Abstract

Neutrinos are the second most abundant known particle in the Universe and are produced abundantly in many extreme astrophysical environments. This gives us the opportunity to study neutrinos in ways not accessible in human made environments. I will begin with the Standard Model and show how neutrinos have shaped its construction, up to the current level where they sit together uneasily. These lectures will focus on neutrino oscillations, how we measure them, how we calculate the observables, and how we relate the calculations to the measurements. They will cover reactor, atmospheric, solar, and accelerator neutrinos, and will also discuss some new physics scenarios.

Neutrino Theory Overview: I

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INSS

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About Me

1. Grew up in Michigan
2. Bachelors in physics and math from Rice, ’10

Visited Fermilab to do accelerator physics

3. PhD from Vanderbilt, ’16
4. Year at Fermilab working with Stephen Parke, ’15-’16
5. Postdoc at the Niels Bohr International Academy, ’16-’18
6. Faculty at Brookhaven, ’18-present

Research interests

- Neutrino oscillations
- New physics in neutrinos
- Astroparticle physics
- Black holes
- Dark matter

Other interests

- Ultimate frisbee
- Hiking
- Piano
- Photography

Resources

"Neutrino Oscillations in the Three Flavor Paradigm" - PBD `2501.08374`

Particle Data Group (PDG) "14. Neutrino Masses, Mixing, and Oscillations" -
M.C. Gonzalez-Garcia, R. Wendel
https://pdg.lbl.gov/2026/reviews/rpp2026-rev-neutrino-mixing.pdf

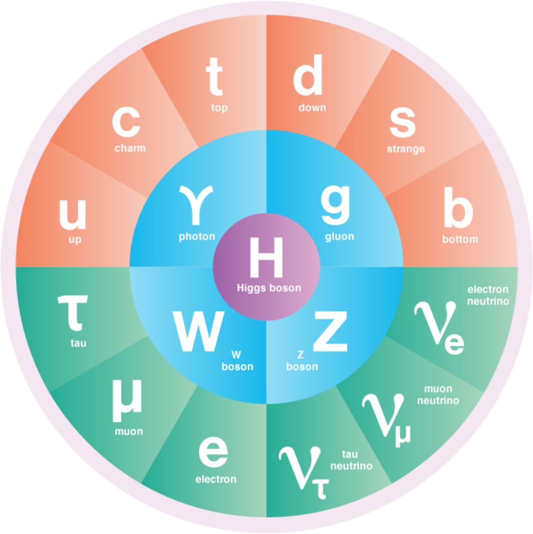
This is designed to be interactive

Outline For Both Lectures

1. Standard Model redux
2. Neutrino properties and results before oscillations
3. Matter effect
4. Reactor oscillations
5. Atmospheric oscillations
6. Solar oscillations
7. Long-baseline oscillations
8. Remaining unknowns in neutrinos
9. New physics in oscillations
10. Neutrinos probe opaque environments

Challenge questions

The Standard Model



Gauge group:

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

Explains all known particle interactions and phenomena except:

- Gravity and the big bang
- The collection of astrophysical and cosmological evidence pointing to dark matter
- The evidence for accelerated expansion (dark energy)
- Neutrino oscillations

Neutrinos Solve an Anomaly

1910s-1920s: β decay spectrum should be monocrhomatic;
improved measurements show continuous spectrum
Wolfgang Pauli, 1930:
Momentum could be conserved if there is an invisible particle

Niels Bohr: Momentum is conserved only statistically

Enrico Fermi, 1934:

$$n \to p^+ + e^- + \nu$$

Developed a successful effective field theory

It fails at $E \gtrsim 100$ GeV ⇒ must be a particle around thre ⇒ $W^\pm$ at 80 GeV

Cross section with regular matter is very small

Nuclear Beta Decay: 1911-1934

Helicity and Chirality

1957, Goldhaber experiment at Brookhaven:
Every neutrino has left helicity: spin and momentum are antiparallel

Antineutrinos have right helicity

Chirality: left/right is how a particle transforms (γ^5)

Helicity is a constant of motion, not Lorentz invariant
Chirality is Lorentz invariant, but not a constant of motion:
left spinors can oscillate into right spinors

Helicity and chirality are identical for massless fermions and when $\gamma \gg 1$

Neutrinos only interact via the weak interaction which respects chirality

Fermion Mass Generation

Fermions: quarks, charged leptons, neutrinos all experience the weak interaction
Thus left and right chiral states are different

This is equivalent to saying that the weak interaction is $V-A$
a result that comes from $\pi^+\to\mu^++\nu_\mu$ vs. $\pi^+\to e^++\nu_e$ decays

Mass terms typically look like:

$$\mathcal{L}=-m\bar\psi\psi=-m(\bar\psi_L\psi_R+\bar\psi_R\psi_L)$$

Not $SU(2)_L$ invariant!

