$\pi\pi$ scattering

- Connecting Multi-hadrons to Precision Observables

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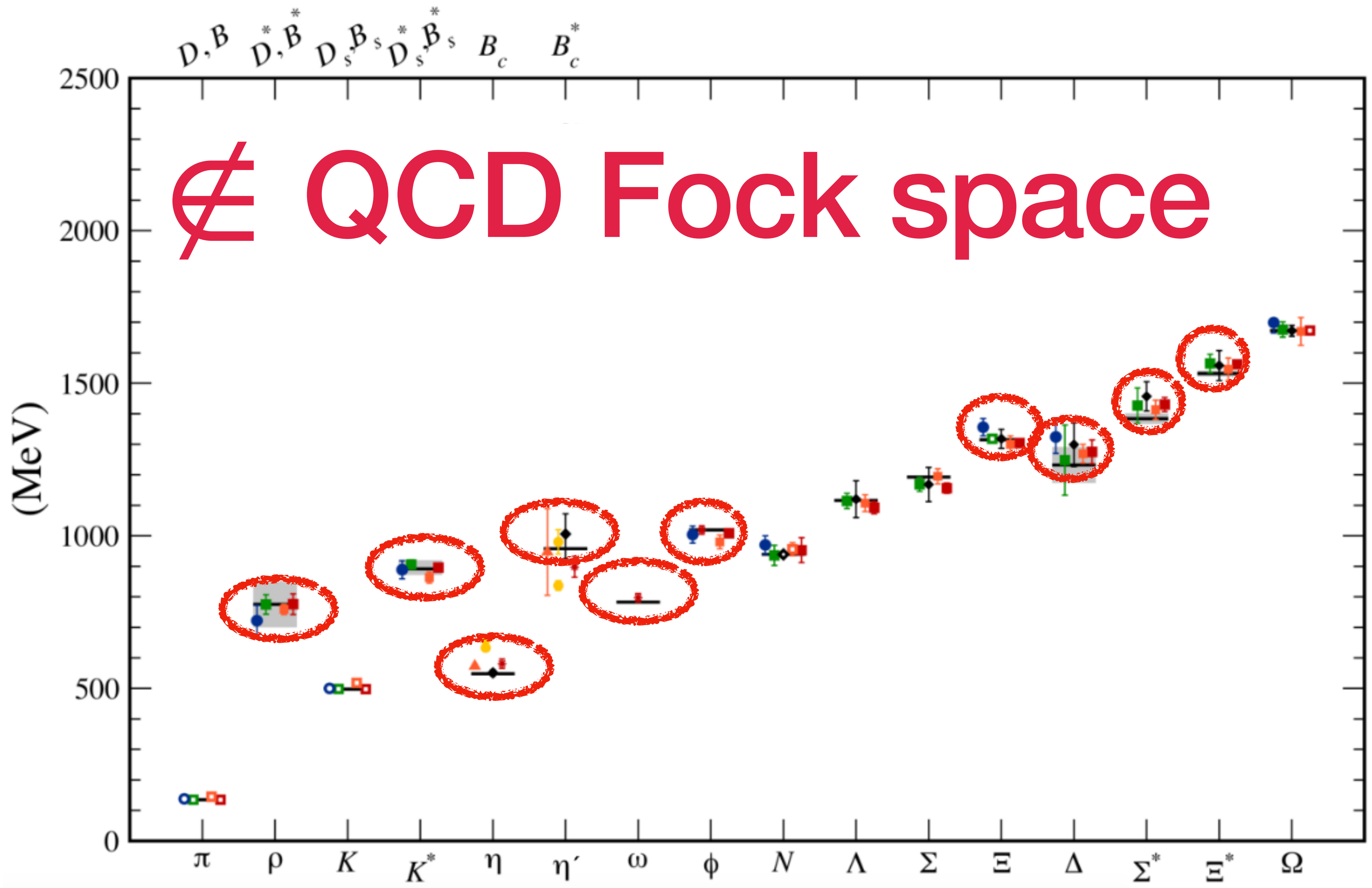
RESEARCH & INNOVATION FOUNDATION
EXCELLENCE/0524/0459

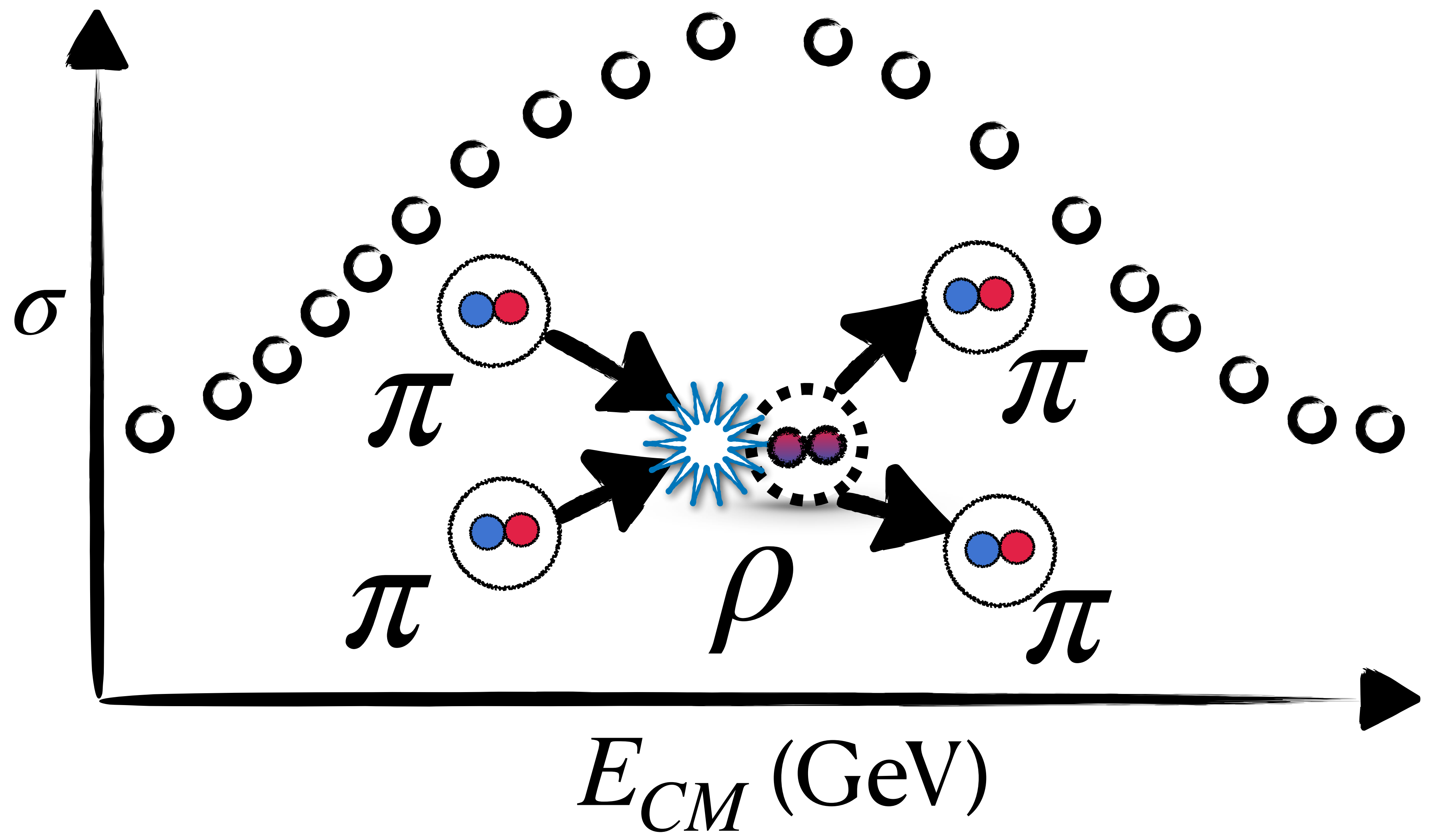
University of Cyprus

Extended Twisted Mass Collaboration

Hadron spectrum



A. S. Kronfeld, Ann. Rev. Nucl. Part. Sci. 62, 265–284 (2012) [1203.1204]

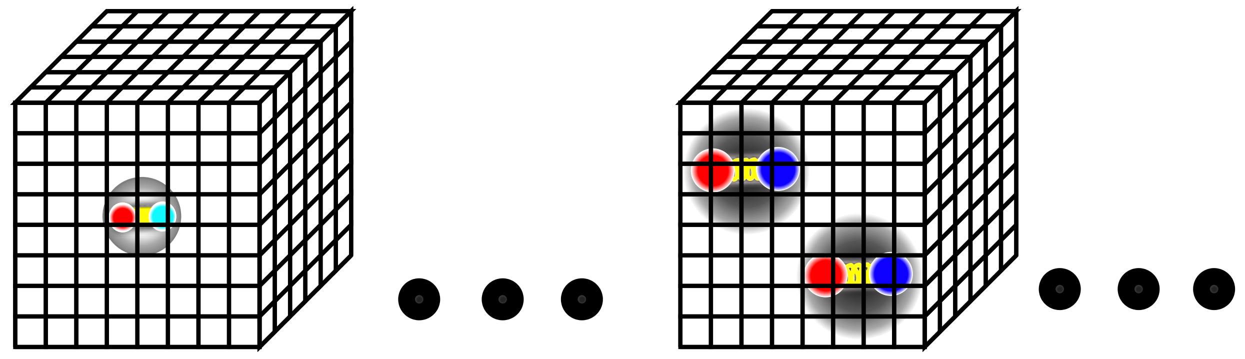


- Dynamical Observable $\not\longrightarrow$ Lattice [NOT Asymptotic]
- Most hadrons are unstable

Hadron spectrum with excited states

Motivated from the scattering Experiments

Multihadron Op.



Same Quantum no.?

Hadron spectrum with excited states

Motivated from the scattering Experiments

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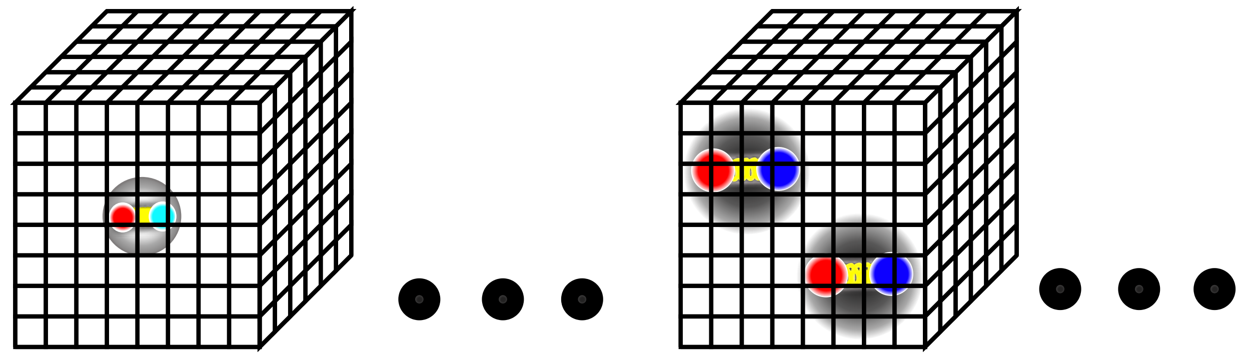
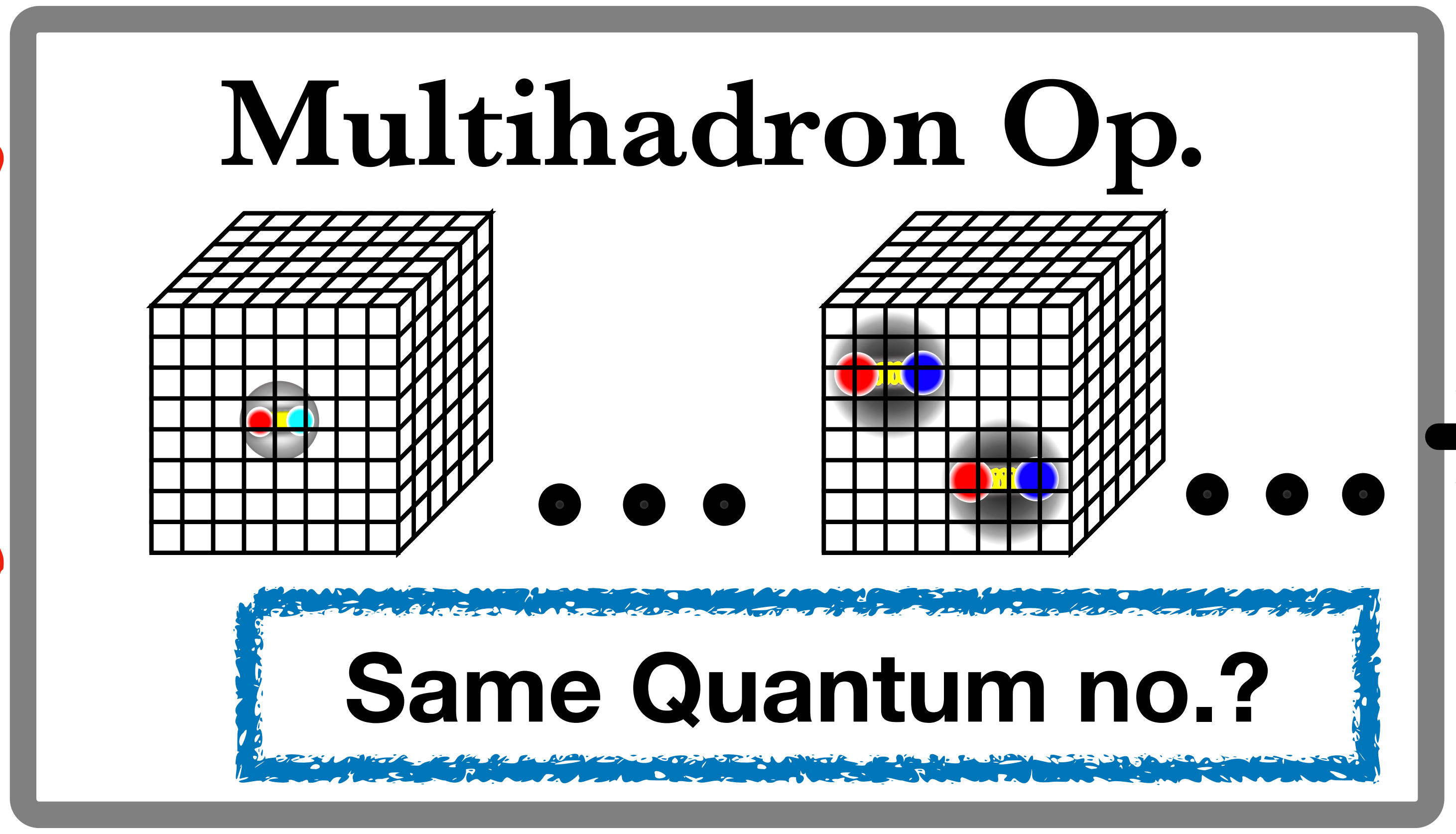


