

The Δ -Self-Consistent-Field Method

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Core Electron Binding Energies

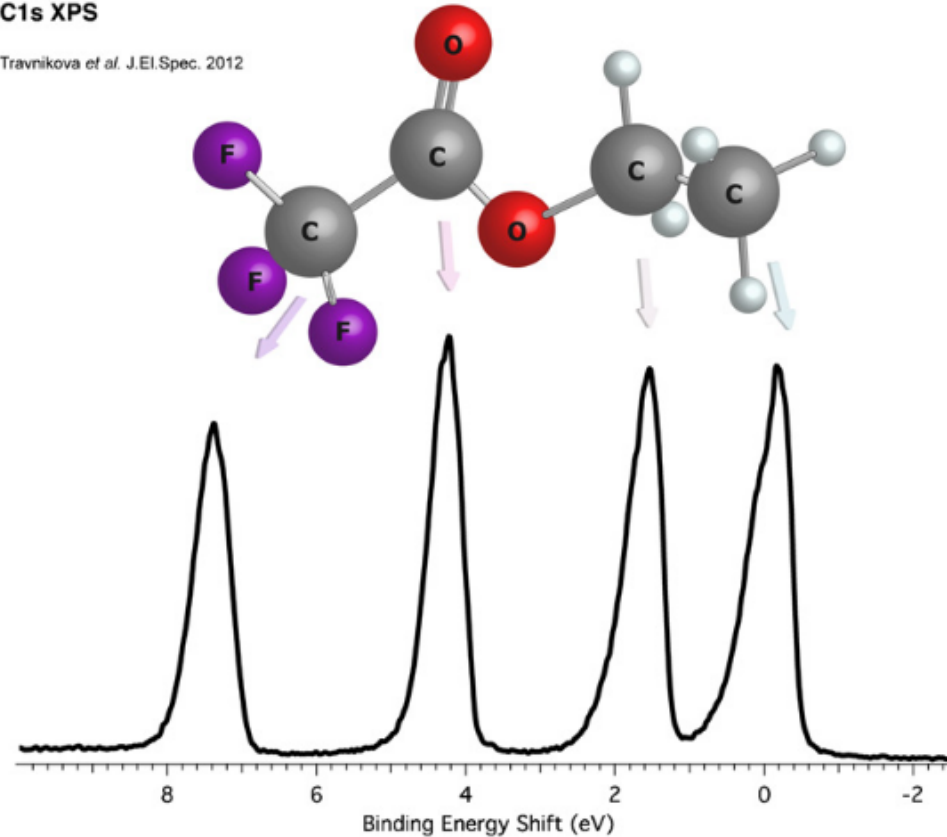
- How much work do we need to do, to remove a core electron from a given atom?
- E.g. the C 1s binding energy in gaseous CH₄ is 290.9 eV
- Measure experimentally, and find out about
 - Elemental composition of the sample
 - Chemical composition (?)

Core electron binding energies

- Small “chemical shifts” depending on the local environment of the atom
- XPS is used to study the chemical composition of the sample

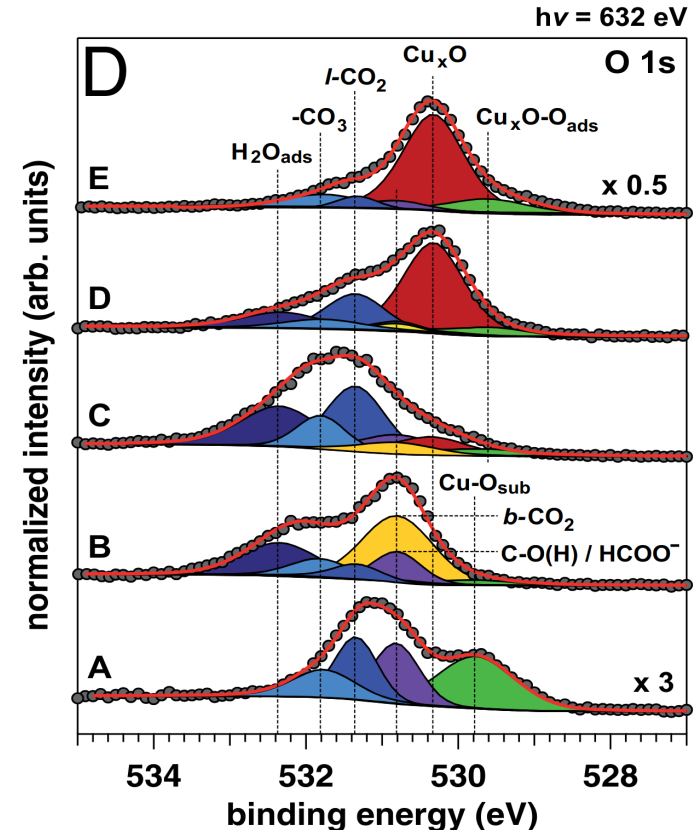
Ethyl trifluoroacetate
C1s XPS

Travnikova *et al.* J.EI.Spec. 2012



The peak-assignment problem

- Real spectra look messy
- How to disentangle the overlapping peaks?
- Reference data?



Reproduced from Favaro et al., PNAS 144, 6706 (2017)

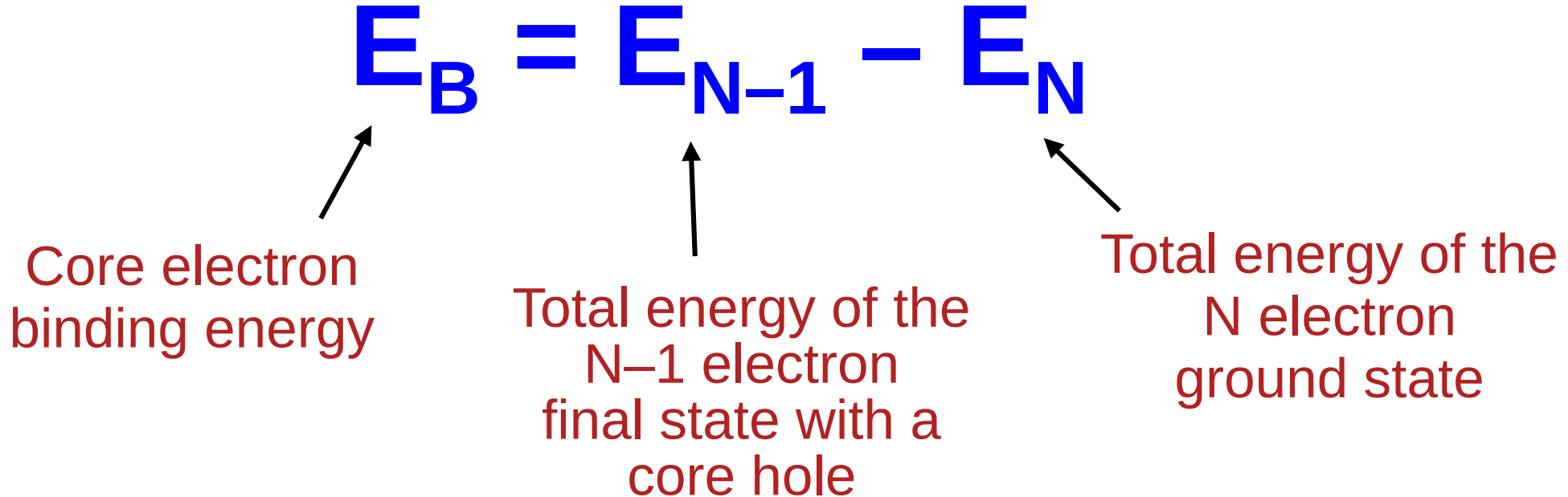
Why do we need theory?

- Let's say we've measured a spectrum, but we are not sure, what the peaks mean.
- Can we work in the opposite direction?
 - Start with a structural model
 - Calculate, what the spectrum should look like
 - Then see, if the measured spectrum is consistent with our theoretical result

A computational approach to core level XPS

$$E_B = E_{N-1} - E_N$$

Core electron
binding energy



The diagram illustrates the computational approach to core level XPS by defining the binding energy E_B as the difference between the total energy of the $N-1$ electron final state with a core hole (E_{N-1}) and the total energy of the N electron ground state (E_N). Arrows point from the text labels to the corresponding terms in the equation.

Total energy of the
 $N-1$ electron
final state with a
core hole

Total energy of the
 N electron
ground state

How to calculate E_B ?

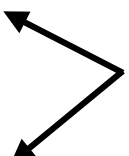
- Several approaches are possible
 - Koopmans' theorem and related methods
 - “Response” theories: GW, IP-CCSD, ...
 - Total energy difference methods

How to calculate E_B ?

