Quantum chaos with and without OTOCs

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02/07/2026

Contours 2026 @ Cambridge

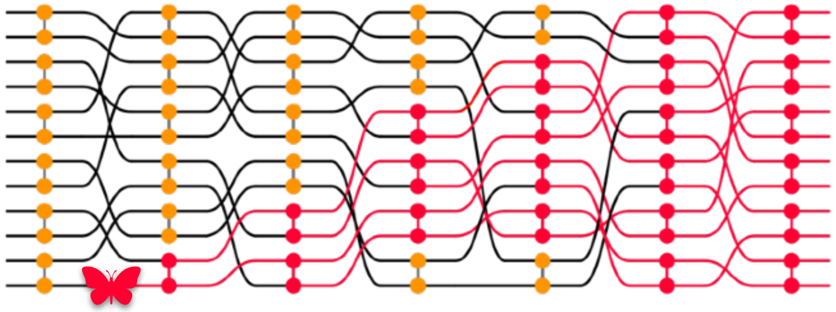
Part 1

Motivation and some recent developments

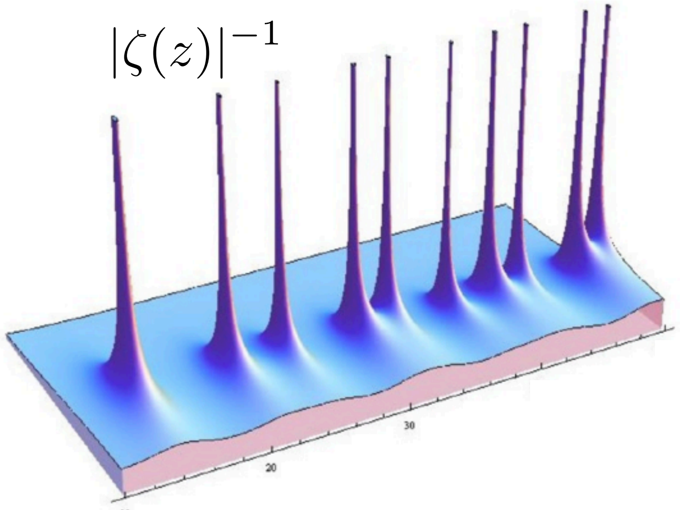
Thermalization & chaos

- Consider time-dependent, complex systems, esp. at finite temperature
- Different observables and timescales require different **contours**
- Via holography: relations to **black holes & quantum gravity**

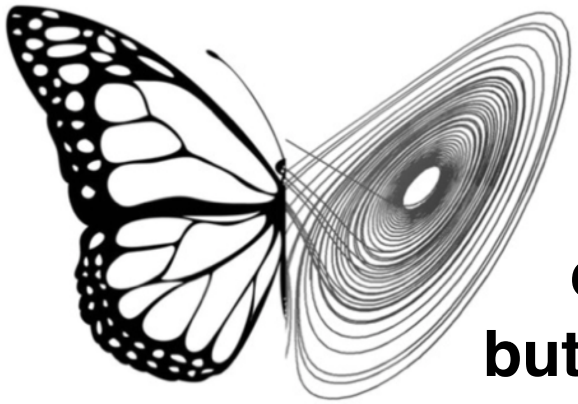
transport & dissipation



operator growth



random matrix universality



quantum butterfly effect

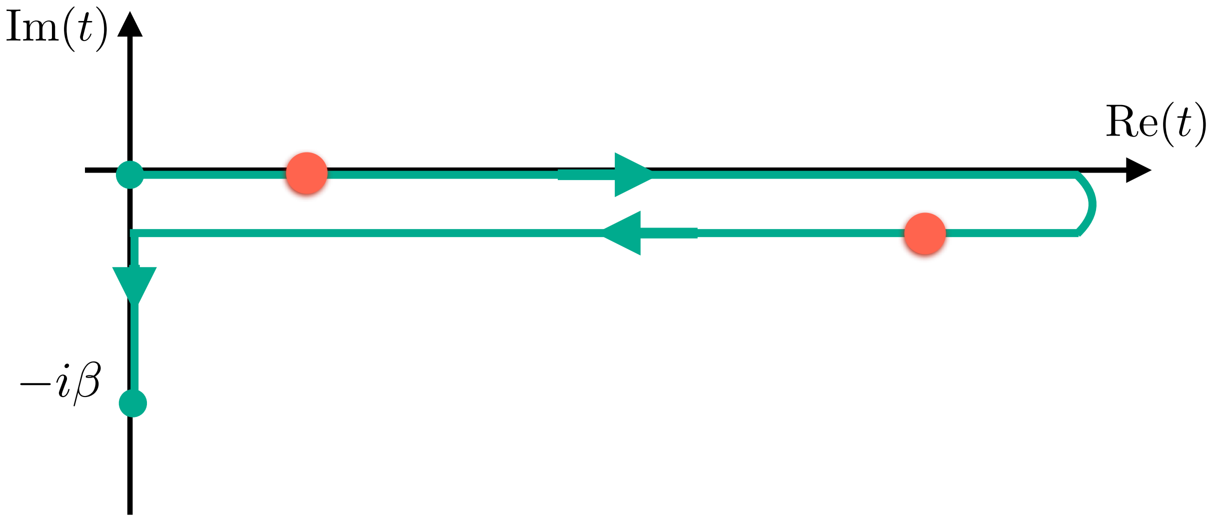
Effective field theory for quantum chaos

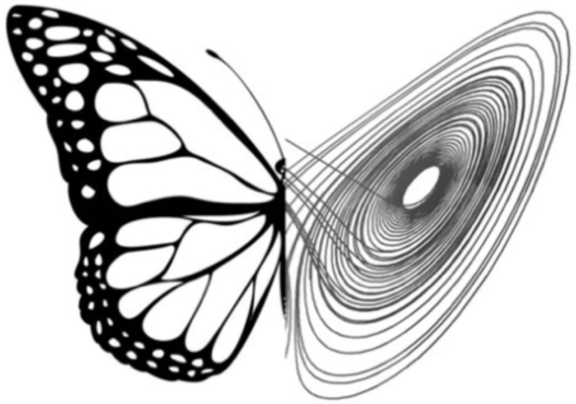
Transport & Dissipation

Hydrodynamics: paradigm for universal EFT, slow relaxation of conserved quantities, non-equilibrium path integrals

- Thermal Schwinger-Keldysh EFT recently understood systematically

  [Nickel/Son] [Dubovsky/Hui/Nicolis/Son] [Kovtun/Moore/Romatschke] …
  [**FMH**/Loganayagam/Rangamani] [Crossley/Glorioso/Liu] … [Jensen/Pinzani-Fokeeva/Yarom] …

