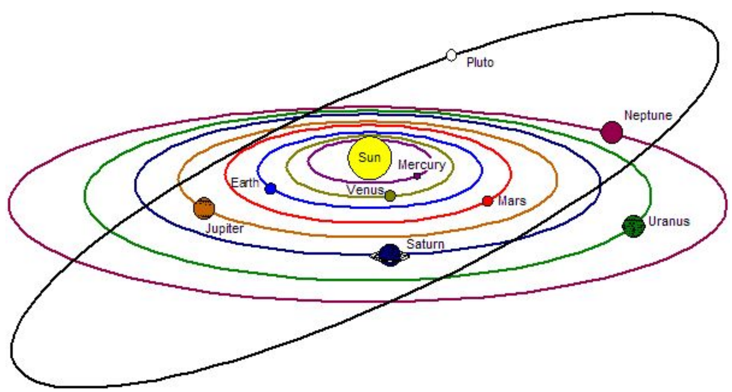
CMB anisotropies with PLANCK Satellite $(\Delta T/T \simeq 10^{-4})$



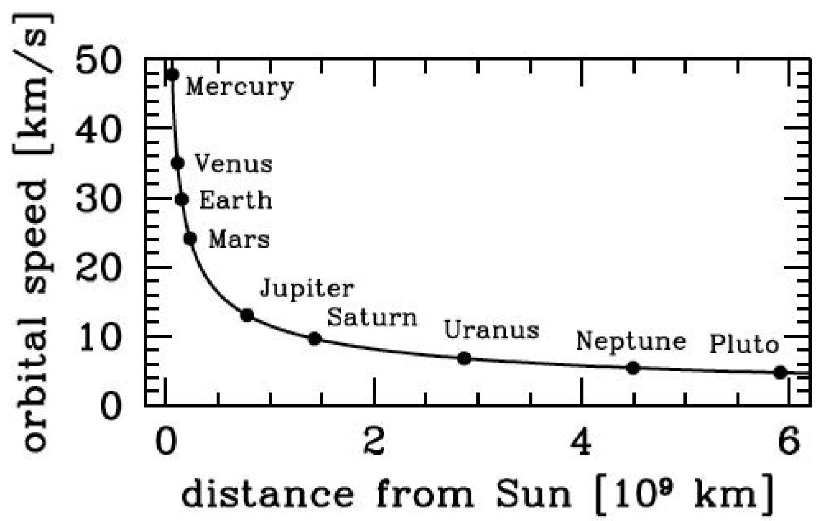
‘Weighing’ galaxies, galaxy clusters and the Universe

Start by ‘weighing’ the Sun

$$\frac{GM_\odot m}{r^2} = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{GM_\odot}{r}}$$



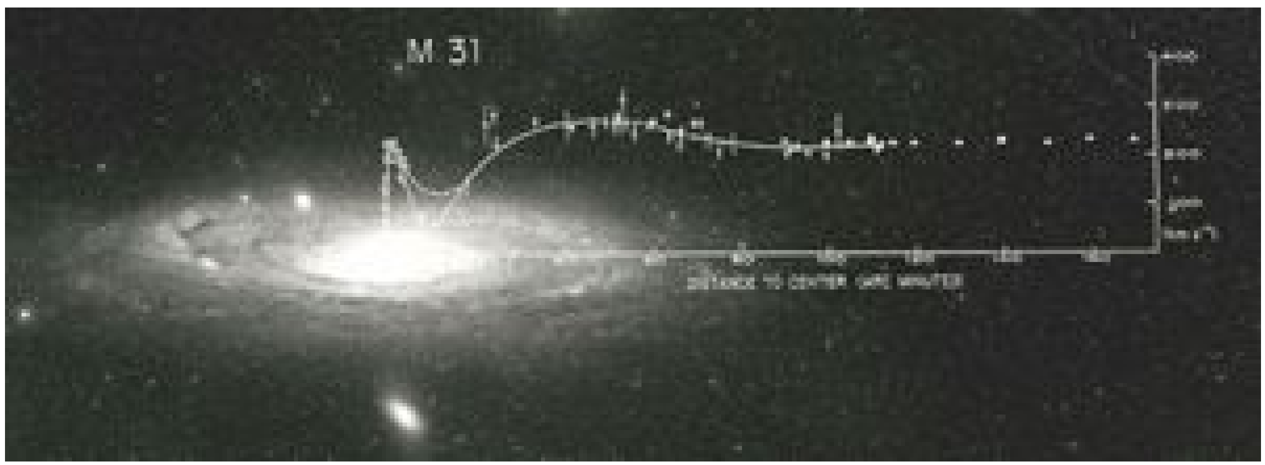
Rotation curve



$$\Rightarrow M_\odot = 1.9889 \times 10^{30}\,\mathrm{kg}$$

‘Weighing’ galaxies:
In the 1970’ Vera Rubin tried using the same method of rotation curves in spiral galaxies, expecting $v \sim 1/\sqrt{r}$ at r larger than the disk radius

She found...

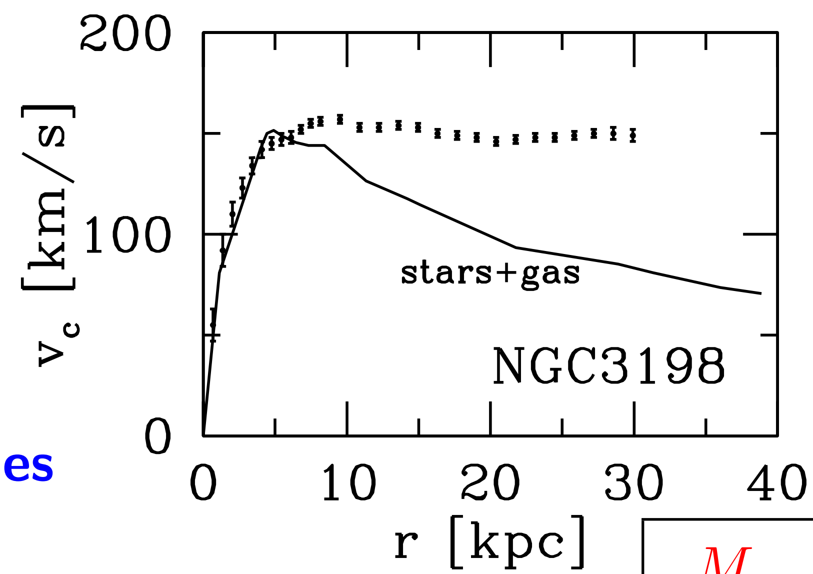


Rotation curves of galaxies ARE FLAT!

$$v = \sqrt{\frac{GM(r)}{r}} = \text{const.}$$

$$\Rightarrow M(r) \sim r$$

even where there is no light!



Dark Matter dominates in galaxies

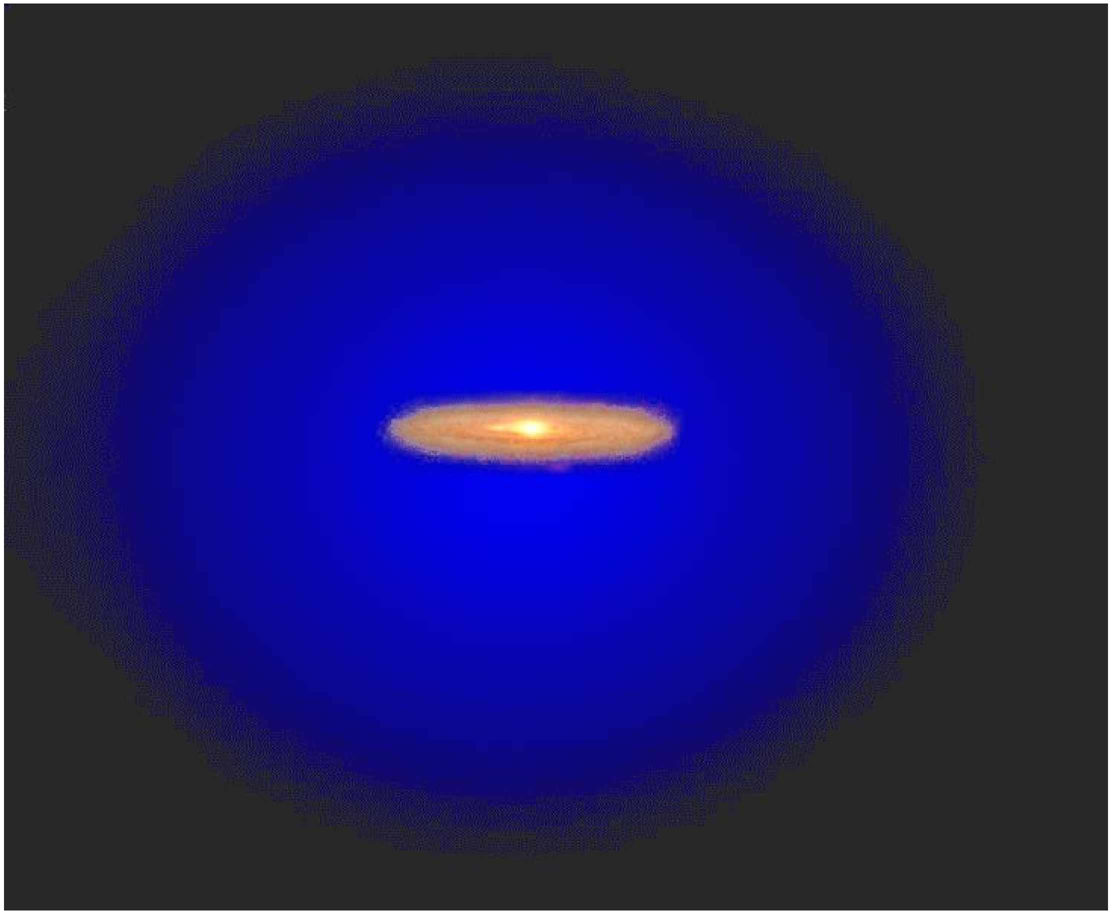
e.g. in NGC3198 $(1\text{pc} = 3.2\ell\text{y})$

$$M = 1.6 \times 10^{11} M_{\odot} (r/30\text{kpc})$$

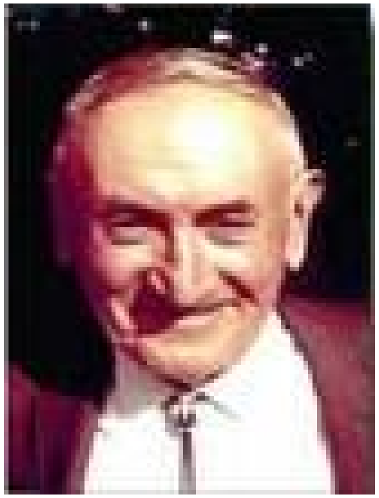
$$M_{\text{stars+gas}} = 0.4 \times 10^{11} M_{\odot}$$

$$\frac{M}{M_{\text{vis}}} > 4$$

Galaxies have a Dark Halo with ≃80% of the total mass



‘Weighing’ galaxy clusters?



In 1933 Fritz Zwicky found DM was needed in clusters
Used the “Virial Theorem”

Example: for planets $\frac{GM_{\odot}m}{r} = mv^2$

|Gravitational Potential Energy|=2×Kinetic Energy

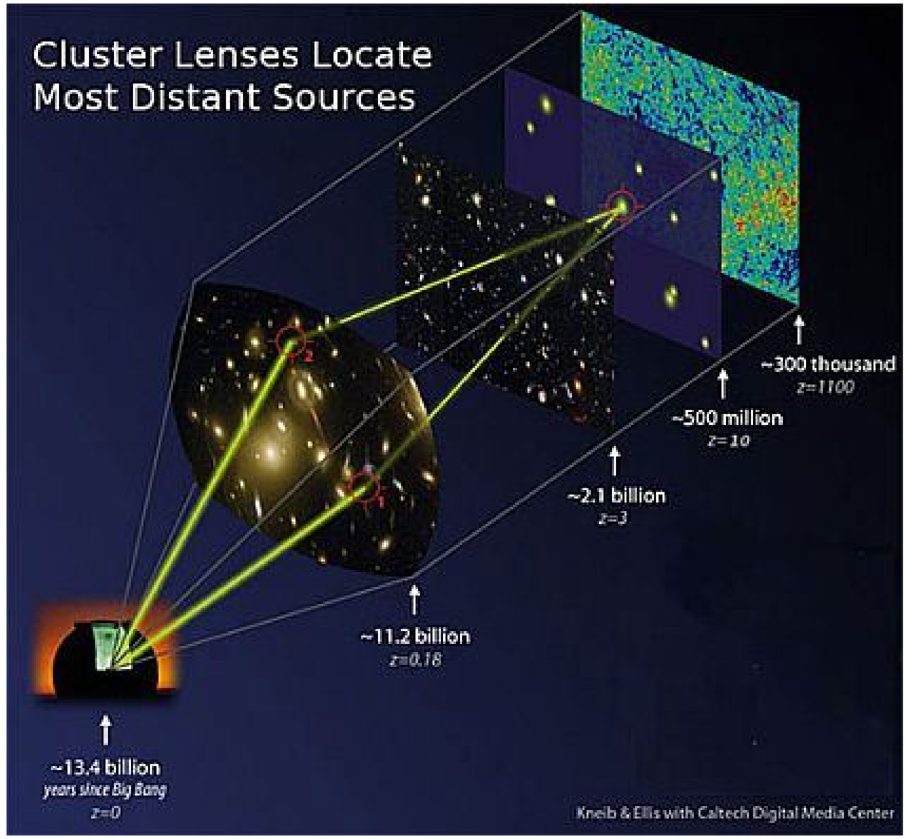
in the Coma Cluster, and found its galaxies move too fast to remain bounded by the visible mass only (needed invisible mass)

Later: also gas in clusters moves too fast (is too hot - as measured in X-rays) to remain in it, unless there is DM.

Another method to 'Weigh' galaxy clusters: gravitational lensing

Gravitational attraction
bends light as does
a particle trajectory

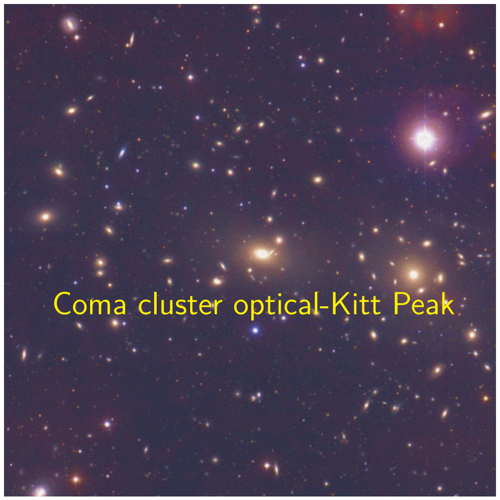
Gravitational lens:
depends on ALL
the intervening mass

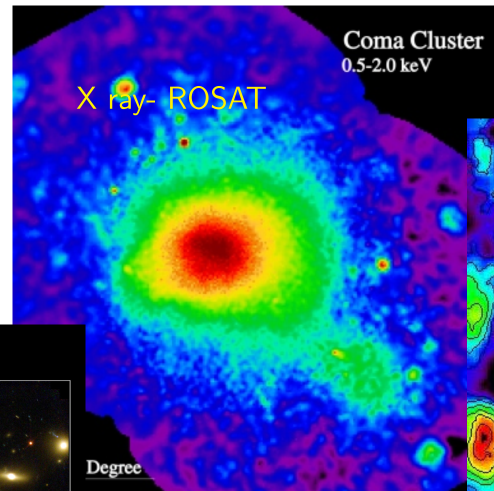


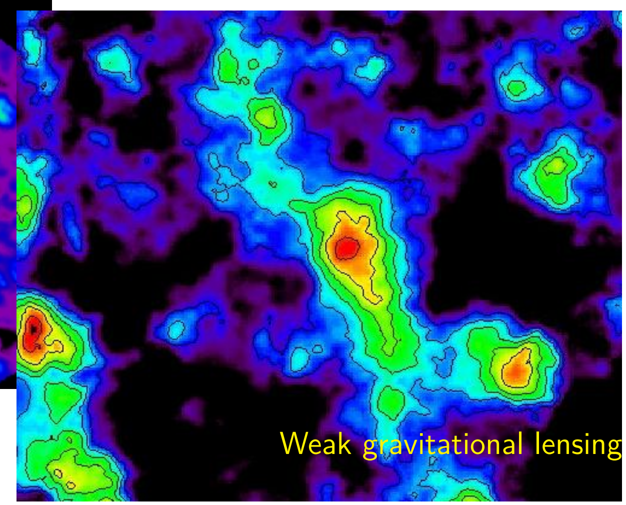
Conclusion...

DM dominates in galaxy clusters

$$\frac{M}{M_{\rm vis}} \simeq 6$$



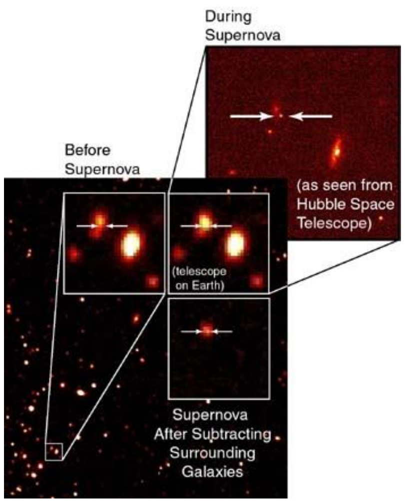


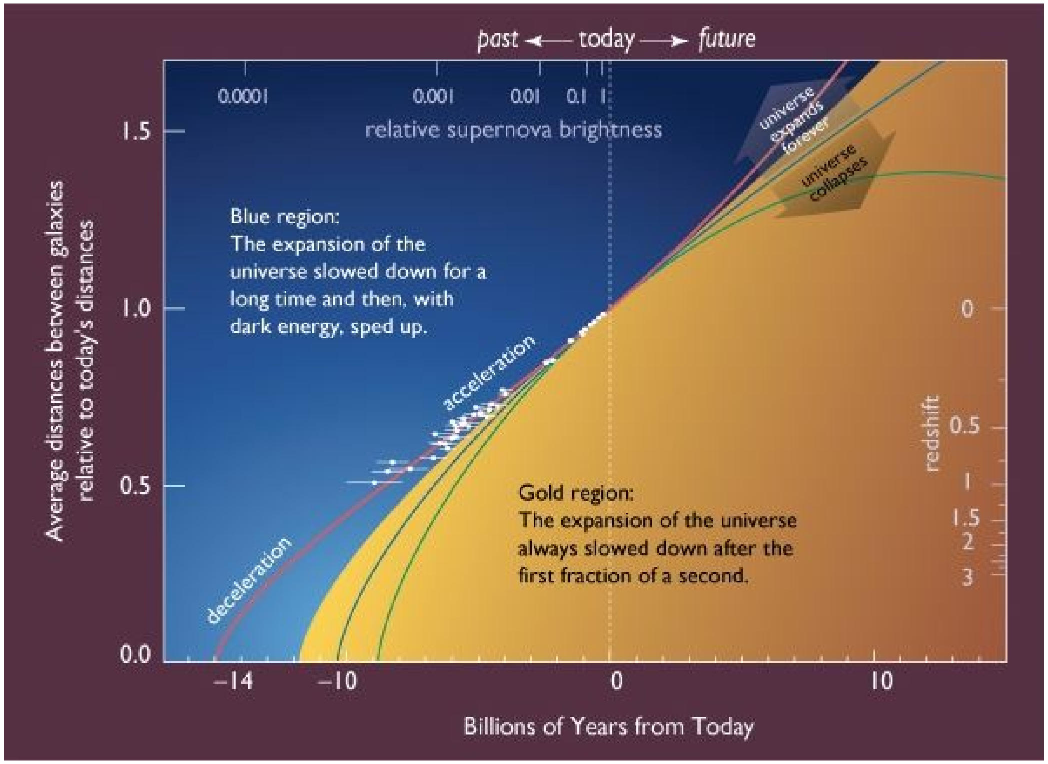


1998 SN-Ia data: The expansion is accelerating!

The main component of the Universe is **Dark Energy**

(Λ-like, with repulsive gravitational interactions)





At the largest scales:

