Abstract

Neutrinos are the second most abundant known particle in the Universe and are produced abundantly in many extreme astrophysical environments. This gives us the opportunity to study neutrinos in ways not accessible in human made environments. I will begin with the Standard Model and show how neutrinos have shaped its construction, up to the current level where they sit together uneasily. These lectures will focus on neutrino oscillations, how we measure them, how we calculate the observables, and how we relate the calculations to the measurements. They will cover reactor, atmospheric, solar, and accelerator neutrinos, and will also discuss some new physics scenarios.

Neutrino Theory Overview: I

Peter B. Denton

INSS

June 29-30, 2026

Brookhaven™
National Laboratory

About Me

1. Grew up in Michigan
2. Bachelors in physics and math from Rice, '10

   Visited Fermilab to do accelerator physics
3. PhD from Vanderbilt, '16
4. Year at Fermilab working with Stephen Parke, '15-'16
5. Postdoc at the Niels Bohr International Academy, '16-'18
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Research interests

- Neutrino oscillations
- New physics in neutrinos
- Astroparticle physics
- Black holes
- Dark matter

Other interests

- Ultimate frisbee
- Hiking
- Piano
- Photography

Resources

"Neutrino Oscillations in the Three Flavor Paradigm" - PBD 2501.08374

Particle Data Group (PDG) "14. Neutrino Masses, Mixing, and Oscillations" - M.C. Gonzalez-Garcia, R. Wendel
https://pdg.lbl.gov/2026/reviews/rpp2026-rev-neutrino-mixing.pdf

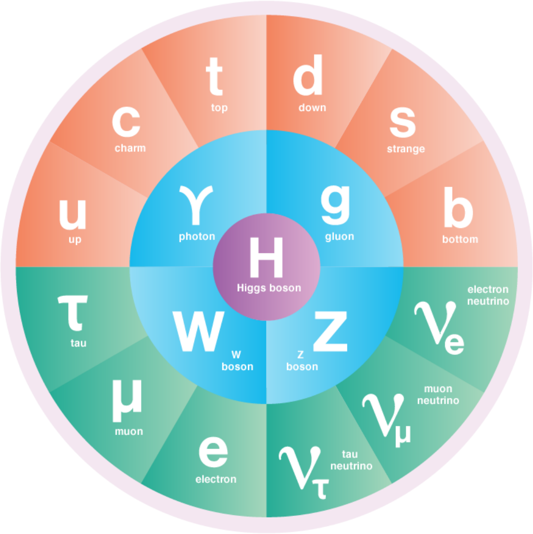
This is designed to be interactive

Outline For Both Lectures

1. Standard Model redux
2. Neutrino properties and results before oscillations
3. Matter effect
4. Reactor oscillations
5. Atmospheric oscillations
6. Solar oscillations
7. Long-baseline oscillations
8. Remaining unknowns in neutrinos
9. New physics in oscillations
10. Neutrinos probe opaque environments

Challenge questions

The Standard Model



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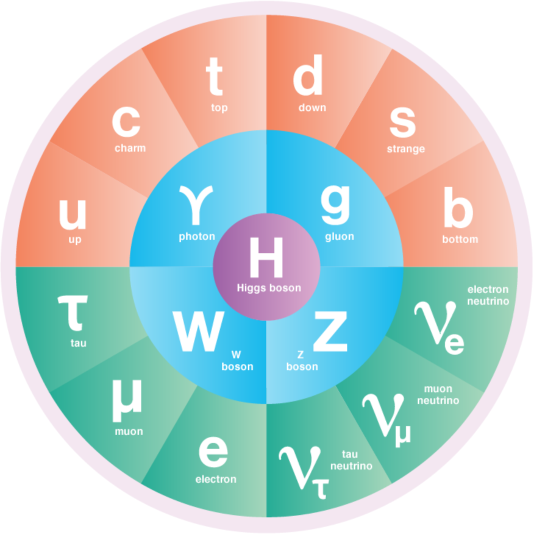