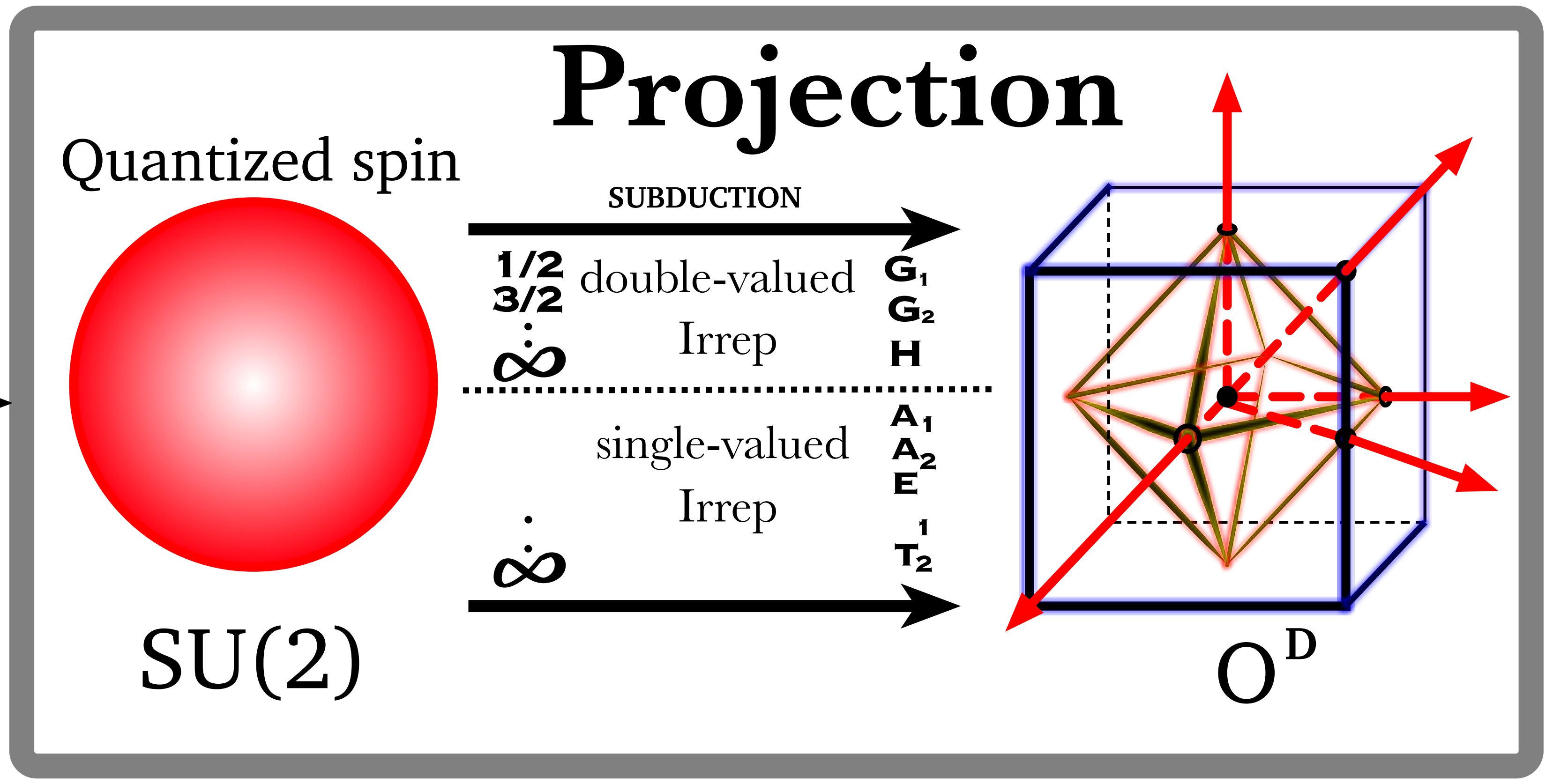
Same Quantum no.?

Same Quantum no.?
But the two hadrons can't move freely on the lattice!
Orbital Angular momentum constrained!

Hadron spectrum with excited states

Motivated from the scattering Experiments





Hadron spectrum with excited states

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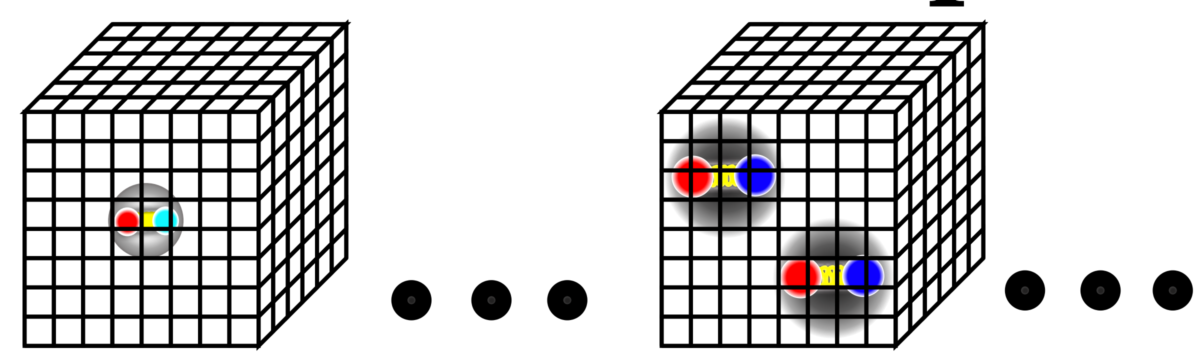
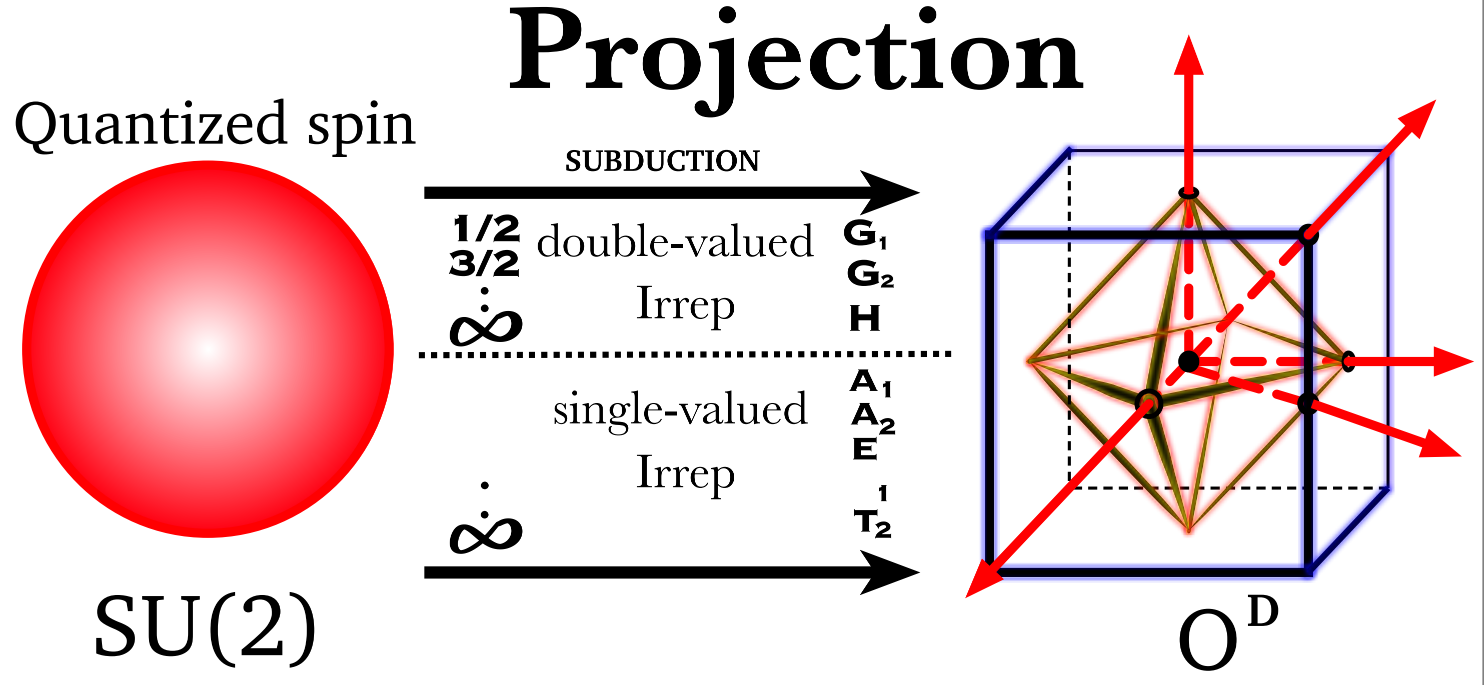
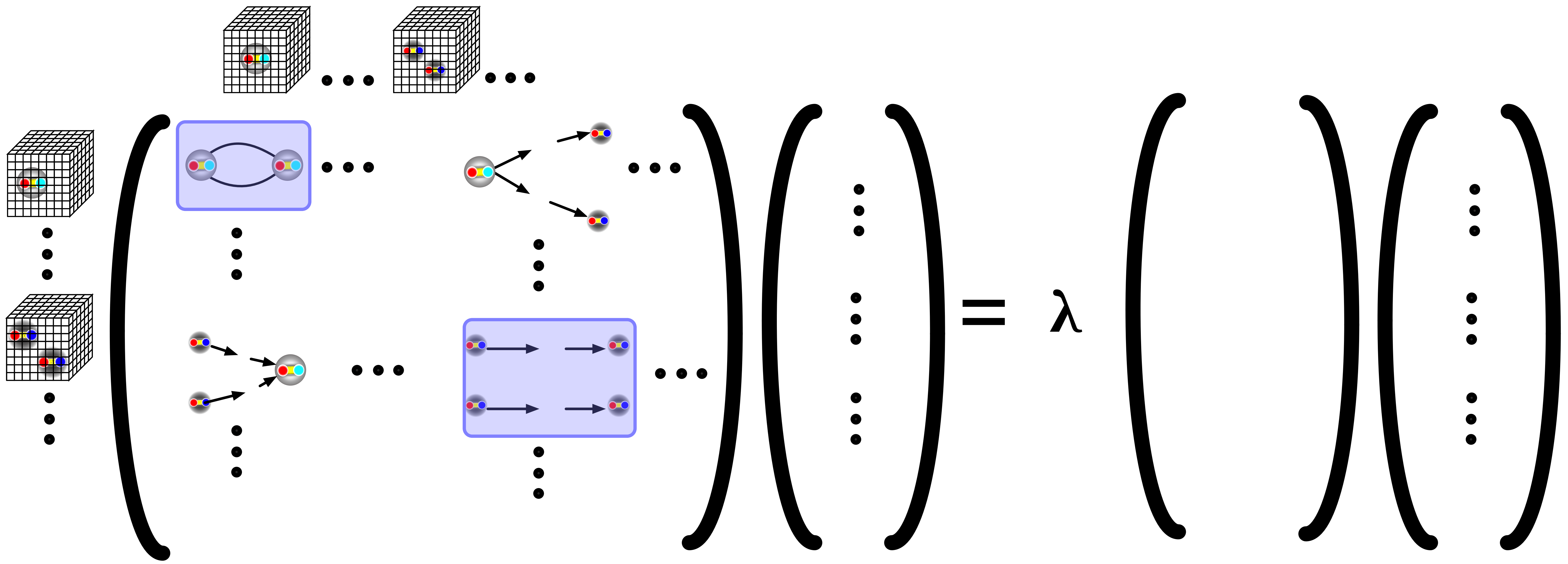