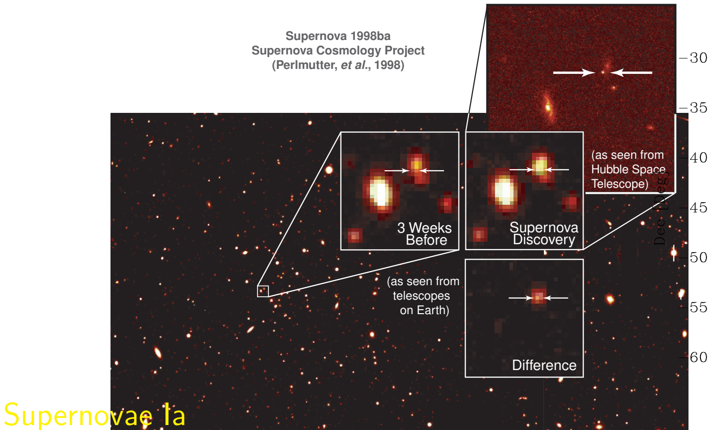
Use General Relativity

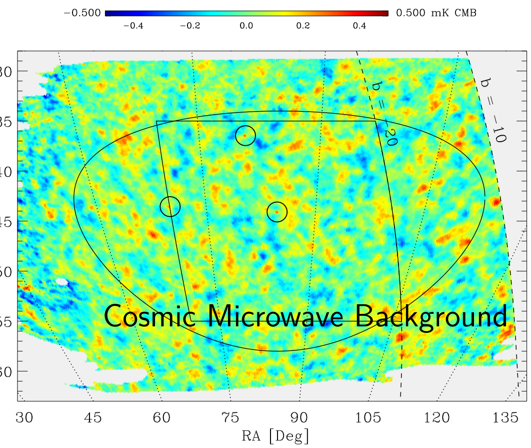
$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \left(-\Lambda g_{\mu\nu}\right)$$

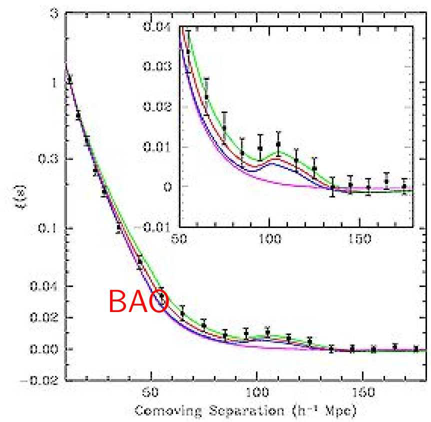
To relate:

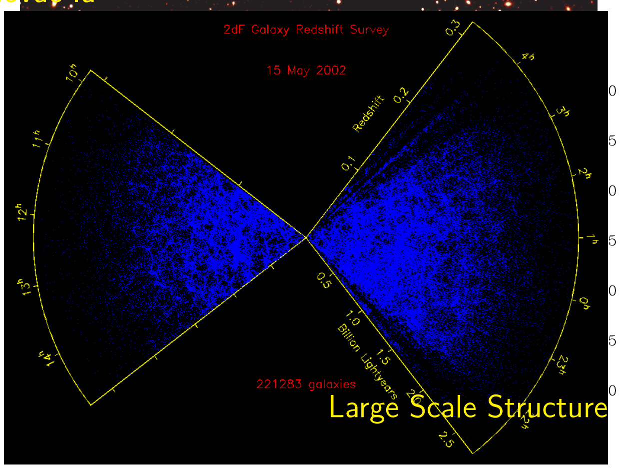
Spacetime geometry $\longleftrightarrow$ **Mass-energy density**

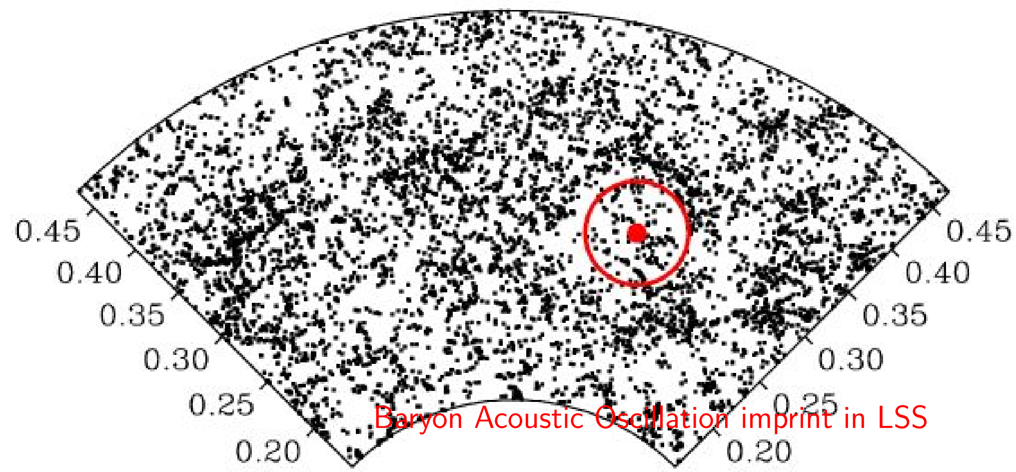
At the largest scales





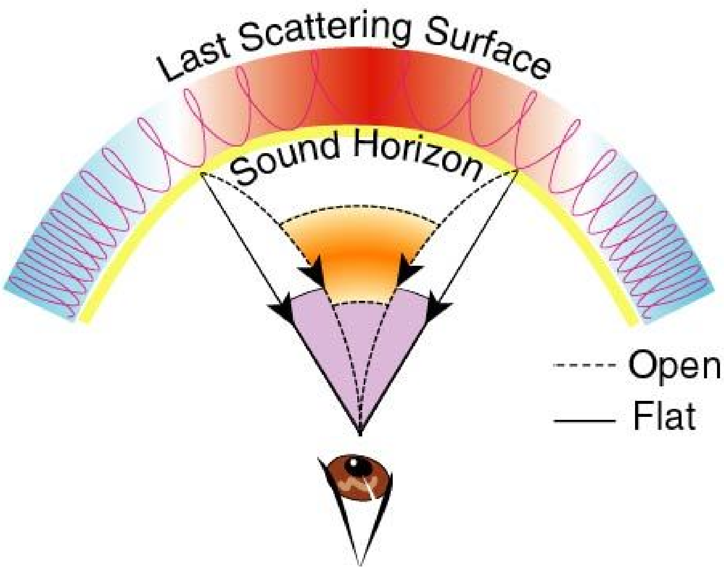


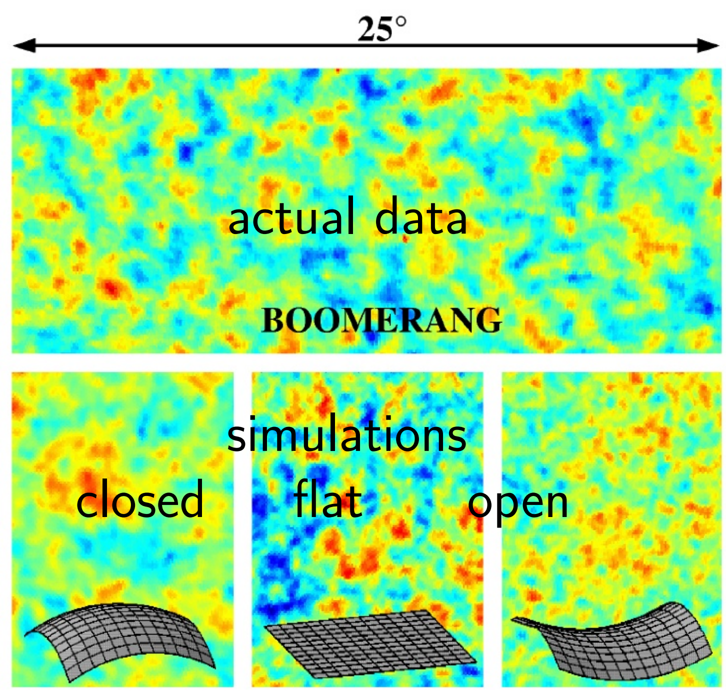




At the largest scales the Universe is spatially FLAT

The angular size of largest spots in CMB anisotropies tells geometry (sum of the internal angles of a triangle is $180^0 \pm 0.5$)





At the largest scales the Universe is spatially FLAT

In cosmology this means that the energy density has a critical value ρ_c, $\rho = \rho_c$

$$\Omega_{\text{total}} \equiv \frac{\rho}{\rho_c} = 1$$

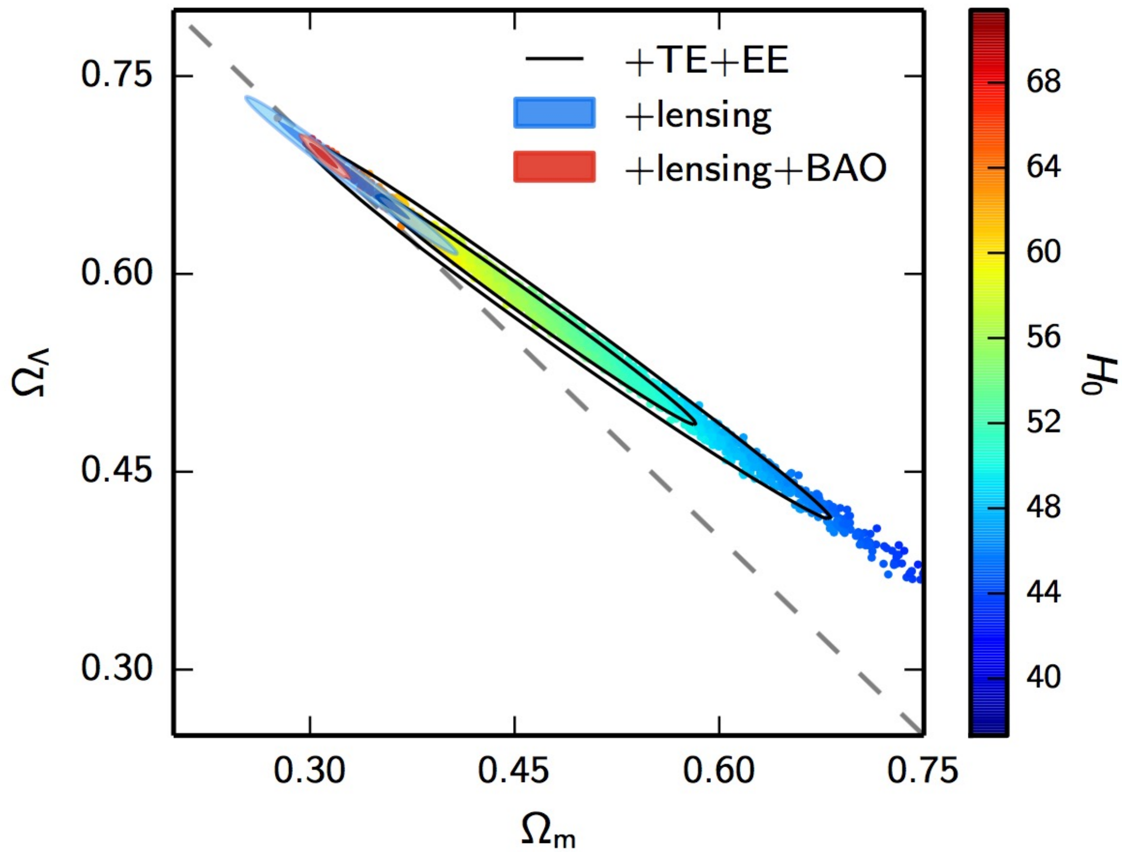
$$\rho_c = 3H_0^2/(8\pi G) = 1.9 \times 10^{-29}\ h^2\ \frac{\text{g}}{\text{cm}^3} = 10.5\ h^2\ \frac{\text{keV}}{\text{cm}^3} \simeq 5\ \frac{\text{keV}}{\text{cm}^3}$$

$h \simeq 0.7$ (Planck measured 0.674$\pm$0.005) is the reduced Hubble constant, $H = 100\ h$ km/(Mpc s)

Planck 2018 data (combined with Planck polarization, lensing and BAO, for $h = 0.6787 \pm 0.0087$ to allow for spatial curvature) gives $\Omega_k = 1 - \Omega_{\text{total}} = 0.0007 \pm 0.0037$ (95%CL) Planck [1807.06209]

$$\Omega_{\text{total}} = 0.9993 \pm 0.0037$$

At the largest scales: concordance cosmology Planck 2015

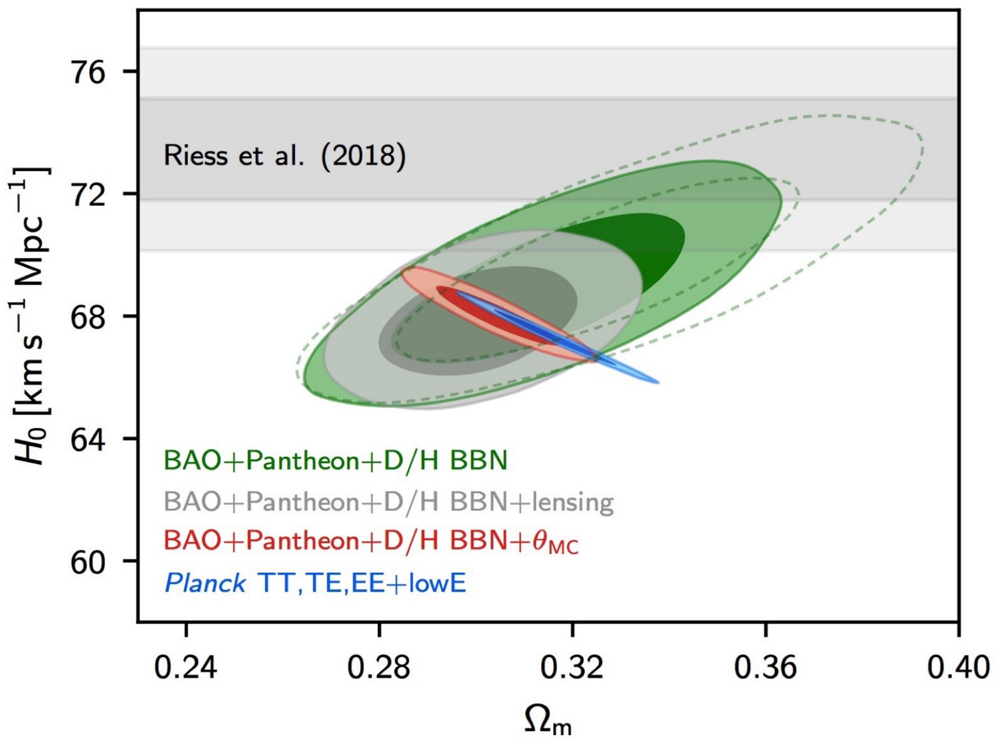


Planck 2018: $H_0 = 67.36 \pm 0.54$ km/Mpc s

But discrepancy in H_0 measurements:

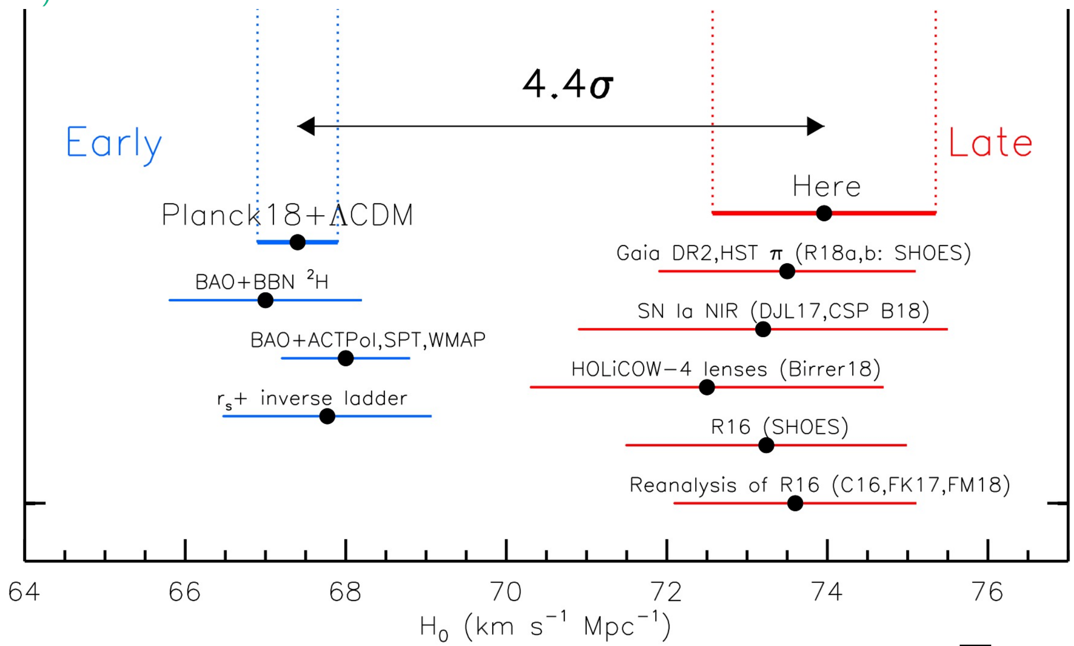
Recent Universe measurements, SN1a and other) Reiss et al 2018: $H_0 = 73.48 \pm 1.66$ km/Mpc s

Early Universe measurements, CMB,BAO, Planck 2018: $H_0 = 67.36 \pm$ 0.54 km/Mpc s



H_0 Tension

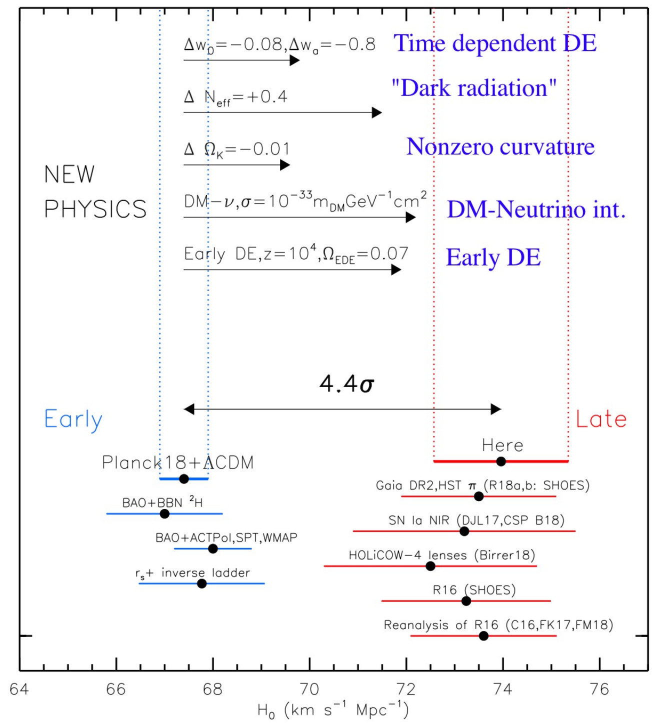
The Hubble constant, H_0 can be measured locally and can be derived from the angle subtended by the sound horizon as observed in CMB temperature fluctuations and BAO.There is a tension between measurements of H_0. (Riess et al. 1903.07603 and 1604.01424)



Present:Planck+DESI:67.58$\pm$0.24 Pantheon-SH0ES+DESI:73.6$\pm$1.0 Tension 5.85σ(2603.28391)

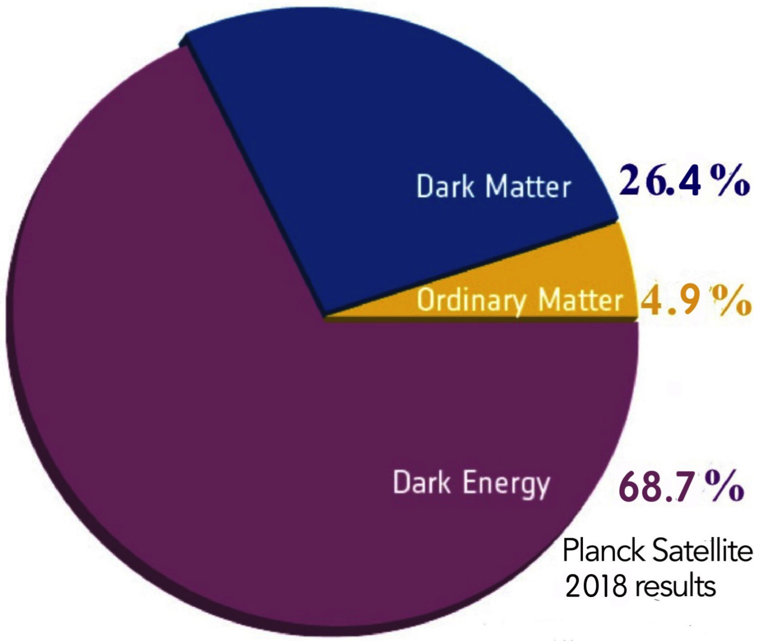
H_0 Tension Many New Physics solutions proposed

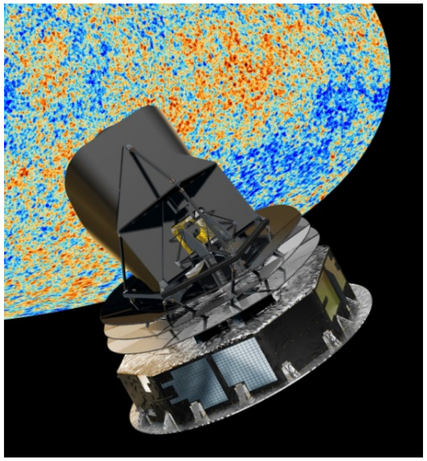
either to modify cosmology at CMB emission so that the CMB/BAO inferred value increases, or at late-times, so that the expansion rate matches the CMB at decoupling and the local rate today.