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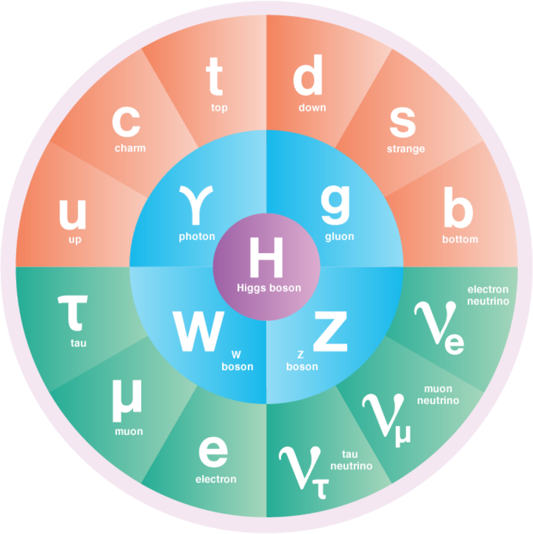

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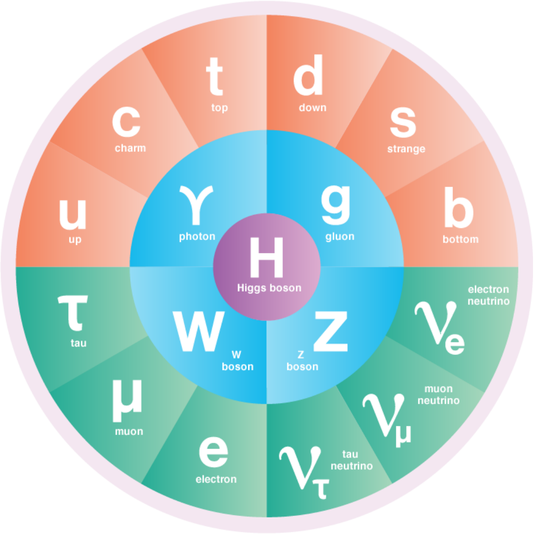

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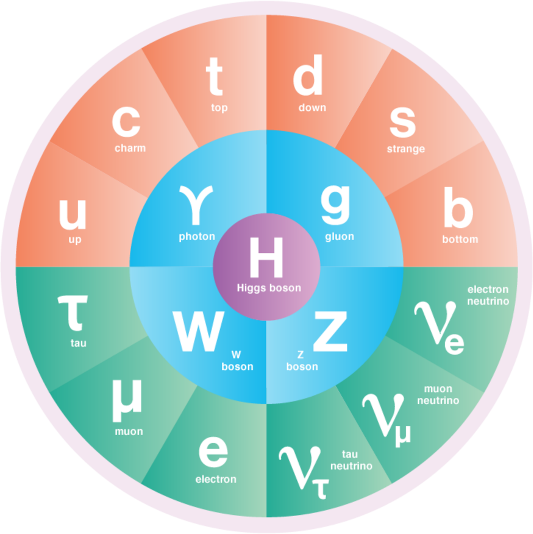

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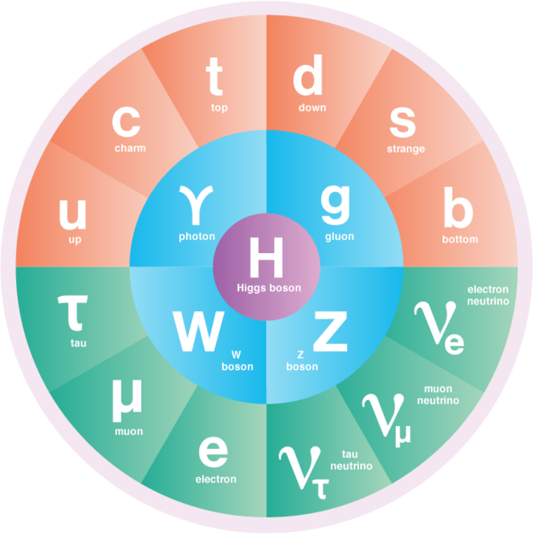