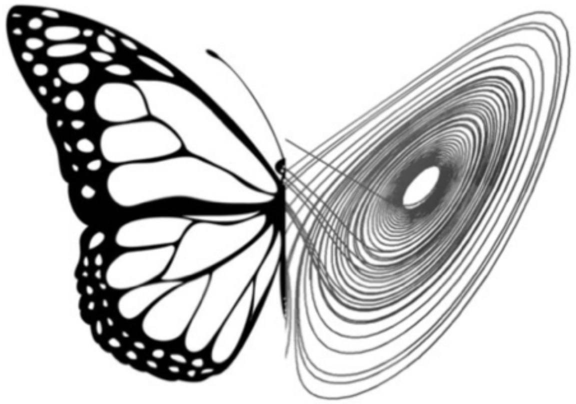




Quantum Butterfly Effect

- $X(t) = e^{iHt} X\, e^{-iHt}$ is ‘complicated’ even if X was ‘simple’:

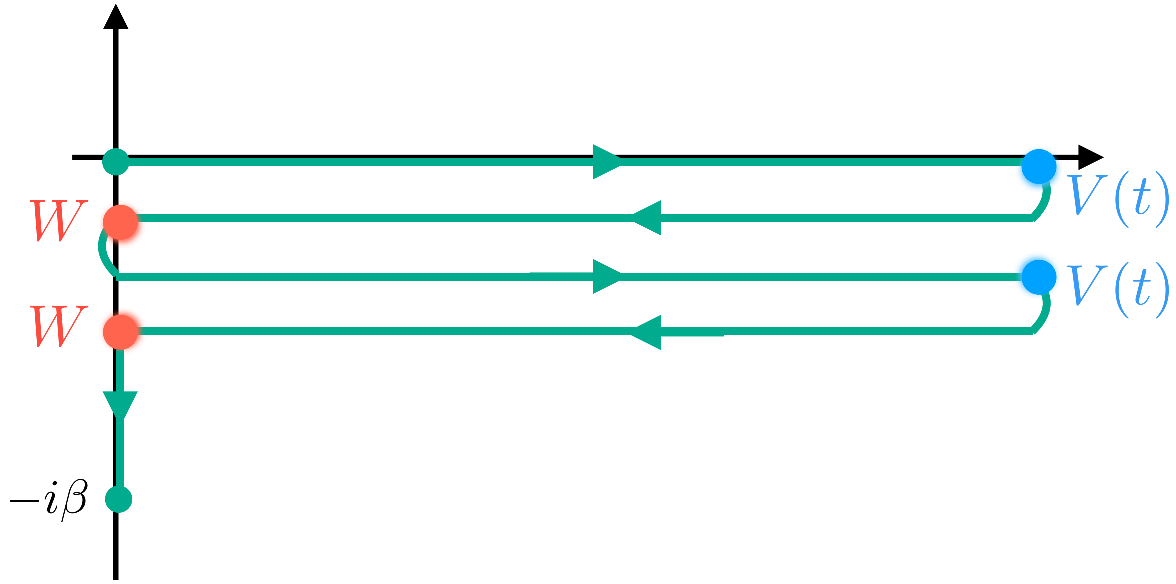
$$X(t) = e^{iHt} X e^{-iHt} = X + it[H,X] - \frac{t^2}{2}[H,[H,X]] + \ldots$$



Quantum Butterfly Effect

- OTOC = overlap of $|V(t)W(0)\rangle$ and $|W(0)V(t)\rangle$:

$$\langle W(0)V(t)|W(0)V(t)\rangle_\beta \sim \langle VV\rangle_\beta\langle WW\rangle_\beta\left(1-\frac{c_0}{N}\,e^{\kappa t}+\dots\right)$$



quantum Lyapunov exponent κ

[Larkin/Ovchinnikov] [Kitaev] [Shenker/Stanford] ...

- Chaos bound: $\kappa \leq 2\pi T$

  [Maldacena/Shenker/Stanford]

- Holography: $\kappa^{(\text{black hole})} = 2\pi T$

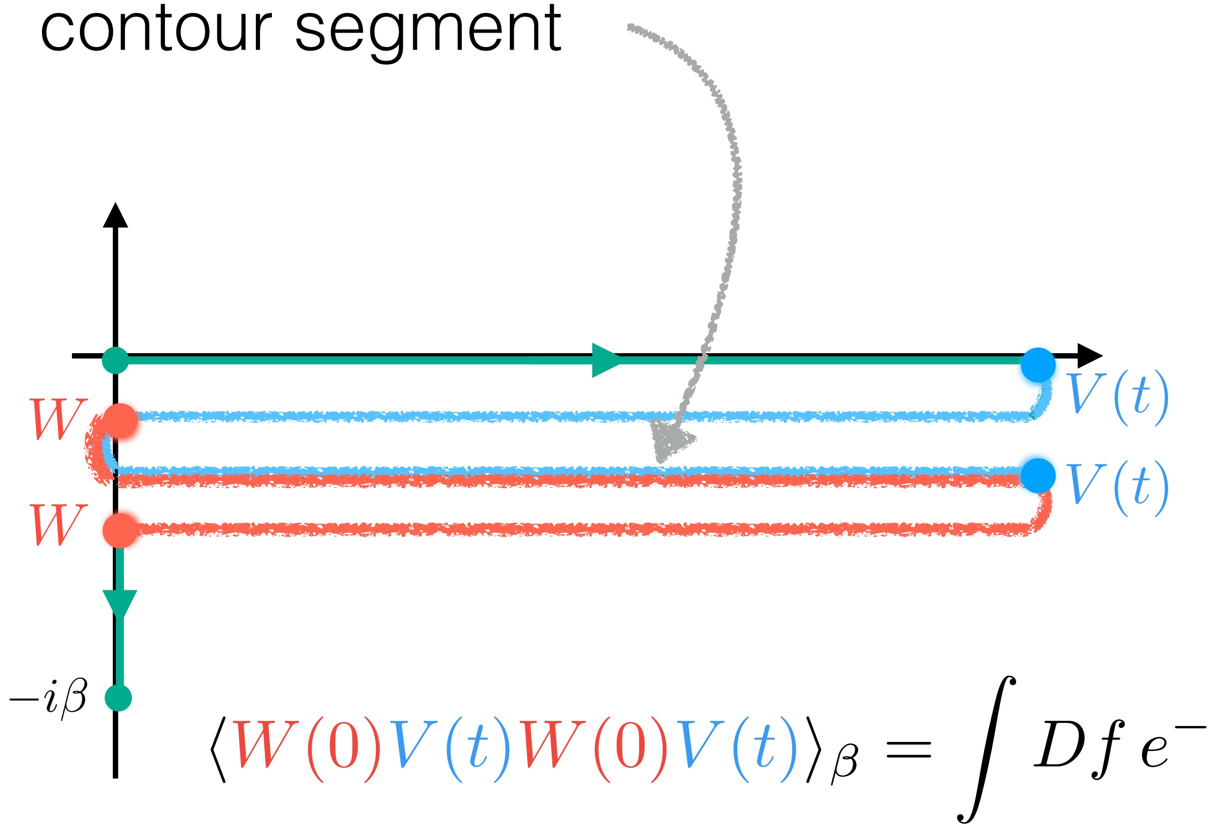
  [Sekino/Susskind] [Shenker/Stanford] ...

