Quantum dynamics of perfect fluids

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Motivation

- Idealized classical system → quantum description (L.H.O.)
- Can we quantize the perfect fluid effective field theory in a way that manifestly preserves its classical symmetries?
    - [Landau, '50s], . . ., [Endlich, Nicolis, Rattazzi, Wang, '10.], . . . [Dersy, Khmelnitsky, Rattazzi, '22], [Cuomo, Eustachon, Firat, Henning, Rattazzi, '24]
- Vacuum is ill-defined, but theory can be quantized around physically preparable semiclassical states.
- Phenomenological motivation:
- Can *ordinary* fluids exist at $T = 0K$?

Lagrange space formulation of the perfect fluid EFT

- Classic hydrodynamics as the field theory, with comoving coordinate of the fluid element:

$$\phi^I = \phi^I(t, \vec{x}), \quad I = 1, 2, 3.$$

- Non-linear sigma model: $\phi^I : \mathrm{spacetime}\,(t, \vec{x}) \to \mathcal{M}^{D-1}$
- Perfect fluid requires volume-preserving diffeomorphisms:

$$\phi^I \to \xi^I(\phi^J), \qquad \det \frac{\partial \xi^I}{\partial \phi^J} = 1.$$

[Carter, '73],[Soper, '75],. . . ,[Dubovski, Gregoire, Nicolis, Rattazzi, '05]

- The action is given by $S = \int d^4x\, F(B)$, $B = \det\,(\partial_\mu \phi^I)(\partial^\mu \phi^J)$, for arbitrary function F that determines the equation of state.

Classical dispersion relations

- The classical ground state of the fluid at rest (solution of the hydro e.o.m.)

$$\langle \phi^I \rangle = x^i$$

  spontaneously breaks all symmetries except the diagonal shifts and rotations.

- With $\phi^I = x^i + \pi^I$, one finds the action of Goldstone modes π^I

$$S = w_0 \int d^4x \left(\frac{1}{2}\dot{\vec{\pi}}^2 - \frac{1}{2}c_s^2(\nabla\vec{\pi})^2 + \mathcal{O}(\pi^3) \right),$$

  where $c_s^2 = \frac{2F''(1)+F'(1)}{F'(1)}$.

- Dispersion relations:
  - Longitudinal: $\vec{\pi} \propto \vec{p}: \quad \omega^2 - c_s^2 p^2 = 0$
  - Transverse: $\vec{\pi}\cdot\vec{p} = 0, \quad \vec{\pi} = \nabla \times (\vec{a}(\vec{x}) + \vec{b}(\vec{x})t): \quad \underline{\omega = 0}$

Quantization

- L-modes: No problem, standard multi-particle Fock state of asymptotic states:

$$\mathcal{H}_L = \{|0_L\rangle, |\vec{k}\rangle = a^+(\vec{k})|0_L\rangle, \ldots, |\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_n\rangle, \ldots\}$$

- T-modes: Flat dispersion relation $\omega = 0$ rules out stabile vacuum and normalizable stationary states.
  - No energy cost for production of T-modes.
  - Perturbative vacuum is not well-defined $\leftrightarrow$ observables are IR singular.

Quantization

- We study the correlators in normalizable state $|\psi_{\rm in}\rangle$ at $t = 0$.
- We compute the retarded stress tensor correlator

$$G_R^{ij,kl}(x_1, x_2) = -i\theta(x_1^0 - x_2^0)\langle\psi_{\rm in}|[T^{ij}(x_1), T^{kl}(x_2)]|\psi_{\rm in}\rangle$$

  where $x_1^0, x_2^0 > 0$.
- We find non-zero contribution from the vortex modes to the stress tensor response function!

