

Basic concepts in track reconstruction in HEP experiments

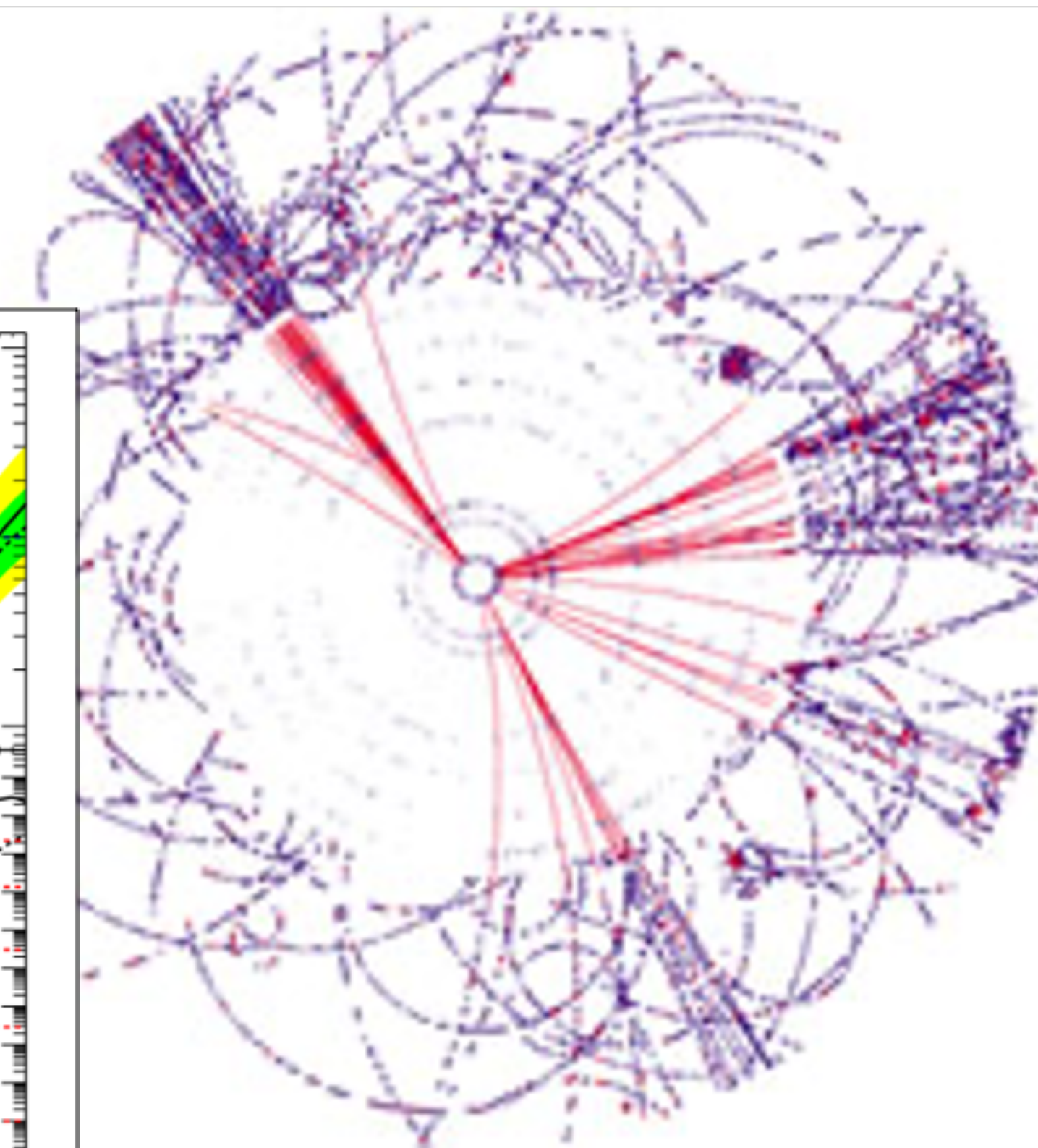
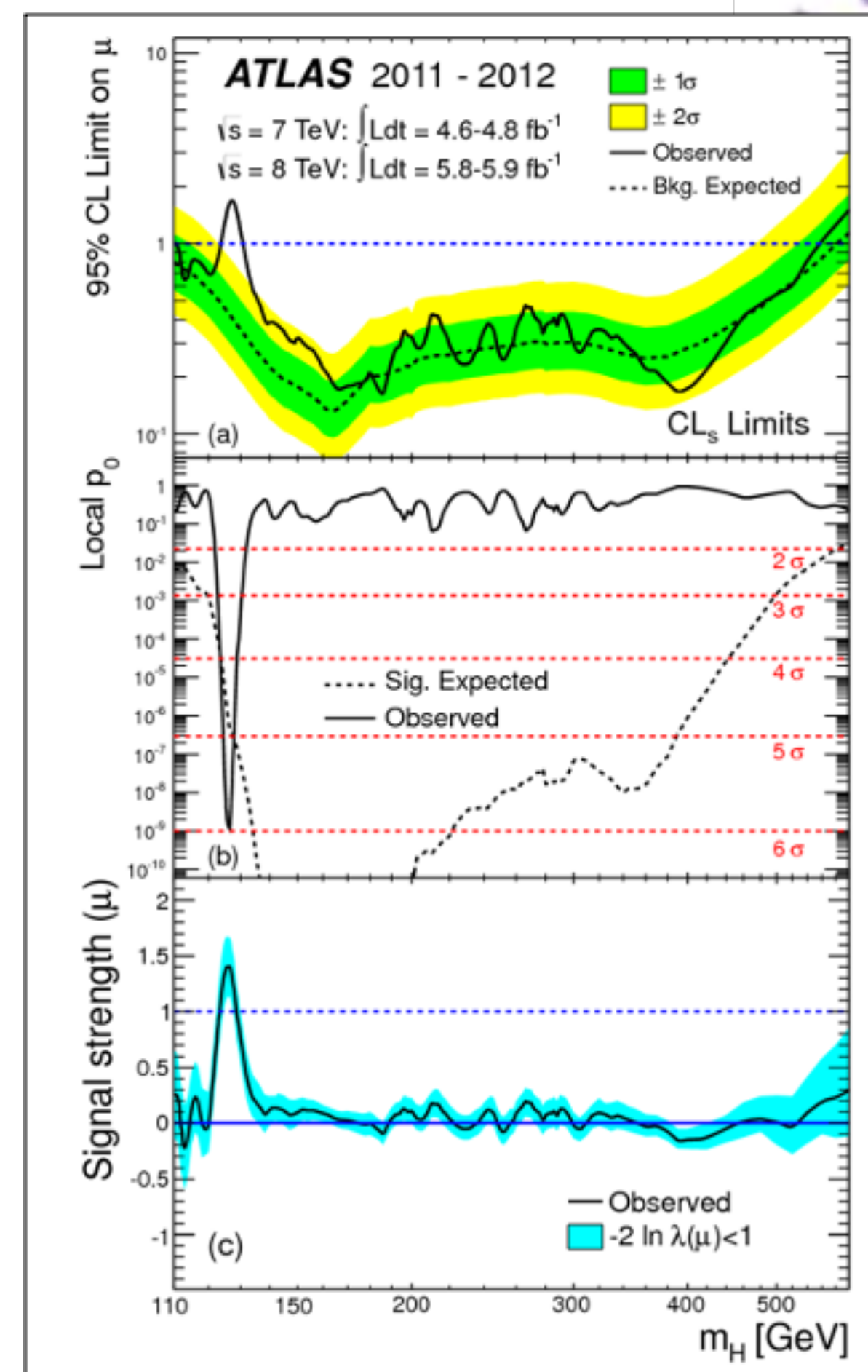
Salvador Martí García
TIDPAN 2026 València

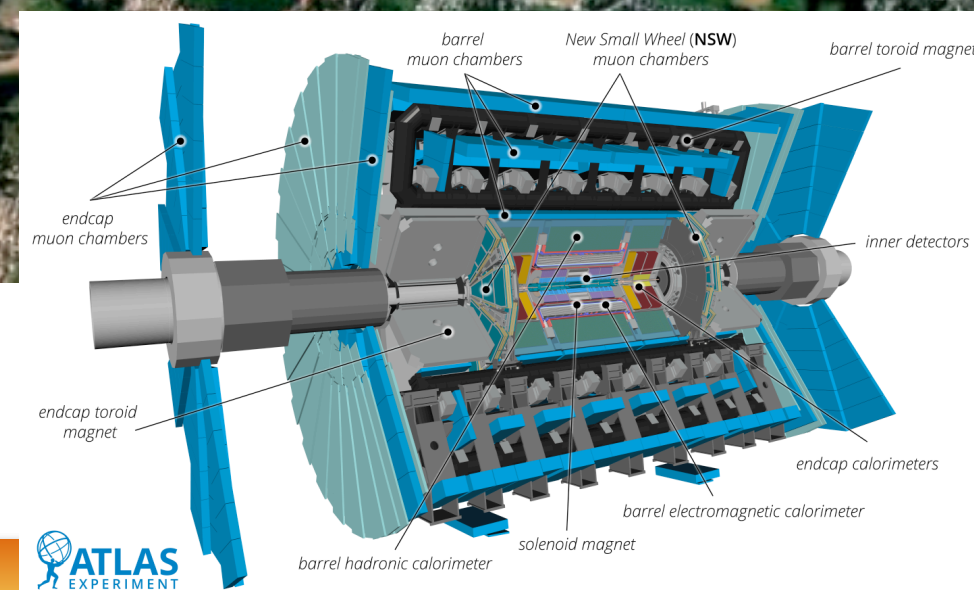
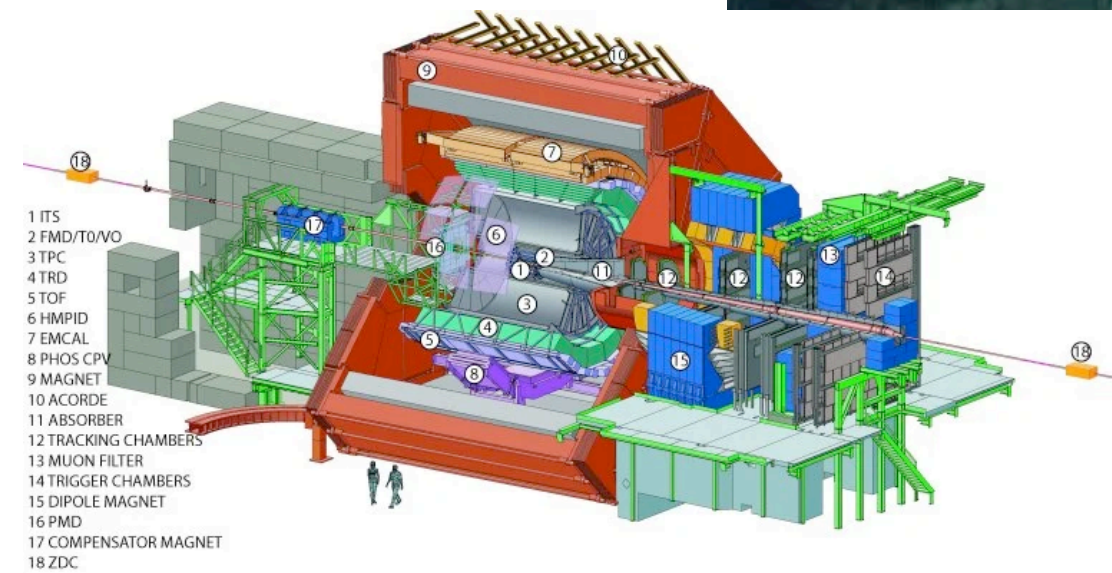
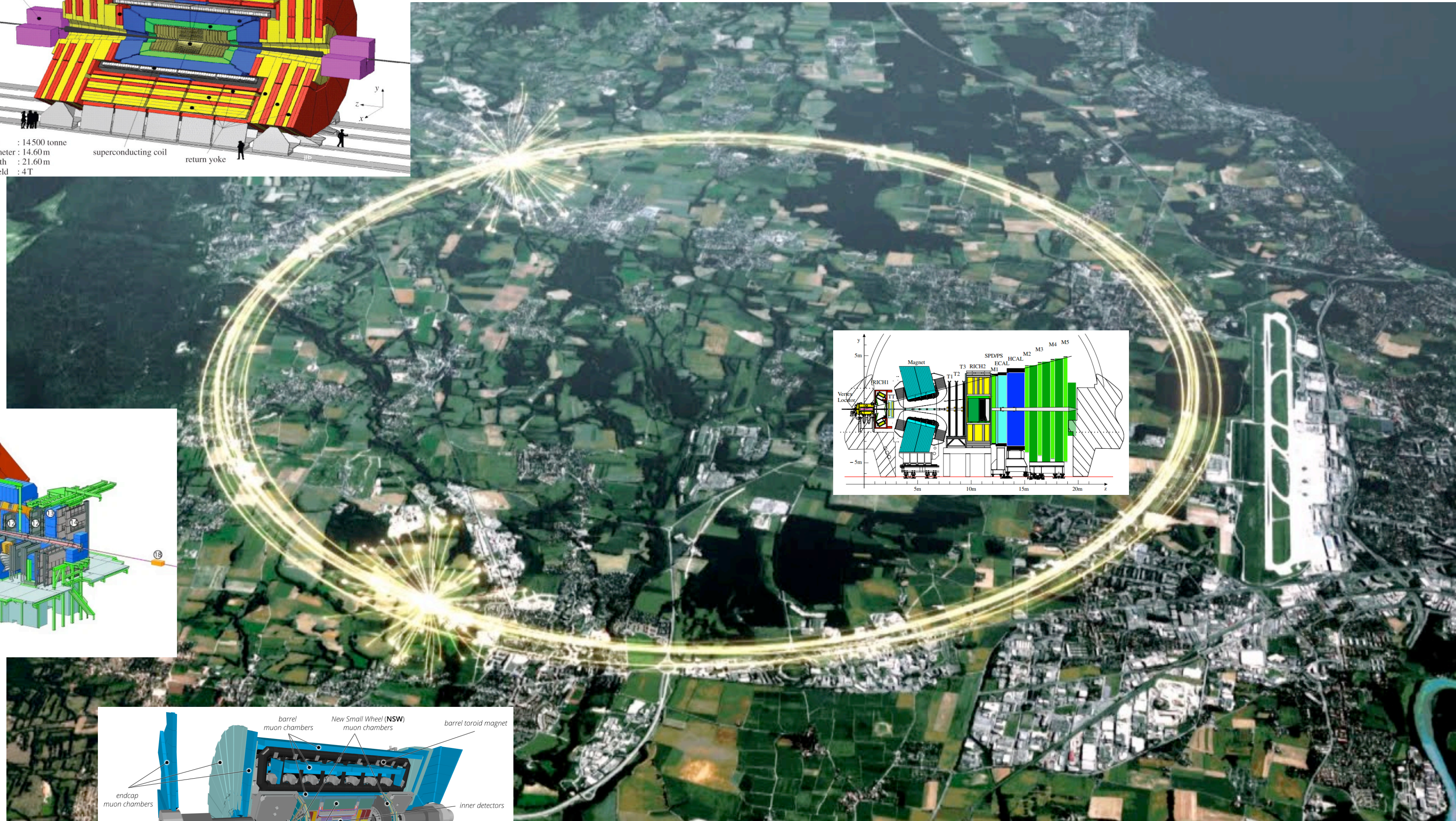


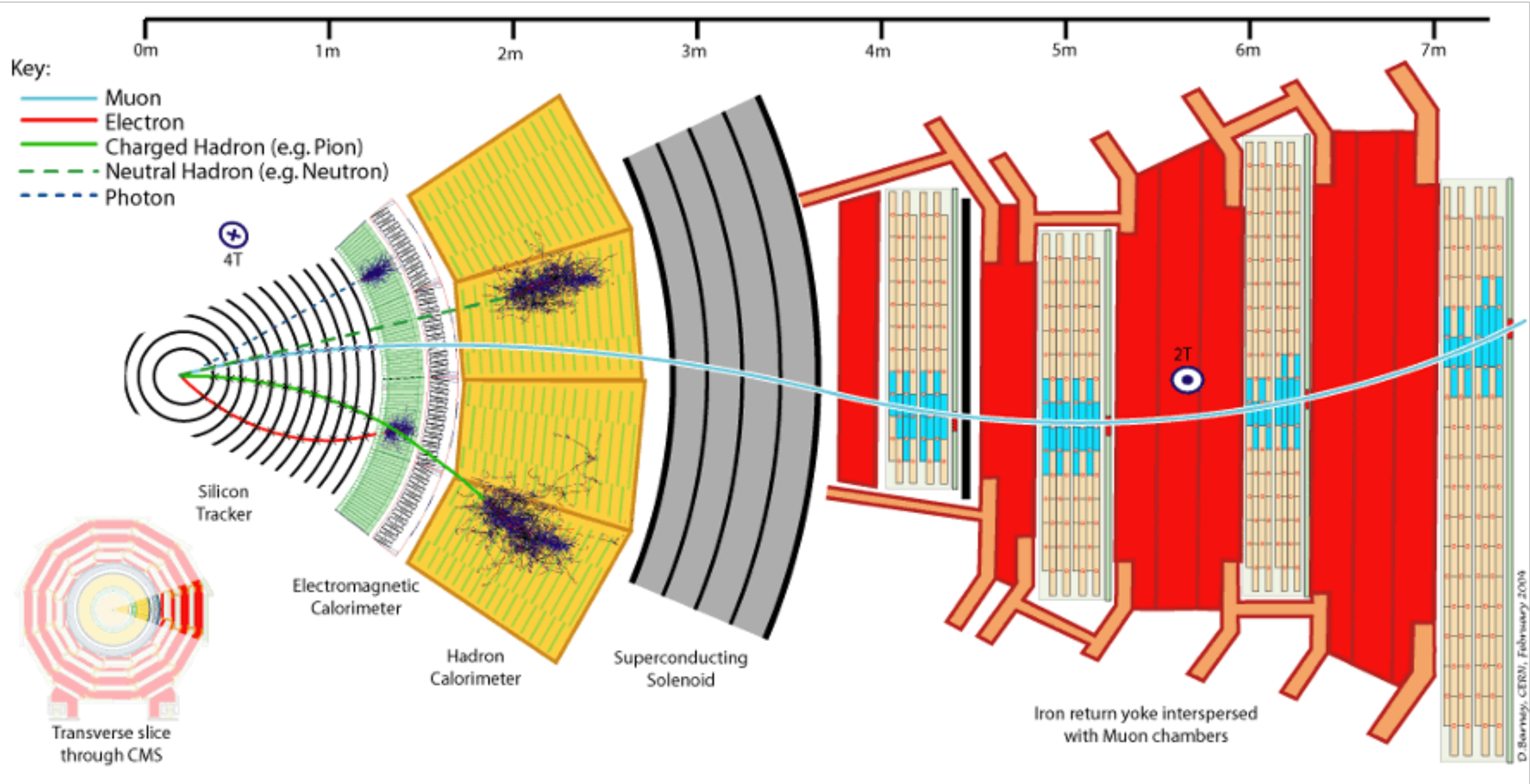
- **Track fitting**
 - Basic ideas & concepts
 - Basic formulae
 - Signal handling and processing
 - Global and Local reference frames
 - Pattern recognition
 - Track fitting with χ^2 and Kalman techniques
 - Multiple Coulomb Scattering
- **Vertex fitting**
 - Basic ideas & concepts
 - Billoir vertex fitting
 - Adaptative vertex fitting
- **Detector alignment**
 - Basic ideas & concepts
 - Basic formulae
 - Alignment strategy

Disclaimer: the detector geometry description is an important issue that is not treated in this lectures

- **Sensors react to the passage of particles and produce signals**
 - Usually as electric pulses
 - Digitization: convert those pulse into digits
- **Trigger**
 - Whenever an interesting event happens
 - Whatever “interesting” means
- **Record the data**
 - In digital format
 - In disk or tape
- **Event reconstruction**
 - Tracker hits → tracks
 - Calorimetry → energy deposition
 - Bear in mind the calibration, geometry, etc.
 - Vertexing, particle identification, jet clustering,...
- **Event analysis & selection**
 - According to the reconstructed objects
- **Physics results**
 - Eureka !



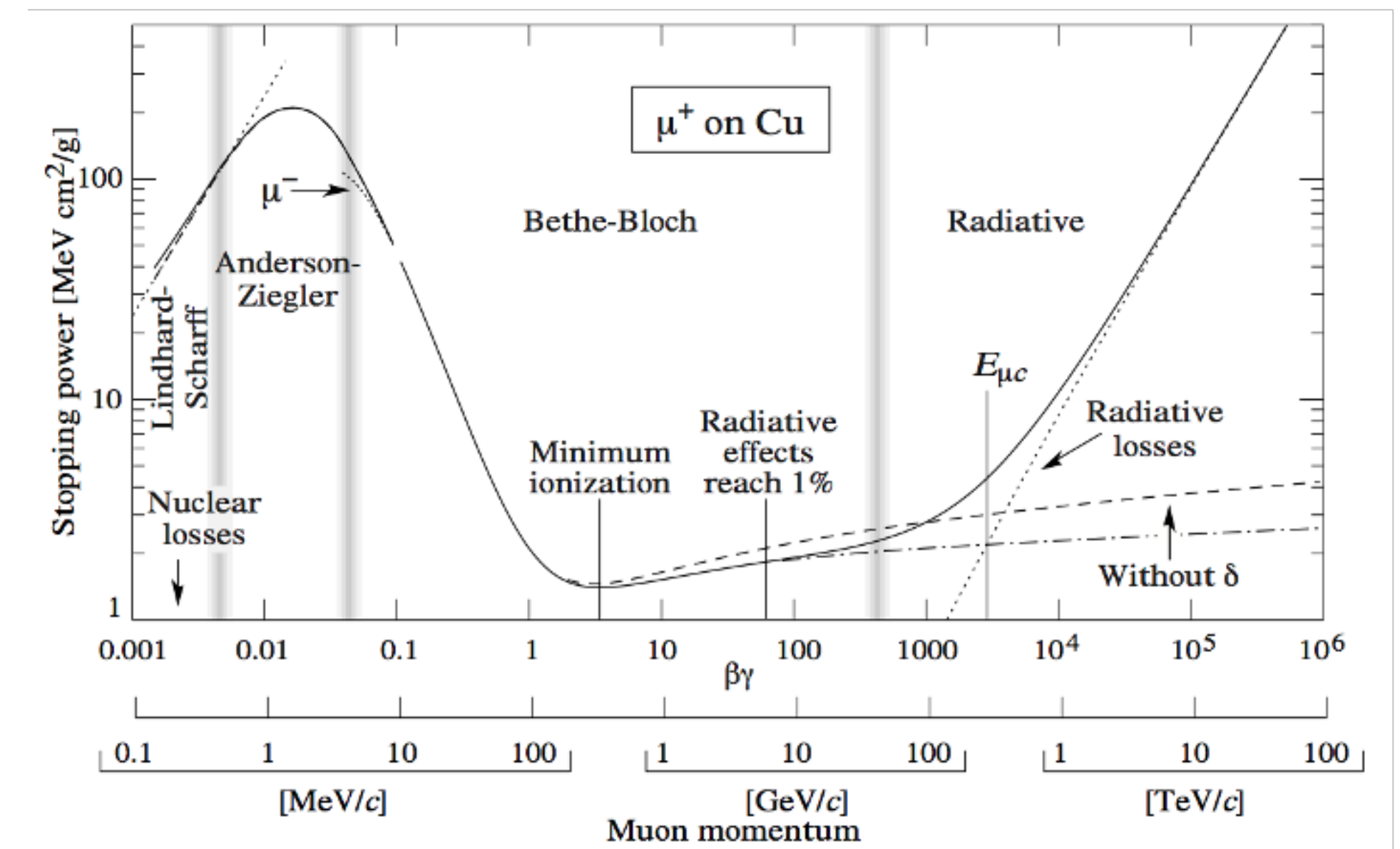
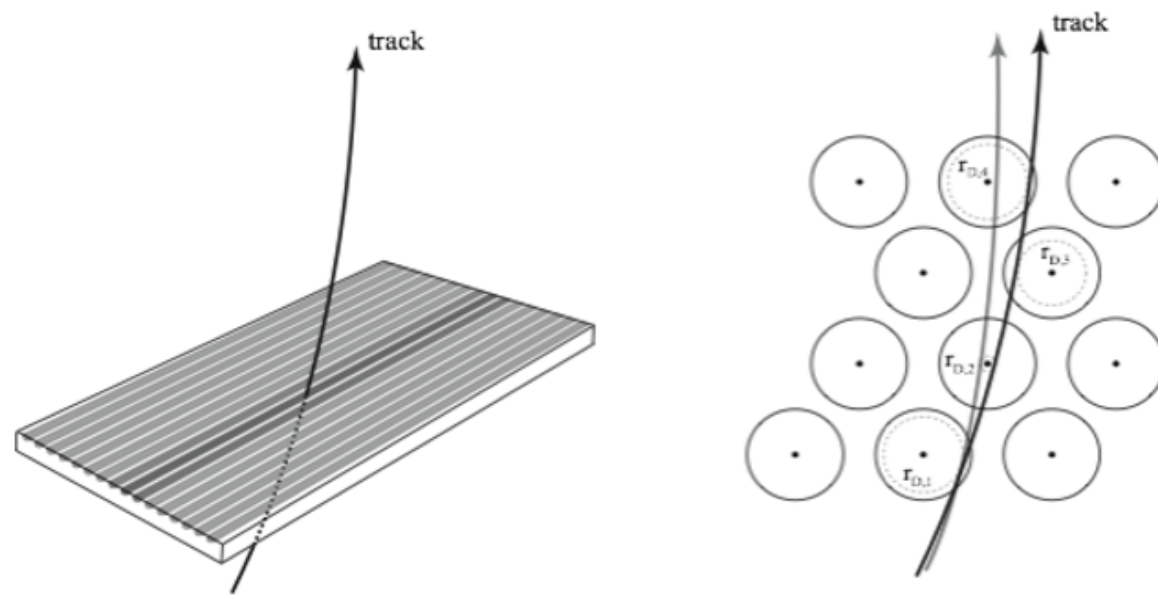




Tracking... what for?

- Tracking allows to determine the properties of those charged particles present in an experiment
 - Where is the particle ?
 - From where does it come from?
 - Where does it go ?
 - Which is its velocity ?
- Tracking is possible because charged particles interact with detector material
 - Energy loss by ionization: radiation detection
 - Bethe-Bloch formula

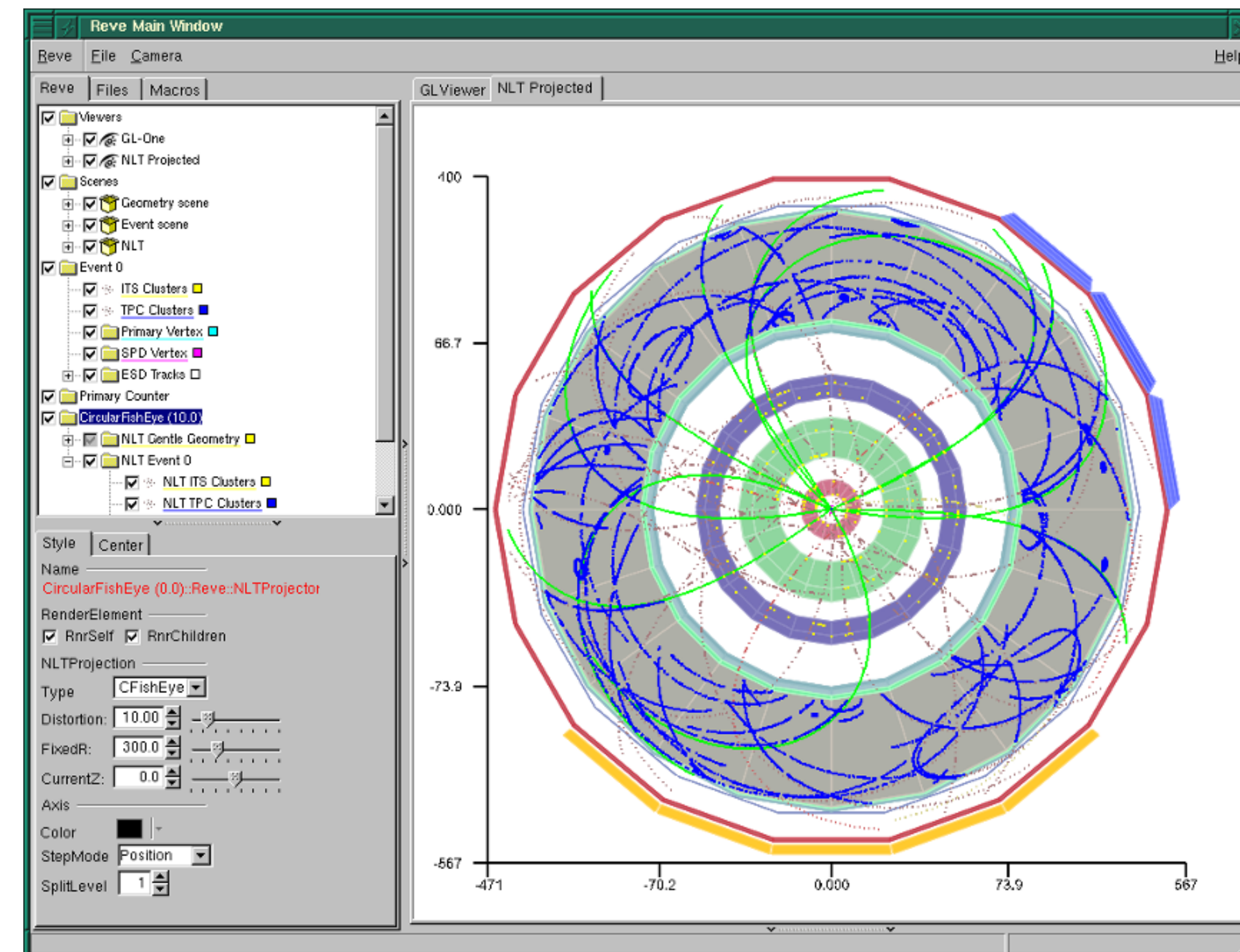
$$\left\langle \frac{dE}{dx} \right\rangle = -Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - 2\beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$



- Side effect: the interaction with the sensor alters the particle properties
 - Energy loss → this can be use for particle identification
 - Scattering angles → altering the trajectory direction

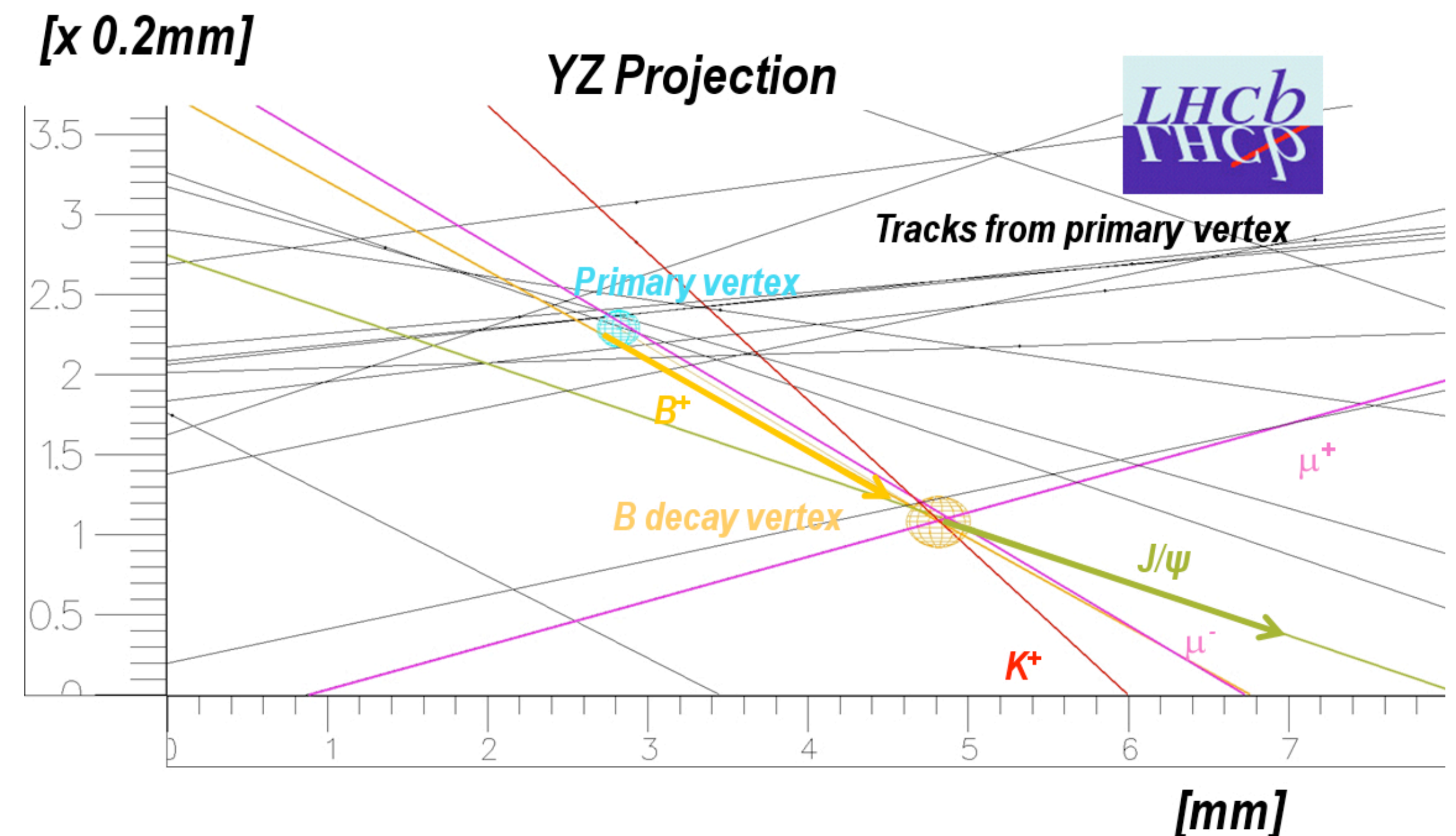
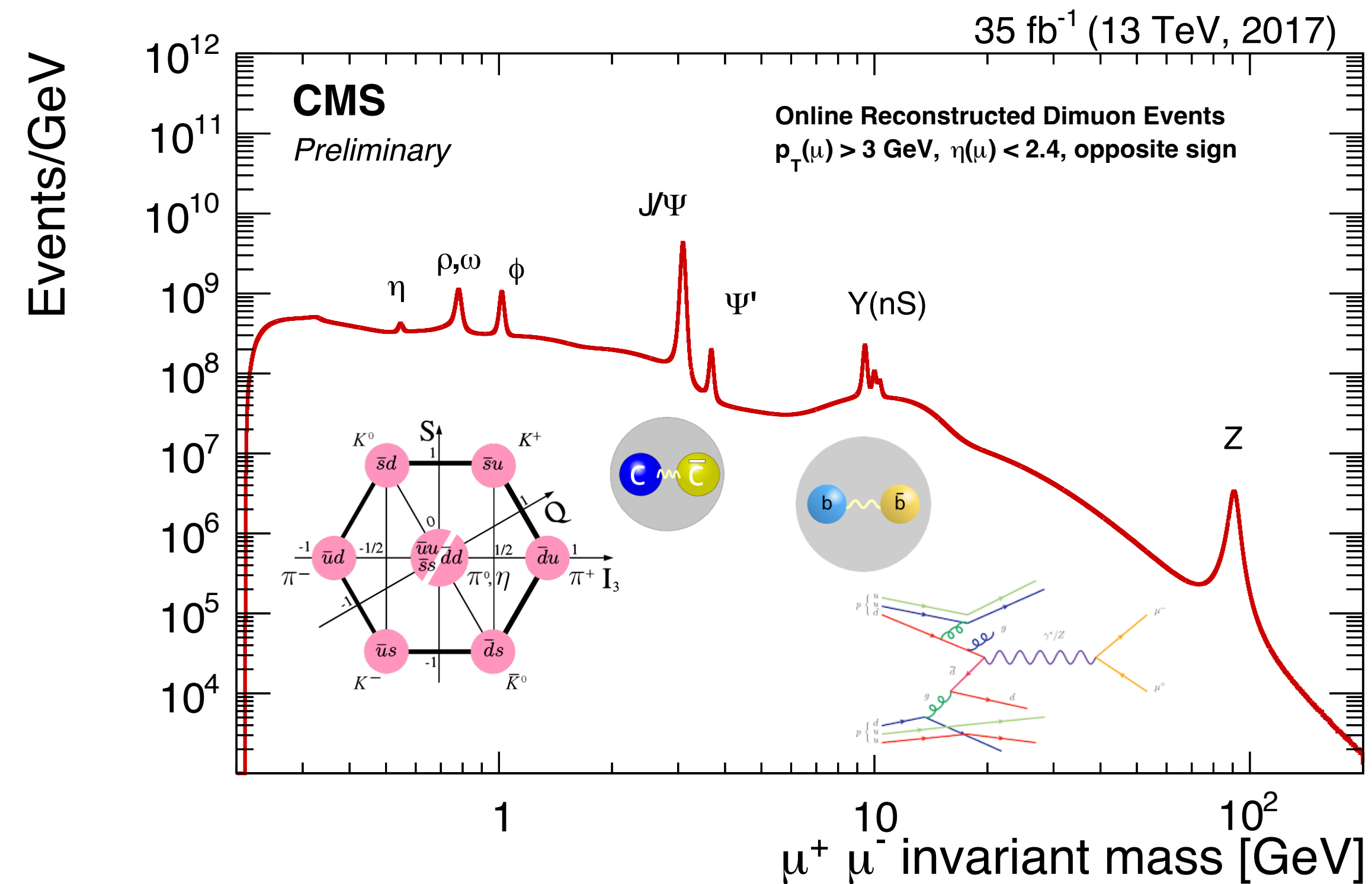
Tracking... what for?

- Tracking allows to determine the properties of those charged particles present in an experiment
 - Where is the particle ?
 - From where does it come from?
 - Where does it go ?
 - Which is its velocity ?
- Need to match the reconstructed points of a particle trajectory
 - (x, y, z) set of reconstructed points
- Extract the track parameters
 - And their uncertainties!



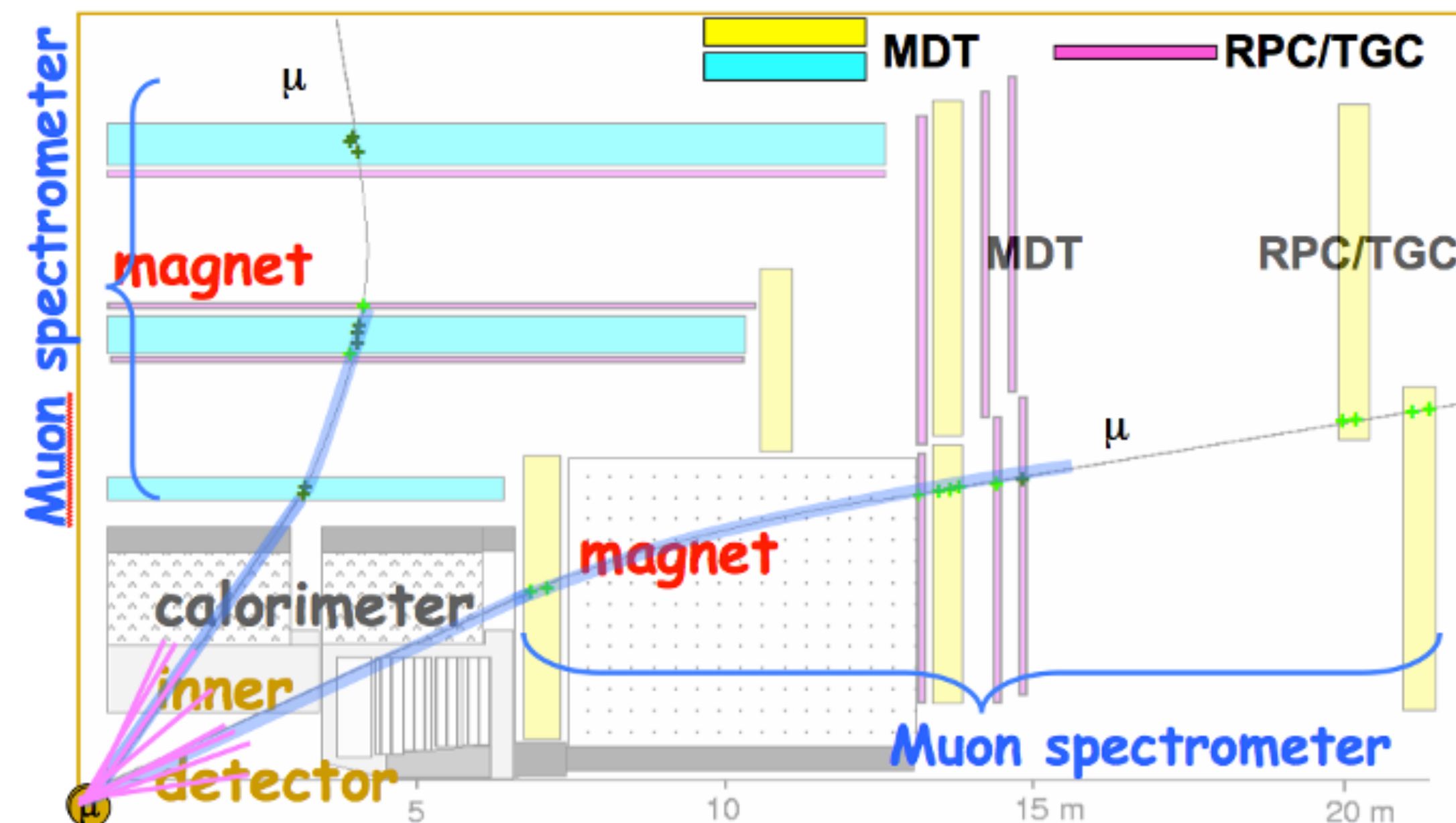
Tracking... what for?