- **Hydro-like chaos EFT on four-fold contour:** effective chaos mode (“scramblon”)
  [Maldacena/Stanford/Yang] [Jensen] … [Blake/Lee/Liu] [**FMH**/Rozali] …

- Can sometimes derive explicitly from microscopics, e.g. in SYK-type models
  [Stanford/Yang/Yao] [Gu/Kitaev/Zhang] [Gao/Liu] [Choi/**FMH**/Mezei/Sarosi] …

- **Effective d.o.f.:** soft modes of the mean-field action that interact on shared contour segment

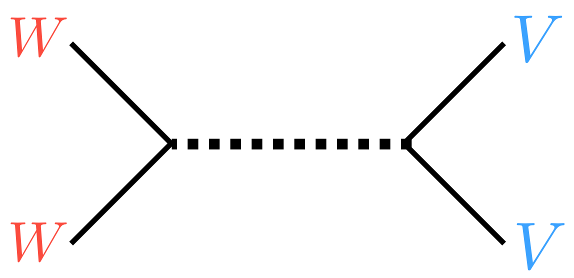
  - Large-N saddle point breaks down when $t \sim \log(N)$
  - Exact path integral over soft modes leads to **eikonal effective action**



$$\langle W(0)V(t)W(0)V(t)\rangle_\beta = \int Df\, e^{-S_{\text{eff}}[f]}\, \langle VV\rangle_\beta^{(f)}\, \langle WW\rangle_\beta^{(f)}$$

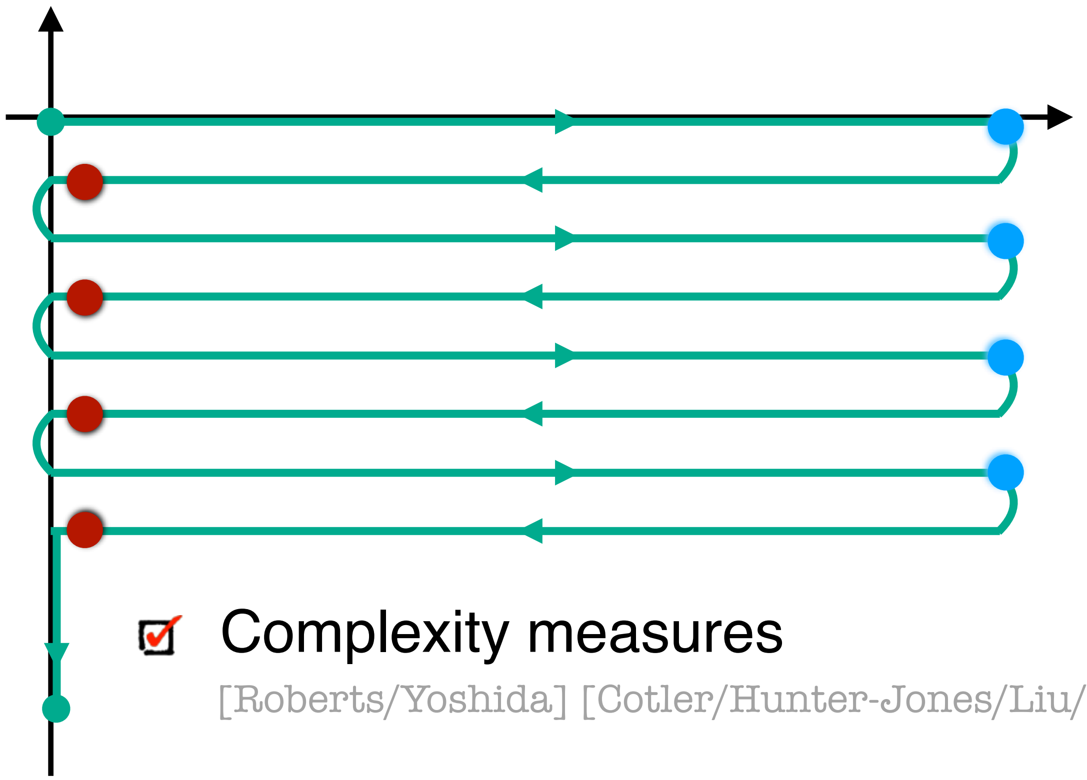
$$\Big\downarrow \; f(t,x) \sim f_0(t,x) + X^+(x)\, e^{-\kappa t} + X^-(x)\, e^{\kappa t} + \ldots$$

$$S_{\text{eik}} \sim N \int dk\, e^{\kappa(\frac{i\pi}{2}-t)} (\cdots)\, X^+(k) X^-(-k)$$