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- Neutrino oscillations

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Cross section with regular matter is very small

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Neutrinos only interact via the weak interaction which respects chirality

Fermion Mass Generation

Fermions: quarks, charged leptons, neutrinos all experience the weak interaction

Thus left and right chiral states are different

This is equivalent to saying that the weak interaction is $V - A$
a result that comes from $\pi^+ \to \mu^+ + \nu_\mu$ vs. $\pi^+ \to e^+ + \nu_e$ decays

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Seems like neutrinos should have masses too?

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Adding neutrino masses is worse than that

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The Higgs-neutrino Yukawa couplings would be $< 10^{-12}$
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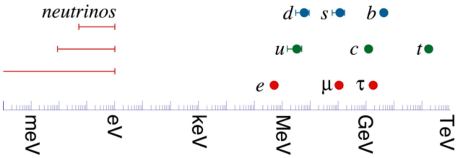

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Some claim this is motivation to avoid this scenario, but I see no problem
Small Yukawa couplings are "technically natural" in the 't Hooft sense

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Majorana masses

$$\mathcal{L}_\nu^{\rm Maj} = -\frac{m}{2}\bar{\nu}_R(\nu_R)^c$$

- Often alongside Dirac masses
- A new kind of mass generation
- Requires several new particles
- Violates L by 2
- Parameters feel good: Yukawa couplings ~ 1 and heavy states connected to GUTs ($\sim 10^{16}$ GeV)
- Heavy states impossible to probe
- Testable via $0\nu\beta\beta$; hard!

Dirac - Majorana Differences

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Dirac's life is not an unsolved mystery

Seesaw

Majorana mass term does not forbid Dirac mass term
Many different seesaw realizations

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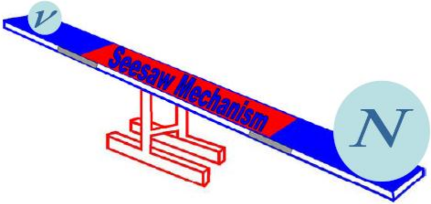

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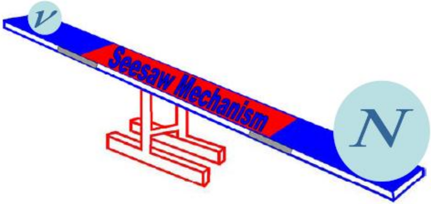

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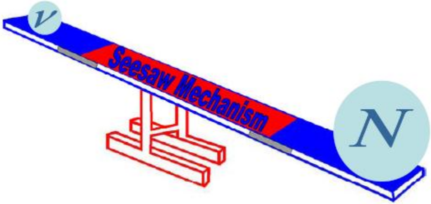

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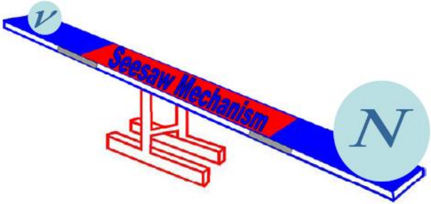

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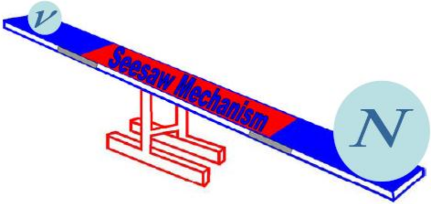

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$$\mathbb{N}^\dagger \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \mathbb{N}^* = \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix} , \quad \begin{pmatrix} \nu \\ N^c \end{pmatrix}_L = \mathbb{N} \begin{pmatrix} \nu_m \\ N^c_m \end{pmatrix}$$

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6. Physical mass terms for $M_R \gg m_D$:

$$m_\nu \approx -\frac{m_D^2}{M_R}, \quad M_N \approx M_R$$

Mass Generation Ideas

Three tree-level minimal realizations of the seesaw: type-I, type-II, type-III

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Interesting mass ranges are often 10^{13} GeV, 10^{3} GeV, or 10^{-26} GeV, not 10^{-9} GeV