- A good performance of the Track Fitting is a key ingredient of the success of the physics program of the HEP experiments
 - An accurate determination of the charged particles properties is necessary
- Invariant masses have to be determined with precision and well estimated errors
- Secondary vertices must be fully reconstructed: evaluate short lifetimes
 - Kink reconstruction: on flight decays



Tracking... what for?

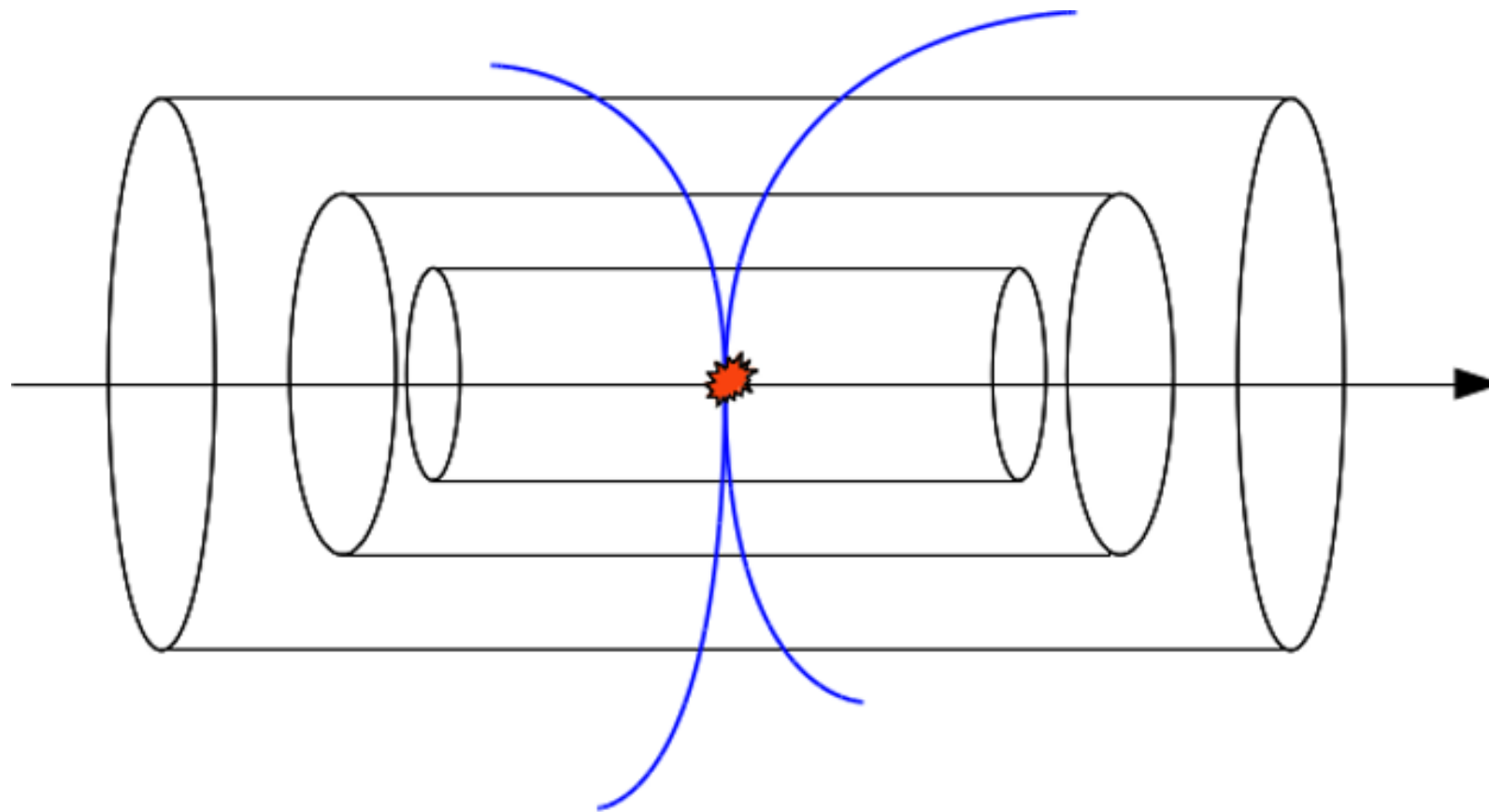
- Challenges for the tracking systems of the LHC detectors
 - Momenta of particles in the final state ranging from MeV to TeV
 - High multiplicity of charged particles (up to 1000 for $\mathcal{L} \sim 10^{34} \text{cm}^{-2}\text{s}^{-1}$)
 - Even higher for heavy ion collisions and expect even more at HL-LHC
 - Large background from secondary activities of the particles
 - Multiple Coulomb Scattering in detector frames, supports, cables, pipes...
 - Complex modular tracking systems combining different detecting technologies, different resolutions
 - Resolutions that vary as a function of the momentum (p), polar angle (θ) or pseudorapidity (η)
 - Very high event rates leading to large amount of data with very demanding requirements of CPU and storage
 - Tracking CPU budget



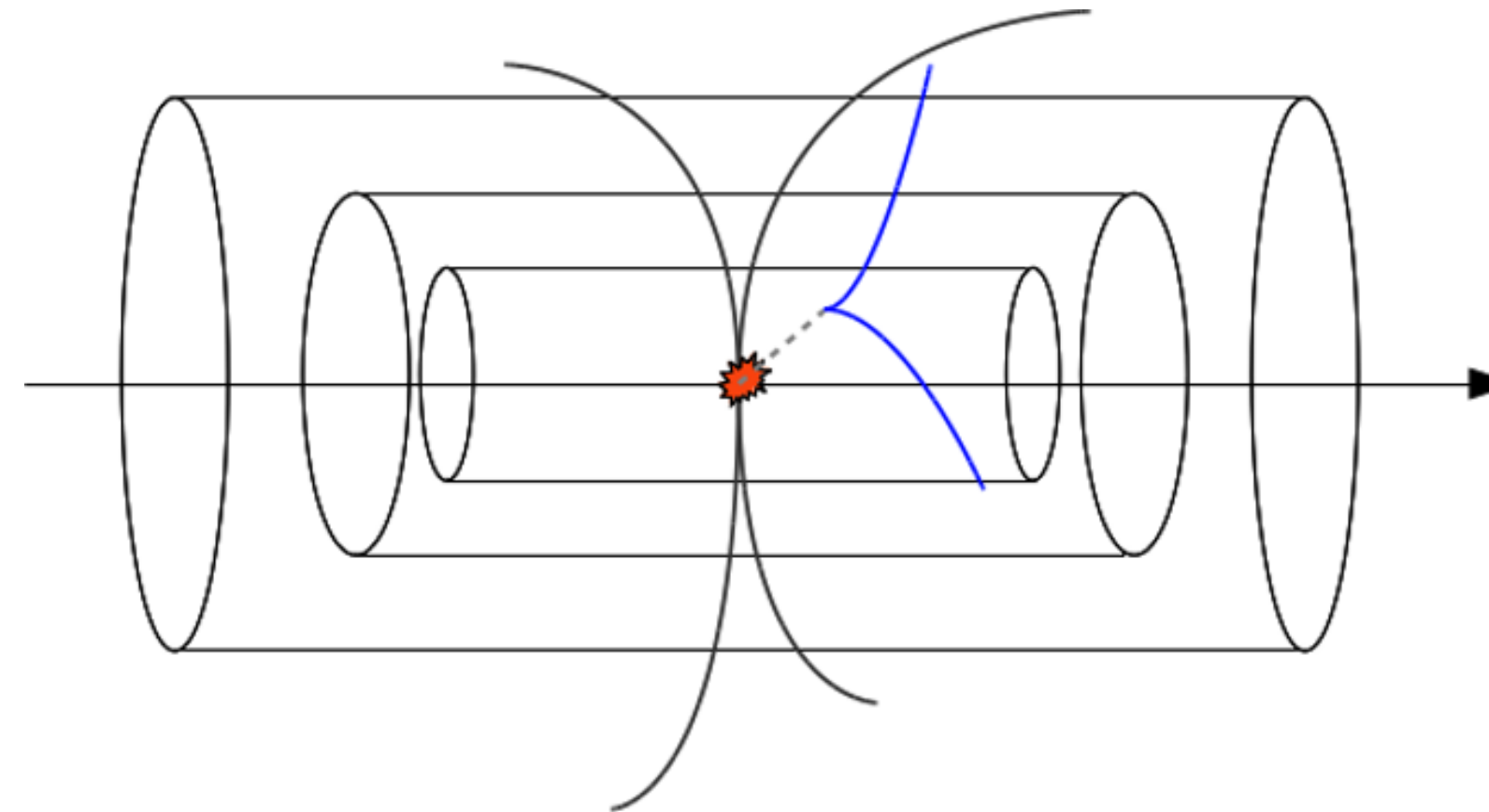
Tracking... what for?

- The vertexing capabilities of an experiment strongly depend on the tracking performance
 - specially in impact parameter and space point resolution
- Finding where the particle was originated tell us much about the physics and the detector
 - primary vertex (hard scattering processes)
 - secondary vertex (long lived particles)
 - material interactions (material distribution in the detector)

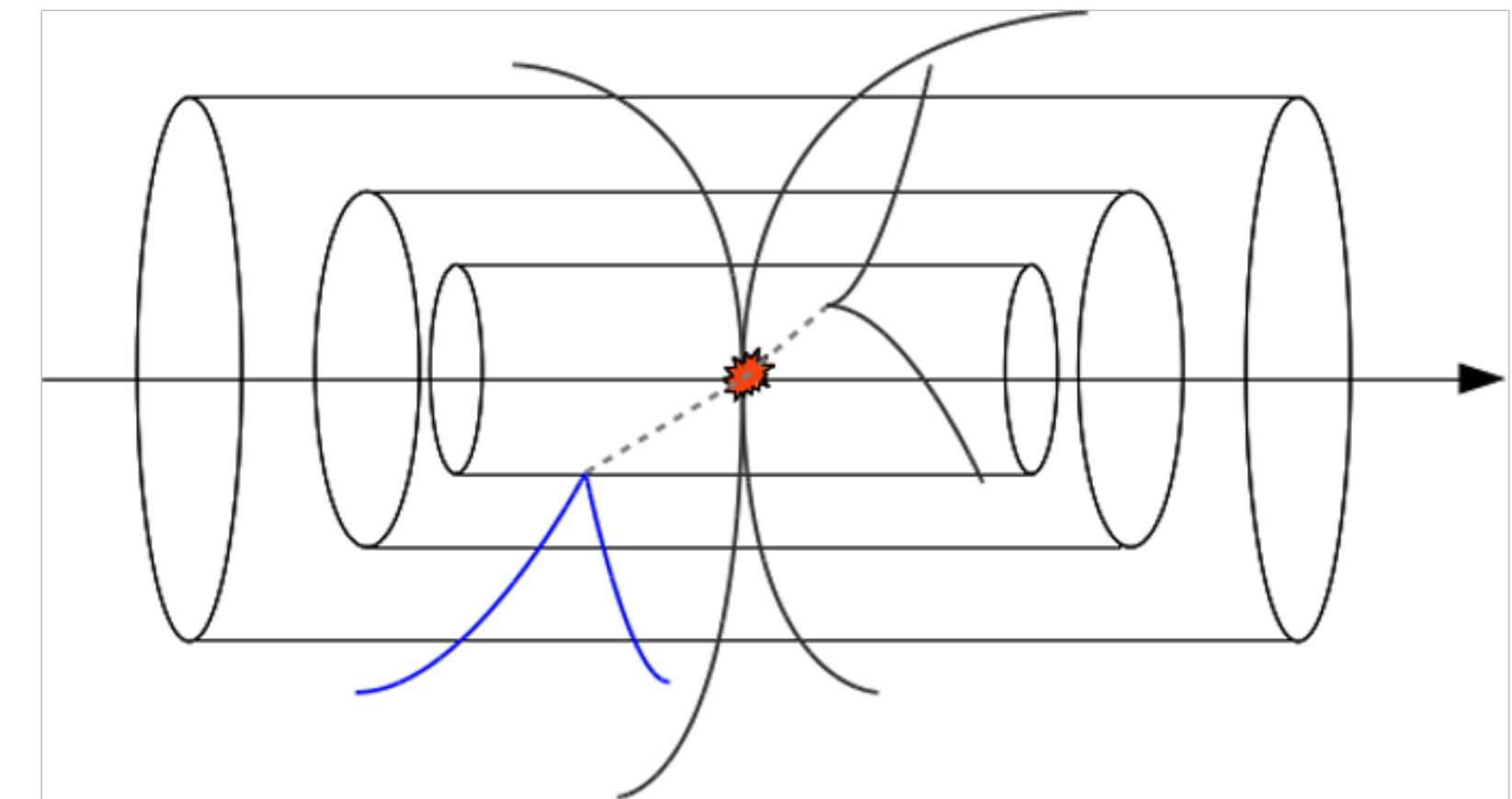
primary vertex



secondary vertex

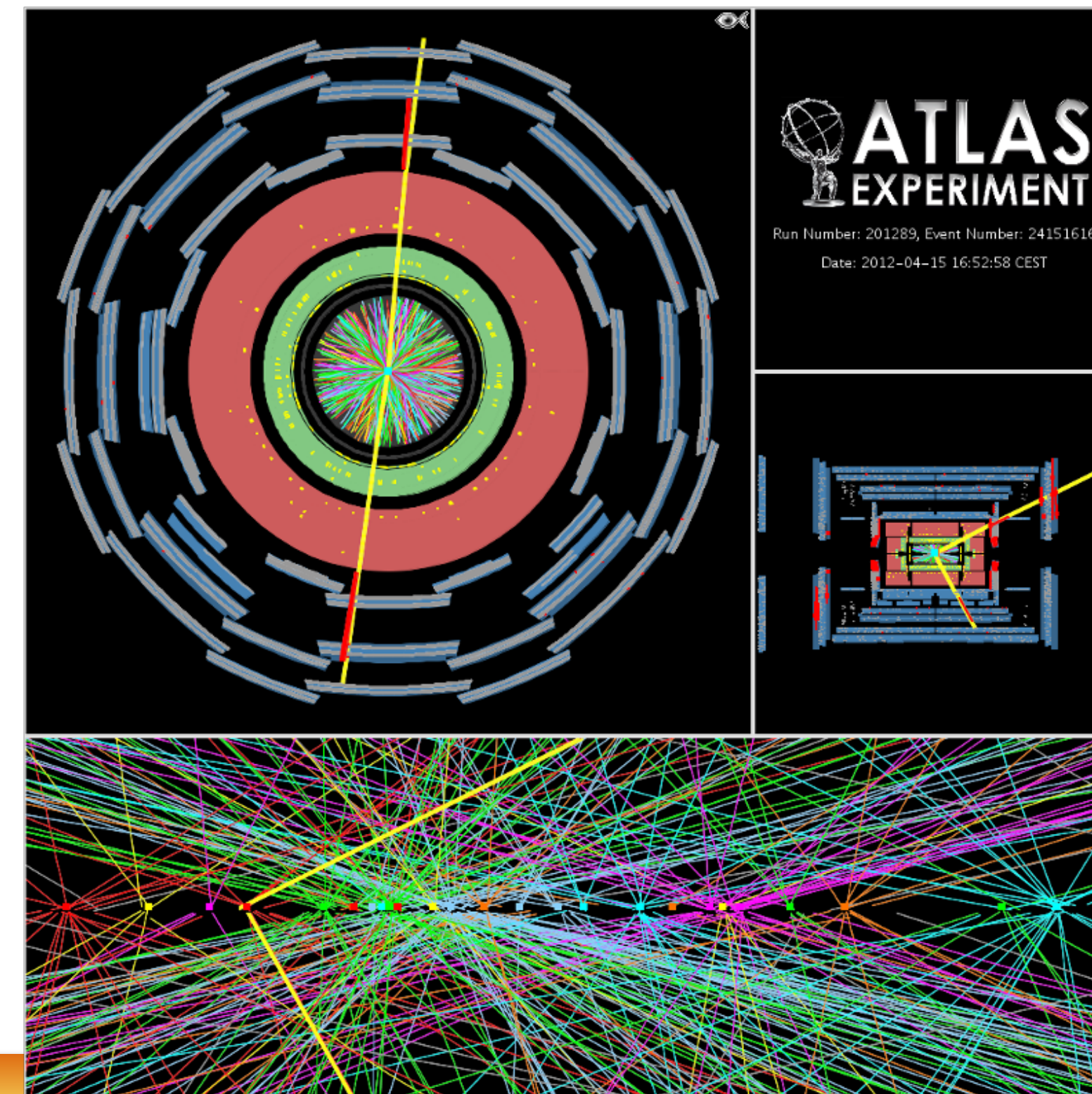
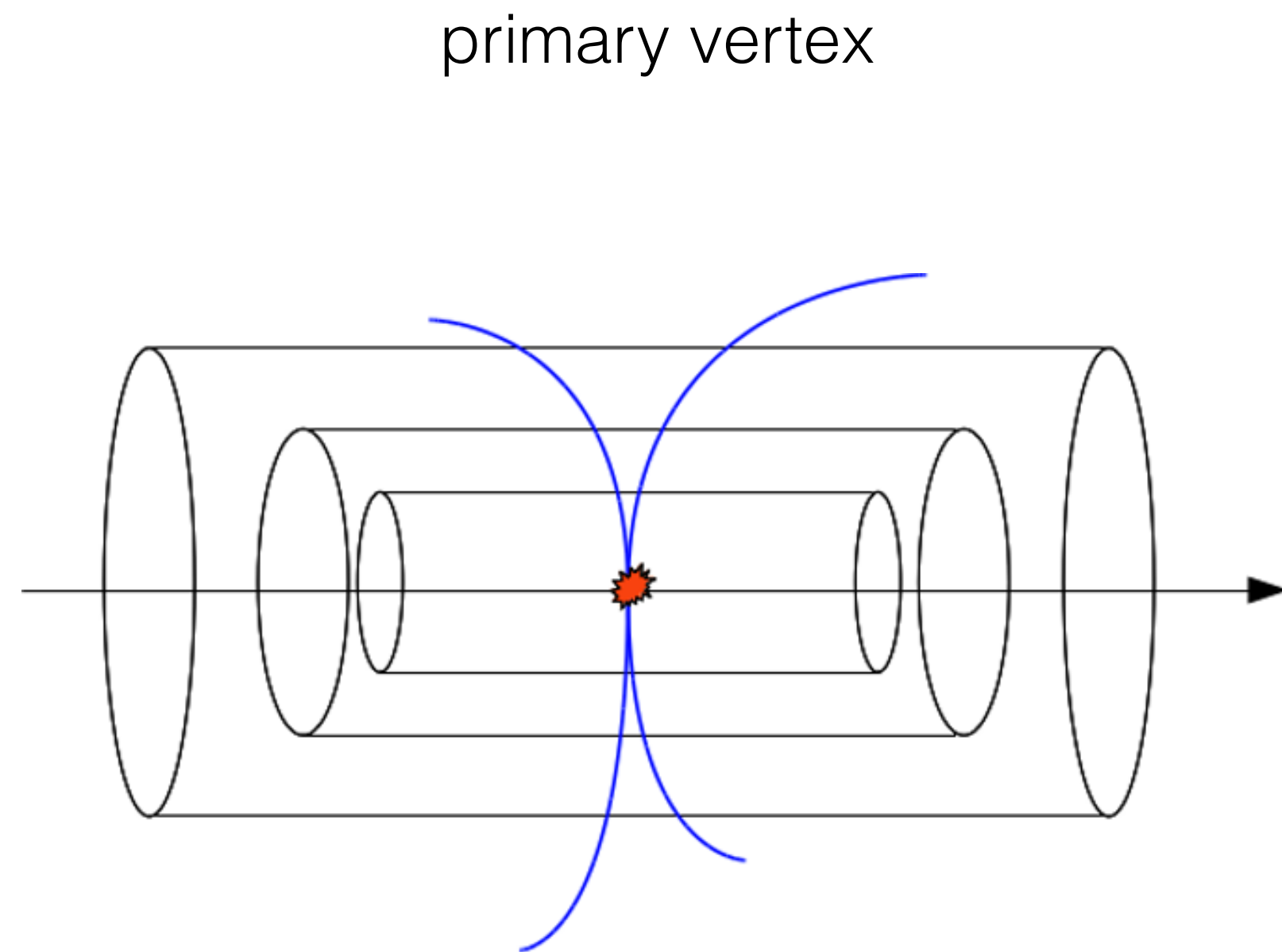


material interaction



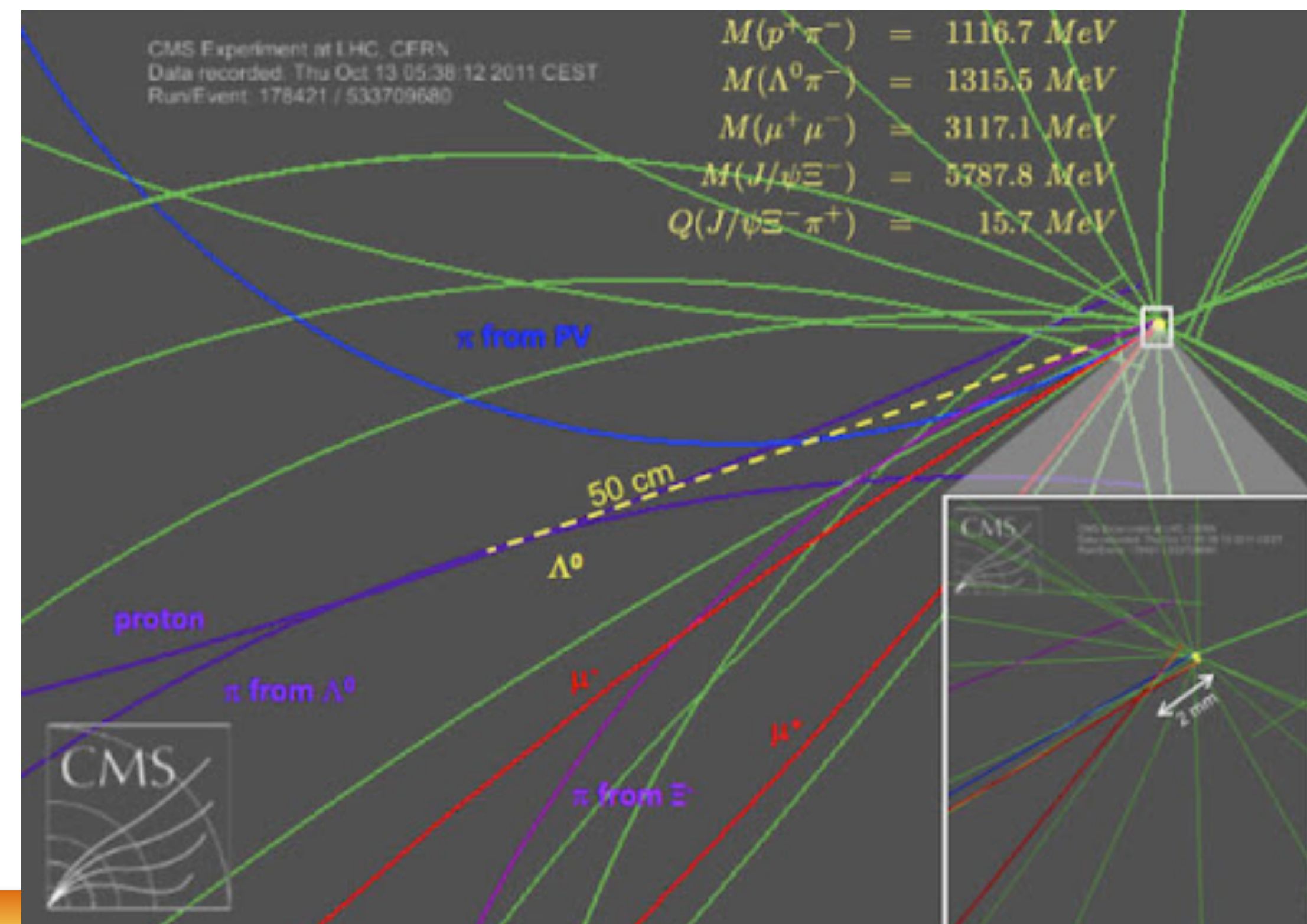
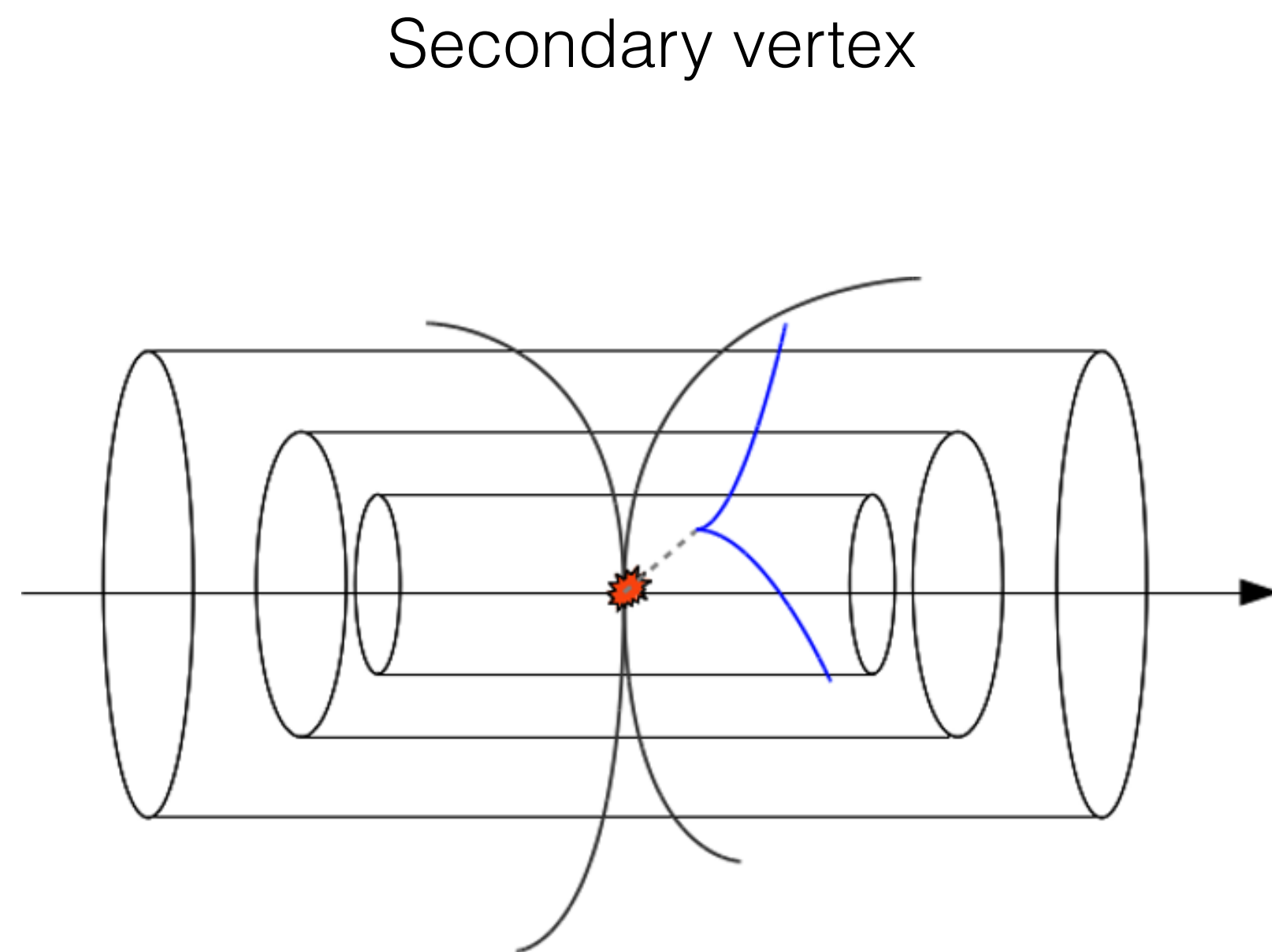
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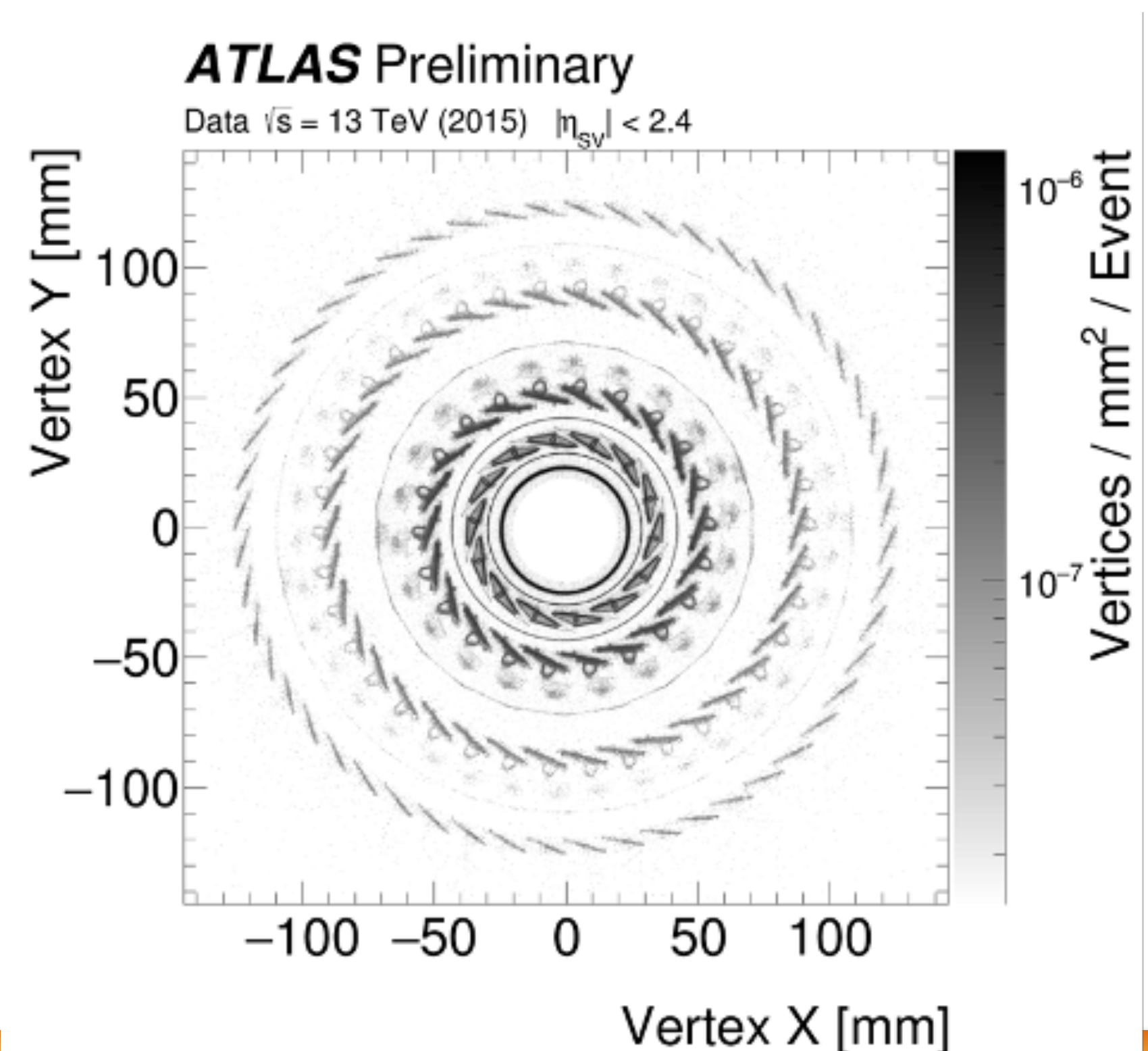
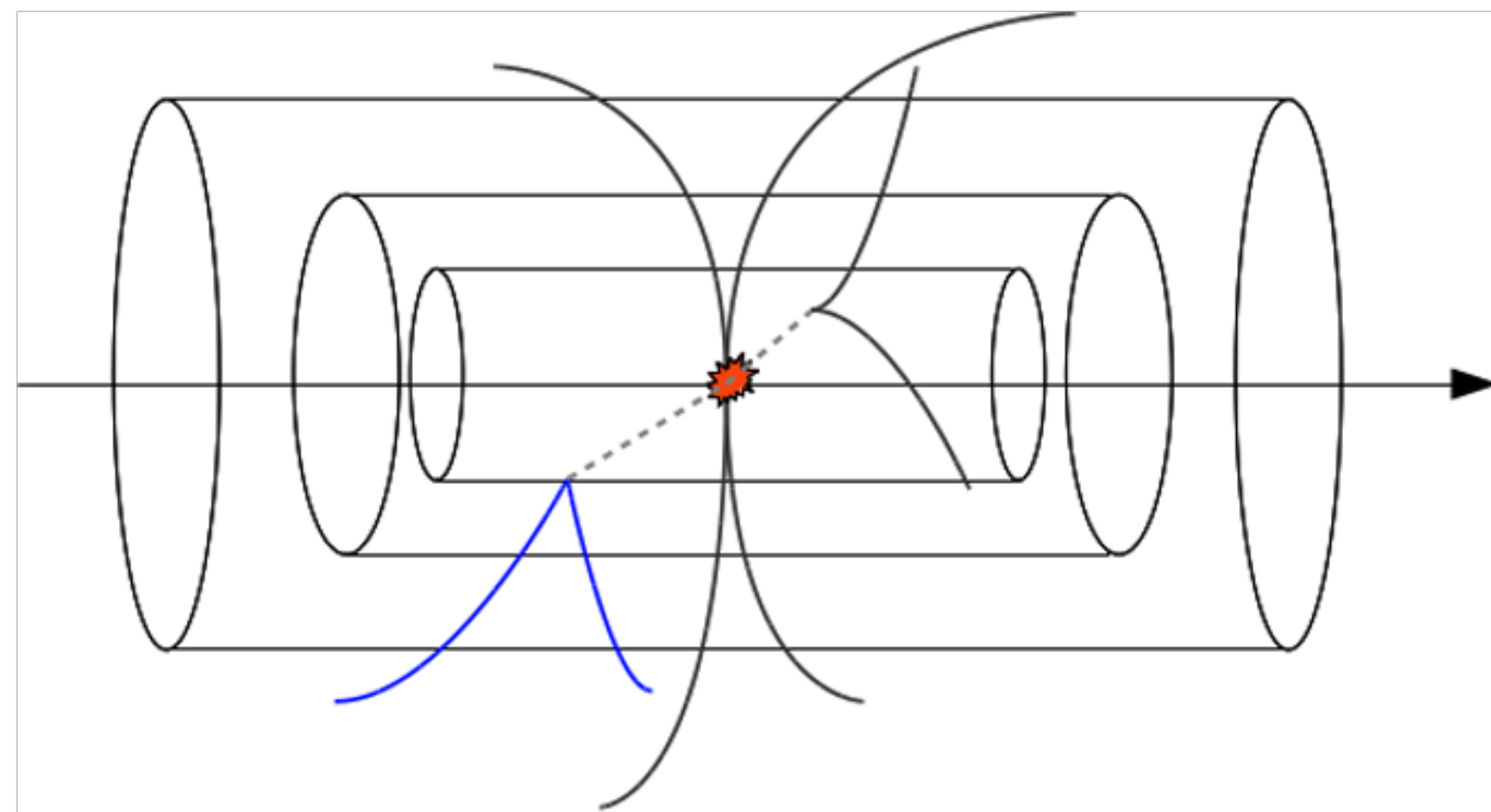
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Material interaction



Basic ingredients of the track reconstruction

• Basic ingredients of the tracking system

- Charged particles (+ve or -ve)
 - $|q| = 1, 2$ (e, μ , π , k, p, α , d,...)

