Neutrino Theory Overview: II

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INSS

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Brookhaven National Laboratory

Connecting experiments to measurements: Reactors

Medium-baseline Reactors

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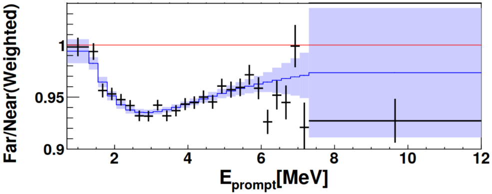
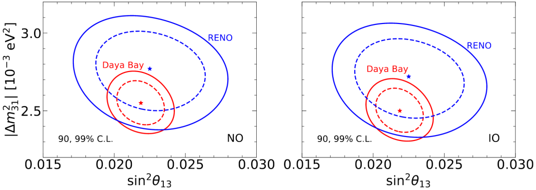


Daya Bay 1809.02261



P. F. de Salas, et al. 2006.11237

Medium-baseline Reactors

The solar Δm^2_{21} oscillation is too slow to develop on these distances

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Δm^2_{ee} is the ν_e weighted average of Δm^2_{31} and Δm^2_{32}:

$$\Delta m^2_{ee} = c^2_{12} \Delta m^2_{31} + s^2_{12} \Delta m^2_{32}$$

This is what they actually measure

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

Long-baseline Reactors

Dominated by solar oscillations:

$$P_{ee}^{\rm LBL} \simeq 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

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JUNO will also measure:

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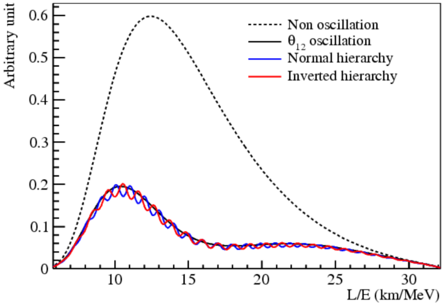
Given that $s_{12}^2 < c_{12}^2$, determining if the larger ampltiude oscillation (Δm_{31}^2) is faster/slower than the smaller ampltiude oscillations, tells if $|\Delta m_{31}^2| <> |\Delta m_{32}^2|$ and thus the atmospheric mass ordering, as solar has determined $\Delta m_{21}^2 > 0$

Vacuum measurements can never determine the mass orderings;
this relies on solar's determination: $\Delta m_{21}^2 > 0$ to work

JUNO's Atmospheric Mass Ordering Determination Method



JUNO 1507.05613

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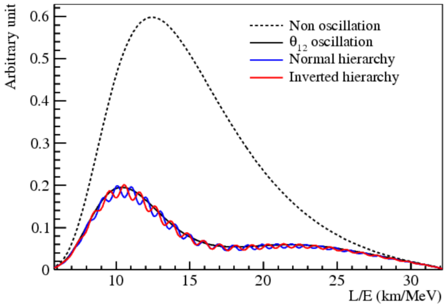

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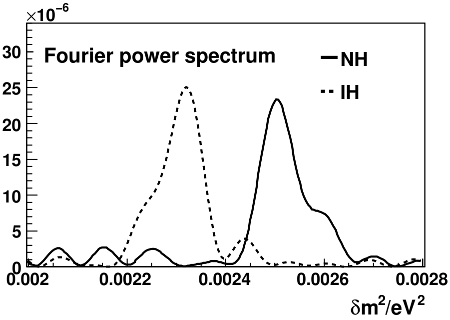

L. Zhan, et al. 0807.3203

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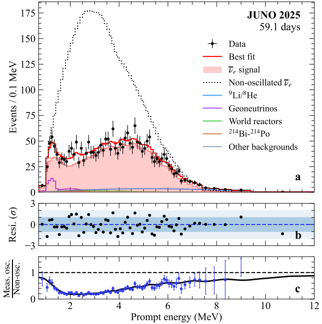
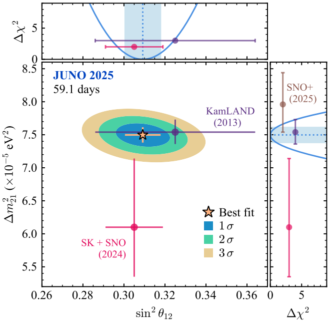
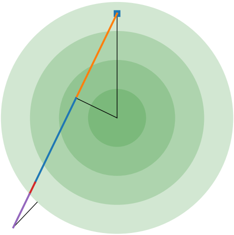




JUNO 2511.14593

Atmospheric Neutrinos

1. Production
2. Propagation
3. Detection



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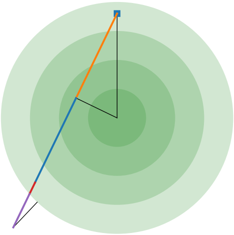

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We often refer to this energy loss as dE/dx
See Brooke Russell's slides
$\pi^\pm \to e + \nu_e$ happens 0.01%
Kaons also produced, $K^\pm \to \nu_e + \cdots = 0.08 K^\pm \to \nu_\mu + \ldots$

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Q: What non-pion production mechanisms
for high energy neutrinos are interesting?

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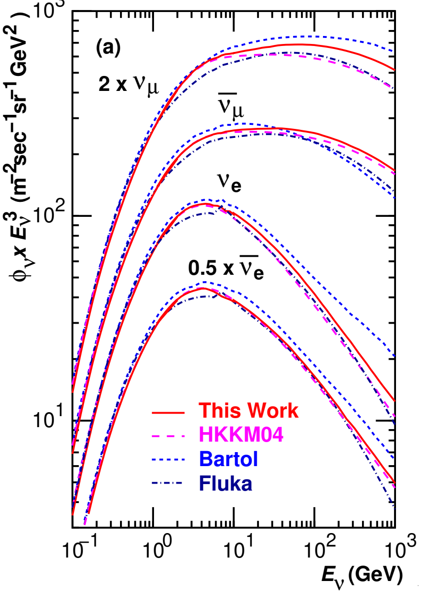

M. Honda, T. Kajita,
K. Kasahara, S. Midorikawa,
T. Sanuki astro-ph/0611418

Open Atmospheric Neutrino Flux Problems

1. Heavier mesons produce neutrinos with a different spectrum: predict this theoretically and measure it

   See work by Mary Hall Reno+, e.g. 2212.07865

2. There are fewer muons at very high energies than predicted by the models

   See e.g. K. Cheminant, et al. 2302.07932

In addition to astroparticle experiments (IceCube, KM3NeT, Auger, Telescope Array, . . .), the LHC plays an important role too

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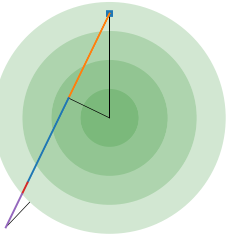

Propagation in Sharply Varying Matter

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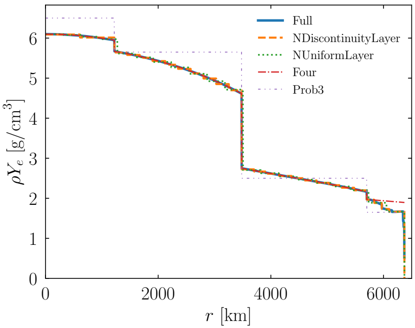
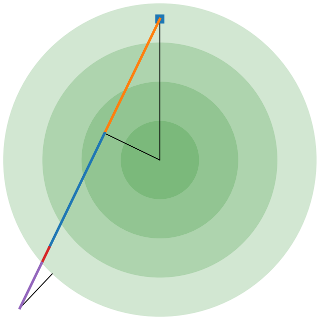
Standard libraries for matrix exponentiation work
Can be much more clever

PBD, S. Parke 2511.04735
github.com/PeterDenton/NuFast-Earth

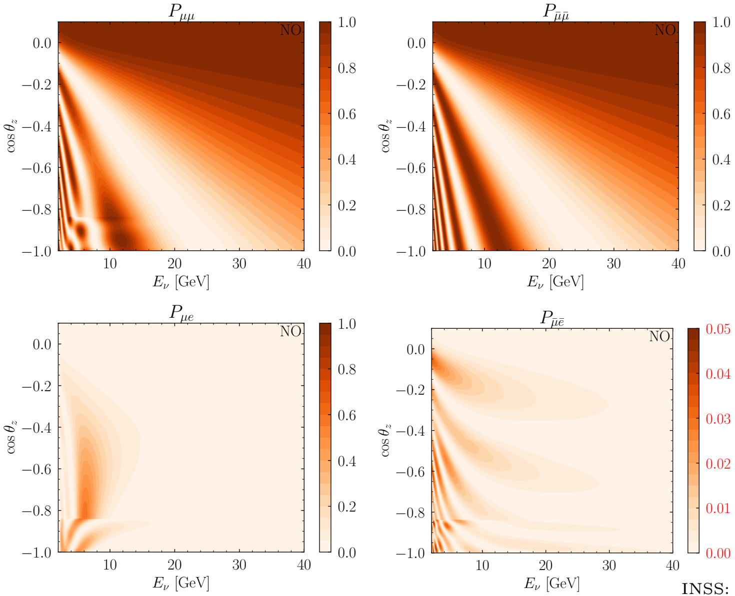
Earth Trajectories



Preliminary Reference Earth Model (PREM)



Atmospheric Neutrino Oscillations: NO



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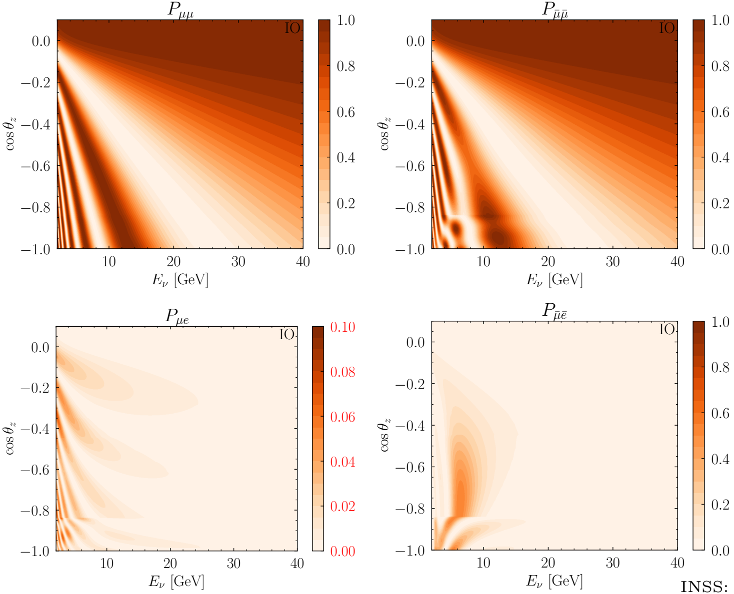

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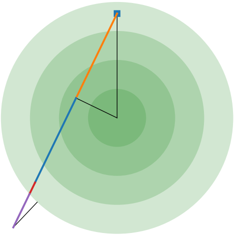


Atmospheric Neutrino Detection

1. Need large volume

Event rate falls off rapidly with energy

2. Need large overburden

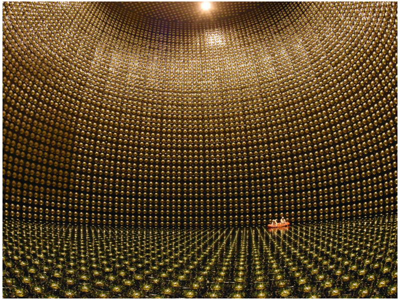
Reduce down-going atmospheric muon background

3. Need direction information: $L \simeq -2R\cos\theta_z$

$\cos\theta_z = -1 \Rightarrow$ core-crossing
$\cos\theta_z = 0 \Rightarrow$ horizontal
$\cos\theta_z = 1 \Rightarrow$ down-going

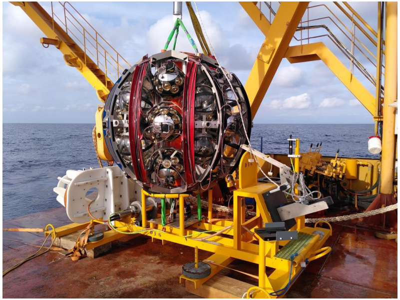
Atmospheric Neutrino Detectors Today and Tomorrow

SuperKamiokande → HyperKamiokande



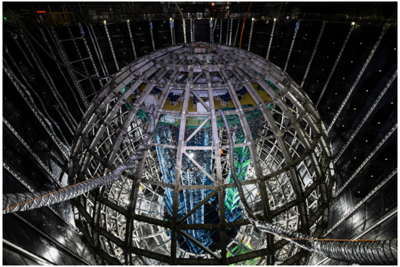
Huge water Cherenkov
Good overburden
Good PMT coverage

IceCube & KM3NeT



Massive water Cherenkov
Good overburden
Poor PMT coverage

DUNE & JUNO



"Small" LArTPC/liquid scintillator
Excellent overburden
Excellent PID/PMT

Atmospheric Neutrino Results

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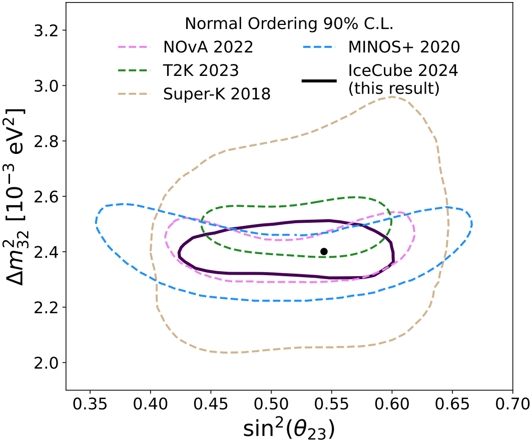

IceCube 2405.02163

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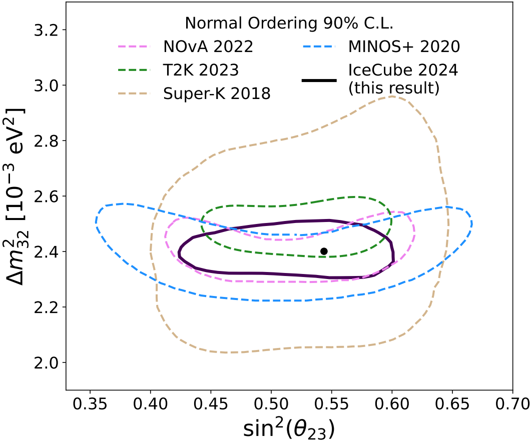

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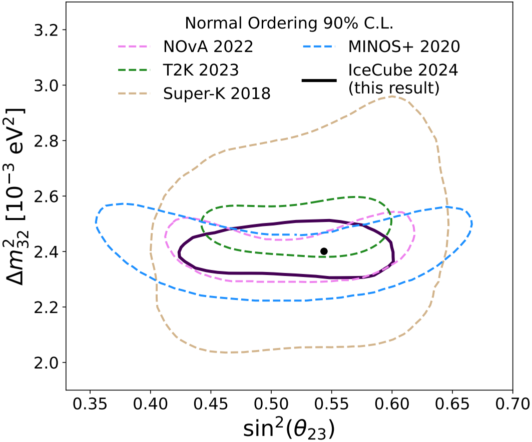
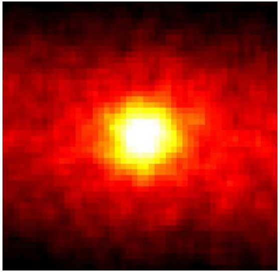
There is also information about the mass ordering and potentially CP violation



IceCube 2405.02163

The Sun produces a lot of neutrinos; exactly how many?