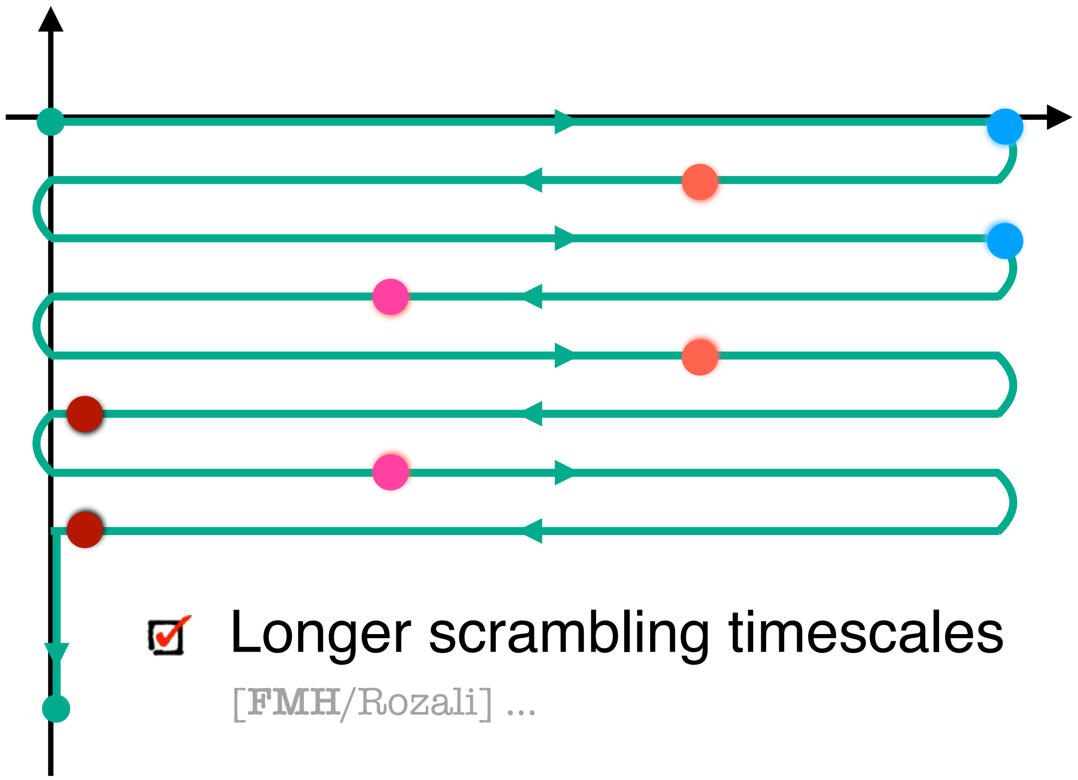
Chaos beyond the standard OTOC

- One way to generalise $\langle WVWV \rangle$: higher-order OTOCs



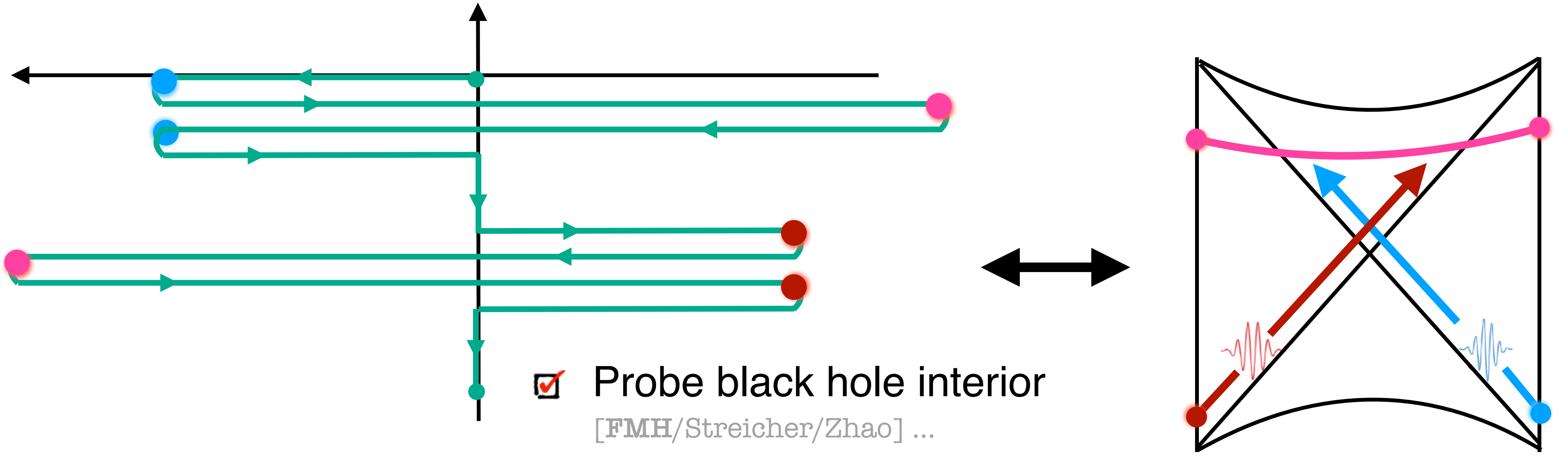
☑ Complexity measures

[Roberts/Yoshida] [Cotler/Hunter-Jones/Liu/Yoshida] ...



☑ Longer scrambling timescales

[**FMH**/Rozali] ...



☑ Probe black hole interior

[**FMH**/Streicher/Zhao] ...

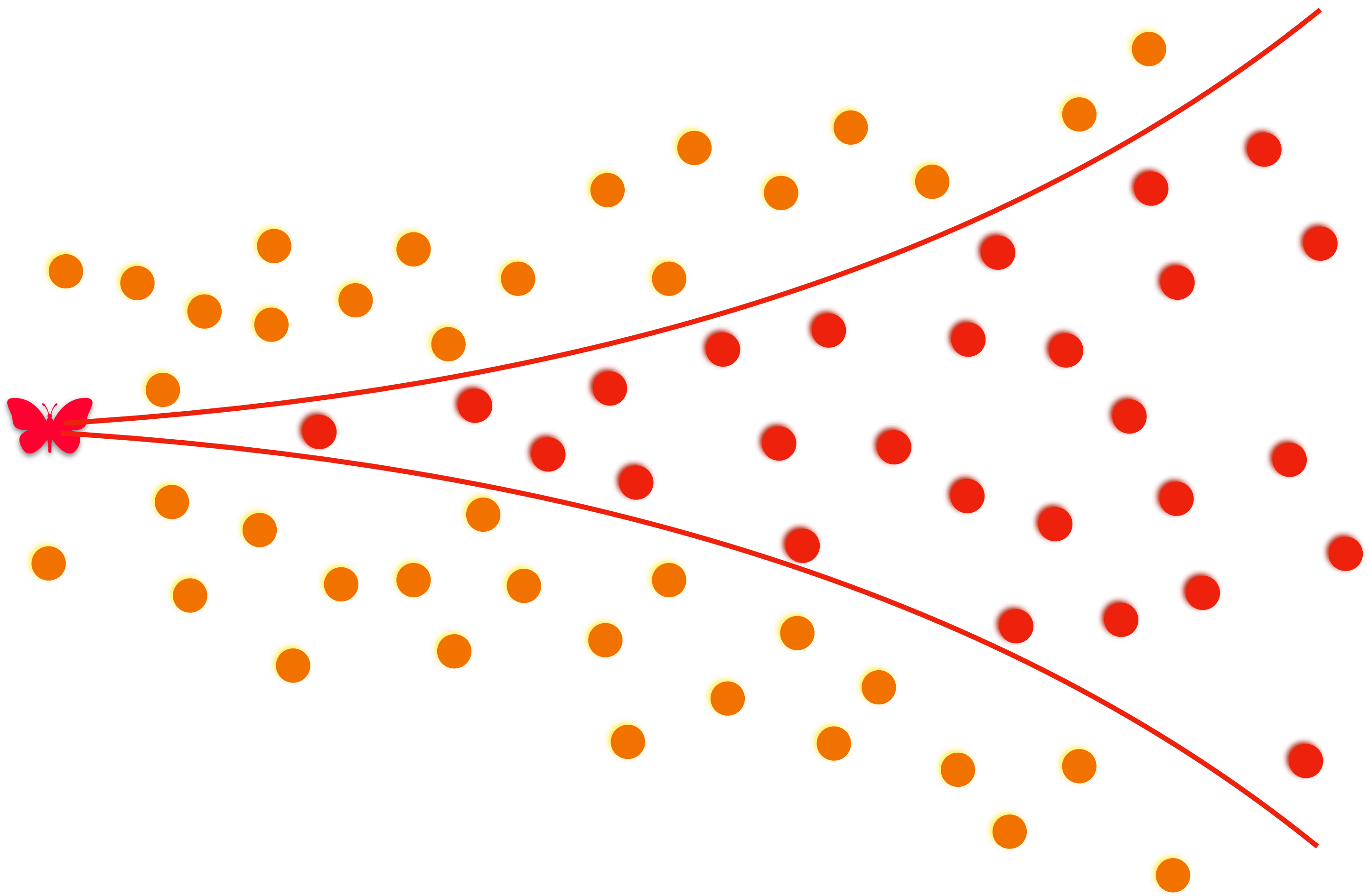
☑ Higher order fluctuation-dissipation theorems