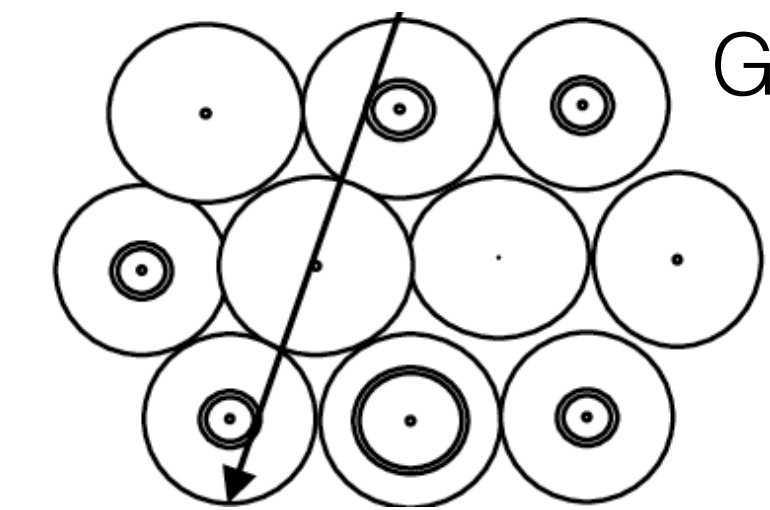
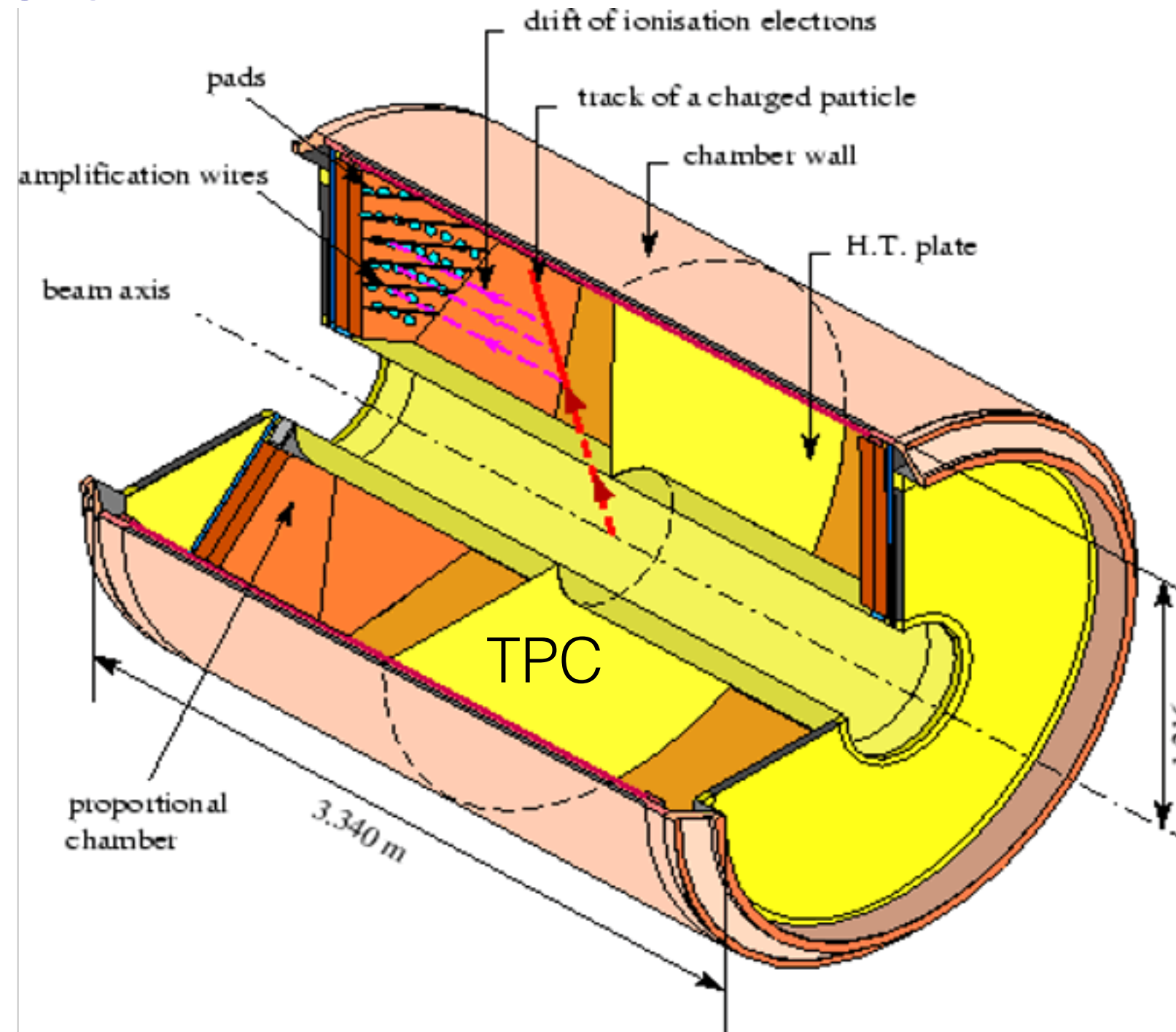
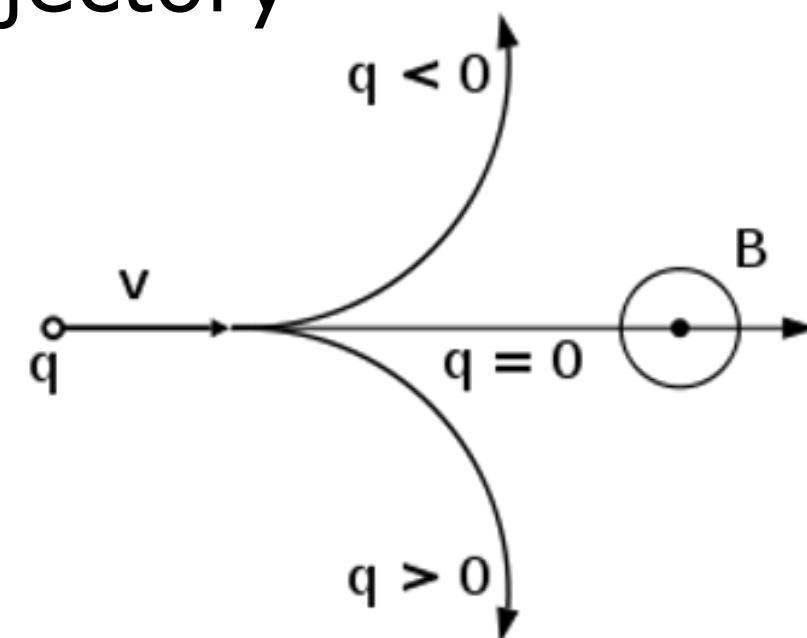
• Ionization detector

- Continuous (e.g.: gas detectors)
- Discrete (e.g.: silicon planar detectors)

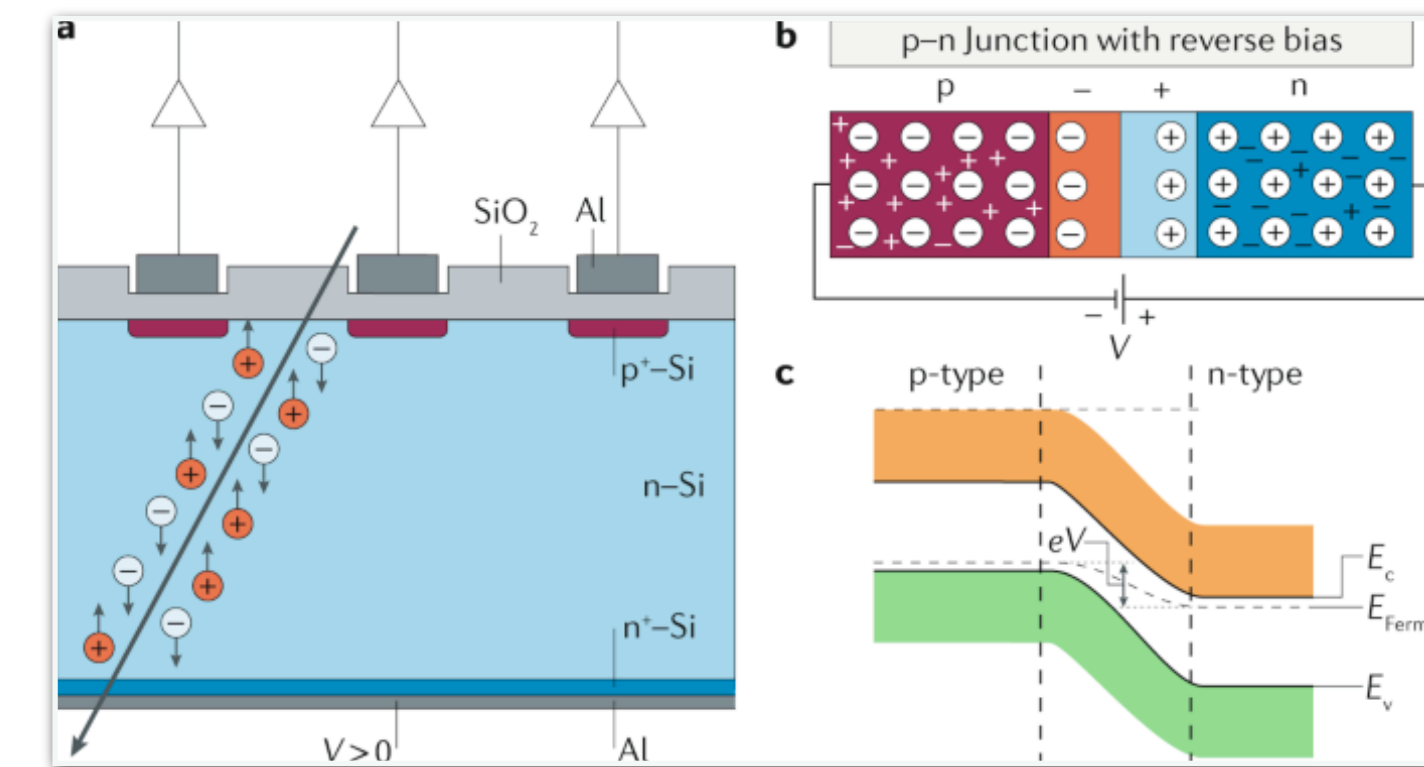
• Magnetic field

- No strictly necessary
- Necessary if momentum determination is required
- Lorentz force: bending of the trajectory

$$\mathbf{F} = q (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$



Gas drift tubes



Silicon sensors

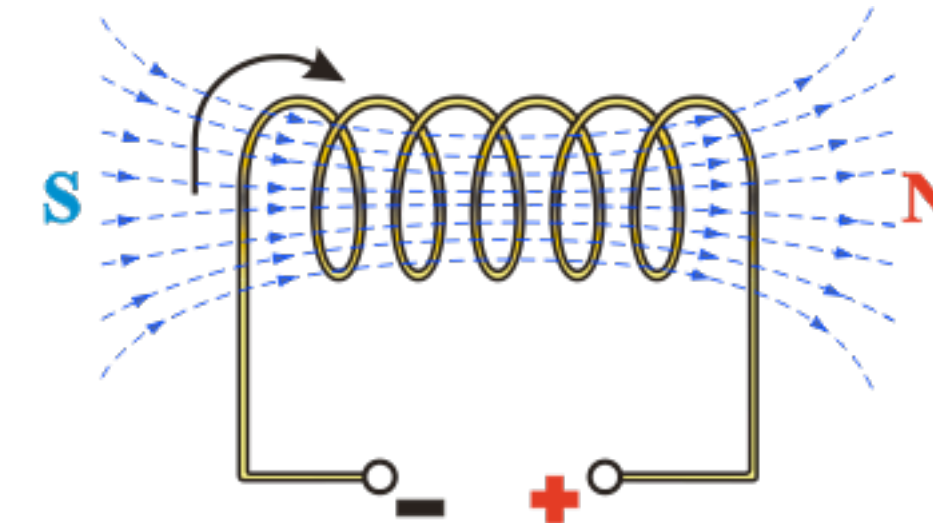
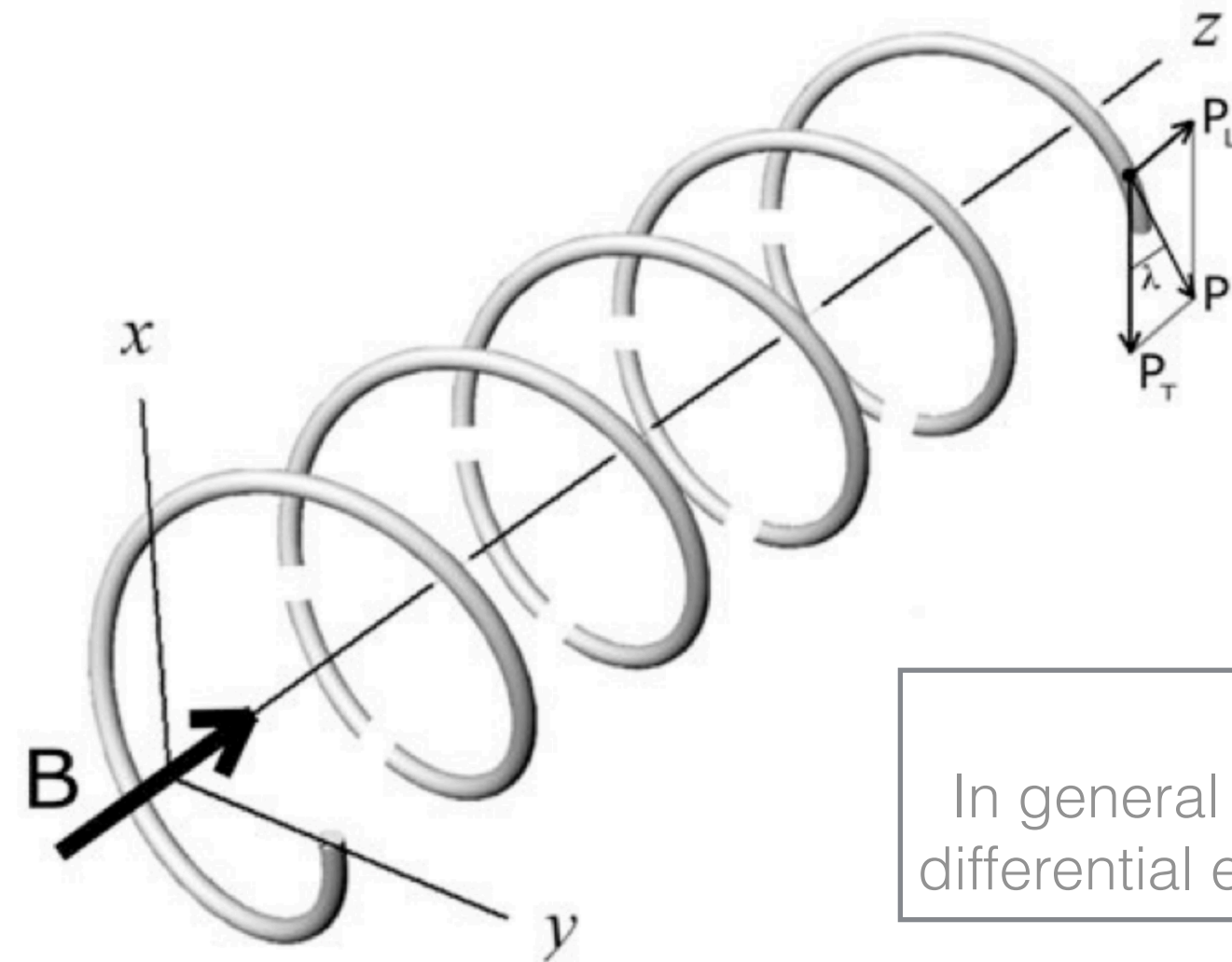
Warning:
usually $E \neq 0$ inside detectors but quite small
as it is necessary for ionization charge collection.
Negligible effects on tracks

Motion of a charged particle in a magnetic field

- Most of HEP experiments at colliders use a solenoid to generate a uniform axial B field
- Charged particle trajectory follows a helicoidal path
 - Describe circles in the XY (transverse plane)
 - Due to the Lorentz force
 - Move uniformly along Z axis

$$\mathbf{F} = m \frac{d\mathbf{v}}{dt} = q \mathbf{v} \times \mathbf{B}$$

$$\mathbf{F} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ 0 & 0 & B \end{pmatrix}$$



Warning:
In general B is non uniform. Therefore the differential equations are solved numerically

- Helix circle radius or curvature radius (ρ) related with transverse momentum

$$p_T [\text{GeV}/c] = 0.3 B[\text{T}] \rho[\text{m}]$$

Helix equation

- Parametric form of the trajectory equation using the path length:

- Apologize for using again s in the notation
- v considered constant: no electric field & neglect energy loss
- $s=0$ is an arbitrary choice

$$s = vt$$

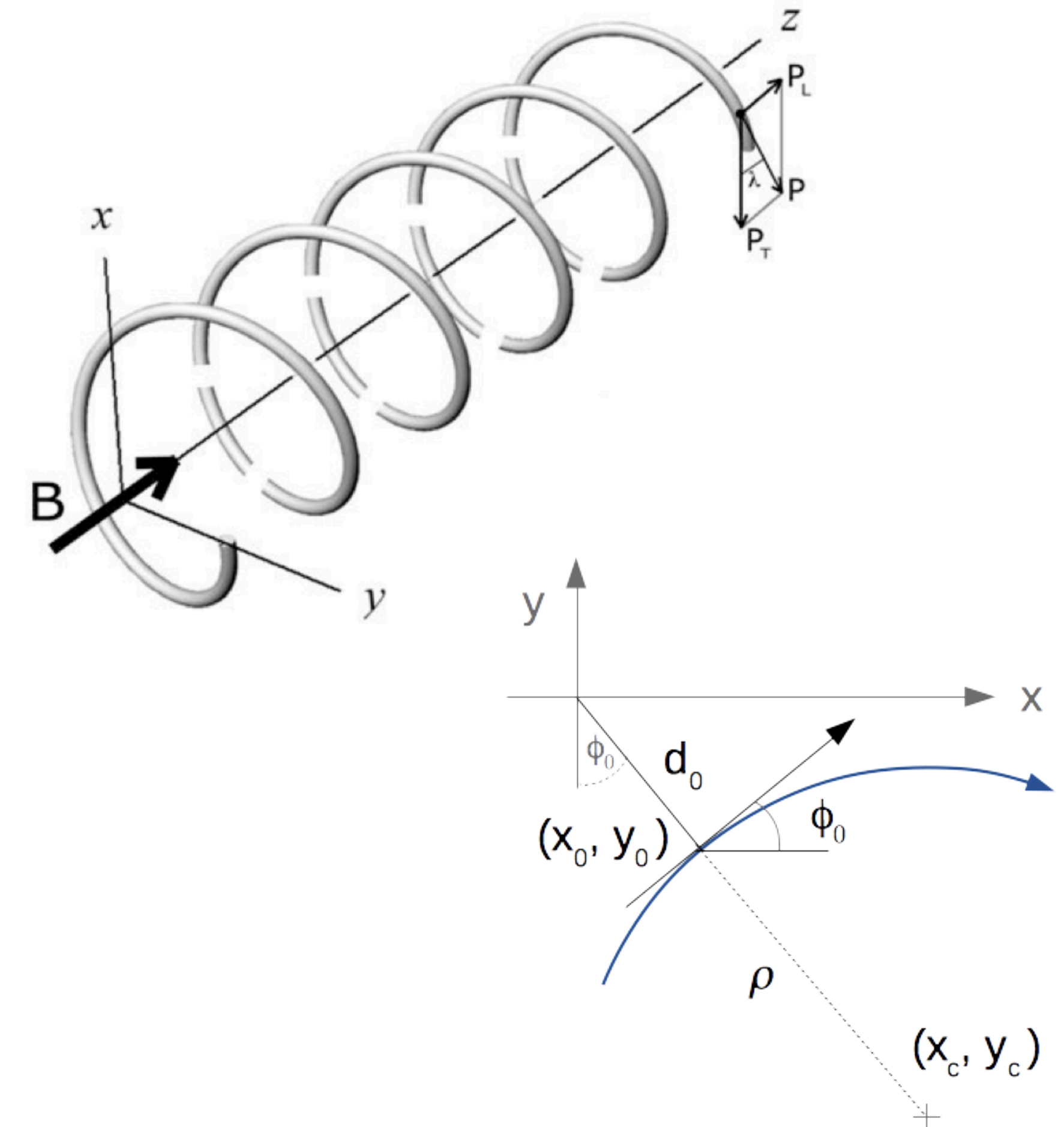
$$ds = v dt$$

$$x(s) = x_0 + \rho \left[\cos \left(\phi_0 + \frac{qs \cos \lambda}{\rho} \right) - \cos \phi_0 \right]$$

$$y(s) = y_0 + \rho \left[\sin \left(\phi_0 + \frac{qs \cos \lambda}{\rho} \right) - \sin \phi_0 \right]$$

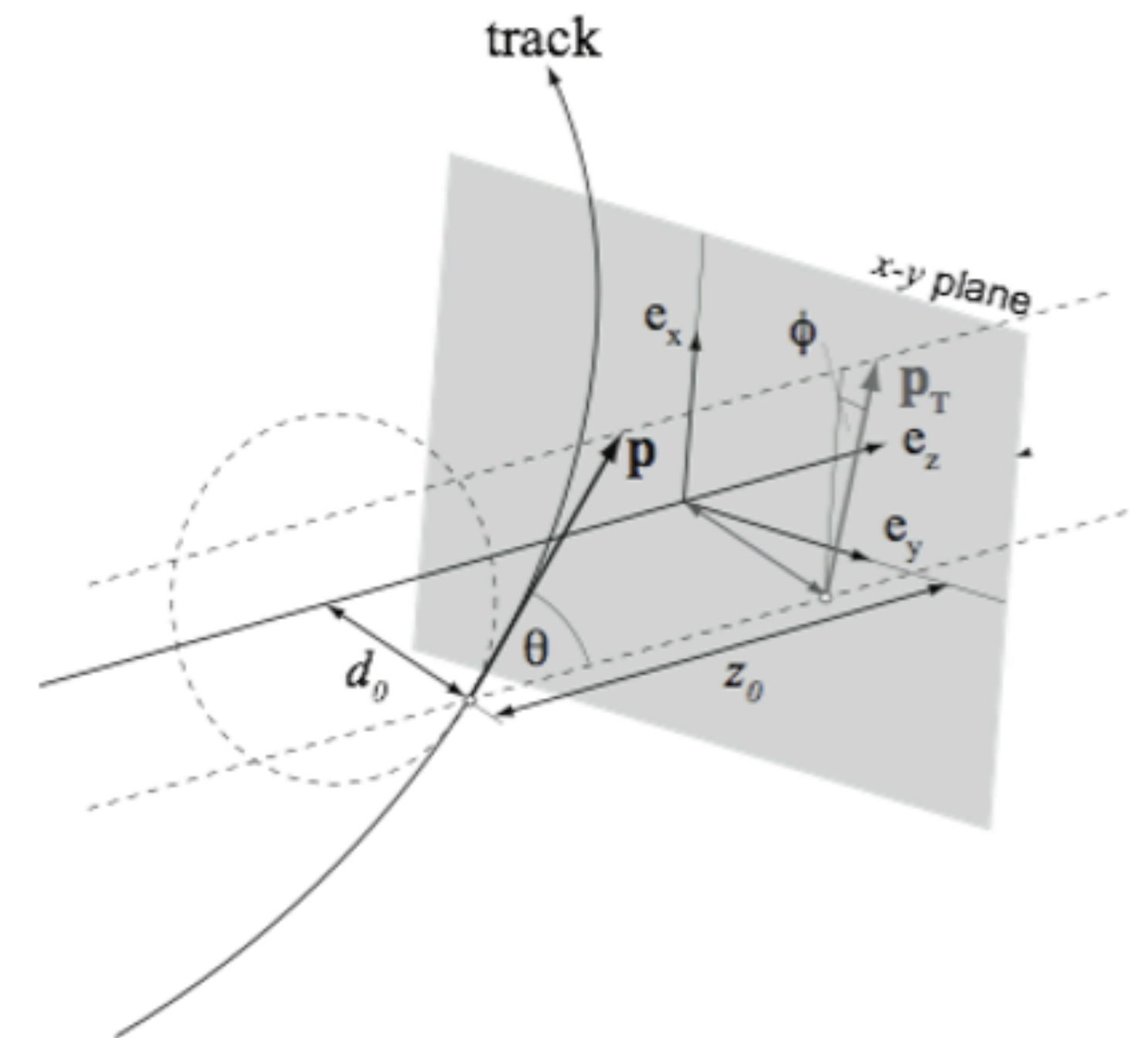
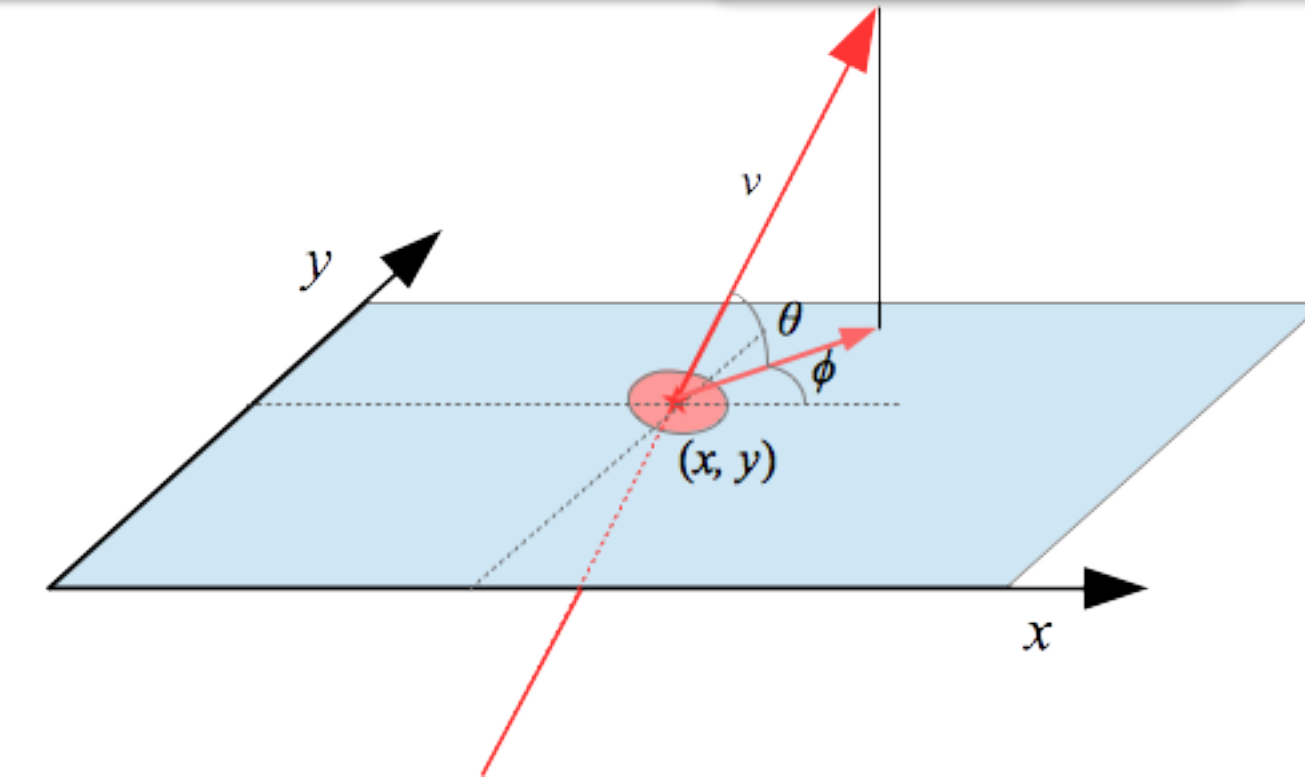
$$z(s) = z_0 + s \sin \lambda$$

$$p_T = p \cos \lambda$$

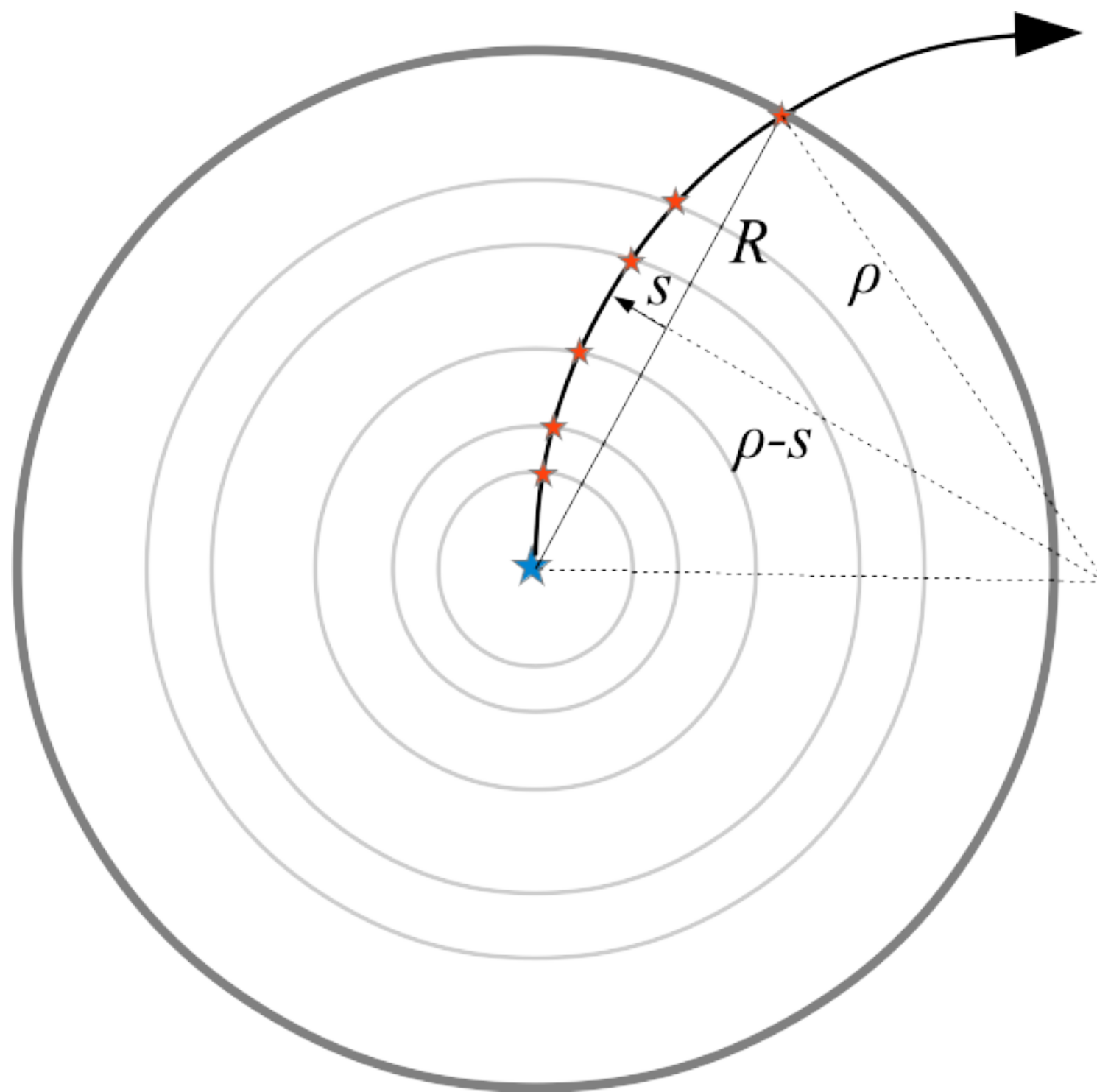


Track parameters

- A trajectory can be parametrized with just 5 parameters at a surface
 - x, y, φ, θ, v
- The track is characterized by its 5 parameters as given at the “perigee surface” & using the global reference coordinate system
 - $d_0, z_0, \varphi_0, \theta_0, q/p \quad \eta = -\ln \left(\tan \frac{\theta}{2} \right)$
 - $d_0, z_0, \varphi_0, \cot \theta_0, q \cdot pT$
 - $d_0, z_0, \varphi_0, \eta, q/p$
- The track extrapolation to detector surfaces usually requires a different parametrization
 - Optimization
- Track parameters given in the local reference frame of the surface
 - Error matrix propagation !
- Track extrapolation
 - Heavily used in tracking code and alignment code



- The basic way to determine the momentum of a particle is by measuring the Sagitta
 - It is a measure of the bending of the particle trajectory
 - It tell us how much the particle path deviated from a straight trajectory
 - Actually, the sagitta gives information of the transverse momentum with respect to the B field axis



$$\rho = \left(\frac{R^2}{8s} + \frac{s}{2} \right) \rightarrow \frac{R^2}{8s} \quad dp_T = -0.3 B \frac{R^2}{8s^2} ds = -0.3 B \frac{R^2}{8s} \frac{ds}{s} = p_T \frac{ds}{s}$$

$$p_T = 0.3 B \rho = 0.3 B \frac{R^2}{8s} \quad \frac{\sigma_{p_T}}{p_T} = \frac{\sigma_s}{s}$$

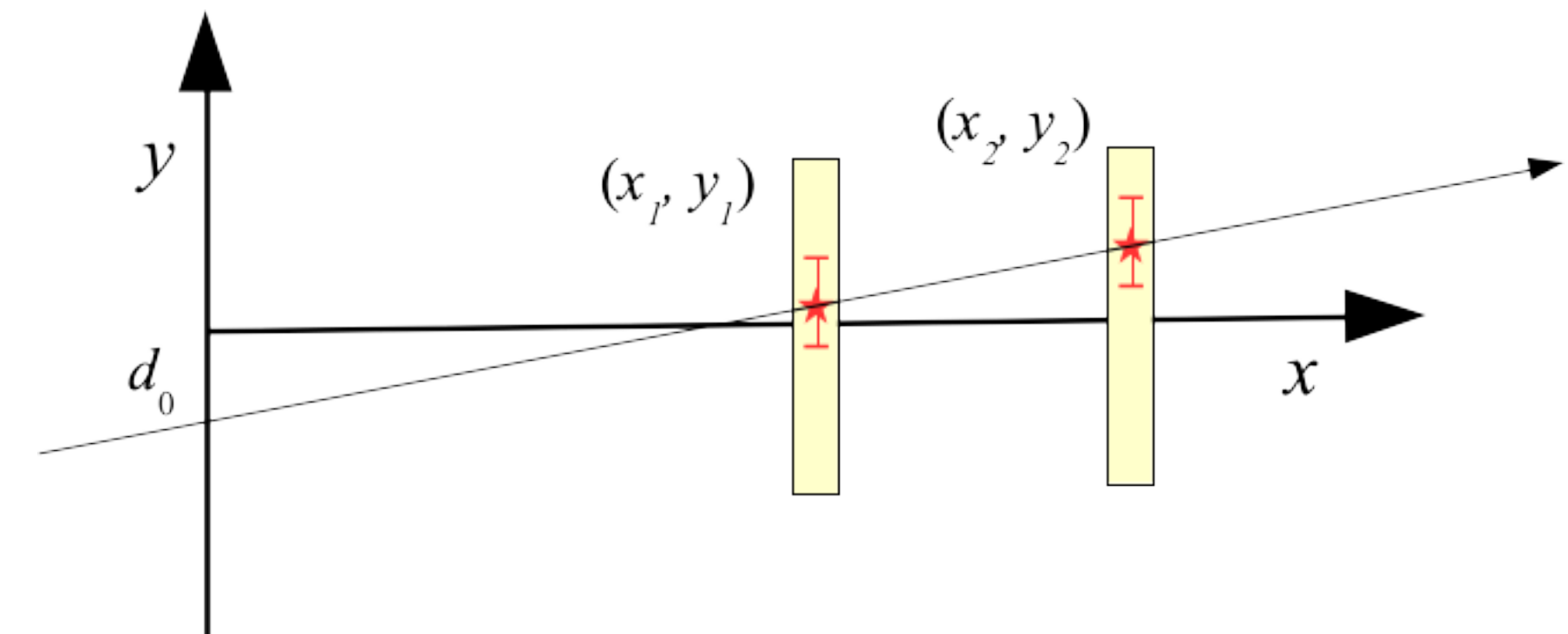
$$\frac{\sigma_{p_T}}{p_T^2} = \frac{8}{0.3 BR^2} \sigma_s$$

Gluckstern formula:

Relative uncertainty of the p_T for N equidistant measurements on sensors with resolution σ_y

$$\frac{\sigma_{p_T}}{p_T^2} \approx \sqrt{\frac{720}{N+4}} \frac{\sigma_y}{BR^2}$$

- Impact parameter resolution is an important figure of merit that has an impact in the physics goals of the experiment
 - Secondary vertex reconstruction, B-tagging, life-time, ...
- In practice impact parameter resolution depends on
 - Sensor resolution (σ_y)
 - Extrapolation distance
 - Material interactions with ancillary structures and sensors
 - Supporting structures, beam pipe, cooling pipes, power cables,...
 - Multiple Scattering effects



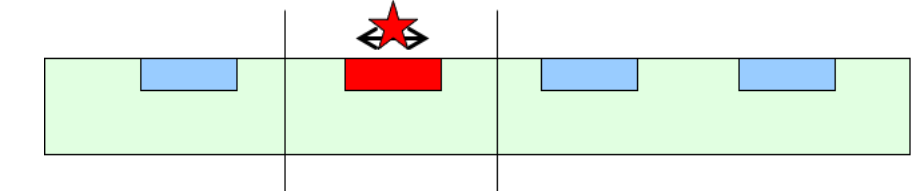
$$\sigma_{d_0} = \frac{\sqrt{x_1^2 + x_2^2}}{x_2 - x_1} \sigma_y \quad \sigma_{d_0} = \sqrt{\frac{n^2 + 1}{(n - 1)^2}} \sigma_y \quad \text{with } n = \frac{x_2}{x_1}$$

- As factor in front σ_y is $>1 \rightarrow$ the spatial resolution of our sensors must be better than the targeted impact parameter resolution

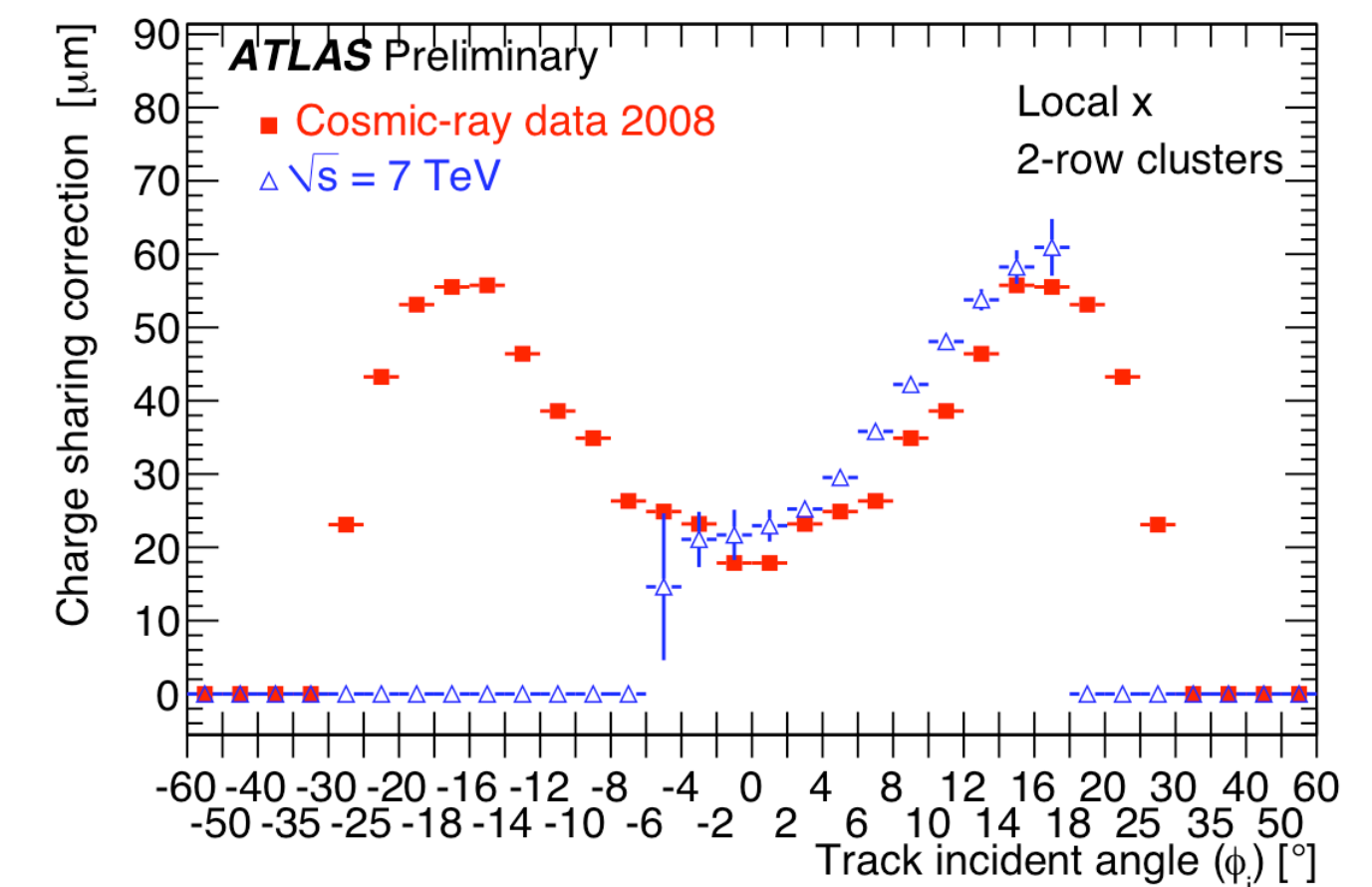
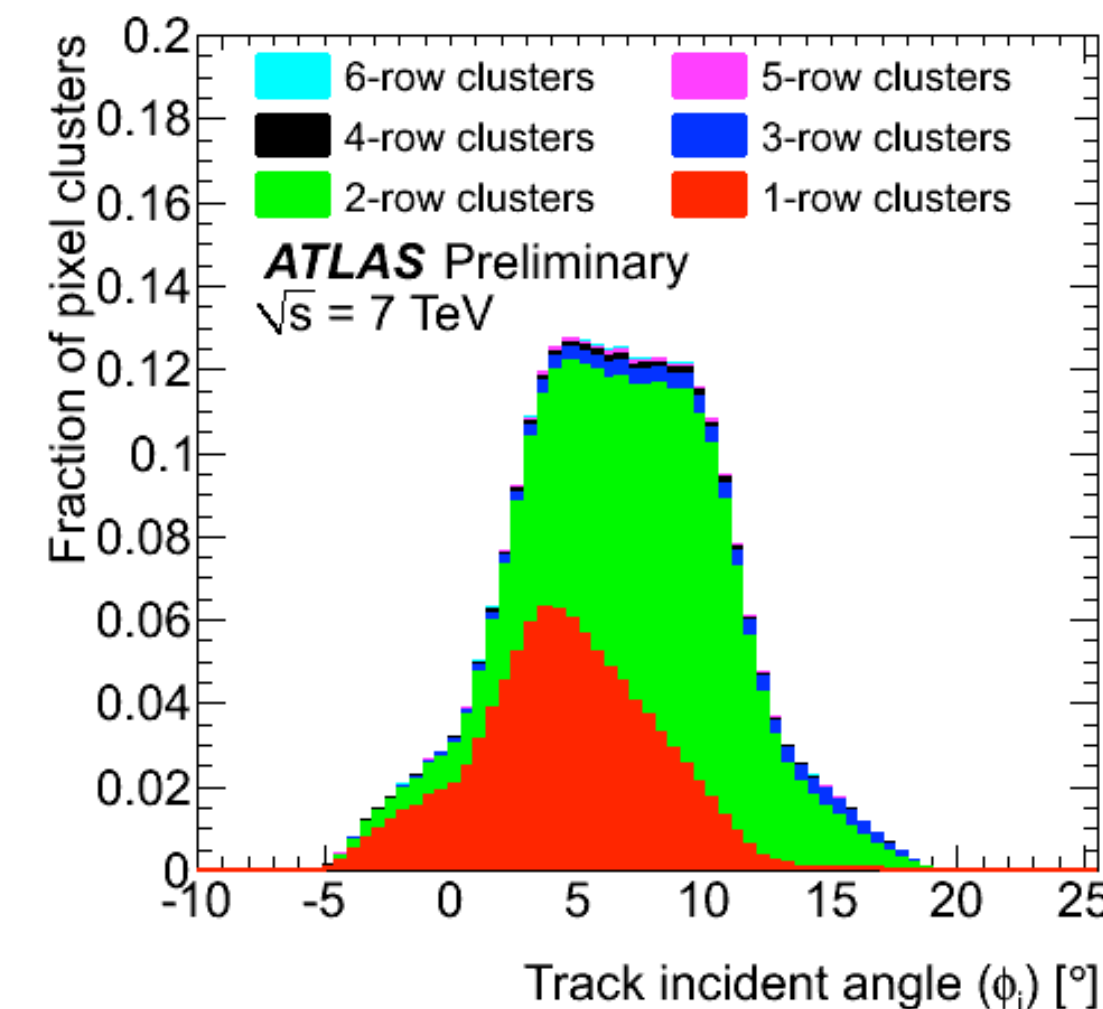
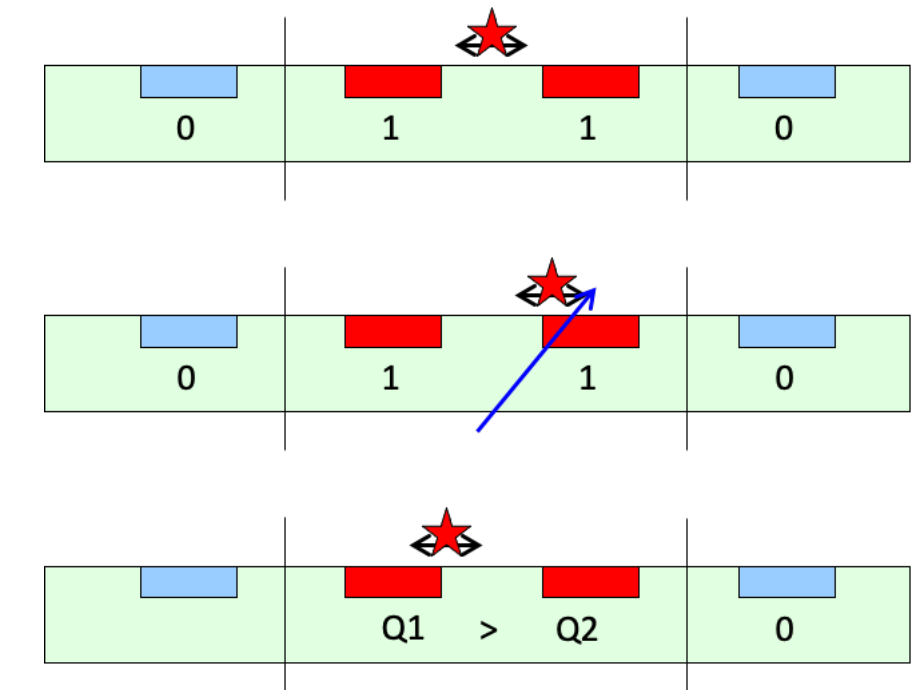
Signal processing for track fitting

- Transform recorded signals into a space points to be used by track fitter
- Hit → sensor channel with signal
 - -Detector specific
 - -Channel ID & pulse height
- Cluster → group of channels
 - From 1 channel to many
 - 3D information:
- Global or local coordinates
 - Position: (x, y, z)
 - Error / uncertainty: $(\sigma_x, \sigma_y, \sigma_z)$ in a covariance matrix form
- Cluster position may depend:
 - Hit data: binary or analog
 - Center of gravity & charge sharing
 - Lorentz angle corrections
 - Charge carriers drift
 - Track incident angle
 - MCS corrections
 - Radiation damage effects
 - ...