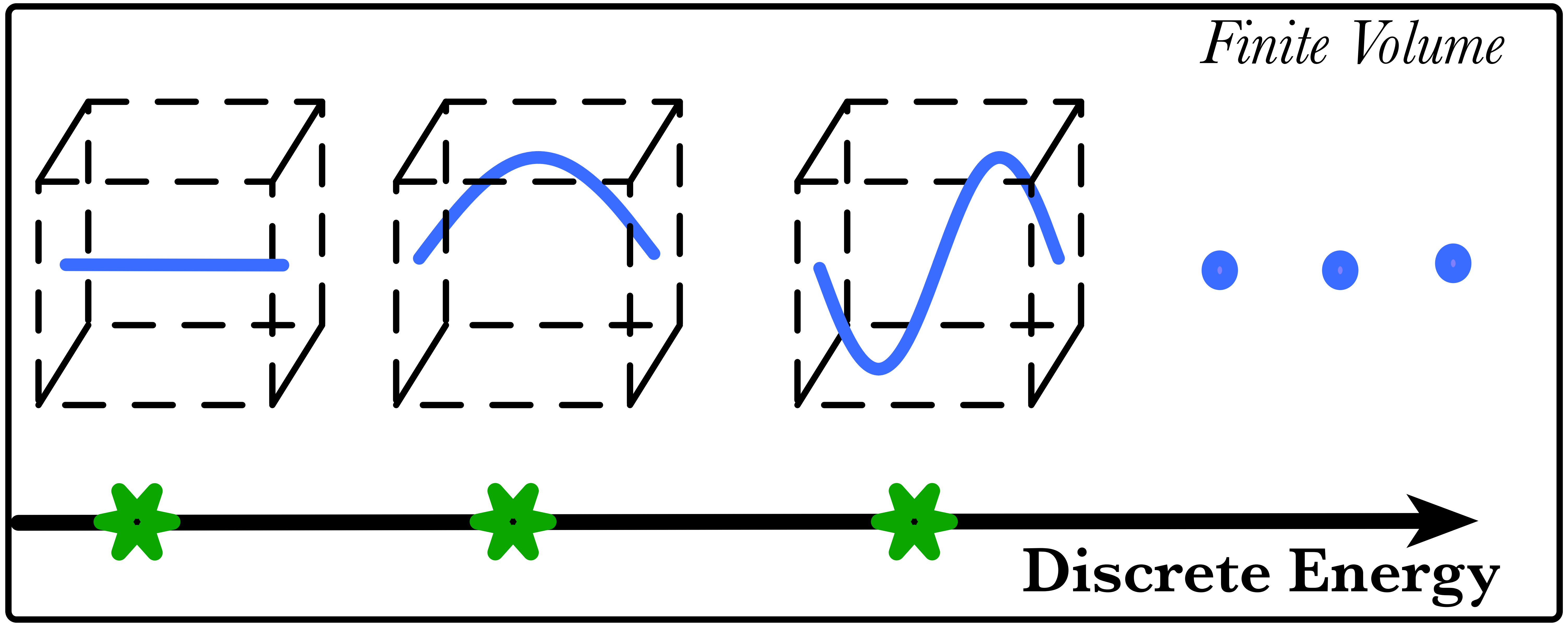
Projection

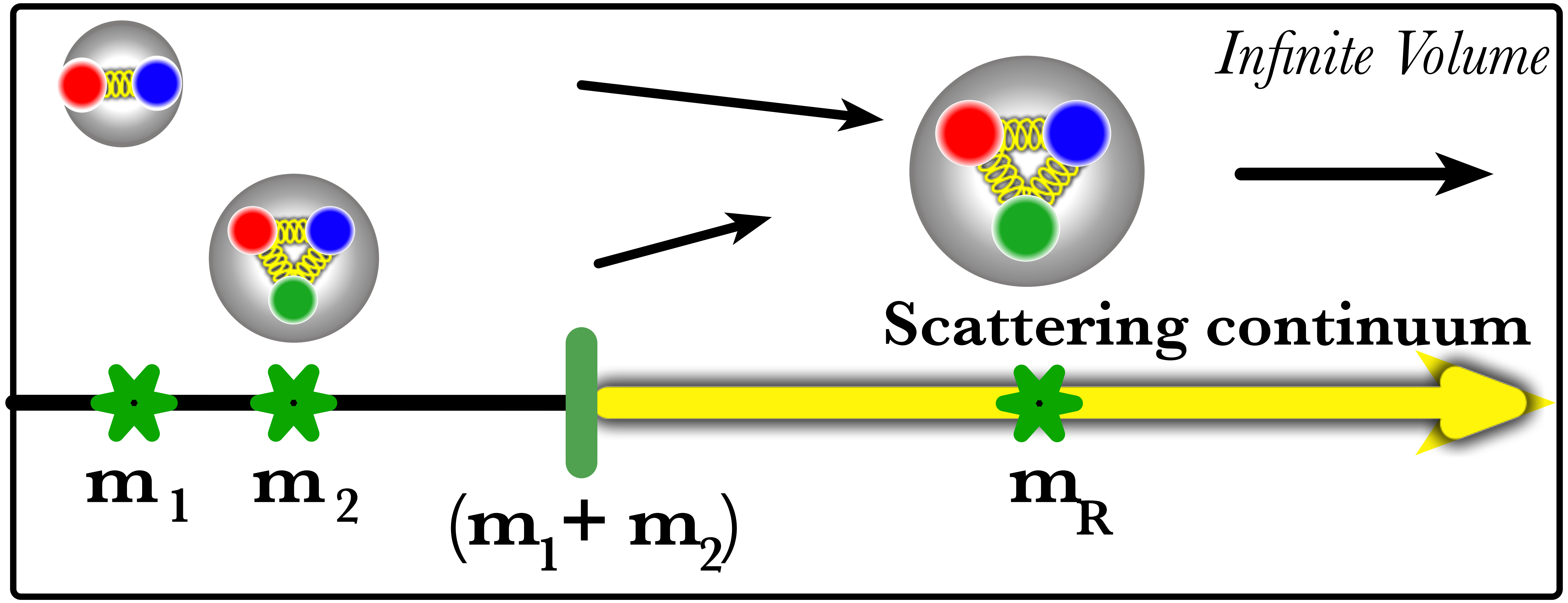




GEVP: $C(t)\, v(t,t_0) = \lambda(t,t_0)\, C(t_0)\, v(t,t_0)$

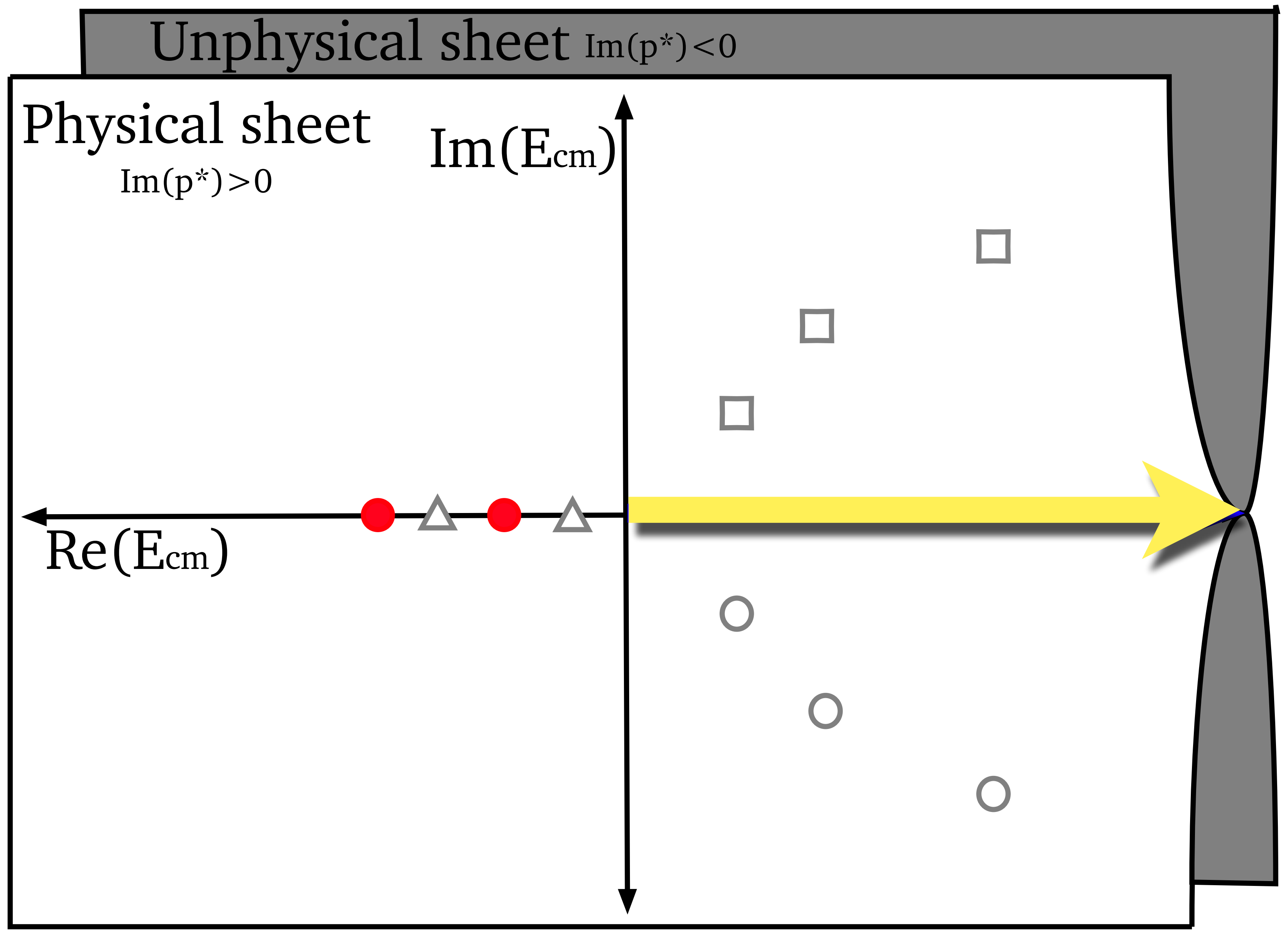
Hadron spectrum with scattering states



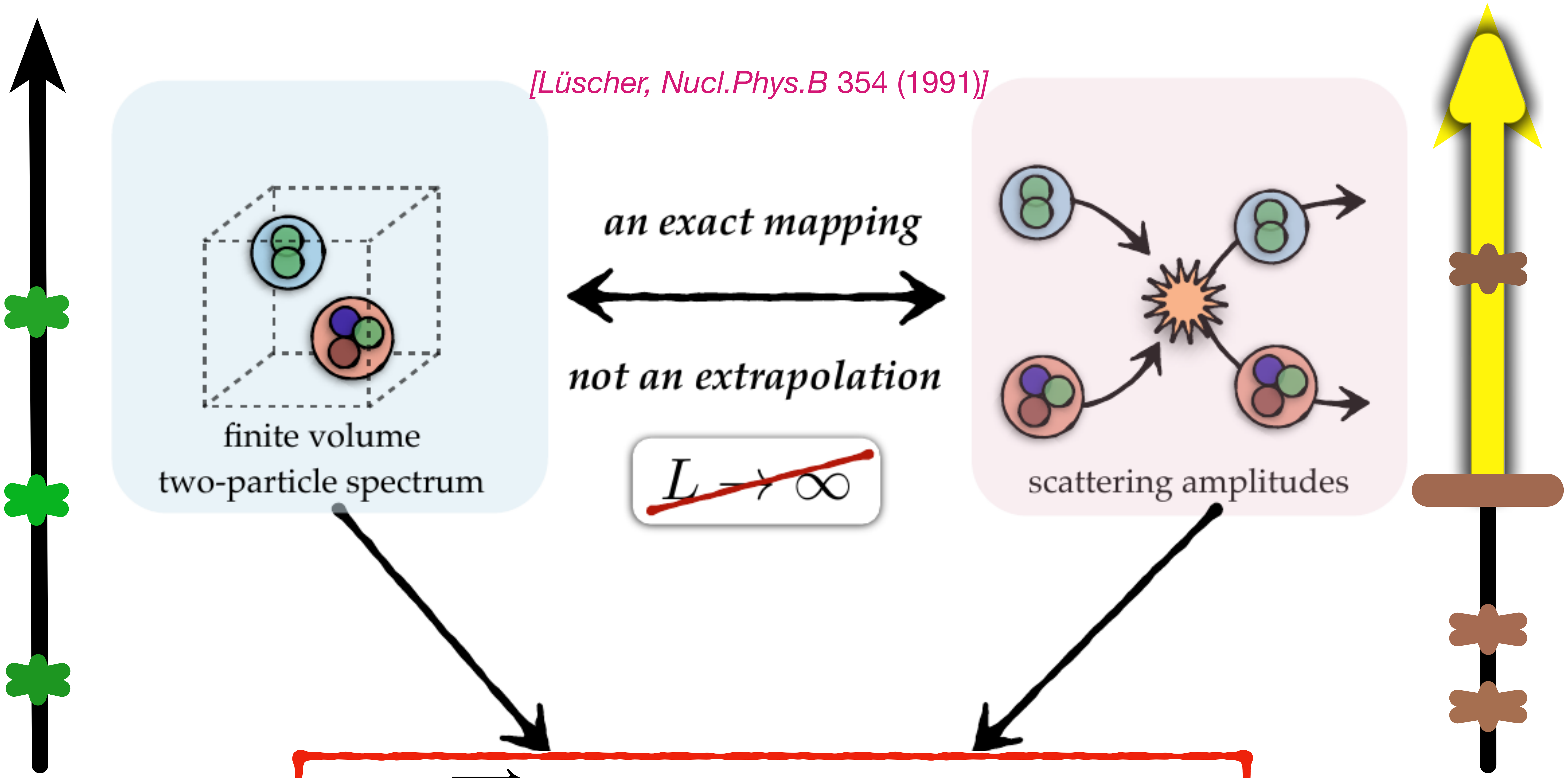


Resonances/unstable states
: Not eigenstates of the system
$\Longrightarrow$ Imaginary energies

How to access information about unstable particles in scattering continuum?



Elastic scattering states



$$\mathscr{M}_{\ell}^{\overrightarrow{P},\Lambda}(E,L) = \cot\delta_{\ell}$$

Elastic scattering states

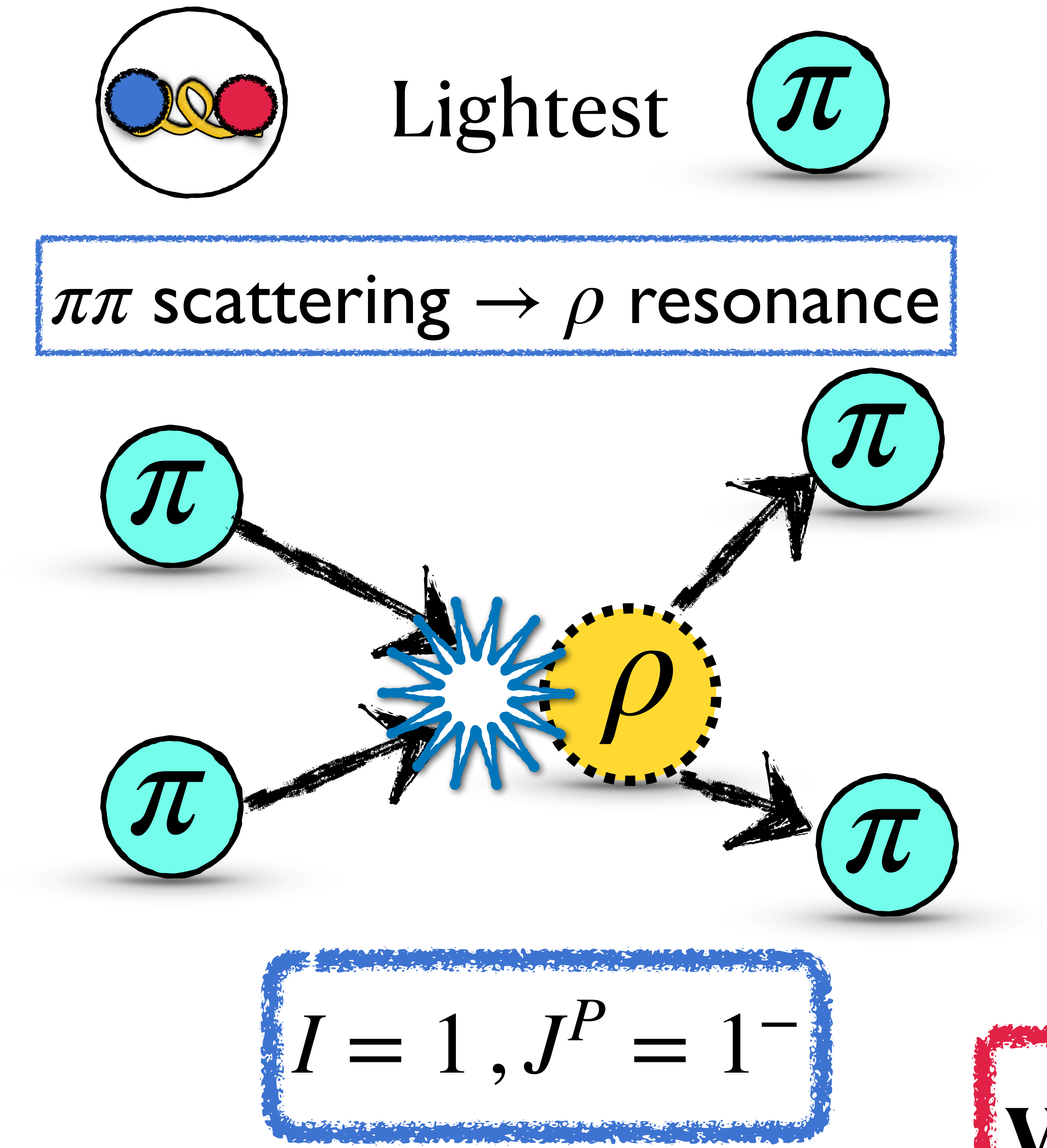
Lüscher Quantization Condition

[Lüscher, Nucl.Phys.B 354 (1991)]

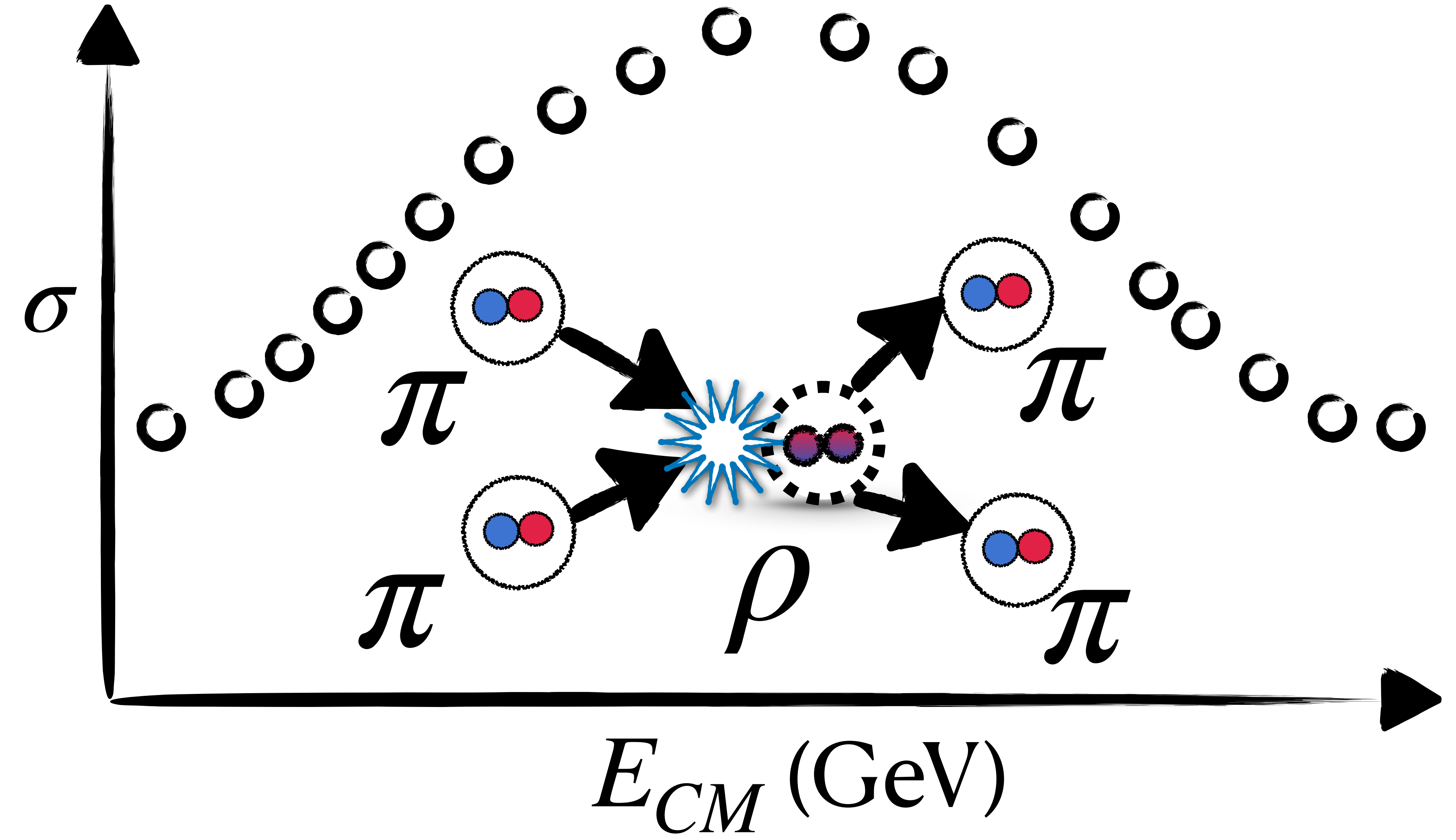
- Interaction range $< L$
- Elastic scattering
- Periodic box
- Upto Exponential corrections
- $a = 0$