[Tsuji/Shitara/Ueda] [**FMH**/Loganayagam/Narayan/Nizami/Rangamani] ...

- Can also discover **quantum chaos signatures** in **lower-order** (Schwinger-Keldysh) correlators and related observables. E.g.:
  - **Pole skipping:** signatures of quantum Lyapunov physics in $\langle TT \rangle_{\rm ret}$

    [Grozdanov/Schalm/Scopelliti] [Blake/Davison/Grozdanov/Liu] ... [**FMH**/Rozali] ...
  - **Operator growth:** spread of support over a given operator basis

$$X(t) = X + it[H,X] - \frac{t^2}{2}[H,[H,X]] + \ldots$$



[Shenker/Stanford] [Roberts/Stanford/Streicher] [Parker/Cao/Avdoshkin/Scaffidi/Altman] [Swingle et al.] [Brown et al.] ...
[Sanchez-Garrido/Rabinovici/Shir/Sonner] [Dymarsky/Smolkin] [Balasubramanian/Caputa/Magan/Wu] ...

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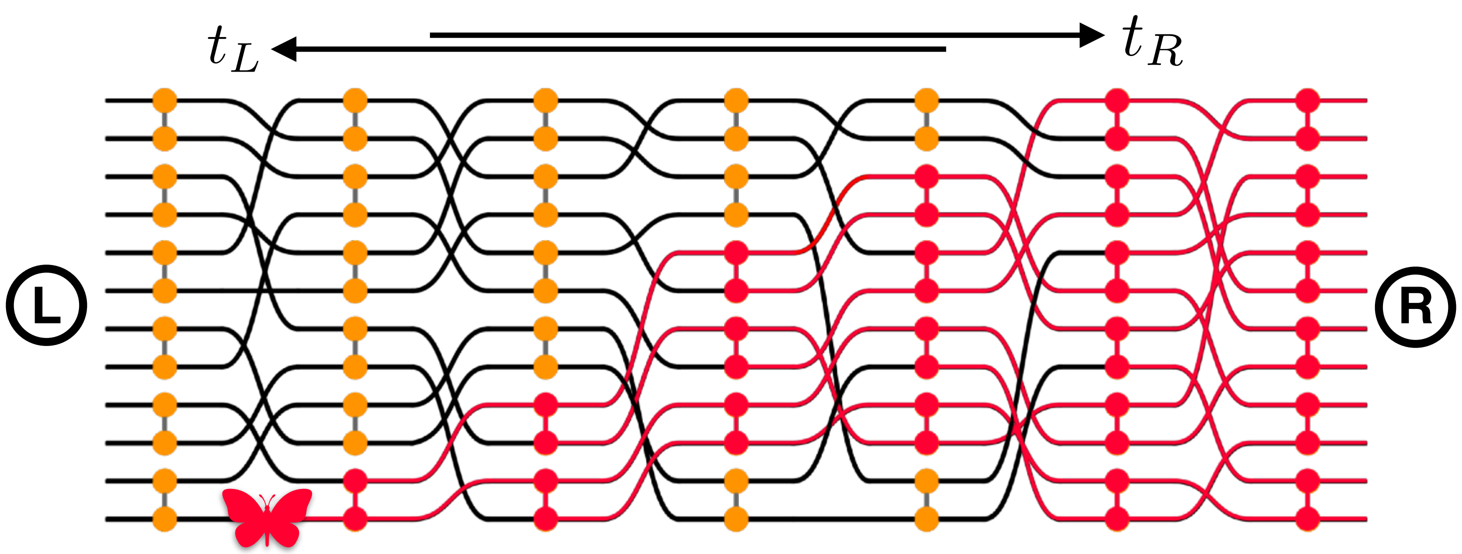
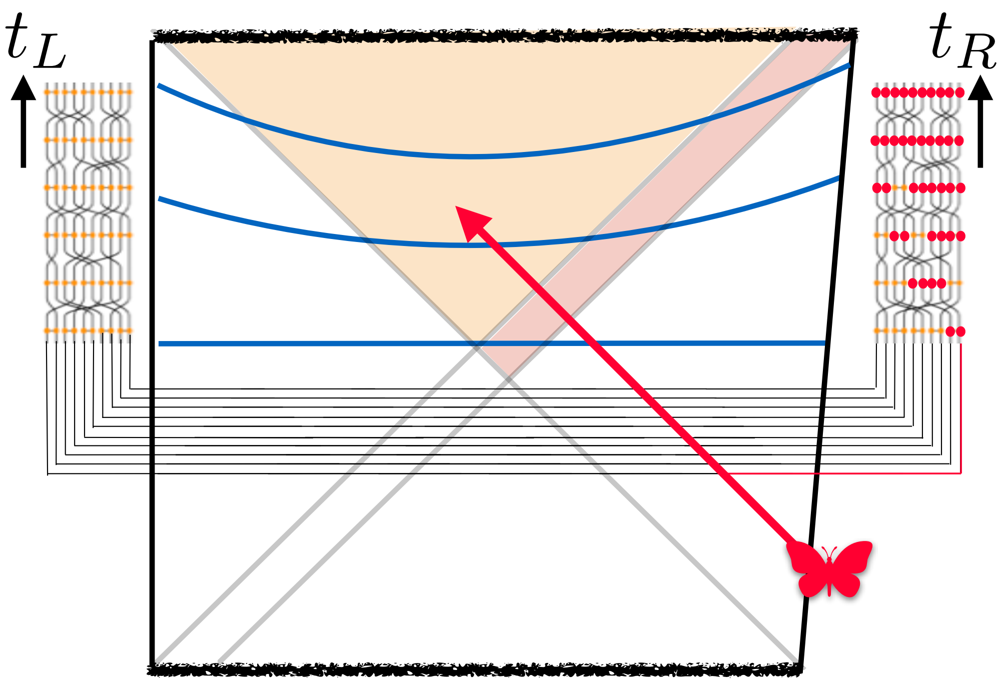
  - **Operator growth:** spread of support over a given operator basis