Single channel: position → channel centre
error: width / $\sqrt{12}$



Many channels: position & error depend on clustering algorithm
hit info (analog or binary), strategy and conditions,



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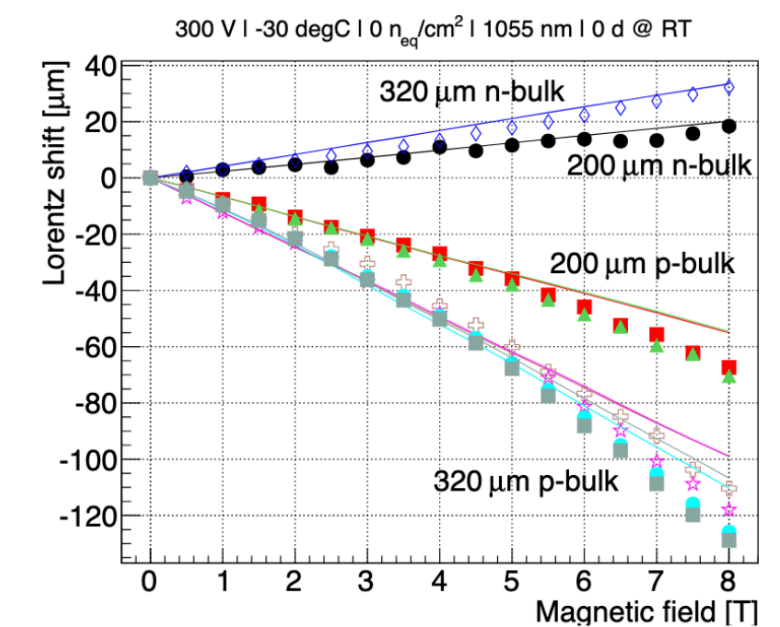
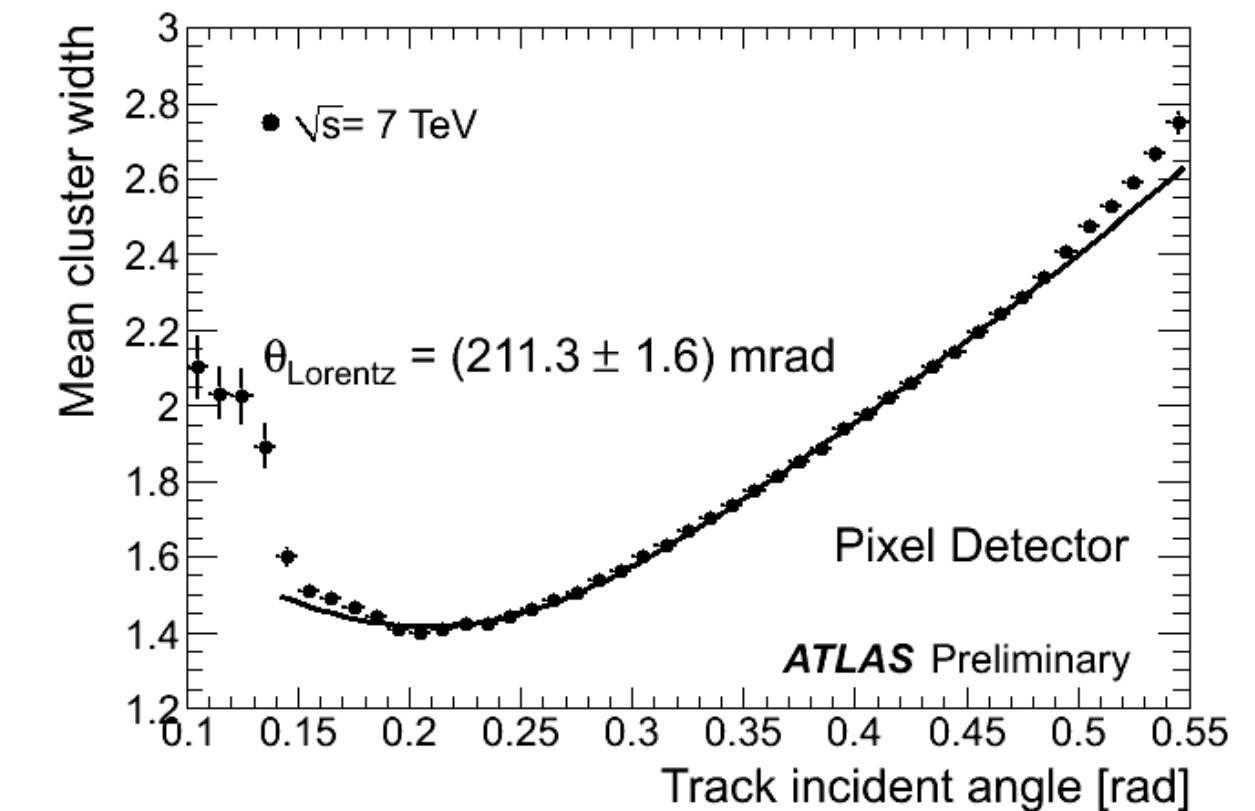
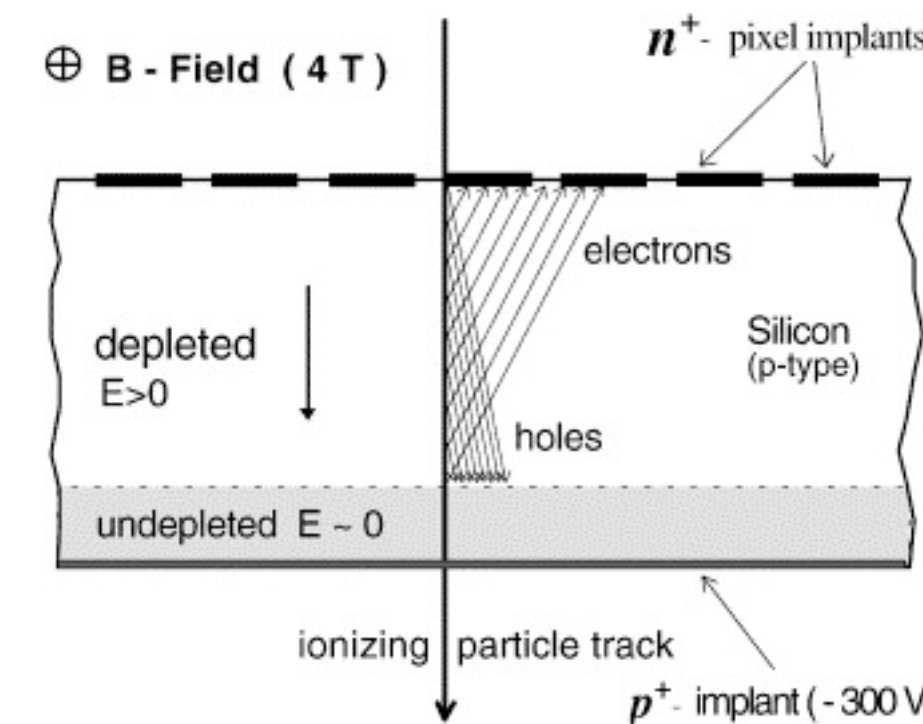


Figure 6. Lorentz shift as a function of the magnetic field obtained in unirradiated sensors with different bulk doping and different active thickness. For electrons, sensors with either p-stop or p-spray strip isolation are shown. The solid lines show the shifts obtained by the simulation model outlined in 3.2.

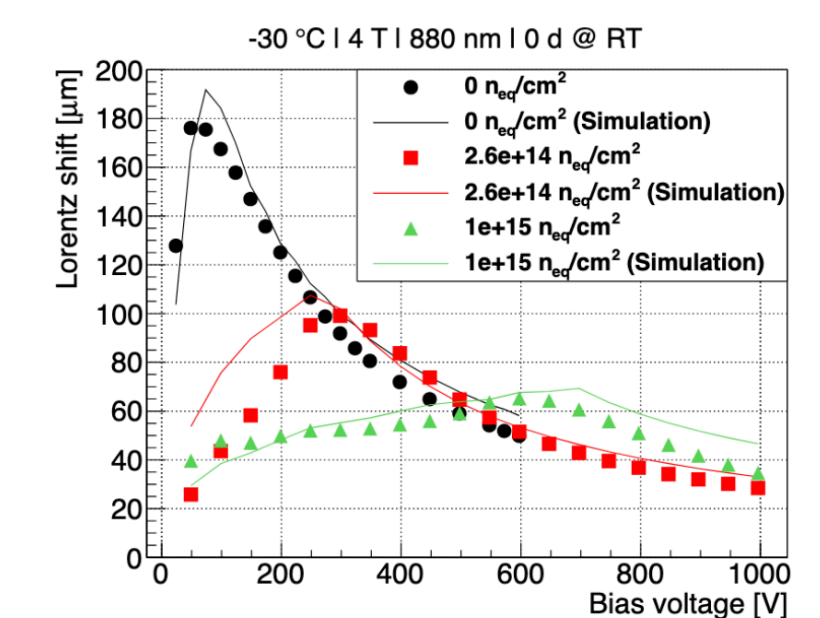


Figure 7. Lorentz shift of electrons in 320 μm thick sensors as a function of the applied bias voltage for three different irradiation fluences. The solid lines are representing the simulation model.

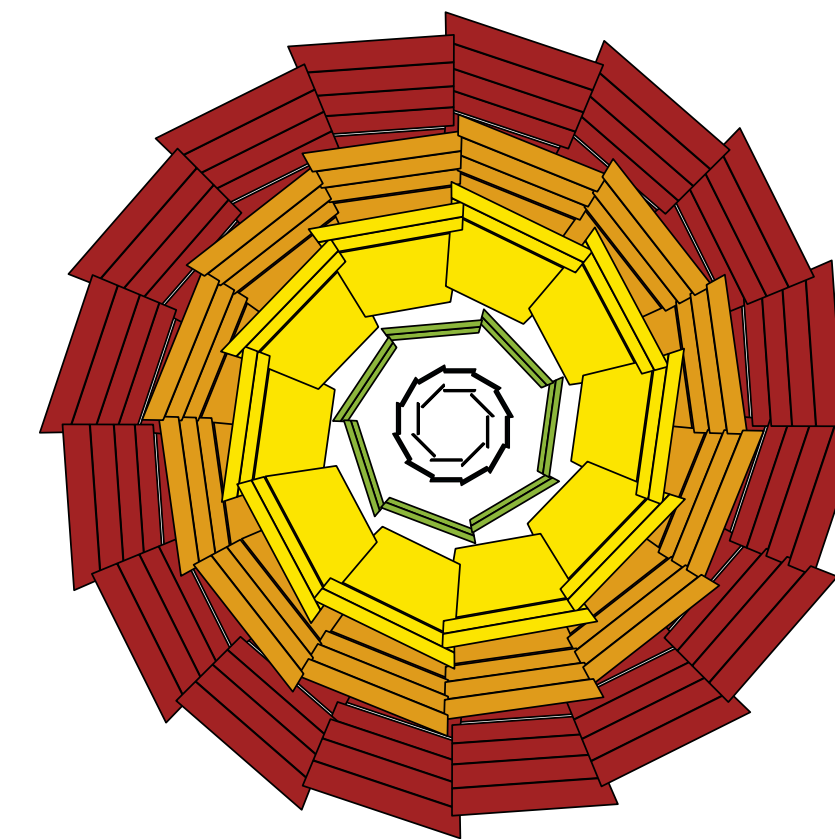
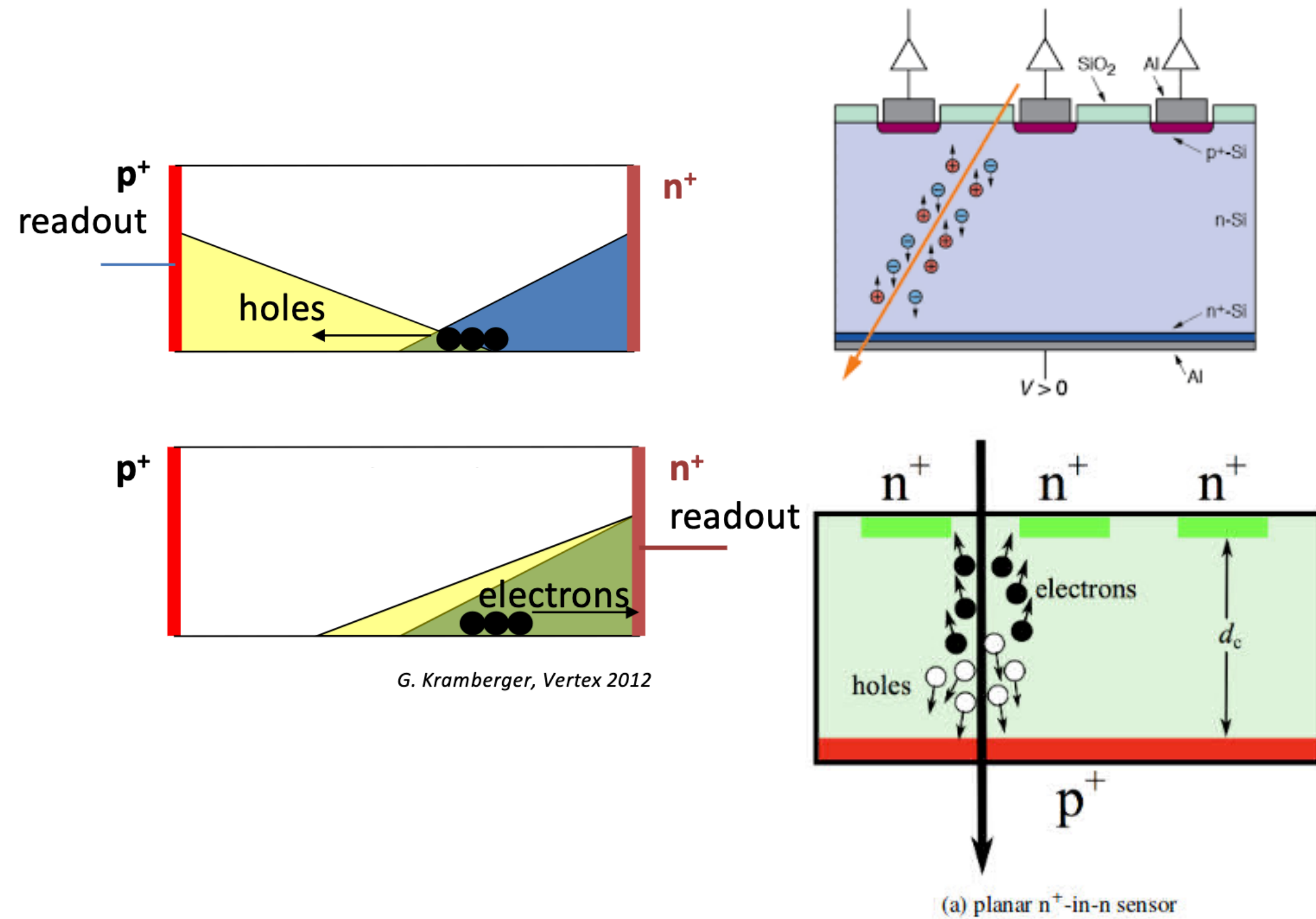
From: Lorentz angle measurements as part of the sensor R&D for the CMS Tracker upgrade
https://cds.cern.ch/record/1478135/files/CR2012_224.pdf

Signal processing for track fitting

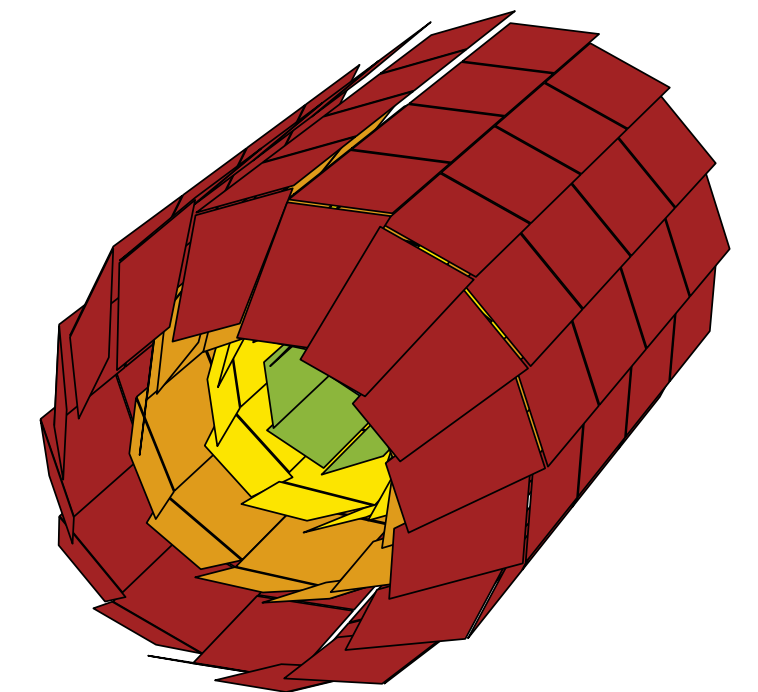
- Lorentz angle corrections example:

- ATLAS strip detector: p-in-n
- ATLAS Pixel detector: n-in-n

BELLE II



Three dimensional view along the z axis



Three dimensional side view

Signal processing for track fitting

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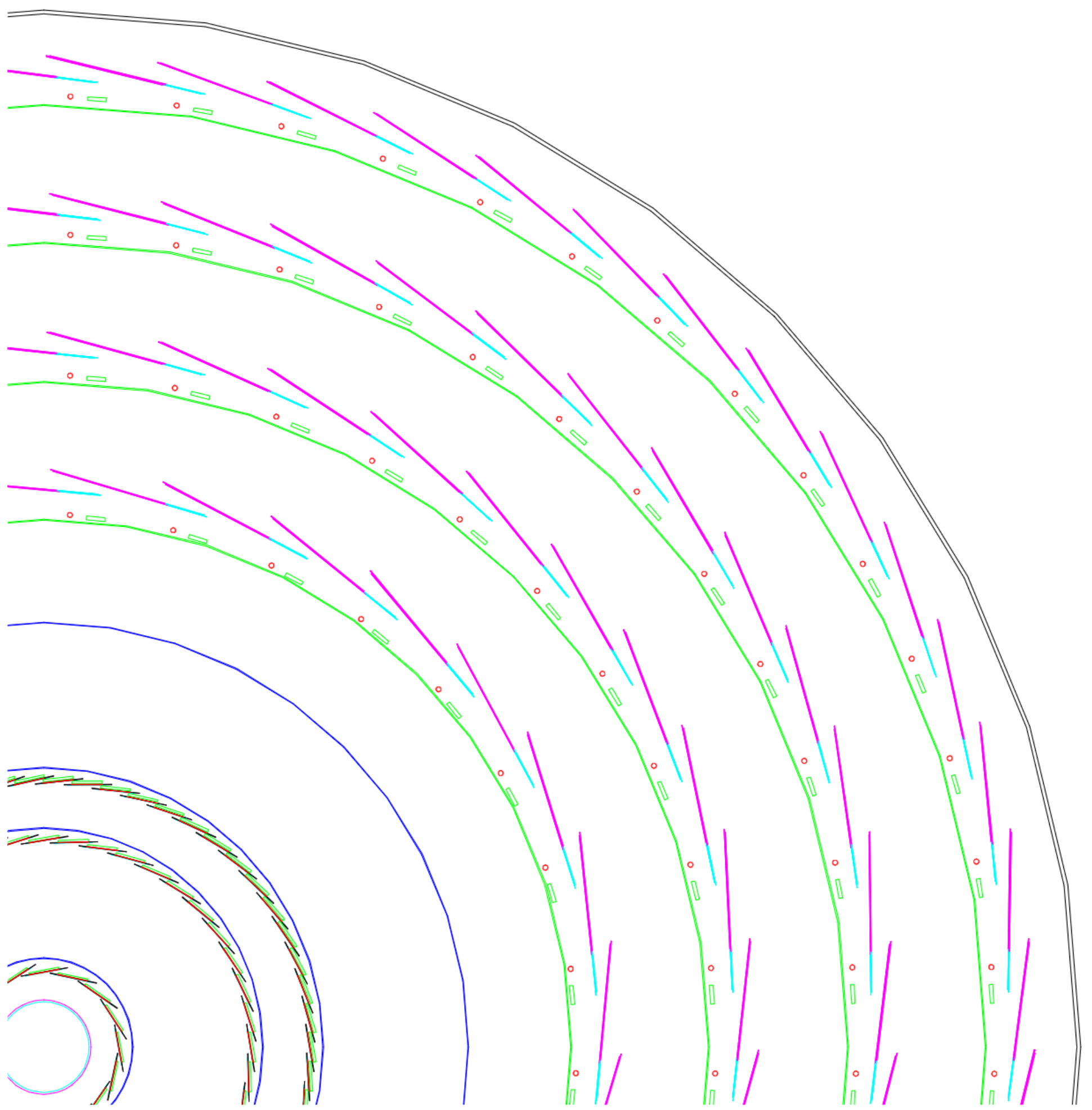
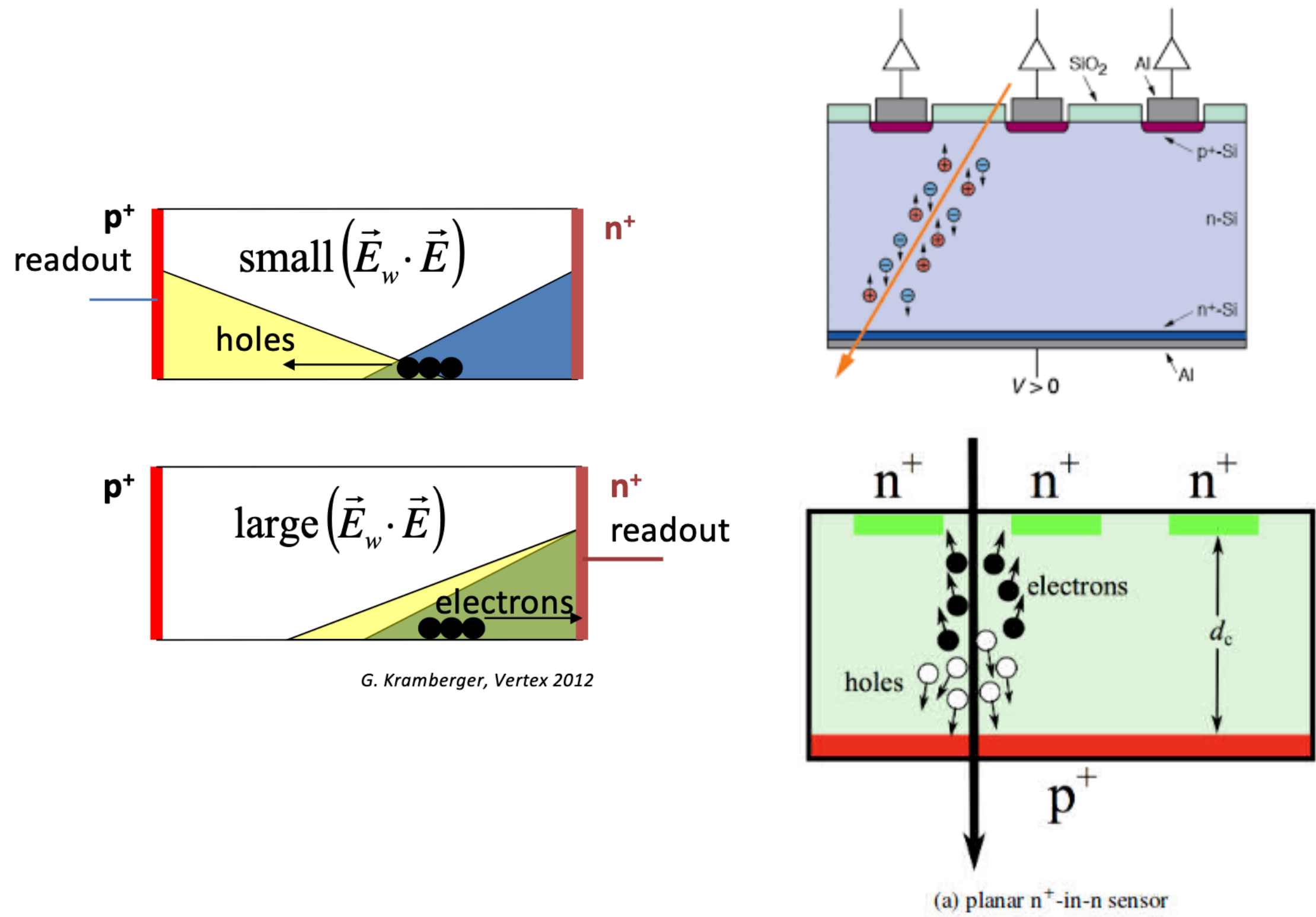
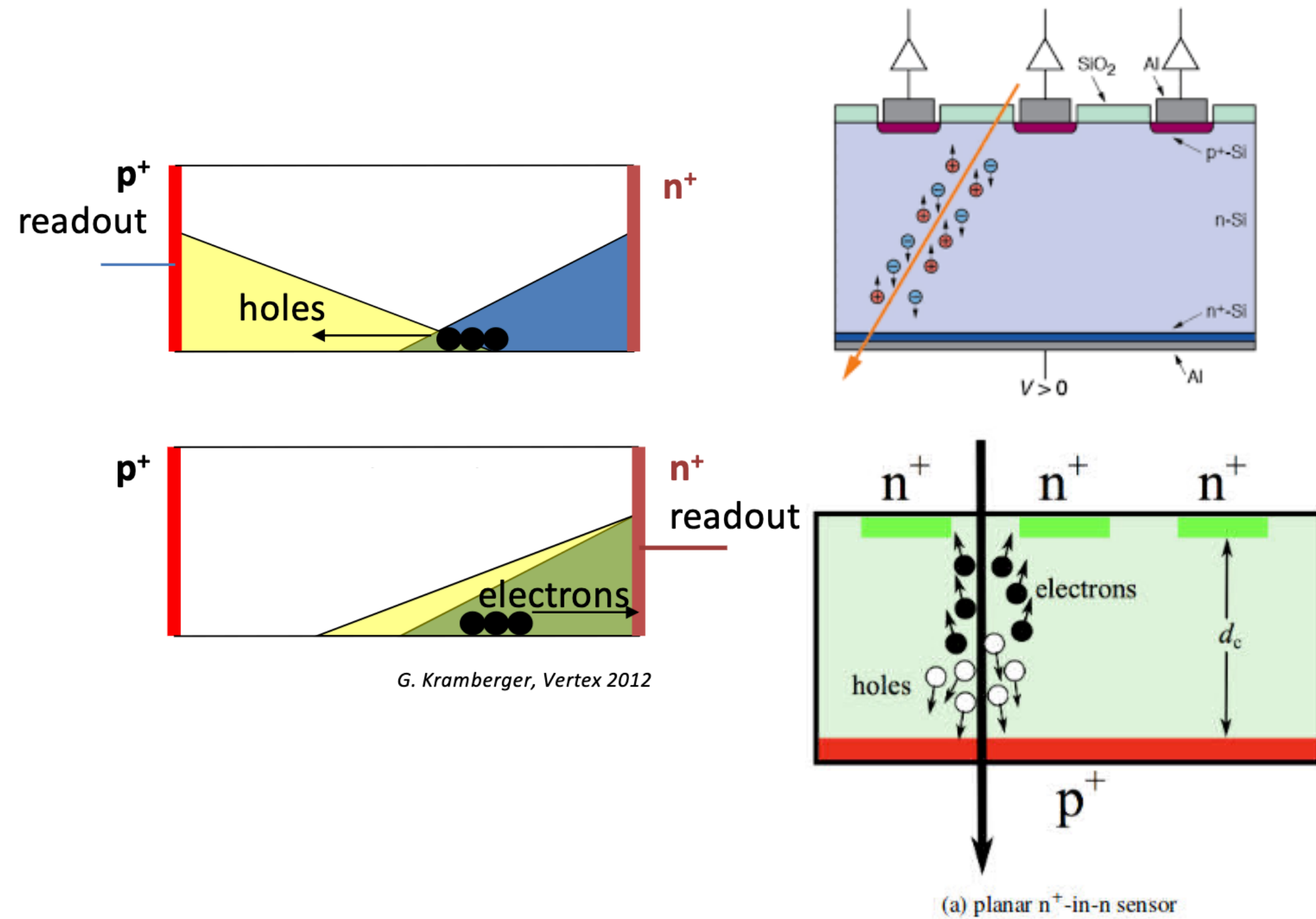


Figure 3-iv Transverse view of a quadrant of the ATLAS Inner Detector (precision layers only). From the centre (lower left-hand corner) are: a) the beam pipe, b) three layers of the barrel pixel tracker: ladder support structure (green), silicon crystal (black) with the active regions (red) and support cylinders (dark blue), c) the overall cylindrical stiffener of the barrel pixel detector (dark blue), d) four layers of the SCT tracker: support (green), lumped power cables and cooling (red circles), active silicon (pink) and electronics boards (pale blue), and e) the SCT's insulating layer (black).

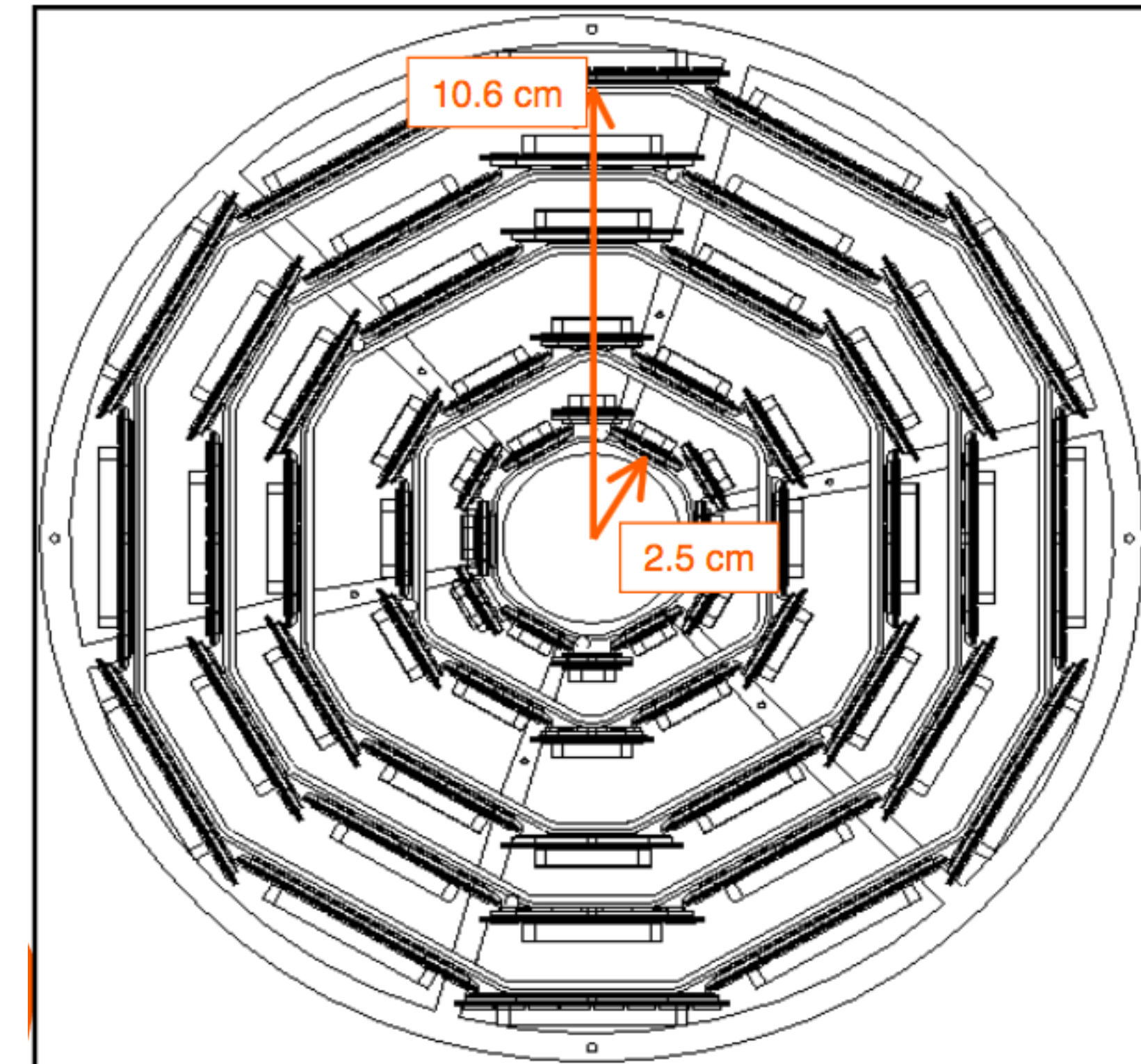
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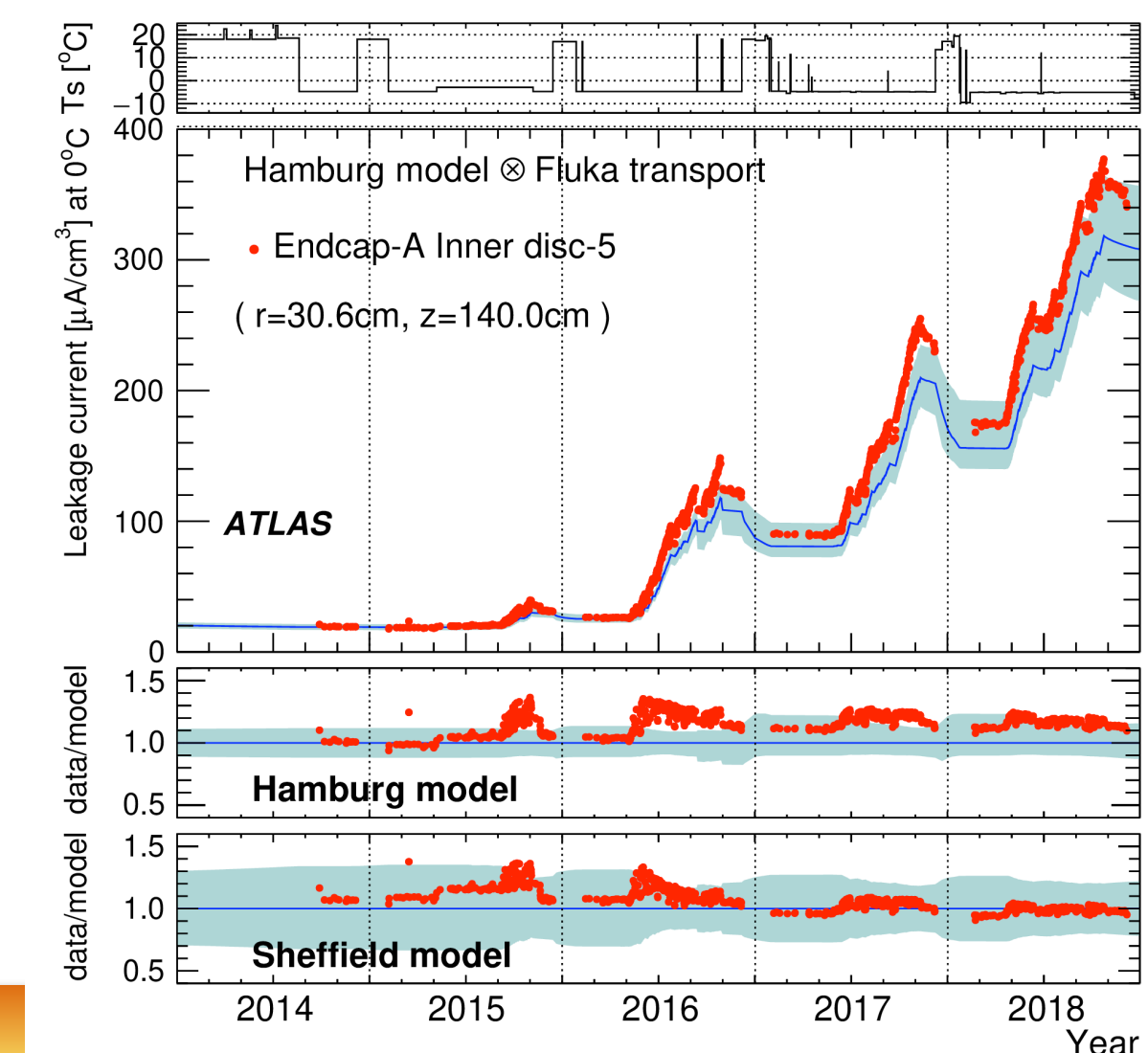
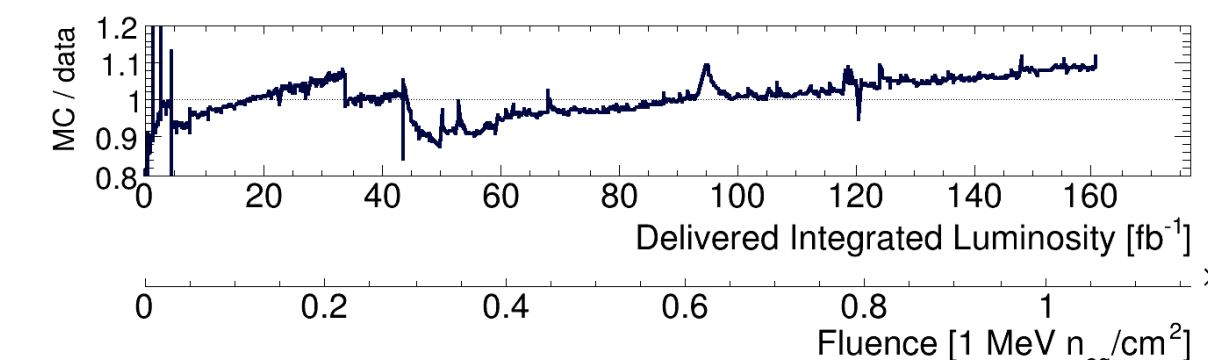
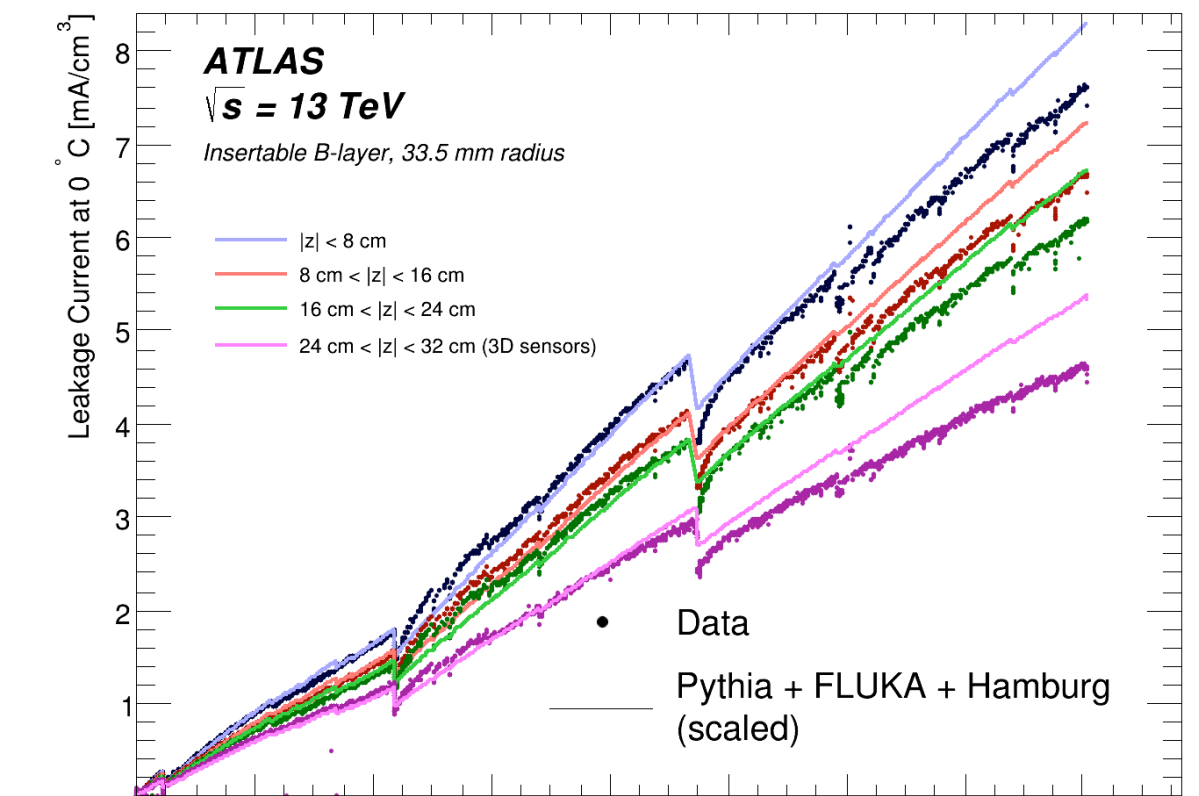


CDF SVT



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Use of NN (tracking in dense environments)

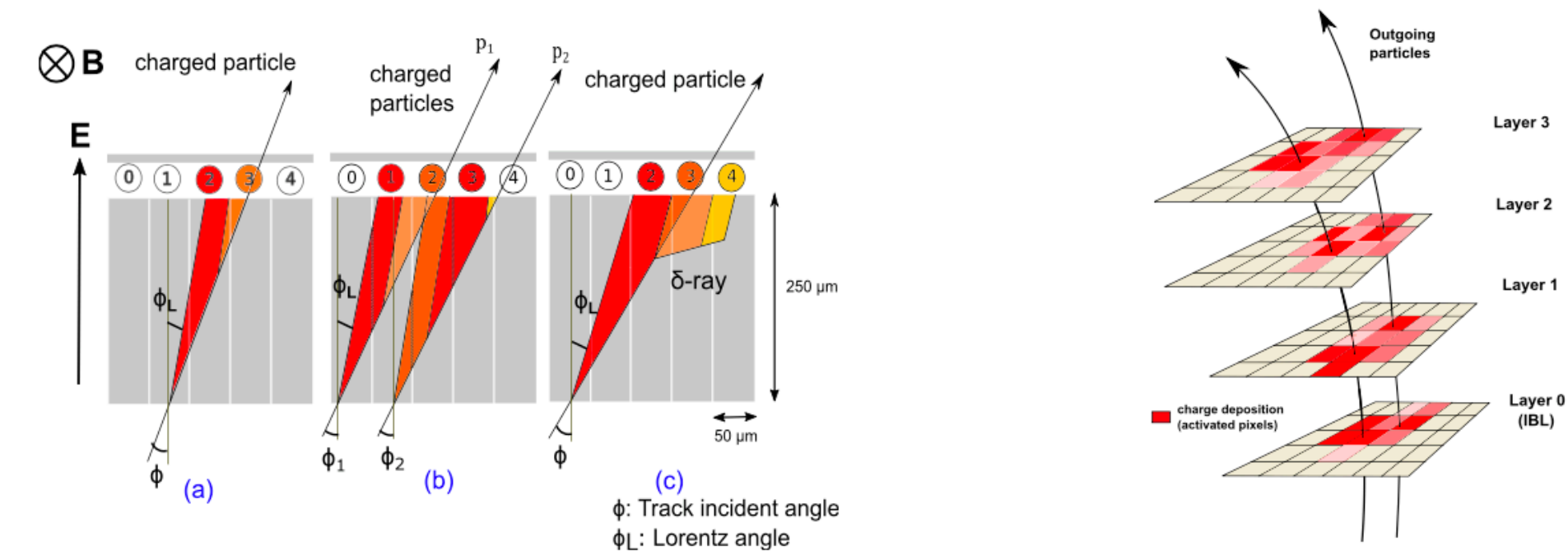


Figure 1: Left:(a) In presence of ATLAS solenoid magnet the charges get diffused and drifted to nearby pixels. (b) Charge clusters of two very close charge particles get merged. (c) δ rays extend the charge cluster size [5]. Right: a schematic diagram where the clusters from two nearby tracks get merged.