$$\mathscr{M}_{\ell}^{\vec{P},\Lambda}(E, L) = \cot \delta_{\ell}$$

I=1 $\pi\pi$ scattering



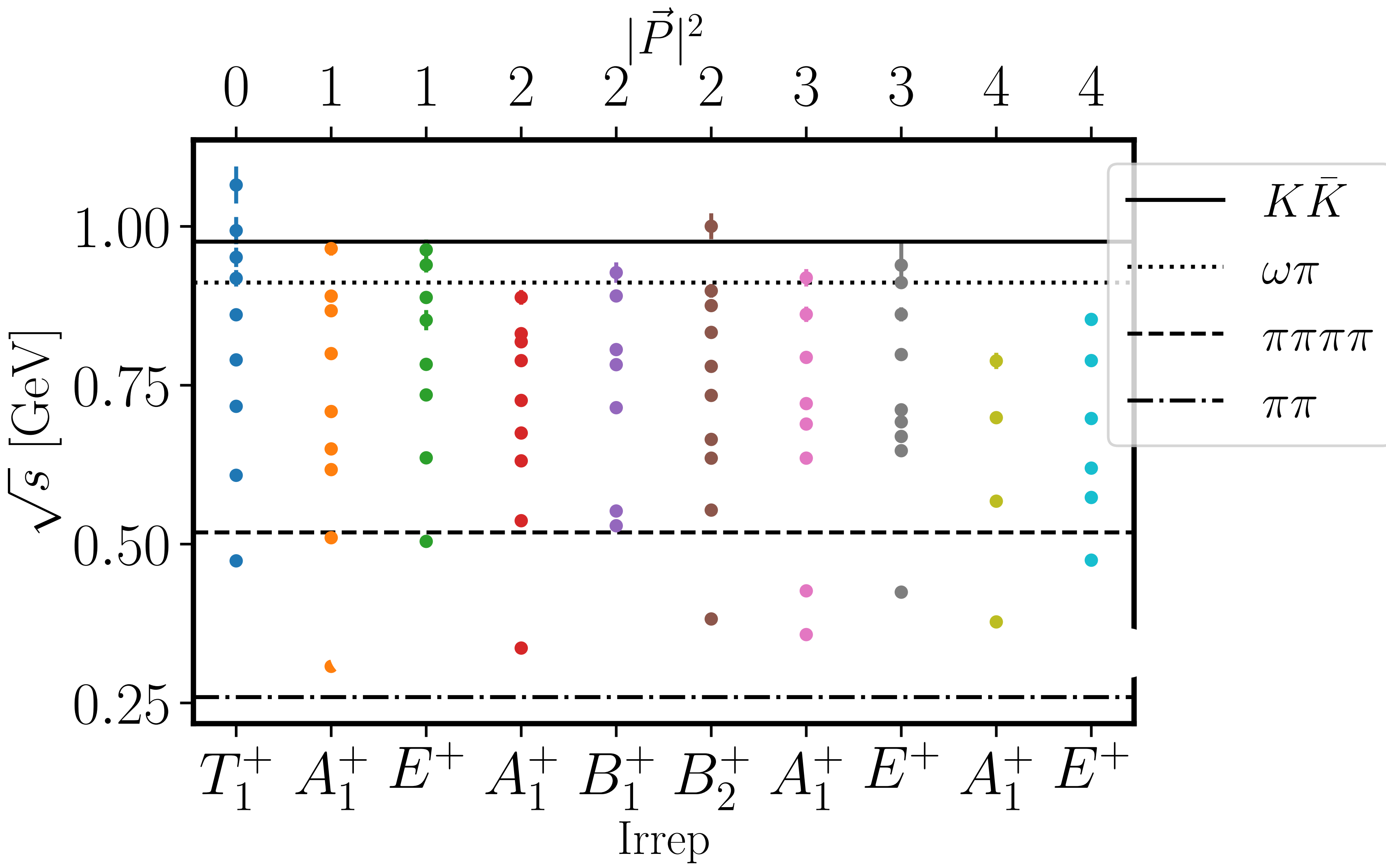
Lightest Meson **Resonance** $\rho(770)$



<u>**Vector Meson Dominance**</u>**:** $\pi\pi$ affects ALL calculations with vector current at <u>**long distances.**</u>

What we can do now...

Finite Volume Spectrum

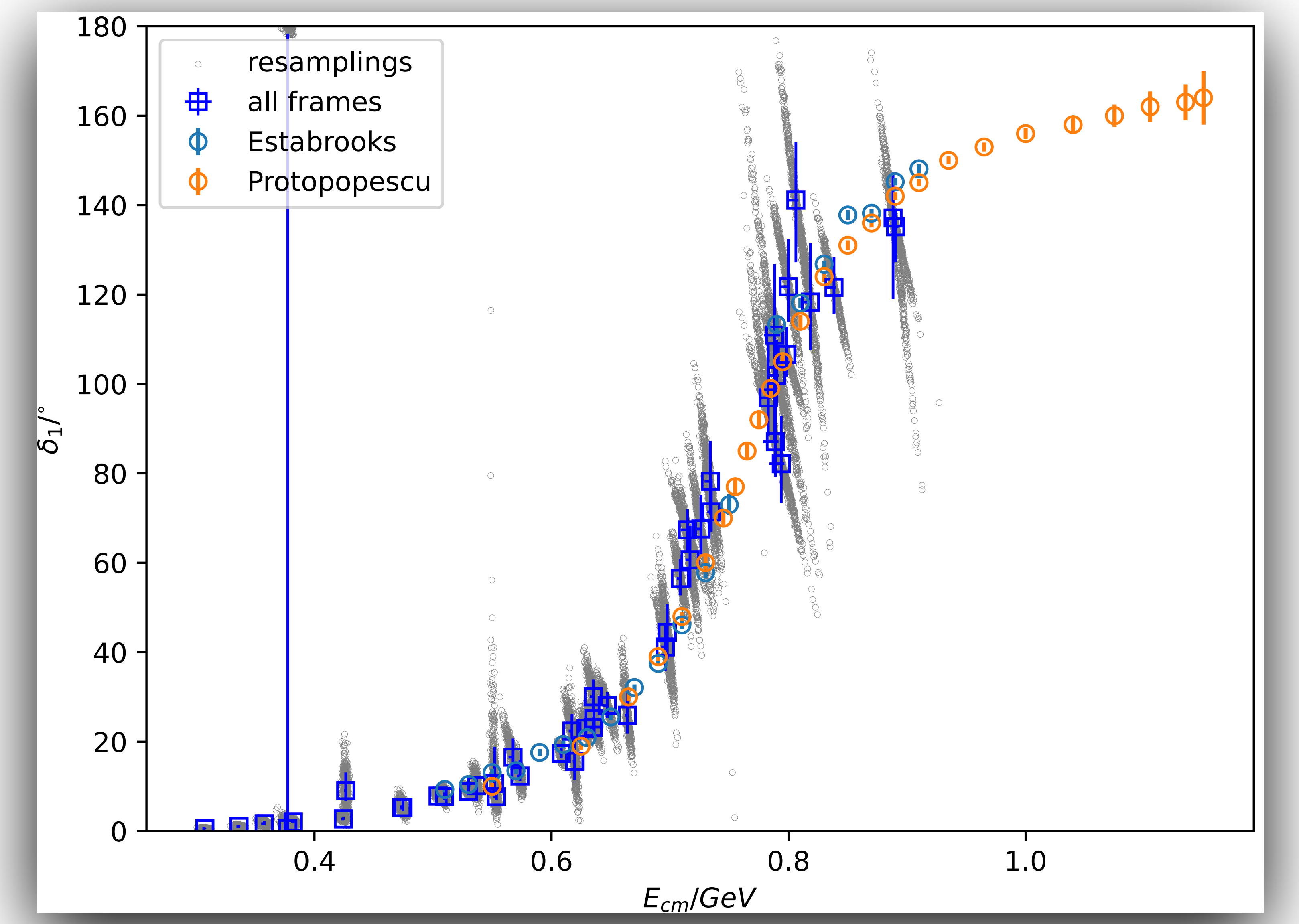


Lattice Box : $96^3 \times 192$
L : 6.2 fm
a : 0.06426 fm
$m_\pi L$: 4.1
Number of configurations: 505
m_π ~ Physical

Gauge Action: Lüscher Wiese
Gauge $\mathcal{O}(a^2)$
Fermion Action: Wilson clover
$\mathcal{O}(a)$
Boundary: Periodic

~~$L \to \infty$~~ ~~$m_{quark} \to$ Phys.~~

$E_n(L) \to$ Lüscher $\to \delta_1(E)$



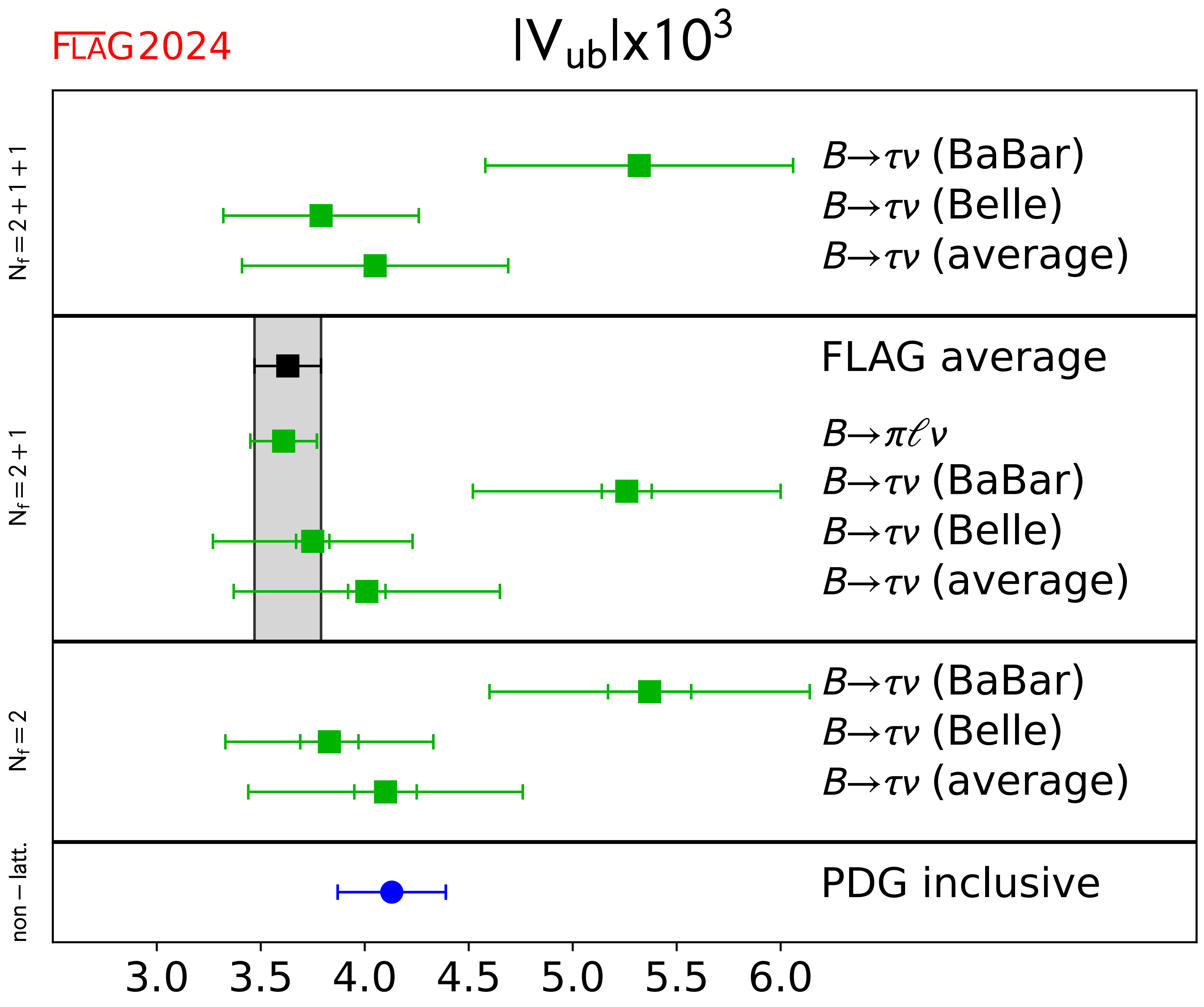
$a \to 0$

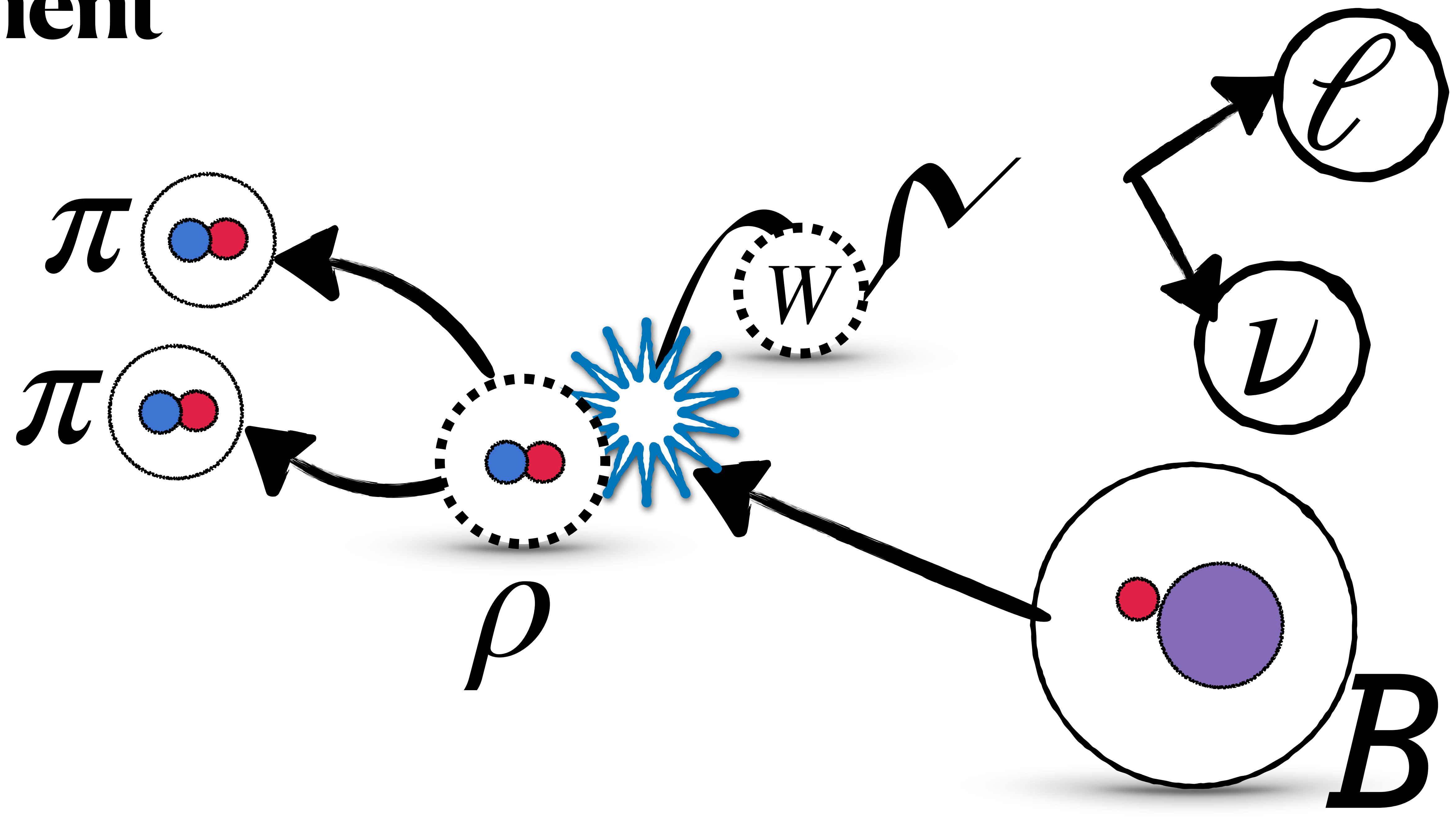
Finite Volume Spectrum is a feature: Scattering $\rightarrow$ Spectroscopy

LHPC + Mainz

[*PoS* LATTICE2021 (2022)]