SuperKamiokande image of the Sun in neutrinos

Problem: Too few neutrinos from the Sun

Solar Neutrinos

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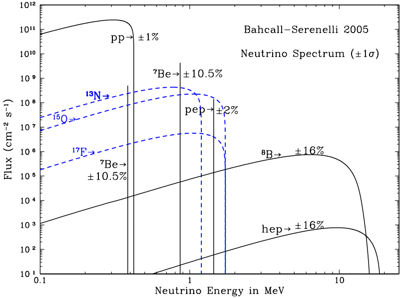
1. John Bahcall predicted the solar neutrino flux



^{8}B flux $\propto T^{24-25}$

J. Bahcall et al. nucl-th/9601044

Solar Neutrinos

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Homestake ApJ. 496 (1998) 505-526

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3. 1990s: SNO used heavy water

Leased the water from a reactor for 1CAD (+ lots of insurance)

SNO nucl-ex/0204008

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SNO nucl-ex/0204008

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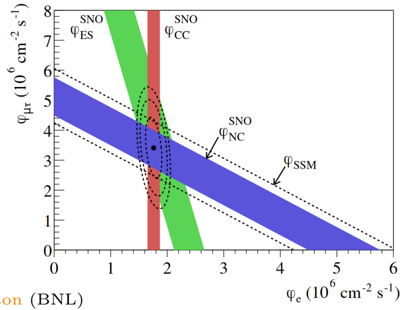

SNO nucl-ex/0204008

Solar Neutrino Probability

1. Compute Hamiltonian at production point

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4. Ensure mass eigenstates remain mass eigenstates. Adiabaticity parameter:

$$\gamma = \frac{\sin^2 2\theta_{12}}{\cos 2\theta_{12}} \frac{\Delta m^2_{21}/2E}{|\dot{N}_e/N_e|}$$

Probability of jump at resonance: $P_j \simeq \exp(-\frac{\pi}{2}\gamma)$

Ensure that $\gamma \gg 1$ at resonance: $a = \Delta m^2_{21} \cos 2\theta_{12}$

S. Parke 2212.06978 (1986)
See also: A. Dighe, A. Smirnov hep-ph/9907423

Solar Neutrino Probability

5. Propagate to Earth: $P_{ij} = \delta_{ij}$

Solar neutrinos decohere:
it is mass states that are relevant

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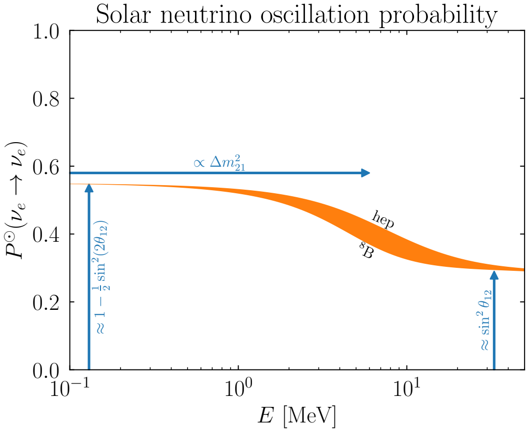

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PBD 2501.08374

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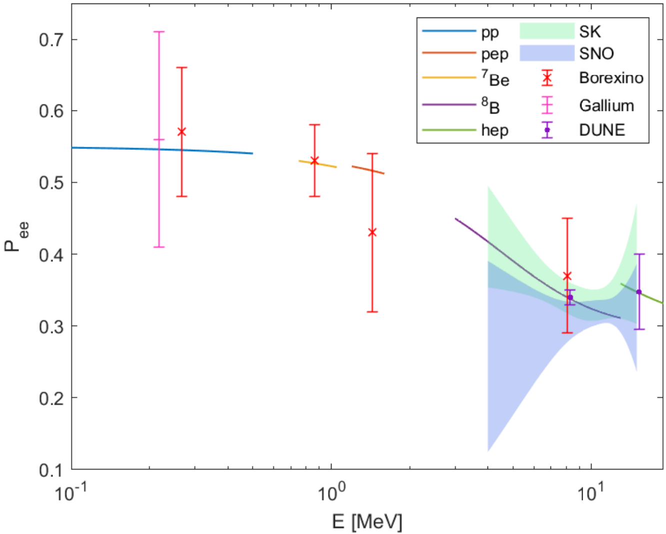
Solar Neutrino Measurements

PBD, C. Gourley 2502.17546



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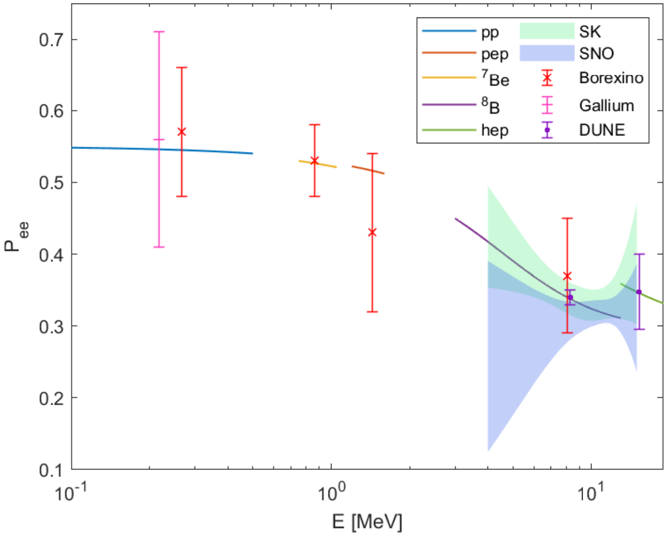
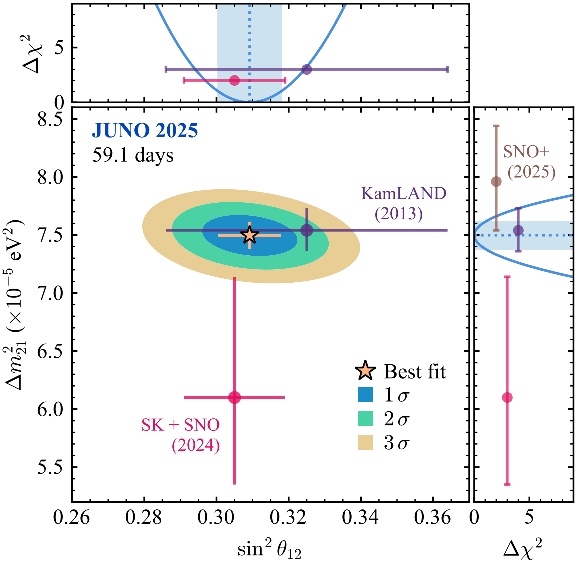


Why discontinuities?

Solar Parameters: SuperK, SNO, KamLAND, JUNO



JUNO 2511.14593

Challenge Questions: Solar Neutrinos

Q: How big is the jump probability at 5 MeV?

Q: Do we see more solar neutrinos during the day or the night?

Q: SNO measured a CC to NC ratio of $\sim 1/3$ of solar neutrinos. KamLAND and JUNO's measurements of reactor neutrinos confirmed the measurement of solar neutrinos by SNO and others. Given the reactor measurements, what other values, if any, could SNO have measured?

Nighttime Solar Neutrinos

At night solar neutrinos experience partial regeneration:
There are more ν_e's from the Sun at night than during the day!

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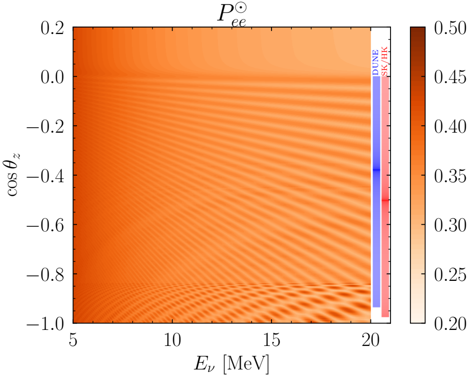

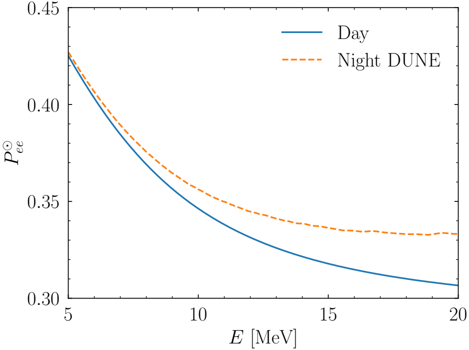

Figs. made with **NuFast-Earth** (github.com/PeterDenton/NuFast-Earth) from PBD, S. Parke 2511.04735

Solar Neutrino Open Questions

- Measure the day-night effect, confirm matter effect in the Earth.
- Detect the hep flux.
- New physics: sterile neutrinos, vector/scalar non-standard interactions, unitarity violation, ...
- Build on Borexino's CNO measurements; understand metalicity discrepancy.

$\vdots$

Long-baseline accelerator experiments:
the most sophisticated neutrino oscillation experiments

Constructing a Long-Baseline Accelerator Experiment

1. Source: proton accelerator, fixed target, focusing horns, decay pipe, absorber

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7. To date provide the only clear evidence for appearance: T2K and NOvA, each in both ν, $\bar\nu$

   Some evidence of ν_τ appearance in atmospherics at SuperK and IceCube

Long-Baseline Accelerator Status

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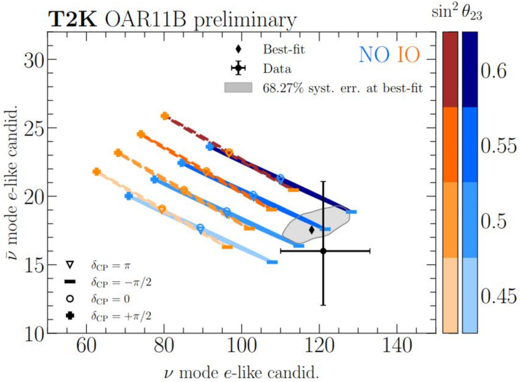
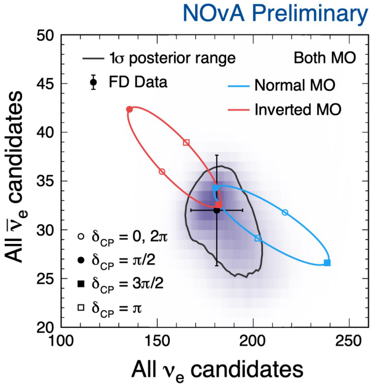
Long-Baseline Accelerator Data



S. King for T2K, Neutrino 2026



Z. Vallari for NOvA, Neutrino 2026

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     Now have >1 good measurement of this
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     know the magnitude, not the sign
  3. θ_{12}: solar & reactor: good
  4. θ_{13}: reactor: good
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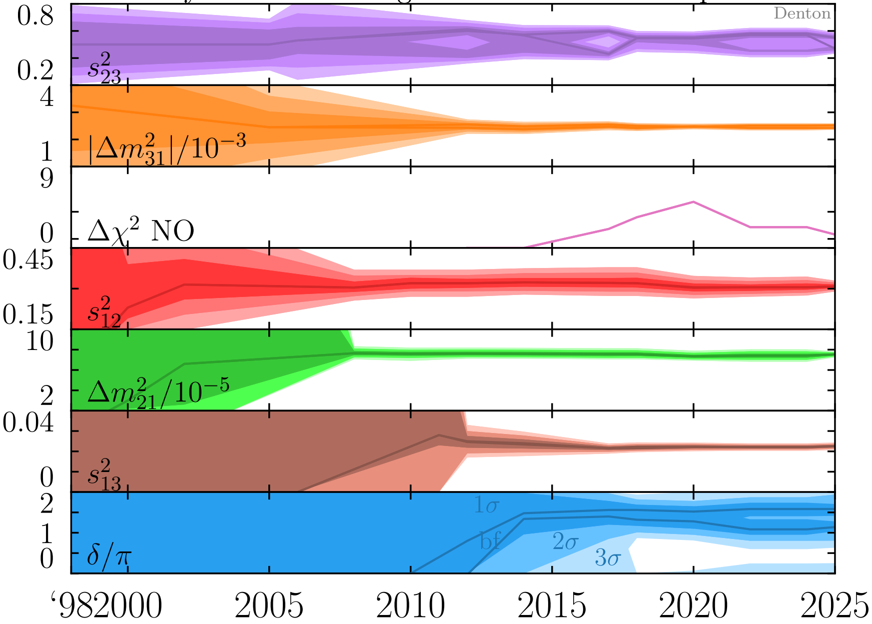
Four remaining known unknowns in particle physics: all neutrinos!

History of our knowledge of the six oscillation parameters



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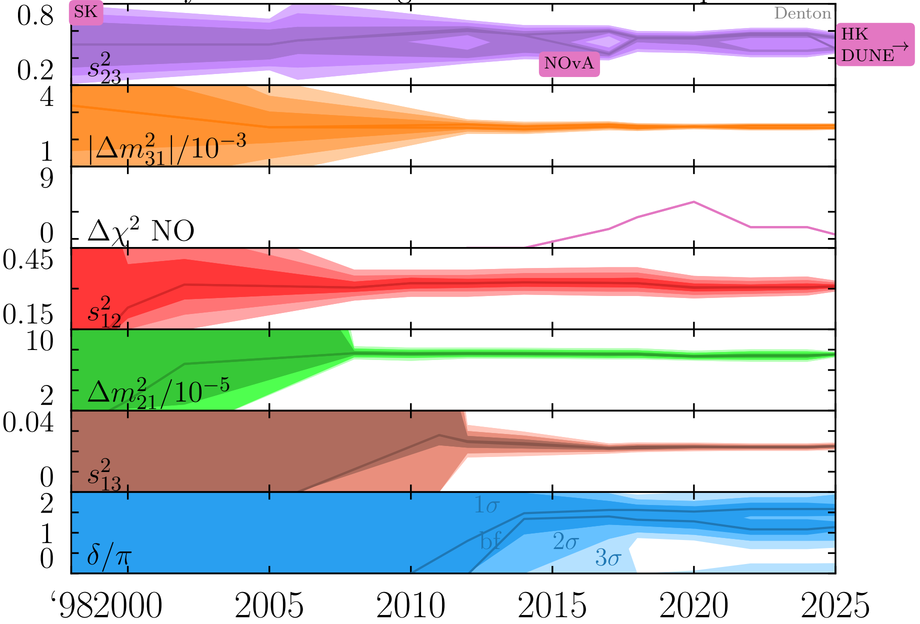

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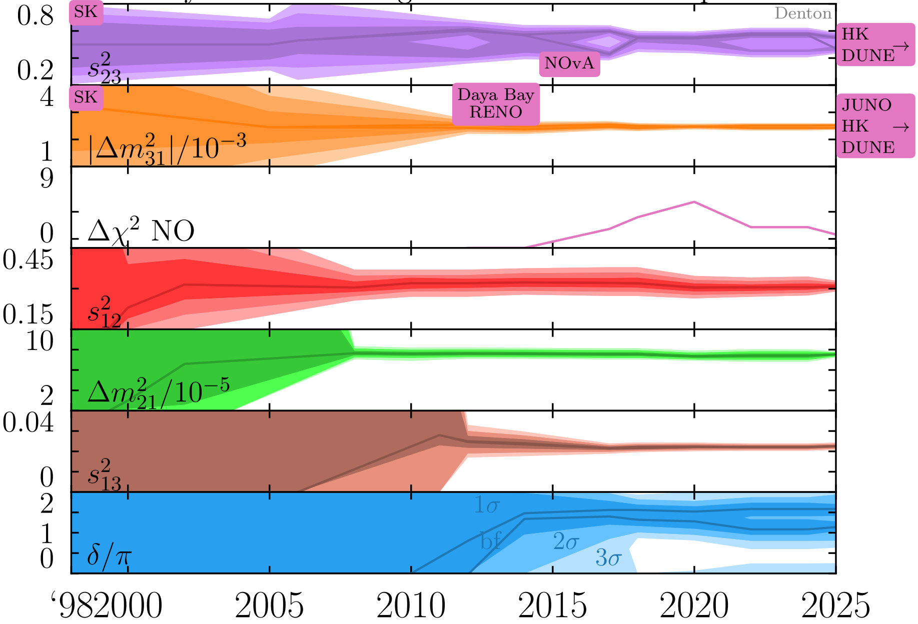