$$X(t) = X + it[H,X] - \frac{t^2}{2}[H,[H,X]] + \dots$$

    - Toy model: random unitary **quantum circuit**
      [Hayden/Preskill] [Susskind] ... [Zhao] [**FMH**/Zhao] ...





- Perturbation spreads exponentially via all-to-all interactions ("gates") until saturation

- Toy model for holographic black hole evolution: perturbation = shockwave

Part 2

Operator growth from the OPE

[Chowdhury/**FMH**/Sanchez-Garrido/Zhao, PRL 136 (2026) 12]

Motivation

- Another aspect of operator growth:

“How complicated has a simple perturbation become?”

“How do two growing perturbations interact over time?”

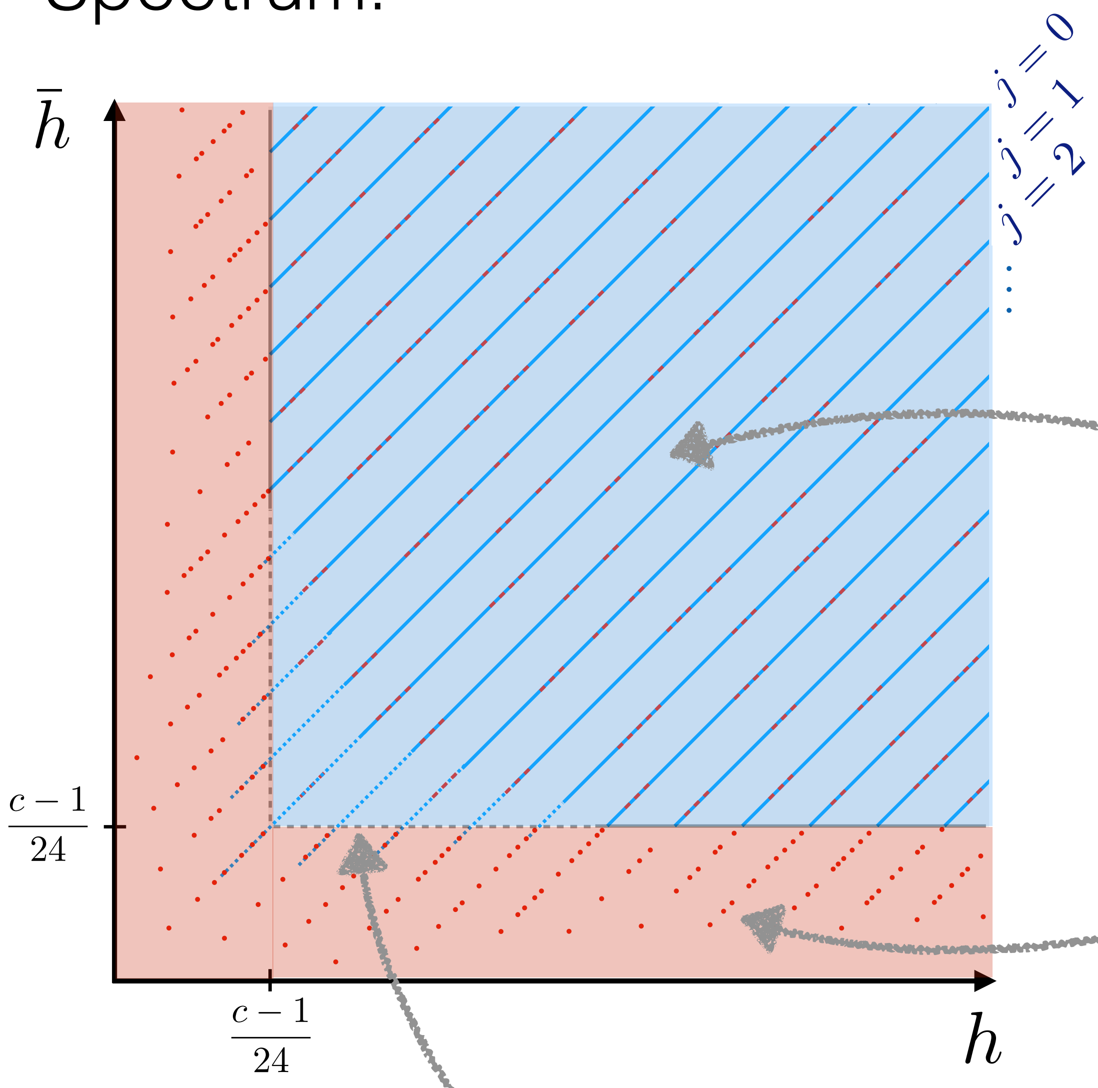
“What operator best describes their combined effect?”