Final $\pi\pi$ states : Resonance enhancement





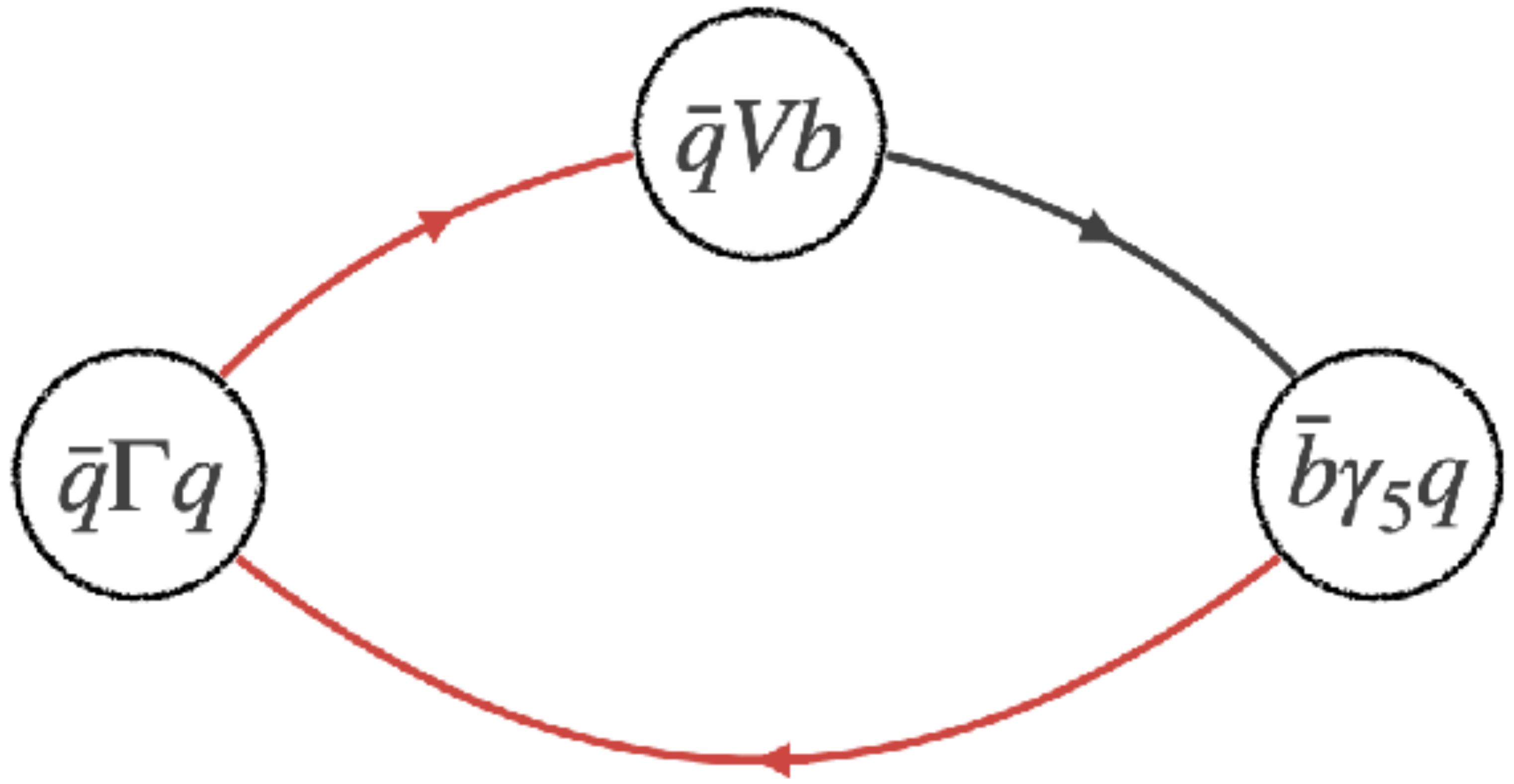
W Boson changes Quark flavor

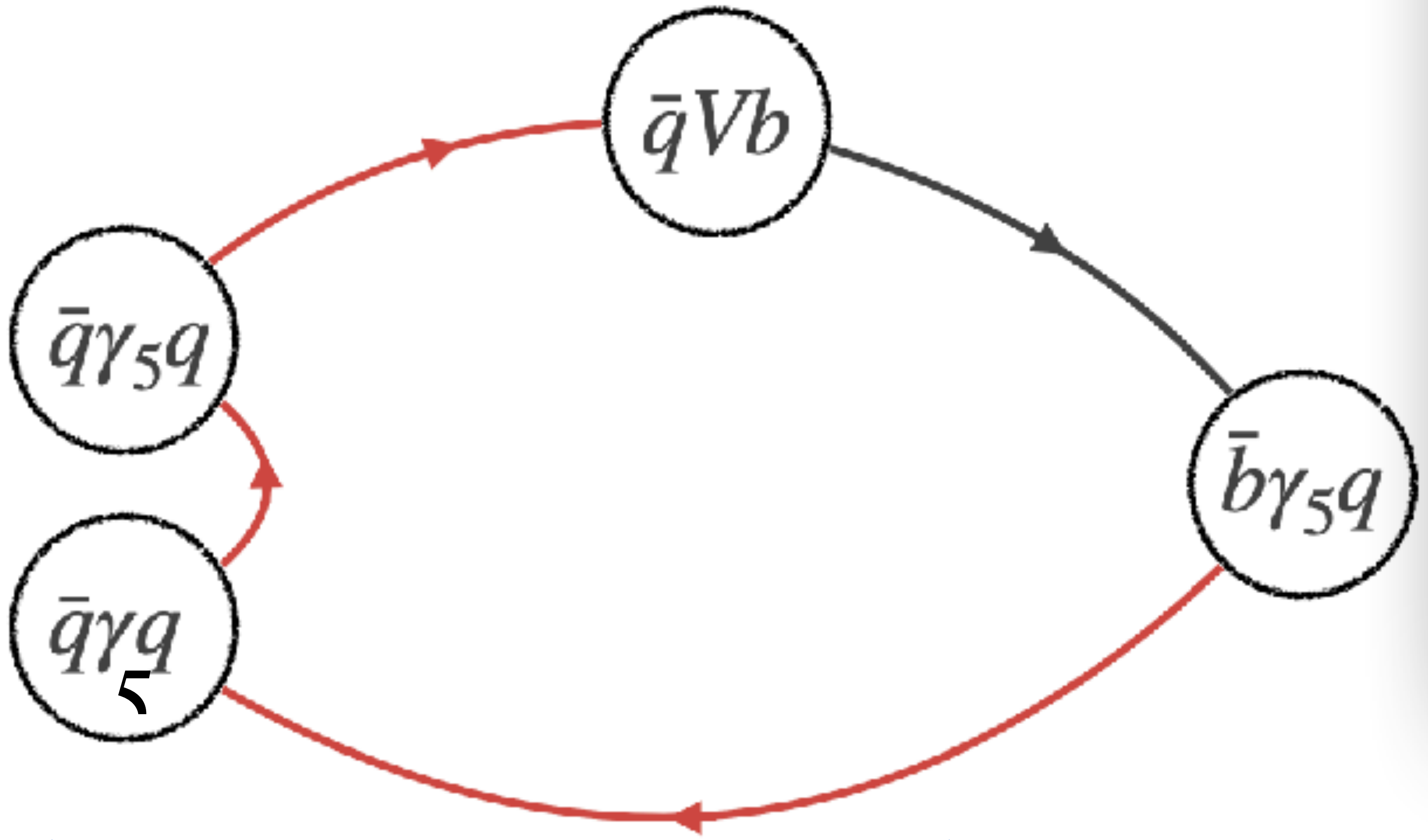
[Exp.] = [Known] x $[V_{ub}]^2$ x **[Hadronic Matrix element]**^{2}

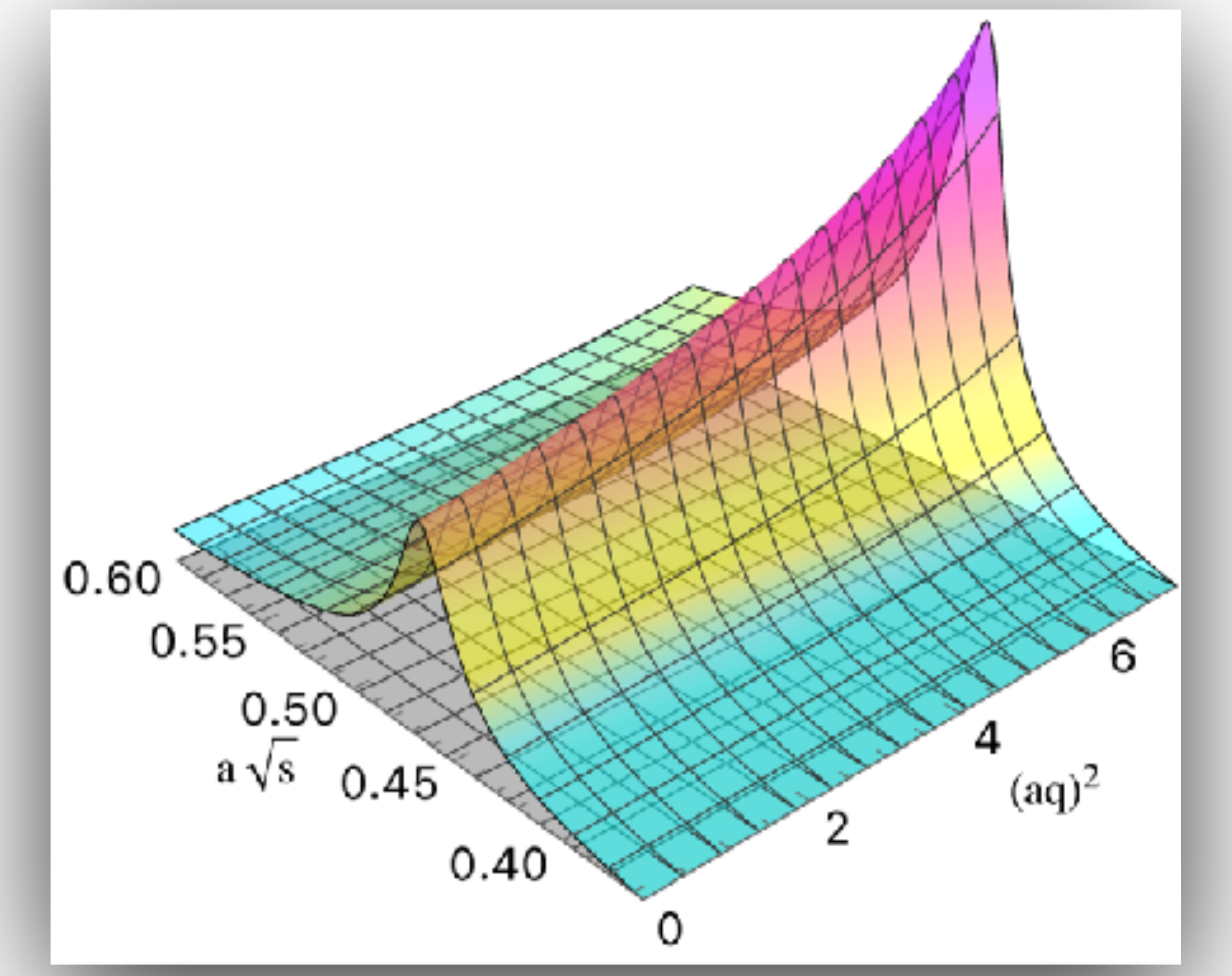
unitary Cabibbo-Kobayashi-Maskawa quark mixing matrix

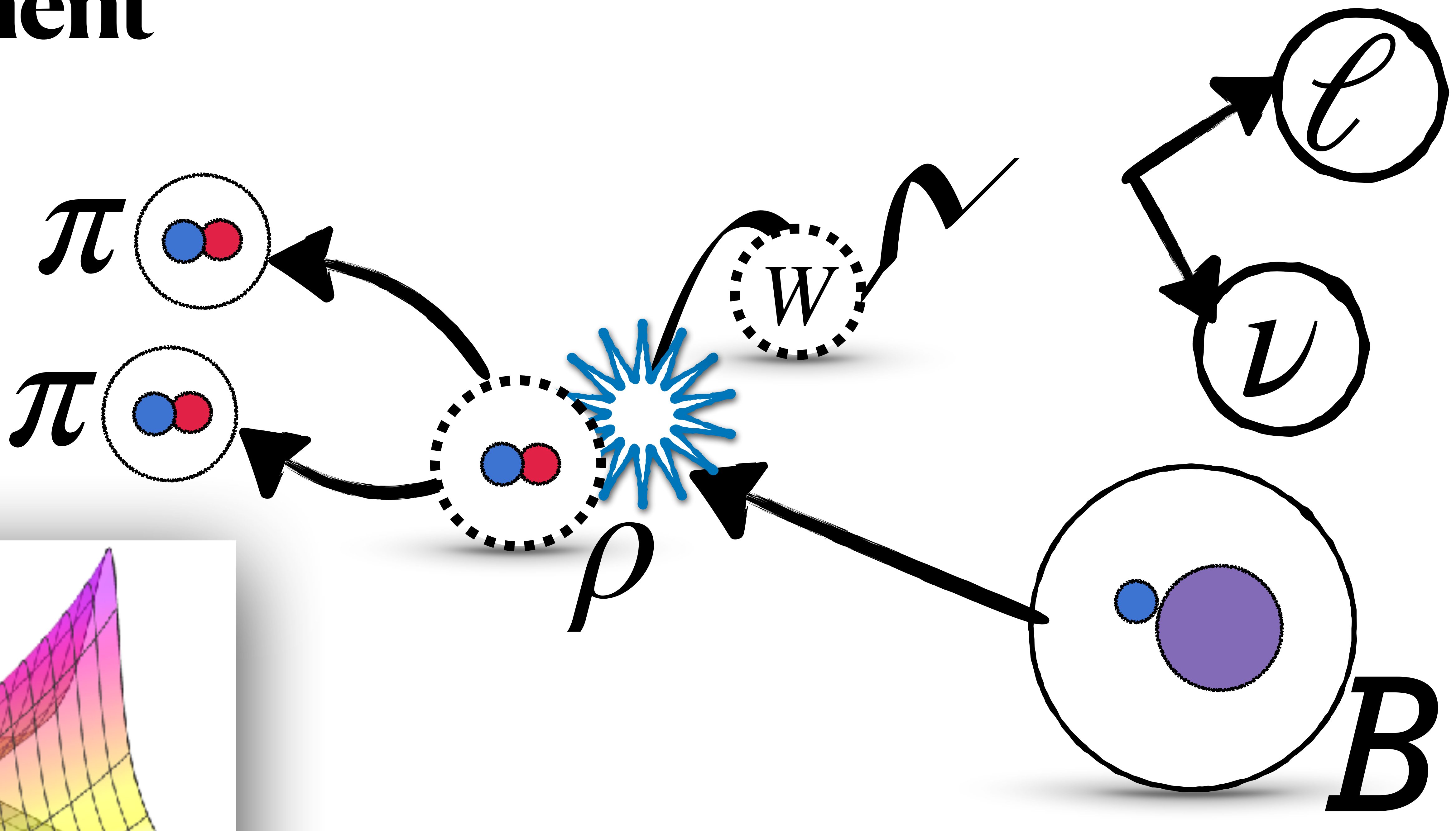
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

Final $\pi\pi$ states : Resonance enhancement









[Exp.] = [Known] x $[V_{ub}]^2$ x [**Hadronic Matrix element**]^2

LHPC Cyprus

From Amplitudes to Flavor Physics

$B \to (\rho)\pi\pi\ell\nu \quad \to |V_{ub}|$

✓ Optimized $\pi\pi$ basis for $B \to \pi\pi\ell\nu$ 3-pt functions [*Phys. Rev. Lett.* (2025)]

Muon Anomalous Magnetic Moment $(g-2)$: Precision Observable

Muon $(g-2)$ Exp.