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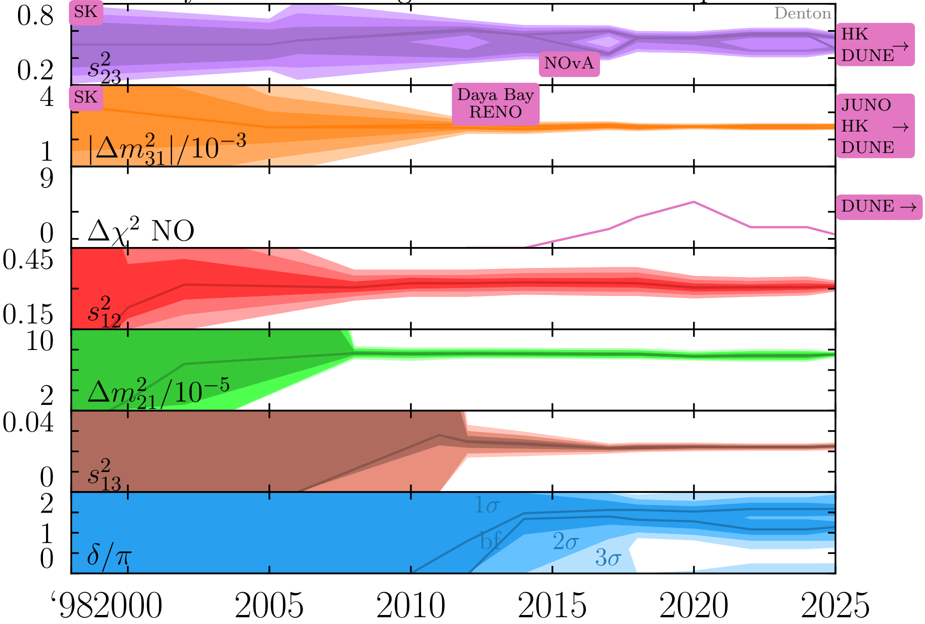

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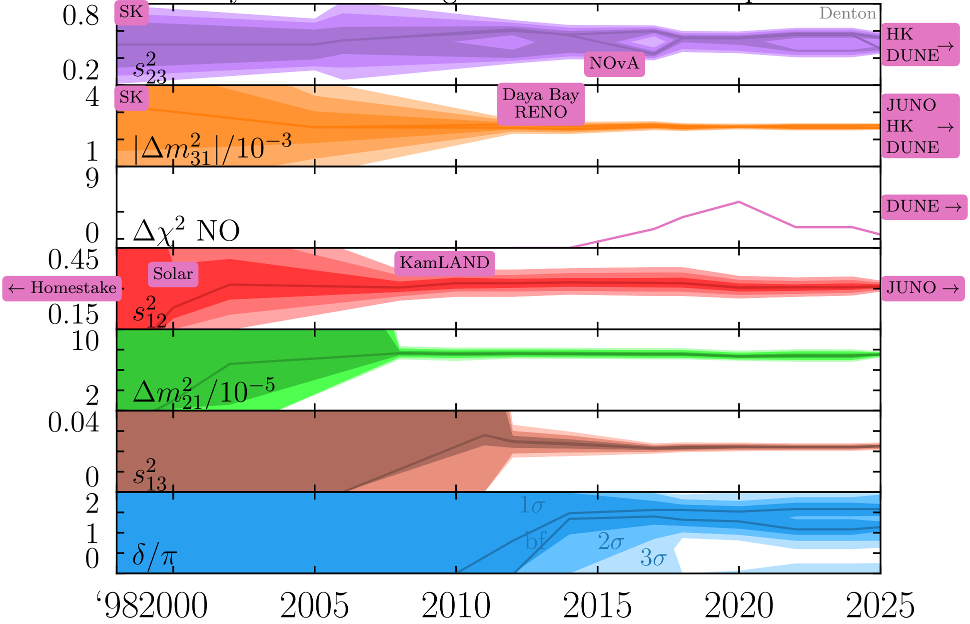

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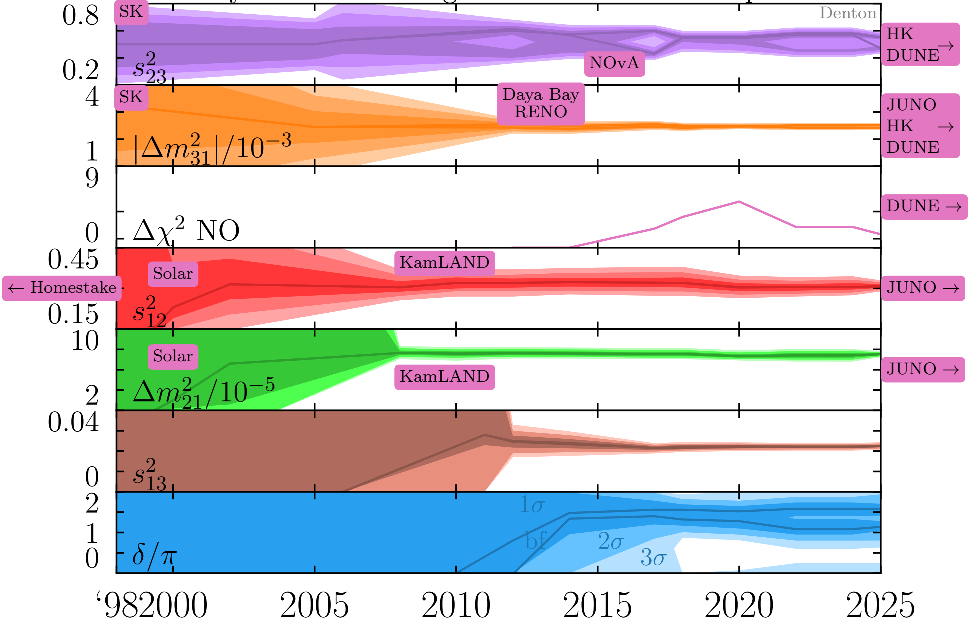

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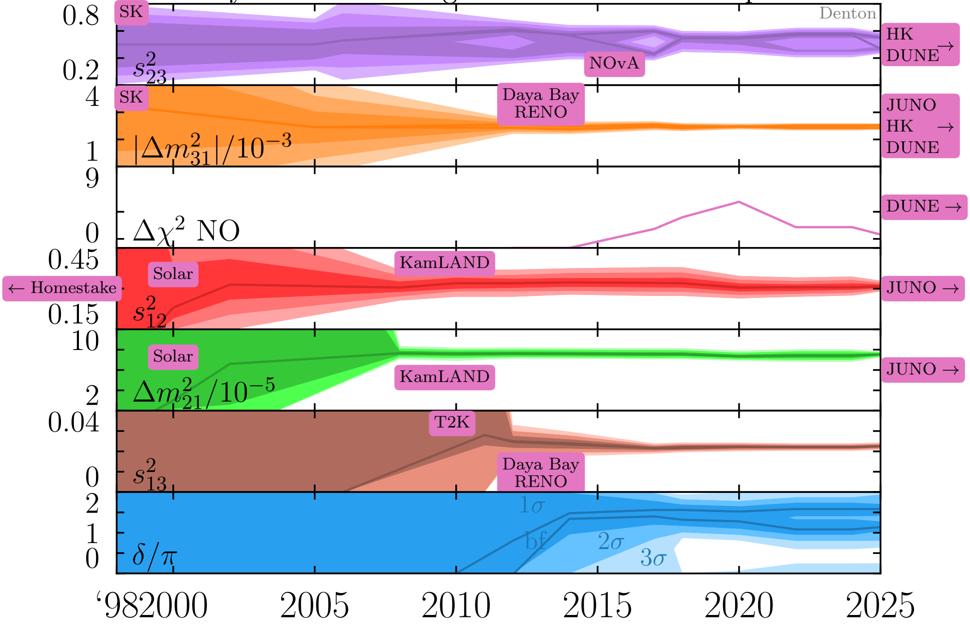
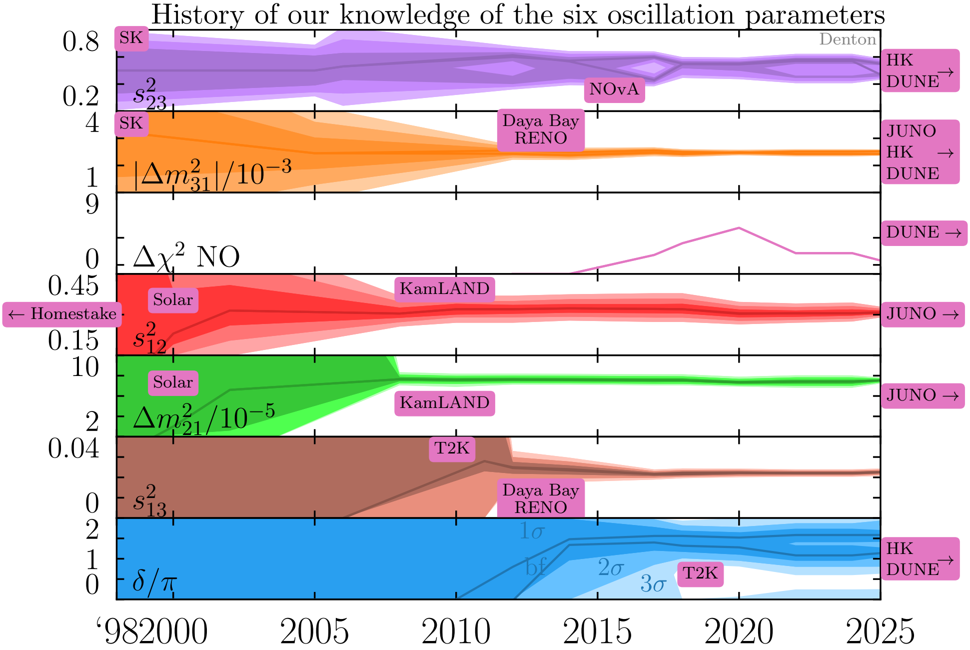
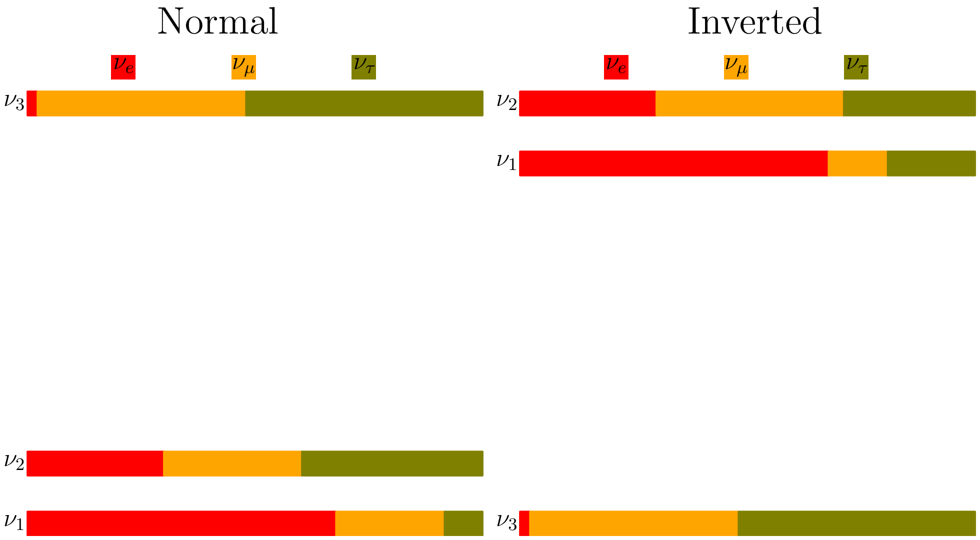


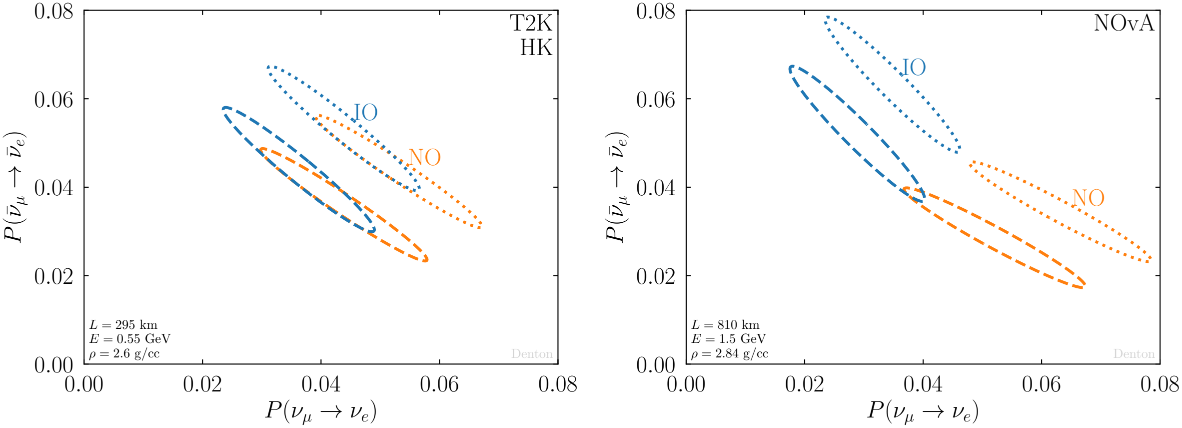

History of our knowledge of the six oscillation parameters
Denton
SK
NOvA
HK
DUNE →
s^2_{23}
0.8
0.2
SK
Daya Bay
RENO
JUNO
HK →
DUNE
$|\Delta m^2_{31}|/10^{-3}$
4
1
9
0
$\Delta\chi^2$ NO
DUNE →
0.45
0.15
Solar
KamLAND
← Homestake
JUNO →
s^2_{12}
10
2
Solar
KamLAND
JUNO →
$\Delta m^2_{21}/10^{-5}$
0.04
0
T2K
Daya Bay
RENO
s^2_{13}
2
1
0
1σ
bf
2σ
3σ
T2K
HK
DUNE →
δ/π
'98
2000
2005
2010
2015
2020
2025

Atmospheric mass ordering

Atmospheric Mass Ordering: What Is It?



Atmospheric Mass Ordering: What Is It Really?



Atmospheric Mass Ordering Current Status: Oscillations

1. NOvA and T2K both prefer NO over IO
2. NOvA+T2K prefers IO over NO
3. JUNO slightly prefers NO
4. JUNO+NOvA enhances NO preference, JUNO+T2K decreases NO preference
5. SuperK prefers NO over IO
6. NOvA+T2K+SuperK still prefers NO over IO
7. + Daya Bay, RENO, JUNO adds slight preference NO
8. $\sim 2.5\sigma$ towards NO

Depends on SuperK's statistical signficiance

Atmospheric Mass Ordering Current Status: Non-Oscillations

Cosmology: $m_1 + m_2 + m_3 < 90$ meV at 95% CL

E. Valentino, S. Gariazzo, O. Mena 2106.15267
$\rightarrow$ 20 meV precision in future

From oscillations:

$$\text{Normal}: \quad m_1 + m_2 + m_3 > 60 \text{ meV} \qquad \text{Inverted}: \quad m_1 + m_2 + m_3 > 100 \text{ meV}$$

See also KATRIN 2105.08533

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PRIORS?