In the following, use the standard (ΛCDM) model and Planck's H

At the largest scales: the "Double-Dark" model





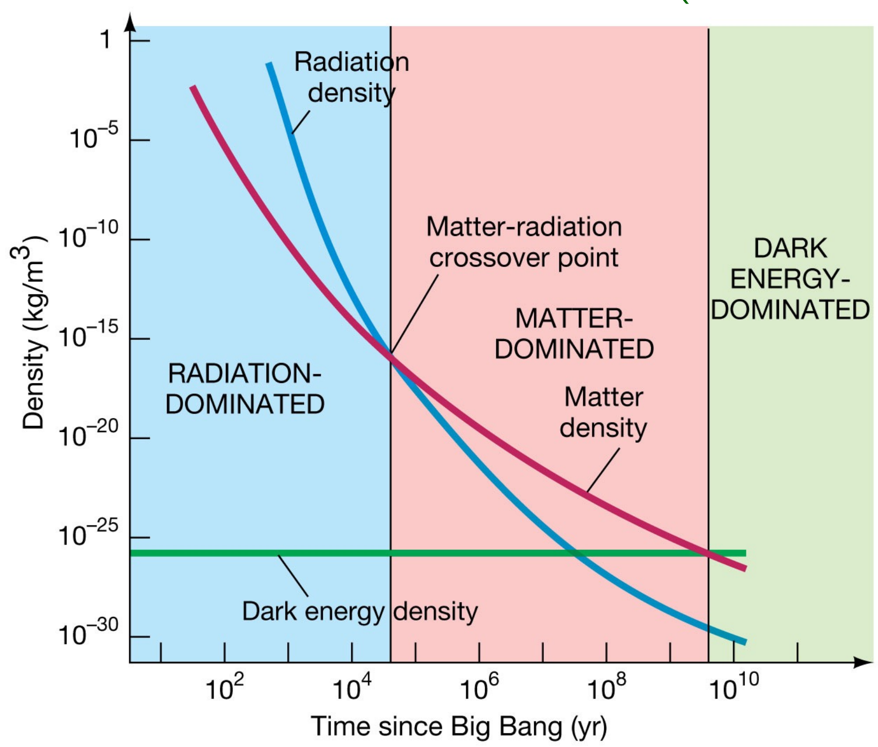
"DARK ENERGY" 69%(with repulsive gravitational interactions)
"MATTER" 31% (with usual attractive gravitational interactions- forms gravitational bound objects) and most of it is "DARK MATTER"
Baryonic matter $< 5\%$ — Radiation $\sim 10^{-5}$ very subdominant at present.

Radiation to Matter to DE Domination

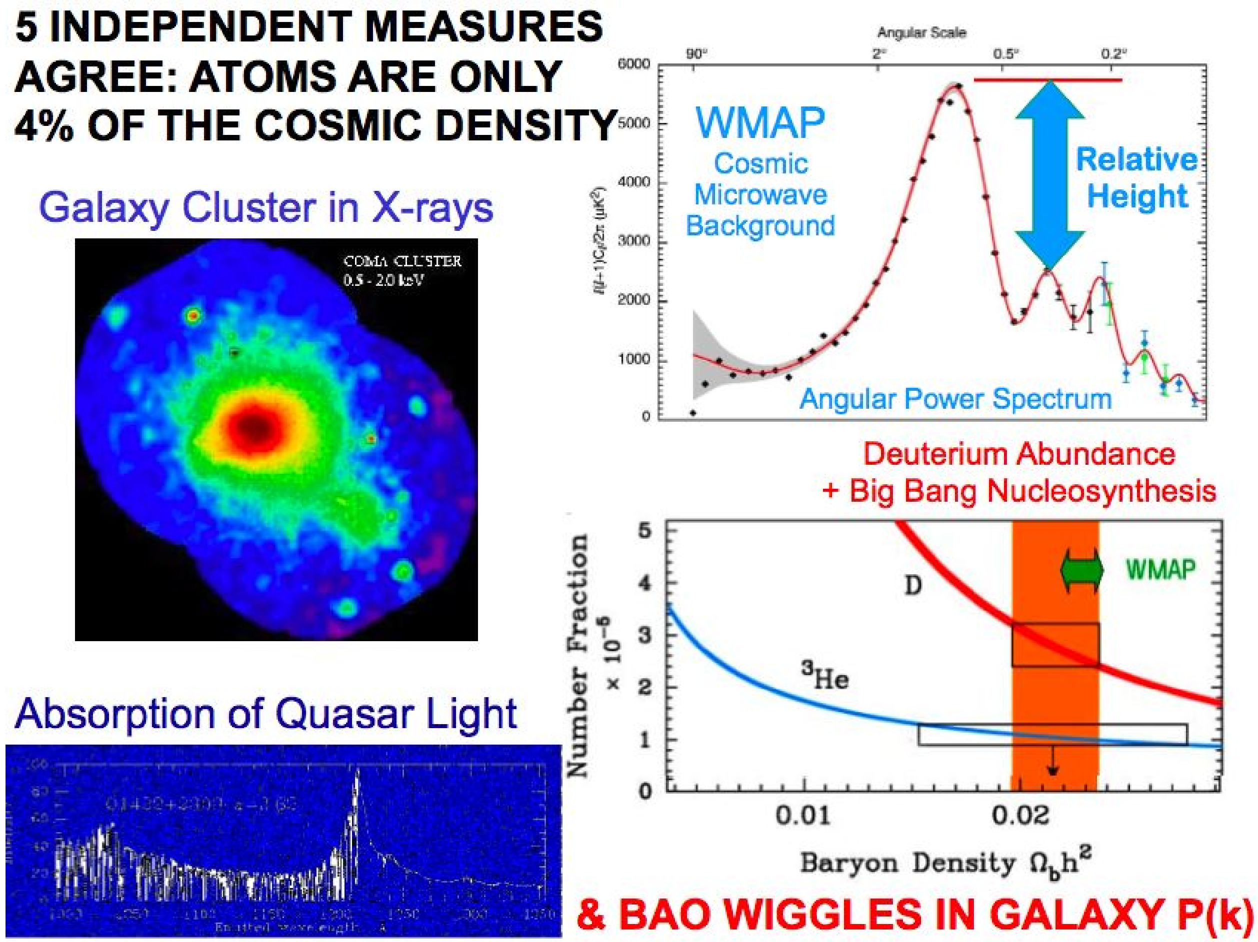
$$\rho_{\rm rad} \sim T^4, \qquad \rho_{\rm m} = \frac{\text{Number} \times \text{Mass}}{\text{Volume}} \sim \frac{1}{a^3} \sim T^3, \qquad \rho_\Lambda = \frac{\Lambda/8\pi}{G} = \text{const.}$$

(almost const for other DE)

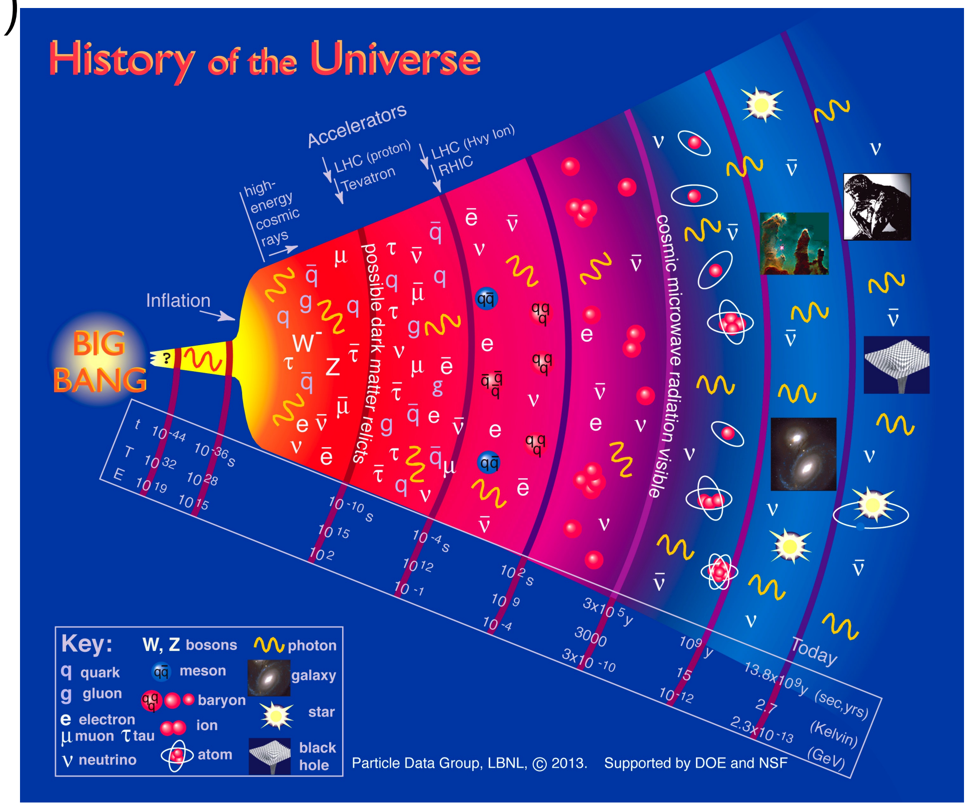
.



Our type of matter is only $< 5\%$.... Fig: from J. Primack 2010



All data confirm the Big-Bang Model of a hot early Universe expanding adiabatically ($T \sim 1/a$)



Earliest data (D, ^{4}He and ^{7}Li):
BBN (Big-Bang Nucleosynthesis)
t$\simeq$3-20min T$\simeq$MeV(blue line)

Radiation domination to Matter domination
t$\simeq$66kyr T$\simeq$1 eV

CMB emitted (atoms form)
(Cosmic Microwave Background)
t$\simeq$380kyr T$\simeq$ 0.3 eV

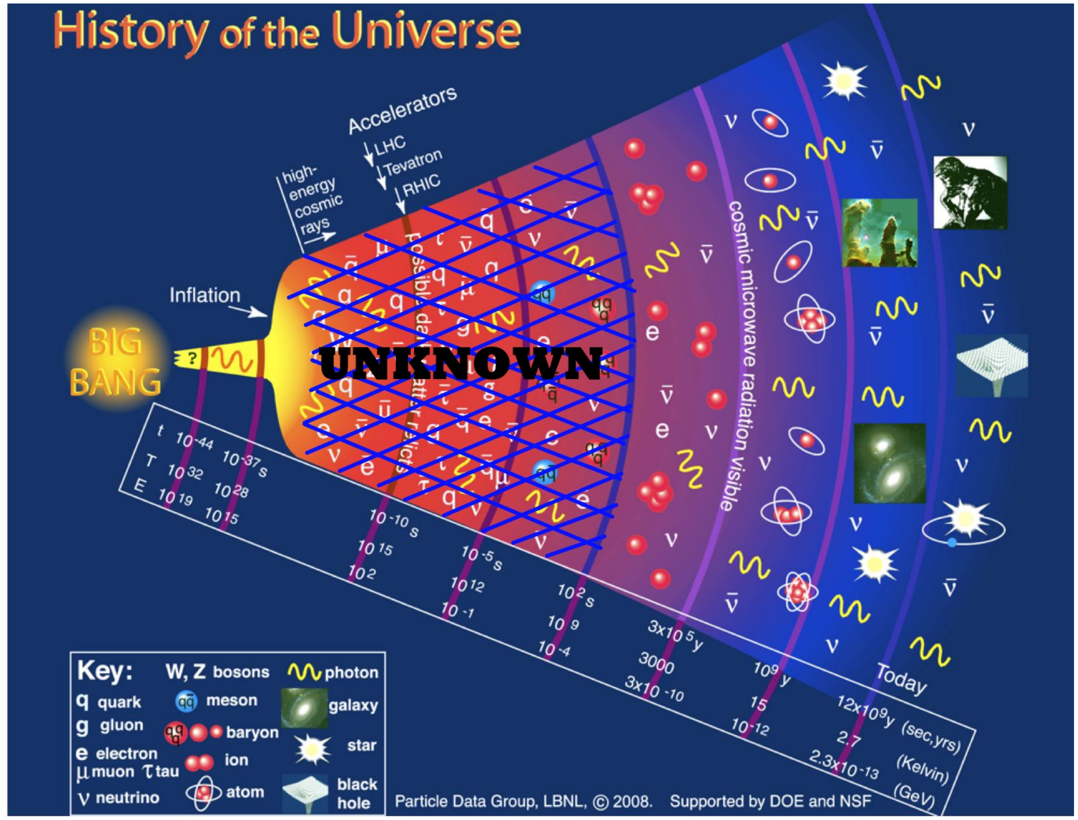
Now t=13.787$\pm$0.020 Gys (Planck2018 + TT,TE,EE+lowE+lensing+BAO-68% limit arXiv:1807.06209v4)

Before BBN?

INFLATION?

period of exponential expansion,
$a \sim e^{Ht}$ drives all densities $\to 0$

"Reheating" at end of inflation produces a Radiation Dominated Universe with temperature T_{RH} expanding adiabatically with $a \sim 1/T \sim t^{1/2}$



BBN implies $T_{RH} >$ 5 MeV

($T_{\rm RH} \geq$ 5.96 MeV Barbieri etal 2025,
$T_{\rm RH} \geq$ 1.8 MeV Hasegawa et al 2019,
De Salas et al 2015, De Bernardis et al 2008, Hannestad 2004, Kawasaki et al 1999 and 2000)

We will start at $T \simeq$ few MeV, when the Universe is radiation dominated populated by e^-, e^+, ν's, and small (10^{-10}) amount of non-relativistic p and n

Basic Cosmology Cosmology, from the greek $\kappa o \sigma \mu o \varsigma$, “order”, is the study of the origin and evolution of the Universe.

For a Universe homogeneous and isotropic, Einstein’s GR lead to two independent equations, the Friedmann and acceleration equations which simplify in the Early Universe (radiation or matter dominated),

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} = \frac{8\pi G}{3}\rho \quad \text{(for a spatially flat Universe)}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3} = \frac{-4\pi G}{3}(\rho + 3P) \quad \text{(notice opposite sign for } \Lambda)$$

To solve for the scale factor, $a(t)$, density $\rho(t)$ and pressure $P(t)$ of matter and radiation, need a 3rd independent eq., Thermodynamics eq. of state (ES): $P = \mathrm{w}\rho$ where w= 1/3 or w= 0 for “radiation” or“matter” (i.e. relativistic or non-relativistic particles)

ES + Energy Continuity (EC) $T^{\mu\nu}_{,\nu} = 0$ for $\mu = 0$ (not independent of the 1st two)
$\dot{\rho} = -3(\dot{a}/a)(\rho + P) \Rightarrow \rho_R \sim 1/a^4$ and $\rho_M \sim 1/a^3$

which in the 1st eq. $\Rightarrow a_R \sim t^{1/2}, H_R = \dfrac{1}{2t}, a_M \sim t^{2/3}, H_M = \dfrac{2}{3t}$

Energy Continuity equation (EC) $T^{\mu\nu}_{,\nu} = 0$ for $\mu = 0$ can be written as

$a^3 d\rho + (\rho + P) d(a^3) = 0$ + local thermal equilibrium $\Rightarrow$

$V d\rho + (\rho + P) dV = T dS = 0$, conservation of entropy per "comoving volume" S,

$V = a^3 V_0$ (that only changes due to the expansion), V_0 is fixed at present $a_0 = 1$,
[use $U = V\rho$ and Thermodynamics First Law $dU = V d\rho + \rho dV = T dS - P dV$]

S= s V, where s is the entropy density.
Using that chemical potentials are negligible in the Early Universe, the internal energy equation $U = TS - PV$ (Euler equation) is $\rho = U/V = -T(S/V) - P$ i.e.

$$s = \frac{\rho + P}{T} \simeq \frac{4}{3}\frac{\rho}{T}$$

since the entropy is dominated by relativistic particles,
Thus sa^3 is conserved while in local thermal equilibrium.

Particle content

In thermal equilibrium, integrating Fermi-Dirac (+sign) or Bose-Einstein (+sign) distributions,

$$f(\mathbf{p}) = \frac{1}{e^{E(\mathbf{p})/T} \pm 1}$$

one obtains: number density n_i, energy density ρ_i, pressure P_i, $\sim$number of spin states g_i (2 for γ, Weyl or Majorana fermions, 3 for W, Z) -$\zeta(3) \approx 1.202$ is the Riemann zeta function

	Relativistic Bosons	Relativistic Fermion	Non-relativistic Either
n_i	$\frac{\zeta(3)}{\pi^2} g_i T^3 = \frac{g_i}{2} n_\gamma = \frac{g_i}{2} 0.244 T^3$	$\frac{3}{4}\frac{\zeta(3)}{\pi^2} g_i T^3 = \frac{3}{4}\frac{g_i}{2} n_\gamma$	$g_i \left[\frac{m_i T}{2\pi}\right]^{3/2} e^{-m_i/T}$
ρ_i	$\frac{\pi^2}{30} g_i T^4 = \frac{g_i}{2} \rho_\gamma = \frac{g_i}{2} 0.658 T^4$	$\frac{7}{8}\frac{\pi^2}{30} g_i T^4 = \frac{7}{8}\frac{g_i}{2} \rho_\gamma$	$m_i n_i$
P_i	$\frac{1}{3}\rho_i$	$\frac{1}{3}\rho_i$	$0 \ (n_i T \ll \rho_i)$
$\langle E_i \rangle$	$2.70T$	$3.15T$	$m + 3T/2$

Energy and entropy densities

Summing over all relativistic particles

$$\rho = \frac{\pi^2}{30} g_* T^4 \text{ where } g_* = \sum_{\rm bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\rm fermions} g_i \left(\frac{T_i}{T}\right)^4$$

g_*: effective number of energy degrees of freedom, and

$$s = \frac{2\pi}{45} g_{*S} T^3 \text{ where } g_{*S} = \sum_{\rm bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{\rm fermions} g_i \left(\frac{T_i}{T}\right)^3$$

g_{*S}: effective number of entropy degrees of freedom.

If particle i is in thermal equilibrium $T_i = T$.

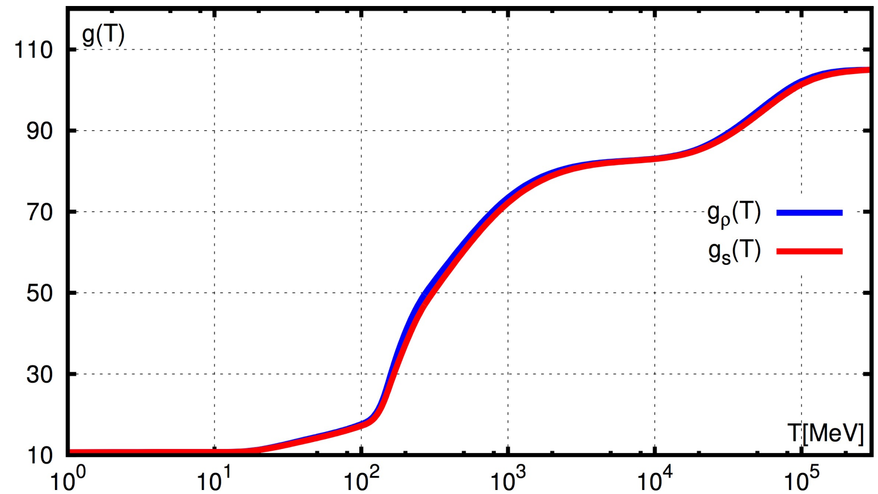
If all $T_i = T$, $g_* = g_{*S}$

After decoupling (next) could have $T_i < T$.