$$a_\mu = \frac{(g-2)_\mu}{2}$$

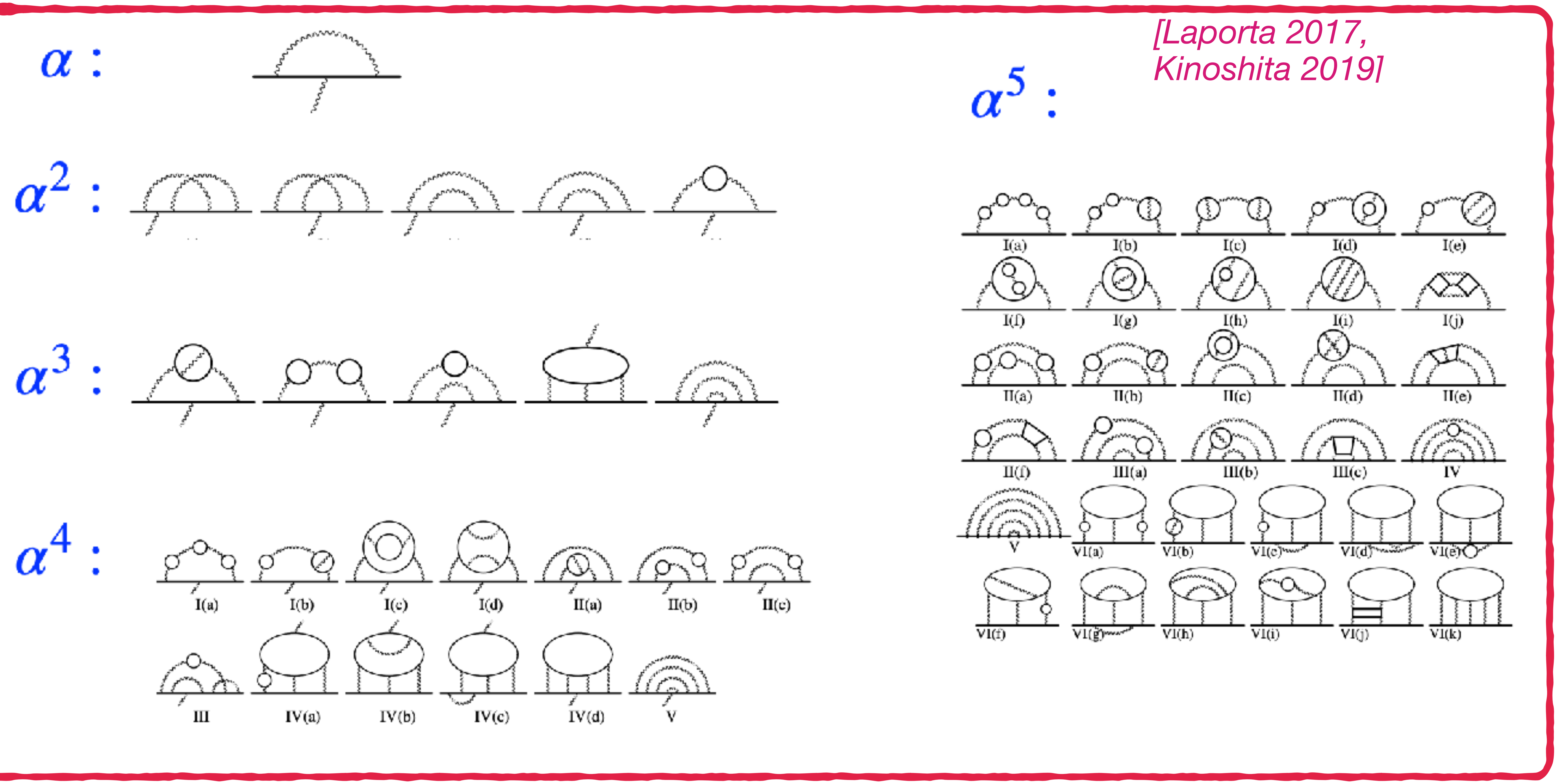
✓ Unambiguous
✓ Dimensionless
✓ Test Standard Model

Extreme Precision

Exp.

Theory

QED
+
Electroweak
+
Hadronic light-by-light



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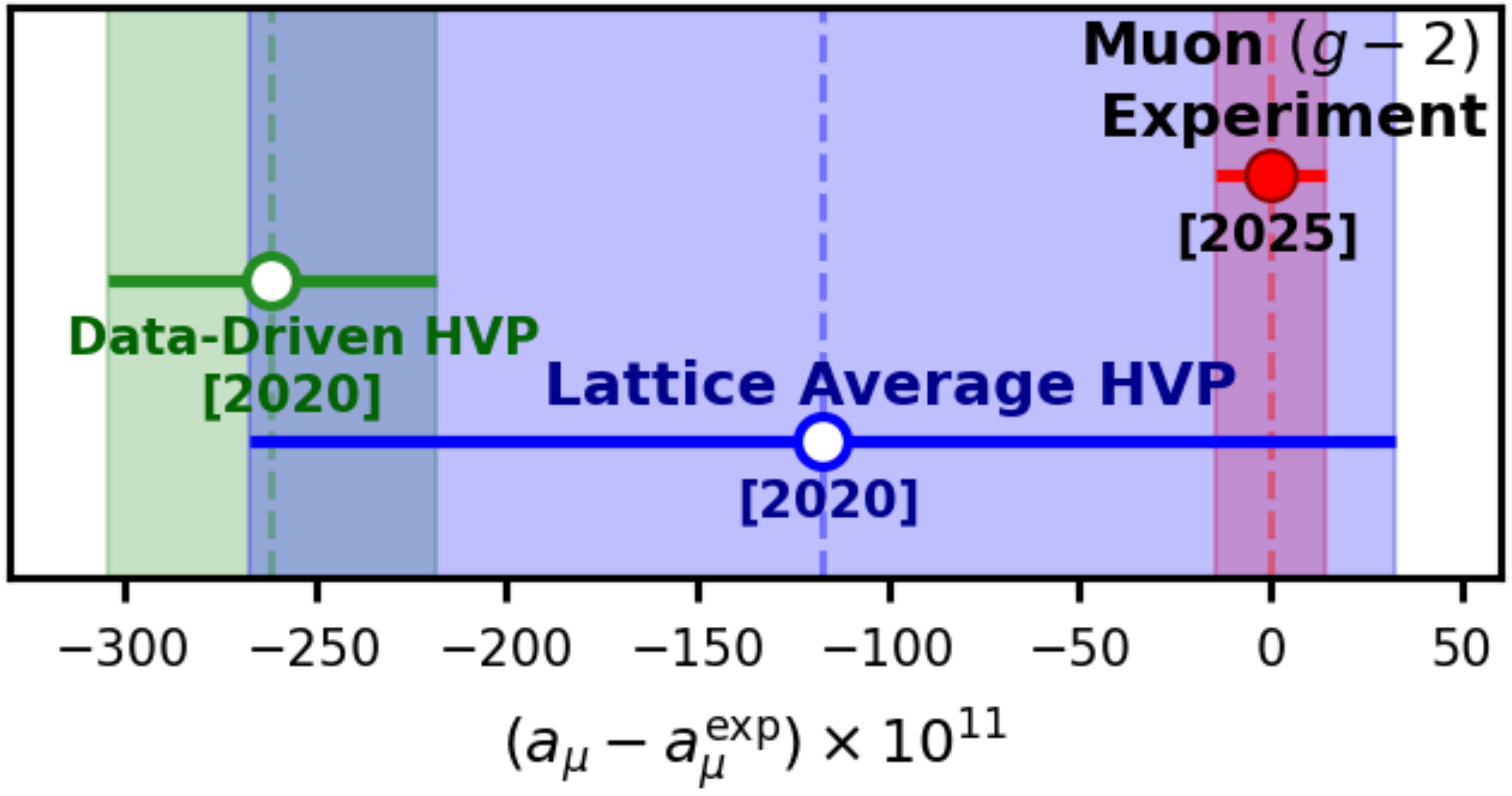
Extreme Precision