- Koopmans' theorem
 - If all other electrons remain frozen, when one electron is removed from a system, then

$$E_{N-1} - E_N = -\epsilon_k$$

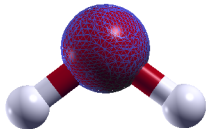
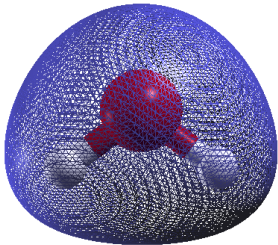
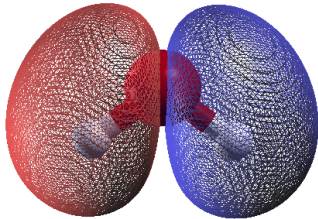
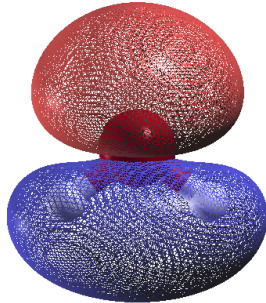
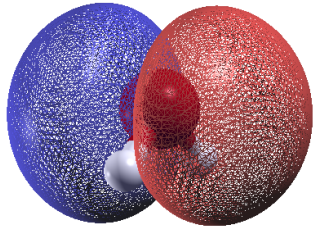
$$E_B = -\epsilon_k$$



Orbital eigenvalue of
the relevant core
orbital from Hartree-
Fock theory

Hartree-Fock theory: H₂O

- 10 electrons, 5 occupied orbitals

#	1	2	3	4	5
label	1a ₁	2a ₁	1b ₁	3a ₁	1b ₂
ε (eV)	-560.24	-37.11	-19.94	-15.93	-13.94
χ					

Koopmans' theorem: H₂O

- H₂O: orbital eigenvalues vs binding energies from photoelectron spectroscopy
- Sources of error: correlations, neglect of orbital relaxation.

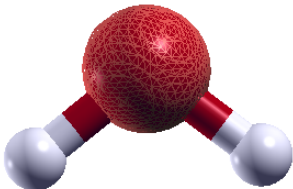
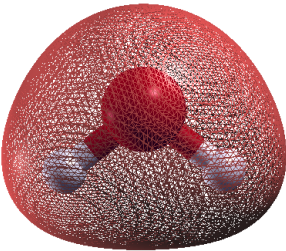
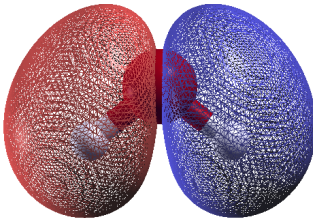
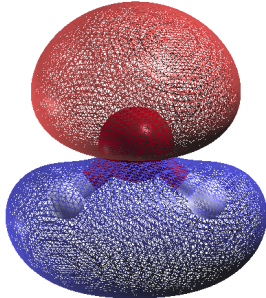
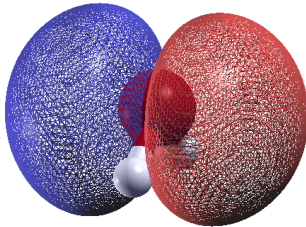
Orbital	1a ₁	2a ₁	1b ₁	3a ₁	1b ₂
- ϵ (eV)	560.24	37.11	19.94	15.93	13.94
I.E. (eV)	539.9	32.2	18.5	14.7	12.6

Could we use orbital eigenvalues from DFT instead of Hartree-Fock theory?

Strictly, Koopmans' theorem does not hold for DFT, but we can still try...

Kohn-Sham DFT: H₂O

(using PBE xc functional)

#	1	2	3	4	5
label	1a ₁	2a ₁	1b ₁	3a ₁	1b ₂
-ε (eV)	511.21	25.26	13.07	9.31	7.20
B.E. (expt.)	539.9	32.2	18.5	14.7	12.6
χ					

How to calculate E_B ?

- Several approaches are possible
 - ~~Koopmans' theorem and related methods~~
 - “Response” theories: GW, IP-CCSD, ...
 - Total energy difference methods

The remaining electrons do NOT remain frozen when a core electron is removed

Sometimes, Koopmans' theorem gives accurate binding energy shifts, but sometimes it doesn't (e.g. when final state screening is important)

How to calculate E_B ?

- Several approaches are possible
 - Koopmans' theorem and related methods

Cheap, but unreliable
 - “Response” theories: GW, IP-CCSD, ...

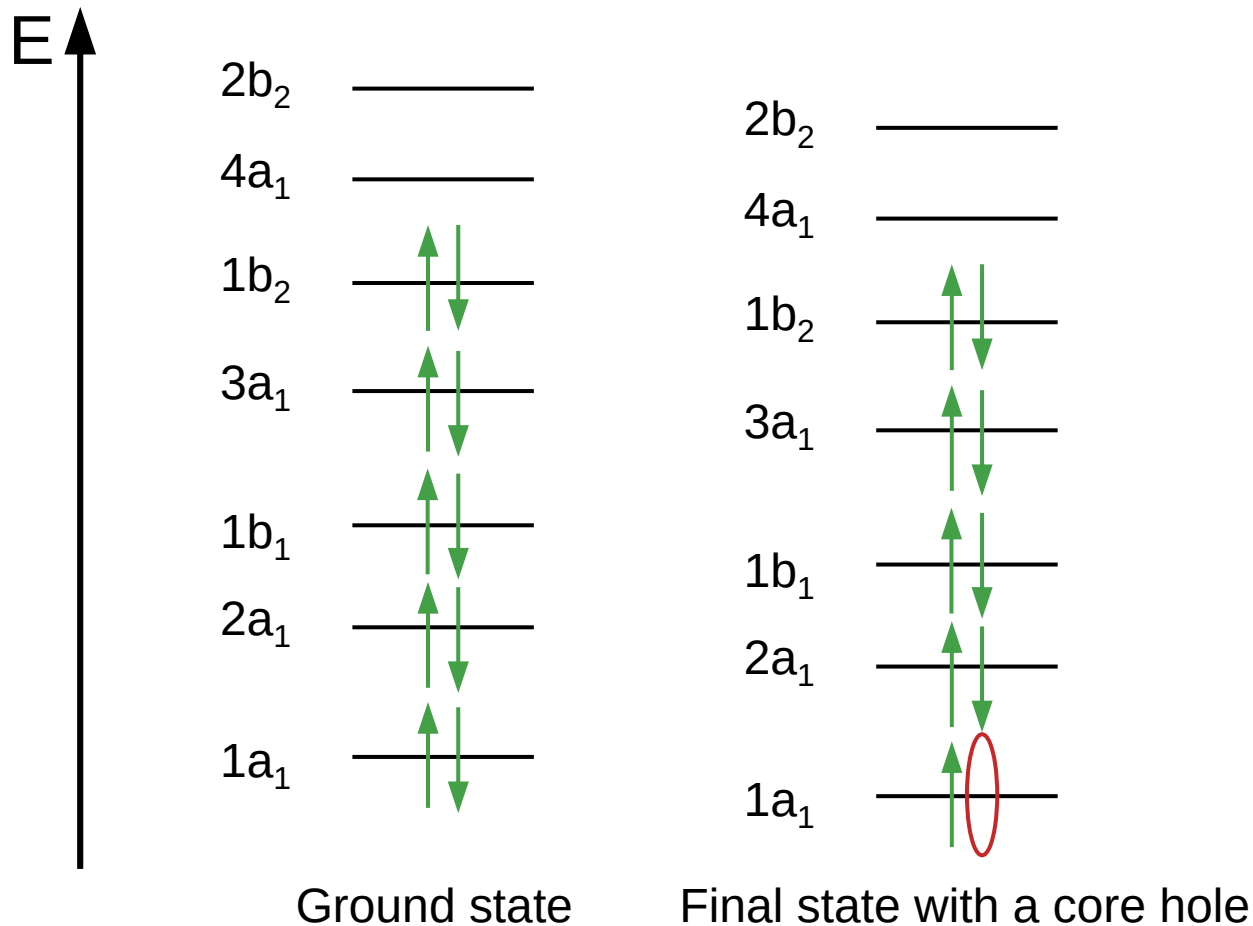
In principle accurate, but expensive
 - Total energy difference methods

The main topic of this talk

The Δ -Self-Consistent-Field method

- Evaluate $E_B = E_{N-1} - E_N$ by “brute force”, i.e. calculate E_N and E_{N-1} directly
- E_N is simply the ground state total energy
- How to calculate E_{N-1} ?

Δ SCF method: H_2O example



The final state with a core hole is a

- high energy excited state
- of a radical cation ($\text{H}_2\text{O}^{+\bullet}$)

It would be challenging to calculate its energy using conventional excited state methods, e.g. CIS, TDDFT, EOM-CCSD, ...