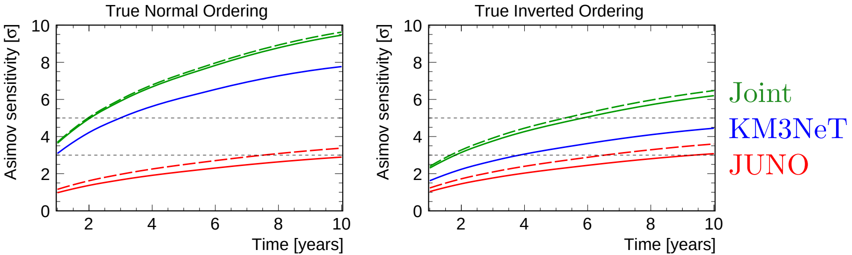
Some claim “decisive” Bayesian evidence for normal

R. Jimenez, et al. 2203.14247
R. Jimenez, et al. 26-6.18987

More general prior assumptions $\Rightarrow$ no significant information from cosmology

S. Gariazzo, et al. 1801.04946
S. Gariazzo, et al. 2205.02195

Atmospheric Mass Ordering: Future Sensitivities



JUNO, KM3NeT 2108.06293
JUNO, IceCube 1911.06745
Note: if lower octant, KM3NeT is less sensitive

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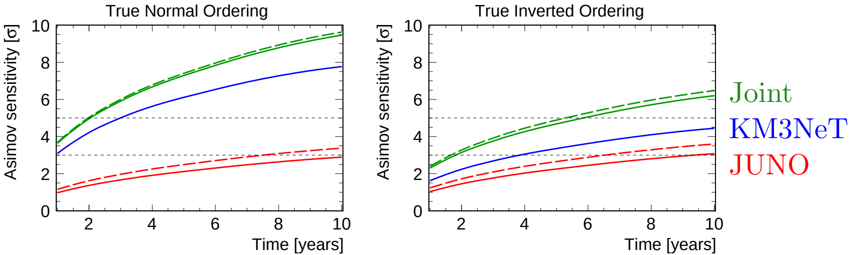

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Differ by $\pm \sim 1.1\%$ in each mass ordering

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

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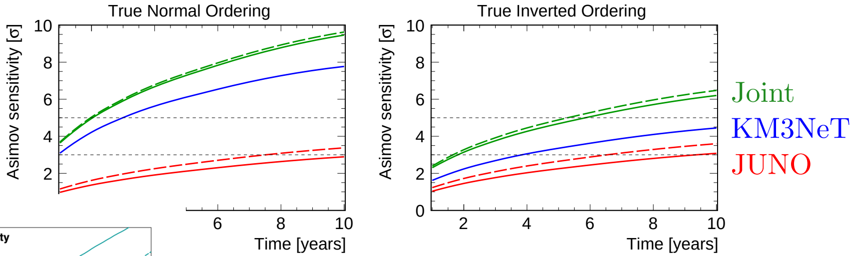

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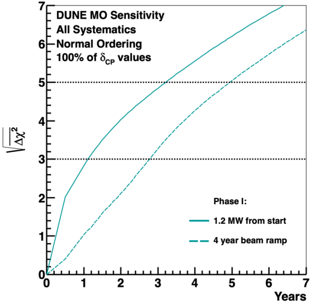
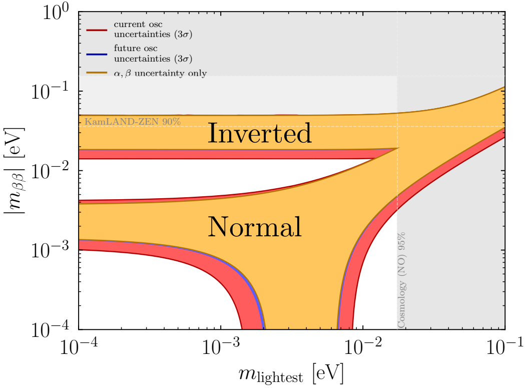


Matter effect ⇒ DUNE 2203.06100

H. Nunokawa, S. Parke, R. Funchal hep-ph/0503283

Atmospheric Mass Ordering: Broad Implications

- Affects cosmology
- Affects $0\nu\beta\beta$
- Affects end point measurements
- Affects $C\nu B$



PBD, J. Gehrlein 2308.09737

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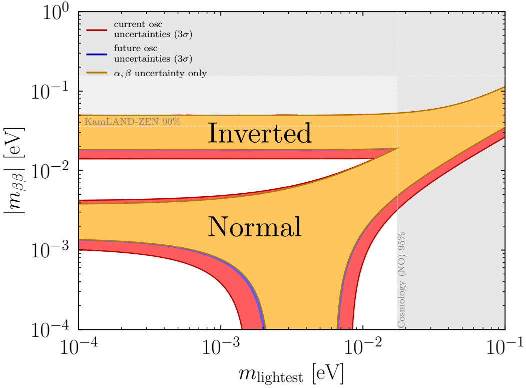

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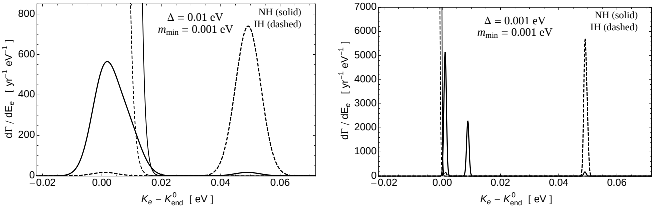
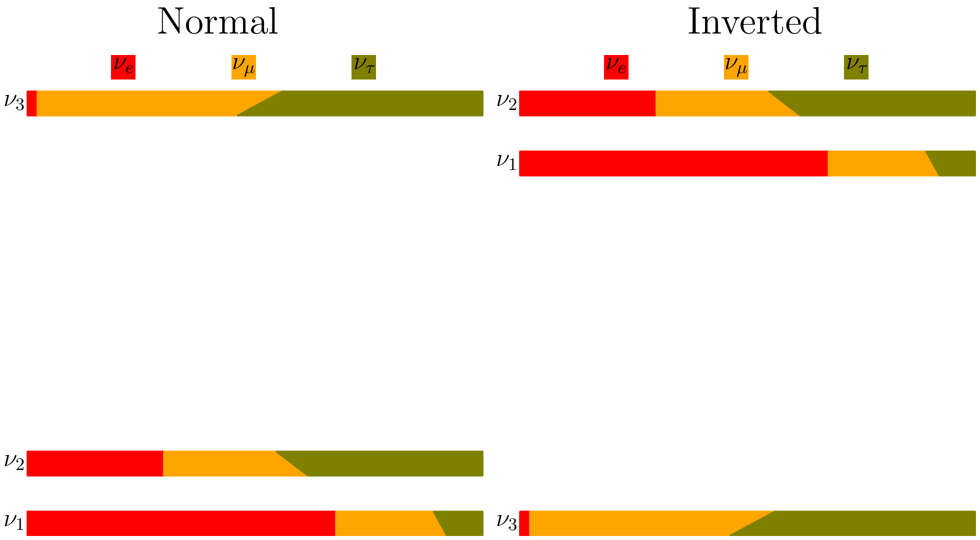


A. Long, C. Lunardini, E. Sabancilar 1405.7654

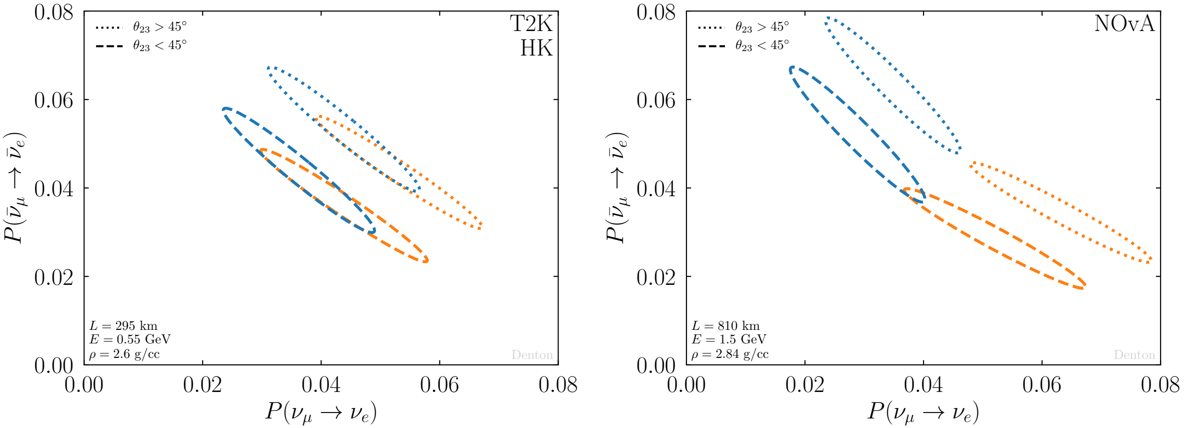
θ_{23} Octant

Not the mixing of ν_2 and ν_3
Not the mixing of ν_μ and ν_τ

θ_{23} Octant: What Is It?

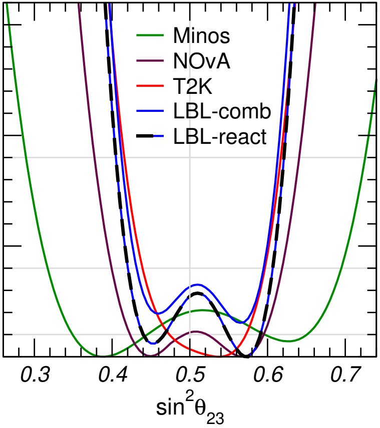


θ_{23} Octant: What Is It Really?

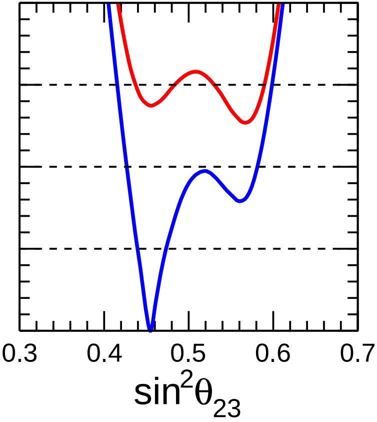


Lower octant more "normal" than upper octant

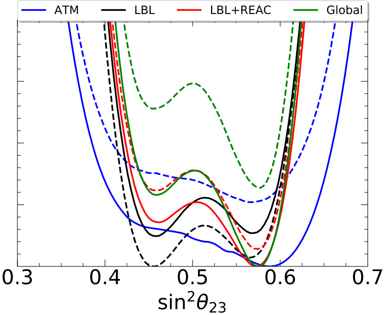
θ_{23} Octant: Status



I. Esteban, et al. 2007.14792



F. Capozzi, et al. 2107.00532



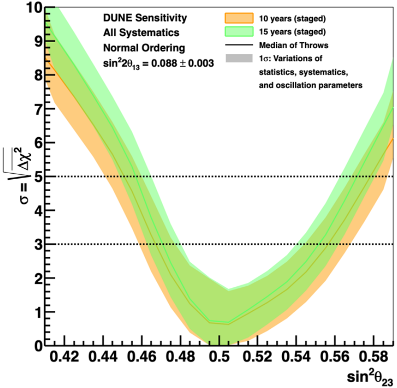
P. de Salas, et al. 2006.11237

Prefers **upper** at $< 1\sigma$

Prefers **lower** at $\sim 1.5\sigma$

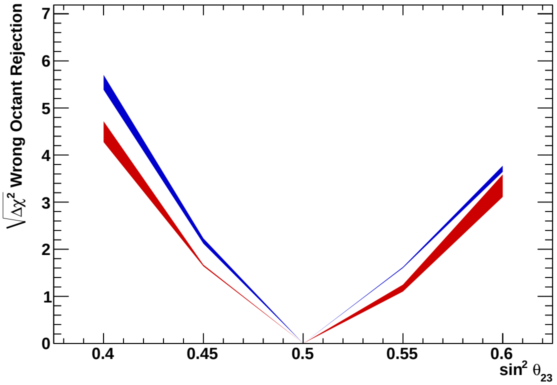
Prefers **upper** at $> 2\sigma$

θ_{23} Octant: Future Sensitivities



$\sim 3-5\sigma$

DUNE 2002.03005



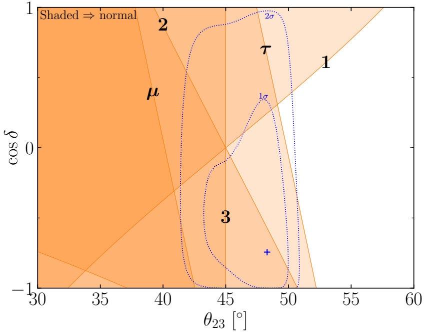
Beam+Atm$\Rightarrow\sim 3-6\sigma$

HK 1805.04163

θ_{23}: Broader Implications

Normalcy

Is the heaviest neutrino mostly ν_τ?
Is the lightest neutrino least ν_τ?



Quarks easily satisfy normalcy PBD 2003.04319

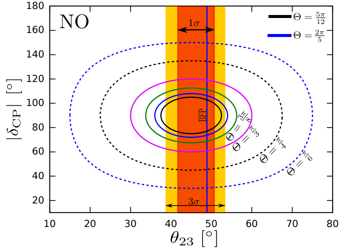
μ-τ interchange/reflection symmetry

$$\nu_\mu \leftrightarrow \nu_\tau$$

$$M_\nu^* = X M_\nu X^T \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$M_\nu \equiv U D_\nu U^\dagger$$

Predicts: $\theta_{23} = 45^\circ$, often $\theta_{13} = 0$



P. Chen, et al. 1512.01551

Parameter Interplay

The majority of the free parameters in particle physics: fermion masses and mixings

Can we predict these?

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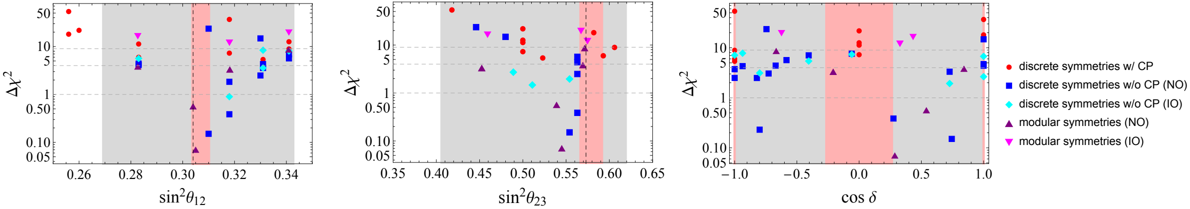
Models predict specific correlations among the parameters



J. Gehrlein, et al. 2203.06219

Complex Phase

δ and CP Violation

$$J_{CP} = s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta$$

C. Jarlskog PRL 55, 1039 (1985)

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$$\frac{\bar{\theta}}{2\pi} < 10^{-11}$$

J. Pendlebury, et al. 1509.04411

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J. Pendlebury, et al. 1509.04411

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CKMfitter 1501.05013

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CKMfitter 1501.05013
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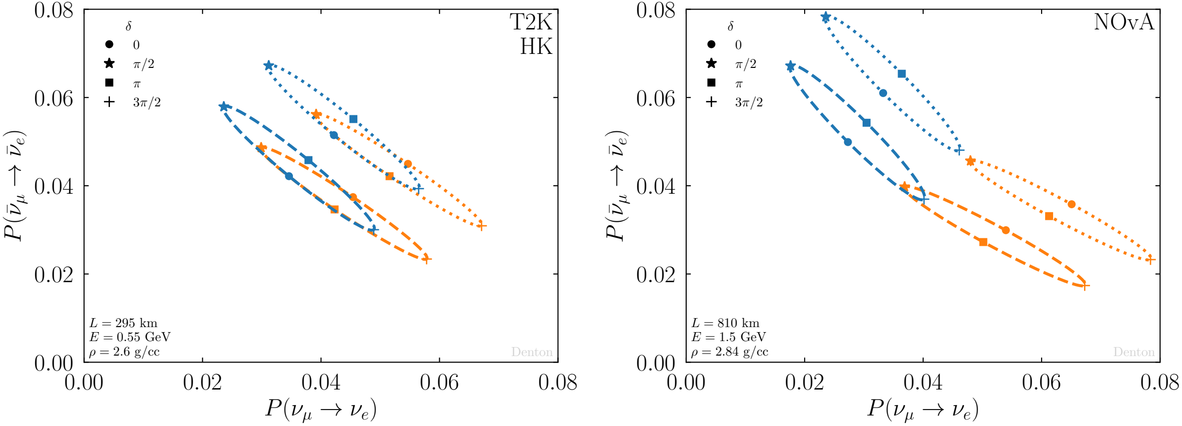
3. Lepton mass matrix: **?**

$$\frac{|J_{\rm PMNS}|}{J_{\max}} < 0.34$$

PBD, J. Gehrlein, R. Pestes 2008.01110

δ: What Is it Really?