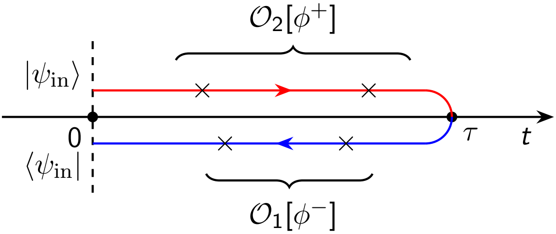
Schwinger-Keldysh contour

- We use Schwinger-Keldysh formalism to compute correlators in fixed initial state by the path-integral methods:

$$\langle\psi_{\rm in}|\bar{T}[\mathcal{O}_1[\phi]]\,T[\mathcal{O}_2[\phi]]|\psi_{\rm in}\rangle = \int^{\phi^I_+(\tau,\vec{x})=\phi^I_-(\tau,\vec{x})}\mathcal{D}\phi^I_-(x)\mathcal{D}\phi^I_+(x)\Psi^*_{\rm in}[\phi_-]\Psi_{\rm in}[\phi_+]e^{iS[\phi_+]-iS[\phi_-]}\mathcal{O}_1[\phi_-]\mathcal{O}_2[\phi_+]$$

- Scheamtically:

$$\langle\psi_{\rm in}|\bar{T}[\mathcal{O}_1[\phi]]\,T[\mathcal{O}_2[\phi]]|\psi_{\rm in}\rangle =$$



Initial state preparation

- We use a minimum uncertainty (Gaussian) wavepacket at $t = 0$ centred at "classical" field configuration of fluid at rest

$$\langle \phi^I(\vec{x}) \rangle = x^i,$$

$$\Psi_{\rm in}[\phi^I] = \exp\left[-\tfrac{1}{2} \int d^3\vec{x} d^3\vec{y} \pi^I(\vec{x}) K_{IJ}(\vec{x}, \vec{y}) \pi^J(\vec{y}) + i \int d^3\vec{x} v_I(\vec{x}) \pi^I(x) \right],$$

  where $\pi^I = \phi^I - \langle \phi^I \rangle$ and $\Psi_{\rm in}[\phi^I] \equiv \langle \phi^I | \psi_{\rm in} \rangle$.

- This state breaks SDiff invariance.
- Functions $K_{IJ}(\vec{x}, \vec{y})$ and $v_I(\vec{x})$ are in principle arbitrary, but can be determined "experimentally" from the fluid properties at $t = 0$.

Initial state preparation

- Correlators in the initial state with $\langle \phi^I(\vec{x}) \rangle = x^i$ are given by

$$\begin{aligned} \langle v_i(\vec{x}) \rangle_{\rm in} &= \frac{1}{w_0} v_i(\vec{x}), \\ \langle \delta v_i(\vec{x}) \delta v_j(\vec{y}) \rangle_{\rm in} &= \frac{1}{2w_0^2} K_{ij}(\vec{x}, \vec{y}), \\ \langle \delta\rho(\vec{x}) \delta\rho(\vec{y}) \rangle_{\rm in} &= \frac{1}{8} w_0^2 \partial_i^{\vec{x}} \partial_j^{\vec{y}} \left[K^{-1} \right]_{ij} (\vec{x}, \vec{y}). \end{aligned}$$

- Correlators at $t = 0$ fix the functions v_I and K_{IJ} that are used to study the evolution of the initial state.
- Correlators $\langle \psi_{\rm in} | \mathcal{O}_1(t_1) \ldots \mathcal{O}_n(t_n) | \psi_{\rm in} \rangle$, $t_i > 0$ are perturbatively calculable.
- For simplicity, we consider $v_I(\vec{x}) = 0$ initial states.

K-parametrization

- K-function ($D = 3+1$) can be parametrized in momentum space as

$$K_{ij}(\vec{p}) = \frac{p_i p_j}{p^2} K_L(p) + \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) K_T(p) + i\varepsilon_{ijk} \frac{p_k}{p} K_A(p),$$

  where K_L is the longitudinal width, K_T the transverse width, and K_A the parity-breaking term.

- We use

$$\begin{aligned} K_L(p) &= w_0 c_s |\vec{p}|, \\ K_T(p) &= w_0 \hat{c}_T \mu \left(\frac{|\vec{p}|}{\mu}\right)^{\Delta}, \\ K_A(p) &= 0. \end{aligned}$$

- μ is the energy scale of the initial state and $\hat{c}_T$ is "transverse speed".
- For $\Delta > -3$ the correlators in the initial state decay at large separation.