Calculating E_{N-1}

For the core-hole state, we can instead perform
a self-consistent field calculation
with an excited electronic configuration

P.S. Bagus, *Phys. Rev.* **139**, A619 (1965)

Calculating E_{N-1}

For the core-hole state, we can instead perform
a self-consistent field calculation DFT or
Hartree-Fock
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Calculating E_{N-1}

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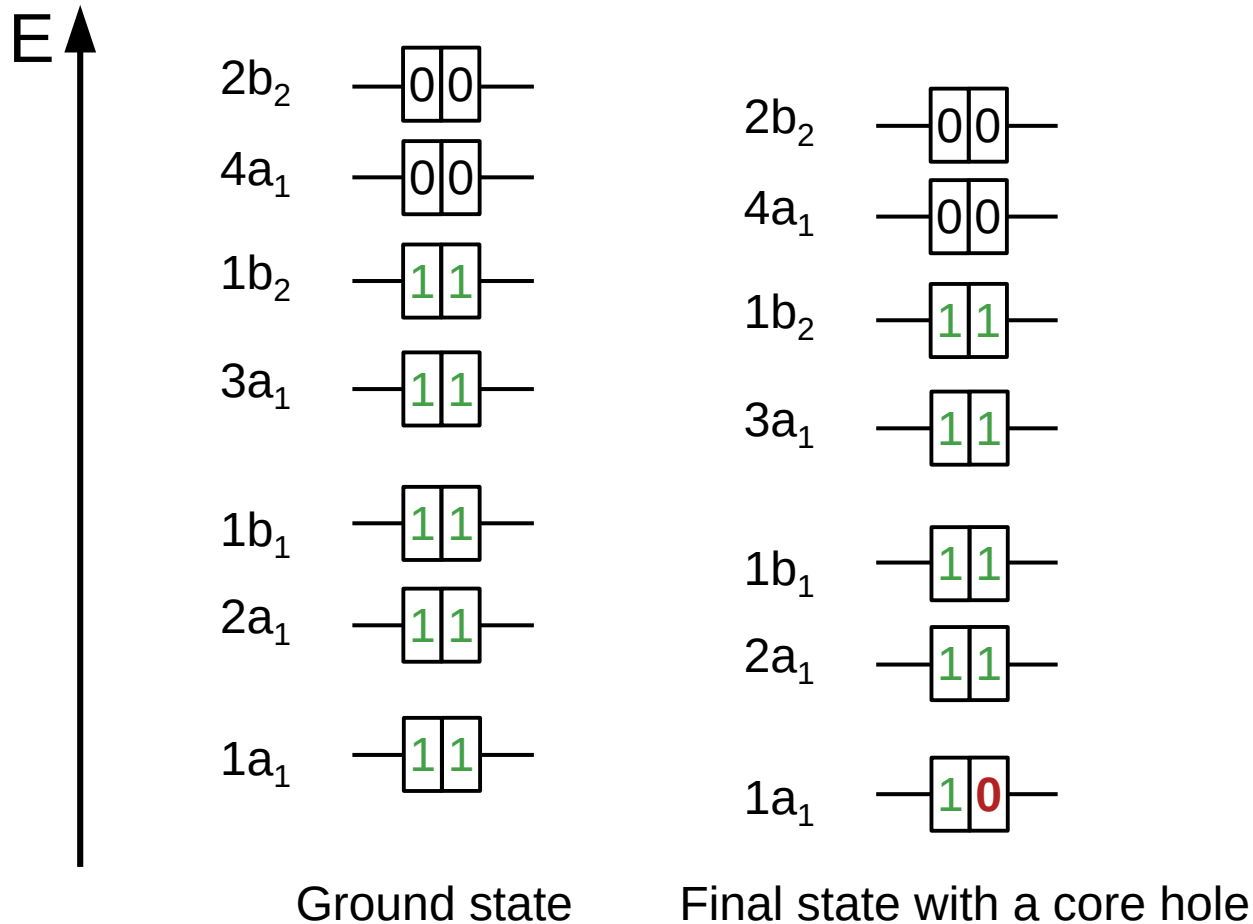
DFT or
Hartree-Fock

with an excited electronic configuration

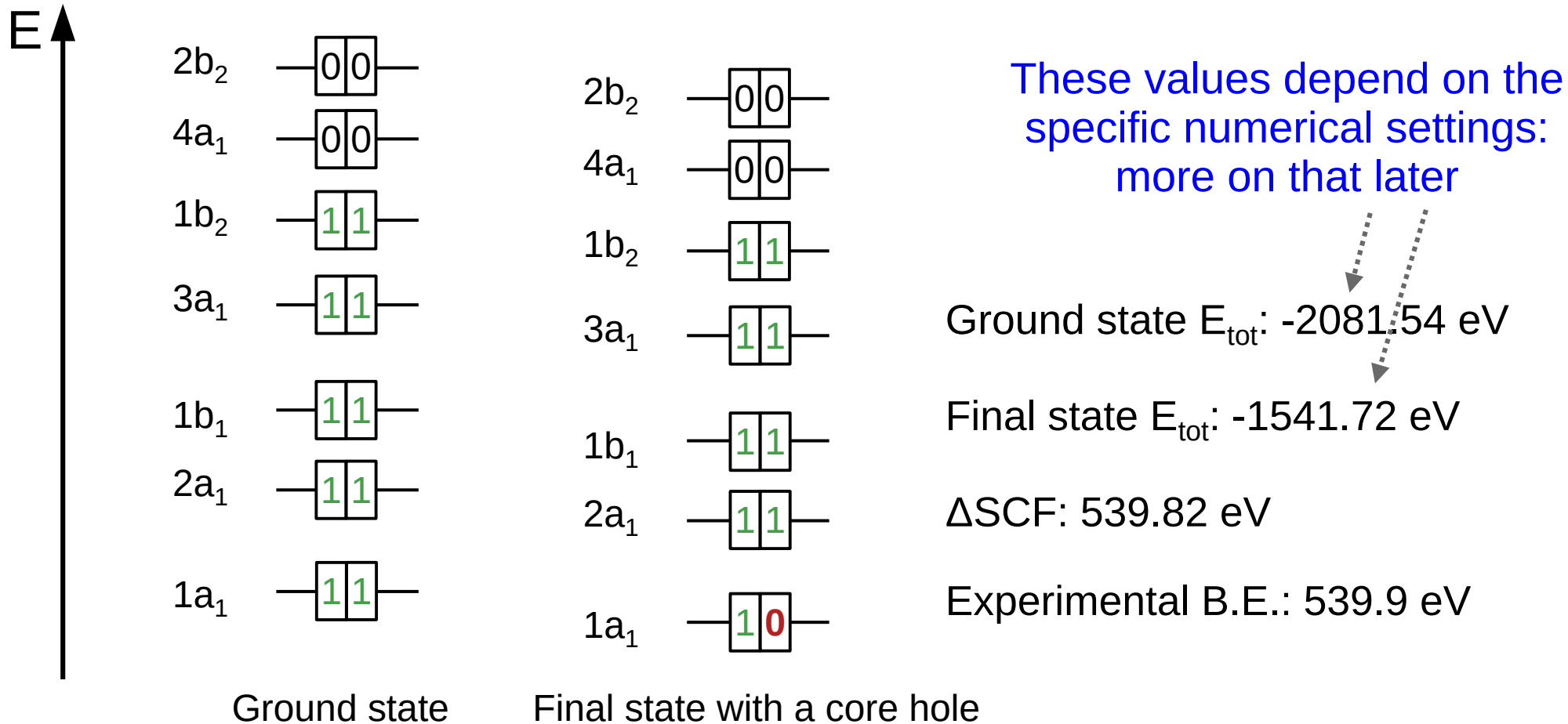
i.e. non-Aufbau-principle
occupation numbers

P.S. Bagus, *Phys. Rev.* **139**, A619 (1965)

Δ SCF method: H₂O example



Δ SCF method: H₂O example



Δ SCF method

- Calculate core electron binding energies at the cost of 1 ground state DFT calculation per core level
 - How to run these calculations?
 - How well does it work?
 - Is it justified?

Δ SCF method

- Calculate core electron binding energies at the cost of 1 ground state DFT calculation per core level
 - How to run these calculations?
 - How well does it work?
 - Is it justified?
- This lecture: molecules (~ aperiodic systems)
- Tomorrow: solids and surfaces

How can this be justified?

- W. Yang and P.W. Ayers, “Foundation for the Δ SCF Approach in Density Functional Theory”, arXiv:2403.04604
- The Hohenberg-Kohn theorem does not hold for excited states but, they show that equivalent and exact excited state theories can be formulated in terms of
 - The excitation quantum number and the noninteracting potential
 - The noninteracting wave function
 - The noninteracting one-electron reduced density matrix

How can this be justified?