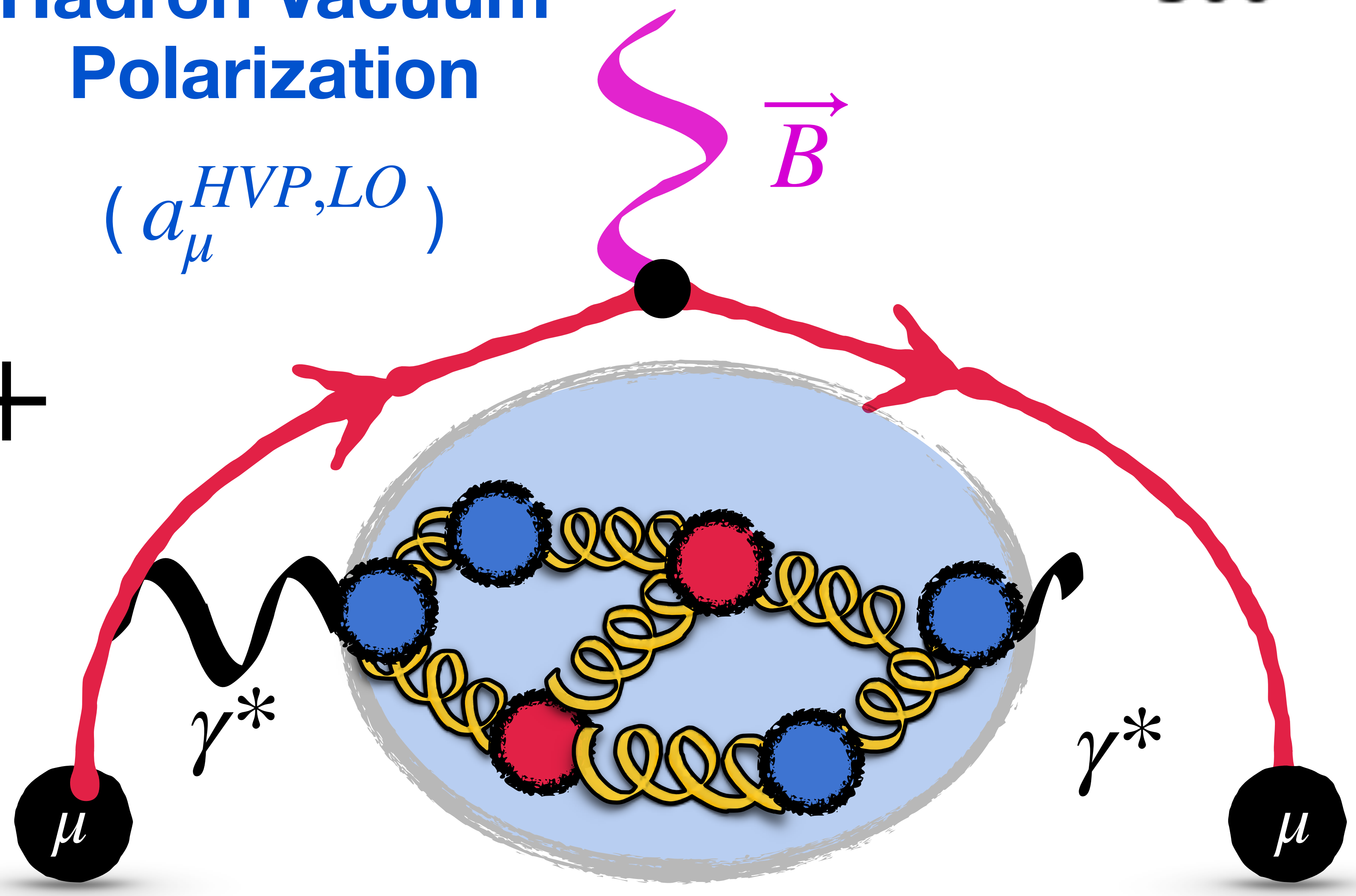
Theory

QED
+
Electroweak
+
Hadronic light-by-light

+

Hadron Vacuum Polarization

$(a_\mu^{HVP,LO})$



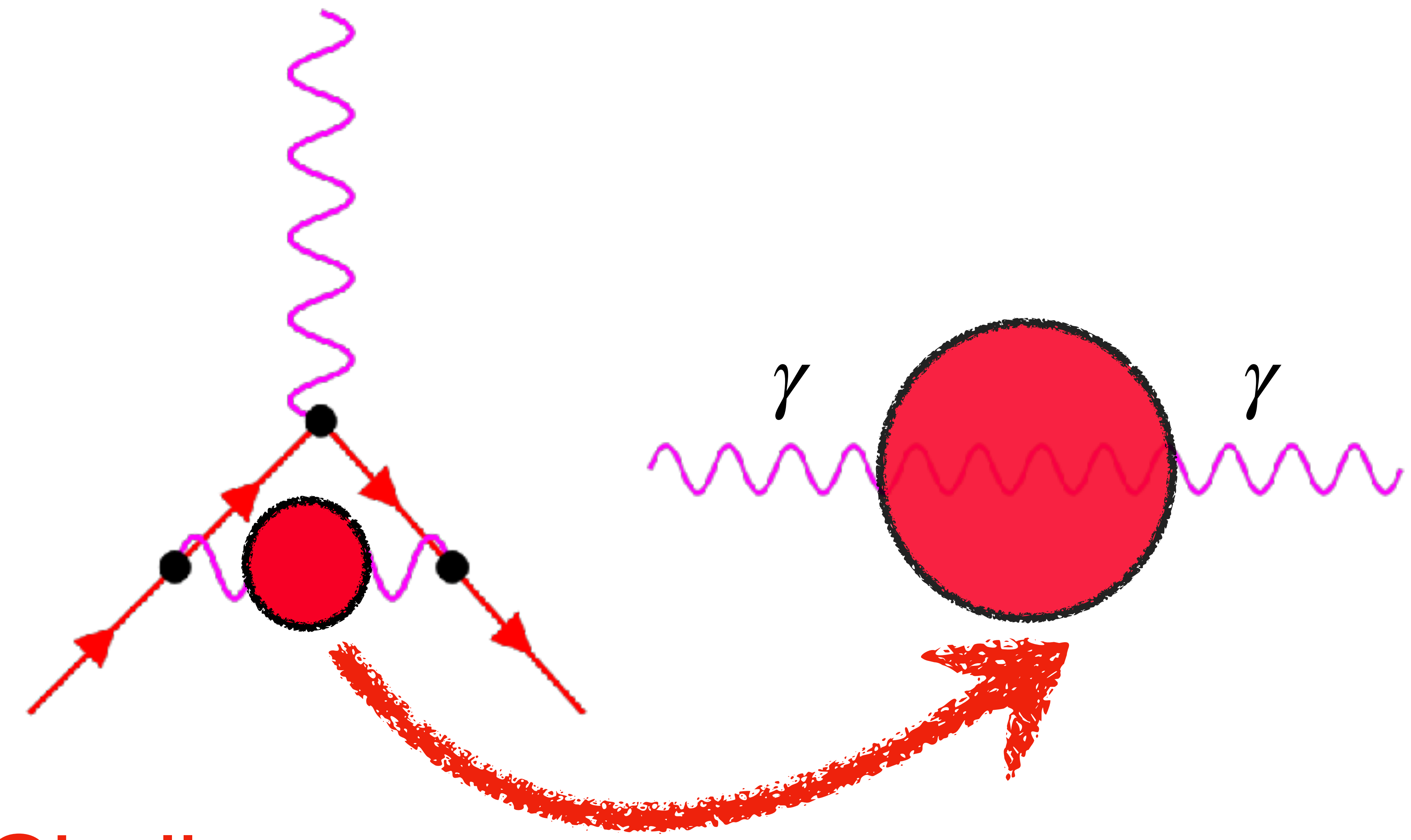
Lattice HVP dominates Error

↓

Window to control and disentangle

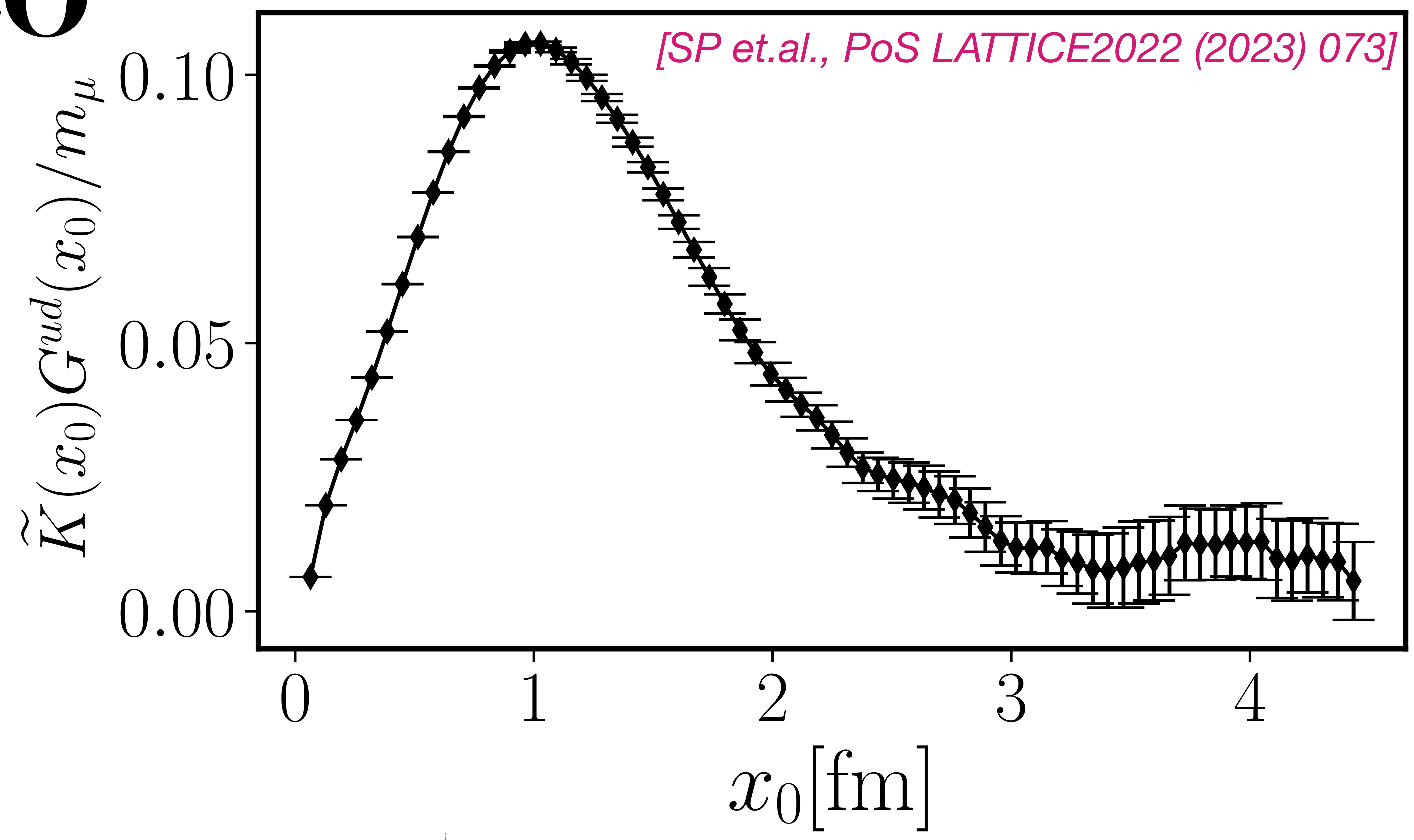
short • **intermediate** • **long**

Computing the $a_\mu^{\mathrm{HVP,\,LO}}$



Challenge

- Exponentially decreasing Signal-to-Noise $x_0 \to \infty$
- Correct Finite Volume effects
- Correct discretization effects
- Quark disconnected diagrams: stat & stoch noise
- Isospin breaking



$$\left(a_\mu^{\mathrm{hvp}}\right)^f = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dx_0 \frac{G\left(x_0\right)\,\widetilde{K}\left(x_0\right)}{m_\mu}$$

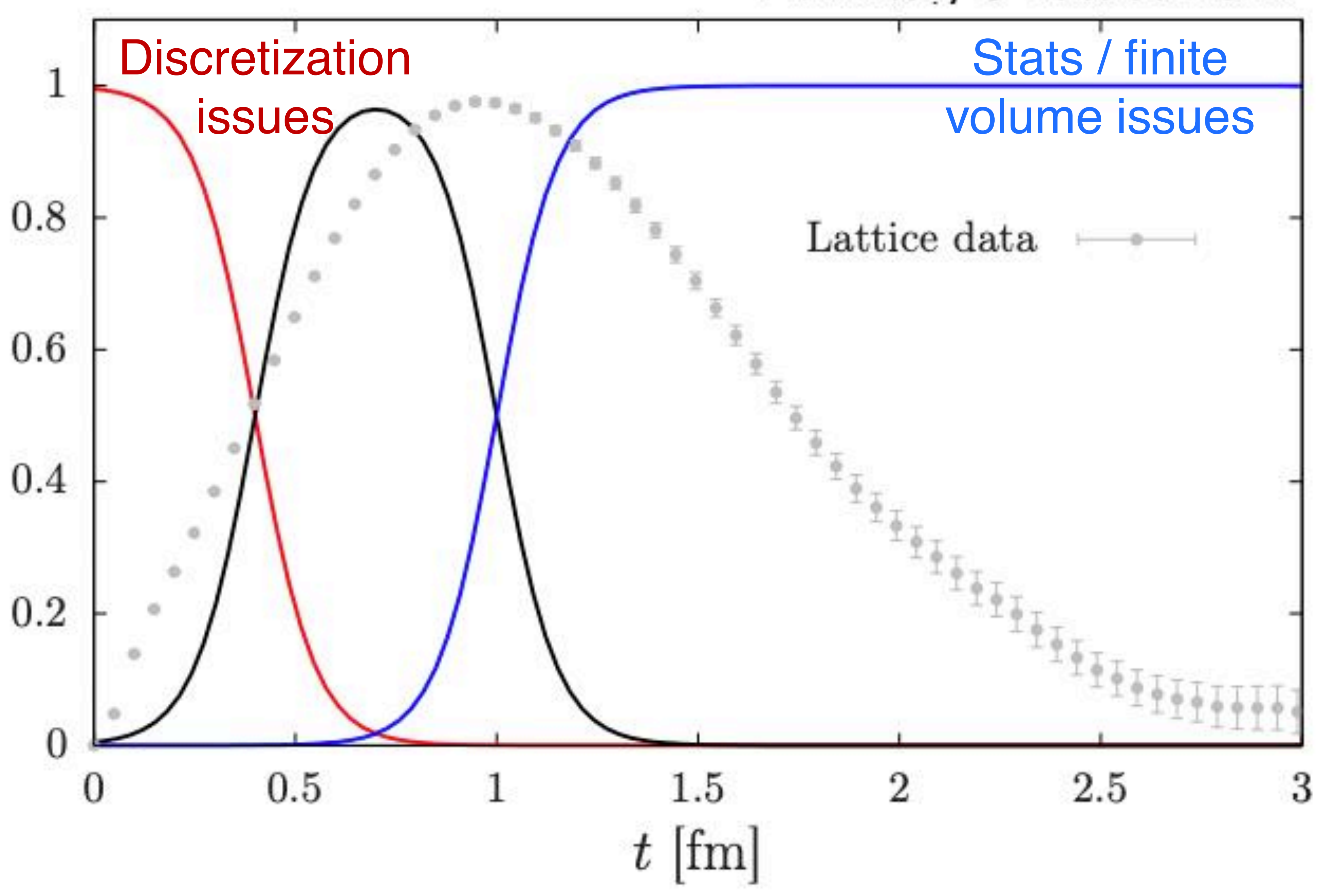
$$G(x_0) = \sum_k \sum_{\vec{x}} \left\langle j_k^{\mathrm{em}}(x) j_k^{\mathrm{em}}(0) \right\rangle$$

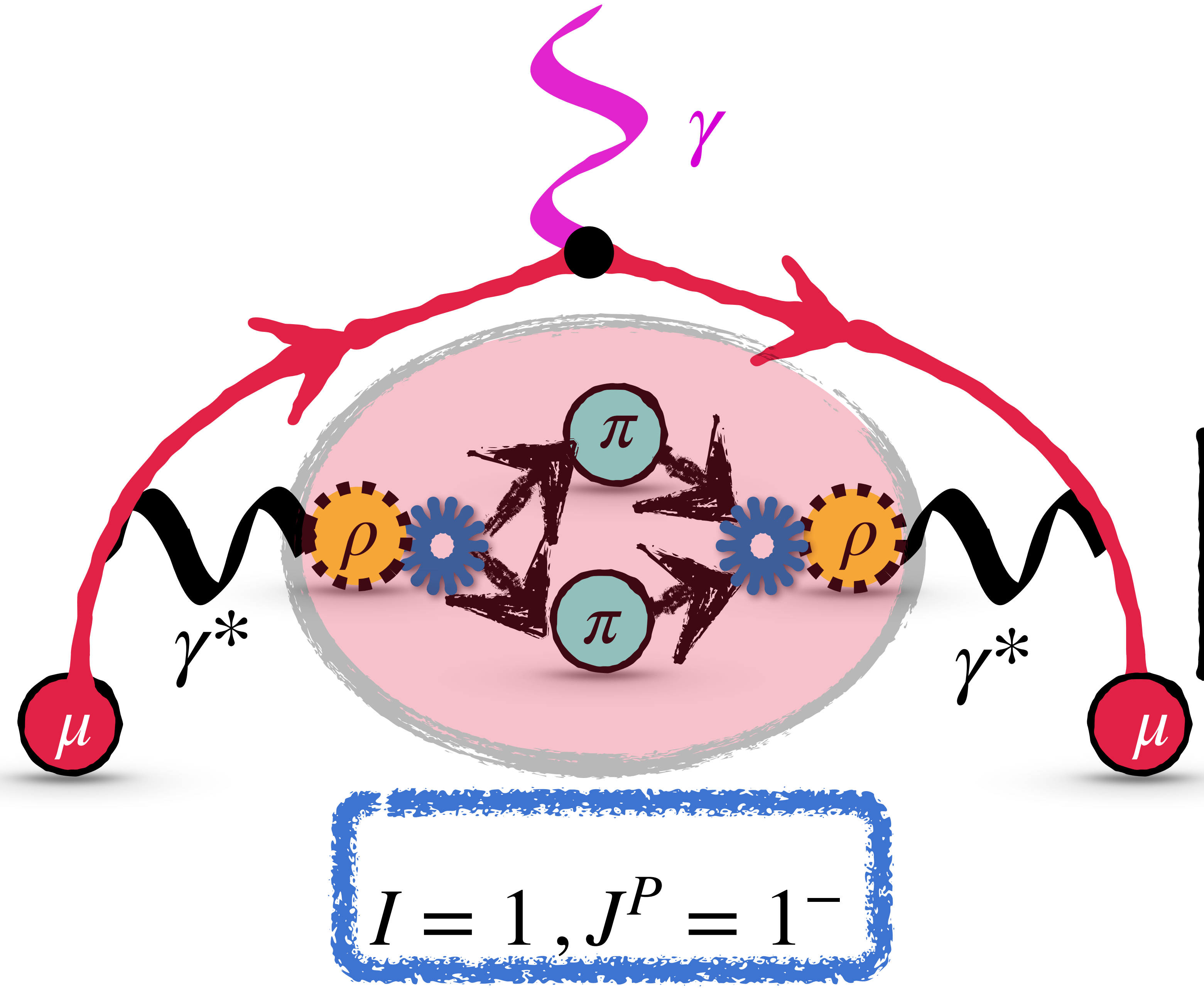
HVP Windows: Where the uncertainty lives

- Short distance window.
- Intermediate window
- Long Distance window

[UKQCD, arXiv:1801.07224]

H. Wittig @ Lattice 2021





$$G^{I=1}(x_0) = \sum_{\vec{p}} \langle j^{\rm em} \,|\, \pi(\vec{p})\pi(-\vec{p}) \rangle \, e^{-E_{\pi\pi}(\vec{p})x_0}$$

Bernecker and Meyer, [1107.4388]

Why the $\pi\pi$ states have the most overlap in the tail?