Entropy conservation, $sa^3 =$ const,

$a \sim \frac{1}{g_{*S}^{1/3} T}$ but in SM $1 < g_{*S}^{1/3} < 4.7$

to a very good approximation $a \sim \frac{1}{T}$



Husdal, Galaxies 4 (2016) 4, 78 [1609.04979]

Borsanyi etal, Nature 539 (2016) 7627, 69 [1606.07494];

Equilibrium and decoupling(or freeze-out)

The temperature T is decreasing at a rate

$$\frac{\dot{T}}{T} = -\frac{\dot{a}}{a} = -H$$

and to maintain thermodynamic equilibrium, reaction rates must exceed the rate of change of T

$$\Gamma \simeq n(T)\sigma v > H \quad \text{or} \quad t_{\rm Reaction} \simeq 1/\Gamma < t_U \simeq 1/H$$

When Γ decreases faster than H as T decreases, thermal equilibrium is lost at **Decoupling** $\Gamma(T_D) = H(T_D)$ (and $\Gamma < H$ for $T < T_D$)

During radiation domination the reaction rate needs to be compared with

$$H = \sqrt{\frac{8\pi G\rho(T)}{3}} = \frac{T^2}{M_{\rm Pl}}\sqrt{\frac{8\pi^3 g_*(T)}{90}} = 1.66\sqrt{g_*(T)}\frac{T^2}{M_{\rm Pl}}$$

where $M_{\rm Pl} = G^{-1/2} = 1.22 \times 10^{19}$ GeV

Decoupling of active neutrinos Back-of-an-envelope calculation (literally!)

Weak interactions: $\nu_i + \nu_j \leftrightarrow \nu_i + \nu_j, \quad \nu_i + e^- \leftrightarrow \nu_i + e^-$, $\nu_i + \bar{\nu}_i \leftrightarrow e^+ + e^-$ ....
n=0.183T^3 per fermion target, and there are 5 (3ν's, e^+,e^-) so n=0.913$T^3 \simeq 1 \times T^3$

Use $n \simeq T^3$, $\sigma \simeq (g_{\rm w}^4/m_Z^4)T^2 \simeq G_F^2 T^2$ Thus, at decoupling

$$\Gamma_{\rm weak} \simeq n\sigma c \simeq T_{\rm D}^3 G_F^2 T_{\rm D}^2 \simeq G_F^2 T_{\rm D}^5 \simeq H = \sqrt{\frac{8}{3}\pi G\rho} \simeq 1.66\sqrt{g_*(T)}\frac{T^2}{M_{\rm Pl}}$$

where g_*(few MeV)=10.75 putting numbers in, $T_{\rm D} \simeq \left(\frac{1.66\sqrt{g_*}}{G_F^2 M_{\rm pl}}\right)^{1/3} \simeq 1.5{\rm MeV}$

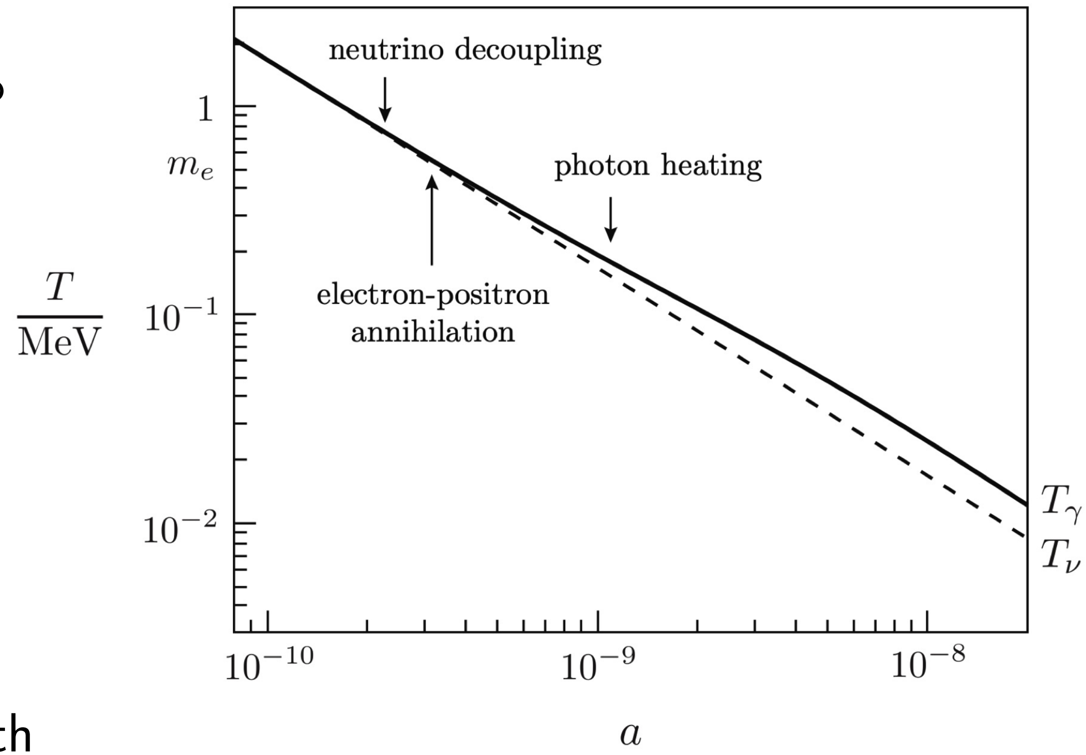
Actual calculations including in-medium neutrino flavor oscillations etc find a continuous decoupling at about 1 MeV for ν_e and 1.5 MeV for ν_μ and ν_τ. (Bond etal, JCAP 09 (2024) 014 [2403.19038]; Akita and Yamaguchi, Universe 8 (2022) 11, 552 [2210.10307]; Froustey, Pitrou, and Volpe, JCAP 12 (2020) 015 [2008.01074]; Salas and Pastor, JCAP 07 (2016) 051) [1606.06986])

After neutrino decoupling, at $T \simeq 2m_e \simeq 1$MeV e^+e^- annihilate, and "heat-up" γ's but NOT neutrinos (actually very little, since decoupling is not instantaneous)

Cosmic Neutrino Background (CνB) Temperature

When T drops to $< 2m_e$, e^+e^- cease to be produced and (dominantly) only annihilate into photons causing T to decrease less fast than the ν's T_ν.

The process is close to equilibrium, so entropy $S = sa^3$ is conserved. In the (good) instantaneous approximation a is constant.



(fig: Baumann TASI2017 (2018) 009 [1807.03098])

Before: e^+e^-: $s_e = 4\frac{7}{8}\frac{2\pi^2}{45}T^3$
and photons: $s_\gamma = 2\frac{2\pi^2}{45}T^3$

After: only the photons remain with
$T = T_{\rm after} > T_{\rm before}$

$(2 + 7/2)T^3_{\rm before} = 2T^3_{\rm after}$ thus $T_\nu = T_{\rm before} = (4/11)^{1/3}\, T$

Cosmic Neutrino Background (CνB) $T_\nu = (4/11)^{1/3}\, T$

Number density of species $i = \nu_e, \nu_\mu, \nu_\tau$

$$\frac{n_i}{n_\gamma} = \frac{3}{4}\frac{g_\nu}{2}\left(\frac{T_\nu}{T}\right)^3 = \frac{3}{11}\frac{g_\nu}{2}$$

Since now $T_{\rm CMB} = 2.72548 \pm 0.00057$ K (COBE FIRAS Fixsen, 2009)

$n_{\rm CMB}$= 410.72/cm^3 $\Rightarrow$ n_i =112.02/cm^3, $\sum_{i=1}^3 n_i$= 336.04/cm ^{3}

Energy density if relativistic, per species $g_*(T_0) = 2(7/8)(T_\nu/T)^4$= 0.454

$\gamma + \nu$'s have $g_*(T_0) = 2 + 3 \times 0.454 = 3.362$, thus the fraction of total density at present in radiation (h is H in units of 100 km/(Mpc sec), $h = 0.674$ from Planck)

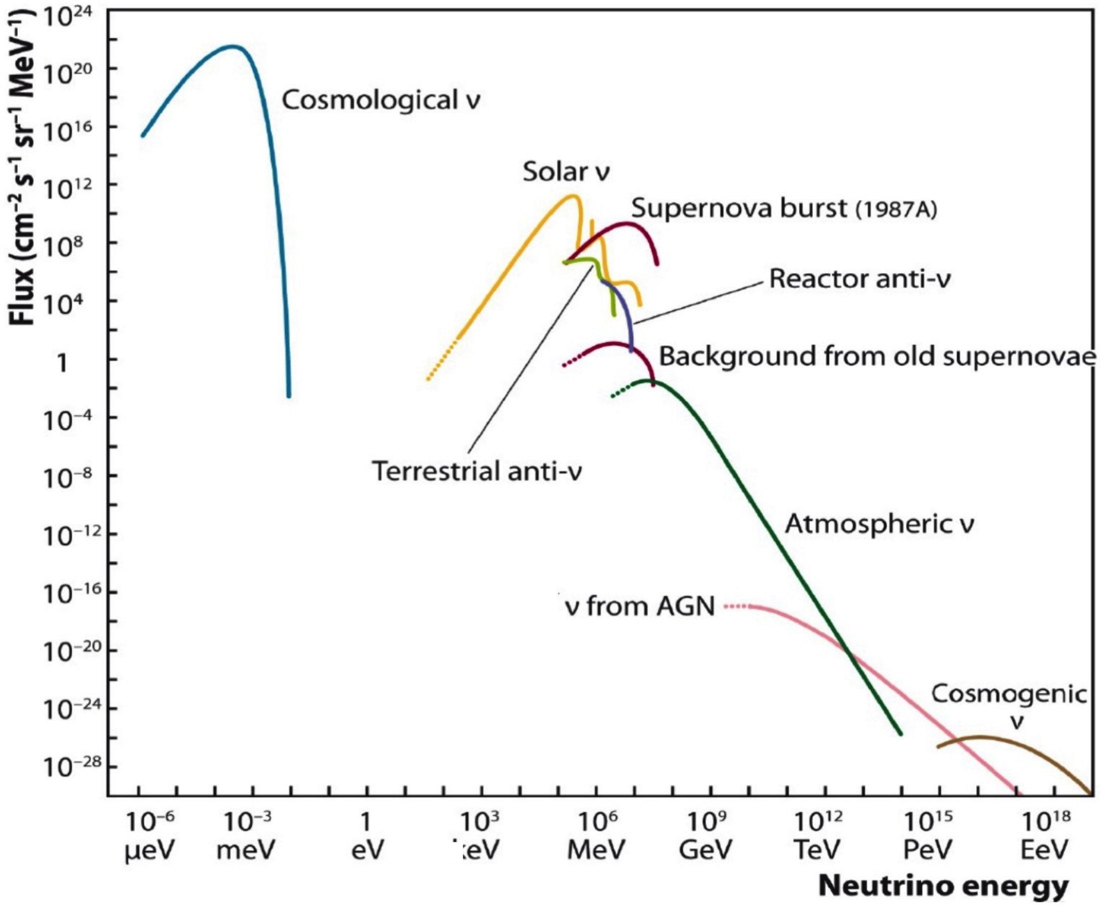
$$\Omega_R = \frac{3.362}{2} \times \Omega_{\rm CMB} = 1.681 \times 5.44 \times 10^{-5}\left(\frac{0.674}{h}\right)^2 = 9.15 \times 10^{-5}\left(\frac{0.674}{h}\right)^2$$

Energy density if non-relativistic:

$\rho_\nu = \sum_{i=1}^3 m_{\nu_i} n_i = \Omega_\nu \rho_c$, $\sum_{i=1}^3 m_{\nu_i} = 94.1{\rm eV} \times \Omega_\nu h^2$

Oscillations+Planck: 0.06 eV< $\sum m_\nu$ <0.12 eV $\Rightarrow 6.4 \times 10^{-4} < \Omega_\nu h^2 < 1.3 \times 10^{-3}$

Cosmic Neutrino Background (CνB)



Effective number of neutrino species $N_{\rm eff}$

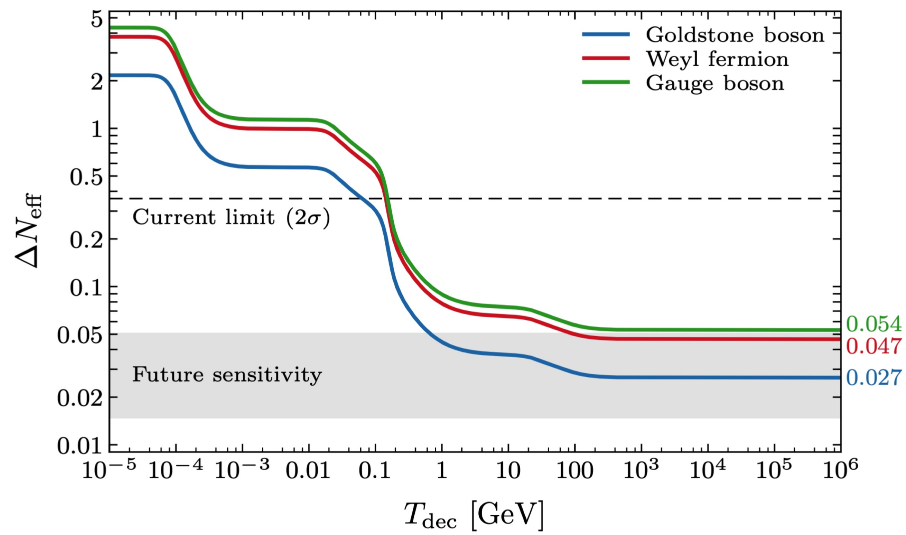
Writing $n_\nu = N_{\rm eff}\left(\frac{3}{4}\right)\left(\frac{4}{11}\right) n_\gamma$, with just 3 species $N_{\rm eff}$ should be 3, but because neutrinos do not decouple instantaneously, they are slightly heated up by e^+e^- annihilations, $N_{\rm eff} = 3.0440 \pm 0.0002$. (Bennett et al, JCAP 04 (2021) 073 [2012.02726]; Froustey, Pitrou and Volpe, JCAP 12 (2020) 015 [2008.0107]; Akita and Yamaguchi,JCAP 08 (2020) 012 [2005.07047]; Mangano et al., NP B729 (2005) 221 [0506164])

However, writing the total radiation density as

$$\rho_R = \left[1 + N_{\rm eff}\left(\frac{7}{8}\right)\left(\frac{4}{11}\right)^{4/3}\right]\rho_\gamma$$

$N_{\rm eff}$ taken as a parameter to be measured, can be due to any BSM "dark radiation" affecting BBN, CMB and LSS, besides neutrinos.

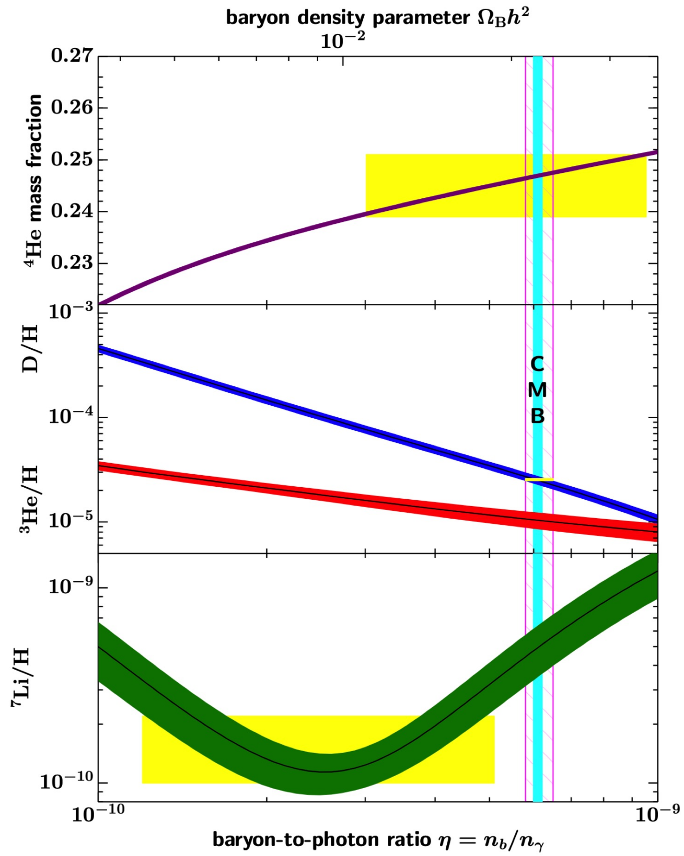
(e.g. figure: contribution $\Delta N_{\rm eff}$ of a single particle decoupled at $T_{\rm dec}$)



(Baumann, PoS TASI2017 (2018) 009 [1807.03098])

Larger $N_{\rm eff}$: increases $H \sim \sqrt{\rho_R}$ and delays radiation-matter equality

Big Bang Nucleosynthesis (BBN) $T \simeq$ 1-0.03 MeV, $t \simeq$ 1-400 sec Predicts the very different observed abundances of D, ^{4}He and ^{7}Li, the earliest relics - D/H etc are number ratios (see e.g. PDG 2025 "BBN" review and refs therein- or Pitrou etal Phys.Rept. 754 (2018) 1-66 [1801.08023])



Primordial nuclear abundances

- Not easy to measure:

D: is most reliable because is only destroyed in stars. Has the lowest binding energy 2.2 MeV. Any observed abundance of D is a lower limit to the primordial abundance. Data comes from high-z, low metallicity QSO absorption line systems.

^{3}He: is produced and destroyed in stars (very complicated evolution)

^{4}He: is produced by H burning in stars, which increases the primordial abundance. Data from ionized extragalactic metal- poor galaxies

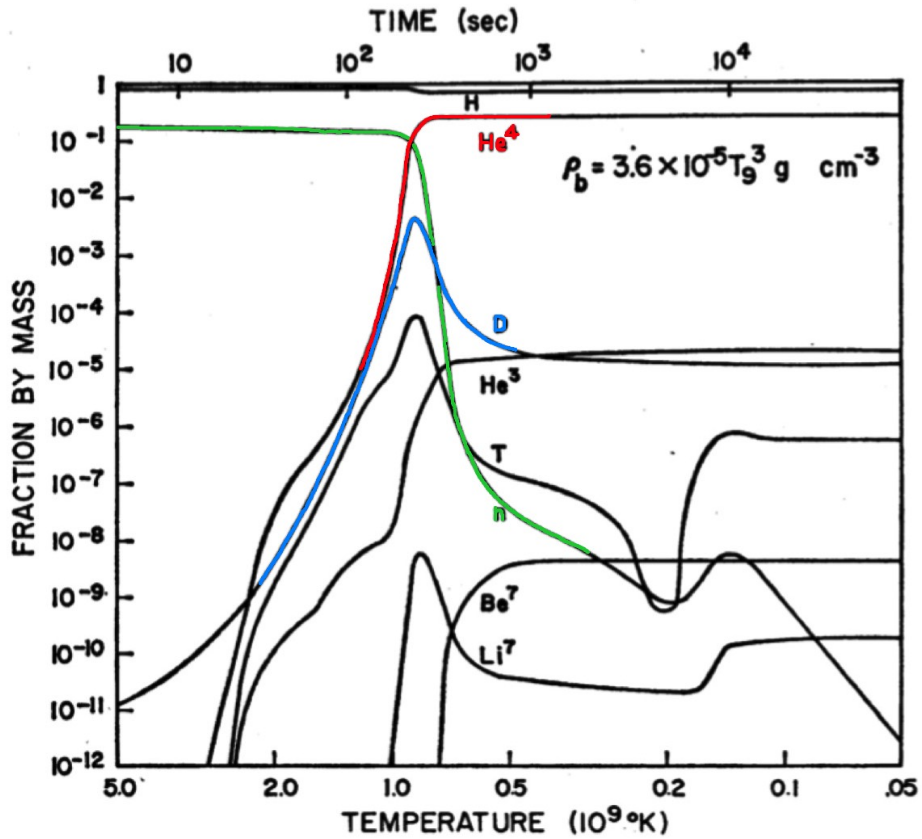
^{7}Li: is destroyed in stars and produced in cosmic ray reactions. Data from oldest, most metal-poor stars in the Galaxy

- Only for ^{4}He $Y_p = \frac{4n_{\rm He}}{\rho_B}$. For all other nuclei: number fractions.