- W. Yang and P.W. Ayers, “Foundation for the Δ SCF Approach in Density Functional Theory”, arXiv:2403.04604
- Importantly, the ground and excited-state exchange-correlation energy use the same universal functional
- Using approximate ground state functionals for excited states ($\sim$ non-Aufbau electronic configurations) might not be such a bad idea

Δ SCF method: H₂O example

- A number of DFT codes could be used, but the examples I will show are run in FHI-aims
 - All electron DFT with numerical atom-centered basis functions
 - Academic license: full source code distributed to all license holders, license fee can be reduced / waived upon request

www.fhi-aims.org



Δ SCF method: H₂O example

- Try 1:
 - “Textbook” nonrelativistic Hamiltonian
 - PBE exchange-correlation functional
 - Default “tight” numerical basis functions

Δ SCF method: H₂O example

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- Ground state E_{tot} : -2078.63 eV

Δ SCF method: H₂O example

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- Final state E_{tot} : -1538.95 eV

Δ SCF method: H₂O example

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Δ SCF method: H₂O example

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This is already quite close to the experimental value of 539.9 eV, but right now we are a bit lucky with error cancellation



Δ SCF method: H₂O example

- Try 1:
 - “Textbook” nonrelativistic Hamiltonian
 - PBE exchange-correlation functional
 - Default “tight” numerical basis functions

- Ground state E_{tot} : -2078.63 eV

- Final state E_{tot} : -1538.95 eV

- Δ SCF: 539.67 eV

Close to the nucleus, electrons travel at speeds approaching the speed of light – for core electrons, relativistic effects matter even for light elements

Δ SCF method: H₂O example

- Try 1:
 - “Textbook” nonrelativistic Hamiltonian
 - PBE exchange-correlation functional
 - Default “tight” numerical basis functions
- Ground state E_{tot} : -2078.63 eV
- Final state E_{tot} : -1538.95 eV
- Δ SCF: 539.67 eV

The default basis set for O only has one basis function for the 1s orbital. This is optimized for the neutral O atom, not the O atom with a core hole.

Δ SCF method: H₂O example

- Try 2:
 - Relativistic effects via the “scaled ZORA” approximation
 - PBE exchange-correlation functional
 - Default “tight” basis for ground state, core-hole optimized basis for the final state
- Ground state E_{tot} : -2080.14 eV
- Final state E_{tot} : -1541.25 eV
- Δ SCF: 538.88 eV

Δ SCF method: H₂O example

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- 

Δ SCF method: H₂O example

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We could try using a different exchange-correlation functional

But now we are further from the experimental value of 539.9 eV!

Δ SCF method: H₂O example

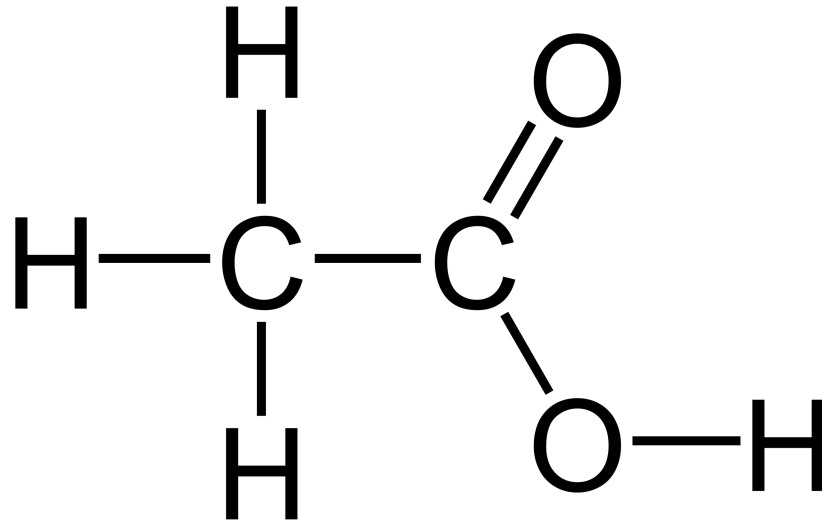
- With proper basis sets, and an adequate treatment of relativity, we get:
 - $E_B = 538.88$ eV (PBE)
 - $E_B = 539.82$ eV (SCAN)
 - $E_B = 539.73$ eV (B3LYP)
 - ...
- C.f. experimental binding energy of 539.9 eV

Δ SCF method: H₂O example

- All reasonable functionals should give us relatively accurate binding energy shifts
 - But only some of them will give us accurate absolute binding energies
 - (And even then, probably not for all core levels of all elements)

Δ SCF method: CH_3COOH example

- Now we have two different C atoms. Let's look at the molecular orbitals.



State	Occupation	Eigenvalue [Ha]	Eigenvalue [eV]	
1	2.00000	-19.027015	-517.75142	} O 1s
2	2.00000	-18.967546	-516.13318	
3	2.00000	-10.182338	-277.07550	} C 1s
4	2.00000	-10.056128	-273.64117	
5	2.00000	-1.079301	-29.36928	
6	2.00000	-0.991741	-26.98665	
7	2.00000	-0.732408	-19.92983	
8	2.00000	-0.581211	-15.81557	
9	2.00000	-0.463889	-12.62306	
10	2.00000	-0.450621	-12.26202	
11	2.00000	-0.444231	-12.08815	
12	2.00000	-0.387741	-10.55097	
13	2.00000	-0.371018	-10.09590	
14	2.00000	-0.367246	-9.99327	
15	2.00000	-0.301673	-8.20893	
16	2.00000	-0.251573	-6.84566	
17	0.00000	-0.028707	-0.78117	← HOMO
18	0.00000	0.009953	0.27083	← LUMO

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Δ SCF method: CH_3COOH example

- To find the C 1s orbital for the $-\text{CH}_3$ group, we can
 - Visualize the orbitals themselves
 - Use Mulliken analysis (which basis functions contribute to which state)

State	Occupation	Eigenvalue [Ha]	Eigenvalue [eV]	
1	2.000000	-19.027015	-517.75142	} O 1s
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Δ SCF method: CH_3COOH example

- Let's run a calculation with a core hole in the fourth eigenstate. ($-\text{CH}_3$ C 1s)

Δ SCF method: CH_3COOH example

- Let's run a calculation with a core hole in the fourth eigenstate. ($-\text{CH}_3$ C 1s)
- Δ SCF result = 291.52 eV
- Experimental B.E. = 291.6 eV

- Look at eigenvalues just in case:

Spin-up eigenvalues:

State	Occupation	Eigenvalue [Ha]	Eigenvalue [eV]
1	1.000000	-19.235958	-523.43705
2	1.000000	-19.190978	-522.21307
3	0.000000	-10.882778	-296.13545
4	1.000000	-10.455453	-284.50736

- Missing electron in eigenstate number 3!

- Look at eigenvalues just in case:

Spin-up eigenvalues:

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- Missing electron in eigenstate number 3!
- Maximum Overlap Method (MOM) was used to keep track of the core hole.

Keeping track of the core hole

- FHI-aims provides two keywords for specifying the eigenstate, whose occupation number you want to constrain

Keeping track of the core hole

- FHI-aims provides two keywords for specifying the eigenstate, whose occupation number you want to constrain
- `deltascf_projector`
 - Provide initial orbitals (e.g. from a ground state calculation)
 - Specify the index of the constrained state
 - Keep track of that orbital throughout the SCF cycle using the maximum overlap method

Keeping track of the core hole

- FHI-aims provides two keywords for specifying the eigenstate, whose occupation number you want to constrain
- `deltascf_basis`
 - Specify a particular basis function, e.g. an atomic “1s” basis function on a specific C atom
 - During each SCF iteration, the occupation constraint is applied to the Kohn-Sham orbital with the highest contribution from that basis function

Δ SCF method: CH_3COOH example

- Δ SCF: 291.52 eV ($-\text{CH}_3$), 295.21 eV ($-\text{COOH}$)
- Experiment: = 291.6 eV ($-\text{CH}_3$), 295.5 eV ($-\text{COOH}$)

Δ SCF method: N₂ example

- Proceed as before:
 - Ground state calculation: write wavefunction to disk
 - Core hole calculation: load wavefunction from disk, create hole in the lowest eigenstate, and keep track of the core hole using the MOM
 - Δ SCF result = 404.76 eV
 - Experimental N 1s B.E. = 409.9 eV
 - What went wrong?

Δ SCF method: N_2 example

- What went wrong?
 - Check Mulliken analysis $\Rightarrow$ the core hole is delocalized across the two N atoms
 - We need to localize the core hole. Breaking the symmetry is usually enough – e.g. by using a core-hole adapted basis set on only one atom
 - Now: Calculated B.E. = 409.97 eV, c.f. experimental B.E. = 409.9 eV

Δ SCF method: core hole localization

- In a Δ SCF calculation, the core hole must always be localized on one atom to get meaningful results
- In almost all systems, this is also physically the correct picture
- However, ironically, in N_2 , the core hole is probably actually delocalized: see Ueda et al., *Eur. Phys. J. Special Topics* **169**, 95 (2009)