δ: What Is It Not?

$$\delta \not\Rightarrow \text{Baryogenesis}$$

The amount of leptogenesis is a function of:

1. δ
2. the heavy mass scale
3. η_1, η_2 (Majorana phases)
4. CP phases in the RH neutrinos
5. ...

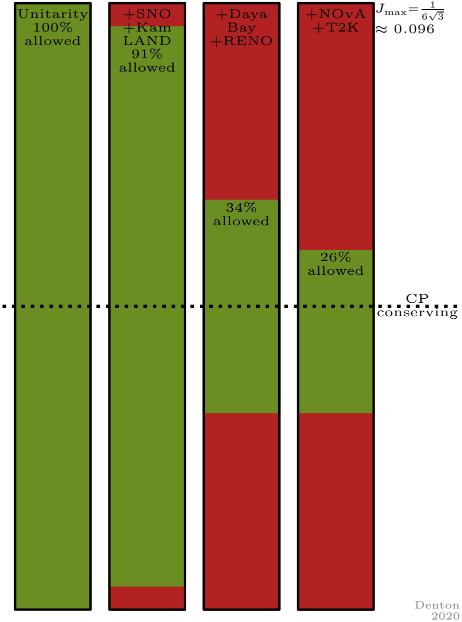
C. Hagedorn, et al. 1711.02866
K. Moffat, et al. 1809.08251

$$\text{Measuring } \delta = 0, \pi \quad \not\Rightarrow \quad \text{no leptogenesis}$$
$$\text{Measuring } \delta \neq 0, \pi \quad \not\Rightarrow \quad \text{leptogenesis}$$

δ, J: Current Status

Maximal CP violation is already ruled out:

1. $\theta_{12} \neq 45^\circ$ at $\sim 15\sigma$
2. $\theta_{13} \neq \tan^{-1}\frac{1}{\sqrt{2}} \approx 35^\circ$ at many (100) σ
3. $\theta_{23} = 45^\circ$ allowed at $\sim 1\sigma$
4. $|\sin\delta| = 1$ allowed



δ: Future Sensitivities

In vacuum at first maximum:

$$P_{\mu e} - \bar{P}_{\mu e} \approx 8\pi J \frac{\Delta m^2_{21}}{\Delta m^2_{32}}$$

$$J \equiv s_{12} c_{12} s_{13} c^2_{13} s_{23} c_{23} \sin\delta$$

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Matter effects are easily accounted for: PBD, S. Parke 1902.07185

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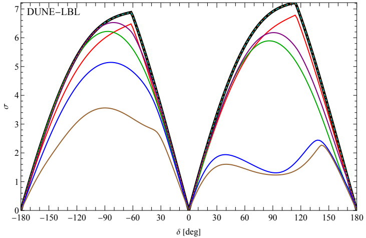
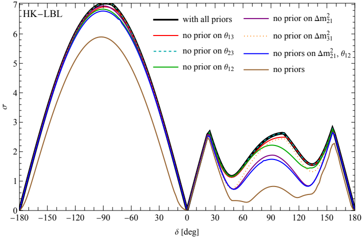
DUNE and HK will make great measurements via appearance $\nu_\mu \to \nu_e$

$\nu+\bar{\nu}$ helps systematics but isn't strictly necessary





PBD, J. Gehrlein 2302.08513

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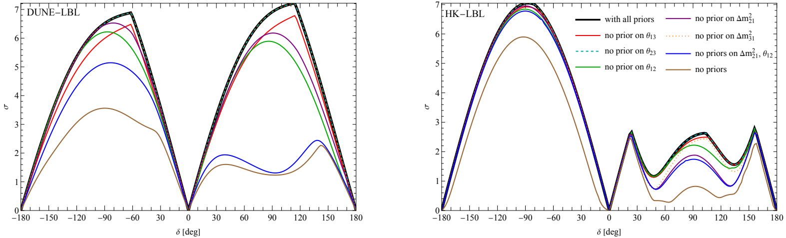

PBD, J. Gehrlein 2302.08513

Need to know solar parameters to measure δ!

δ: Future Sensitivities

Appearance is best
Disappearance works too

PBD 2309.03262

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Disappearance probability:

$$P(\nu_\alpha \to \nu_\alpha) = 1 - 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2 \Delta_{21} - 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{31} - 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2 \Delta_{32}\,,$$

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PBD 2309.03262

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PBD 2309.03262

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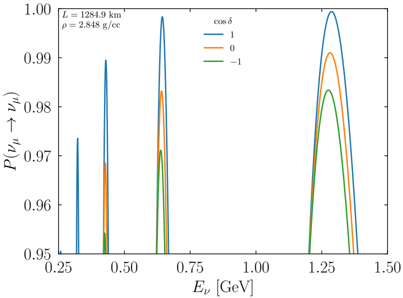

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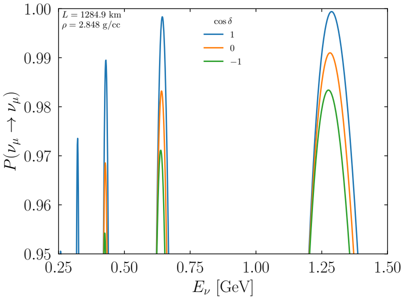

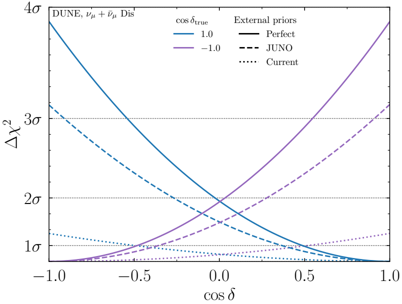

PBD 2309.03262

More new physics?

Neutrino oscillations are BSM but no longer NP;
NP is more BSM beyond neutrino oscillations

Lots of Interesting New Physics Scenarios in Oscillations

1. Sterile neutrinos
2. Non-standard neutrino interactions (NSI)
   with any Lorentz structure: SPVAT
3. Non-standard neutrino SELF interactions
4. Neutrino decay
   with visible or invisible final states
5. Unitarity violation
6. Neutrino – dark matter interactions
7. Decoherence
8. Lorentz invariance or CPT violation

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1. Sterile neutrinos
   PBD, Y. Farzan, I. Shoemaker 1811.01310
   PBD 2111.05793
2. Non-standard neutrino interactions (NSI)
   with any Lorentz structure: SPVAT
   PBD, J. Gehrlein, R. Pestes 2008.01110
   P. Coloma, PBD, M. Gonzalez-Garcia, M. Maltoni, T. Schwetz 1701.04828
   PBD, J. Gehrlein 2008.06062, 2204.09060
   PBD, A. Giarnetti, D. Meloni 2210.00109
3. Non-standard neutrino SELF interactions G. Barenboim, PBD, I. Oldengott 1903.02036
4. Neutrino decay
   with visible or invisible final states
   PBD, I. Tamborra 1805.05950
   PBD, A. Abdullahi 2005.07200
5. Unitarity violation
   PBD 2109.14576
   PBD, J. Gehrlein 2109.14575
6. Neutrino – dark matter interactions
   A. Dev, et al. 2205.06821
   C. Boehm, P. Fayet, R. Schaeffer astro-ph/0012504
7. Decoherence
   T. Stuttard, M. Jensen 2007.00068
   A. Gouvêa, V. Romeri, C. Ternes 2104.05806
8. Lorentz invariance or CPT violation
   S. Ge, H. Murayama 1904.02518

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   - MicroBooNE (2015-2021): built to test MiniBooNE: much more sophisticated detector; no excess of ν_e in ν_μ beam

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3. New flavor state must be sterile: RH, a SM gauge singlet
4. Can oscillate into it depending on Δm^2_{41}, 3 θ_{i4}, and 2 new CPV phases

   ...for 1 new sterile
5. Historical anomalies indicating new frequency $\Delta m^2_{41} \sim 0.1 - 10$ eV^2:
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   - MiniBooNE (2002-2019): built to test LSND, different L, E; similar L/E: $> 4\sigma$ for $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$
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   - Reactor neutrino deficit from theory (RAA), at Daya Bay, ...; could be explained by fuel evolution test
   - Gallium: low-energy solar ν experiments, deficit in callibration, now $> 5\sigma$ $\nu_e \to \nu_e$

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More complicated oscillations scenarios (3+2, 3+3) don't help

Sterile Neutrino Oscillation Formalism

$$U_4 = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}$$

6 angles, 3 complex phases (plus Majorana phases)

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$$\left.+\begin{pmatrix}2\sqrt{2}G_FN_eE-\sqrt{2}G_FN_n&&&\\&-\sqrt{2}G_FN_n&&\\&&-\sqrt{2}G_FN_n&\\&&&0\end{pmatrix}\right]$$

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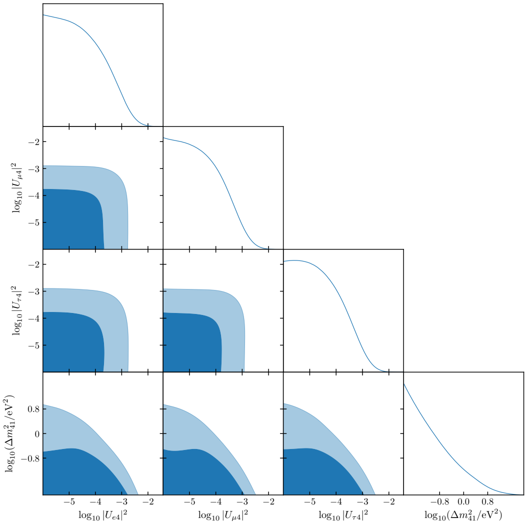
Cosmological Bounds



1σ, 2σ S. Hagstotz, et al. 2003.02289

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- No local H_0 constraint
- Bounds independent of flavor
- To be consistent with data must have small mixing **and** small mass

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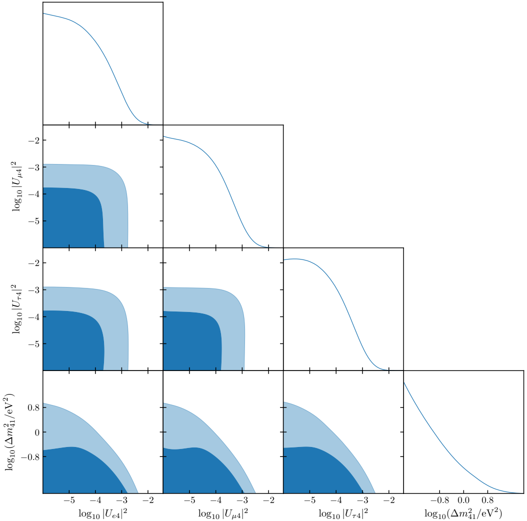

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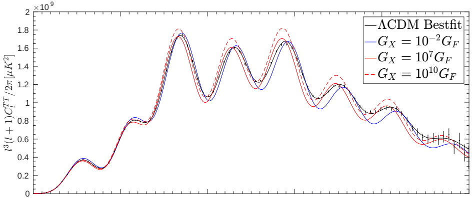
- To be consistent with data must have small mixing **and** small mass
- Much more than just $N_{\rm eff}$ and $\sum m_\nu$
- Just adding a new interaction is not straightforward



N. Song, M. Gonzalez-Garcia, J. Salvado 1805.08218

Gallium Experiments

- Low energy solar neutrino experiments measure the *pp* flux
  - Consistent with KamLAND and JUNO

SAGE 0901.2200
GALLEX 1001.2731

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3.0σ: C. Giunti, M. Laveder 1006.3244
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$> 4\sigma$: BEST 2109.11482

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C. Giunti, et al. 2212.09722
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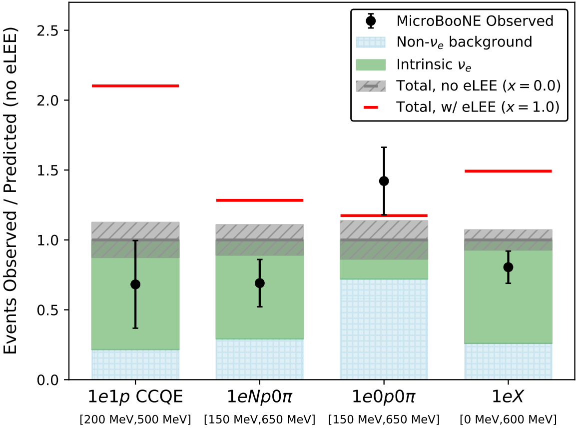
- Prefers:
  - $\Delta m^2_{41} \gtrsim 0.5$ eV^2
  - $\sin^2 2\theta_{ee} = 4|U_{e4}|^2(1-|U_{e4}|^2) \sim 0.4$

MicroBooNE



- Three analysis teams:
  1. Wire-Cell
  2. Deep Learning
  3. Pandora
     - With 0 protons
     - With 1+ protons
- Underfluctuation compared to no-oscillations
- Disfavors MiniBooNE's best fit LEE hypothesis at 3.75σ

MicroBooNE 2110.14054

Current Status of eV Scale Sterile Neutrinos

1. Not a mass range theoretically motivated
2. Gallium seems to be something, no obvious non-sterile explanation
3. Appearance hints in tension with other appearance measurements, ν_e and ν_μ disappearance
4. Cosmology generally disfavors the whole relevant parameter space, although care is required given H_0 and DESI tensions

Two new physics scenarios:

Light sterile neutrinos (~ 1 eV)

Non-standard neutrino interactions (NSI)

Non-Standard Neutrino Interactions Overview

Neutrino mass generation often includes new interactions
Pheno of scattering and oscillations are very different
Connect them in a joint effective field theory framework: NSI

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L. Wolfenstein PRD 17 (1978)

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Affects oscillations via new matter effect

$$H = \frac{1}{2E}\left[UM^2U^\dagger + a\begin{pmatrix} 1+\epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}\right]$$

Matter potential $a \propto G_F \rho E$

B. Dev, K. Babu, PBD, P. Machado, et al. 1907.00991

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Models with large NSIs consistent with CLFV:

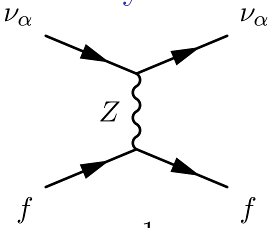
Y. Farzan, I. Shoemaker 1512.09147 Y. Farzan, J. Heeck 1607.07616 D. Forero and W. Huang 1608.04719
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822 PBD, Y. Farzan, I. Shoemaker 1804.03660
U. Dey, N. Nath, S. Sadhukhan 1804.05808 Y. Farzan 1912.09408

NSI Behavior

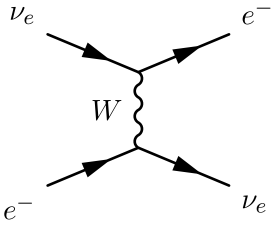
- Vector interaction ⇒ modifies energy
- Can have over Lorentz structures; P, A don't affect oscillations
- Can have NC or CC: both affect oscillations, CC affects production/detection

  CC subject to tight constraints from e.g. μ, $\pi^\pm$ decays
- Can have different couplings to e, u, and d
- Can lead to degeneracies
- Ultimately there must be a mediator; mediator dependence affects scattering constraints

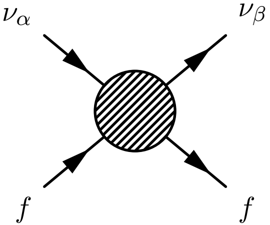
Matter Effects in Feynman Diagrams



$$V_{\rm NC} = \mp\frac{1}{2}\sqrt{2}G_F n_n$$

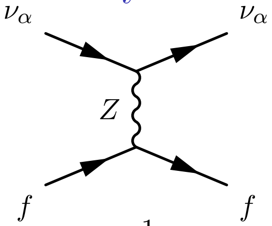


$$V_{\rm CC} = \pm\sqrt{2}G_F n_e$$



$$V_{\rm NSI} = \pm\epsilon_{\alpha\beta}^{f,X}\sqrt{2}G_F n_f$$

Matter Effects in Feynman Diagrams



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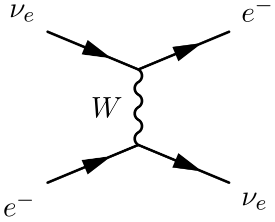

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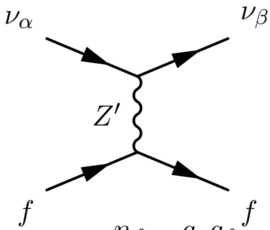

$$V_{\rm NSI} = \frac{n_f}{2}\frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

NSI Behavior

- $\epsilon_{ee}-\epsilon_{\mu\mu}$: Modifies the matter effect, only constrained by solar+LBL reactor

  Can subtract a diagonal part; convention is often $\epsilon_{\mu\mu}$
  DUNE will have modest sensitivity to this as well
  Degeneracy?