Higgs mechanism introduces an $SU(2)_L$ doublet complex scalar that provides an effective mass term that is equivalent in all cases

Except at high temperatures

Seems like neutrinos should have masses too?

Right Handed Fermions

We have detected RH quarks and charged leptons

When I say RH, I refer to particles or LH antiparticles

We should add RH neutrinos, right?

We have never detected a RH neutrino
At the very minimum, including masses to neutrinos requires adding 3 new undetected particles

Given oscillations, in principle 1 ν could be massless, so only 2 RH would be needed

This is already new physics

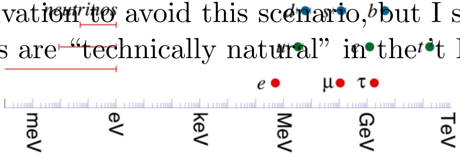
Adding neutrino masses is worse than that

How Small is Small?

The Standard Model of particle physics has evolved in time
Perhaps adding RH neutrinos is the next step?

Before oscillations, $m_\nu \lesssim$ eV
The Higgs-neutrino Yukawa couplings would be $< 10^{-12}$
The top Yukawa coupling is ~ 1, but the electron Yukawa coupling is $\sim 10^{-6}$

Some claim this is motivation to avoid this scenario, but I see no problem
Small Yukawa couplings are "technically natural" in the 't Hooft sense



What Makes a Model Standard?

The problem: there are two “comparable” ways to add neutrino masses:

Dirac masses

$$\mathcal{L}_\nu^{\text{Dir}} = -y_\nu \bar{\ell}_L \phi \nu_R$$

- The same as how other Fermions get mass (via Higgs)
- Requires 2-3 new RH neutrinos
- Requires Yukawa couplings 10^{-14} – 10^{12} or less
- Requires enforcing lepton number as a good symmetry
- Yukawa couplings impossible to probe

Majorana masses

$$\mathcal{L}_\nu^{\text{Maj}} = -\frac{m}{2} \bar{\nu}_R (\nu_R)^c$$

- Often alongside Dirac masses
- A new kind of mass generation
- Requires several new particles
- Violates L by 2
- Parameters feel good: Yukawa couplings ~ 1 and heavy states connected to GUTs ($\sim 10^{16}$ GeV)
- Heavy states impossible to probe
- Testable via $0\nu\beta\beta$; hard!

Dirac - Majorana Differences

Dirac is equivalent to Majorana in ultrarelativistic limit,
differences go like $\sim \left(\frac{m_\nu}{E_\nu}\right)^2 \lesssim 10^{-14}$ in oscillation experiments

Majorana is not "neutrino=antineutrino", but rather that ν and $\bar{\nu}$ mix and can oscillate into each other

Majorana neutrinos only have ν_L and $\bar{\nu}_R$ while Dirac neutrinos also have ν_R and $\bar{\nu}_L$
Majorana neutrinos: ν_L can oscillate into $\bar{\nu}_R$ which are both active $\Rightarrow 0\nu\beta\beta$
Implications for cosmic neutrino background which is mostly nonrelativistic today

Q: Do we detect more CνB neutrinos if they are Dirac or Majorana?

Dirac's life is not an unsolved mystery

Seesaw

Majorana mass term does not forbid Dirac mass term
Many different seesaw realizations

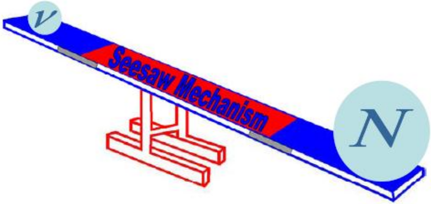
Ingredients:

1. 3 left handed neutrinos ν in a SU(2) doublet
2. 3 right handed neutrinos N with Majorana mass term
3. Write down a Dirac mass term from the Higgs for ν and N
4. After electroweak symmetry breaking

$$\mathcal{L} \supset -m_D\overline{\nu_L}N_R - \frac{1}{2}M_R\overline{(N^c)}_L N_R$$