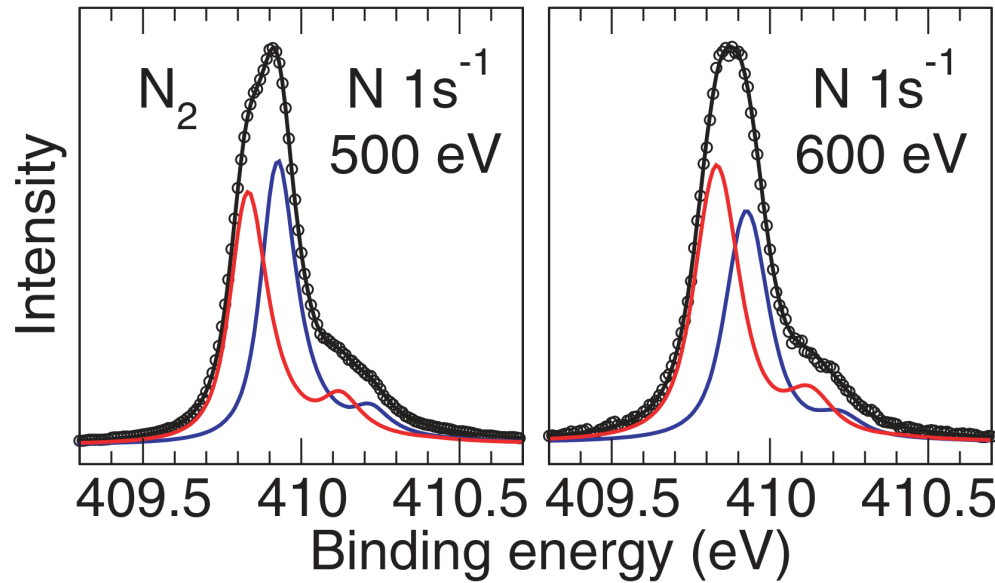


Fig. 2. N 1s photoelectron spectra of N₂ recorded at photon energies of 500 eV and 600 eV. The solid lines through the data points represent the fit results. The blue and red subspectra indicate the contributions from the 1σ_g and 1σ_g components, respectively.

Figure 2 from Ueda et al., Eur. Phys. J. Special Topics 169, 95 (2009)

Δ SCF method: core hole localization

- Δ SCF – final state wavefunction is a single Slater determinant
- Real life – delocalized core hole in N_2 correctly described using configuration interaction theory
- In this case, the need for localization stems from a limitation of the Δ SCF approach
- In most real systems, vibrations decouple core orbitals – core holes really are localized

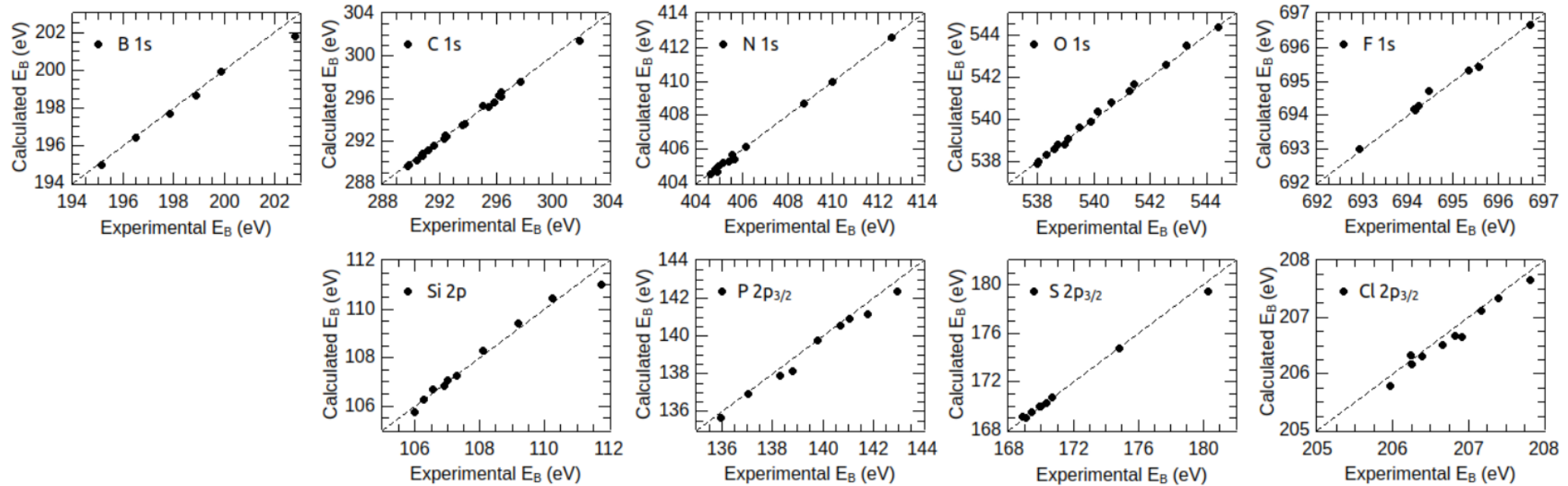
Δ SCF method: H₂S example

- Experimentally reported S 2p_{3/2} binding energy 170.32 eV
 - Most codes (incl. FHI-aims) only permit scalar-relativistic Δ SCF calculations
 - Workaround: calculate scalar relativistic 2p binding energy, then subtract 1/3 of the splitting to get theoretical 2p_{3/2} result
- Spin-orbit splitting of S 2p core level, i.e. separation between the 2p_{3/2} and 2p_{1/2} peaks is 1.16 eV

Δ SCF method: performance

- Numerical settings used for benchmarking:
 - SCAN
 - scaled ZORA relativity
 - FHI-aims with core-enhanced numerical basis sets for the atoms with a core hole

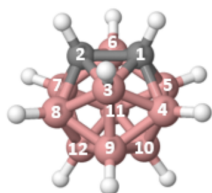
Performance – small molecules



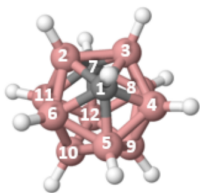
MAE = 0.16 eV for 103 binding energies from gas phase XPS

Kahk et al., Phys. Rev. Materials 3, 100801(R) (2019)

Performance – slightly bigger molecules



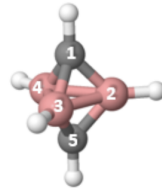
o-Carborane
 $1,2\text{-C}_2\text{B}_{10}\text{H}_{12}$



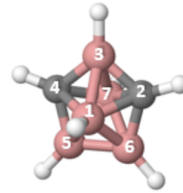
m-Carborane
 $1,7\text{-C}_2\text{B}_{10}\text{H}_{12}$



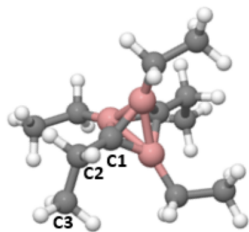
2,3- $\text{C}_2\text{B}_4\text{H}_8$



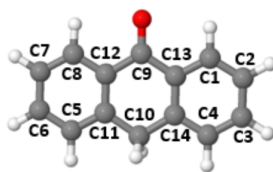
1,5- $\text{C}_2\text{B}_3\text{H}_5$



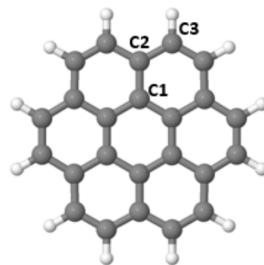
2,4- $\text{C}_2\text{B}_5\text{H}_7$



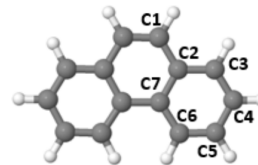
1,5- $\text{C}_2\text{B}_3(\text{C}_2\text{H}_5)_5$



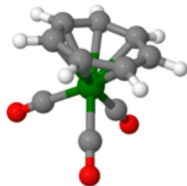
Anthrone
 $\text{C}_{14}\text{H}_{10}\text{O}$



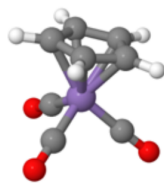
Coronene
 $\text{C}_{24}\text{H}_{12}$



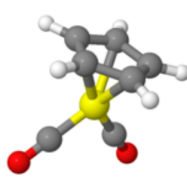
Phenanthrene
 $\text{C}_{14}\text{H}_{10}$



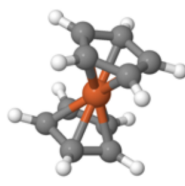
$\text{V}(\text{CO})_3\text{C}_7\text{H}_7$



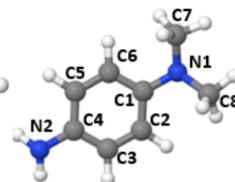
$\text{Mn}(\text{CO})_3\text{C}_5\text{H}_5$



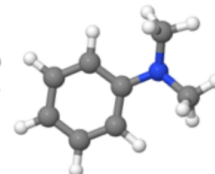
$\text{Co}(\text{CO})_2\text{C}_5\text{H}_5$