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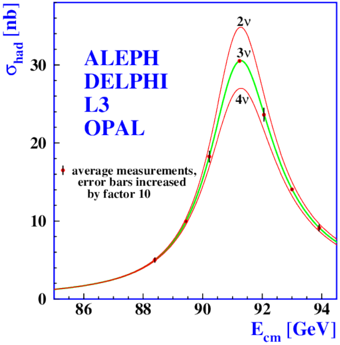
     LEP hep-x/0509008



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Q: Can there be heavier active neutrinos?

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     C. Cowan, F. Reines 1956, L. Lederman, M. Schwartz, J. Steinberger 1962, DONUT 2000
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     LEP hep-x/0509008
   - Number of relativistic degrees of freedom in the early Universe:
     $N_{\rm eff} = 2.99 \pm 0.17$

     PLANCK 1807.06209
3. Lepton number seems to be conserved
4. Lepton flavor number seems to be conserved

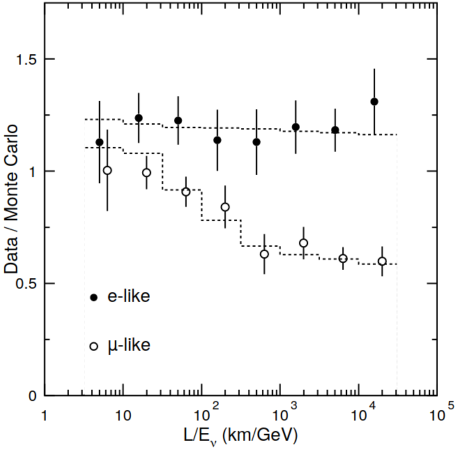
   This one is violated by oscillations

Big Surprise of 1998

- Electroweak understood, mediators (γ, W, Z) found
- Strong understood, mediators (gluon) found
- All fermions detected except tau neutrino (2000), but no surprises expected
- Higgs boson still to be found
- Standard Model looks to be in great shape

Atmospheric Neutrinos Disappear

Cosmic rays hit the atmosphere, produce π^+, μ, and ν_μ



SuperKamiokande hep-ex/9807003

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1. Neutrinos experience time $\Rightarrow$ must have mass
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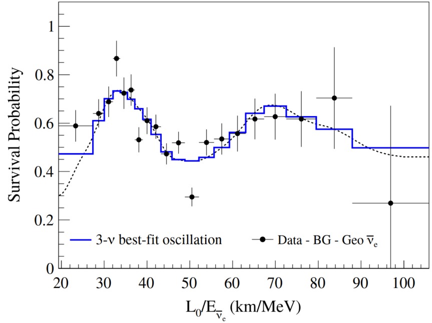
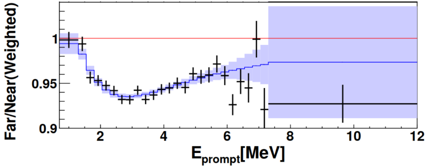


KamLAND 1303.4667



Daya Bay 1809.02261

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Q: What angle leads to maximal oscillations?

How to compute neutrino oscillations

Schrödinger Equation

Neutrinos propagate in eigenstates of the Hamiltonian

$$i\frac{d}{dt}|\nu\rangle = H|\nu\rangle$$

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$$|\nu_i(L)\rangle = e^{-iE_iL}|\nu_i(0)\rangle \to e^{-im_i^2L/2E}|\nu_i(0)\rangle$$

i, j indicates mass eigenstate
Assume ν's are ultrarelativistic: $E_i \to p + m_i^2/2E$, $t \to L$
See also e.g. E. Akhmedov, A. Smirnov 0905.1903

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Q: Why does the p part does not contribute?