5. Diagonalize the m ses

$$\mathbb{N}^\dagger \begin{pmatrix} 0 \\ m_I \end{pmatrix} \quad = \mathbb{N}\begin{pmatrix} \nu_m \\ N^c_m \end{pmatrix}$$



6. Physical mass ter

Mass Generation Ideas

Three tree-level minimal realizations of the seesaw: type-I, type-II, type-III

H. Fritzsch, M. Gell-Mann, P. Minkowski PLB 1975
P. Minkowski PLB 1977
W. Konetschny, W. Kummer PLB 1977
D. Wyler, L. Wolfenstein NPB 1983
R. Foot, H. Lew, X. He, G. Joshi ZPC 1989

Can also consider case where $M_R \ll m_D$: Pseudo-Dirac

L. Wolfenstein NPB 1981
S. Bilenky, S. Petcov RMP 1987

Interesting mass ranges are often 10^{13} GeV, 10^{3} GeV, or 10^{-26} GeV, not 10^{-9} GeV

The neutrino mass generation mechanism is:

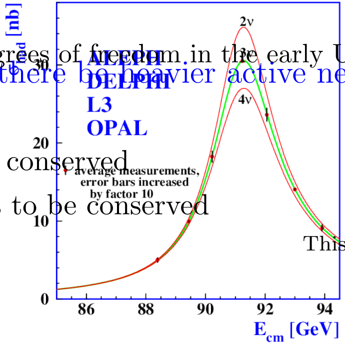
undetermined today,
an active area of research,
and is a top open question in particle physics

Neutrinos Before 1998

1. Tiny cross section, all left handed, only experience weak interaction
2. Only three active generations
    - Detected via charged current (CC)

      C. Cowan, F. Reines 1956, L. Lederman, M. Schwartz, J. Steinberger 1962, DONUT 2000
    - Measured $Z \to$invisible is $3\times$ $Z \to \nu_i$, $\Rightarrow$ 3 neutrinos $m_\nu < M_Z/2 = 46$ GeV

      LEP hep-x/0509008



    - Number of relativistic degrees of freedom in the early Universe: $N_{\rm eff} = 2.99 \pm 0.17$

      PLANCK 1807.06209

Q: Can there be heavier active neutrinos?

3. Lepton number seems to be conserved
4. Lepton flavor number seems to be conserved

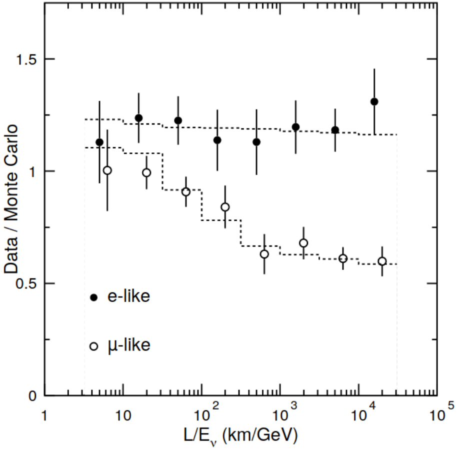
   This one is violated by oscillations

Big Surprise of 1998

- Electroweak understood, mediators (γ, W, Z) found
- Strong understood, mediators (gluon) found
- All fermions detected except tau neutrino (2000), but no surprises expected
- Higgs boson still to be found
- Standard Model looks to be in great shape

Atmospheric Neutrinos Disappear

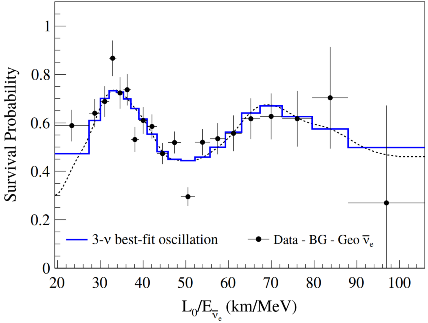
Cosmic rays hit the atmosphere, produce π^+, μ, and ν_μ



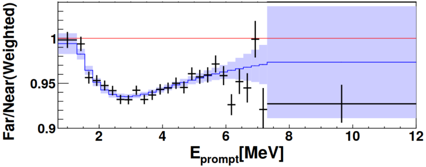
SuperKamiokande hep-ex/9807003

Neutrinos Really Oscillate

1. Neutrinos experience time ⇒ must have mass
2. Neutrino oscillate ⇒ must mix & masses must be different



KamLAND 1303.4667



Daya Bay 1809.02261

Two-flavor Neutrino Oscillation Probability

Only one mixing angle, one $\Delta m^2_{32} \equiv m_3^2 - m_2^2$, no complex phase

$$P(\nu_\alpha \to \nu_\alpha) = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

Same flavor: disappearance

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

New flavor: appearance

Can easily confirm unitarity:

$$\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$$

Q: What angle leads to maximal oscillations?