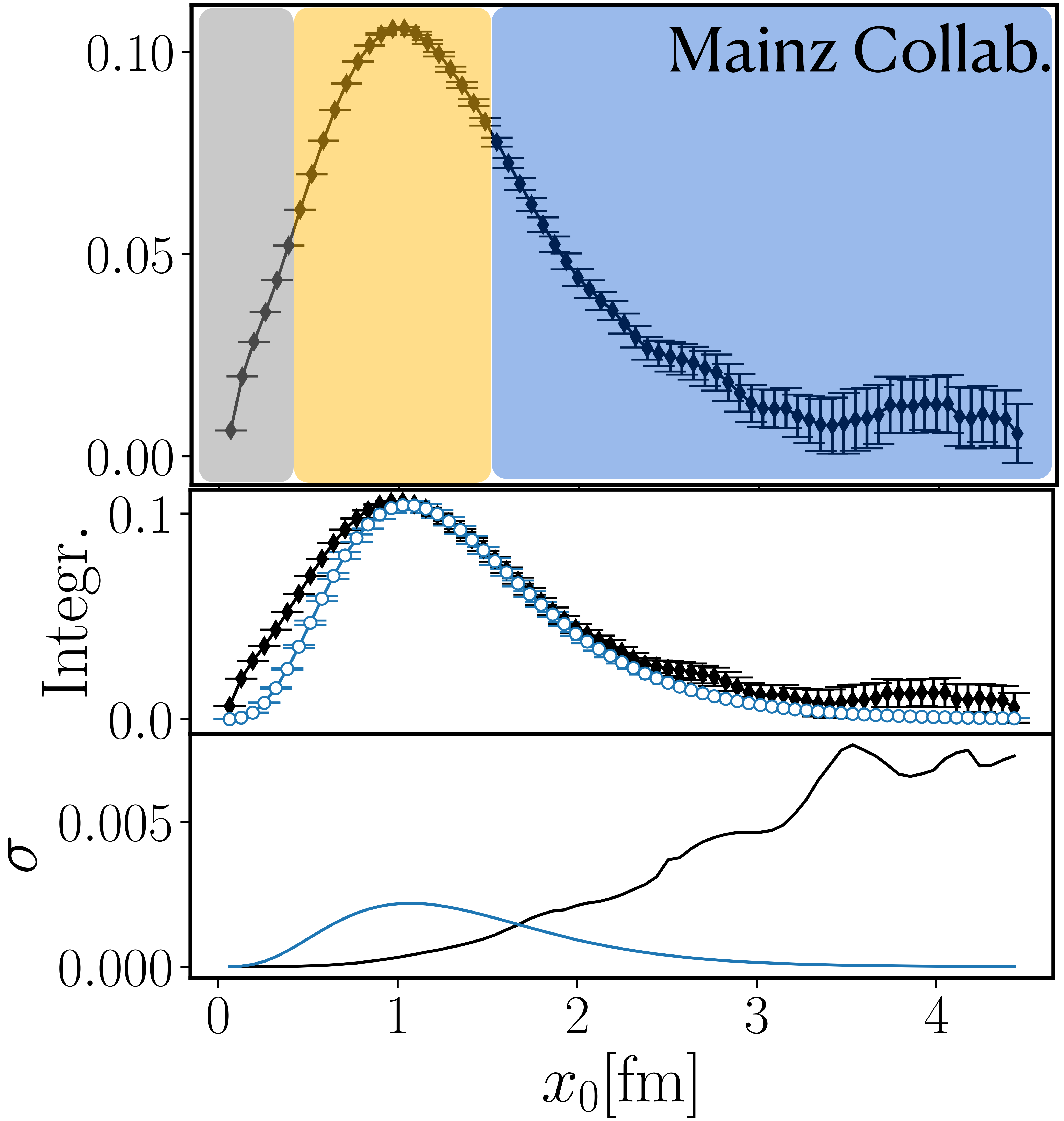
Pions: Compton λ

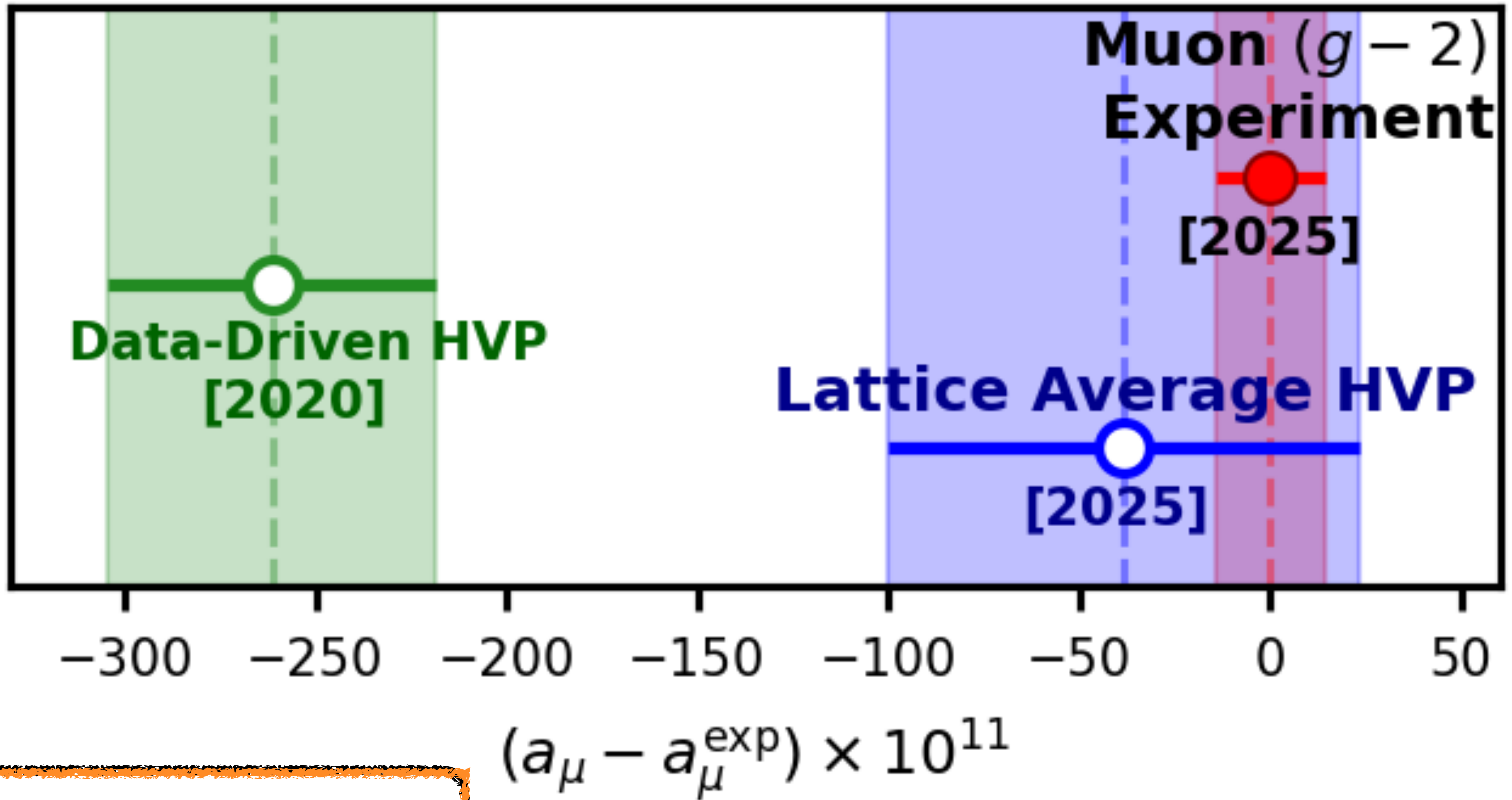
$$\lambda = \frac{1}{m_\pi}$$

Their contributions to long-distance part of all correlators is the most.

HVP Windows: Where the uncertainty lives

Lattice HVP (Area under curve)
short • intermediate • long





Intermediate Window

- ✓ Computed light-quark connected at sub-percent stat. precision on GPUs.
- ✓ Light quark propagators using *Multigrid Solver*.

Long distance Window

- ✓ Exponential error reduction in tail.
- ✓ Reconstructed tail using **multi-hadron** $\pi\pi$ states

Outlook

- π scattering states **contaminate** correlators at **long distances** when **projected** with **correct quantum numbers.**

- **Resonance enhancement** when $\pi\pi$ final state, important for **CKM matrix elements.**

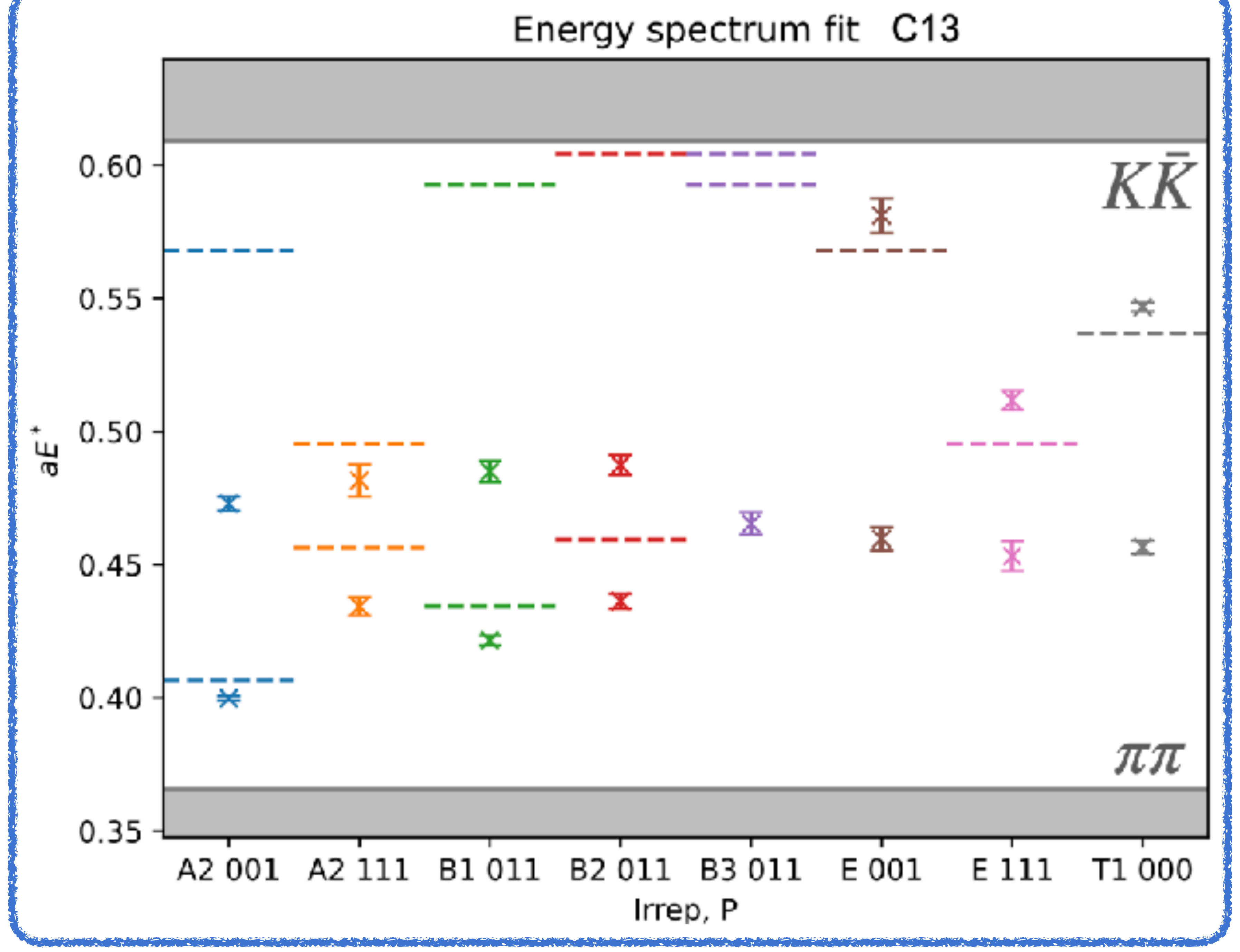
- **Non-resonant maximal isospin** $K \to \pi\pi$ channel(I=2), possible with ETMC lattices

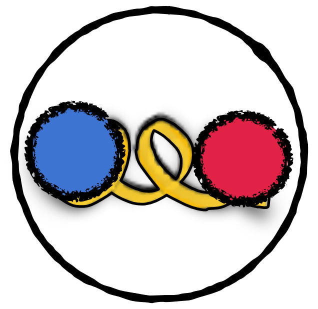
- Variational method (**GEVP**) gives a clean way to obtain **Finite Volume Spectrum**.

- **GEVP optimized eigenvectors** can be used eliminate **excited state contamination**.

What have been done: Benchmark

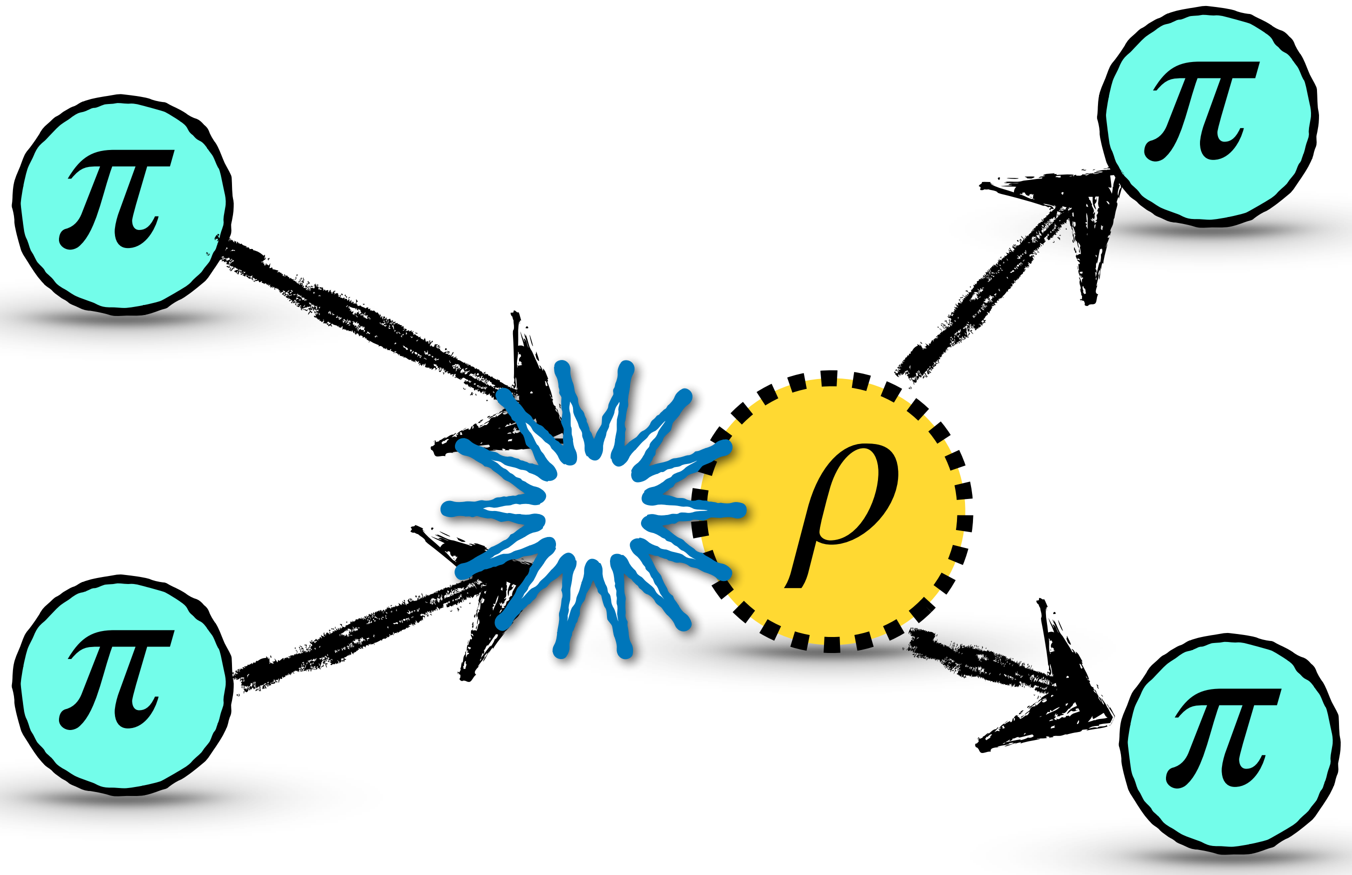
Finite Volume Spectrum





Lightest π

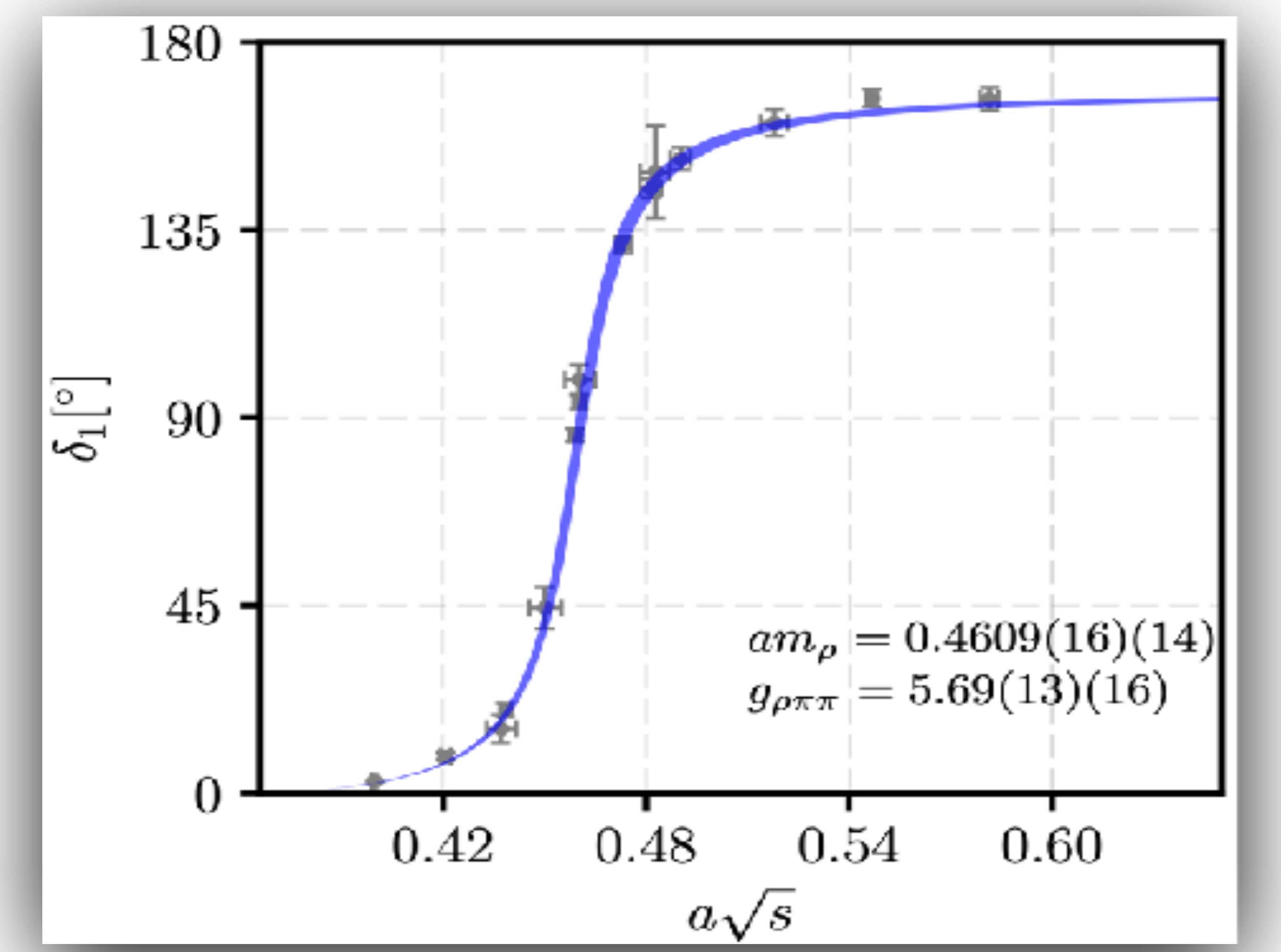
$\pi\pi$ scattering → ρ resonance



$I = 1\,, J^P = 1^-$

~~$L \to \infty$~~

$E_n(L)$ → Lüscher → $\delta_1(E)$



$m_{\text{quark}} \to$ Phys.

$a \to 0$

Finite Volume Spectrum is a feature: Scattering →Spectroscopy