Mass Basis to Flavor Basis

We don't produce neutrinos in eigenstates of the Hamiltonian in vacuum, e.g. mass eigenstates; they are produced in the flavor/interaction/weak basis

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$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For more on parameterizations see: PBD, R. Pestes 2006.09384

Q: Why can we ignore Majorana phases in oscillations?

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The physical observable is the probability: $P(\nu_\alpha \to \nu_\beta; L, E)$

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Assume that E and direction don't change during propagation

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- Decohered probabilities are easy!

$$P(\nu_\alpha \to \nu_\beta) = \sum_{i,j=1}^{3} P_{\alpha i} P_{ij} P_{j\beta} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$$

Mass states are eigenstates of the Hamiltonian $\Rightarrow P_{ij} = \delta_{ij}$
Everything is at the probability level not the amplitude level
Relevant for all astrophysical neutrinos, except solar and supernova
This is the same expression as oscillation averaged probabilities

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Three angles, one complex phase (all in U), and three Δm^2_{ij} (two are very similar)

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It is less easy to show that:

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P(\nu_\alpha \to \nu_\alpha) = 1 &- 4|U_{\alpha 1}|^2|U_{\alpha 2}|^2 \sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right) \\
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Each term is manifestly T even (even in L)
Each term is manifestly CP even (even under $U \to U^*$ or $E \to -E$)

Three Flavor: Appearance

$$
\begin{aligned}
P(\nu_\alpha \to \nu_\beta) = & -4\Re[U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 2}U_{\beta 2}]\sin^2\left(\frac{\Delta m^2_{21}L}{4E}\right) \\
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Final coefficient:

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8\Im[U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 2}U_{\beta 2}] \equiv 8J = 8s_{12}c_{12}s_{13}c^2_{13}s_{23}c_{23}\sin\delta
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This is the same for all appearance channels (up to sign)
C. Jarlskog PRL 55 (1985)
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$s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

First three terms are manifestly T even and CP even

Last term is manifestly T odd and CP odd

Total probability is neither even nor odd under CP or T

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5. This follows from CPT. CP: $\delta \to -\delta$ and T is $L \to -L$

Matter effect causes apparent CPT violation

CPT Implications

Because physics should be invariant under CPT:

- Simultaneously flip $\delta \to -\delta$, $\Delta m^2_{31} \to -\Delta m^2_{31}$, and $\Delta m^2_{21} \to -\Delta m^2_{21}$: physics is unchanged
- Measurements cannot determine the sign of δ, Δm^2_{21}, or Δm^2_{31}
- This degeneracy is broken due to the matter effect
- This is called the "dark side" as it indicates that both the true and incorrect mass orderings fit the data equally

A. de Gouvêa, A. Friedland, H. Murayama hep-ph/0002064
PBD, J. Gehrlein 2204.09060

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How do we even define the mass states? Literature differs

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1. Best: $|U_{e1}| > |U_{e2}| > |U_{e3}|$, or $0.82 > 0.55 > 0.15$
   - Single clear definition of things we have measured well
   - Says that $\theta_{12} \in [0, 45°]$ while $\theta_{13}, \theta_{23} \in [0, 90°]$
   - Means that there are two mass ordering questions: signs of Δm^2_{21} and Δm^2_{31}
   - Solar neutrinos told us $\Delta m^2_{21} > 0$

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PBD 2003.04319

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2. Used sometimes: $|U_{e3}| < |U_{e1}|$, $|U_{e3}| < |U_{e2}|$, and $m_1 < m_2$
   - Makes a plot of solar and LBL-reactor neutrinos continuous
   - Says that all three mixing angles are $\theta_{ij} \in [0, 90°]$
   - Treats solar and atmospheric sectors differently
   - Means that there is one mass ordering question: sign of Δm^2_{31}, and one octant question: $\theta_{12} <> 45°$
   - Solar neutrinos told us that $\theta_{12} < 45°$