Tree-level propagators

- Tree-level Wightman propagators written in $(t, \vec{p})$ space are given by

$$\langle\psi_{\rm in}|\pi_I(t_2,\vec{p})\,\pi_J(t_1,-\vec{p})|\psi_{\rm in}\rangle = \frac{1}{2w_0 c_s|\vec{p}|}e^{-ic_s|\vec{p}|(t_2-t_1)}P^L_{IJ}(\vec{p}) + \frac{(i+\hat{c}_T\mu^{1-\Delta}|\vec{p}|^\Delta t_2)(-i+\hat{c}_T\mu^{1-\Delta}|\vec{p}|^\Delta t_1)}{2w_0\hat{c}_T\mu^{1-\Delta}|\vec{p}|^\Delta}P^T_{IJ}(\vec{p}),$$

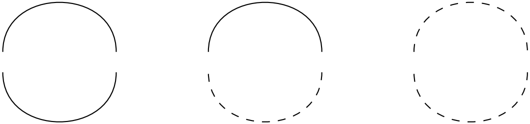
  where $\hat{P}^L$ and $\hat{P}^T$ are longitudinal and transverse projectors.

- Transverse propagators have a linear time dependence (independent of K-functions).

- Time-translation is broken by the initial state at $t = 0$.

Correlators

- We study the correlators of the diffeomorphism-invariant observables.
- Use
$$T_{xy} = w_0 \left(\dot{\pi}_x \dot{\pi}_y + \mathcal{O}\left(\pi^3\right)\right)$$
  as the simple example.
- Leading contribution is at 1-loop.
- Three diagrams contribute LL, LT, TT:



- LL contribution agrees with superfluid calculation, TT contribution vanishes in the retarded correlator.

T_{xy}-correlator: LT contribution

- In LT case master integrals for Wightman correlators are of the form

$$I_{\alpha,\beta}(z,\vec{p}) = \int \frac{d^d\vec{k}}{(2\pi)^d}\frac{e^{-z|\vec{k}|}}{(\vec{k}^2)^{\alpha}(\vec{p}-\vec{k})^{2\beta}}.$$

- Using Mellin-Barnes parametrization

$$e^{-z|\vec{k}|} = \int_{a-i\infty}^{a+i\infty}\frac{ds}{(2\pi i)}(z|\vec{k}|)^{-s}\Gamma(s),$$

one finds

$$I_{\alpha,\beta}(z,\vec{p}) = f_{\rm soft}(z^2\vec{p}^2) + f_{\rm hard}(z,|\vec{p}|),$$

where f's can be found explicitly in terms of ${}_2F_3$.

- $f_{\rm soft} \sim$ poles of $\Gamma(s)$, low loop momenta region in small-c_s expansion.
- $f_{\rm hard} \sim$ poles of other Γ's, large loop momenta region in small-c_s expansion.

T_{xy}-correlator: LT contribution

- For LT contribution, we compute the response function

$$
\begin{aligned}
&-i\theta(t)\,\langle\psi_{\rm in}\,|[T_{xy}(t,\vec{p}),\ T_{xy}(0,-\vec{p})]|\,\psi_{\rm in}\rangle_{LT} = \\
&\theta(t)\frac{\sin\left(\frac{\pi\Delta}{2}\right)\Gamma(\Delta+4)\hat{c}_T\mu^{1-\Delta}}{2\pi^2 c_s^{8+\Delta}\,t^{9+\Delta}\,|p_z|^5\,\Delta(\Delta+2)} \\
&\times\Big(tc_s|p_z|(\Delta+5)\cos(tc_s|p_z|)\left(t^2c_s^2p_z^2(\Delta+4)-6(\Delta+7)\right) \\
&\quad+\sin(tc_s|p_z|)(t^4c_s^4p_z^4(\Delta+2)-3t^2c_s^2p_z^2(\Delta+5)(\Delta+6) \\
&\quad+6(\Delta+5)(\Delta+7))\Big).
\end{aligned}
$$

- Transverse modes do not decouple from physical observables.
- One can compute correlators of any observables at higher loops now!

Looking ahead...

- We are extending the results to the time-dependent backgrounds.
- Relativistic radial Milne flow

  $$\tau = \sqrt{t^2 - r^2}, \qquad \tanh \chi = \frac{r}{t},$$

  fields π^I live at the surface of constant τ.
- Diffeomorphic to this expanding cosmology with hyperbolic slices.
- We use the tools of (Euclidean) AdS/CFT tools to compute interesting observables.

Thank you!