Setup

- Consider chaotic, large c, **holographic CFT$_2$**
- Can make notions of operator growth very precise

[Dymarsky/Smolkin] … [Caputa/Magan/Patramanis] [Caputa/Datta] …

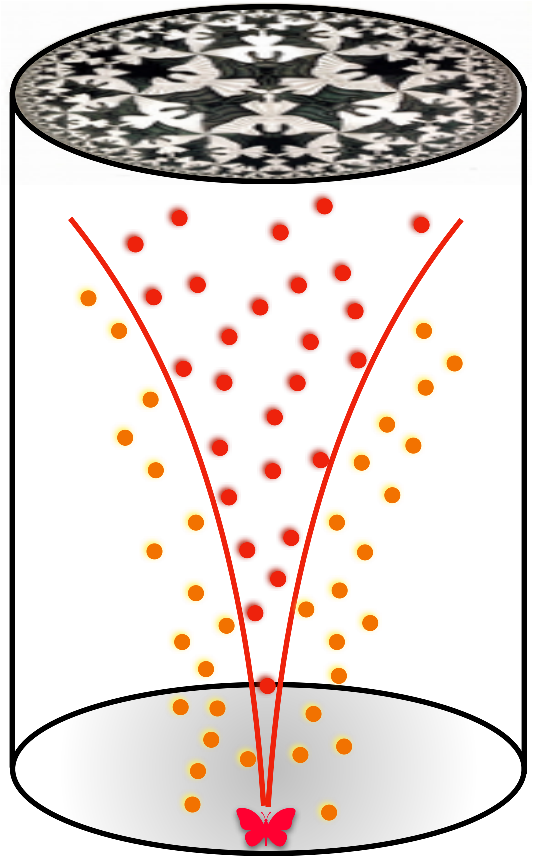
- Spectrum:



“Heavy” states:
—> Dense, chaotic
—> Dual to black holes

“Light” states:
—> Sparse
—> Dual to perturbative excitations, conical defect geometries, …

Black hole threshold

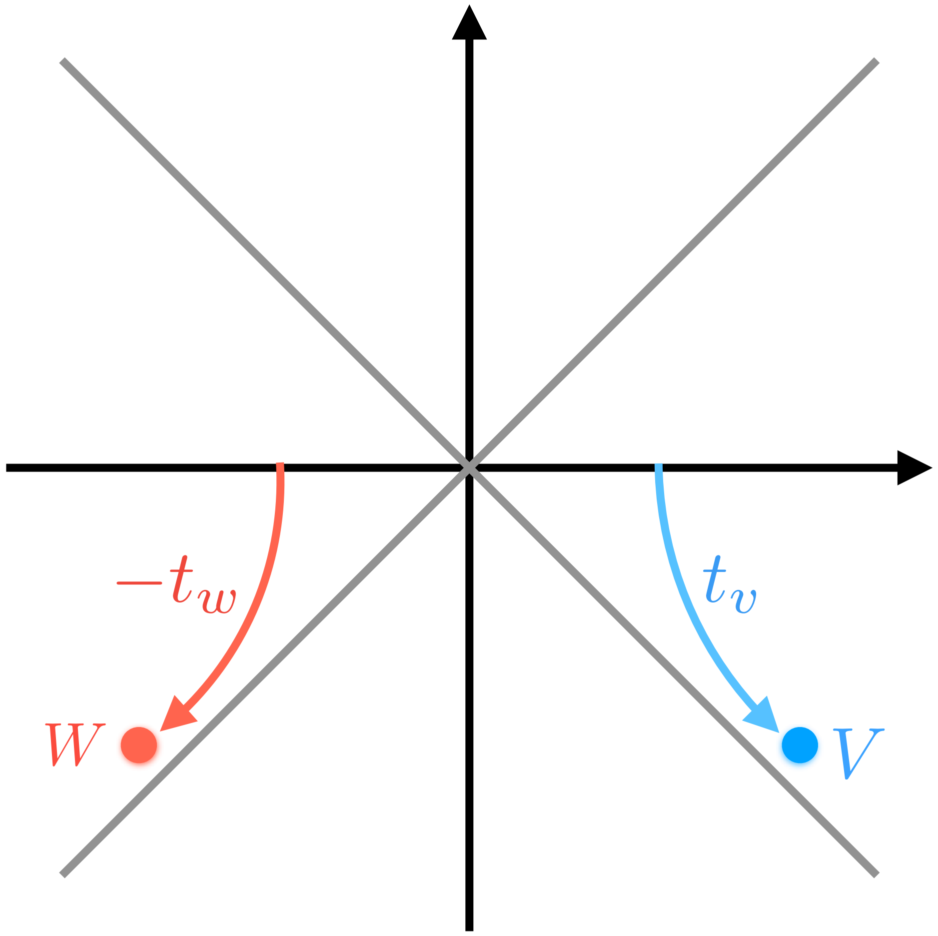


Setup

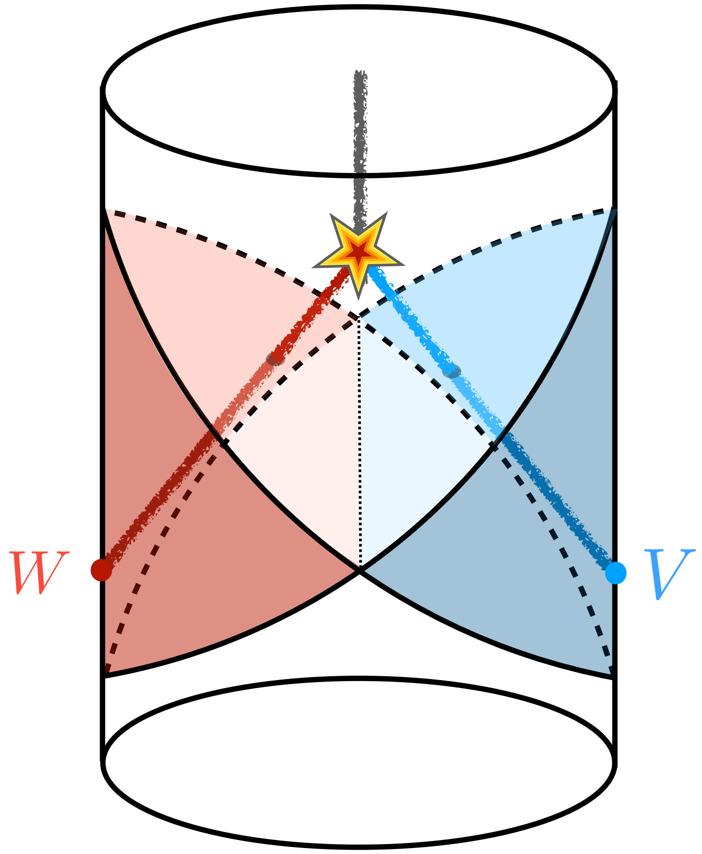
- Consider the two-operator state for light primary operators in CFT2:

$$|\Psi_{W_L V_R}\rangle = W(-t_w - i\pi, x_w)V(t_v, x_v)|0\rangle \qquad (t_v < 0,\, t_w < 0)$$

Rindler time evolution:



AdS3 shockwaves:



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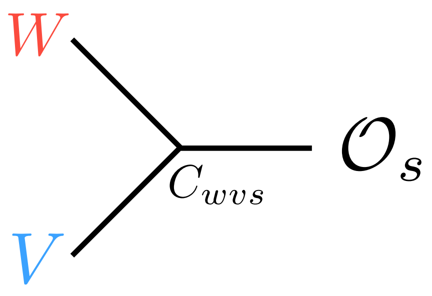
$$|\Psi_{W_L V_R}\rangle = W(-t_w - i\pi, x_w)V(t_v, x_v)|0\rangle \qquad (t_v < 0,\, t_w < 0)$$