- Larger $\eta = n_B/n_\gamma$ leads to shorter "D-bottleneck" (D photodissociation period), so less n's decay and more end up in ^{4}He, and ^{4}He abundance increases. More n and p density leads to more efficient burning of D and ^{3}He into ^{4}He, so D and ^{3}He abundance decreases.

Development of nuclear abundances during BBN

Only after D-bottleneck" BBN proceeds in earnest and free neutrons disappear



$N_{\rm eff}$ effects in BBN

- At $T \simeq$ few MeV: p and n are maintained in equilibrium by weak interactions $\Gamma_W > H$
$n + \nu_e \leftrightarrow p + e^-, \quad p + \bar{\nu}_e \leftrightarrow n + e^+, \quad n \leftrightarrow p + e^- + \bar{\nu}_e$
$\Delta m = m_n - m_p \simeq$ 1.29 MeV thus $n_n/n_p \simeq e^{-\Delta m/T} < 1$ and decreases as T decreases.

- n_n/n_p is fixed at 0.17$\simeq 1/6$ (except for n decays)
when weak interactions freeze out, $\Gamma_W \simeq H$, at $T \simeq$0.8 MeV, $t \simeq$1.5 s.

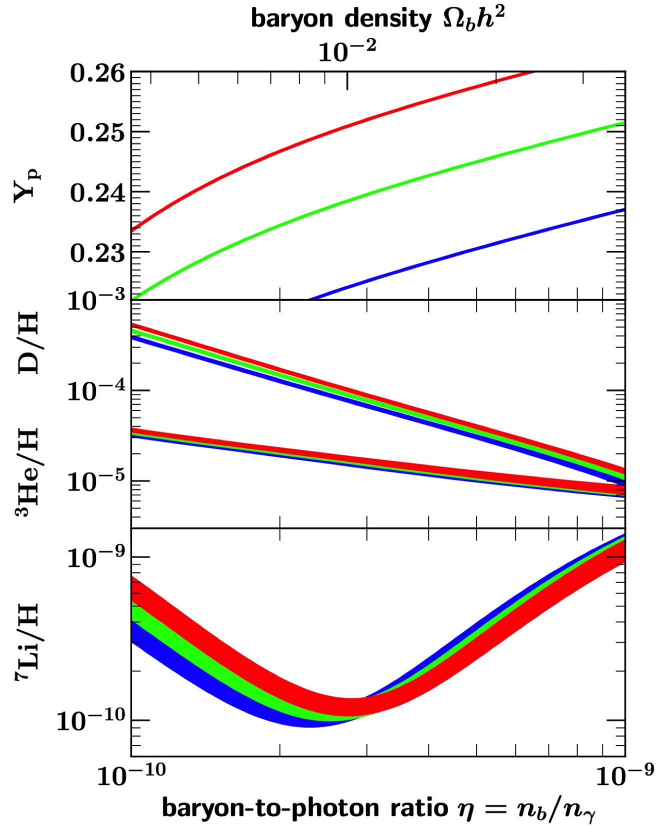
- When BBN finally proceed, all n end up into ^{4}He ($\simeq$ 25% of the mass) together with an equal number of p (the remaining p stay as H ($\simeq$ 75% of the mass).

- As $N_{\rm eff}$ increases, H increases, freeze-out happens earlier when n_n/n_p is larger, leading to more ^{4}He.

- Also, with larger H universe cools down faster, the time window between the end of the "Deuterium bottleneck" (when D becomes stable against photodissociation) $T \simeq$ 0.08 MeV, $t \simeq$ 150 s, and the end of BBN is compressed. Nuclear fusion reactions have less time to burn D into He. So more primordial Deuterium survives unburned, leading to a higher final D/H ratio (Li is more complicated and less well measured)

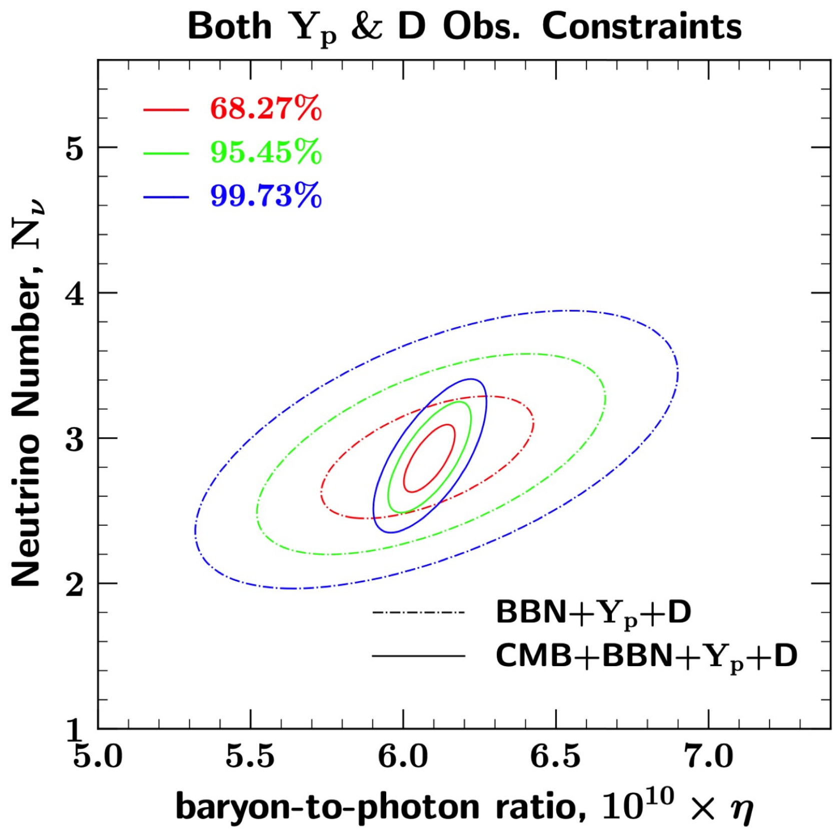
$N_{\rm eff}$ effects in BBN N_ν= 2 (blue), 3 (green) and 4 (red)

(Fields, Olive, Yeh and Young JCAP 03 (2020) 010, JCAP 11 (2020) E02 (erratum) [1912.01132])



Two parameters $N_{\rm eff}$ and η can be measured from two abundances ^{4}He and D

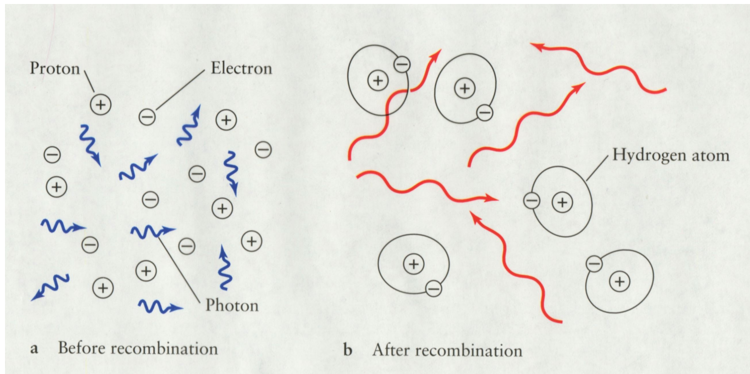
$N_{\rm eff}$ effects in BBN (Fields, Olive, Yeh and Young JCAP 03 (2020) 010, JCAP 11 (2020) E02 (erratum) [1912.01132])



BBN data are now always combined with the more precise CMB data

Cosmic Microwave Background radiation t≃380kyr , T≃ 0.3 eV, z≃ 1090

Emitted when atoms became stable for the first time, at "recombination".



Due to the expansion of the Universe radiation cools to now (COBE FIRAS Fixsen, 2009))

T$= 2.72548 \pm 0.00057^o$K$\simeq 2.35 \times 10^{-4}$eV

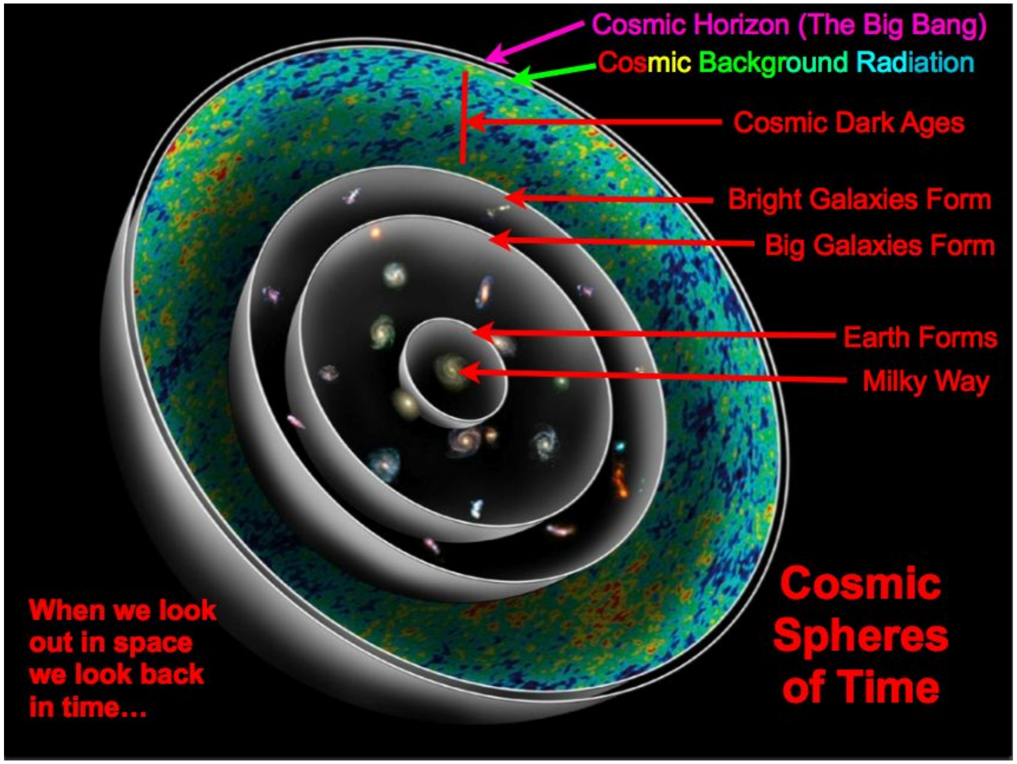
thus now $\rho_{\rm CMB}/\rho_c \equiv \Omega_{\rm CMB} = 2.473 h^{-2} \times 10^{-5} \simeq 5.44 \times 10^{-5} (0.674/h)^2$

Redshift z$\equiv (\lambda_0 - \lambda)/\lambda$ – at emission $\lambda = a\lambda_0$ if $a_0 = 1$, thus **z+1= 1/a**

"Recombination", is also called the "surface of last scattering" of the CMB....

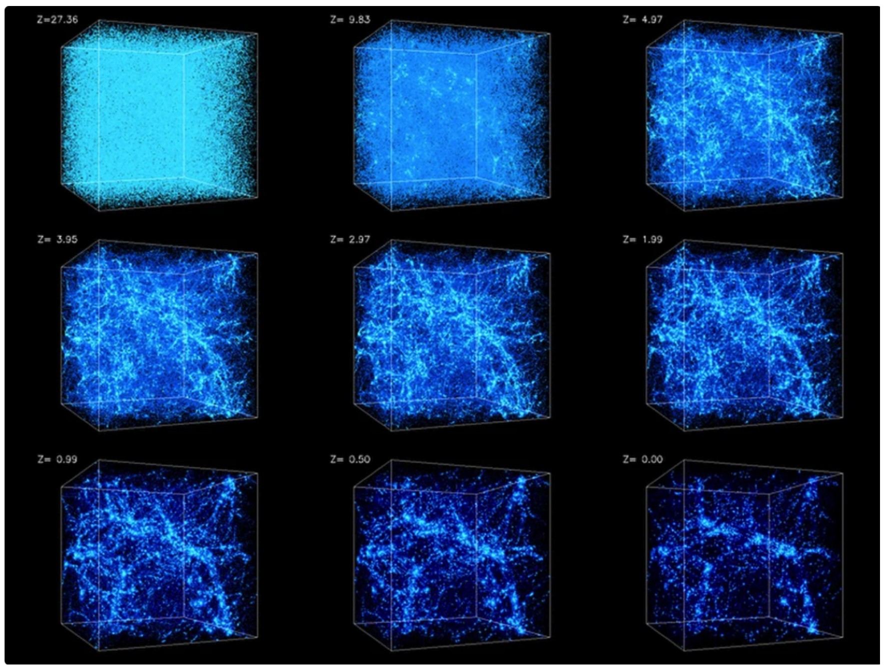
Far away is long ago We see the galaxies within the distance light took to come to us since the first moment galaxies formed, before there was the "Cosmic Dark Age" with no stars, and before then the CMBR was emitted at "recombination", when atoms became stable.

Fig from J. Primack



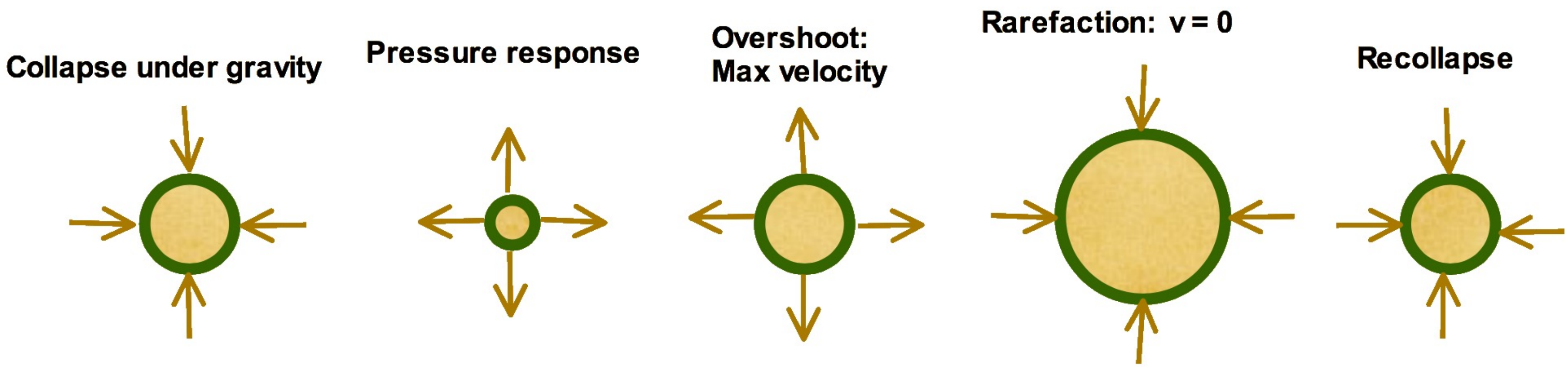
Inhomogeneities in matter lead to Structure Formation

At "recombination" small density inhomogeneities $\frac{\delta\rho}{\rho} = \frac{\rho - \bar{\rho}}{\bar{\rho}}$ produce CMB anisotropies $\frac{\delta T}{T} \simeq 10^{-4}$ and matter inhomogeneities that through gravitational collapse form the Large Scale Structure of the Universe (Fig from A. Kravtsov)



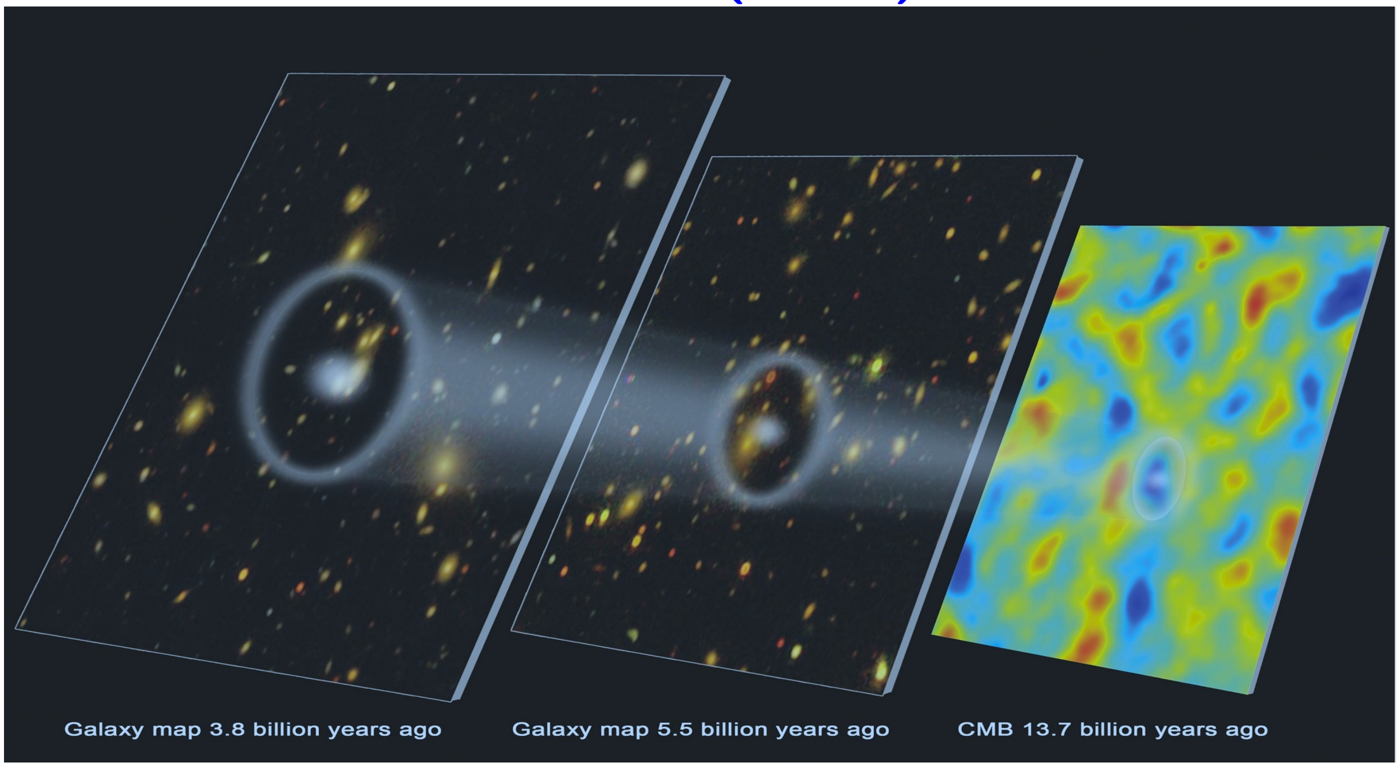
Pressure standing oscillations before recombination

Before recombination, gravity attraction + repulsion due to the pressure in the photon-electron-baryon plasma, produce standing waves, hotter compression zones and cooler rarefaction zones



When atoms become stable, photons escape (and reach us as the CMB radiation) and show us the hotter and cooler regions as CMB anisotropies and baryons remain in spherical shells of predictable radius which are seen as Baryon Acoustic Oscillations (BAO) in the Matter Power Spectrum (SDSS 2005, BOSS 2012)

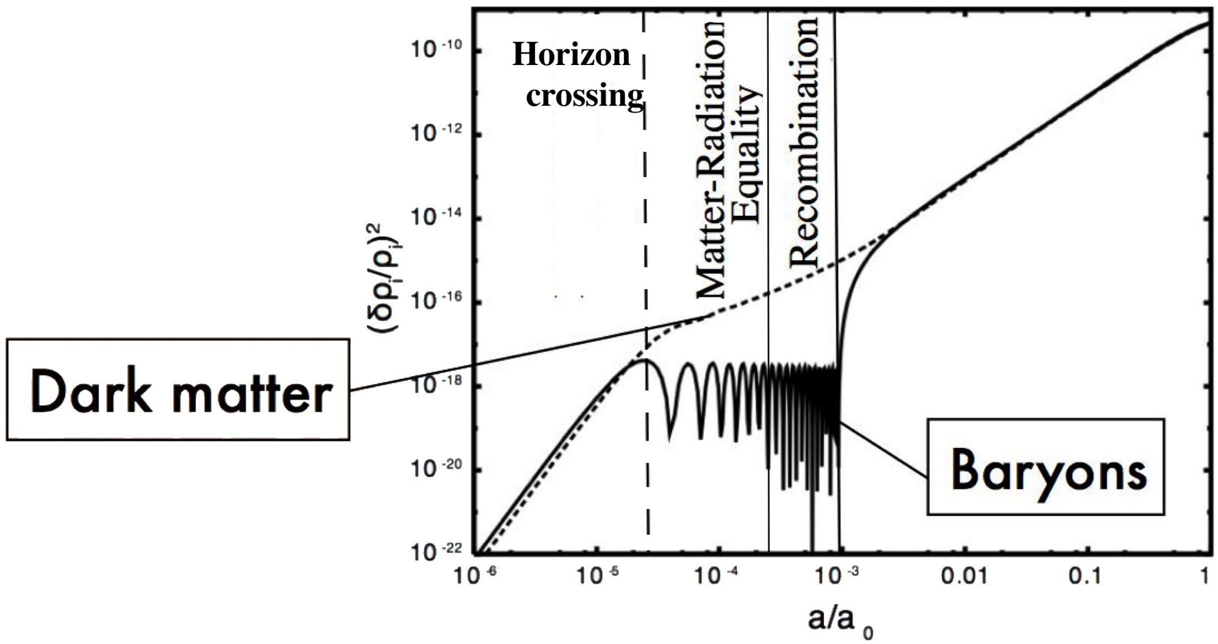
Baryon Acoustic Oscillations (BAO)



In a region with high initial density, there was high pressure in the baryon-photon fluid which propagated as an expanding spherical sound wave. After recombination the photons go off with speed c and baryons are left sitting in a spherical shell around the initial excess density of DM.