- $\epsilon_{\tau\tau}-\epsilon_{\mu\mu}$, $\epsilon_{\mu\tau}$: Strongly constrained by high energy $\sim$ TeV atmospherics
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: hard to constrain: LBL, atmospherics, need appearance

Mass Ordering: New Physics Degeneracies

In the presence of new physics such as NSI we have:

$$[\text{NO}] + [\epsilon = 0] \quad \equiv \quad [\text{IO}] + [\epsilon_{ee} = -2]$$

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Equivalences hold even if all oscillation probabilities are *perfectly* measured

P. Bakhti, Y. Farzan 1403.0744
P. Coloma, T. Schwetz 1604.05772
PBD, S. Parke 2106.12436
PBD, J. Gehrlein 2204.09060

This is known as the **LMA-Dark** solution

Is The Mass Ordering Robust?

Need **scattering** to break

Can probe same NC $\epsilon = -2$ process in scattering, but...

1. CHARM and NuTeV for $M_{Z'} \gtrsim 10$ GeV

   PBD, et al. 1701.04828

2. COHERENT for $M_{Z'} \gtrsim 50$ MeV and cosmology for $M_{Z'} \lesssim 5$ MeV

   PBD, Y. Farzan, I. Shoemaker 1804.03660

3. Dresden-II for any mediator mass

   PBD, J. Gehrlein 2204.09060

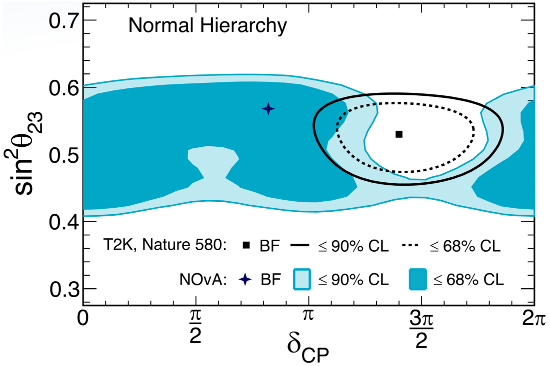
4. Can still evade with specific flavor structures

   $\epsilon_{\mu\mu} = \epsilon_{\tau\tau} = 2$ or certain u / d combinations

5. CCM & COHERENT can close all loopholes

CP Violation at NOvA and T2K?

Example of NSI in data:



A. Himmel for NOvA 10.5281/zenodo.3959581

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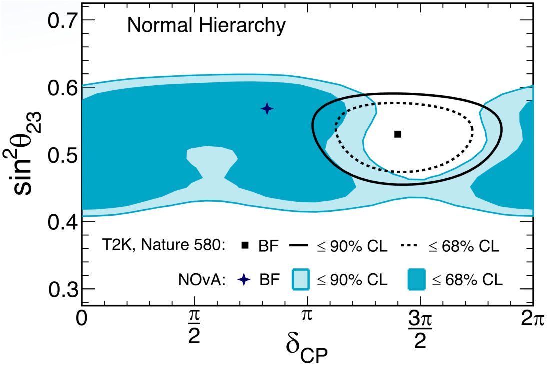

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Difference could be due to off-diagonal (flavor changing) CP violating NSI
NSI effect is large for NOvA, modest for T2K

Estimate size of effect: magnitude

$$|\epsilon_{e\beta}| \approx \frac{s_{12}c_{12}c_{23}\pi\Delta m^2_{21}}{2s_{23}w_\beta}\left|\frac{\sin\delta_{\rm T2K}-\sin\delta_{\rm NOvA}}{a_{\rm NOvA}-a_{\rm T2K}}\right| \approx \begin{cases}0.22 & \text{for } \beta=\mu\\ 0.24 & \text{for } \beta=\tau\end{cases}$$

$a \propto \rho E$

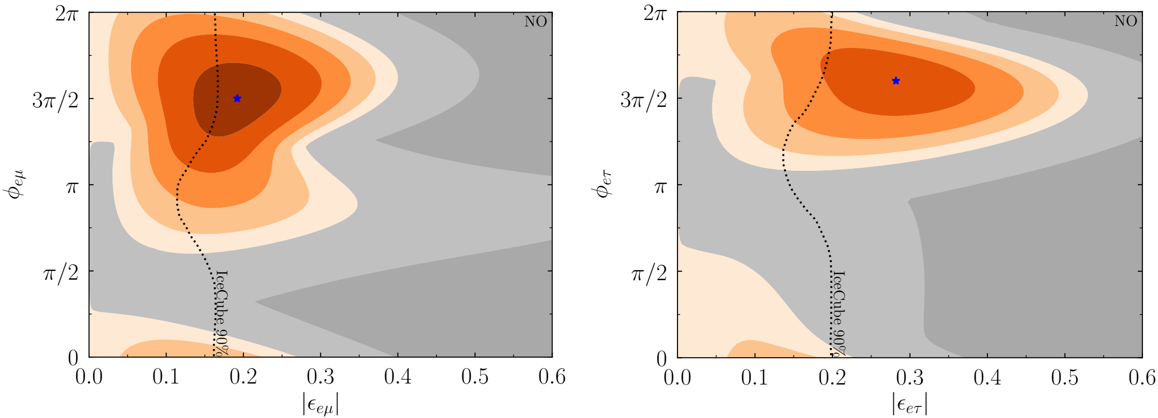
$w_\beta = s_{23},\ c_{23}$ for $\beta=\mu,\tau$

Assumed upper octant $\theta_{23} > 45^\circ$

Consistency checks:

- $\sin\delta_{\rm NOvA} = \sin\delta_{\rm T2K} \Rightarrow |\epsilon| = 0$
- $\sin\delta_{\rm NOvA} \neq \sin\delta_{\rm T2K}$ and $a_{\rm NOvA} = a_{\rm T2K} \Rightarrow |\epsilon| \to \infty$
- Octant:
  1. LBL is governed by ν_3
  2. Upper octant $\Rightarrow \nu_3$ is more ν_μ
  3. More $\nu_\mu \Rightarrow$ need less new physics coupling to ν_μ to produce a given effect

NSI parameters



Orange is preferred over SM at integer values of $\Delta\chi^2$, dark gray is disfavored at 4.61

T. Ehrhardt, IceCube PPNT (2019)

$\epsilon_{\mu\tau}$, IO in backups

Other CP violating NSI constraints

NSI effects grow with energy, density, and distance

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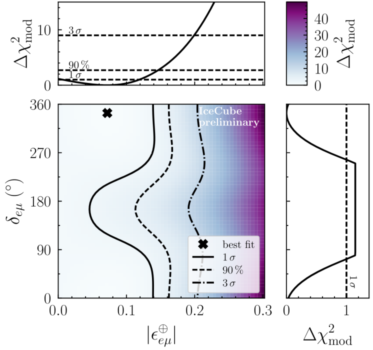
Best probes:

- $\epsilon_{\mu\tau}$: atmospheric
- $\epsilon_{e\mu}$, $\epsilon_{e\tau}$: LBL appearance, atmospheric

- IceCube
  - Constraint is at LBL best fit with 3 yrs

    10 yrs of data in the bank
  - Prefers non-zero $|\epsilon_{e\mu}|$ at $\sim 1\sigma$



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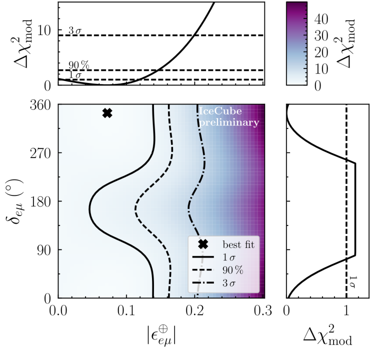


T. Ehrhardt, IceCube PPNT (2019)

- Super-K
  - Only consider real NSI
  - Comparable sensitivity as IceCube

Super-K 1109.1889

- COHERENT
  - Only applies to NSI models with $M_{Z'} \gtrsim 10$ MeV
  - NSI u, d, e configuration matters
  - Comparable constraints

COHERENT 1708.01294

PBD, Y. Farzan, I. Shoemaker 1804.03660

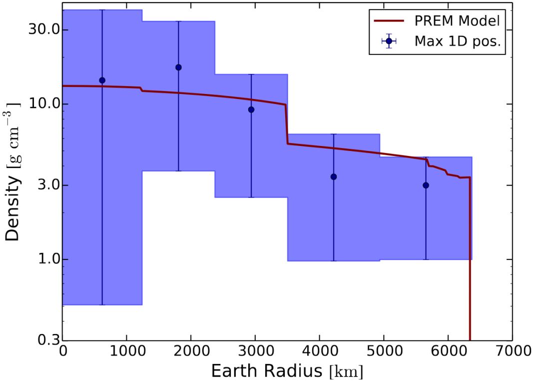
PBD, J. Gehrlein 2008.06062

Neutrinos probe opaque environments

Terrestrial Tomography

HE neutrinos are absorbed in the Earth

Assumes isotropic flux



A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

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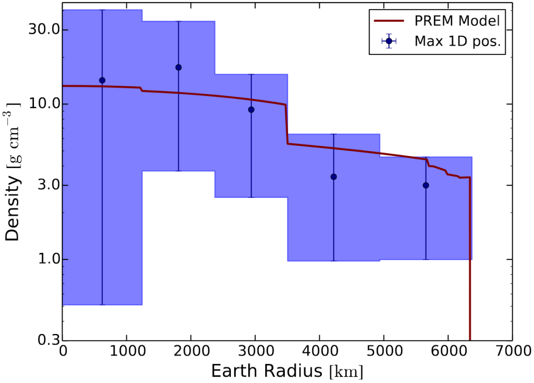
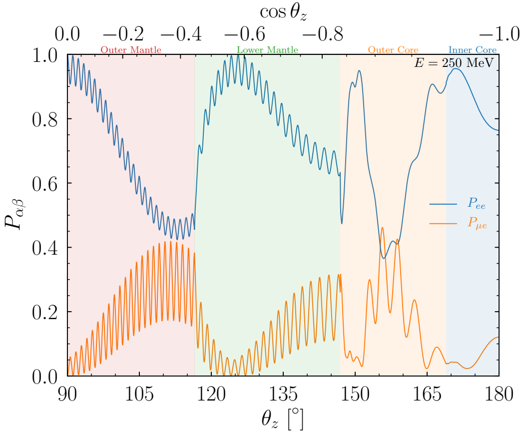


A. Donini, S. Palomares-Ruiz, J. Salvado 1803.05901

Can also do this with oscillations

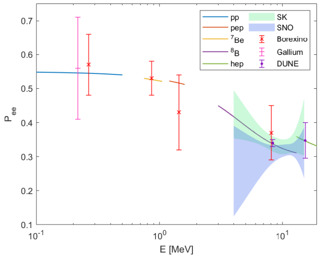


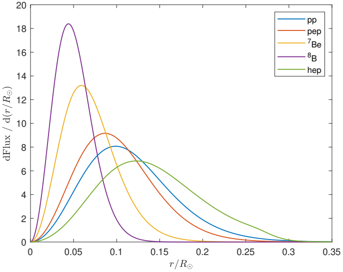
PBD, R. Pestes 2110.01148 (PRD)

DUNE-atm can measure $r^{\oplus}_{\rm core}$ to 9%

See also K. Kelly, et al. 2110.00003

Solar Tomography

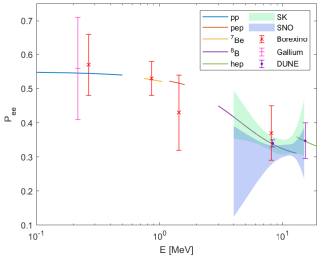


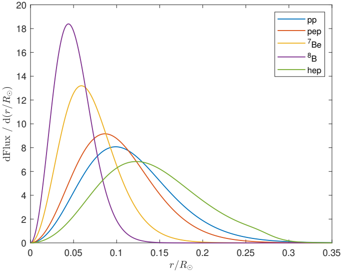


Solar neutrinos constrain Sun's density at different radii

PBD, C. Gourley 2502.17546

Solar Tomography

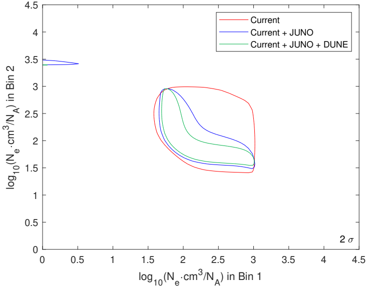




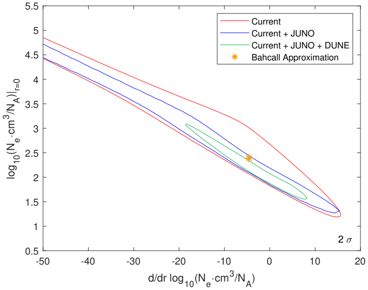
Solar neutrinos constrain Sun's density at different radii

Bin 1: $\frac{r}{R_\odot} \in [0, 0.05]$

Bin 2: $\frac{r}{R_\odot} \in [0.05, 0.1]$



$$\log_{10} N_e = Ar + B$$



PBD, C. Gourley 2502.17546

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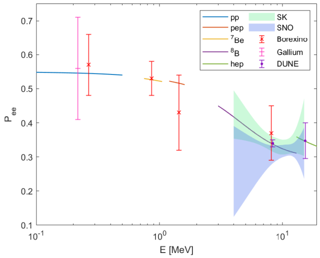

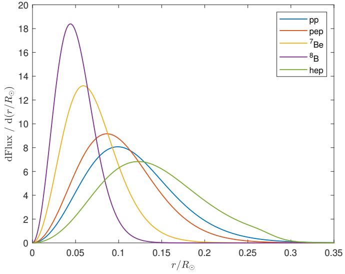

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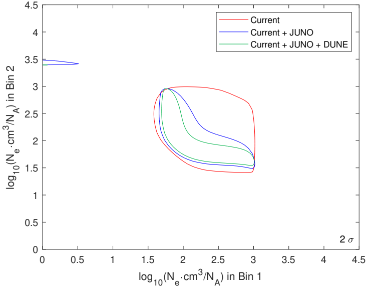
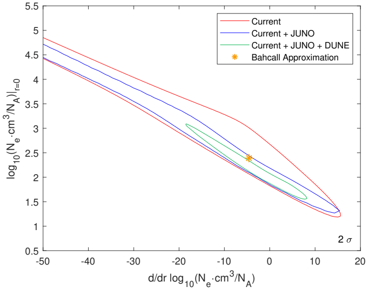


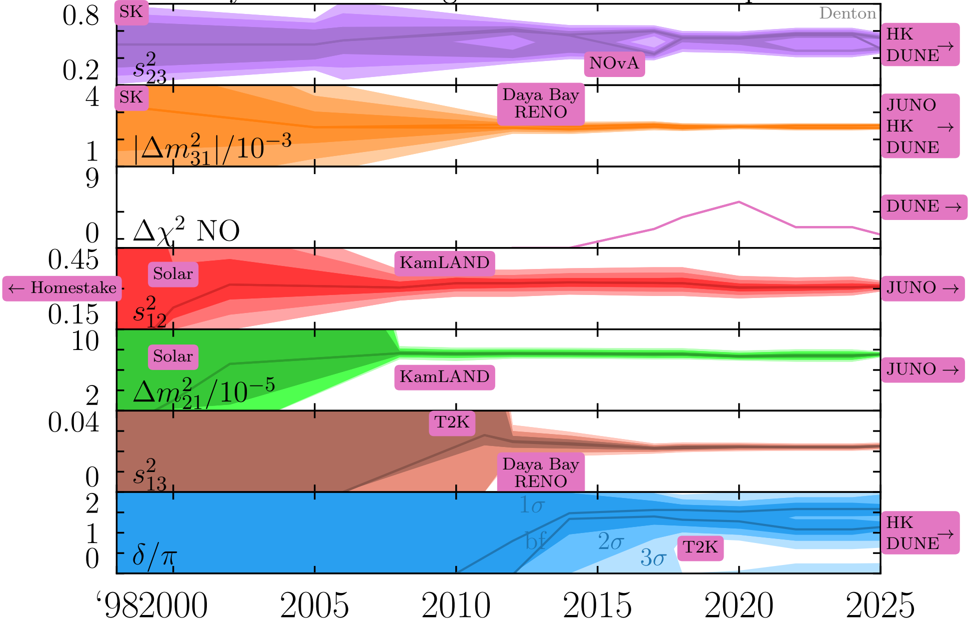


First determination of Sun's density with neutrinos

Only way to probe innermost 5% of the Sun

PBD, C. Gourley 2502.17546

History of our knowledge of the six oscillation parameters



Discussion time!