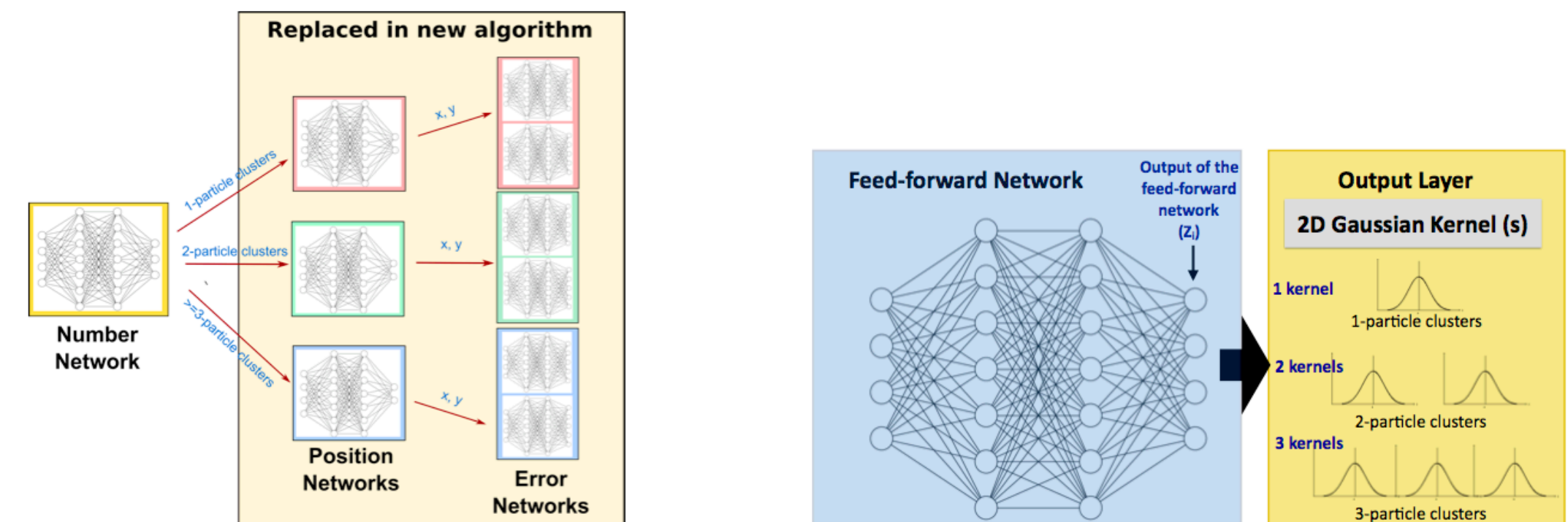
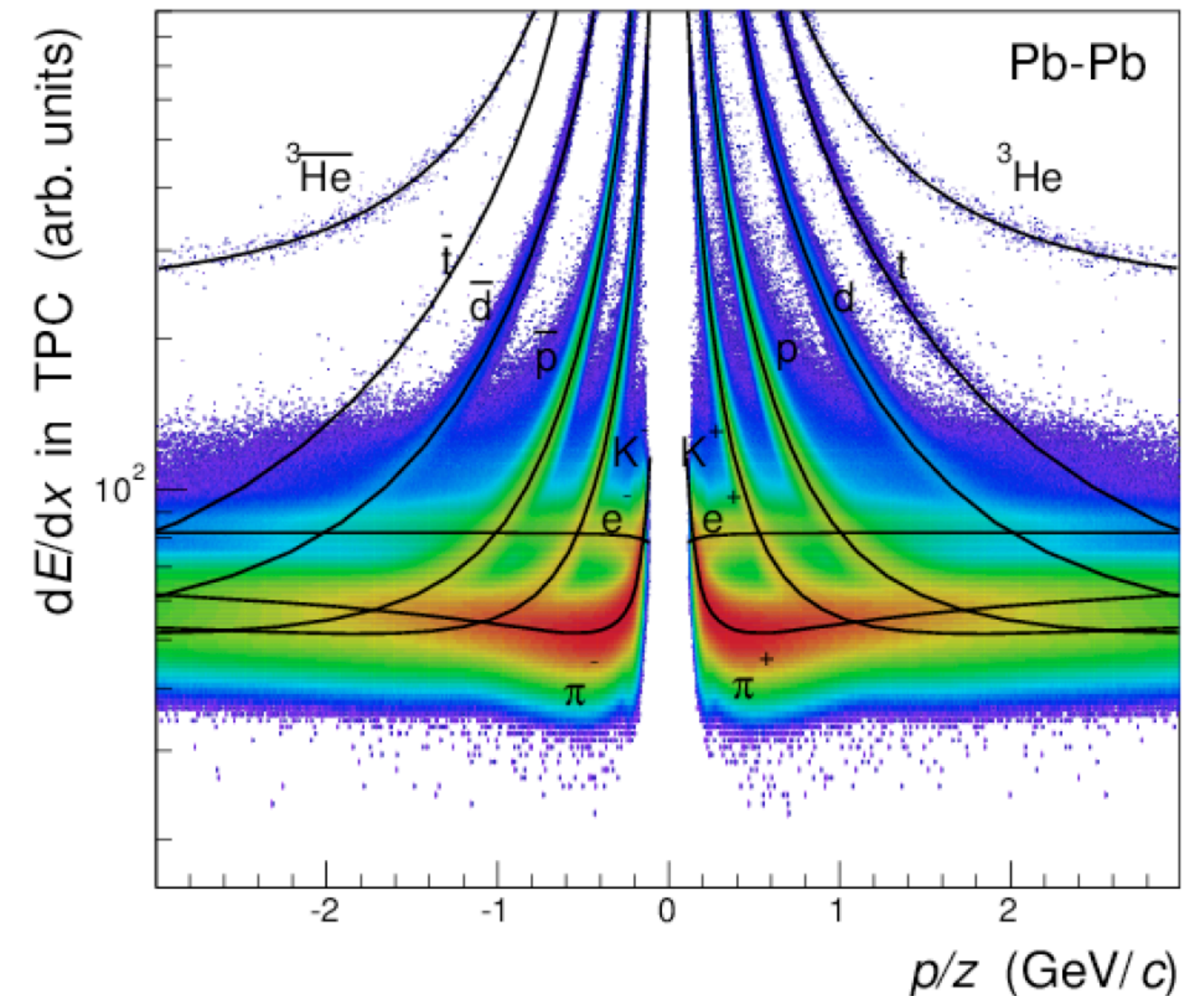


Figure 2: Left: schematic diagram of the current work flow. Right: a schematic diagram of the MDN.

From: ATLAS pixel cluster splitting using Mixture Density Networks
<https://pos.sissa.it/350/009/pdf>

Signal processing for track fitting

- Transform recorded signals into a space points to be used by track fitter
- Hit → sensor channel with signal
 - -Detector specific
 - -Channel ID & pulse height
- Cluster → group of channels
 - From 1 channel to many
 - 3D information:
- Global or local coordinates
 - Position: (x, y, z)
 - Error / uncertainty: $(\sigma_x, \sigma_y, \sigma_z)$ in a covariance matrix form
- Cluster position may depend:
 - Hit data: binary or analog
 - Center of gravity & charge sharing
 - Lorentz angle corrections
 - Charge carriers drift
 - Track incident angle
 - MCS corrections
 - Radiation damage effects
 - ...

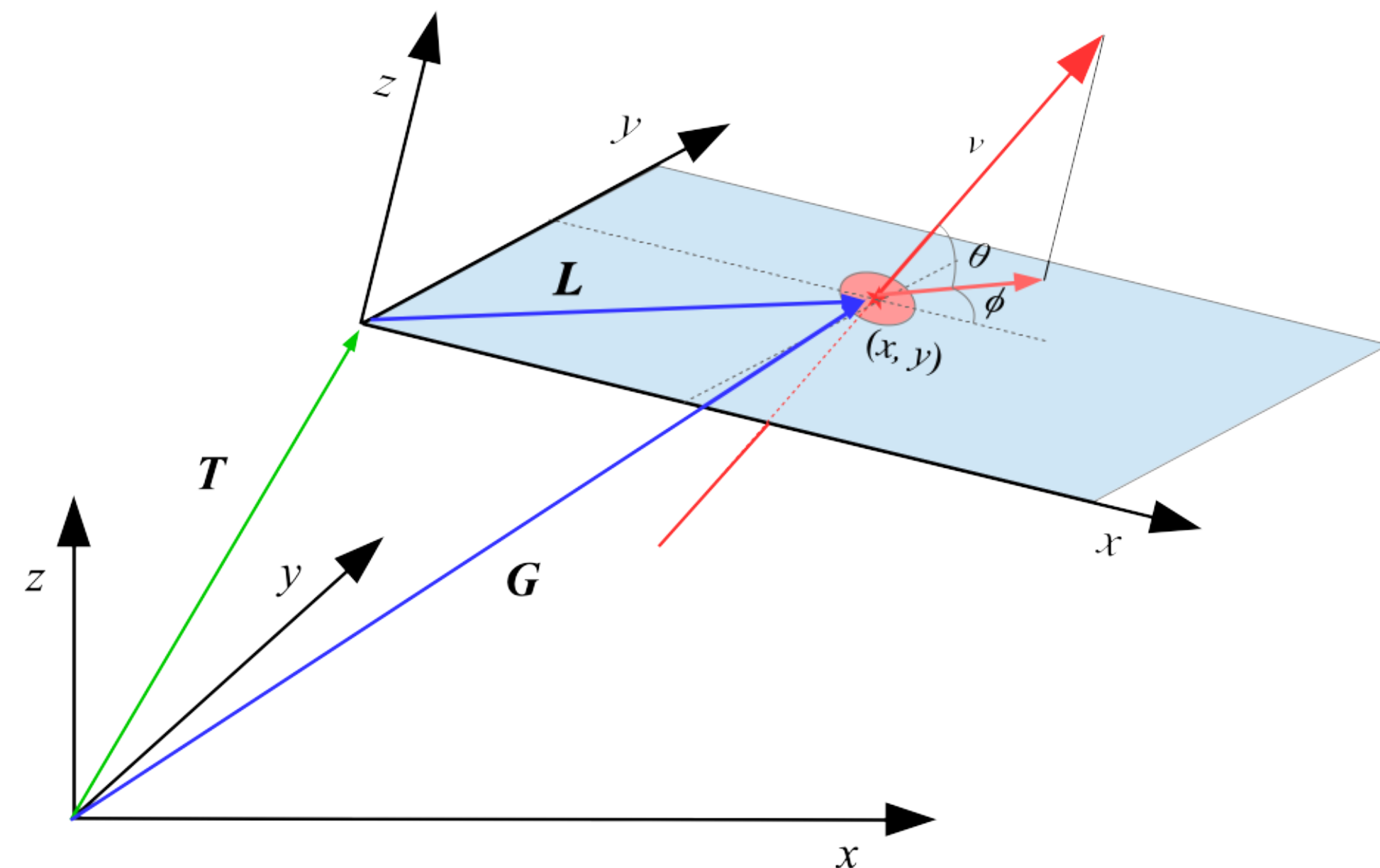


- Reference frame transforms may be heavily used in track fitting
- Points and their uncertainties are needed/computed in the local and global reference frames
 - Measurements (sensor hits)
 - Predictions (track extrapolation)
 - Track-hit residuals are usually given in the local reference frame
- **The transform from the global-to-local reference frame includes:**
 - A translation (of the origin of the local frame w.r.t the global frame)
 - A rotation (orientation of the axis of the local frame w.r.t. the global frame)
 - Still the possibility to use cartesian, cylindric, spherical,... coordinates in either frame

$$H = [T, R]$$

$$G = H L = T + R L$$

$$L = H^{-1}G = R^{-1} (G - T)$$



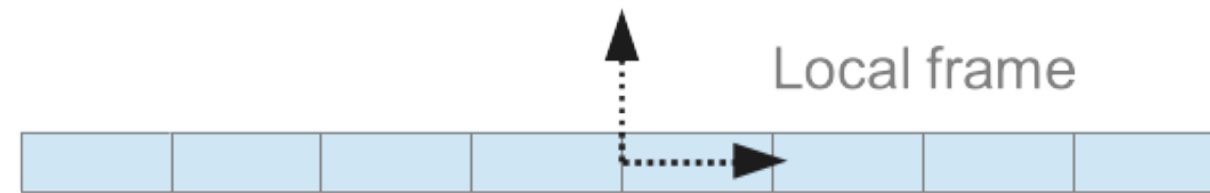
$$T = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix} \quad R = R(\alpha, \beta, \gamma) = R_x(\alpha) R_y(\beta) R_z(\gamma)$$

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix} \quad R_y(\beta) = \begin{pmatrix} -\cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{pmatrix} \quad R_z(\gamma) = \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

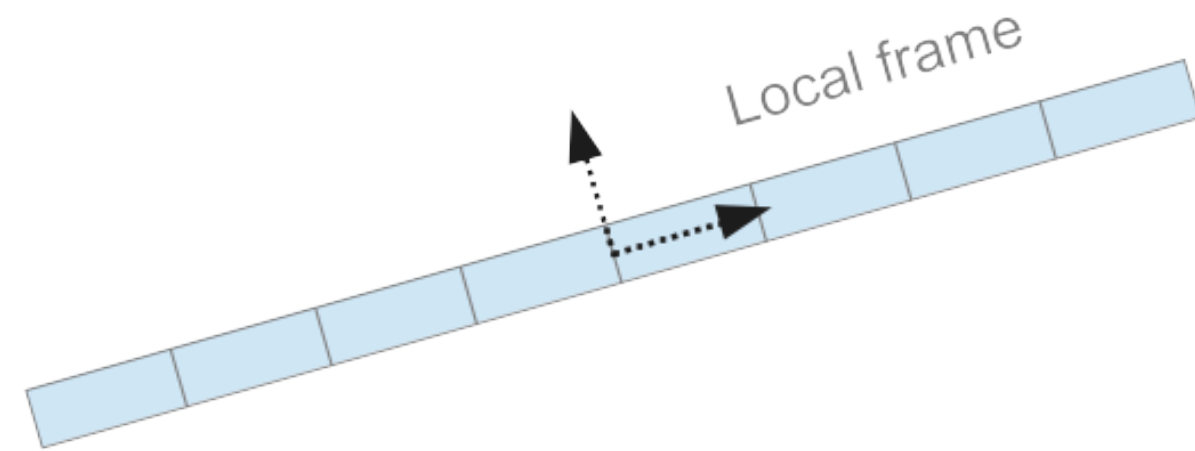
- The R matrices are change of base matrices
 - Unitary matrices $R^\top = R^{-1}$
 - The norm of the vectors is kept
- The translation is given just by a 3D vector
- The covariance matrix of the measurements/extrapolations in the local and global reference frame are related via
 - Usually local covariance matrix is diagonal

$$V_L = \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{pmatrix}$$
$$V_G = R^\top V_L R = R^\top \begin{pmatrix} \sigma_x^2 & 0 & 0 \\ 0 & \sigma_y^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{pmatrix} R$$
$$V_L = R V_G R^\top$$

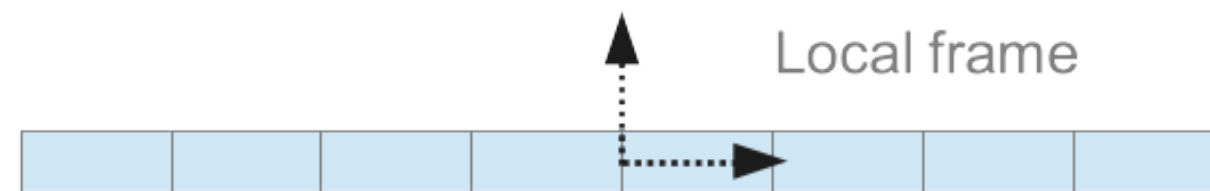
Global and local reference frames



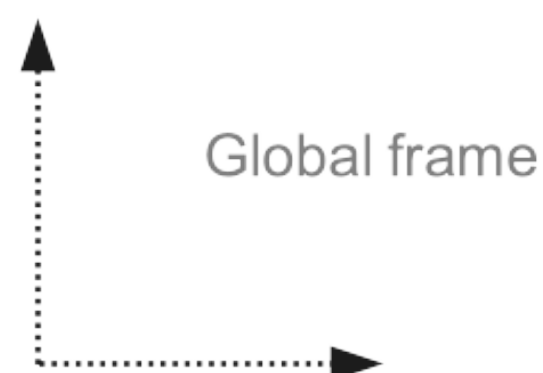
$$V_L = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} \quad V_G = R^\top V_L R = R^\top \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$



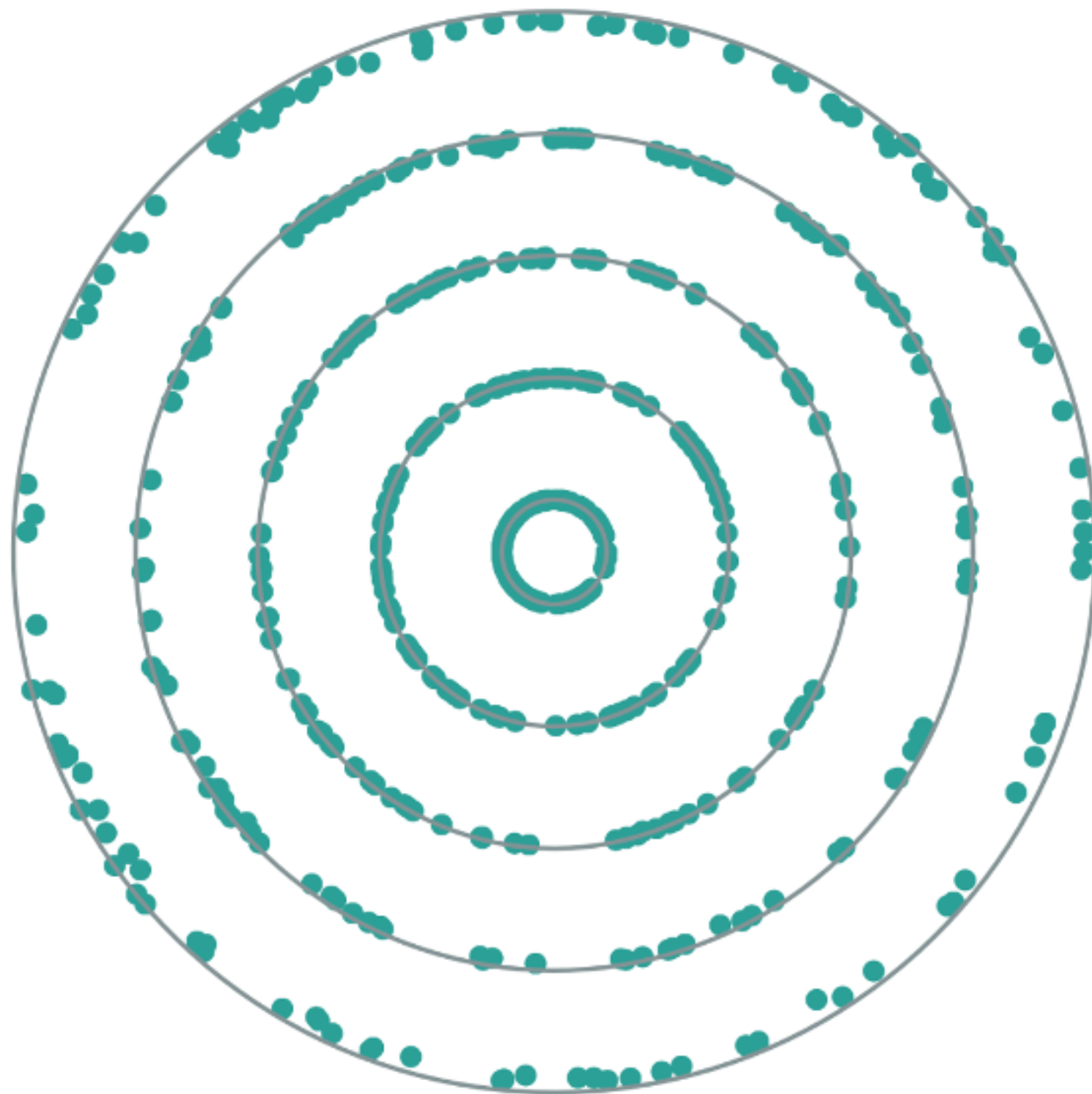
$$V_G = R^\top V_L R = R^\top \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} R = \begin{pmatrix} \cos \gamma & \sin \gamma \\ -\sin \gamma & \cos \gamma \end{pmatrix} \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix} \begin{pmatrix} \cos \gamma & -\sin \gamma \\ \sin \gamma & \cos \gamma \end{pmatrix}$$



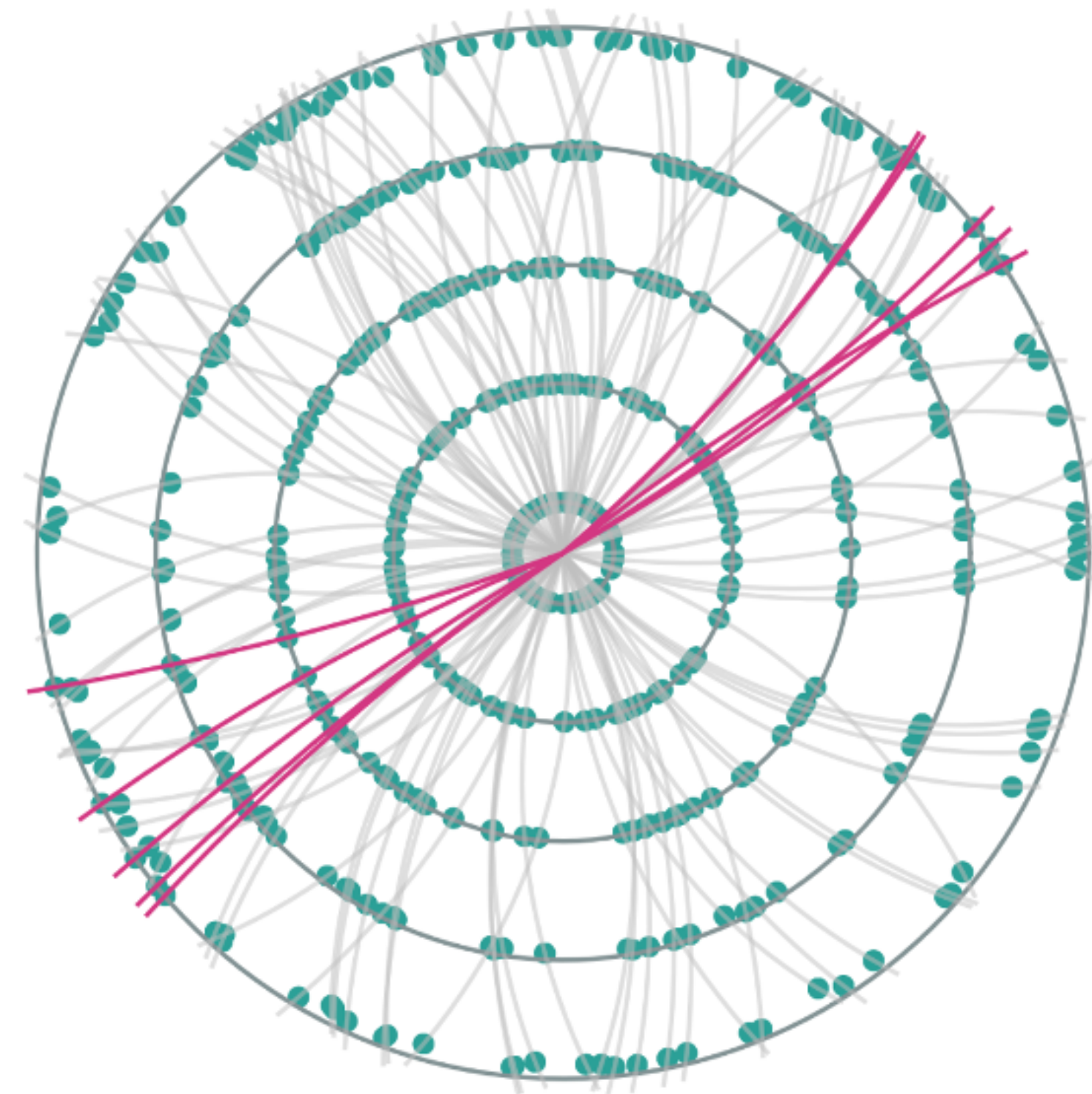
$$V_G = \begin{pmatrix} \sigma_x^2 \cos^2 \gamma + \sigma_y^2 \sin^2 \gamma & -\sigma_x^2 \cos \gamma \sin \gamma + \sigma_y^2 \cos \gamma \sin \gamma \\ -\sigma_x^2 \cos \gamma \sin \gamma + \sigma_y^2 \cos \gamma \sin \gamma & \sigma_x^2 \sin^2 \gamma + \sigma_y^2 \cos^2 \gamma \end{pmatrix}$$



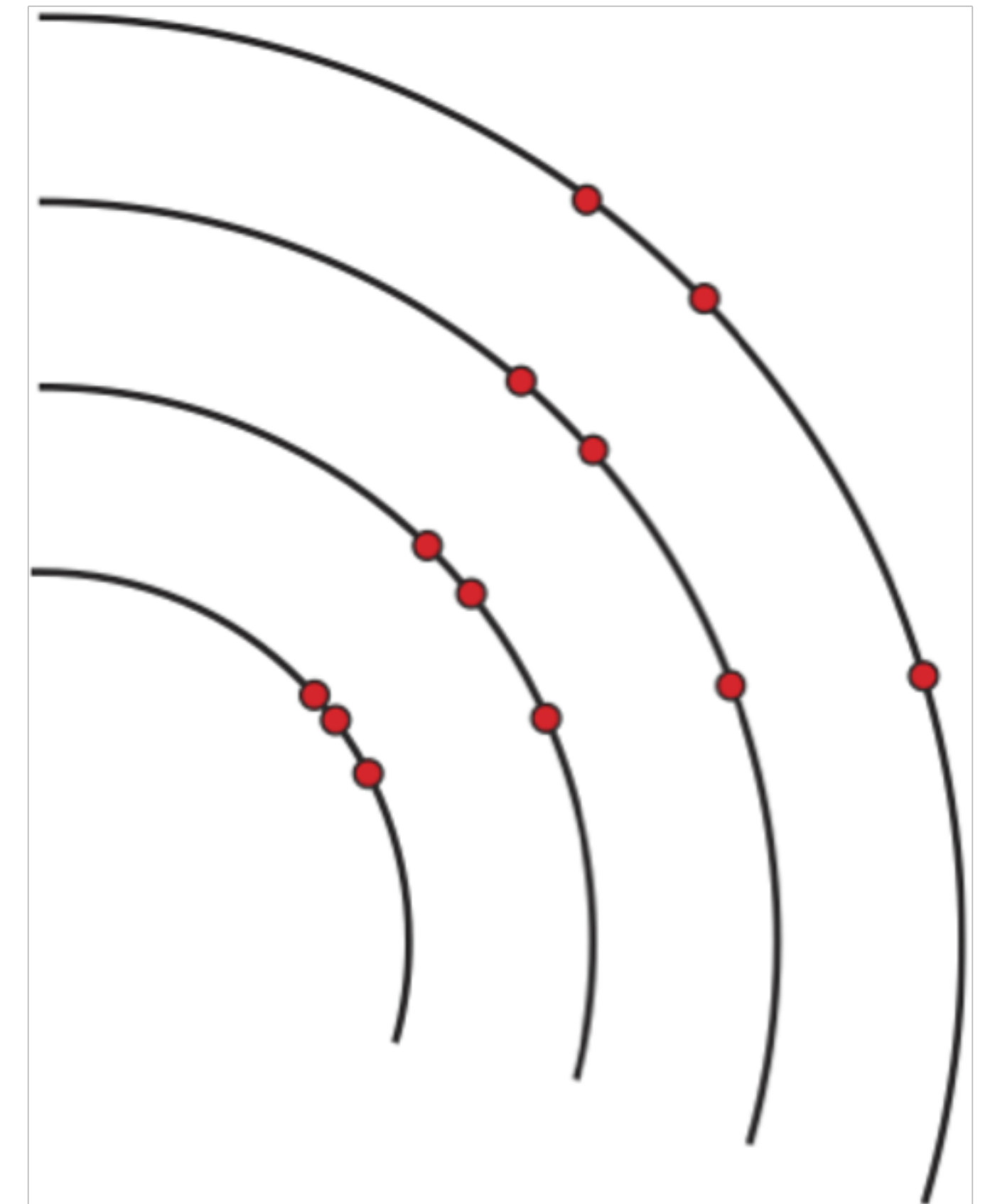
From hits



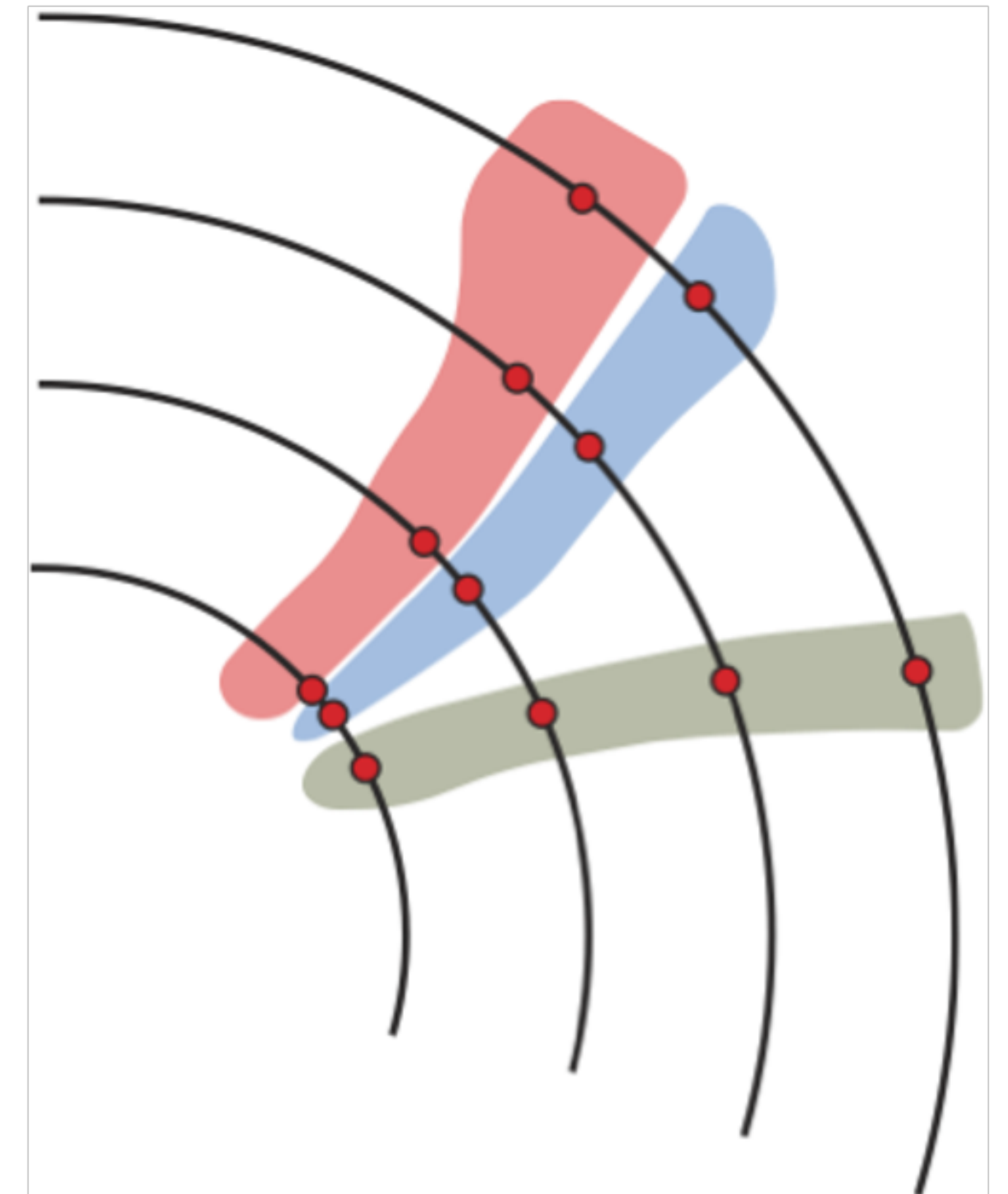
...to tracks



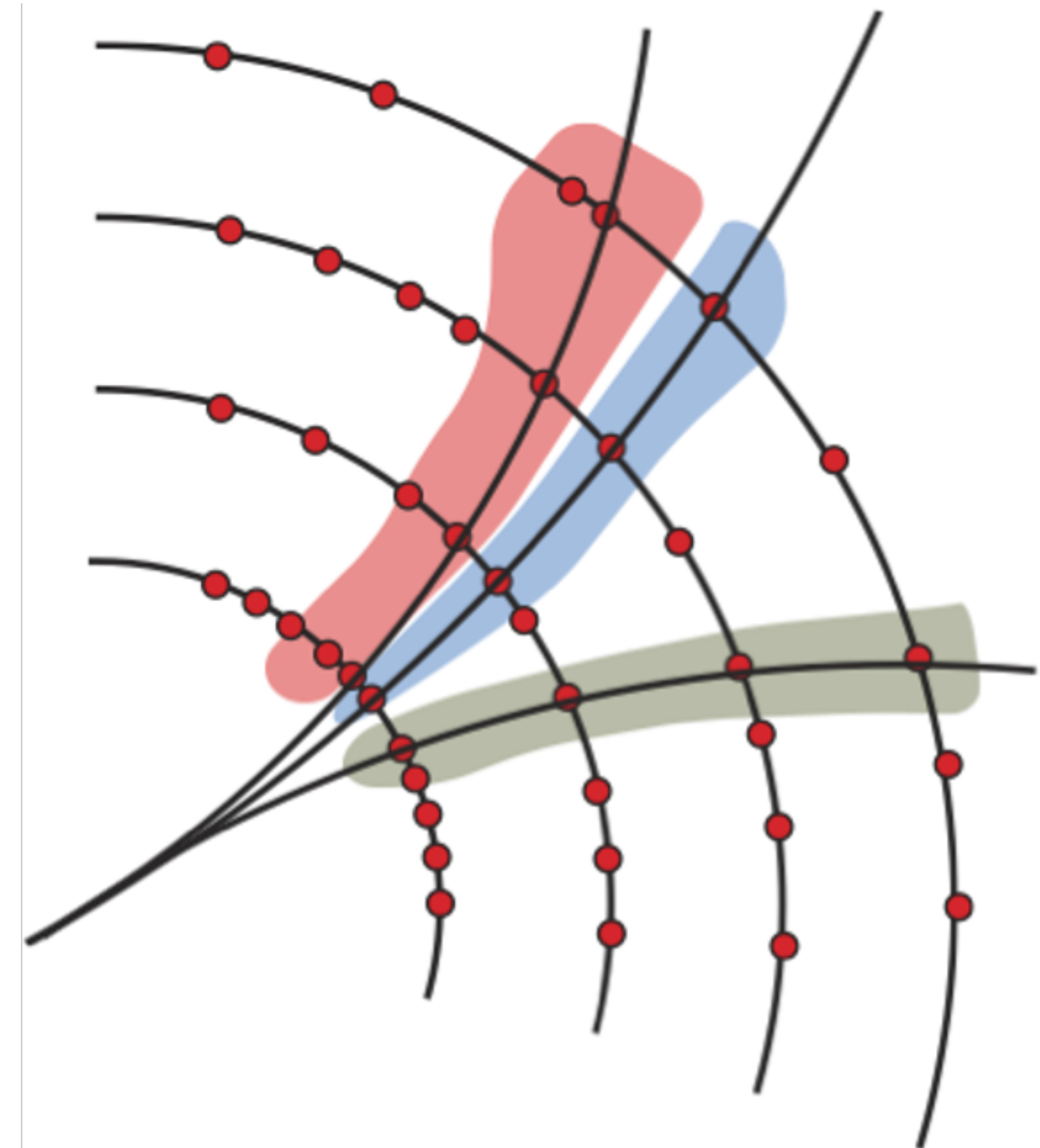
- The main goal of the pattern recognition is to associate hits to tracks
 - Efficient: all hits
 - Robust: no noise and no hits from other tracks
- Pattern recognition is a field of applied mathematics
 - It makes use of statistics, cluster analysis, combinatorial optimization, etc
 - The choice of the algorithm depends heavily in the type of measurements
 - 2D vs 3D points
 - And in the track model
- Detector shape and B field
 - Hough space transform
 - template matching
 - minimum spanning tree
 - local pattern recognition
- Hit-to-track association
 - Defined by pattern recognition
 - Later may be altered by tracking
- Removing bad hits & outliers
 - Noisy channels tend to be the “party spoilers”