How to compute neutrino oscillations

Schrödinger Equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i\frac{d}{dt}|\nu\rangle = H|\nu\rangle$$

In the absence of any interactions $H_{\text{vac}}|\nu_i\rangle = E_i|\nu_i\rangle$.

$$|\nu_i(L)\rangle = e^{-iE_iL}|\nu_i(0)\rangle \to e^{-im_i^2L/2E}|\nu_i(0)\rangle$$

i, j indicates mass eigenstate
Assume ν's are ultrarelativistic: $E_i \to p + m_i^2/2E$, $t \to L$
See also e.g. E. Akhmedov, A. Smirnov `0905.1903`

No oscillations!

Q: Why does the p part does not contribute?

Mass Basis to Flavor Basis

We don't produce neutrinos in eigenstates of the Hamiltonian in vacuum, e.g. mass eigenstates; they are produced in the flavor/interaction/weak basis

$$|\nu_\alpha\rangle = \sum_{i=1}^{3} U^*_{\alpha i} |\nu_i\rangle$$

$\alpha, \beta \in \{e, \mu, \tau\}$ are flavor states

U is a unitary 3×3 complex matrix which has 4 or 6 physical degrees of freedom

18 dofs; unitarity $\Rightarrow 18 - 9 = 9$ dofs
$U(1)$ rephasing $\Rightarrow$ either $9 - 3 = 6$ dofs (Majorana) or $9 - 5 = 4$ dofs (Dirac)

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For more on parameterizations see: PBD, R. Pestes `2006.09384`

Q: Why can we ignore Majorana phases in oscillations?

The Probability

The physical observable is the probability: $P(\nu_\alpha \to \nu_\beta; L, E)$

First calculate the transition amplitude:

1. Project from ν_α to ν_i via U^*
2. Propagate mass state ν_i via phase accumulation
3. Project from ν_i to ν_β via U^T
4. Sum over all intermediate states*

*coherency must apply

$$\mathcal{A}(\nu_\alpha \to \nu_\beta; L, E) = \sum_{i=1}^{3} U_{\alpha i}^* e^{-im_i^2 L/2E} U_{\beta i}$$

$$P(\nu_\alpha \to \nu_\beta; L, E) = |\mathcal{A}(\nu_\alpha \to \nu_\beta; L, E)|^2$$

Discrete symmetries:

$$T : P(\nu_\alpha \to \nu_\beta) \leftrightarrow P(\nu_\beta \to \nu_\alpha) \Leftrightarrow L \leftrightarrow -L$$

$$CP : \nu \leftrightarrow \bar{\nu} \;\Leftrightarrow\; U_{\alpha i} \to U_{\alpha i}^* \;\Leftrightarrow\; E \to -E$$

Assume CPT is conserved: $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$

Assume that E and direction don't change during propagation

Coherent Propagation

- Neutrino oscillations requires all 3 wavefunctions to overlap
- Properly calculating this requires QFT
  - Need to integrate over production region
  - Need to account for detection uncertainties
- Literature is somewhat inconsistent in how to do this
  - All correct approaches give same answer
- Nearly all cases of oscillations are known to be coherent
- Exceptions:
  - Solar neutrinos: decohere from Sun to Earth
  - Astrophysical neutrinos: (galactic or extragalactic) decohere
- Decohered probabilities are easy!

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i,j=1}^{3} P_{\alpha i} P_{ij} P_{j\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Mass states are eigenstates of the Hamiltonian ⇒ $P_{ij} = \delta_{ij}$
Everything is at the probability level not the amplitude level
Relevant for all astrophysical neutrinos, except solar and supernova
This is the same expression as oscillation averaged probabilities

Three Flavor

Three angles, one complex phase (all in U), and three Δm^2_{ij} (two are very similar)

It is less easy to show that:

$$\begin{aligned} P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) \\ &- 4|U_{\alpha 1}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \\ &- 4|U_{\alpha 2}|^2|U_{\alpha 3}|^2 \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right) \end{aligned}$$

Many different ways to write these probabilities

Q: Why do we discuss $\Delta m^2_{ij} = m^2_i - m^2_j$ and not just m^2_i?