Backups

Let's do a direct kinematic search

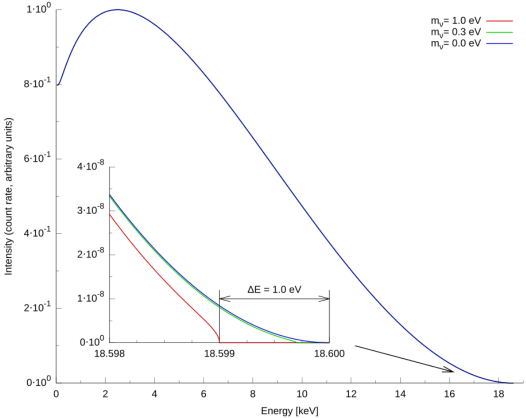
KATRIN 2006

$$^3_1H \to ^3_2 He + e^- + \bar{\nu}_e$$

For massless neutrinos,

what is the maximum electron energy?

Neutrino Masses: Kinematic End Point is Hard



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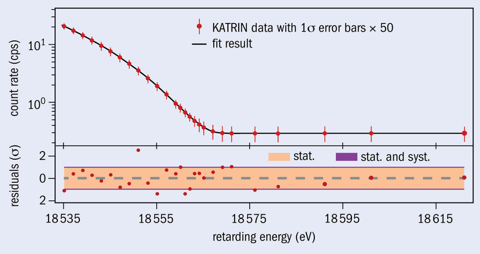

KATRIN 2018

$$m_\nu \lesssim 1 \text{ eV}$$

Maximal Mixing: Atmospheric Neutrinos

Mixing for atmospheric angles seems to be maximal $\theta_{23} \sim 45°$

	θ_{23}	θ_{13}	θ_{12}	δ
Quarks	$2.4°$	$0.20°$	$13°$	$69°$
Leptons	$\sim 45°$	X	X	unknown

Was an expectation that mixing angles should be small

Other atmospheric experiments had hints for oscillations, didn't frame it since "mixing angles should be small"

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Two large angles
Surely θ_{13} will be small?!

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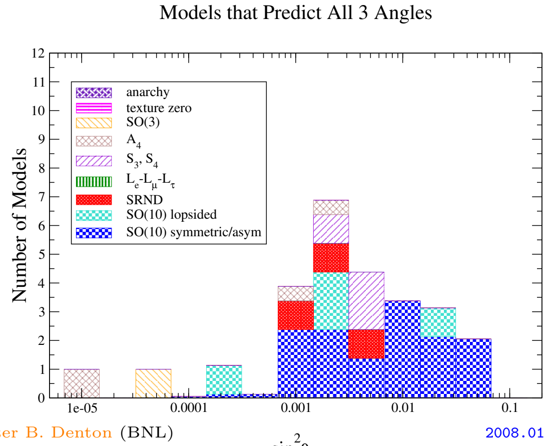

	θ_{23}	θ_{13}	θ_{12}	δ
Quarks	2.4°	0.20°	13°	69°
Leptons	$\sim 45^\circ$	8.5°	33°	unknown

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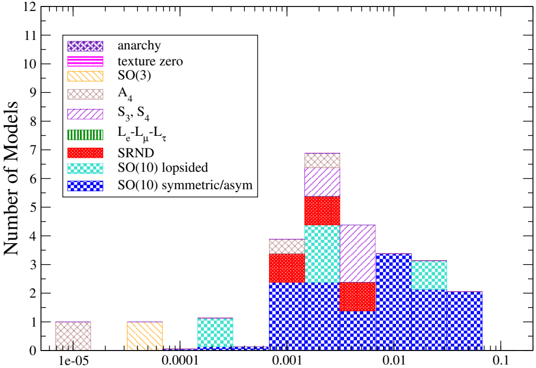
Models that Predict All 3 Angles



True value:

$\sin^2\theta_{13} = 0.02$, $\theta_{13} = 8.5^\circ$

Quite large!

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   - Disappearance can also probe this — PBD 2309.03262

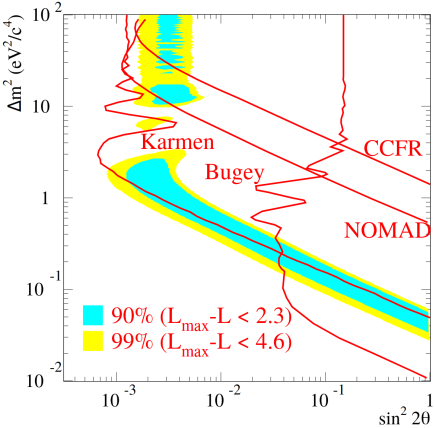
LSND Sees a ~ 1 eV Sterile?

LSND at Los Alamos:

1. $\bar{\nu}_\mu$ from μ^+ decay-at-rest
2. Saw an excess of $\bar{\nu}_e$ events: $87.9 \pm 22.4 \pm 6.0$

3.8σ

LSND hep-ex/0104049



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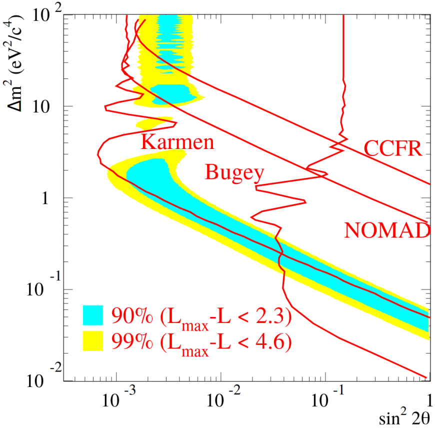

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LSND hep-ex/0104049

Could be a cut problem:
J. Hill hep-ex/9504009

MiniBooNE Results

MiniBooNE 1805.12028

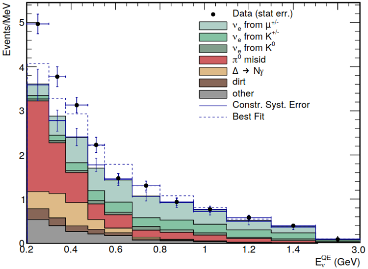

FIG. 1: The MiniBooNE neutrino mode E_ν^{QE} distributions, corresponding to the total 12.84×10^{20} POT data, for ν_e CCQE data (points with statistical errors) and background (histogram with systematic errors). The dashed curve shows the best fit to the neutrino-mode data assuming two-neutrino oscillations. The last bin is for the energy interval from 1500-3000 MeV.

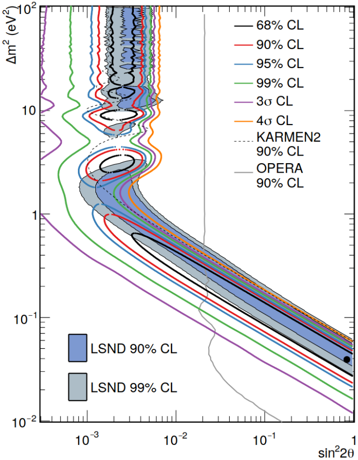

FIG. 3: MiniBooNE allowed regions in neutrino mode (12.84×10^{20} POT) for events with $200 < E_\nu^{QE} < 3000$ MeV within a two-neutrino oscillation model. The shaded areas show the 90% and 99% C.L. LSND $\bar{\nu}_\mu \to \bar{\nu}_e$ allowed regions. The black point shows the MiniBooNE best fit point. Also shown are 90% C.L. limits from the KARMEN [37] and OPERA [38] experiments.

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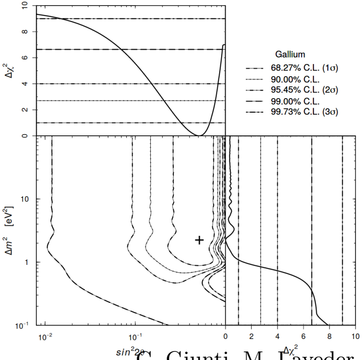

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C. Giunti, M. Laveder 1006.3244

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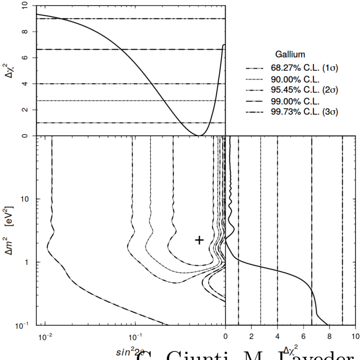

C. Giunti, M. Laveder 1006.3244

3. 3σ deficit consistent with fast oscillations
4. Using improved nuclear shell models: $3.0\sigma \to 2.3\sigma$

J. Kostensalo, et al. 1906.10980

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- Near detectors $\mathcal{O}(100)$ m from cores, expect no oscillations

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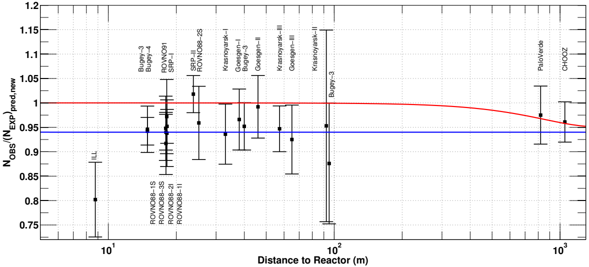

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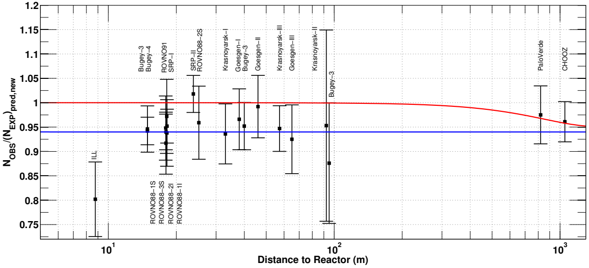

- Deficit compared to theory
  $\Rightarrow \Delta m^2_{41} \gtrsim 1.5$ eV^2 $\sin^2 2\theta_{14} \sim 0.14$

G. Mention, et al. 1101.2755

Reactor Anti-neutrino Anomaly: Fuel Evolution

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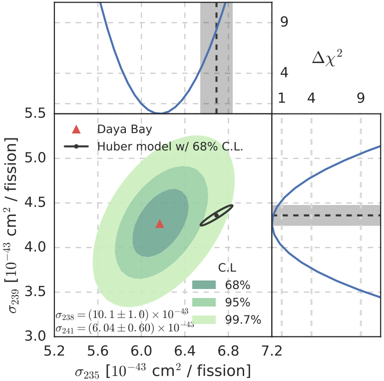
- Data indicates the deficit does evolve with flux



Daya Bay 1704.01082

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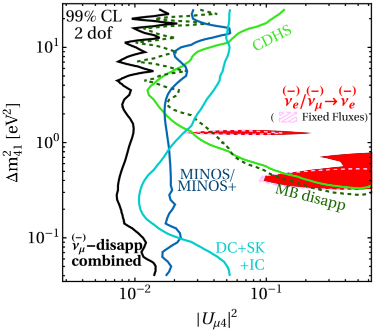

M. Dentler, et al. 1803.10661

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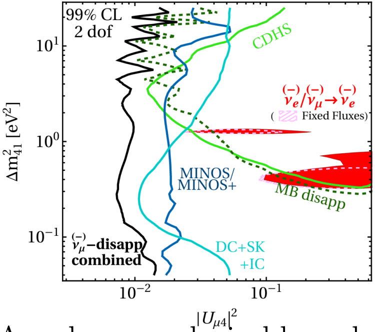

M. Dentler, et al. 1803.10661

- Are also cosmological bounds

Other Anomalies

- ANITA
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ANITA 1803.05088

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E. Bulbul, et al. 1402.2301 & A. Boyarsky, et al. 1402.4119

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PBD, I. Tamborra 1805.05950

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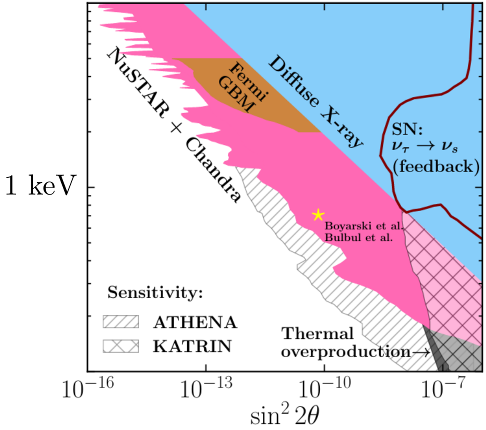
  - IceCube measures tracks (ν_μ) and cascades (ν_e, ν_τ)
  - At $> 3\sigma$ neutrino decay is preferred
- NOvA and T2K slightly disagree PBD, I. Tamborra 1805.05950
  - Flavor changing CP violating non-standard interactions
  - Model preference is slight $\sim 2\sigma$
  - Testable at IceCube and COHERENT PBD, J. Gehrlein, R. Pestes 2008.01110

Steriles: keV

- keV sterile neutrinos can be DM
- Would be a bit high in temperature
- A possible hint of their existence at 7 keV
- Would also affect SNe



A. Suliga, I. Tamborra, M. Wu 2004.11389

Steriles: GeV+

If they are heavy they won't affect oscillations, just kinematics

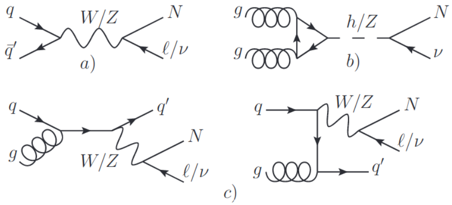
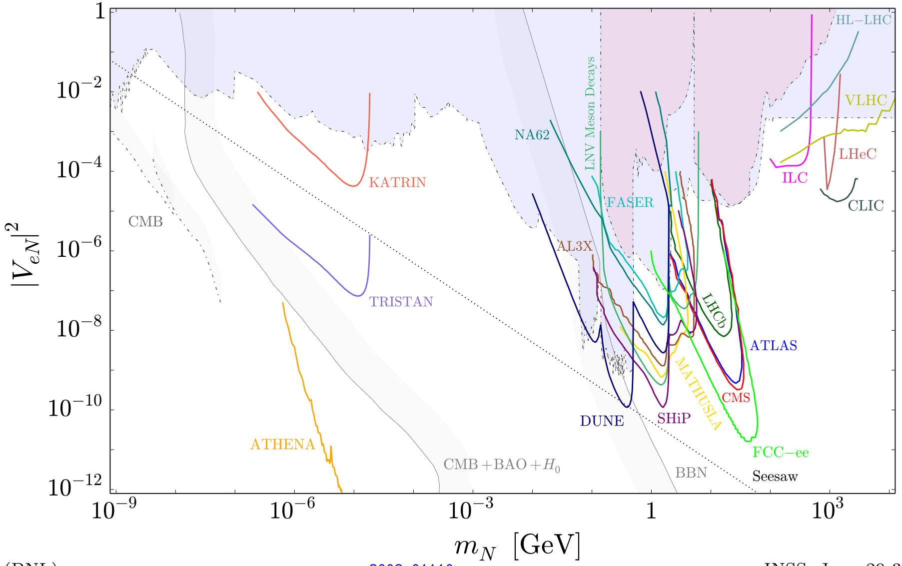


Figure 7. HNL production channels: a) Drell-Yan-type process; b) gluon fusion; c) quark-gluon fusion.