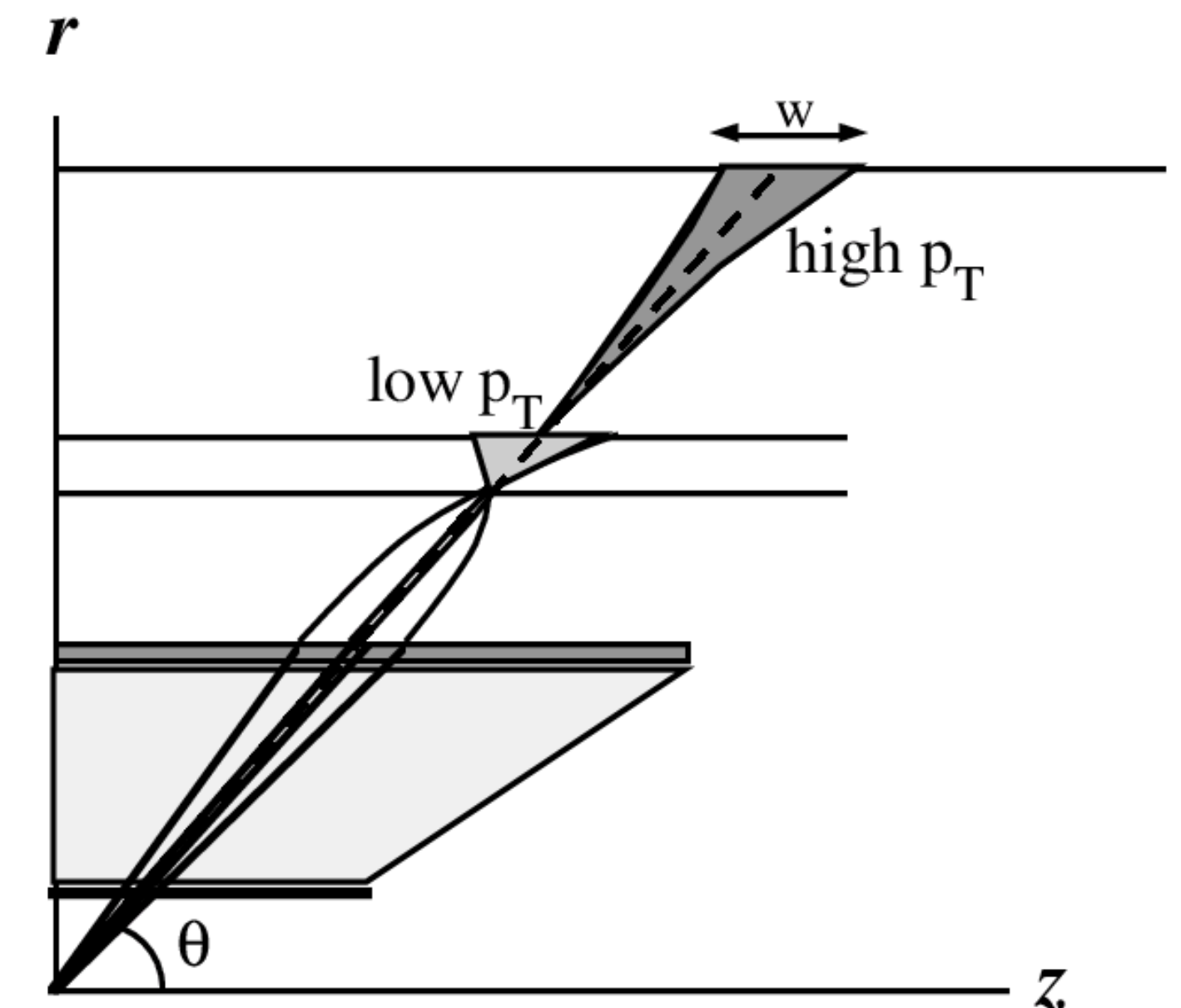
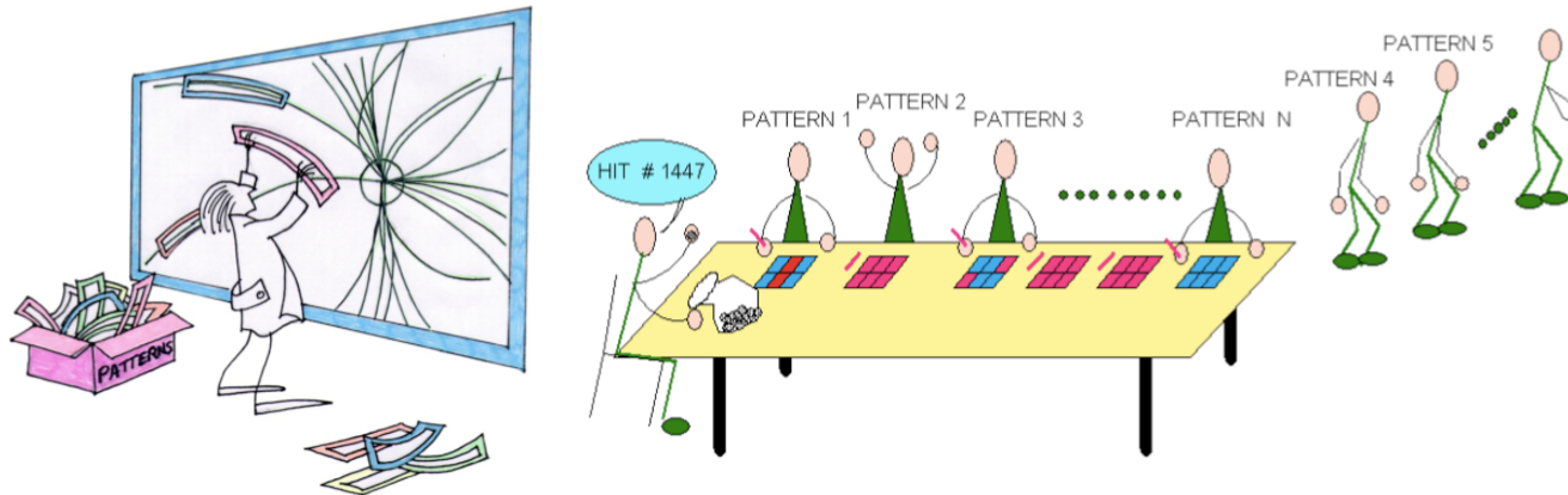
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- It is possible to perform an online pattern recognition for a fast online tracking
- Why fast tracking ?
 - Online one has a limited time to decide if the event is stored or discarded
 - A finite set of track topologies is used
 - Possibility to implement a “fast tracking” based trigger
 - Trigger on secondary vertices → online B-tagging
- Preferred methods
 - Template matching using predefined roads
- This field is evolving fast
 - Use of Machine Learning algorithms



- **Hough transform** it is a technique for digital image detection
 - It can detect the points that belong to a line (straight, circle, ellipse, helix...)
- **So the coordinates of the track points satisfy the line equation**
 - The Hough space has as many dimensions as the number of parameters to determine
 - Straight line (2D): 2. Circle (2D): 3. Helix (3D): 5
 - Then take all possible tuples (of track parameters) that will pass by each point
- **Infinite combinations → discretize &/or use constraints**
 - (e.g.: particles were originated at the center of the detector).
 - Count how many times a given parameter tuple is possible / find intersections
 - Select the most frequent parameter tuple (more intersections)
- **Use the matched points for the track fit**
 - Initial track parameters → use most frequent parameter tuple
- **Example: straight line**
 - Points are given as many available (x, y) tuples.
 - Lines are given as (tanθ, y₀) tuples
 - Solve: draw lines in Hough space and check for intersections

$$\text{2D space (x,y): } y = x \tan\theta + y_0$$

$$\text{Hough space (tan}\theta, y_0\text{): } y_0 = y - x \tan\theta$$

Pattern recognition: Hough transform

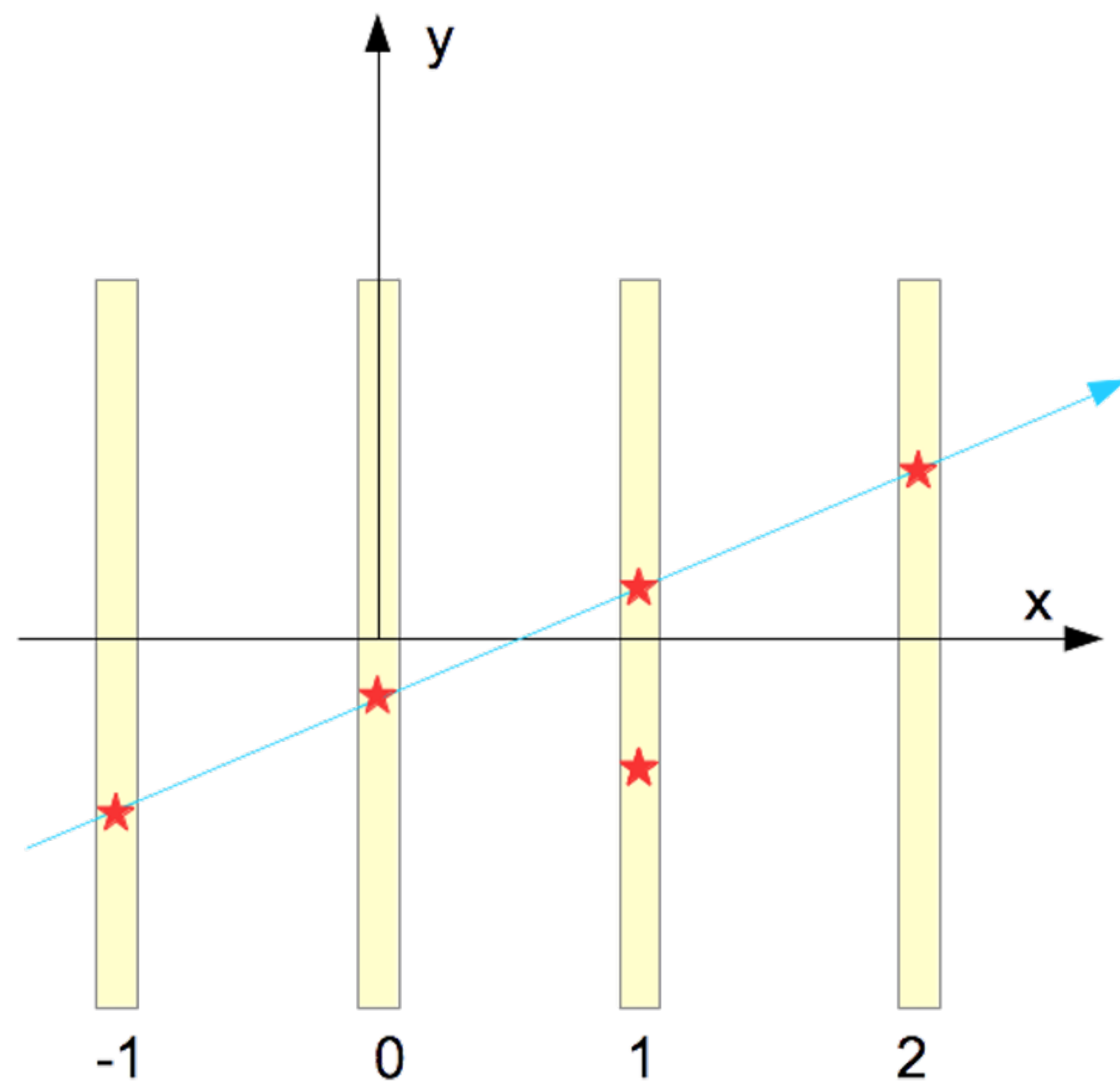


Table 2: List of hits recorded in this event. In total 5 hits: 4 hits from the track plus a noise hit. Units are arbitrary.

x	y
-1	-0.18
0	0.39
1	1.02
1	0.77
2	1.57

Warning: points have been randomised around their real y-coordinate

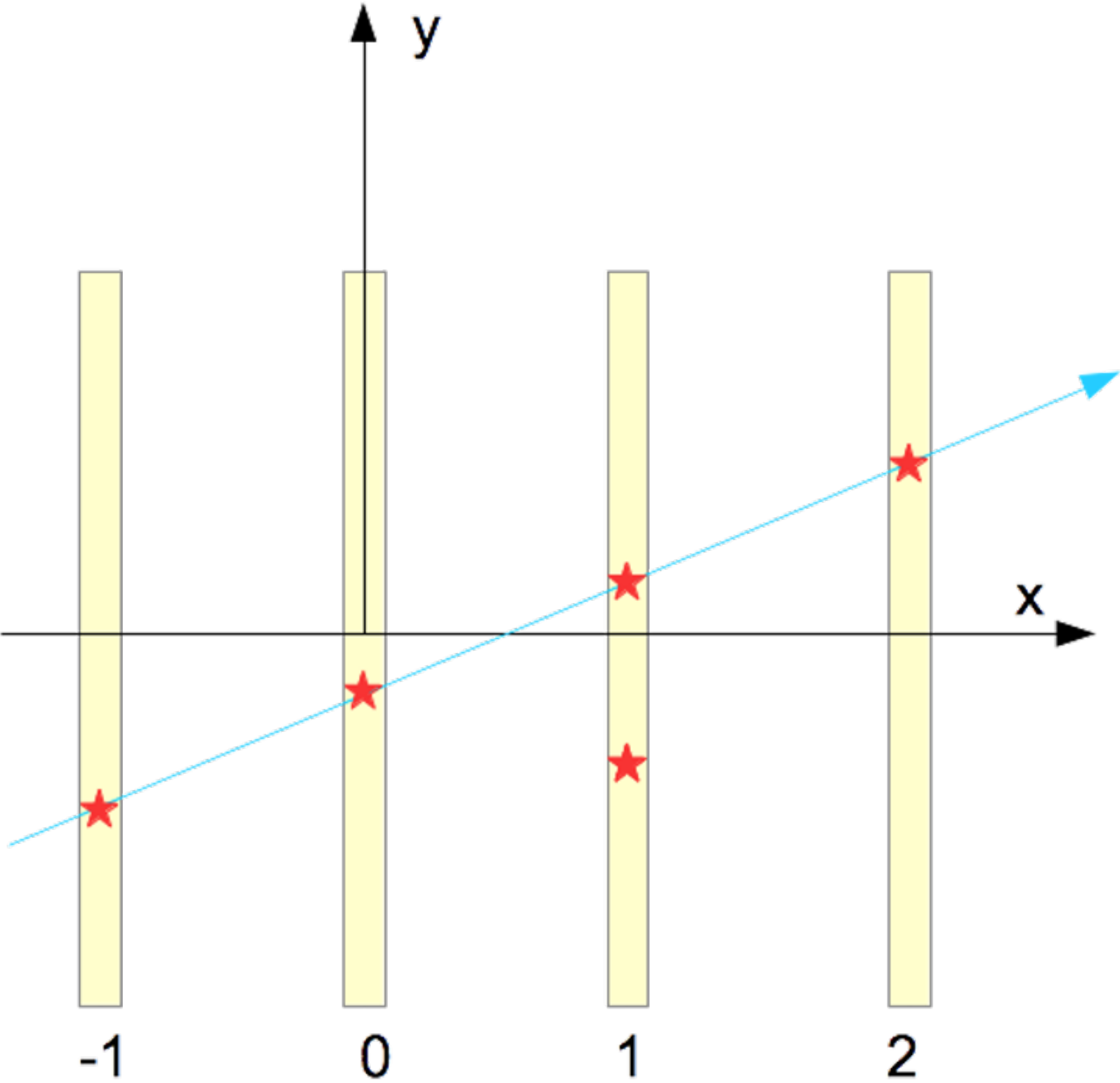
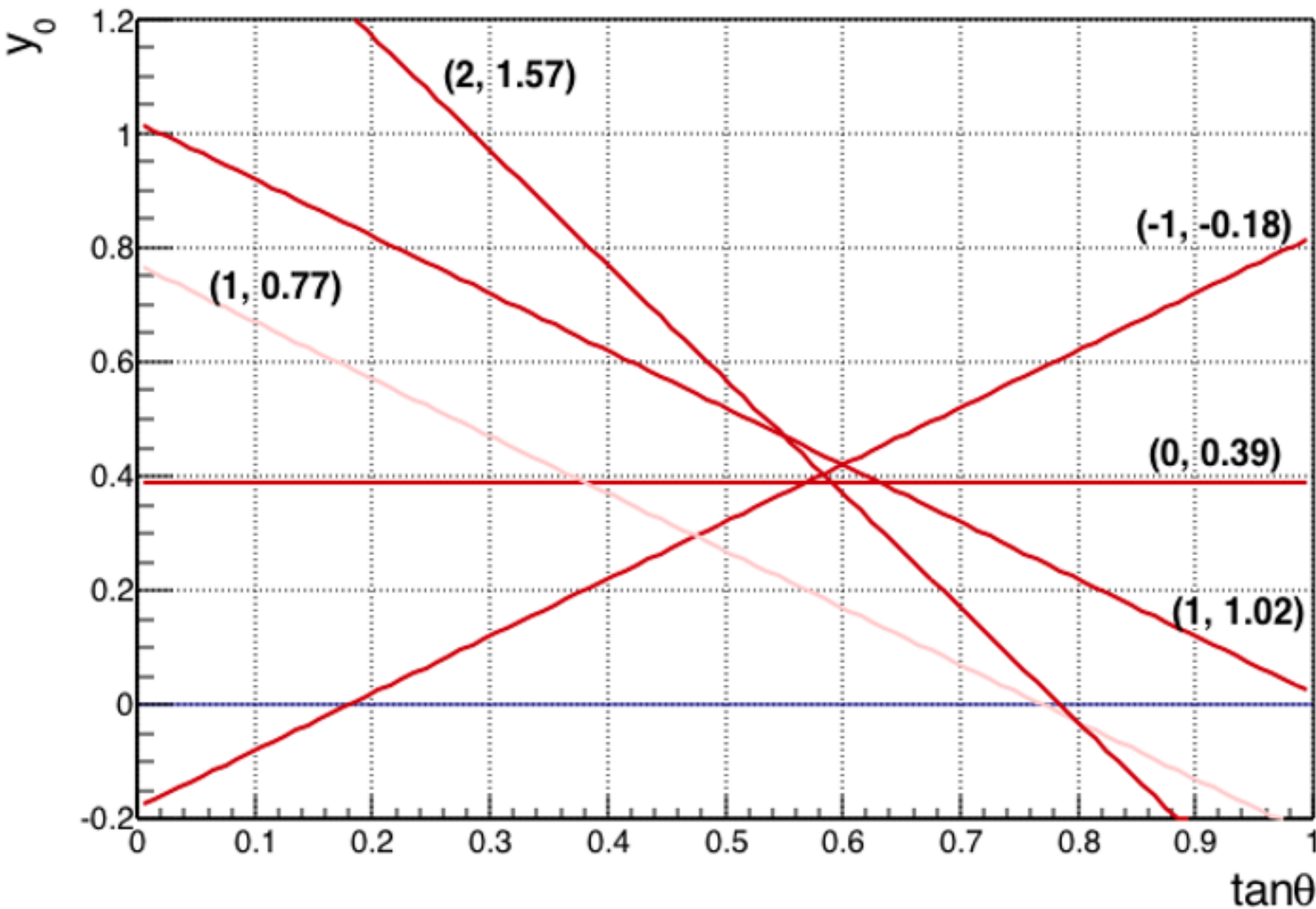


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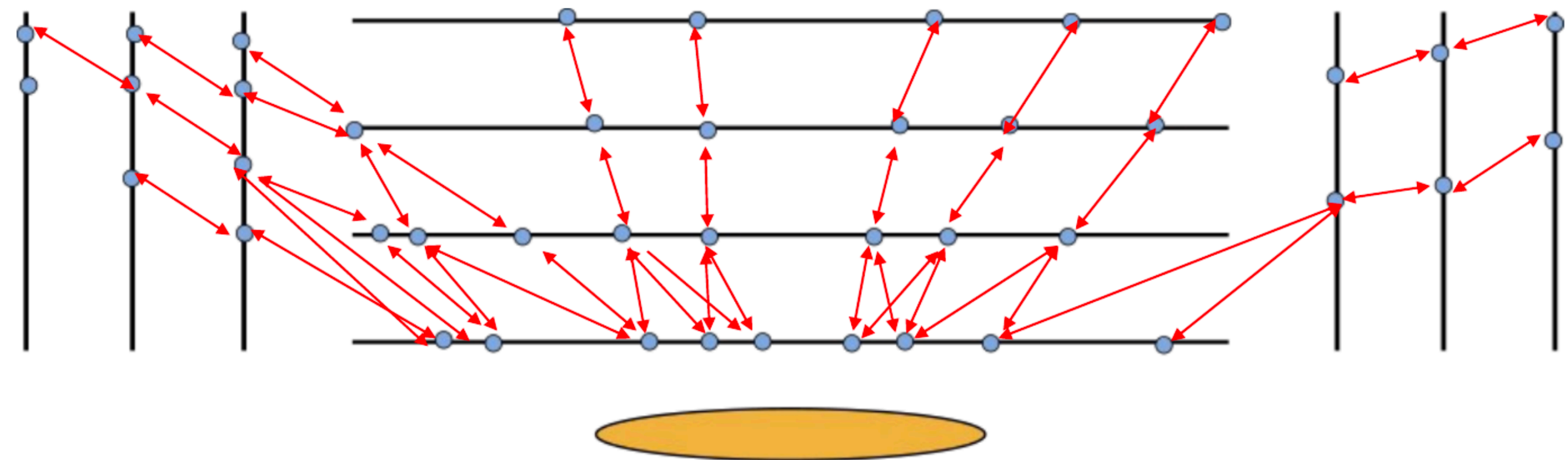
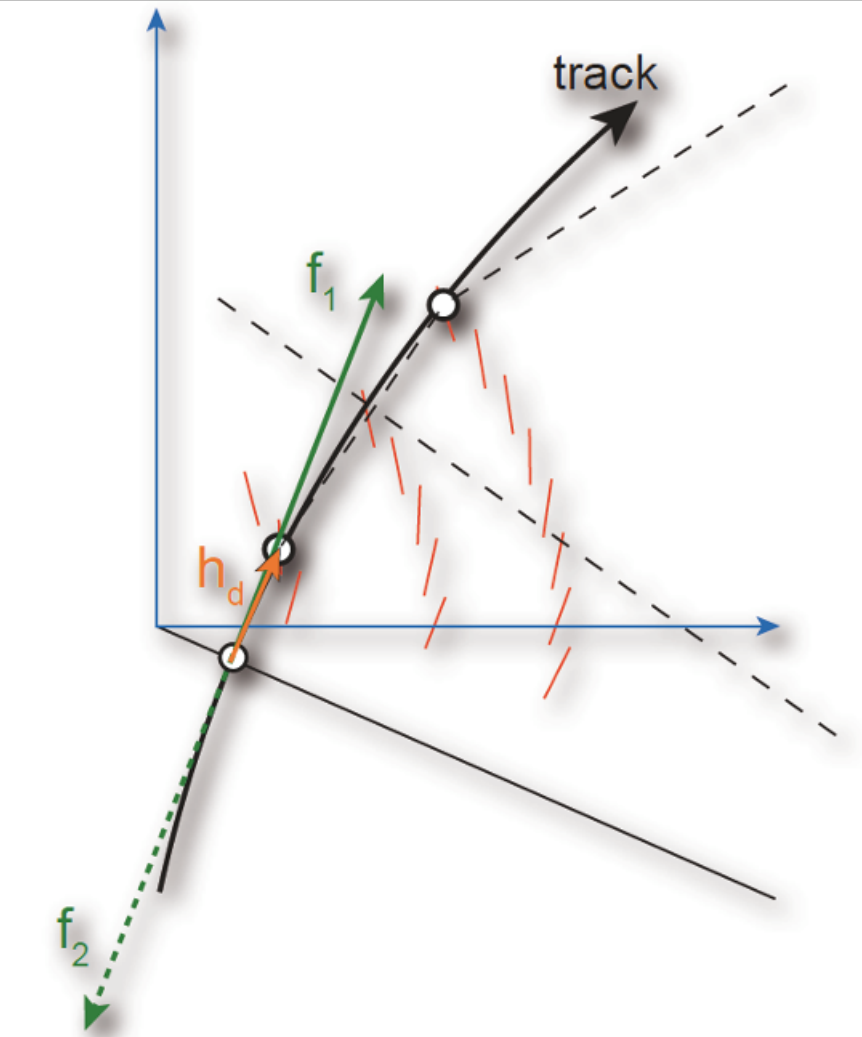
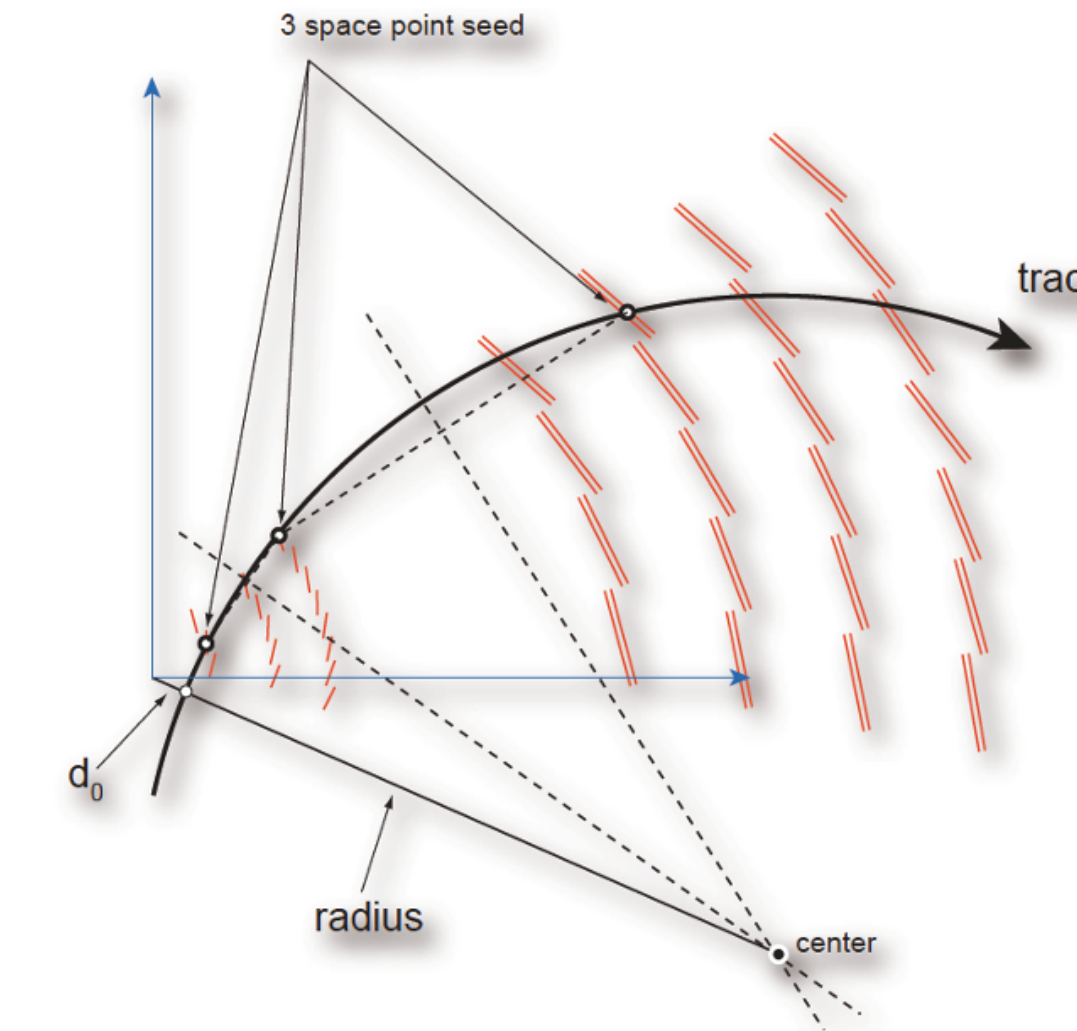
x	y
-1	-0.18
0	0.39
1	1.02
1	0.77
2	1.57



2D space (x,y): $y = x \tan\theta + y_0$

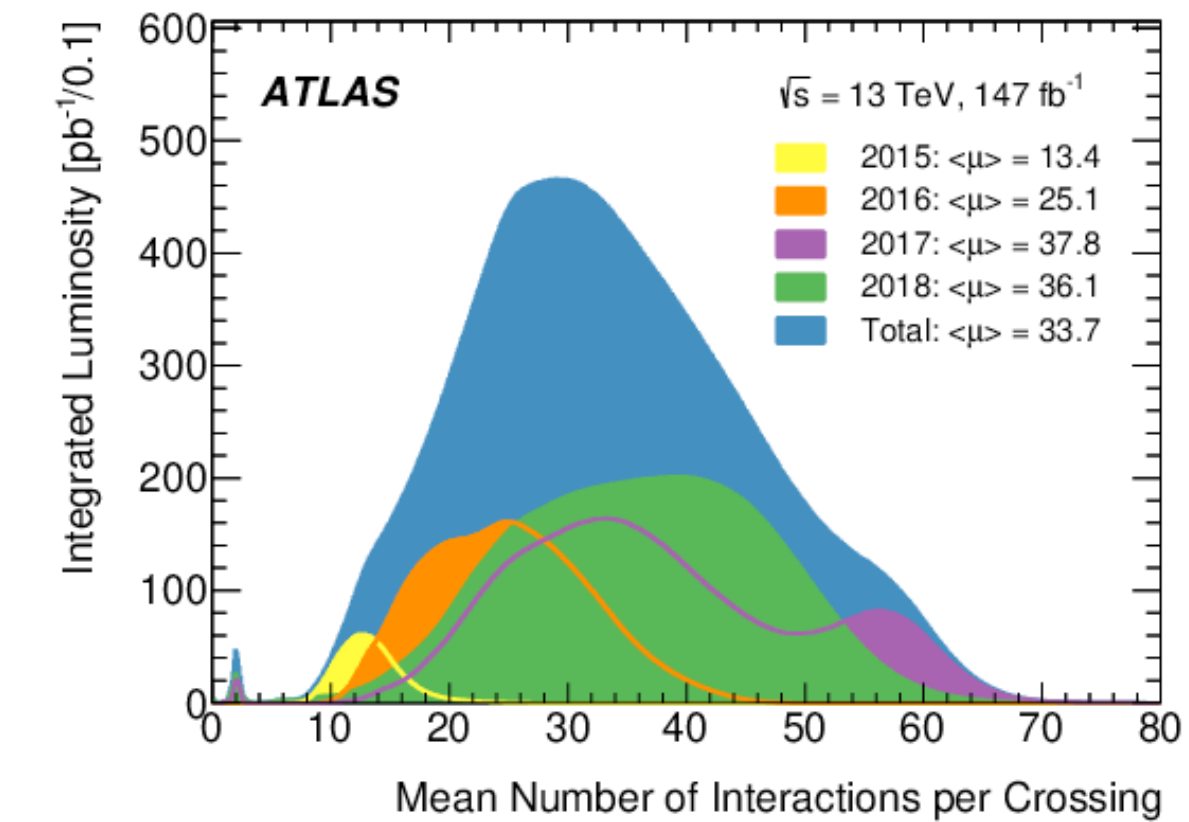
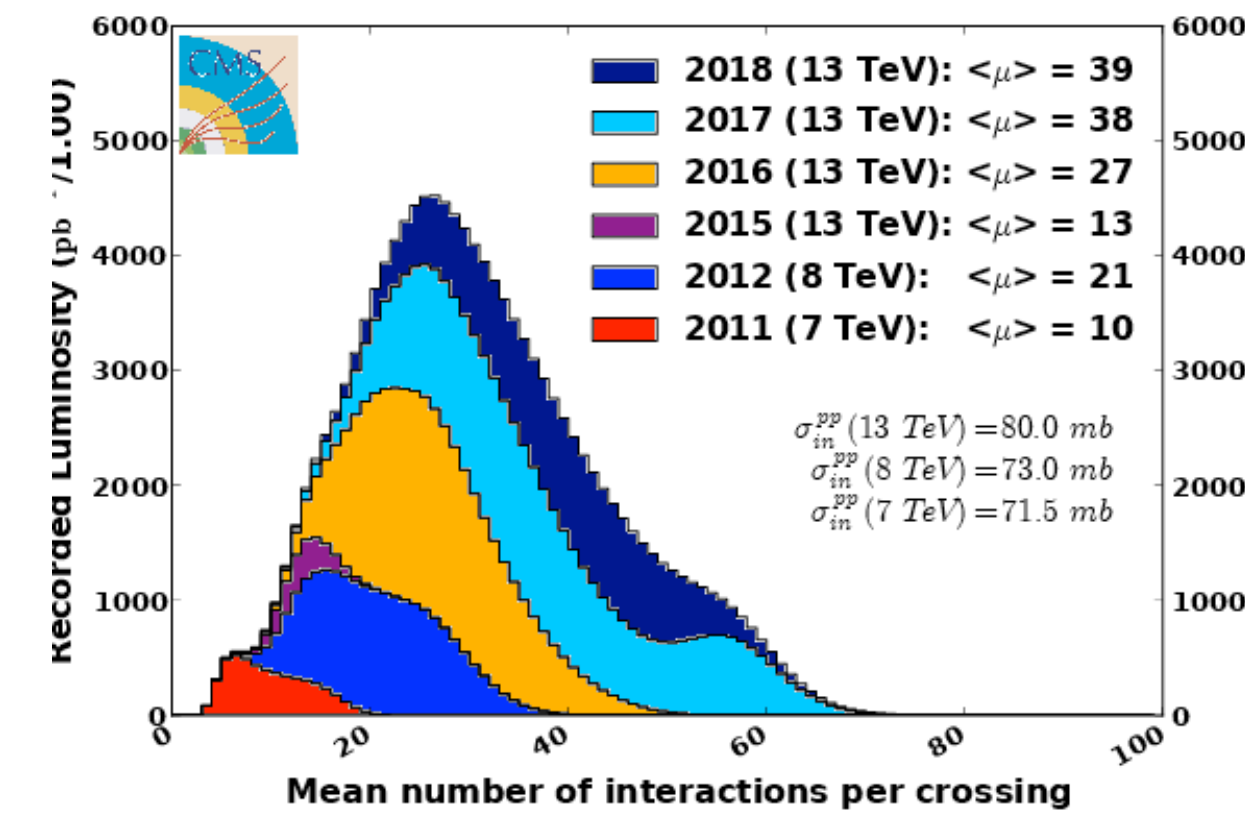
Hough space (tanθ,y₀): $y_0 = y - x \tan\theta$

- **3 points seed**
 - Adding other measurements: (inside-out or outside-in) may use 3 consecutive measurements (compute a circle) and extrapolate the track (outwards or inwards) attaching near-by measurements
 - Extrapolation uses a Kalman filter to define the search area
- **Cellular Automaton**
 - Implemented to cope with the increase of luminosity → quadruplet pixel track seeds
 - Parallelism, keep high efficiency and low fake rate
 - Avoid $\mathcal{O}(n^3)$ combinatorics
 - p_T and interaction region
 - Documentation: F. Pantaleo

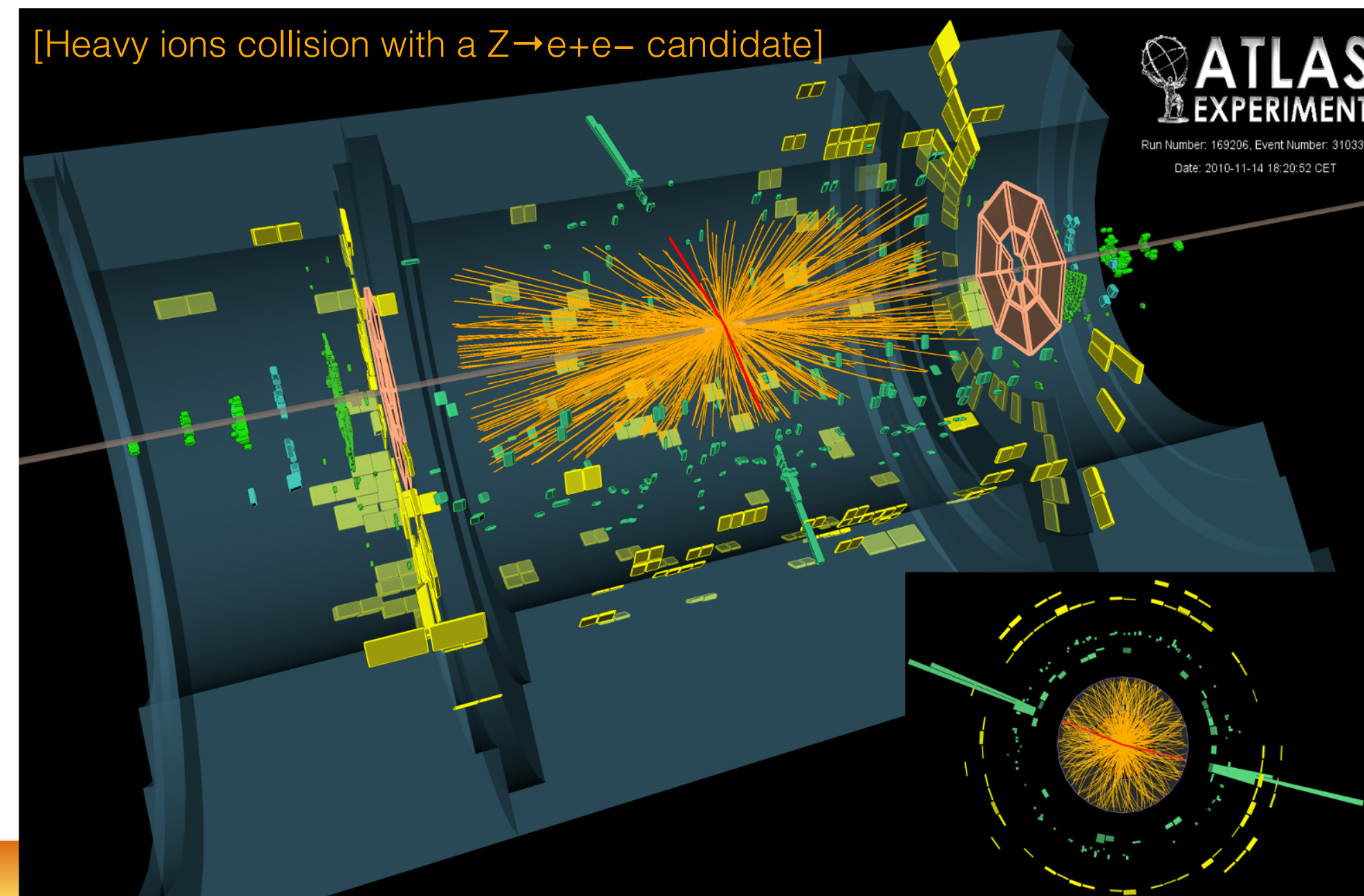


Pattern recognition:

- Pattern recognition algorithms being tuned for HL-LHC
 - Expecting ~ 200 collisions per beam crossing



- Extreme conditions already encountered in Pb-Pb collisions



Track fitting with χ^2 minimization

- **The Least Squares Method best meets the requirements of track fitting:**
 - Rather simple formulation (usual χ^2 definition) and statistical properties
 - Easy implementation of measurements (hits) and their errors
 - Quite fast numerically (even for large number of degrees of freedom)
 - Provides solution to track parameters and their uncertainties
- **An important ingredient is the track model which has to:**
 - Be well approximated by a linear model in the neighborhood of the measurements
 - (→ second and higher order derivatives negligible)
- **The track model requires that (generic to all track fitters):**
 - The equation of motion can be solved with sufficient precision
- **When B is in use, this implies that B is well known → accurate propagation**
 - Remember if B is not uniform, the differential equation of the motion must be solved numerically
 - This adds extra computing time
 - The material traversed by the particles is well known
- **Allow the accurate evaluation of energy loss and multiple scattering**
 - No wrong measurements (noisy hits) have been associated to the track during the pattern recognition

Track fitting with χ^2 minimization

- Use well known technique of residual minimization for track parameters determination via a χ^2 functional
- Usual χ^2 definition
 - Residuals (r) and their errors (σ) & χ^2 minimization w.r.t. track parameters (τ)

$$\chi^2 = \sum_{i=1}^{N_R} \frac{r_i^2}{\sigma(r_i)^2} \quad \frac{d\chi^2}{d\tau} = 2 \sum_{i=1}^{N_R} \frac{r_i}{\sigma(r_i)^2} \frac{dr}{d\tau} = 0$$