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   - Single clear definition of things we have measured well
   - Says that $\theta_{12} \in [0, 45°]$ while $\theta_{13}, \theta_{23} \in [0, 90°]$ Q: What is special about θ_{12}?
   - Means that there are two mass ordering questions: signs of Δm^2_{21} and Δm^2_{31}
   - Solar neutrinos told us $\Delta m^2_{21} > 0$
2. Used sometimes: $|U_{e3}| < |U_{e1}|$, $|U_{e3}| < |U_{e2}|$, and $m_1 < m_2$
   - Makes a plot of solar and LBL-reactor neutrinos continuous
   - Says that all three mixing angles are $\theta_{ij} \in [0, 90°]$
   - Treats solar and atmospheric sectors differently
   - Means that there is one mass ordering question: sign of Δm^2_{31}, and one octant question: $\theta_{12} <> 45°$
   - Solar neutrinos told us that $\theta_{12} < 45°$
3. Bad: $m_1 < m_2 < m_3$
   - Has been used in the past, now regarded as a bad definition
   - Makes the mass ordering question very complicated
   - Once we know both mass orderings, this may be the convention

PBD 2003.04319

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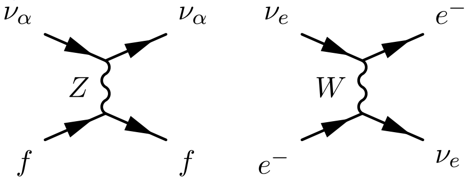
- Neutrinos propagate in a new basis as mass states are no longer eigenstates of the Hamiltonian

Hamiltonian Dynamics: With Matter



Neutrino energy levels are modified by presence of background fermion fields

These diagrams are dominantly forward ($\theta = 0$) elastic (particles in = particles out)

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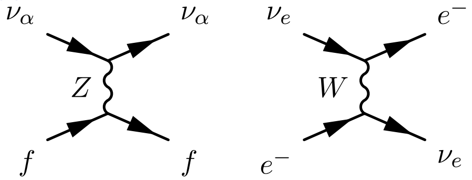

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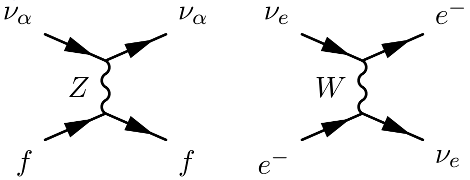

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$$H_{\rm flav}(t) = \frac{1}{2E}\left[U\begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix} a(x) & & \\ & 0 & \\ & & 0\end{pmatrix}\right]$$

Matter effect: $a(x) = 2\sqrt{2}G_F N_e(x) E$
As $a \to 0$, we recover earlier vacuum calculations

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In matter ν's propagate in a new basis that depends on $a \propto N_e E_\nu$.
Assume $H_{\rm flav}(x) \simeq H_{\rm flav}$ is approximately constant:

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Assume $H_{\rm flav}(x) \simeq H_{\rm flav}$ is approximately constant:

$$\mathcal{A} = e^{-iH_{\rm flav}L} = V\Lambda V^\dagger$$

V is the unitary matrix composed of normalized eigenvectors of H
Λ is the diagonal matrix composed of $e^{-i\lambda_i L}$ where λ_i are the eigenvalues of H

Matter Effect: Varying

Solar neutrinos in an adiabatically changing matter potential

Solution = MSW effect

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Neutrinos in supernovae experience MSW effect too,
but they also experience neutrino-neutrino interactions

Propagation in SNe is much more involved

Matter Effect Open Questions

- Matter has only been measured in the Sun, by combining solar with reactor. Confirm it in the Earth.
- New physics that looks like the matter effect takes the form of vector non-standard neutrino interactions: vNSI. Is there evidence for this? Can we constrain it?
- There is a degeneracy within new physics (vNSI) related to measuring the matter effect and the mass ordering ($\operatorname{sgn}\Delta m^2_{31}$) called LMA-Dark. Can we rule it out/discover it?
- How well do we know the density of the Earth/Sun?

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7th parameter: absolute mass scale

Cosmology, KATRIN, $0\nu\beta\beta$