Dark Matter is needed for Structure Formation

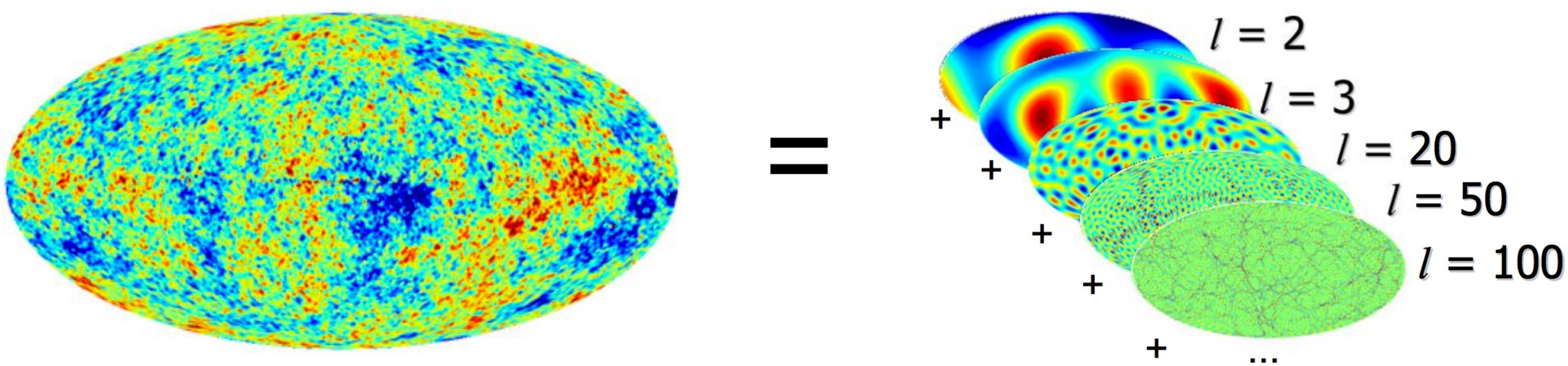
Structure in baryons cannot grow until "recombination" -(before: photon pressure in plasma).Baryons must fall into potential wells of DM, or not enough time for structures to form: in Matt-Dom Universe $(\delta\rho/\rho)_m \sim a$ could go from 10^{-4} to 10^{-1} but need > 1 to form the structures we see



CMB Anisotropies Angular Power Spectrum

The amplitude of the fluctuations as function of scale is quantified by the Power Spectrum, $P(k) =$ square of the Fourier amplitude as function of k.
For functions on a sphere we use an expansion in Spherical Harmonics

$$\frac{\delta T}{T}(\hat{n}) = \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\hat{n})$$



Location: $\hat{n} = (\theta, \phi)$, Angular Power Spectrum:$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell,m}|^2$

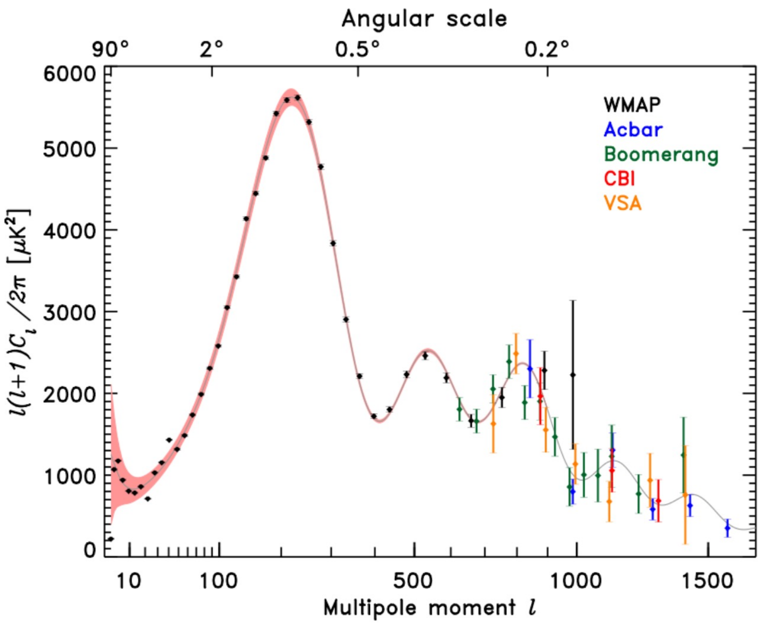
CMB Anisotropies Angular Power Spectrum

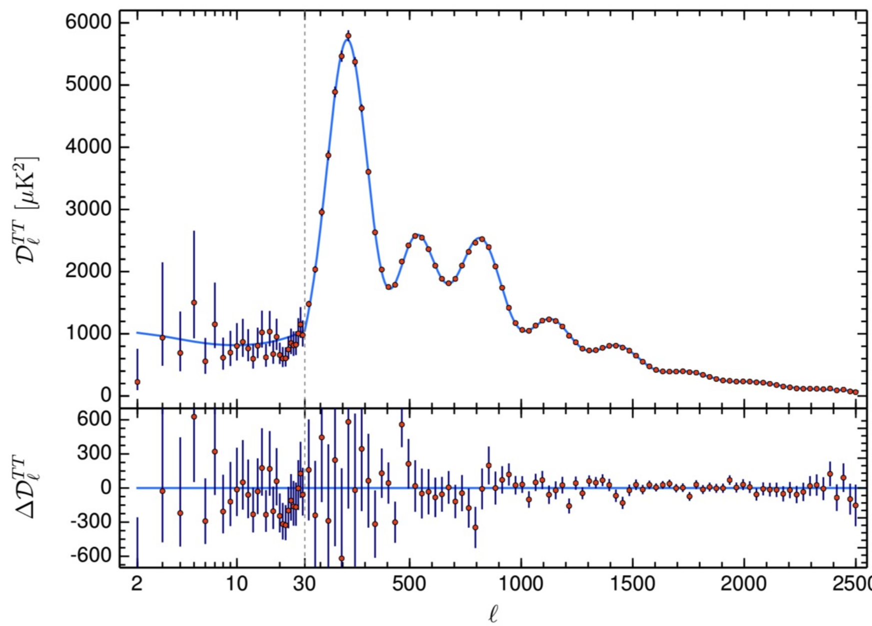
C_ℓ also defines the T-T auto-correlation function ($P_\ell(\cos\theta)$: Legendre Polynomial)

$$C(\theta) = \langle \frac{\delta T}{T}(\hat{n_1})\frac{\delta T}{T}(\hat{n_2})\rangle = \frac{1}{4\pi}\sum_\ell (2\ell+1)C_\ell P_\ell(\cos\theta)$$

Before Planck: Only 3 peaks of the TT angular power spectrum observed

After Planck: 7 TT peaks, E-modes, precision parameter determination.

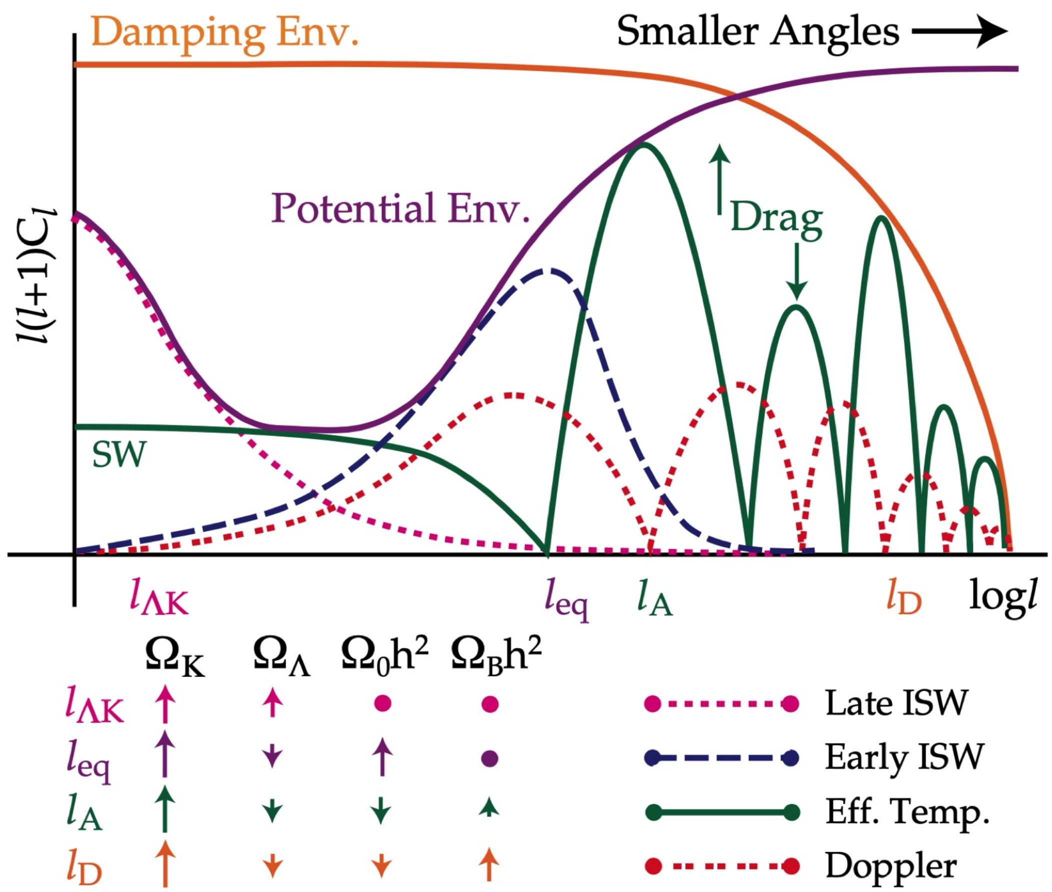




High ℓ modes washed out by photon diffusion mixing of cold and hot regions (Silk Damping)

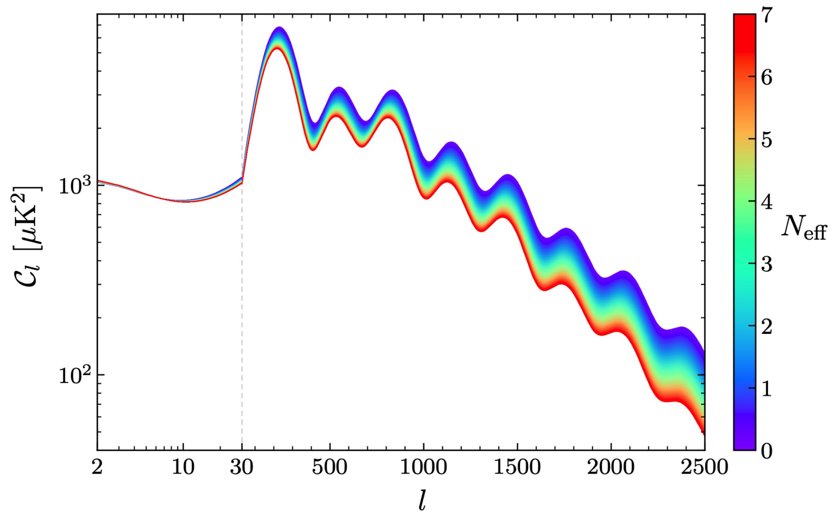
Graciela Gelmini-UCLA

Damping envelope due to photon diffusion homogenizing colder and hotter regions of size smaller than the damping scale (Hu, Sugiyama and Silk [9604166])



$N_{\rm eff}$ effects on the CMB spectrum

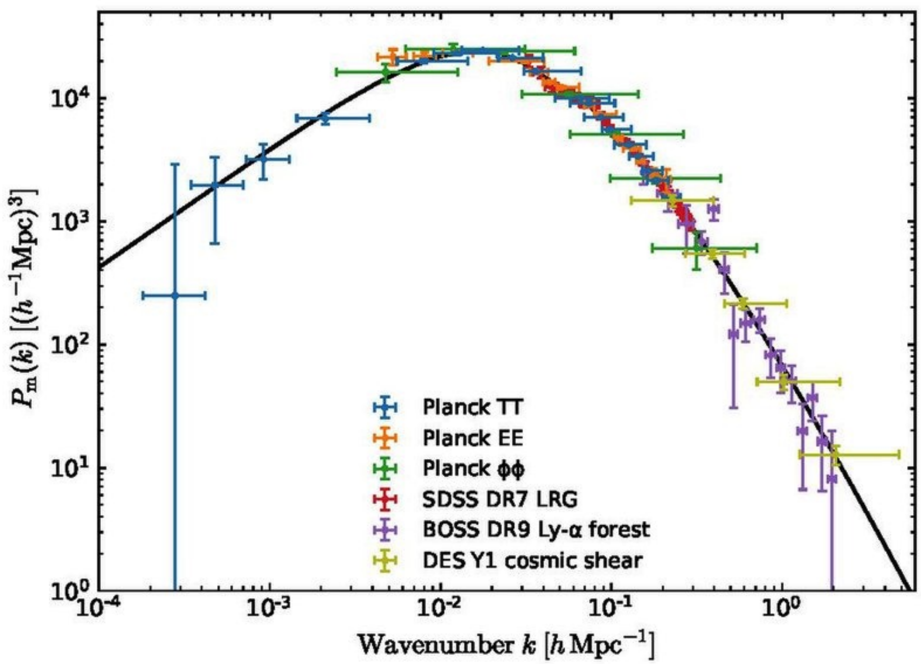
A major effect of increasing $N_{\rm eff}$ is to enhance Silk Damping of small scale anisotropy modes by photon diffusion, by increasing H: the Damping Scale due to photon "random walk" is $r_D \simeq \sqrt{N}\lambda_{\rm mfp}$ where the number of steps is $N = t/\Delta t$, the lifetime of the Universe is $t \simeq H^{-1}$ and the time between steps is $\Delta t = \lambda_{\rm mfp}/c = \lambda_{\rm mfp}$, i.e. $N = H^{-1}/\lambda_{\rm mfp}$. Thus, $r_D \simeq \sqrt{\lambda_{\rm mfp} H^{-1}}$ and its size relative to the (sound) horizon $\simeq ct = H^{-1}$ (to the size of the standing acoustic waves) increases as $r_D/H^{-1} \sim \sqrt{H}$ - Plot keeping fixed peak locations (Baumann TASI2017 (2018) 009 [1807.03098], Wallish thesis [1810.02800])



Other effects: e.g. a phase shift due to ν free streaming

Matter Power Spectrum $P(k)$ Showing BAO

$$\frac{\delta\rho}{\rho}(\vec{x}) = \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}\,, \qquad \langle \frac{\delta\rho}{\rho}(\vec{x}_1)\,\frac{\delta\rho}{\rho}(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} P(k)\; e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}$$



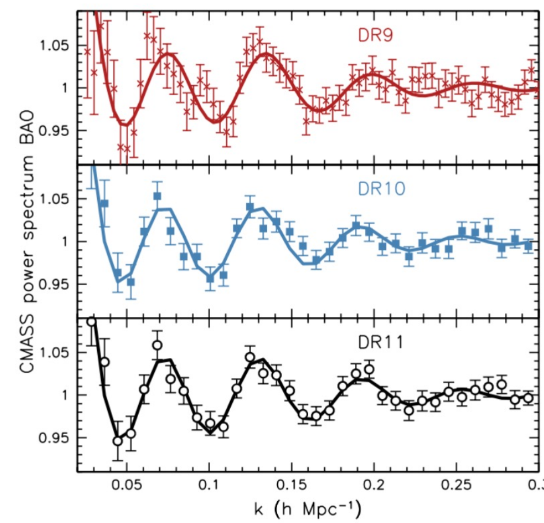
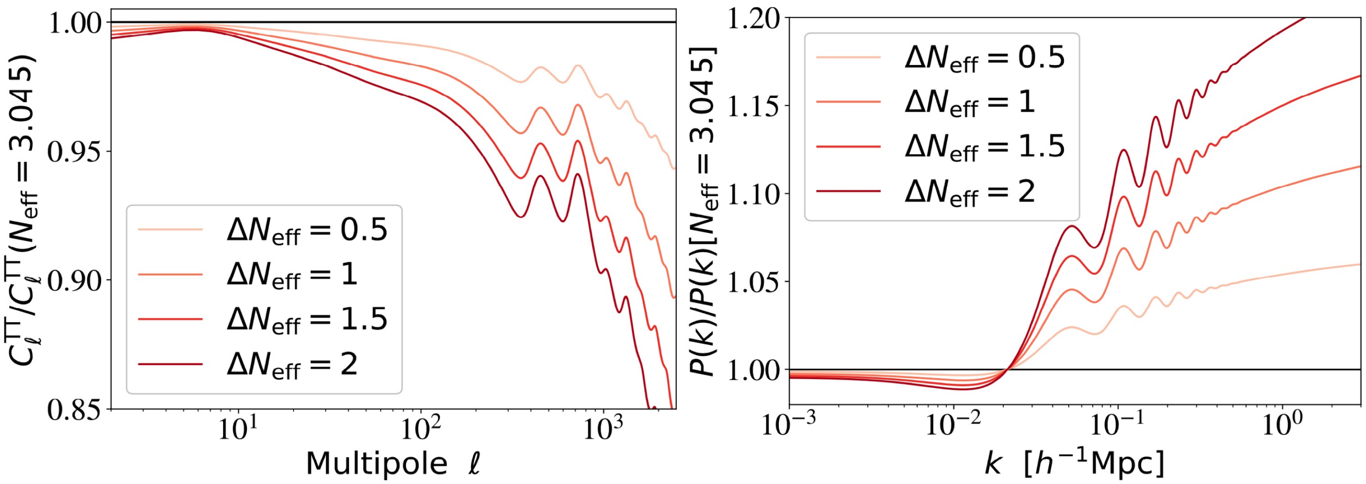


Figure 16. The CMASS BAO feature in the measured reconstructed power spectrum of each of the BOSS data releases, DR9, DR10, and DR11. The data are displayed with points and error-bars and the best-fit model is displayed with the curves. Both are divided by the best-fit smooth model. We note that a finer binning was used in the DR9 analysis.

$N_{\rm eff}$ effects on the matter spectrum (Particle Data Group 2025 Review 26)

Without changing other parameters, major effect of increasing $N_{\rm eff}$ and consequently ρ_R would be to delay radiation-matter equality, and to decrease the horizon size at recombination $\simeq H^{-1}$ (shifting BAO peaks toward smaller physical scales -larger k). However, the limits on $N_{\rm eff}$ are obtained while changing other parameters.