$\text{Fe}(\text{C}_5\text{H}_5)_2$



N,N-dimethyl-*p*-phenylenediamine
 $\text{C}_8\text{H}_{12}\text{N}_2$

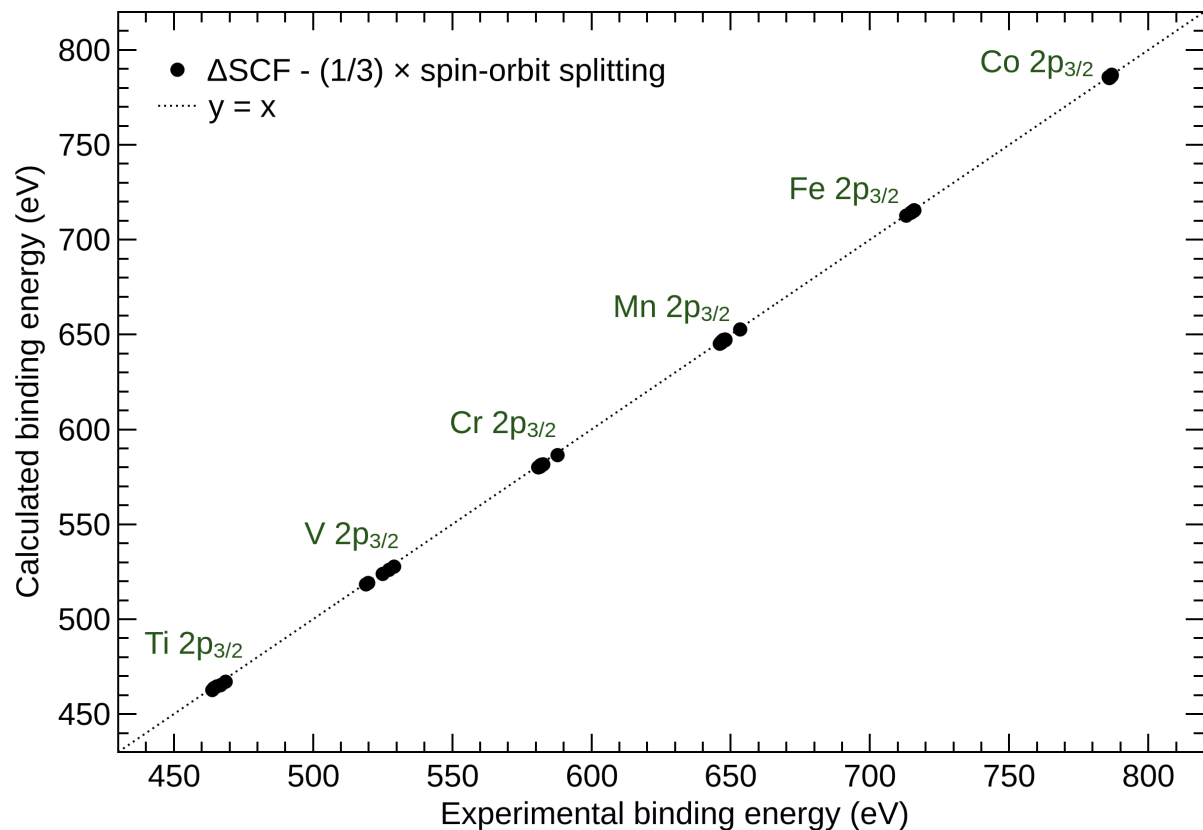


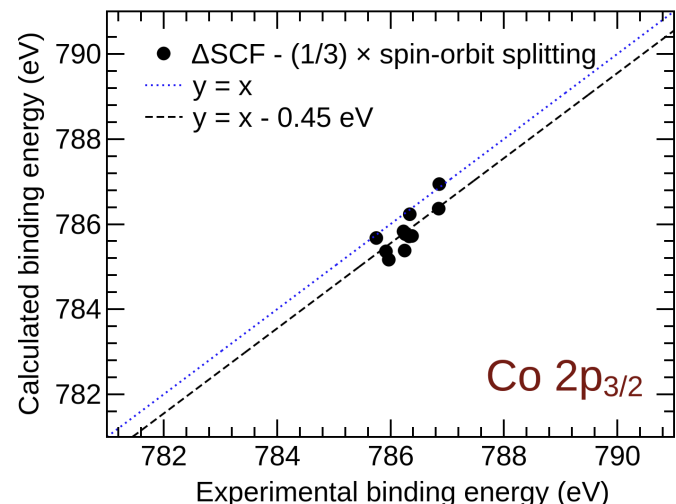
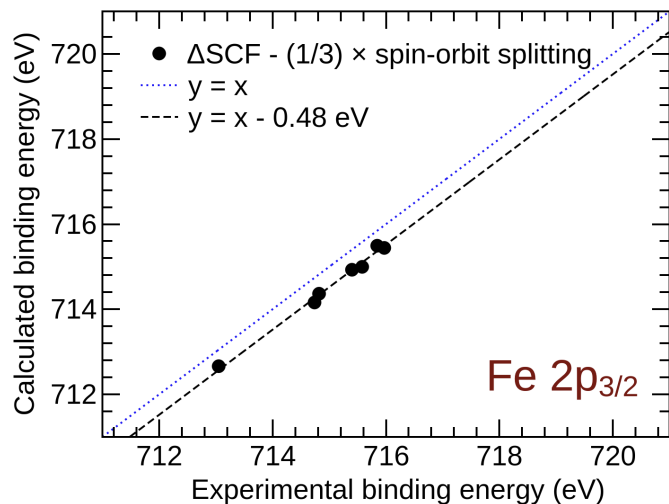
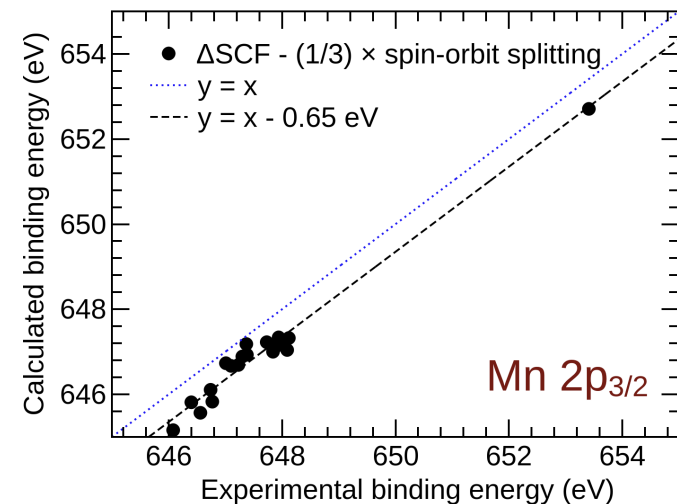
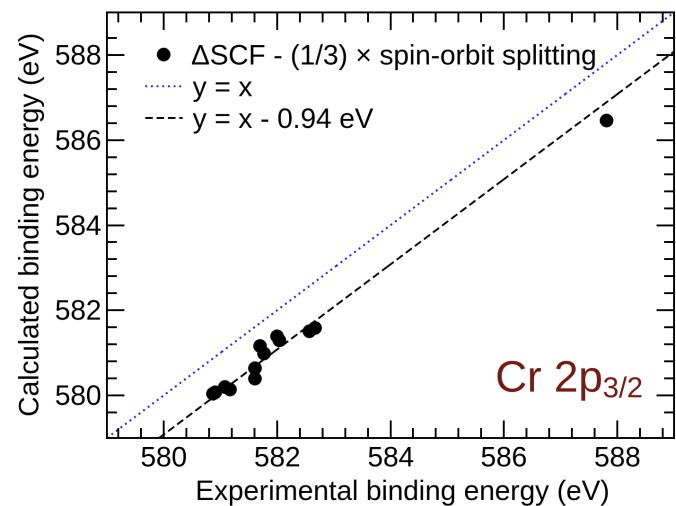
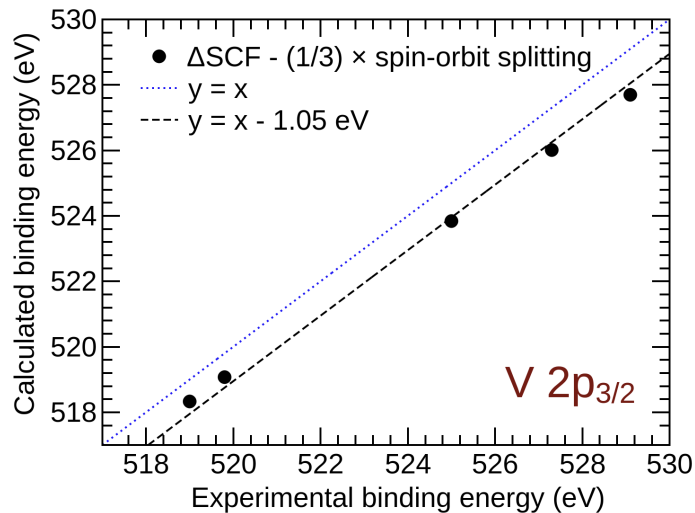
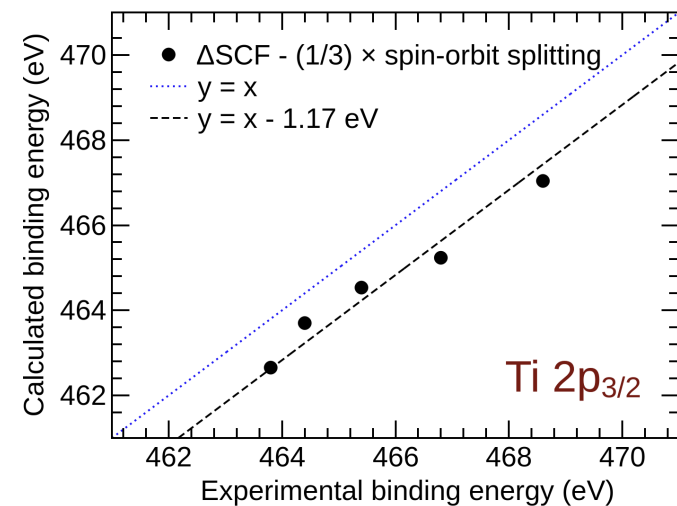
N,N-dimethylaniline
 $\text{C}_8\text{H}_{11}\text{N}$