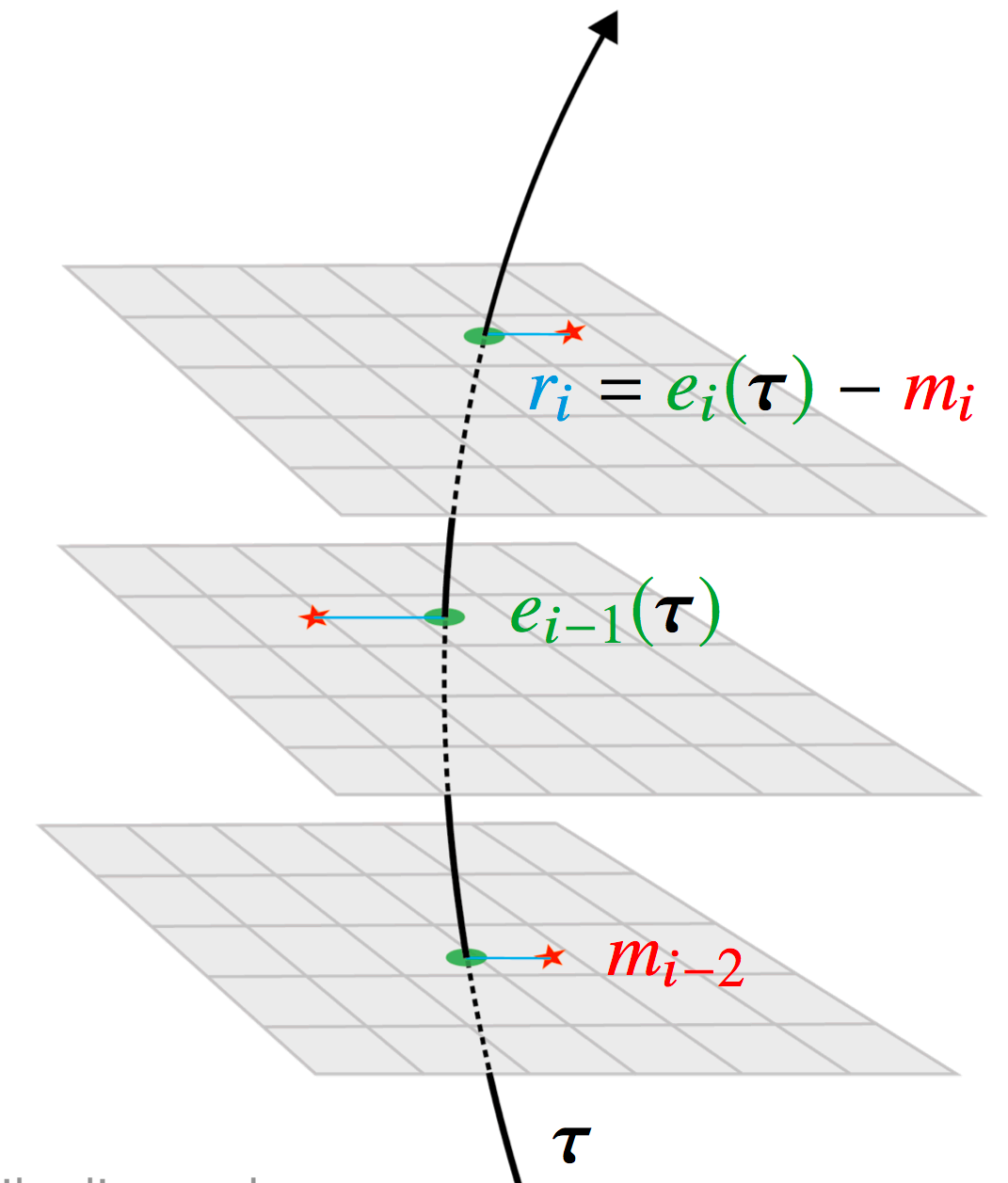
- Rewrite the χ^2 using matrix algebra:

$$\mathbf{r}(\tau) = \mathbf{m} - \mathbf{e}(\tau) = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_{N_R} \end{pmatrix} \quad V = \begin{pmatrix} \sigma(r_1)^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma(r_{N_R})^2 \end{pmatrix}$$

$$\chi^2 = \mathbf{r}^T V^{-1} \mathbf{r}$$

1) V may contain correlation coefficients as well. Therefore V is not necessarily diagonal

2) The residuals errors are taken as the intrinsic uncertainty of the detector elements. Each hit may come from a different tracking device, therefore each one has its own uncertainty



- Apply the χ^2 minimization w.r.t. track parameters (τ)

$$\frac{d\chi^2}{d\tau} = \mathbf{0} \rightarrow 2 \left(\frac{d\mathbf{r}}{d\tau} \right)^T V^{-1} \mathbf{r} = \mathbf{0}$$

$$\tau = \begin{pmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_{N_T} \end{pmatrix} = \begin{pmatrix} d_0 \\ z_0 \\ \phi_0 \\ \theta_0 \\ q/p \end{pmatrix}$$

$$\frac{d\mathbf{r}}{d\tau} = \begin{pmatrix} \frac{\partial r_1}{\partial \tau_1} & \dots & \frac{\partial r_1}{\partial \tau_{N_T}} \\ \vdots & \ddots & \vdots \\ \frac{\partial r_{N_R}}{\partial \tau_1} & \dots & \frac{\partial r_{N_R}}{\partial \tau_{N_T}} \end{pmatrix}$$

Track fitting with χ^2 minimization

- Taylor's expansion up to first order derivatives:

- Computed with the initial track parameters (τ_0) estimation
- Neglect second and higher order derivatives:

$$\mathbf{r}(\tau) = \mathbf{r}(\tau_0) + \left(\frac{d\mathbf{r}}{d\tau} \right)_{\tau=\tau_0} \delta\tau$$

$$\frac{d^2\mathbf{r}}{d\tau_i d\tau_j} = 0$$

- The minimum condition equation becomes (system of linear equations):

$$\frac{d\chi^2}{d\tau} = \mathbf{0} \rightarrow 2 \left(\frac{d\mathbf{r}}{d\tau} \right)^\top V^{-1} \mathbf{r} = \mathbf{0} \rightarrow 2 \left(\underbrace{\left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top_{\tau=\tau_0} V^{-1} \left(\frac{d\mathbf{r}}{d\tau} \right)_{\tau=\tau_0} \right]}_{N_T \times N_T \text{ matrix}} \underbrace{\delta\tau}_{N_T \text{ vector}} + \underbrace{\left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top_{\tau=\tau_0} V^{-1} \mathbf{r}(\tau_0) \right]}_{N_T \text{ vector}} \right) = \mathbf{0}$$

N_T vector: one per track parameter

$\mathbf{r}(\tau) = \mathbf{r}(\tau_0) + \left(\frac{d\mathbf{r}}{d\tau} \right)_{\tau=\tau_0} \delta\tau$

- Solving the above system requires to invert a $N_T \times N_T$ matrix

$$\delta\tau = - \left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top_{\tau=\tau_0} V^{-1} \left(\frac{d\mathbf{r}}{d\tau} \right)_{\tau=\tau_0} \right]^{-1} \left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top_{\tau=\tau_0} V^{-1} \mathbf{r}(\tau_0) \right] \rightarrow \tau = \tau_0 + \delta\tau$$

- Pros & cons:

- Pros:
 - The covariance matrix of the track parameters is just the inverse of the track derivatives matrix. So track parameters uncertainties are computed for free :)
 - If the problem is linear then the solution is exact
- Cons:
 - The derivatives of the residuals wrt track parameters may be hard to compute and suffer from long extrapolation uncertainties
 - If the problem is not linear then one needs to iterate

Track fitting with χ^2 minimization

- Track fit with constrained track parameters
 - Beam spot, secondary vertices, invariant masses, ...

$$\chi^2 = \mathbf{r}^\top V^{-1} \mathbf{r} + \mathbf{R}^\top W^{-1} \mathbf{R}$$

$$\mathbf{R} = \begin{pmatrix} d_0 - \hat{d}_0 \\ z_0 - \hat{z}_0 \\ \vdots \\ q/p - \hat{q}/p \end{pmatrix} \quad W = \begin{pmatrix} \sigma(d_0)^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma(q/p)^2 \end{pmatrix} \quad \frac{d\mathbf{R}}{d\tau} = \begin{pmatrix} \frac{\partial R_1}{\partial \tau_1} & \dots & \frac{\partial R_1}{\partial \tau_{N_T}} \\ \vdots & \ddots & \vdots \\ \frac{\partial R_{N_T}}{\partial \tau_1} & \dots & \frac{\partial R_{N_T}}{\partial \tau_{N_T}} \end{pmatrix} = \begin{pmatrix} \frac{\partial(d_0 - \hat{d}_0)}{\partial d_0} & \dots & \frac{\partial(d_0 - \hat{d}_0)}{\partial q/p} \\ \vdots & \ddots & \vdots \\ \frac{\partial q/p}{\partial d_0} & \dots & \frac{\partial(q/p - \hat{q}/p)}{\partial q/p} \end{pmatrix} = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix} = \mathbf{1}$$

$$\delta\tau = - \left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top V^{-1} \left(\frac{d\mathbf{r}}{d\tau} \right) + \left(\frac{d\mathbf{R}}{d\tau} \right)^\top W^{-1} \left(\frac{d\mathbf{R}}{d\tau} \right) \right]^{-1} \left[\left(\frac{d\mathbf{r}}{d\tau} \right)^\top V^{-1} \mathbf{r}(\tau_0) + \left(\frac{d\mathbf{R}}{d\tau} \right)^\top W^{-1} \mathbf{R}(\tau_0) \right] \rightarrow \tau = \tau_0 + \delta\tau$$

- Compacted notation and calculation

$$\mathbf{r}' = \begin{pmatrix} \mathbf{r} \\ \mathbf{R} \end{pmatrix} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_R} \\ R_1 \\ \vdots \\ R_{N_T} \end{pmatrix} \quad V' = \begin{pmatrix} V & 0 \\ 0 & W \end{pmatrix} = \begin{pmatrix} \sigma(r_1)^2 & \dots & 0 & & & \\ \vdots & \ddots & \vdots & & & \\ 0 & \dots & \sigma(r_{N_R})^2 & & & \\ & & & \sigma(d_0)^2 & \dots & 0 \\ & & & \vdots & \ddots & \vdots \\ & & 0 & 0 & \dots & \sigma(q/p)^2 \end{pmatrix}$$

$$\chi^2 = \mathbf{r}^\top V^{-1} \mathbf{r} + \mathbf{R}^\top W^{-1} \mathbf{R} = \mathbf{r}'^\top V'^{-1} \mathbf{r}'$$

Track fitting with χ^2 minimization

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 - Beam spot, secondary vertices, invariant masses, ...

$$\chi^2 = \mathbf{r}^\top V^{-1} \mathbf{r} + \mathbf{R}^\top W^{-1} \mathbf{R}$$

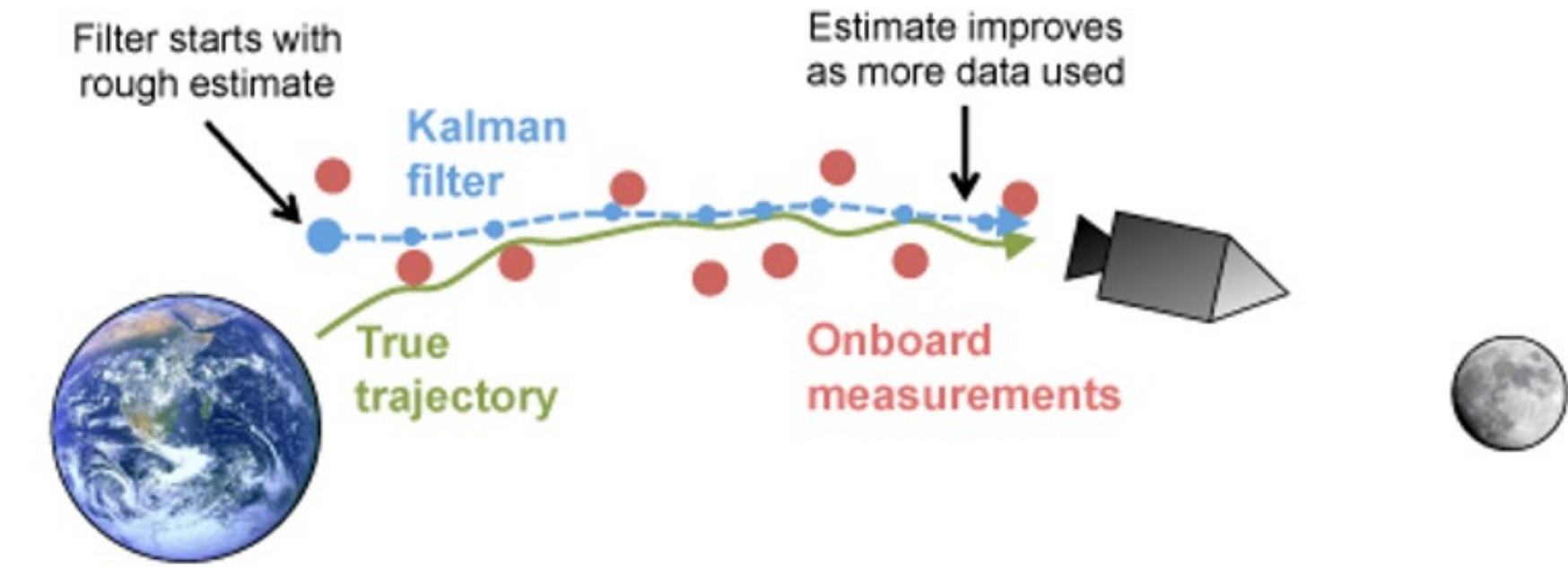
$$\mathbf{R} = \begin{pmatrix} d_0 - \hat{d}_0 \\ z_0 - \hat{z}_0 \\ \vdots \\ q/p - \hat{q/p} \end{pmatrix} \quad W = \begin{pmatrix} \sigma(d_0)^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma(q/p)^2 \end{pmatrix} \quad \frac{d\mathbf{R}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{\partial R_1}{\partial \tau_1} & \dots & \frac{\partial R_1}{\partial \tau_{N_T}} \\ \vdots & \ddots & \vdots \\ \frac{\partial R_{N_T}}{\partial \tau_1} & \dots & \frac{\partial R_{N_T}}{\partial \tau_{N_T}} \end{pmatrix} = \begin{pmatrix} \frac{\partial(d_0 - \hat{d}_0)}{\partial d_0} & \dots & \frac{\partial(d_0 - \hat{d}_0)}{\partial q/p} \\ \vdots & \ddots & \vdots \\ \frac{\partial q/p}{\partial d_0} & \dots & \frac{\partial(q/p - \hat{q/p})}{\partial q/p} \end{pmatrix} = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \end{pmatrix} = \mathbf{1}$$

$$\delta\boldsymbol{\tau} = - \left[\left(\frac{d\mathbf{r}}{d\boldsymbol{\tau}} \right)^\top V^{-1} \left(\frac{d\mathbf{r}}{d\boldsymbol{\tau}} \right) + \left(\frac{d\mathbf{R}}{d\boldsymbol{\tau}} \right)^\top W^{-1} \left(\frac{d\mathbf{R}}{d\boldsymbol{\tau}} \right) \right]^{-1} \left[\left(\frac{d\mathbf{r}}{d\boldsymbol{\tau}} \right)^\top V^{-1} \mathbf{r}(\boldsymbol{\tau}_0) + \left(\frac{d\mathbf{R}}{d\boldsymbol{\tau}} \right)^\top W^{-1} \mathbf{R}(\boldsymbol{\tau}_0) \right] \rightarrow \boldsymbol{\tau} = \boldsymbol{\tau}_0 + \delta\boldsymbol{\tau}$$

- Goodness of the fit: evaluate the pull quantities
 - When fit is correct: pulls follow a Normal distribution ($\mu=0$, $\sigma=1$)
 - Three conditions must be fulfilled
 - 1) The track model must be correct
 - 2) The covariance matrix of the measurement uncertainties must be correct
 - 3) The reconstruction software must work properly

Track fitting with Kalman filter

- The Kalman filter was developed by R.E. Kalman during the 1950's
 - To solve differential matrix equations without large matrix inversions
 - It is a method of estimating the states of dynamic systems
- Applied by the NASA in the rocket trajectory control for the Apollo program
 - Military applications: compute plane trajectory by radar tracking

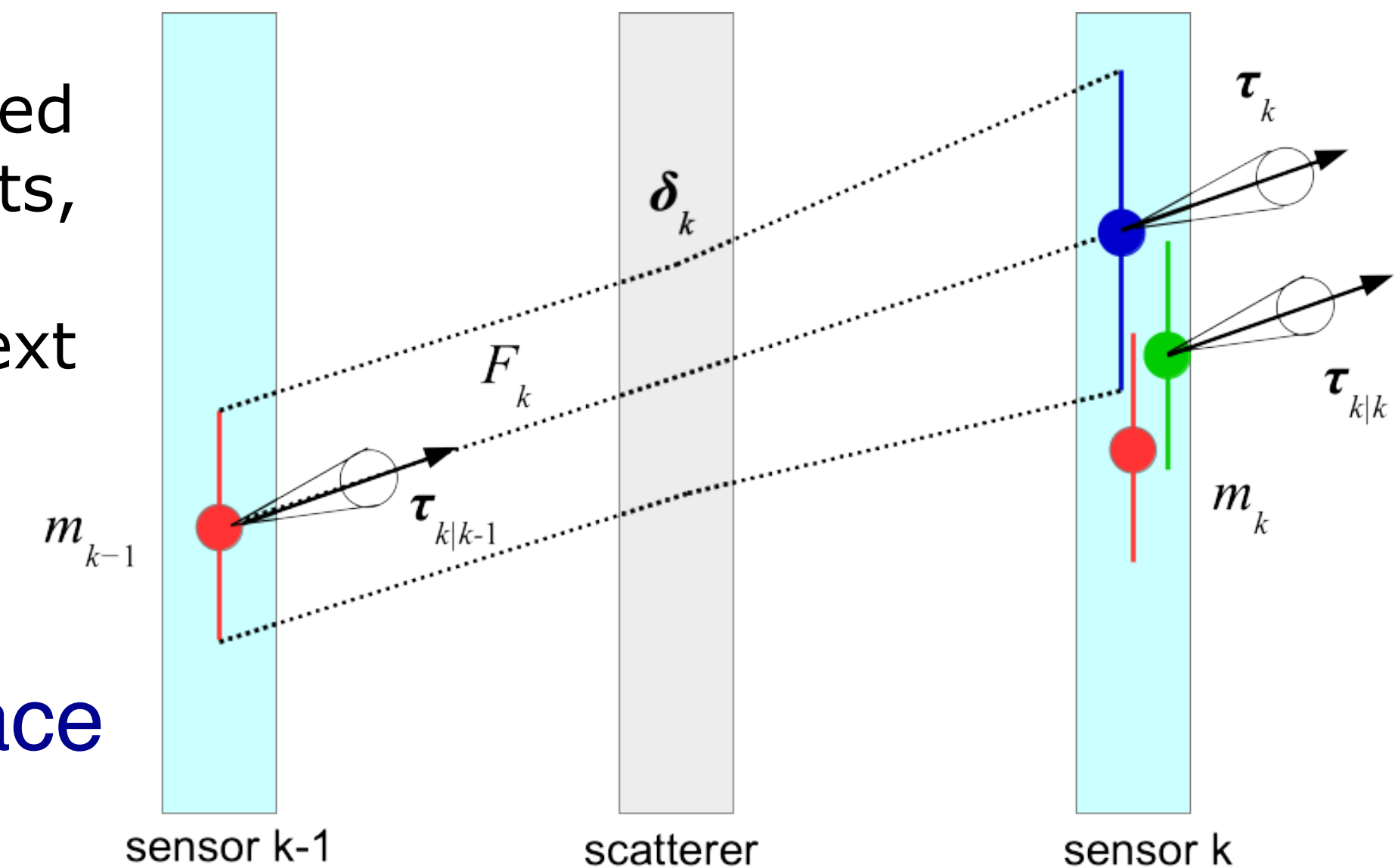


- **Assumption:**
 - The trajectory of a particle between two adjacent surfaces is described by a deterministic function plus random disturbances (material effects, etc)
 - The system equation: propagates the estate in one surface to the next surface

$$\boldsymbol{\tau}_k = F_k \boldsymbol{\tau}_{k|k-1} + P_k \boldsymbol{\delta}_k \quad \langle \boldsymbol{\delta}_k \rangle = \mathbf{0} \quad Q_k = Cov(\boldsymbol{\delta}_k)$$

- The measurement equation: mapping the track in the surface and considers some measurement error

$$\mathbf{m}_k = H_k \boldsymbol{\tau}_k + \boldsymbol{\epsilon}_k \quad \langle \boldsymbol{\epsilon}_k \rangle = \mathbf{0} \quad V_k = Cov(\boldsymbol{\epsilon}_k)$$



Track fitting with Kalman filter

- The aim is to estimate the track parameters from the observations

- From $k-1$ observations and τ_{k-1} estimate plus a k -th measurement: obtain a new k estimate

$$\left\{ \{m_1, m_2, \dots, m_{k-1}\}, \tau_{k-1} \right\} + m_k \rightarrow \tau_k$$

- Prediction:**

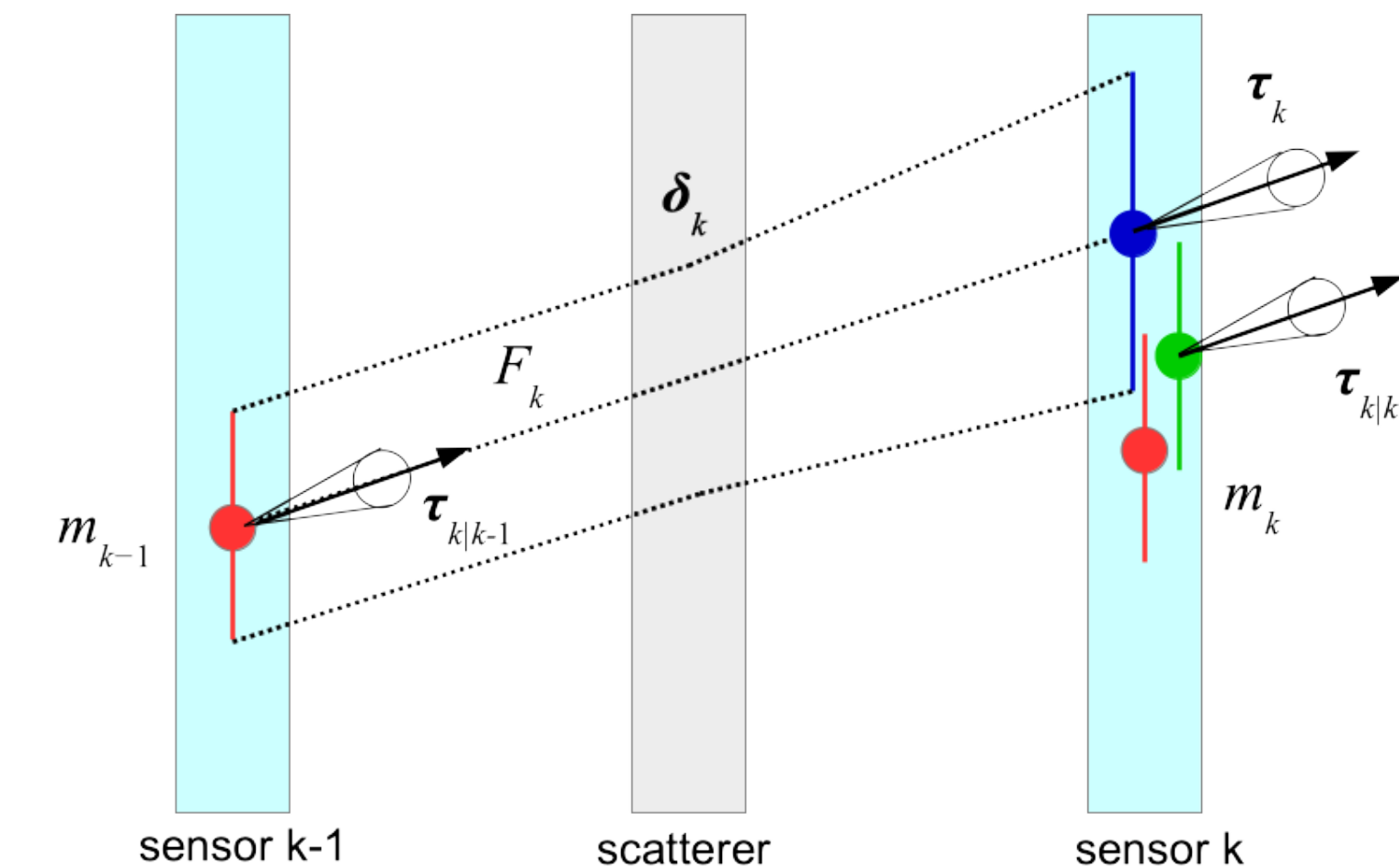
- Propagate the previous state to the new surface
- and its covariance matrix (error):

$$\tau_{k|k-1} = F_k \tau_{k-1} + P_k \delta_k$$

$$C_{k|k-1} = F_k C_{k-1|k-1} F_k^\top + P_k Q_k P_k^\top$$

- Filtering:**

- In the k surface one has two estimates of the particle track
- 1) the prediction $\tau_{k|k-1}$ (blue point)
- 2) the measurement m_k (red point)



- The Kalman filter consists in combining both: prediction and measurement

- minimizing the following:

$$L(\tau_k) = (m_k - H_k \tau_k)^\top V_k^{-1} (m_k - H_k \tau_k) + (\tau_{k|k-1} - \tau_k)^\top C_{k|k-1} (\tau_{k|k-1} - \tau_k)$$

$$\chi^2 = \mathbf{r}^\top V^{-1} \mathbf{r} + \mathbf{R}^\top W^{-1} \mathbf{R}$$

- The solution of the track parameters should be well known by now:

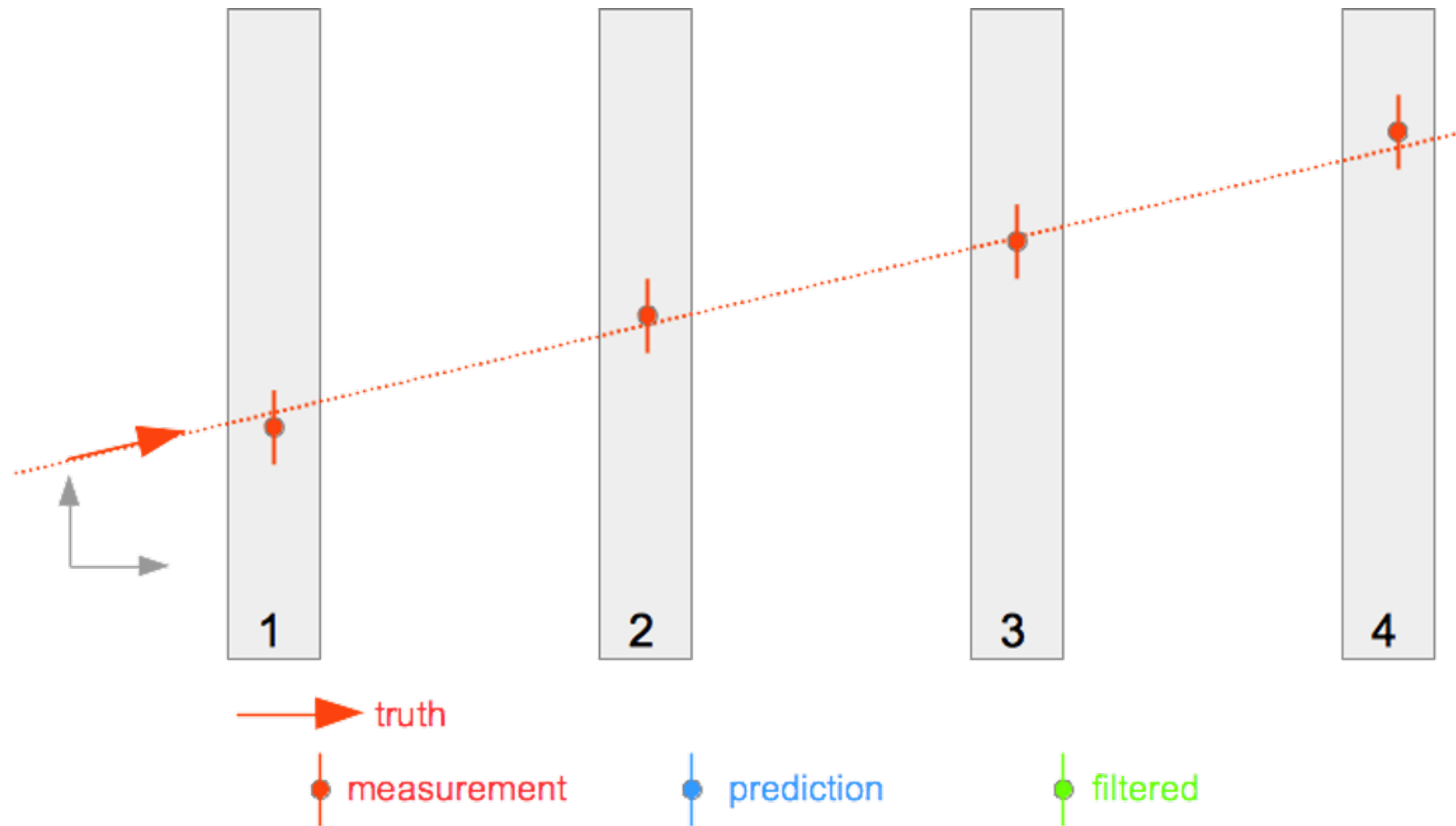
- And its covariance matrix (track parameter uncertainties):

$$\tau_{k|k} = \tau_{k|k-1} - \left[H_k^\top V_k^{-1} H_k + C_{k|k-1} \right]^{-1} \left[H_k^\top V_k^{-1} (m_k - H_k \tau_{k|k-1}) \right]$$

$$C_{k|k} = \left[H_k^\top V_k^{-1} H_k + C_{k|k-1} \right]^{-1}$$

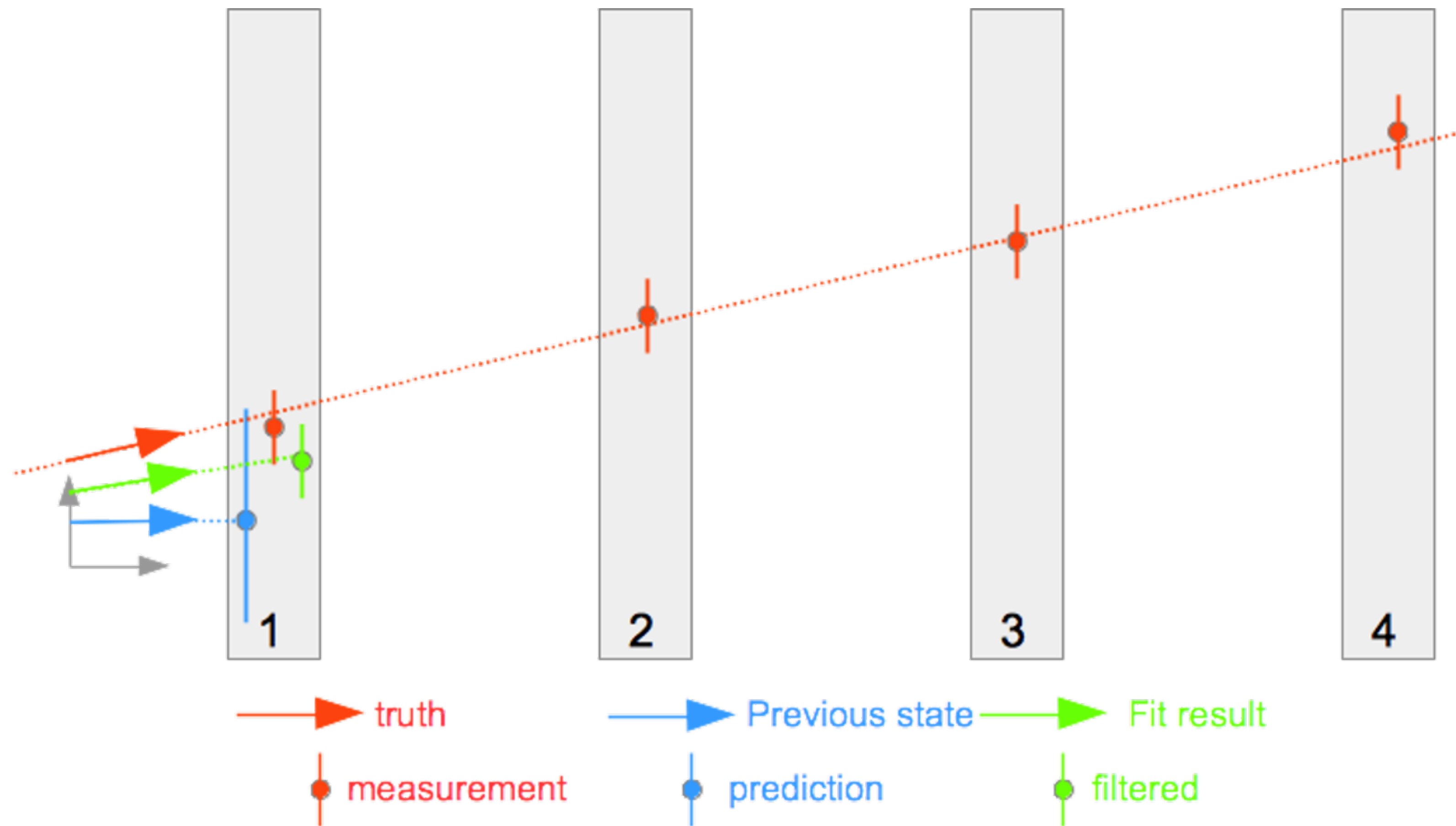
Track fitting with Kalman filter

- Estimate of the track parameters and state at the detector surfaces
 - Filtering from estimate $k-1$ to k



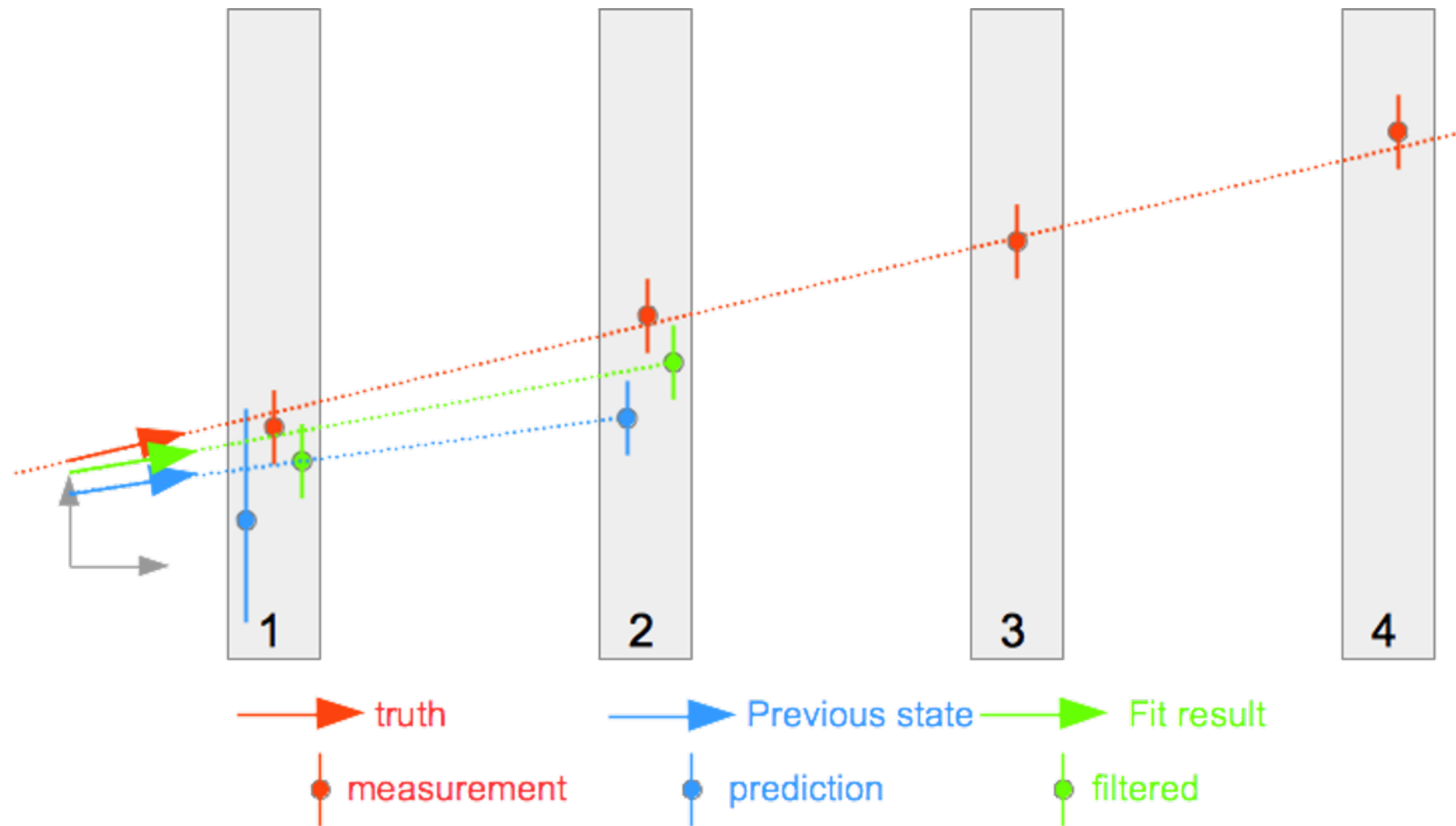
Track fitting with Kalman filter

- Estimate of the track parameters and state at the detector surfaces
 - Filtering from estimate $k-1$ to k



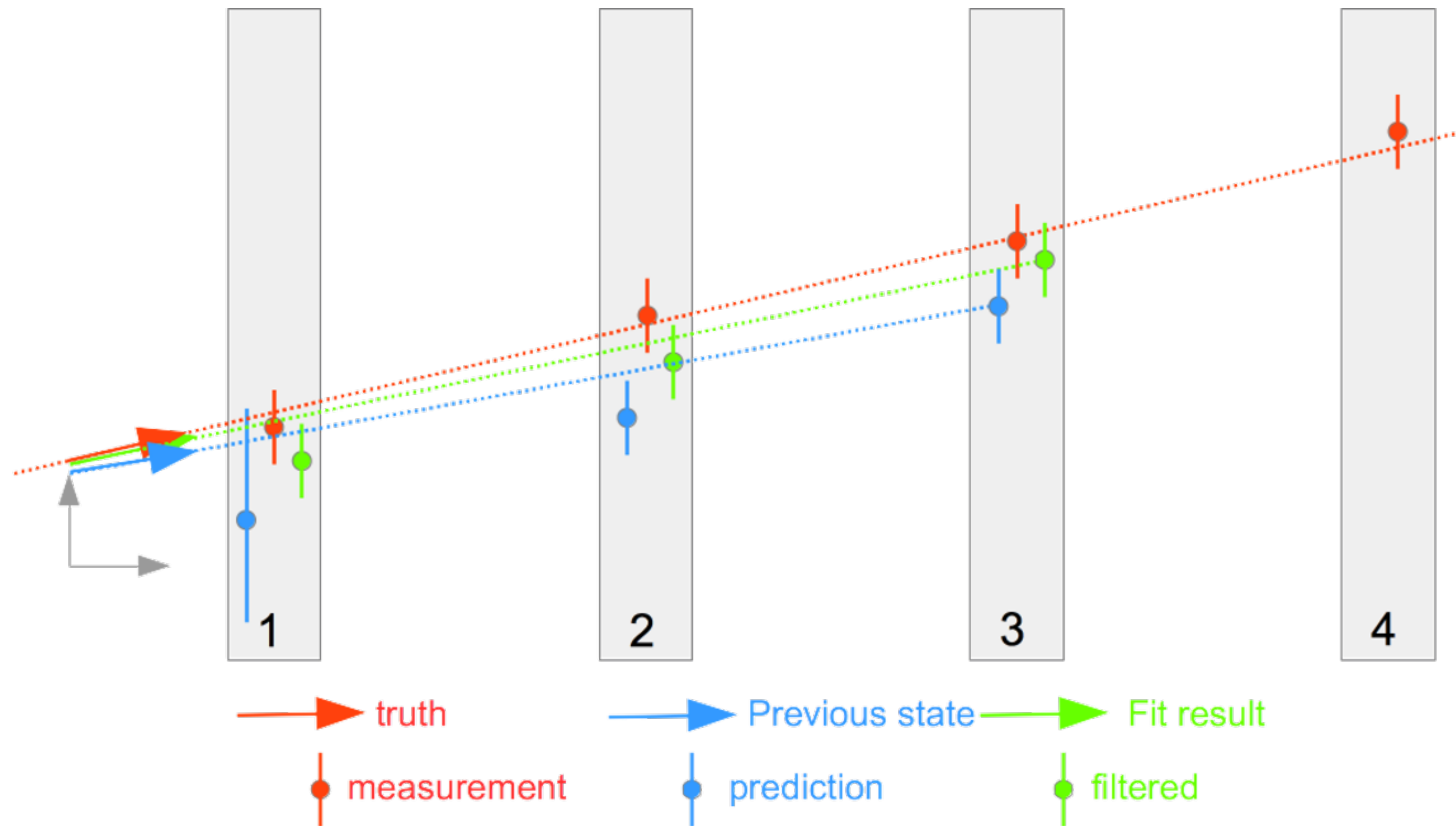
Track fitting with Kalman filter

- Estimate of the track parameters and state at the detector surfaces
 - Filtering from estimate $k-1$ to k