$z_{\rm equality}$ (and ω_b) is very constrained by CMB data, $N_{\rm eff}$ can be changed while keeping $z_{\rm equality}$, z_Λ (the redshift of matter-λ equality) and the baryon density $\omega_b = \Omega_b h^2$. (Bashinsky and Seljak, PRD69, 083002 (2004), [0310198]) This implies increasing $\rho_{\rm DM}$ and ρ_Λ by the same amount as ρ_R.



Limits on N_{eff} from CMB and BAO Table from Particle Data Group 2025 Review 26

Using a ΛCDM+N_{eff} model with 7 parameter- limits still compatible with SM ([53] Aghanim et al. (Planck) [1807.06209], [54] Calabrese et al. (ACT) [2503.14454], [55] Camphuis et al. (SPT-3G) [2506.20707])

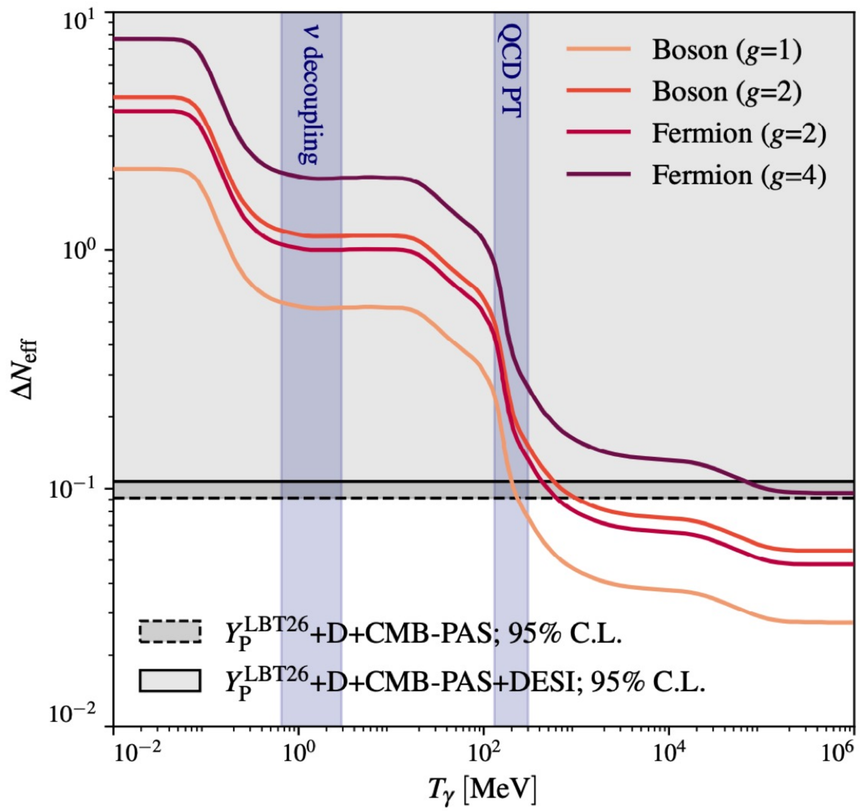
	Model	N_{eff}	Ref.
CMB alone			
Pl18[T&E]	ΛCDM+N_{eff}	2.92 ± 0.18 (68%CL)	[53]
Pl18[T&E]+ACT[T&E]	ΛCDM+N_{eff}	2.73 ± 0.14 (68%CL)	[54]
Pl18[T&E]+ACT[T&E]+SPT-3G[T&E,lensing]	ΛCDM+N_{eff}	2.81 ± 0.12 (68%CL)	[55]
CMB + BAO			
SPT-3G[T&E,lensing]+DESI-DR2	ΛCDM+N_{eff}	3.52 ± 0.23 (68% CL)	[55]
Pl18[T&E]+ACT[T&E,lensing]+DESI-DR1	ΛCDM+N_{eff}	2.86 ± 0.13 (68% CL)	[54]
Pl18[T&E]+ACT[lensing]+DESI-DR2	$w_0 w_a$CDM+N_{eff}	2.96 ± 0.18 (68% CL)	[56]

N_{eff}=2.81 ? Only in specific models (e.g. generating entropy after ν decoupling so lowering T_ν/T) -Future CMB target $\sigma(N_{\mathrm{eff}})$ <0.06 could confirm. (Escudero, Ovchynnikov, Weiner 2603.22391)

Lower limit from Low T_{RH} (BBN+Planck+CMB lensing+DESI BAO): $N_{\mathrm{eff}} > 2.98$ for $T_{\mathrm{RH}} \geq 5.96$ MeV- Without BBN: $N_{\mathrm{eff}} > 2.58$ for $T_{\mathrm{RH}} \geq 3.78$ MeV (Barbieri etal PRL135 (2025) 18, 181003 [2501.01369])

Limit on $\Delta N_{\rm eff}$ from CMB and BAO (Goldstein et al [2603.13226])

Limits potential extra dark energy particles, as function of their decoupling temperature.



Simons Observatory LAT goal by mid 2030s $\sigma(N_{\rm eff})$=0.050 (Ade et al (SO), JCAP 02 (2019) 056 [1808.07445])

Neutrino masses

- At least two of neutrino mass eigenstates are non-relativistic at present (likely all three):

$$|\Delta m^2_{31}|^{1/2} \simeq 0.05 \text{ eV} > |\Delta m^2_{21}|^{1/2} \simeq 0.009 \text{ eV} >> T_{\nu 0} \simeq \times 1.7 \times 10^{-4} \text{ eV}$$

- Before neutrinos become non-relativistic they contribute to the radiation and to the matter density after. They would become non-relativistic:

- at radiation-matter equality if $m_\nu \simeq 3.15 T_\nu^{\rm eq} \simeq 1.5$ eV

- at CMB emission if $m_\nu \simeq 0.6$ eV (affecting CMB and BAO)

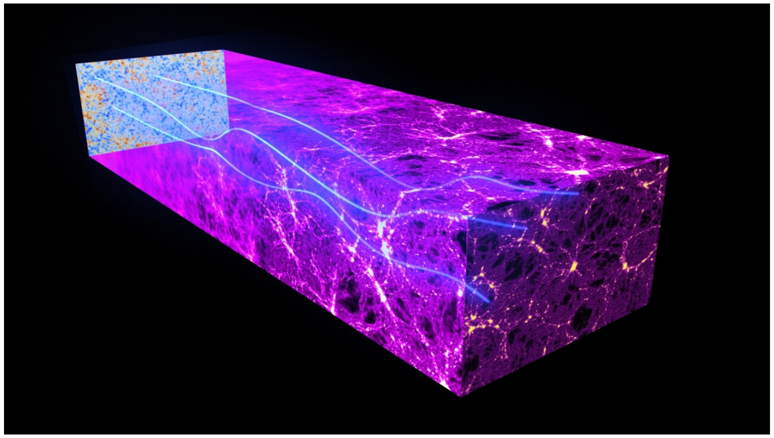
- during structure formation if $m_\nu << 0.6$ eV (affecting CMB, BAO and LSS). This is the scenario allowed now for Standard Model neutrinos.

Effects of neutrino masses on the CMB (see e.g. PDG 2025 Review 26)

- Increasing $\sum m_\nu$ enhances the non-relativistic density at late times, which changes the (angular diameter) distance to recombination and $z_{\rm equality}$ (changing h and Ω_Λ can fix one but not both)

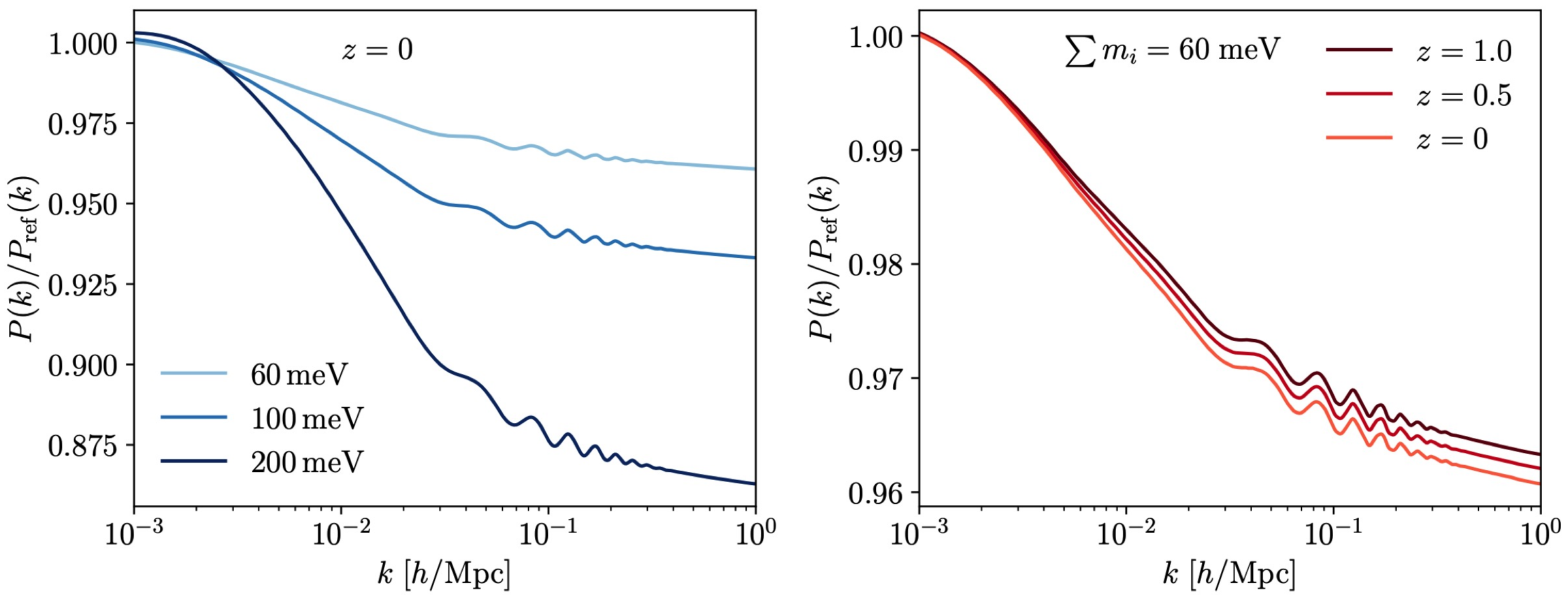
- Early ISW: When neutrinos become non-relativistic they change the gravitational potentials through which the CMB photons pass coming to us. Entering a well photons are accelerated by gravity gain energy, exiting the same well they lose energy. If the well does not change much during the photon passage, the gain and loss of energy is the same. If it does change while the photon is inside there is a net energy shift. This effect integrated from emission to now is the Integrated Sachs-Wolfe Effect (ISW). Early ISW occurs shortly after recombination.

- CMB Lensing: measures the integrated mass distribution back to the last scattering surface. As $\sum m_\nu$ increases, $\rho_{\rm DM}$ must be a smaller share of the "late" (after CMB emission) total $\rho_{\rm matter}$ (that makes the Universe spatially flat) this means less structure thus suppressed lensing.



Effects of neutrino masses on the matter spectrum

Neutrinos escape freely from inhomogeneities (which moving with c) decreasing their density and suppressing their growth while relativistic, i.e. at scales smaller than their "free-streaming length" $\lambda_{\rm fs} \simeq 100{\rm Mpc}\,(1{\rm eV}/m_\nu)$. Here $\rho_\nu = n_\nu \sum m_\nu$ **assuming a common SM number density for all three neutrino species** (see e.g. Gerbino etal Snowmass 2021 [2203.07377] and references therein)



Limits on $\sum m_\nu$ from CMB, BAO and LSS (Table from PDG2025 Review 26)

Using a ΛCDM+$\sum m_\nu$ model with 7 parameter- 95% constraints

([53] Aghanim et al. (Planck) [1807.06209]; [55] Camphuis et al. (SPT-3G) [2506.20707]; [58] Adame et al. (DESI) [2411.12022]; [77] Hinshaw et al. (WMAP) [arXiv:1212.5226]; [78] Garcia-Quintero et al. (DESI) [2504.18464]; [79] D. Chebat et al., 01, [2507.12401])

	Model	(eV)	Ref.
CMB alone			
WMAP[T&E]	ΛCDM+$\sum m_\nu$	< 1.3	[77]
Pl18[T&E]	ΛCDM+$\sum m_\nu$	< 0.26	[53]
Pl18[T&E+lensing]	ΛCDM+$\sum m_\nu$	< 0.24	[53]
Pl18[T&E]+ACT[T&E]+SPT-3G[T&E,lensing]	ΛCDM+$\sum m_\nu$	< 0.17	[55]
CMB + BAO			
Pl18[T&E]+ACT[T&E,lensing]+DESI-DR2	ΛCDM+$\sum m_\nu$	< 0.077	[78]
Pl18[T&E]+ACT[T&E]+SPT-3G[T&E,lensing]+DESI-DR2	ΛCDM+$\sum m_\nu$	< 0.048	[55]
Pl18[T&E]+ACT[T&E,lensing]+DESI-DR2	w_0w_aCDM+$\sum m_\nu$	< 0.186	[78]
CMB + BAO + LSS			
Pl18[T&E]+ACT[lensing]+DESI-DR1[BAO,FS]	ΛCDM+$\sum m_\nu$	< 0.071	[58]
PR4[T&E]+ACT[lensing]+DESI-DR1[BAO] + Lyman-α	ΛCDM+$\sum m_\nu$	< 0.053	[79]

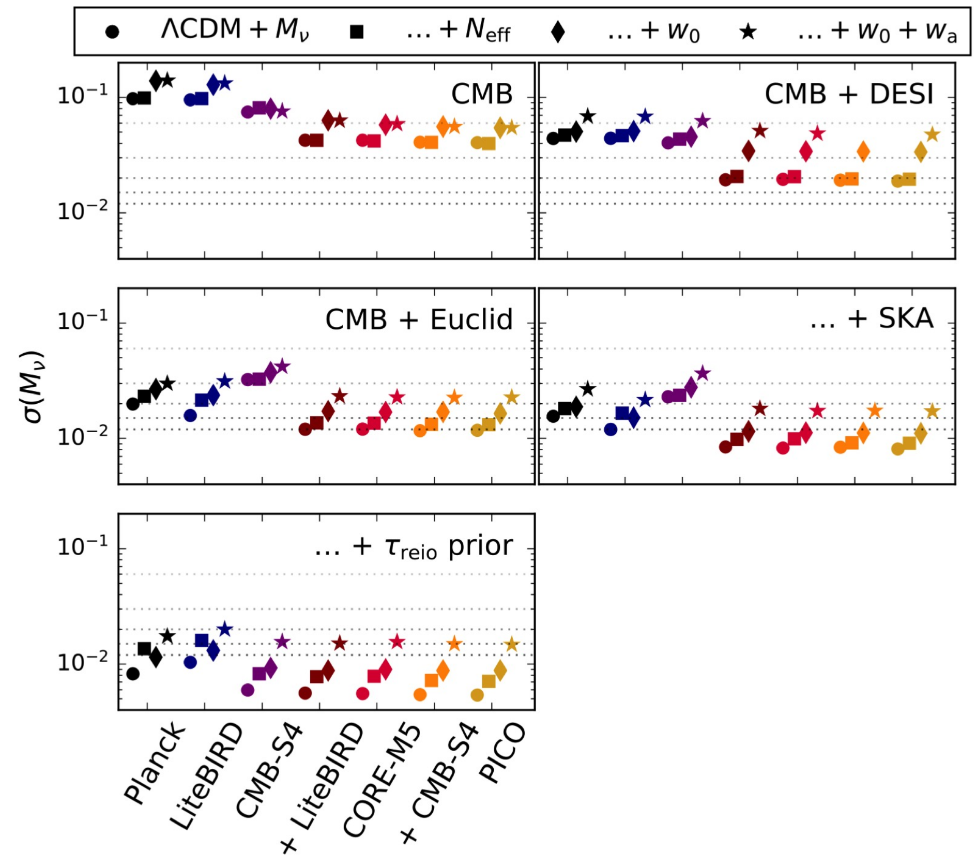
These limits show that neutrino masses are hierarchical (not large and almost degenerate), but even put the inverted hierarchy under pressure, within the ΛCDM model (see e.g. Jimenez et al "... Decisive cosmological evidence for the normal mass hierachy" 2606.18987)

Inverted Hierarchy: $\sum m_\nu > 0.099$ eV, Normal Hierarchy: $\sum m_\nu > 0.059$ eV

Future sensitivity to $\sum m_\nu$ from CMB, BAO and LSS

will detect $\sum m_\nu$, lines are 1-5σ detection thresholds for 0.06 eV Brinckmann et al JCAP 01 (2019) 059

- EUCLID: ESA satellite-operating in L2 since 2023 (LSS, BAO)
- DESI (DE Spectroscopic Inst): in US since 2021 (LSS, BAO)
- SKA (Square Km Array): giant radio telescope under construction South Africa+Australia (LSS, BAO)
- LiteBIRD: JAXA satellite (funded) will launch in 2032 (CMB)
- PICO (Probe of Inflat and Cosmic Origins): NASA study funded (CMB)
- CORE (Cosmic Origin Explorer) ESA satellite not selected proposal (CMB)



- CMB-S4 radio telescope (Chile+Antarctica) NSF-DOE cancelled in 2025 (CMB)

Sterile neutrinos

The 3 (left-handed) neutrinos of the SM are called "active neutrinos" because they have full strength weak interactions, but others with no weak interactions (right-handed) thus called **"sterile"** ν_s (Bruno Pontecorvo- 1967)

ν_s, can be easily added to the SM with a Majorana mass since they are singlets (one or more), leading to a see-saw mechanism for neutrino masses.

ν_s can be created via active-sterile neutrino oscillations, either without (Dodelson & Widrow 1994) or with (Shi & Fuller 1998) a large Lepton Asymmetry L (L-driven MSW conversion), and are the prime Warm DM candidates (SF produces "colder" ν_ss than DW)

For two-neutrino active-sterile mixing where $|\nu_{\alpha,s}\rangle$ are interaction eigenstates (α left handed, s right-handed) and $|\nu_{1,2}\rangle$ are mass eigenstates, $m_1 << m_2 \equiv m_s$
$|\nu_\alpha\rangle = \cos\theta \; |\nu_1\rangle + \sin\theta \; |\nu_2\rangle$;
$|\nu_s\rangle = -\sin\theta \; |\nu_1\rangle + \cos\theta \; |\nu_2\rangle$

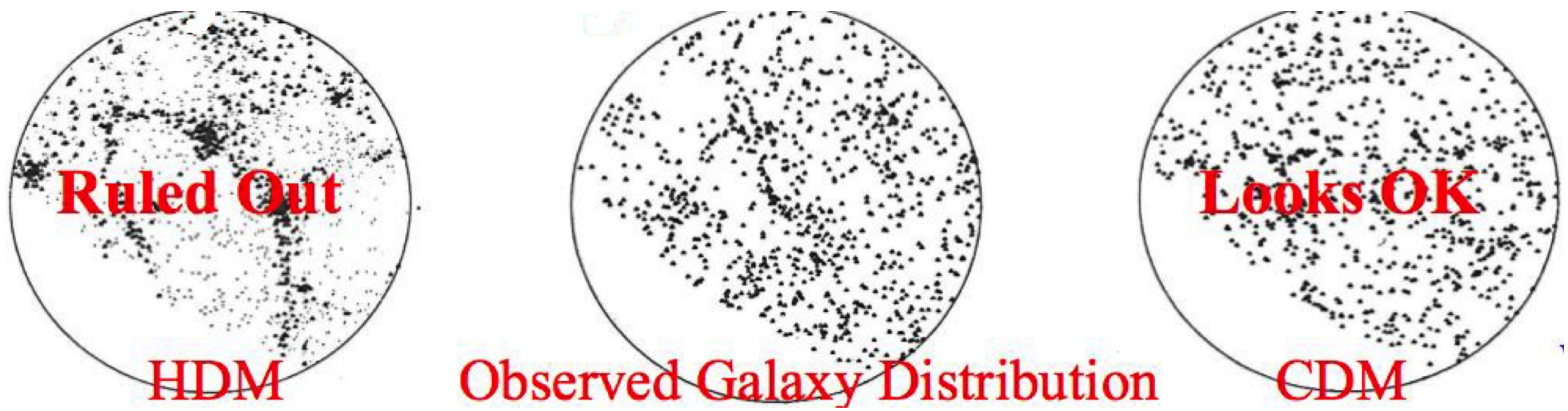
ν_s can also be produced in the decay of other particles, e.g. new scalar fields or heavier sterile neutrinos, a case I will not consider here.