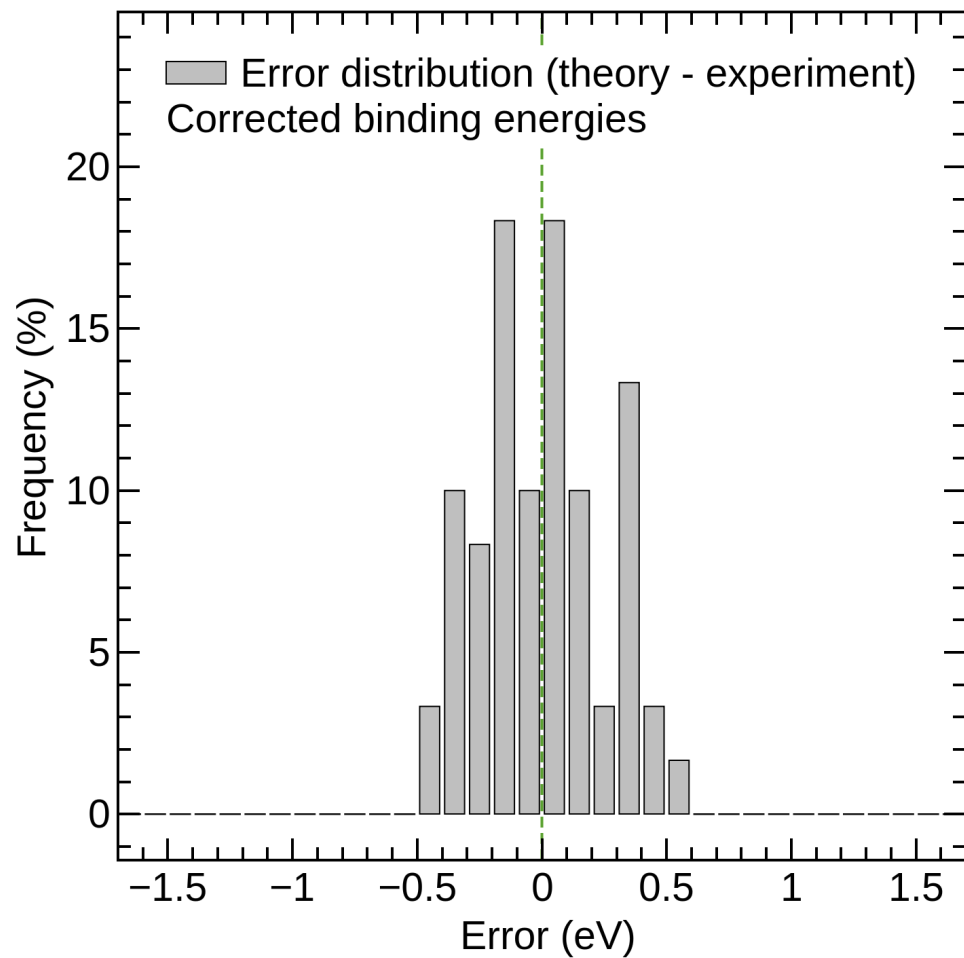
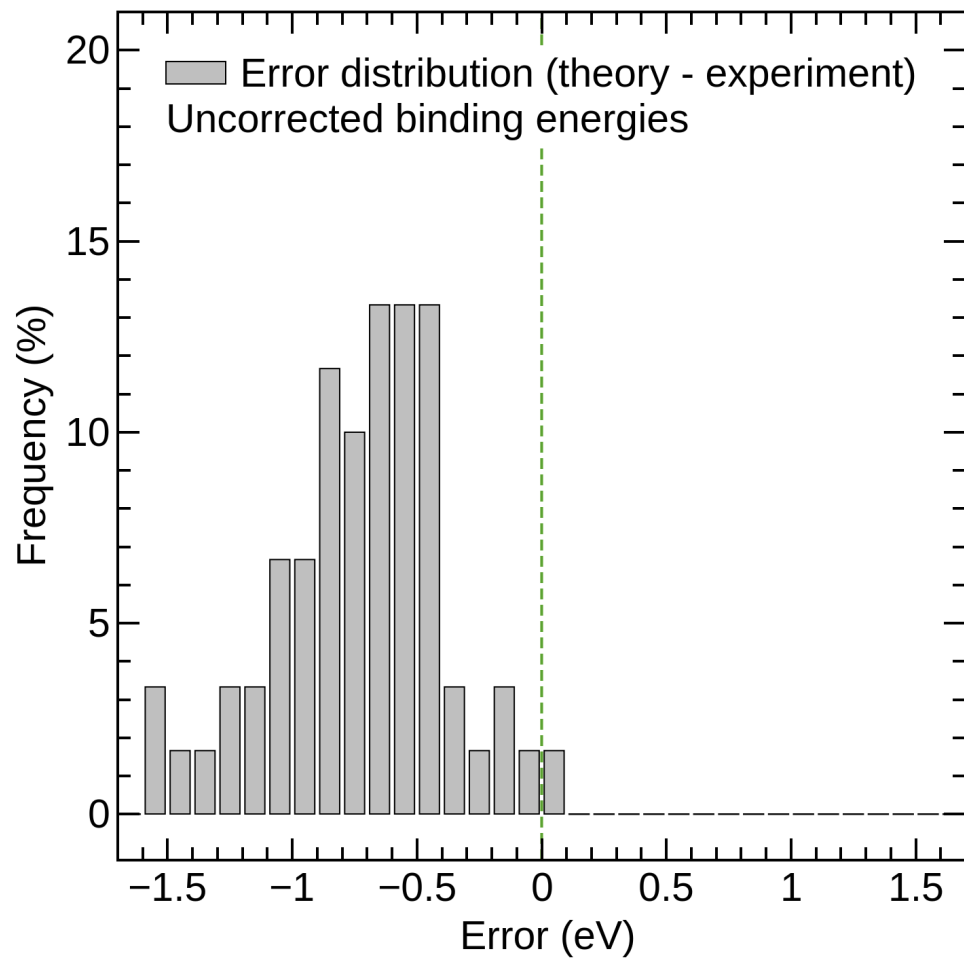
Performance – slightly bigger molecules

- Mean absolute error (SCAN) = 0.19 eV
- Mean error (SCAN) = -0.05 eV
- Mean absolute error (B3LYP) = 0.19 eV
- Mean error (B3LYP) = 0.06 eV