Track fitting with Kalman filter

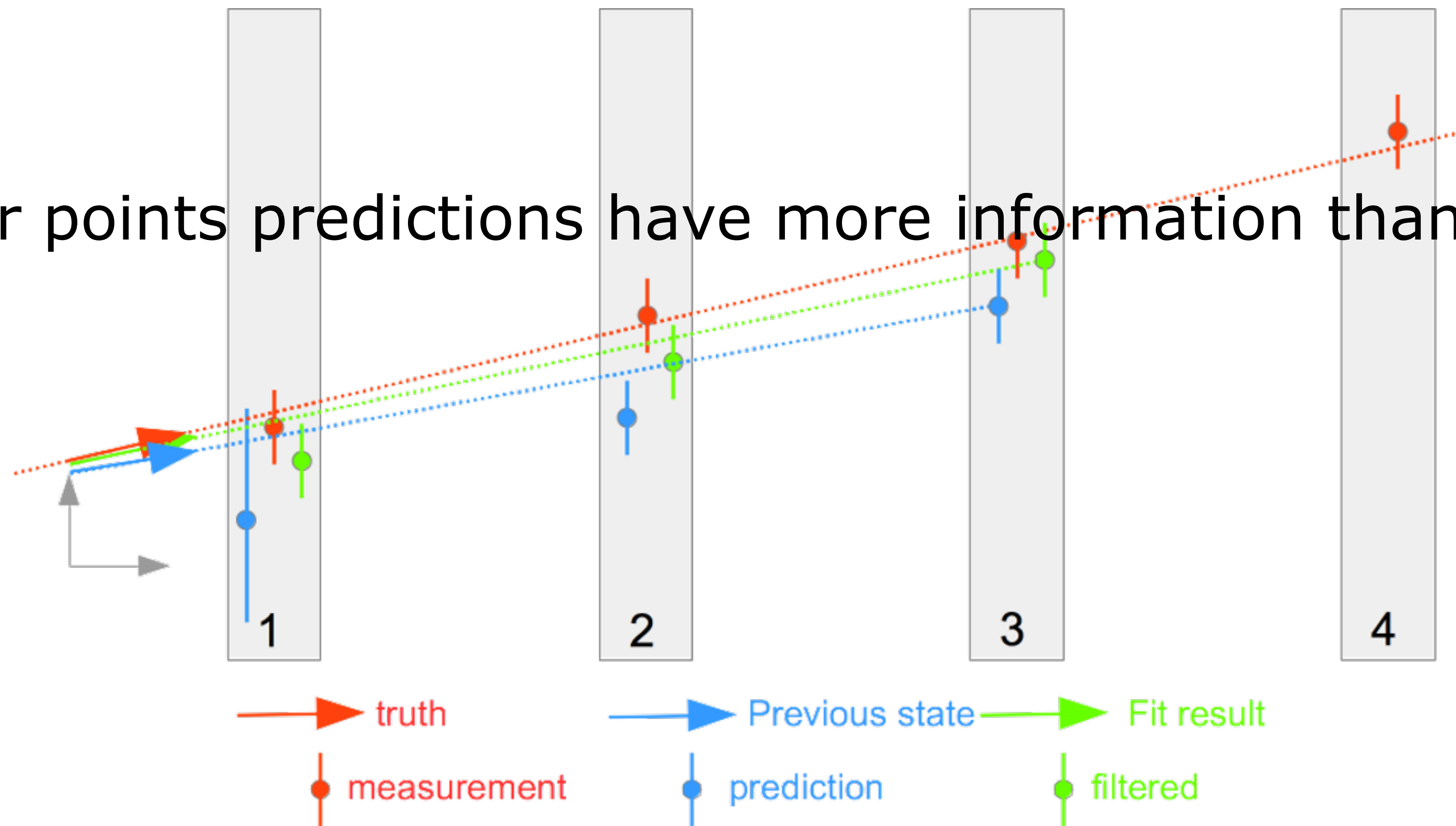
- Estimate of the track parameters and state at the detector surfaces
 - Filtering from estimate $k-1$ to k



Track fitting with Kalman filter

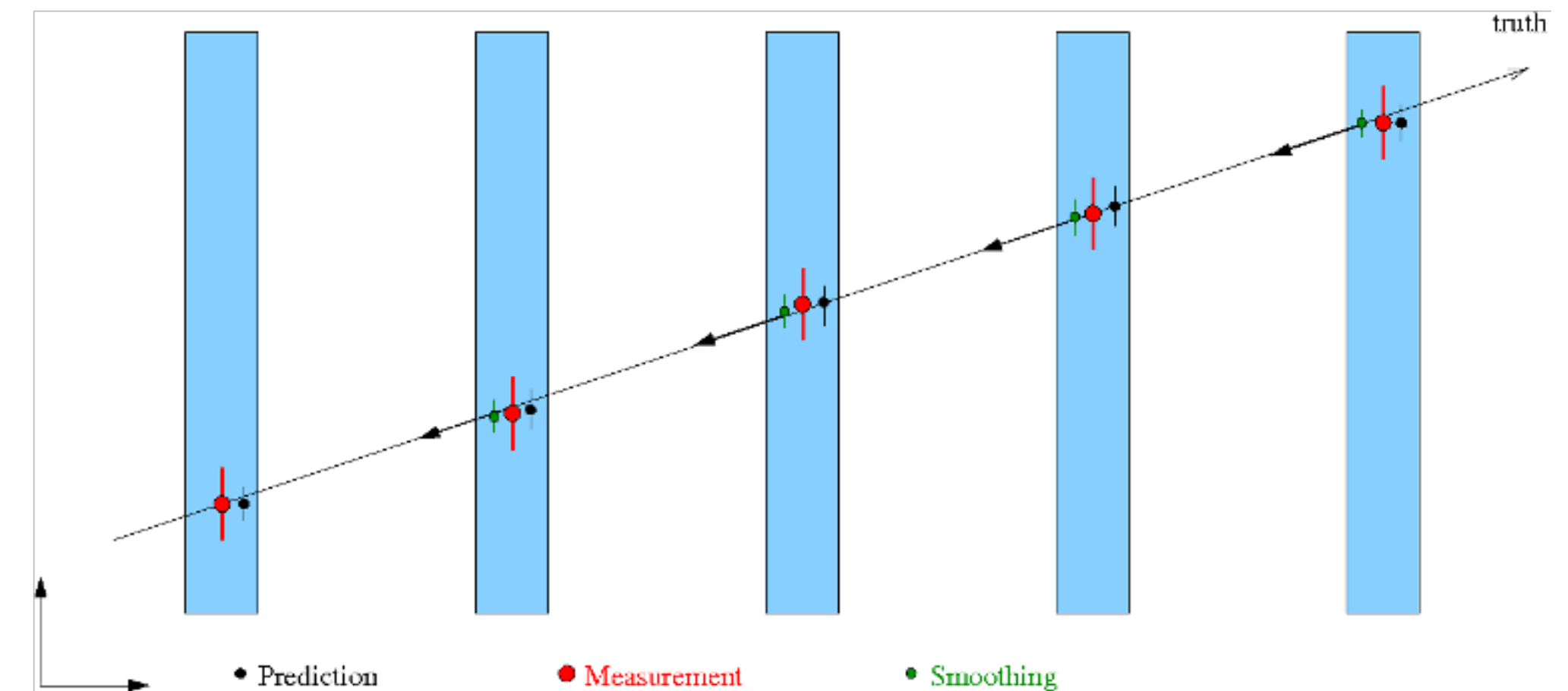
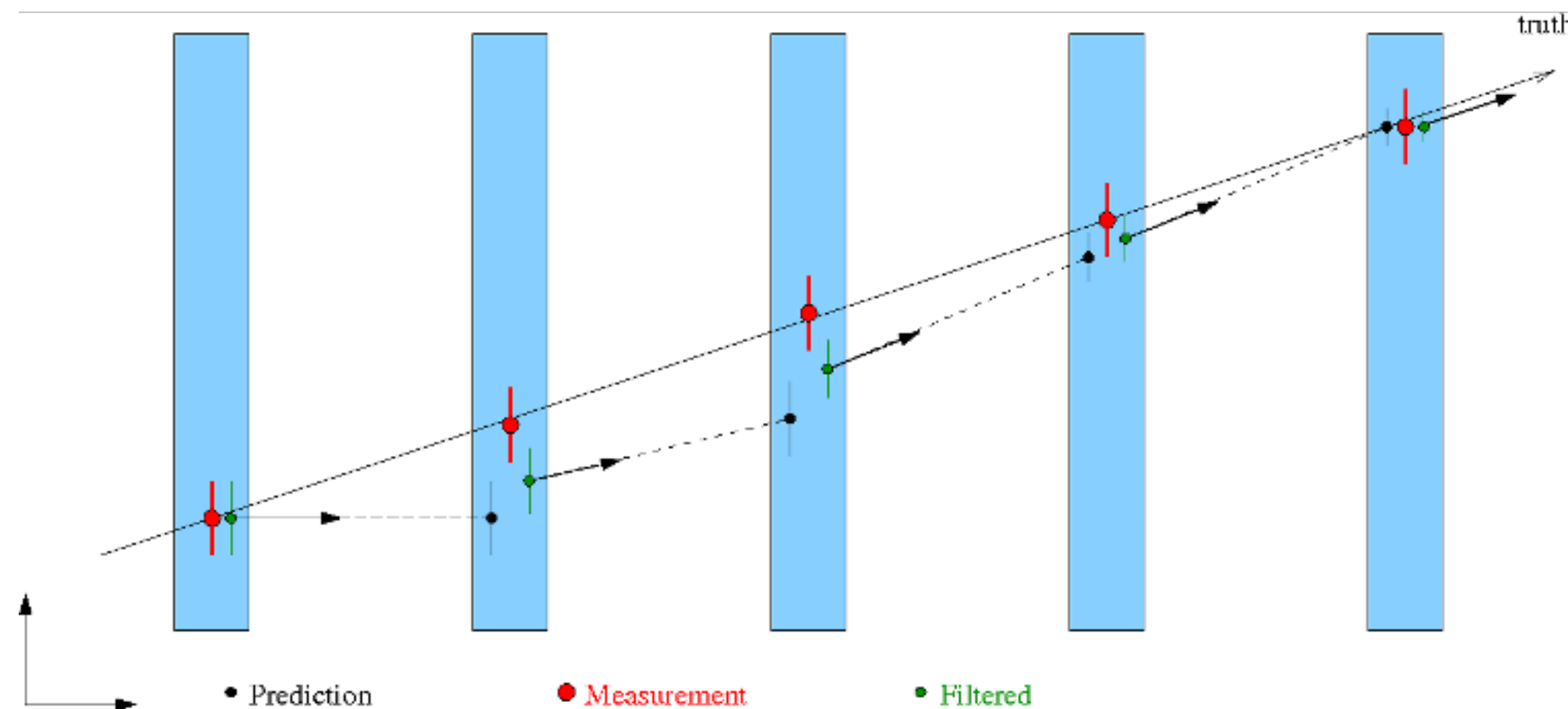
- Estimate of the track parameters and state at the detector surfaces
 - Filtering from estimate $k-1$ to k

Issue: Outer points predictions have more information than inner points

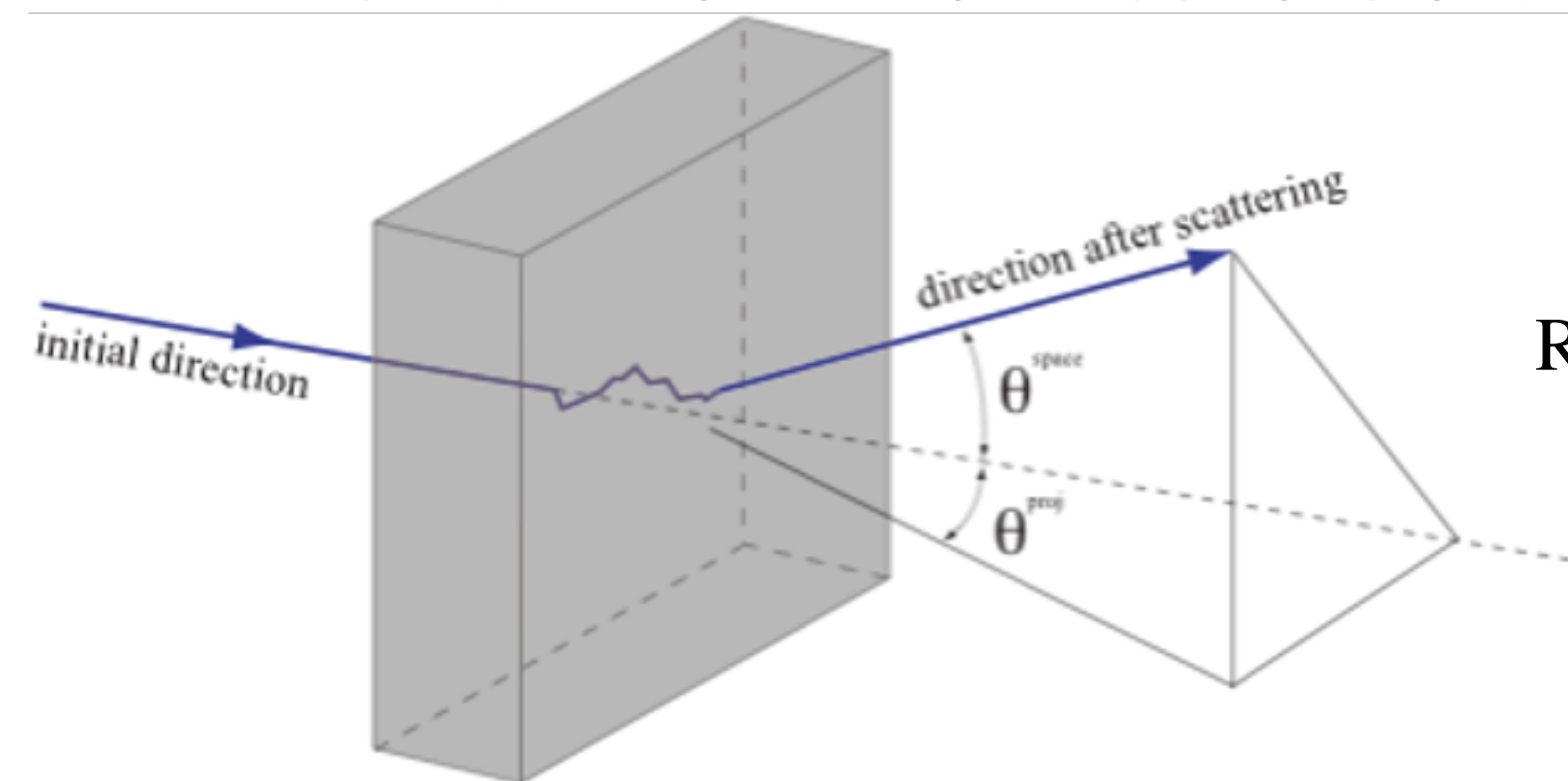


Track fitting with Kalman filter

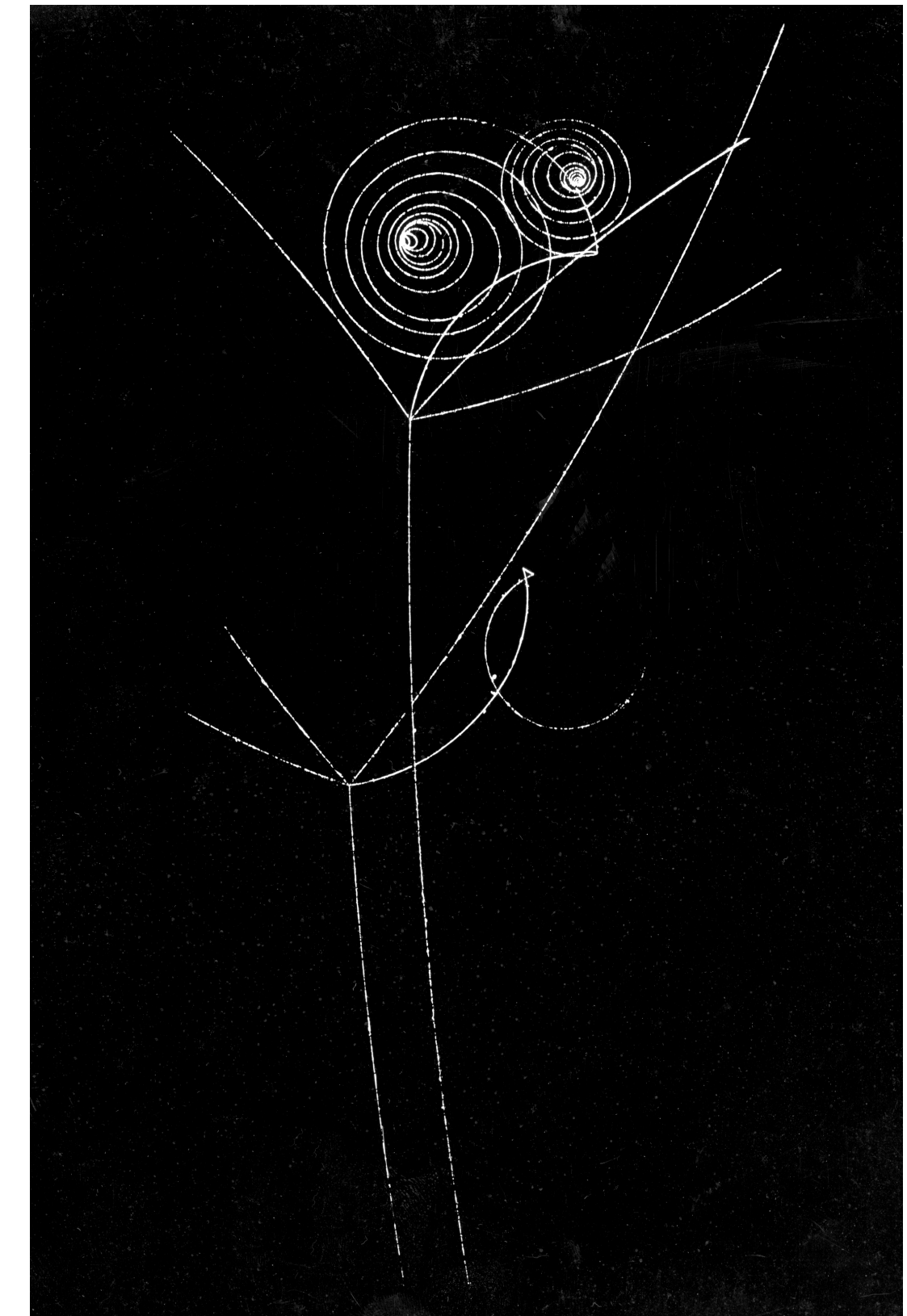
- Filtering
 - Moving outwards from estimate $k-1$ to k



- When travelling in the interior of a detector a particle interacts with the material of the sensors and the ancillary structures
 - Coulomb scattering of the incident particles with the sensors and structures material
 - The particles undergoes successive deflections altering the particle trajectory
 - The energy loss (dE/dx),
 - If severe, may affect the curvature radius (mainly for electrons)
 - It also helps to particle identification
- Assumption: MCS angles follow a Gaussian distribution
 - It is well known that MCS tails are larger than those of the Gaussian



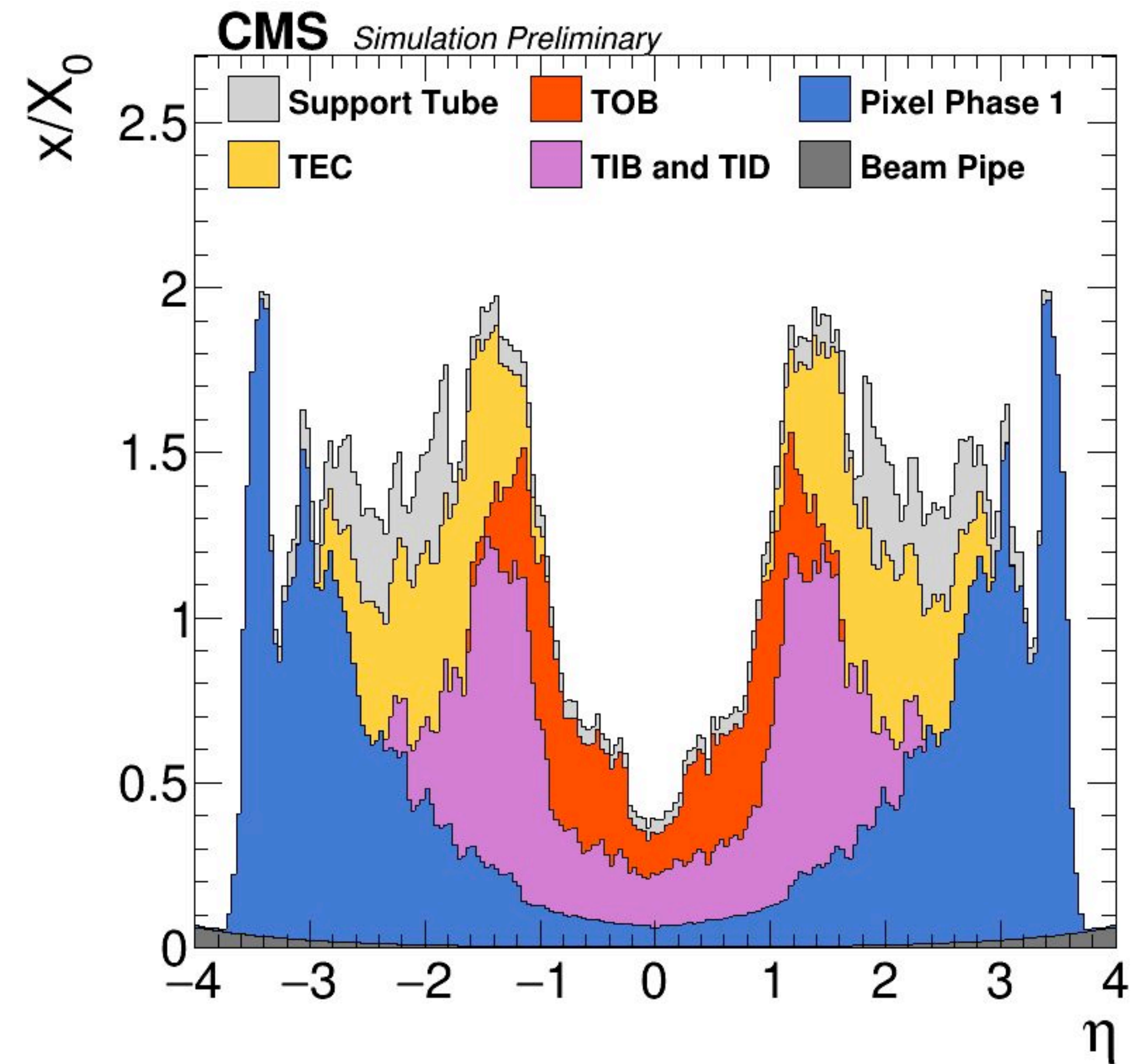
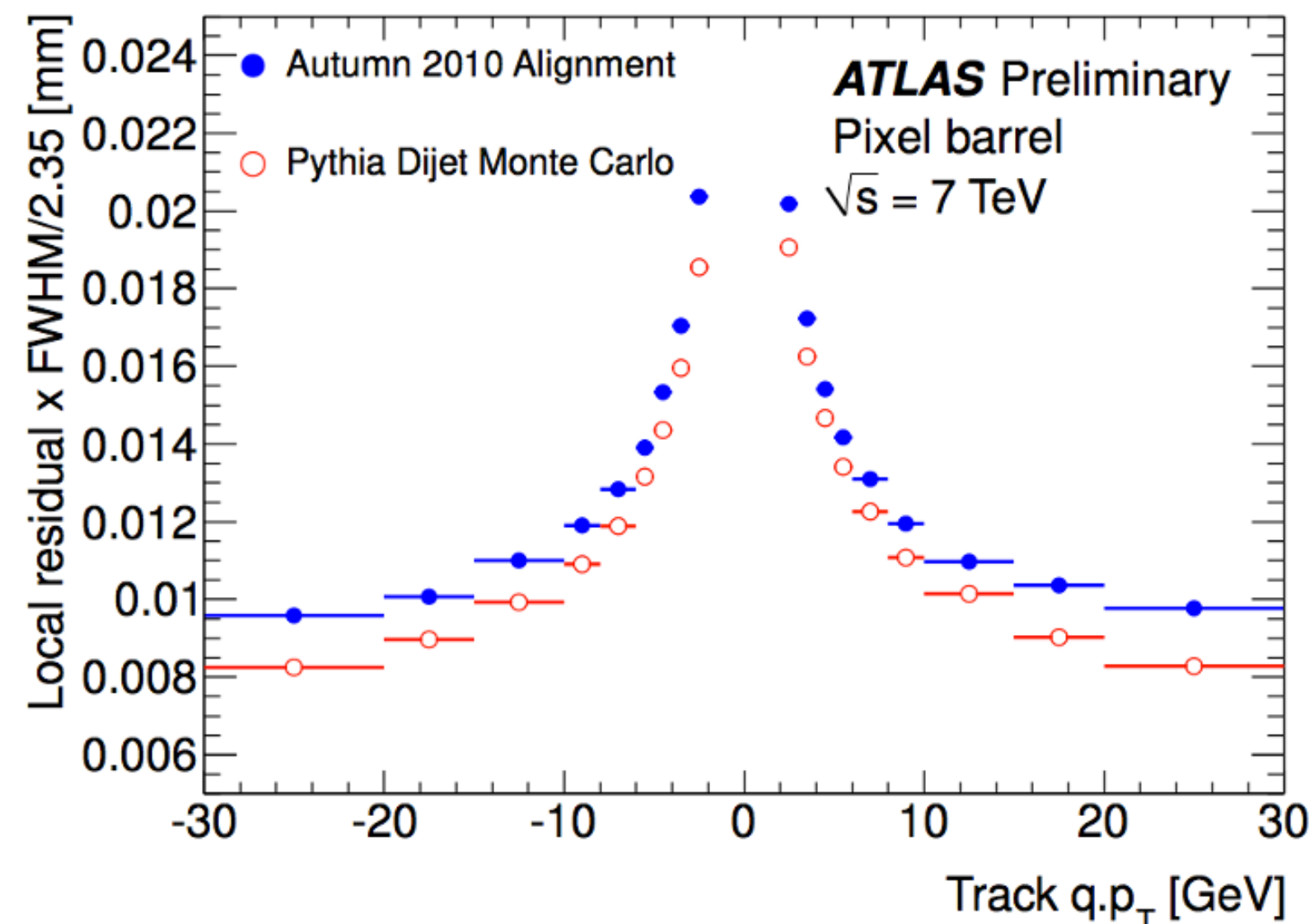
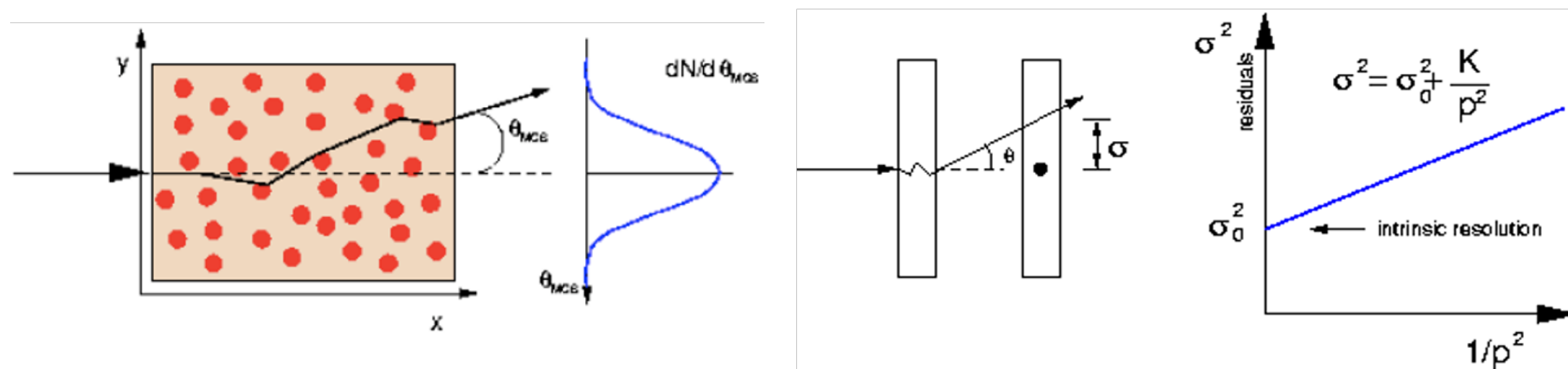
$$\text{RMS } (\theta_{MCS}) \approx \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{\frac{x}{x_0}} \left[1 + 0.038 \ln \left(\frac{x}{x_0} \right) \right]$$



- The interaction of the particles with the detector has to be included in the tracking model

Multiple Coulomb Scattering

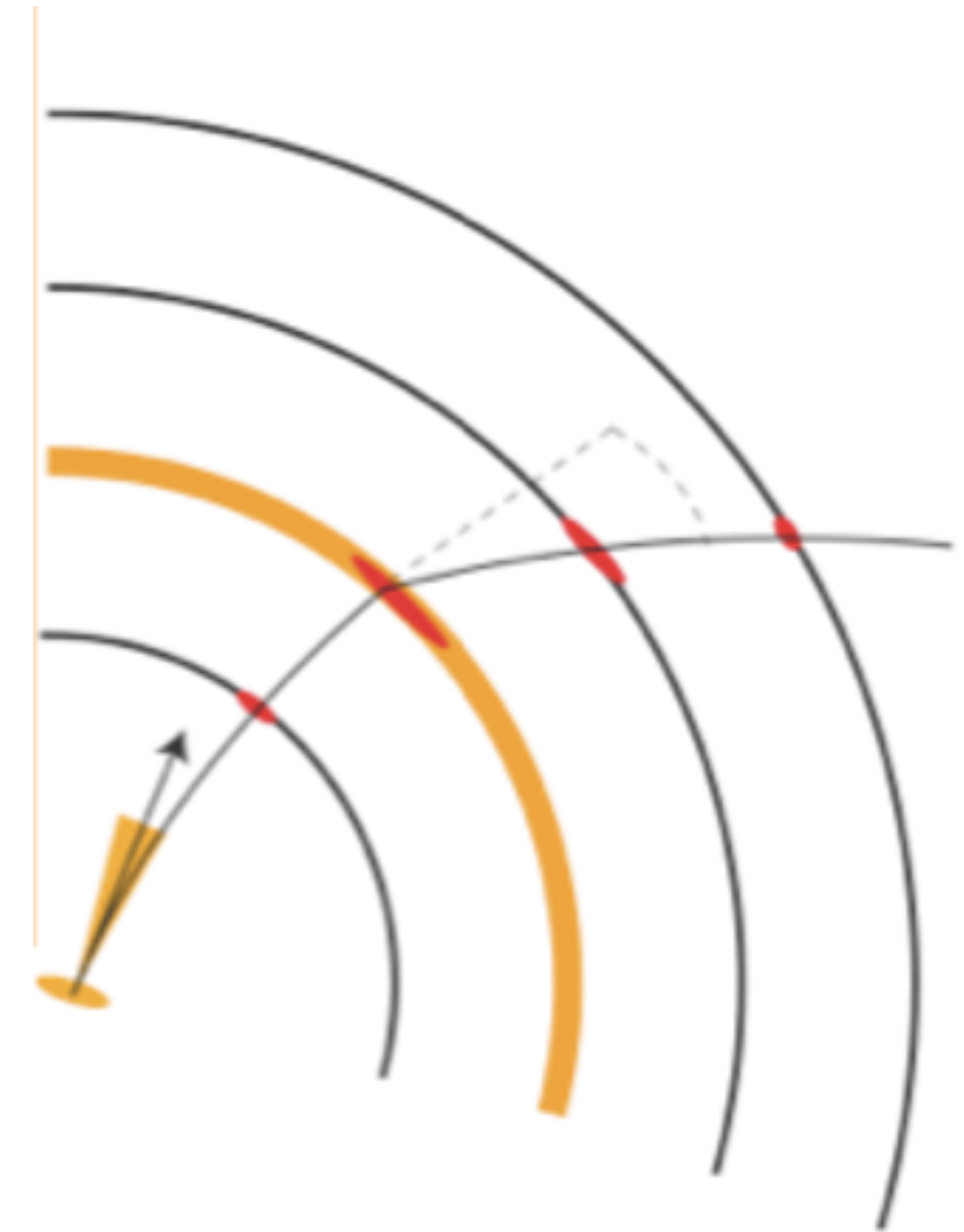
- The MCS deflects the tracks. Therefore it affects the detector residuals
 - The scattering angles depend on particle momentum
 - Residuals become momentum dependent
 - Residuals depend also on the detector structure



Multiple Coulomb Scattering

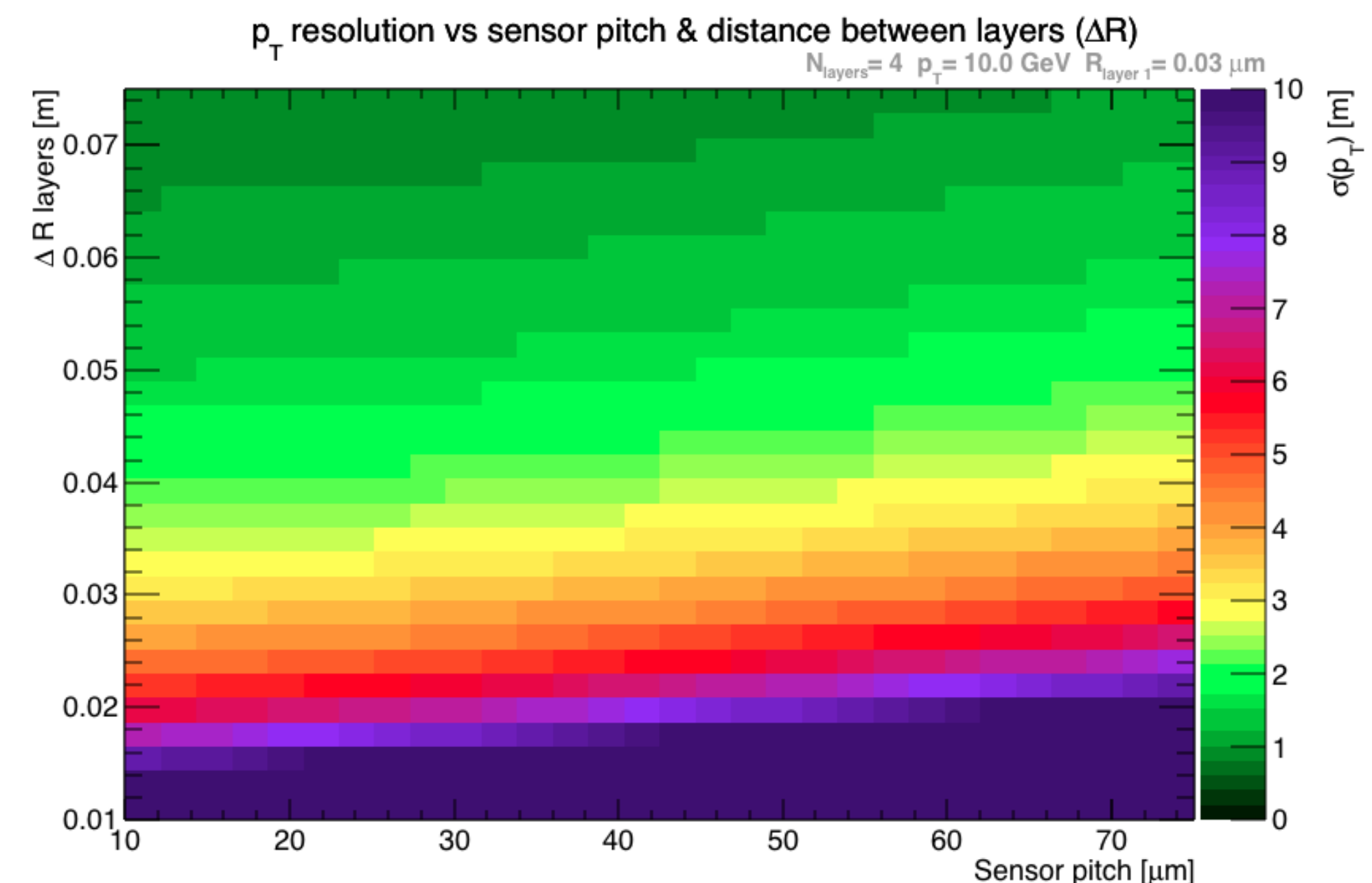
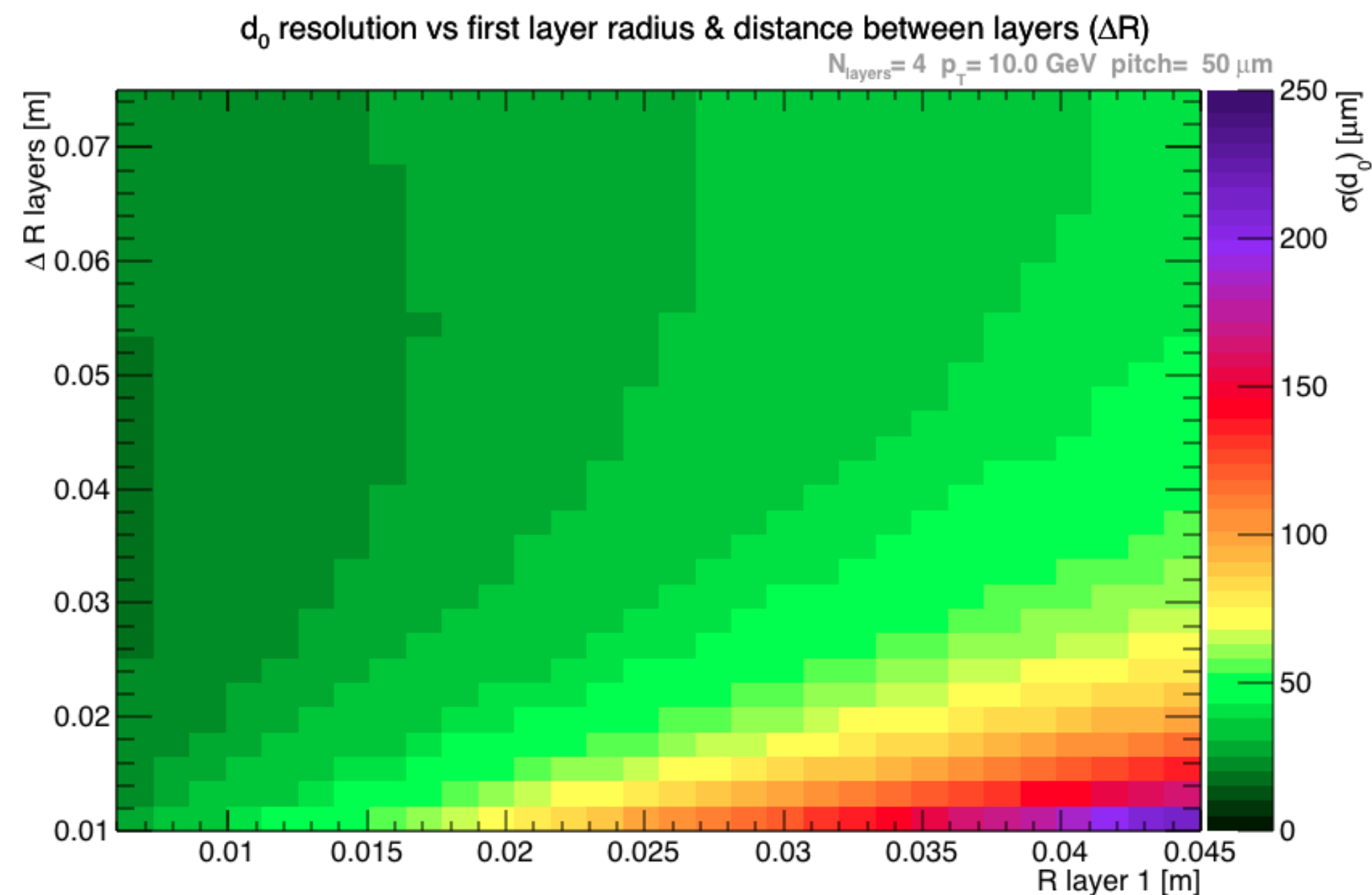
- Inclusion of the MCS angles in the tracking
 - Extra parameters to be fitted
 - 2 angles per tracking surface
 - In case of tracking with χ^2 , matrix size may be an issue
 - Example: 10 surfaces $\rightarrow$ 20 extra degrees of freedom
 - Larger inversion CPU times and error propagation
 - For Kalman filter, 2 extra degrees of freedom per step

$$\mathbf{r}_{\text{MCS}} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N_s} \end{pmatrix} \quad \chi^2 = \mathbf{r}^T V^{-1} \mathbf{r} + \mathbf{r}_{\text{MCS}}^T V_{\text{MCS}}^{-1} \mathbf{r}_{\text{MCS}}$$



Example: circular tracks

- Consider circular tracks in 2D (x, y)
- Detector formed by 4 circular sensor layers centred at (0,0)
 - First layer at R1. Rest of layers uniformly spaced by ΔR
- Spatial resolution: σ (same for all layers)
- Track parameters: d_0 , ρ & φ_0
- Estimate track parameters uncertainties by inverting the track fit matrix of the χ^2 fit



- Track reconstruction is a key ingredient of the High Energy Physics experiments
- Reaching the Physics goals of the experiment depend on the tracking
 - Invariant masses, life-time, flavour tagging, particle identification... and triggering
- Tracking starts with performant detectors
 - Converting sensor signals in 3D space points
 - Signal processing: clustering, Lorentz angle, radiation damage, ...
 - Sensors and supporting structure: MCS
- Tracking can be quite complex as experiment combine
 - Different detection technologies
 - Different sensor resolutions
- The challenge of extracting the most of information of the track particles is a field in evolution
 - Machine Learning and AI algorithm for pattern recognition
 - Specially important for HL-LHC and track trigger
- In todays experiments every aspect of particles interaction with the detectors is considered in the tracking
 - Aim: reach best performance as possible