Each term is manifestly T even (even in L)
Each term is manifestly CP even (even under $U \to U^*$ or $E \to -E$)

Three Flavor: Appearance

$$\begin{aligned}P(\nu_\alpha \to \nu_\beta) = &-4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}]\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right)\\ &-4\Re[U_{\alpha1}U^*_{\beta1}U^*_{\alpha3}U_{\beta3}]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)\\ &-4\Re[U_{\alpha2}U^*_{\beta2}U^*_{\alpha3}U_{\beta3}]\sin^2\left(\frac{\Delta m^2_{32}L}{4E}\right)\end{aligned}$$

$$+8\Im[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}]\sin\left(\frac{\Delta m^2_{21}L}{4E}\right)\sin\left(\frac{\Delta m^2_{31}L}{4E}\right)\sin\left(\frac{\Delta m^2_{32}L}{4E}\right)$$

Final coefficient:

$$8\Im[U_{\alpha1}U^*_{\beta1}U^*_{\alpha2}U_{\beta2}] \equiv 8J = 8s_{12}c_{12}s_{13}c^2_{13}s_{23}c_{23}\sin\delta$$

This is the same for all appearance channels (up to sign)

C. Jarlskog PRL 55 (1985)

$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

First three terms are manifestly T even and CP even

Last term is manifestly T odd and CP odd

Total probability is neither even nor odd under CP or T

More on Probabilities

1. The Jarlskog term is the only term $\propto \sin\delta$
2. The Jarlskog term is the only term $\propto L^3$

all others $\propto L^2$
$\Rightarrow \delta$ is hard to measure

3. $\nu \to \bar{\nu} \Rightarrow U \to U^*$ which is $\delta \to -\delta$
4. $\delta \to -\delta \Rightarrow L \to -L$ or $E \to -E$

Care is required because of the matter effect

5. This follows from CPT. CP: $\delta \to -\delta$ and T is $L \to -L$

Matter effect causes apparent CPT violation

CPT Implications

Because physics should be invariant under CPT:

- Simultaneously flip $\delta \to -\delta$, $\Delta m^2_{31} \to -\Delta m^2_{31}$, and $\Delta m^2_{21} \to -\Delta m^2_{21}$: physics is unchanged
- Measurements cannot determine the sign of δ, Δm^2_{21}, or Δm^2_{31}
- This degeneracy is broken due to the matter effect
- This is called the "dark side" as it indicates that both the true and incorrect mass orderings fit the data equally

A. de Gouvêa, A. Friedland, H. Murayama `hep-ph/0002064`
PBD, J. Gehrlein `2204.09060`

Mass Eigenstate Definitions

How do we even define the mass states? Literature differs

1. Best: $|U_{e1}| > |U_{e2}| > |U_{e3}|$, or $0.82 > 0.55 > 0.15$
   - Single clear definition of things we have measured well
   - Says that $\theta_{12} \in [0, 45°]$ while $\theta_{13}, \theta_{23} \in [0, 90°]$
   - Means that there are two mass ordering questions: signs of Δm^2_{21} and Δm^2_{31}
   - Solar neutrinos told us $\Delta m^2_{21} > 0$
2. Used sometimes: $|U_{e3}| < |U_{e1}|$, $|U_{e3}| < |U_{e2}|$, and $m_1 < m_2$
   - Makes a plot of solar and LBL-reactor neutrinos continuous
   - Says that all three mixing angles are $\theta_{ij} \in [0, 90°]$
   - Treats solar and atmospheric sectors differently
   - Means that there is one mass ordering question: sign of Δm^2_{31}, and one octant question: $\theta_{12} <> 45°$
   - Solar neutrinos told us that $\theta_{12} < 45°$
3. Bad: $m_1 < m_2 < m_3$
   - Has been used in the past, now regarded as a bad definition
   - Makes the mass ordering question very complicated
   - Once we know both mass orderings, this may be the convention

Q: What is special about θ_{12}?