Dark Matter is "Cold" or "Warm"

Dark Matter is classified as "HOT" or "WARM" of "COLD" if it is

RELATIVISTIC (moves with c), SEMI-RELATIVISTIC or NON-RELATIVISTIC

at the moment dwarf galaxy core size structures start to form (when $T \sim$ keV).
We know since the 1980's (Fig. S. White 1986) that these structures (or smaller ones) form first and structure cannot form with relativistic matter.



Dark Matter is “Cold” or “Warm”

Both work well at scales larger than dwarf galaxies.

The differences are at smaller scales where observations and their interpretation are still not conclusive.

With WDM only structures of dwarf-galaxy cores size and larger survive.

With CDM structures much smaller than galaxy size survive. Galaxies form “bottom-up”, by coalescence of smaller structures. Some of the small structures remain in the larger ones (many DM mini-haloes within galactic haloes).

"Double-Dark" model works well with CDM or WDM above galactic scales, distinction at sub-galactic scales

Fig: from Tegmark ("Standard model" with ΛCDM: with Cold DM)

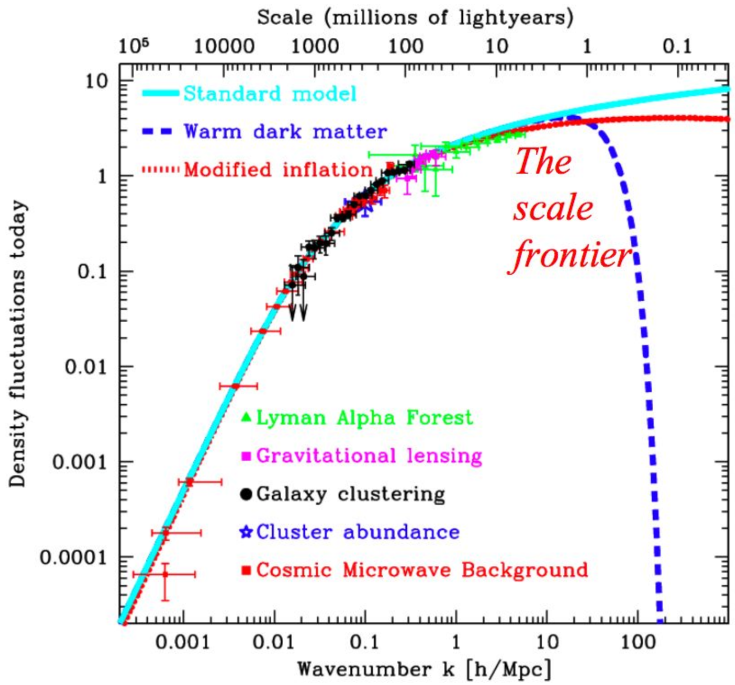
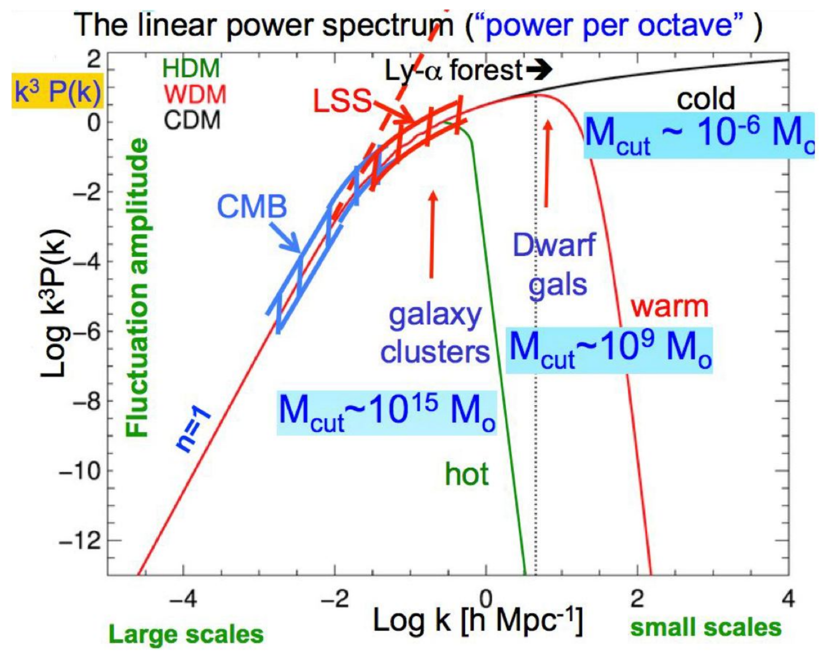


Fig: from Carlos Frenk

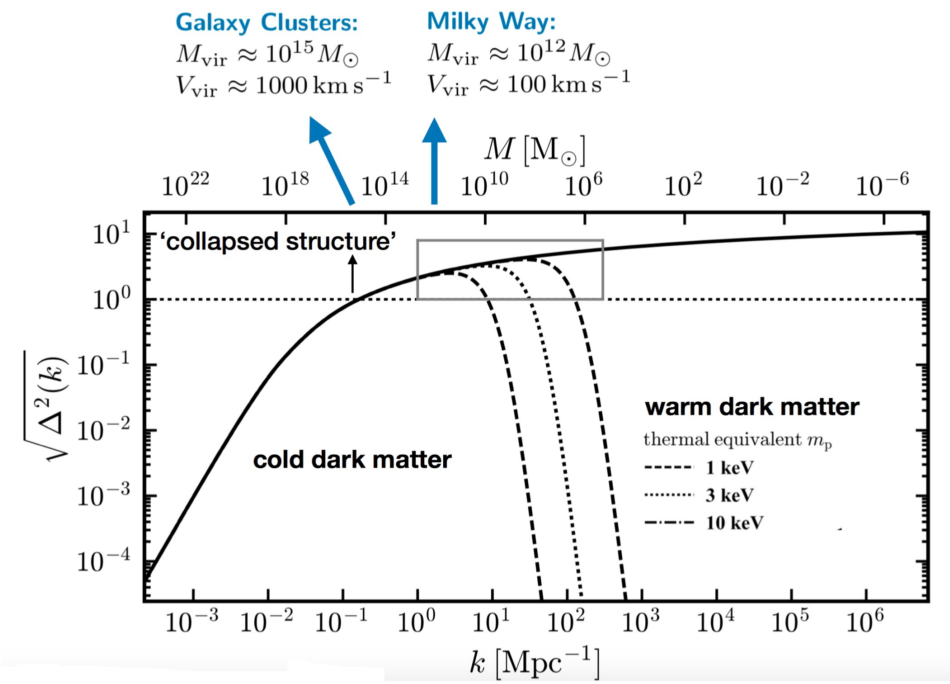


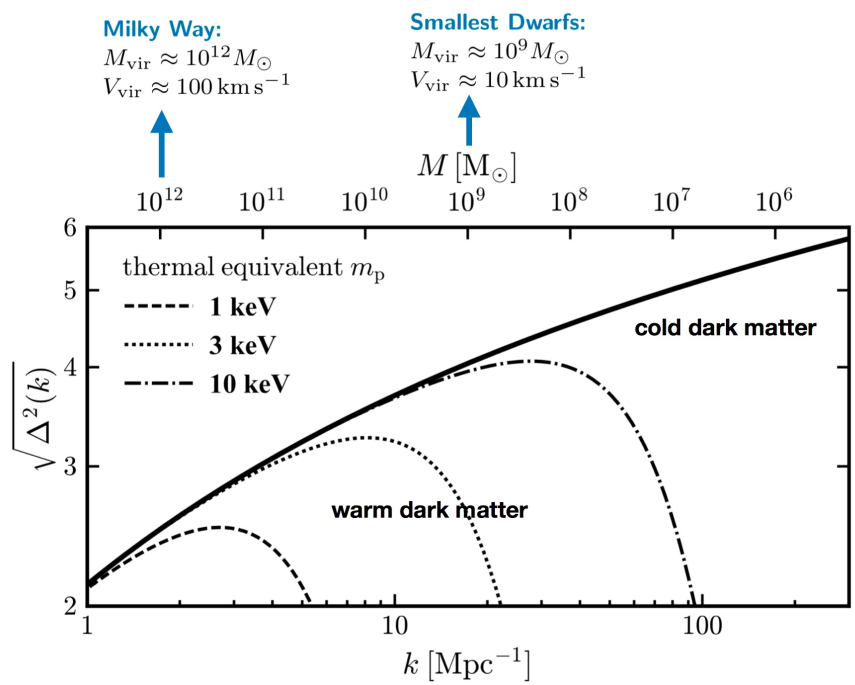
Distinguishing CDM-WDM-SIDM-mixed DM and baryonic effects at sub-galactic scales is where most of the structure formation simulations and observational efforts are directed at present.

“Double-Dark” model works well with CDM or WDM above galactic scales, distinction at sub-galactic scales

Figs: from James Bullock, Boylan-Kolchin, ARAA, 2017

Only it has a thermal spectrum or similar, $\bar{E} \simeq 3T$, WDM requires $m \simeq$ keV





Distinguishing CDM-WDM-SIDM-mixedDM and baryonic effects at sub-galactic scales is where most of the structure formation simulations and observational efforts are directed at present.

Non thermal mechanism: Freeze-in of sterile neutrinos

Production of sterile neutrinos with no-extra SM interactions via active-sterile oscillations:

- At $t = 0$: produce $\nu_\alpha = \cos\theta \nu_1 + \sin\theta \nu_2$; $\nu_{1,2}$ evolve with different phases, $\approx e^{-itm_i^2/2E}$ for $E >> m_i$.

- At $t > 0$: $\nu(t) = a(t)\nu_\alpha + b(t)\nu_s$, thus $P(\nu_\alpha \to \nu_s) = \sin^2 2\theta \sin^2\left(\frac{t}{\ell}\right)$

$\ell = \dfrac{\Delta m^2}{2E}$ = vacuum oscillation length.

- Matter effects: ℓ, $\sin^2 2\theta \to \ell_m$, $\sin^2 2\theta_m = \left(\frac{\ell_m^2}{\ell^2}\right) \sin^2 2\theta$,

- Collisions: act as measurements, so $t = t_{coll}$

- "Average regime": $t_{coll} >> \ell_m$ so $\langle \sin^2\left(\frac{t_{coll}}{\ell_m}\right)\rangle = \frac{1}{2}$.

Thus, rate of production of sterile neutrinos:

$\Gamma_s \simeq P(\nu_\alpha \to \nu_s)\ \Gamma_\nu \simeq \left(\frac{\ell_m^2}{\ell^2}\right) \sin^2 2\theta\ \Gamma_\nu$, where for negligible L_ν ($\simeq 10^{-10}$)

$$\ell_m \simeq \frac{\ell}{\left\{\sin^2 2\theta + \left[\cos 2\theta - \frac{2E\ V^T}{\Delta m^2}\right]^2\right\}^{1/2}}$$

$V^T \sim T^5$: thermal potential due to finite temperature effects

- Low T: $\Gamma_s \simeq \Gamma_\nu \simeq n\sigma \sim T^5$ (V^T term negligible, $\ell_m \simeq \ell$ as in vacuum)
- High T: $\Gamma_s \simeq \left(\frac{\Delta m^2}{V^T 2E}\right)^2 \Gamma_\nu \sim T^{-7}$ (V^T term dominates $\left(\frac{\ell_m}{\ell}\right) \simeq \frac{\Delta m^2}{V^T 2E}$)

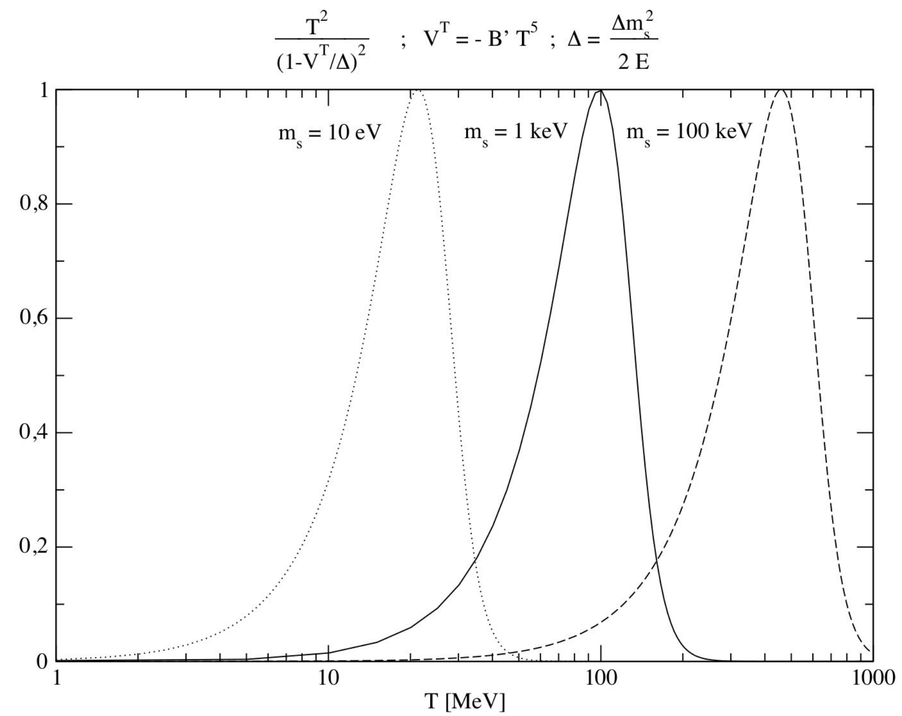
So Γ_s is max. at $T_{\rm max} \approx 130\,{\rm MeV}\left(\frac{m_s}{1\,{\rm keV}}\right)^{1/3}$

(Dodelson and Widrow, Phys. Rev. Lett. **72**, 17 (1994))

With standard cosmological assumptions Sterile neutrinos rate of production through non-resonant active-sterile oscillations (DW scenario) has a sharp peak at

$T_{\max} \simeq 130\text{MeV}\left(\frac{m_s}{1\,\text{keV}}\right)^{1/3} > 5$ MeV for $m_s > 0.057$ eV (Dodelson, Widrow 1994)

$\frac{1}{f_\nu}\left(\frac{df_s}{dT}\right)_{E/T}$



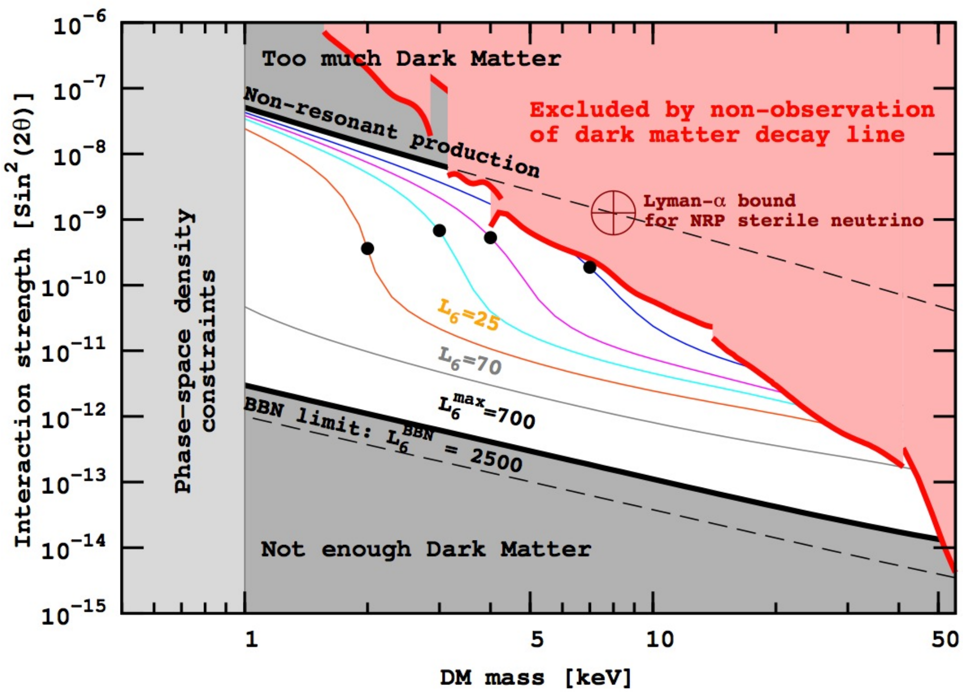
Sterile Neutrinos (Abazajian et al. hep-ph/1204.5379; fig from 0901.0011)

Non-resonantly produced (Dodelson-Widrow 1994) sterile neutrinos cannot constitute the whole of the DM ($< 10\%$ OK)

Dwarf-Spheroidal Galaxies phase space: m_s>1keV

$\nu_s \to \nu\gamma$ would produce X-rays in galaxies and galaxy clusters

Early structures (Ly-α clouds): m_s>5keV but not with large L (Cool DM)-black dots are allowed by the Ly-α limits



If ν_s are part of the DM, $\nu_s \to \nu\gamma$ would produce a monochromatic X-ray line in galaxies and galaxy clusters. This line may have been seen at 3.5 keV! (XRISM, a JAXA/NASA mission launched Sept 7 2023 could confirm or reject it)

Non thermal mechanism: Freeze-in of sterile neutrinos

When propagating in a medium with large L_ν the neutrino mixing changes to

$$\sin^2 2\theta_m = \frac{\Delta^2(p)\sin^2 2\theta}{\Delta^2(p)\ \sin^2 2\theta + \left[\Delta(p)\cos 2\theta - V^L + |V^T(p)|\right]^2}$$

Where $\Delta(p) = \frac{|m_s^2 - m_\alpha^2|}{2p}$, and the potential $V^L \sim L_\nu$,
L_ν is the lepton asymmetry

$$L_{\nu_\alpha} \equiv \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}$$

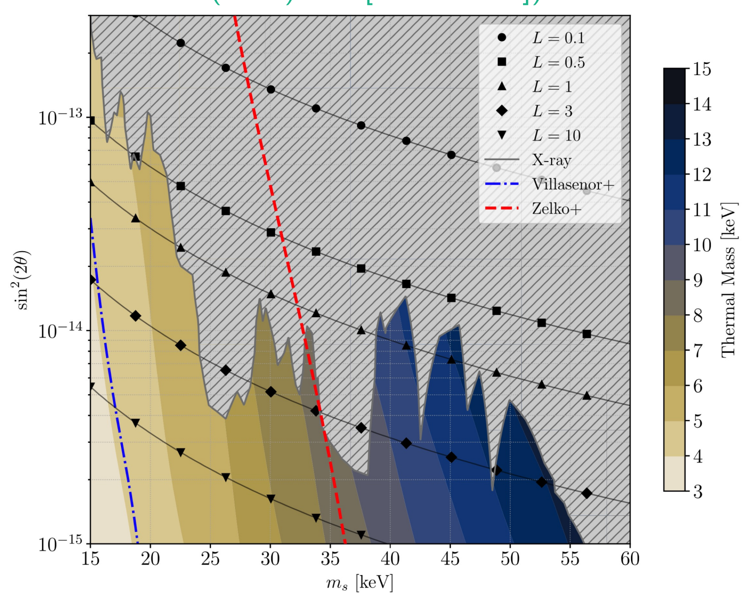
When $\Delta(p)\cos 2\theta - V^L + |V^T(p)| \simeq 0$, $\sin^2 2\theta_m = 1$: resonant production
(similar to the Mikheev-Smirnov-Wolfenstein (MSW) mechanism for active neutrinos in the Sun)

See e.g. Abazajian 1705.01837

Graciela Gelmini-UCLA

Resonantly produced sterile neutrinos Can constitute the whole of the dark matter if $L > 0.5$ in colored region (Vogel et al PRD 112 (2025) 12, 123508 [2507.18752])

Neutrino flavor oscillation physics locks-in L_{ν_α} to potentially large and unequal early, here $T = 10$ GeV, so that $L_{\nu_\alpha} \simeq 10$ can exist during ν_s production (2 orders relaxation of limit $L_{\nu_\alpha} < 0.05$ that assumed equilibration of ν flavors from oscillations) (Froustey and Pitrou, PRD 110, 103551 (2024) [2405.06509]; Domcke et al JCAP 02 (2026) 017 [2510.02438])



Important: ν_s would be relics of the unknown pre-BBN era- so different cosmological assumptions lead to very different relic abundance and spectrum (see my Neutrino 2026 talk)