K. Bondarenko, et al. 1805.08567

- Look in colliders, beam dumps
- Battle between energy and intensity

Sterile Neutrinos: Where are they Hiding?



NSI at the Hamiltonian Level

$$H^{\text{vac}} = \frac{1}{2E} U \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} U^\dagger$$

$$H^{\text{mat,SM}} = \frac{a}{2E} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}$$

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$$H^{\text{mat,NSI}} = \frac{a}{2E} \begin{pmatrix} \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon^*_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}$$

$$H = H^{\text{vac}} + H^{\text{mat,SM}} + H^{\text{mat,NSI}}$$

NSI at the Lagrangian Level

EFT Lagrangian:

$$\mathscr{L}_{\rm NSI} = -2\sqrt{2}G_F \sum_{f,P,\alpha,\beta} \epsilon_{\alpha,\beta}^{f,P} (\bar\nu_\alpha \gamma^\mu P_L \nu_\beta)(\bar f \gamma_\mu P f)$$

with $\Lambda = \frac{1}{\sqrt{2\sqrt{2}\epsilon G_F}}$.

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Simplified model Lagrangian:

$$\mathscr{L}_{\rm NSI} = g_\nu Z'_\mu \bar{\nu}\gamma^\mu \nu + g_f Z'_\mu \bar{f}\gamma^\mu f$$

which gives a potential

$$V_{\rm NSI} \propto \frac{g_\nu g_f}{q^2 + m_{Z'}^2}$$

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$$V_{\rm NSI} \propto \frac{g_\nu g_f}{q^2+m^2_{Z'}}$$

Models with large NSIs consistent with CLFV:

Y. Farzan, I. Shoemaker 1512.09147 Y. Farzan, J. Heeck 1607.07616
D. Forero and W. Huang 1608.04719 U. Dey, N. Nath, S. Sadhukhan 1804.05808
K. Babu, A. Friedland, P. Machado, I. Mocioiu 1705.01822 Y. Farzan 1912.09408
PBD, Y. Farzan, I. Shoemaker 1804.03660

Neutrino Decay

Since neutrinos have different masses, they decay

- Loop suppressed
- Long lifetime: $\tau \gtrsim 10^{35}$ years

Test this!

Typical Lagrangian for $\nu_i \to \nu_j + \phi$ with $m_i > m_j$

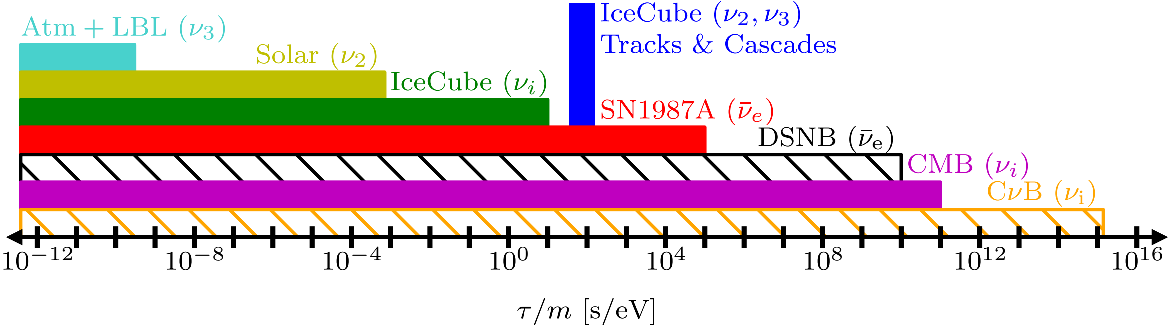
$$\mathcal{L} \supset \frac{g_{ij}}{2}\bar{\nu}_j \nu_i \phi + \frac{g'_{ij}}{2}\bar{\nu}_j i\gamma_5 \nu_i \phi$$

Neutrino Decay Phenomenology

Neutrino decay is phenomenologically classified into:

- **Invisible decay**:
  - The decay products are sterile or too low energy to be detected
  - Results in a *depletion* of the flux below the relevant energy
- **Visible decay**:
  - Decay products are detected
  - In addition to depletion, there is *regeneration*
  - Regeneration happens at a lower energy than depletion

Invisible ν Decay Constraints and Evidence



M. Gonzalez-Garcia and M. Maltoni 0802.3699

J. Berryman, A. de Gouvea, D. Hernandez 1411.0308

G. Pagliaroli, et al. 1506.02624

PBD, I. Tamborra 1805.05950

Kamiokande-II, PRL 58 1490 (1987)

S. Ando hep-ph/0307169

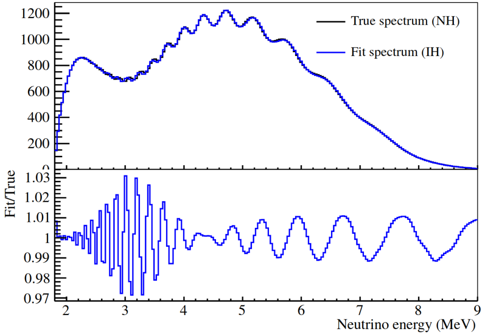
S. Hannestad, G. Raffelt hep-ph/0509278

A. Long, C. Lunardini, E. Sabancila 1405.7654

Next Generation Oscillation Experiments

JUNO: KamLAND 2.0, coming online in ~ 1 year

1. Improved measurement of solar parameters θ_{12}, Δm^2_{21}
2. Measurements of MBL reactor parameters θ_{13}, Δm^2_{31}
3. Mass ordering measurement by Δm^2_{31} vs. Δm^2_{32} discrimination



JUNO 1508.07166

Flavor Models

Popular early models: Bimaximal, tri-bimaximal, & golden ratio

All predicted $U_{e3} = 0 \Rightarrow \theta_{13} = 0$

Now know $\theta_{13} = 8.5^\circ$

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

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Need more degrees of freedom: sum rules

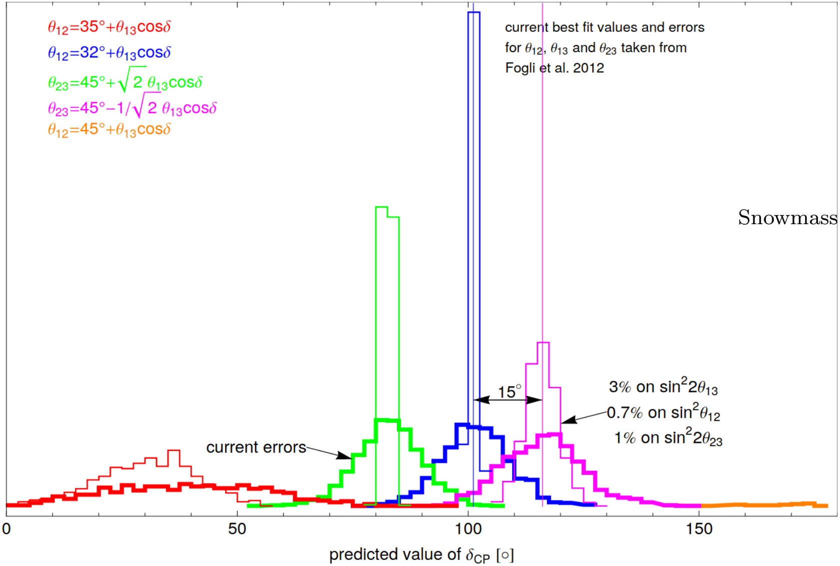
Perhaps:

$$U = \begin{pmatrix} c_\phi & s_\phi e^{-i\psi} & 0 \\ -s_\phi e^{i\psi} & c_\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} U_{TBM}$$

which predicts:

$$\cos\delta \approx \frac{\theta_{12} - \sin^{-1}\frac{1}{\sqrt{3}}}{\theta_{13}}$$

Flavor Models



Snowmass 1310.4340

2008.01110

Values and Vectors

Probability amplitude:

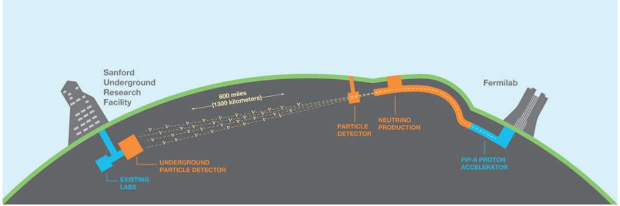
$$\mathcal{A}_{\alpha\beta} = \sum_i \widehat{U}^*_{\alpha i}\, e^{-i\widehat{m^2}_i L/2E}\, \widehat{U}_{\beta i}$$

- Eigen**values** give the frequencies of the oscillations

Where should DUNE be?

- Eigen**vectors** give the amplitudes of the oscillations

How many events will DUNE see?



Exact Neutrino Oscillations in Matter: Mixing Angles

$$s^2_{\widehat{12}} = \frac{-\left[(\widehat{m^2}_2)^2 - \alpha\widehat{m^2}_2 + \beta\right]\Delta\widehat{m^2}_{31}}{\left[(\widehat{m^2}_1)^2 - \alpha\widehat{m^2}_1 + \beta\right]\Delta\widehat{m^2}_{32} - \left[(\widehat{m^2}_2)^2 - \alpha\widehat{m^2}_2 + \beta\right]\Delta\widehat{m^2}_{31}}$$

$$s^2_{\widehat{13}} = \frac{(\widehat{m^2}_3)^2 - \alpha\widehat{m^2}_3 + \beta}{\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}}$$

$$s^2_{\widehat{23}} = \frac{s^2_{23}E^2 + c^2_{23}F^2 + 2c_{23}s_{23}c_\delta EF}{E^2 + F^2}$$

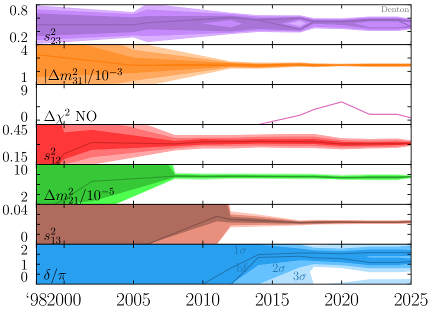
$$e^{-i\widehat{\delta}} = \frac{c_{23}s_{23}\left(e^{-i\delta}E^2 - e^{i\delta}F^2\right) + \left(c^2_{23} - s^2_{23}\right)EF}{\sqrt{\left(s^2_{23}E^2 + c^2_{23}F^2 + 2EFc_{23}s_{23}c_\delta\right)\left(c^2_{23}E^2 + s^2_{23}F^2 - 2EFc_{23}s_{23}c_\delta\right)}}$$

$$\alpha = c^2_{13}\Delta m^2_{31} + \left(c^2_{12}c^2_{13} + s^2_{13}\right)\Delta m^2_{21}\,, \quad \beta = c^2_{12}c^2_{13}\Delta m^2_{21}\Delta m^2_{31}$$

$$E = c_{13}s_{13}\left[\left(\widehat{m^2}_3 - \Delta m^2_{21}\right)\Delta m^2_{31} - s^2_{12}\left(\widehat{m^2}_3 - \Delta m^2_{31}\right)\Delta m^2_{21}\right]$$

$$F = c_{12}s_{12}c_{13}\left(\widehat{m^2}_3 - \Delta m^2_{31}\right)\Delta m^2_{21}$$

References



SuperK hep-ex/9807003

M. Gonzalez-Garcia, et al. hep-ph/0009350

M. Maltoni, et al. hep-ph/0207227

SuperK hep-ex/0501064

SuperK hep-ex/0604011

T. Schwetz, M. Tortola, J. Valle 0808.2016

M. Gonzalez-Garcia, M. Maltoni, J. Salvado 1001.4524

T2K 1106.2822

D. Forero, M. Tortola, J. Valle 1205.4018

D. Forero, M. Tortola, J. Valle 1405.7540

P. de Salas, et al. 1708.01186

The CPV Term in Matter

The amount of CPV is

$$P_{\alpha\beta} - \bar{P}_{\alpha\beta} = \pm 16 J \sin\Delta_{21} \sin\Delta_{31} \sin\Delta_{32} \qquad \alpha \neq \beta$$

where the Jarlskog is

$$J \equiv \Im[U_{\alpha i} U_{\beta j} U^*_{\alpha j} U^*_{\beta i}] \qquad \alpha \neq \beta,\ i \neq j$$

$$J = c_{12} s_{12} c^2_{13} s_{13} c_{23} s_{23} \sin\delta$$

C. Jarlskog PRL 55 (1985)

The exact term in matter is known to be

$$\frac{\widehat{J}}{J} = \frac{\Delta m^2_{21} \Delta m^2_{31} \Delta m^2_{32}}{\Delta\widehat{m^2}_{21} \Delta\widehat{m^2}_{31} \Delta\widehat{m^2}_{32}}$$

V. Naumov IJMP 1992

P. Harrison, W. Scott hep-ph/9912435

CPV in Matter

CPV in matter can be written sans $\cos(\frac{1}{3}\cos^{-1}(\cdots))$ term.

$$\frac{\widehat{J}}{J} = \frac{\Delta m^2_{21}\Delta m^2_{31}\Delta m^2_{32}}{\Delta\widehat{m^2}_{21}\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}}$$

$$\left(\Delta\widehat{m^2}_{21}\Delta\widehat{m^2}_{31}\Delta\widehat{m^2}_{32}\right)^2 = (A^2-4B)(B^2-4AC)+(2AB-27C)C$$

$$A \equiv \sum_j \widehat{m^2}_j = \Delta m^2_{31} + \Delta m^2_{21} + a$$

$$B \equiv \sum_{j>k} \widehat{m^2}_j \widehat{m^2}_k = \Delta m^2_{31}\Delta m^2_{21} + a(\Delta m^2_{ee}c^2_{13} + \Delta m^2_{21})$$

$$C \equiv \prod_j \widehat{m^2}_j = a\Delta m^2_{31}\Delta m^2_{21}c^2_{13}c^2_{12}$$

This is the only oscillation quantity in matter that can be written exactly without $\cos(\frac{1}{3}\cos^{-1}(\cdots))$!

H. Yokomakura, K. Kimura, A. Takamura hep-ph/0009141

CPV Factorizes

Thus $\widehat{J}^{-2}$ is fourth order in matter potential:
only two matter corrections are needed.

$$\frac{\widehat{J}}{J}=\frac{1}{|1-(a/\alpha_1)e^{i2\theta_1}||1-(a/\alpha_2)e^{i2\theta_2}|}$$

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CPV in matter can be well approximated:

$$\frac{\widehat{J}}{J}\approx\frac{1}{|1-(a/\Delta m^2_{ee})e^{i2\theta_{13}}||1-(c^2_{13}a/\Delta m^2_{21})e^{i2\theta_{12}}|}$$

PBD, Parke 1902.07185

See also X. Wang, S. Zhou 1901.10882

Precise at the $<0.04\%$ level!

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PBD, Parke 1902.07185

See also X. Wang, S. Zhou 1901.10882

Precise at the $< 0.04\%$ level!

One Caveat in Support of δ

If the goal is **CP violation** the Jarlskog should be used

however

If the goal is **measuring the parameters** one must use δ

Given θ_{12}, θ_{13}, θ_{23}, and J, I can't determine the sign of $\cos\delta$ which is physical

e.g. $P(\nu_\mu \to \nu_\mu)$ depends on $\cos\delta$ a tiny bit