PBD 2003.04319

Neutrinos propagating in matter feel the effects of the matter

- Relevant for the Earth's crust and interior, the Sun, and supernova
- Atmosphere is low enough density that it can be ignored
- Pointed out by L. Wolfenstein PRD 17 (1978)
- Usually only the electron density N_e contributes

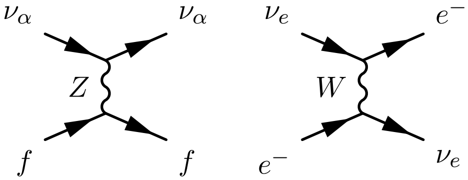
See sterile neutrino discussion

- Neutrinos propagate in a new basis as mass states are no longer eigenstates of the Hamiltonian

Hamiltonian Dynamics: With Matter

Neutrino energy levels are modified by presence of background fermion fields

These diagrams are dominantly forward ($\theta = 0$) elastic (particles in = particles out)



L. Wolfenstein PRD 17 (1978)

Q: Why isn't there a term in the Hamiltonian for the Z boson diagram?

Schrödinger equation:

$$i\frac{d}{dt}|\nu_\alpha\rangle = H_{\rm flav}(t)|\nu_\alpha\rangle$$

$$H_{\rm flav}(t) = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0\end{pmatrix}\right]$$

Matter effect: $a(x) = 2\sqrt{2}G_F N_e(x) E$

As $a \to 0$, we recover earlier vacuum calculations

Matter Effect: Constant

In matter ν's propagate in a new basis that depends on $a \propto N_e E_\nu$.
Assume $H_{\rm flav}(x) \simeq H_{\rm flav}$ is approximately constant:

$$\mathcal{A} = e^{-iH_{\rm flav}L} = V\Lambda V^\dagger$$

V is the unitary matrix composed of normalized eigenvectors of H
Λ is the diagonal matrix composed of $e^{-i\lambda_i L}$ where λ_i are the eigenvalues of H

Matter Effect: Varying

Solar neutrinos in an adiabatically changing matter potential
Solution = MSW effect

S. Mikheev, A. Smirnov Nuovo Cim. C9 (1986) 17-26
The terminology: “MSW” applies for solar (and SN) neutrinos, but not LBL or atmospheric

- Potential changes slowly ⇒ neutrinos remain in same state
- In center of the sun $\nu_e \approx \widehat{\nu}_2$ in matter: nearly an eigenstate
- Eigenstates of the Hamiltonian don’t oscillate; but the Hamiltonian is changing
- So long as the Hamiltonian changes “slow enough” it remains in the same state
- $\widehat{\nu}_2 \to \nu_2$ and neutrinos leave the Sun as ν_2
- Probability to detect ν_e is simply:

$$P_{ee} \simeq P^{\odot}_{e2} P^{\rm vac}_{22} P^{\rm det}_{2e} \approx 1 \times 1 \times |U_{e2}|^2 \approx \sin^2\theta_{12}$$

Neutrinos in supernovae experience MSW effect too,
but they also experience neutrino-neutrino interactions

Propagation in SNe is much more involved

Matter Effect Open Questions

- Matter has only been measured in the Sun, by combining solar with reactor. Confirm it in the Earth.
- New physics that looks like the matter effect takes the form of vector non-standard neutrino interactions: vNSI. Is there evidence for this? Can we constrain it?
- There is a degeneracy within new physics (vNSI) related to measuring the matter effect and the mass ordering ($\operatorname{sgn}\Delta m^2_{31}$) called LMA-Dark. Can we rule it out/discover it?
- How well do we know the density of the Earth/Sun?

Experiment to Oscillation Parameters

Six oscillation parameters: θ_{12}, θ_{13}, θ_{23}, δ, Δm^2_{21}, Δm^2_{31}

- Atmospheric ν_μ disappearance $\to \sin 2\theta_{23}$, $|\Delta m^2_{31}|$

  SuperK, IMB, IceCube, KM3NeT

- Solar ν_e disappearance $\to s^2_{12}$, $\sin^2 2\theta_{12}$, $\pm\Delta m^2_{21}$

  SNO, Borexino, SuperK

- Reactor ν_e disappearance:
  - LBL ($\gtrsim 50$ km) $\to \sin^2 2\theta_{12}$ and $|\Delta m^2_{21}|$

    KamLAND, SNO+, JUNO

  - Next-gen LBL $\to \pm\Delta m^2_{31}$

    JUNO

  - MBL (~ 1 km) $\to \sin^2 2\theta_{13}$, $|\Delta m^2_{31}|$

    Daya Bay, RENO, Double Chooz

- Accelerator LBL ν_e appearance: $\pm\Delta m^2_{31}$, s^2_{23}, s^2_{13}, $\sin\delta$

  T2K, NOvA, HK, DUNE

7th parameter: absolute mass scale

Cosmology, KATRIN, $0\nu\beta\beta$