Performance – small molecules, heavier elements

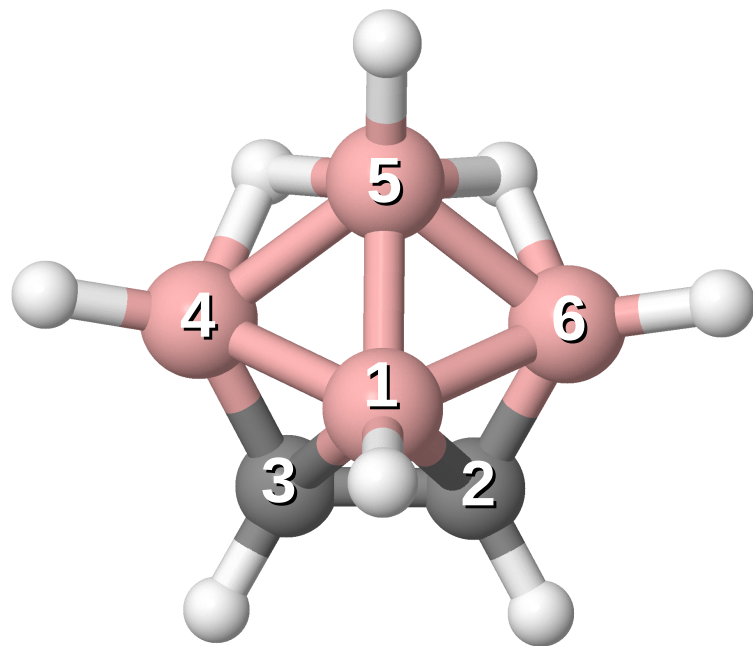






Molecules – some interesting results

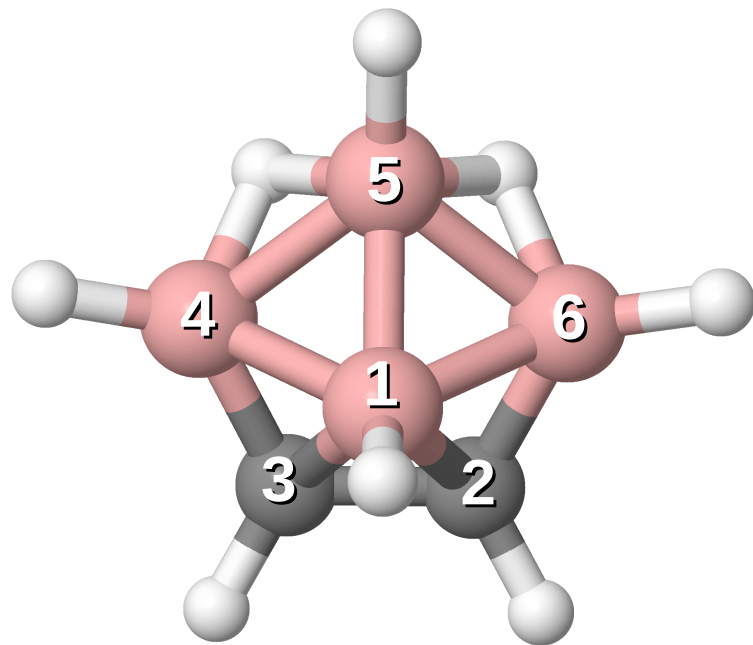
Core electron binding energies in 2,3-C ₂ B ₄ H ₈			
Core level	Theory	Experiment*	Error
B 1s (1)	195.47	196.0	-0.53
B 1s (5)	195.92	195.7	0.22
B 1s (4, 6)	195.17	195.2	-0.03
C 1s	290.83	290.9	-0.07



* Allison et al., *J. Electron Spectrosc. Relat. Phenom.* **1**, 269 (1972)

Molecules – some interesting results

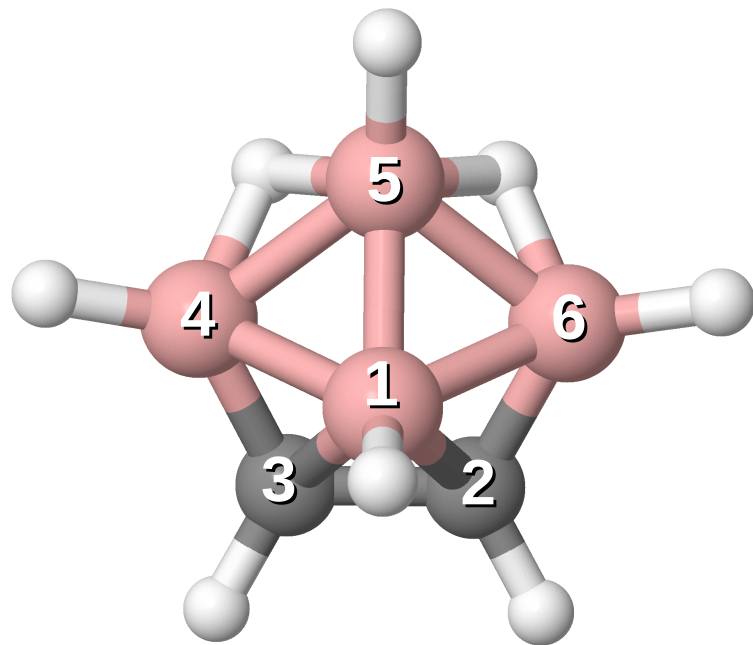
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Molecules – some interesting results

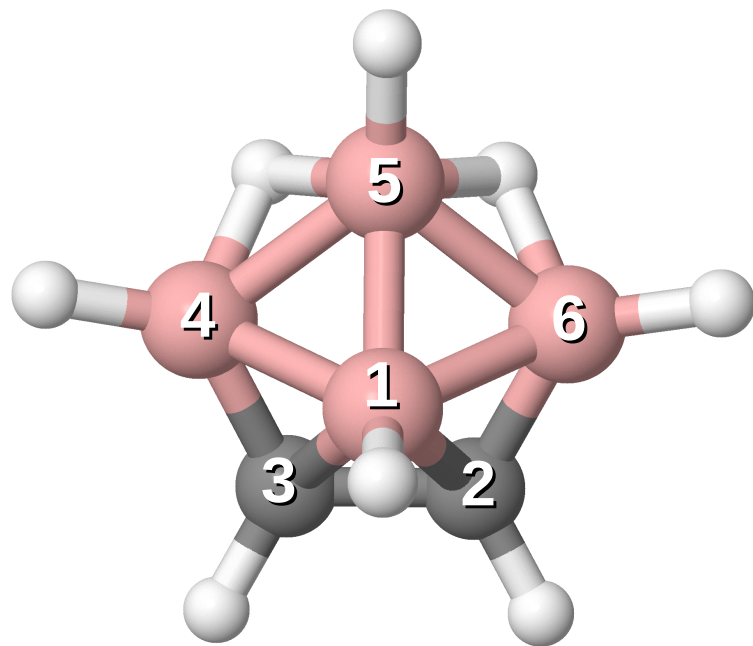
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Molecules – some interesting results

Core electron binding energies in 2,3-C ₂ B ₄ H ₈			
Core level	Theory	Experiment*	Error
B 1s (1)	195.47	195.7	-0.23
B 1s (5)	195.92	196.0	-0.08
B 1s (4, 6)	195.17	195.2	-0.03
C 1s	290.83	290.9	-0.07



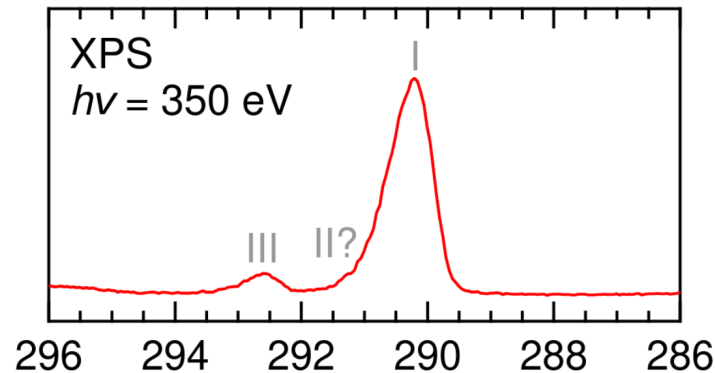
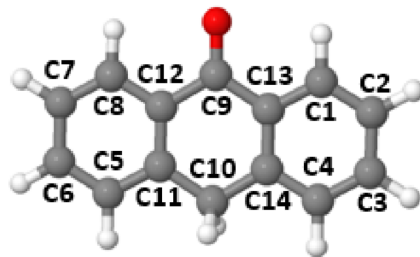
* Allison et al., *J. Electron Spectrosc. Relat. Phenom.* **1**, 269 (1972)

Molecules – some interesting results

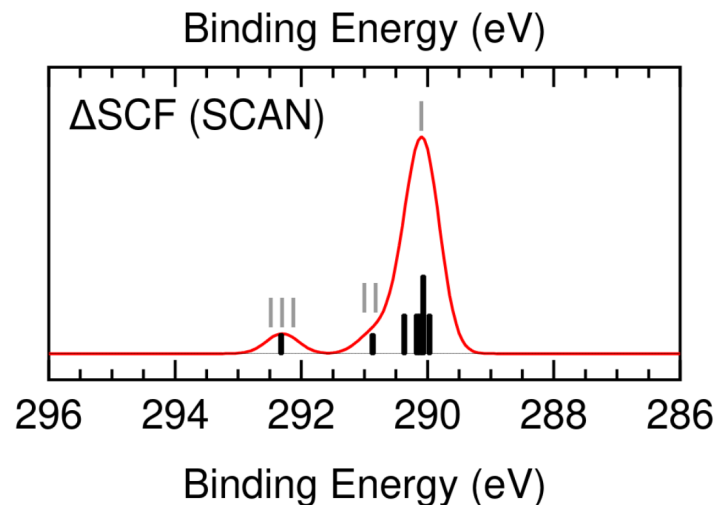
- Errors of > 1 eV reported for calculated C 1s and O 1s binding energies in anthrone.
 - *J. Chem. Theory Comput.* **14**, 4856 (2018)
- “For medium-sized to large molecules, the accuracy of Δ SCF can quickly reduce by an order of magnitude”
 - *J. Phys. Chem. Lett.* **11**, 1840 (2020)

Molecules – some interesting results

Remeasured gas phase
XPS on anthrone at
FinEstBeAMS



Core level	Theory	Experiment	Error
C 1s (1-8, 11-14)	290.11	290.3	-0.19
C 1s (10)	290.85	290.9	-0.05
C 1s (9)	292.28	292.7	-0.42
O 1s	536.25	536.6	-0.35



Summary - molecules

- Calculate core electron binding energies at the cost of two ground state DFT calculations
- Binding energy shifts: relatively easy
- Absolute binding energies: possible
 - 1s core level in 2nd row elements, 2p_{3/2} in 3rd row elements
 - Heavier elements – so far only if an element- and method-specific empirical correction is known
- Localization of the core hole onto the correct atom is critical
- Very little work thus far on “strongly correlated” systems