- Decompose into irreps of the Virasoro algebra (**operator product expansion**):

$$|\Psi_{W_L V_R}\rangle \propto \sum_{\mathcal{O}_s} C_{wvs}\, |\mathcal{B}_{WV\mathcal{O}_s}(t_w, x_w; t_v, x_v)\rangle$$

OPE coefficients

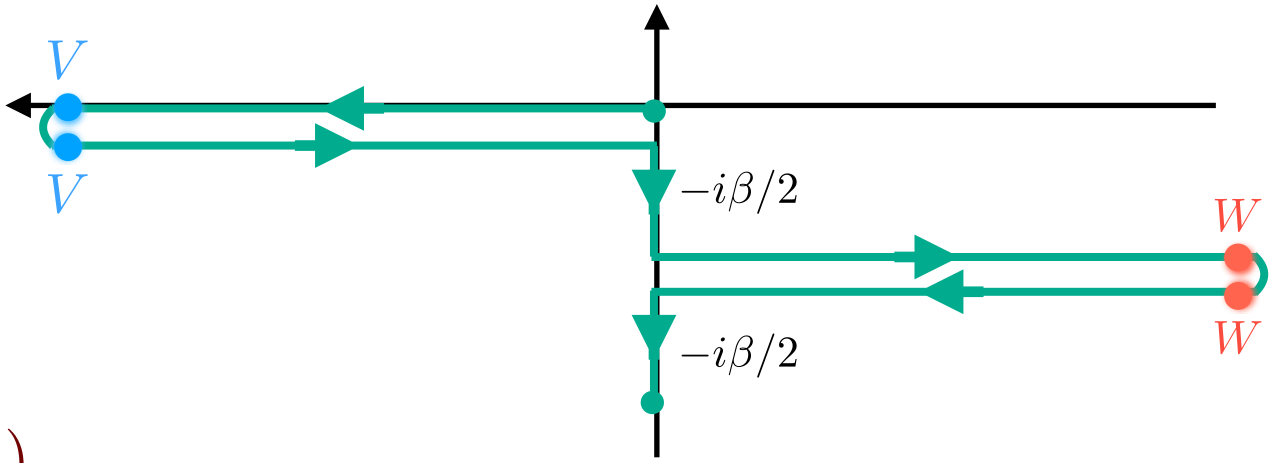
OPE blocks



- How does the “wavefunction” on the space of $\mathcal{B}_{WV\mathcal{O}_s}$ depend on $t \equiv -t_v - t_w$?

Self-overlap

- Compute self-overlap of $|\Psi_{W_L V_R}\rangle$:



$$\mathcal{F} = \frac{\langle \Psi_{W_L V_R} | \Psi_{W_L V_R} \rangle}{\langle V V \rangle \langle W W \rangle} = \frac{\mathrm{tr}\left(W^\dagger W \, \rho^{\frac{1}{2}} \, V^\dagger V \, \rho^{\frac{1}{2}} \right)}{\mathrm{tr}\,(V^\dagger V \, \rho)\, \mathrm{tr}\,(W^\dagger W \, \rho)}$$

—> not an OTOC!

—> time ordered

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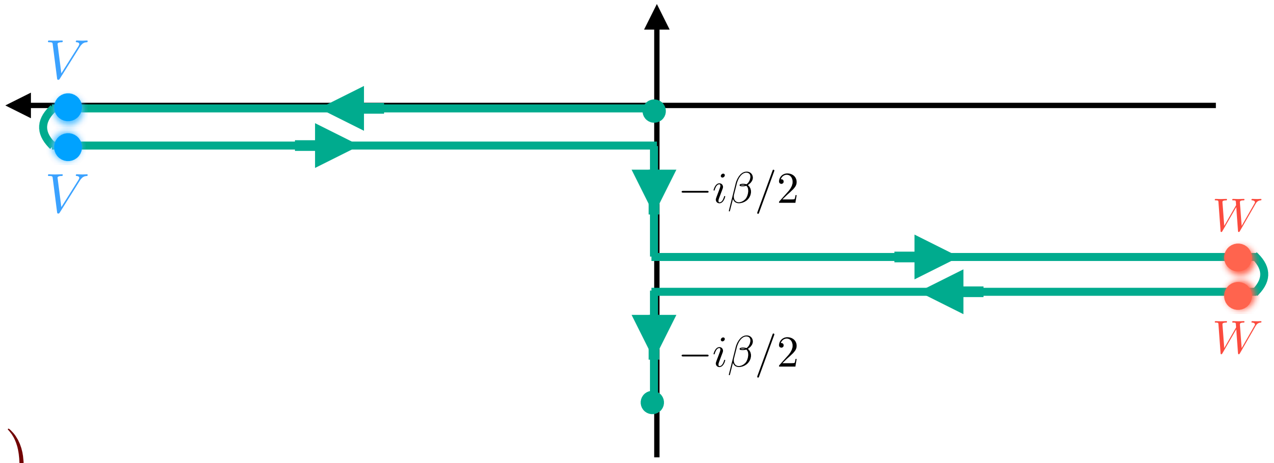

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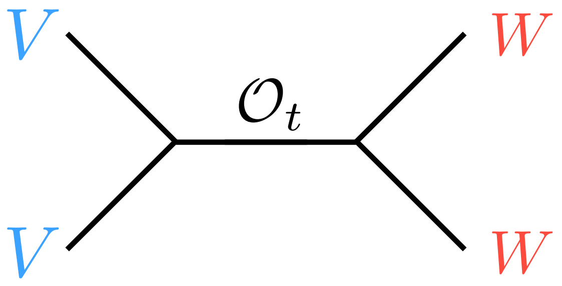
“t-channel”

“s-channel”

$$= \sum_{\mathcal{O}_t}$$



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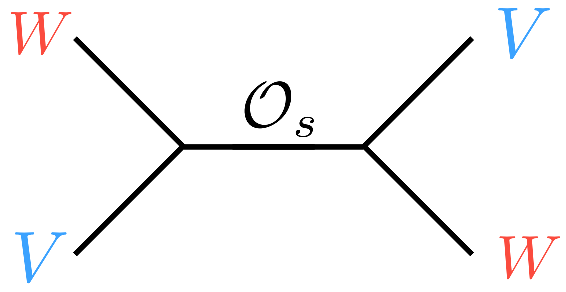

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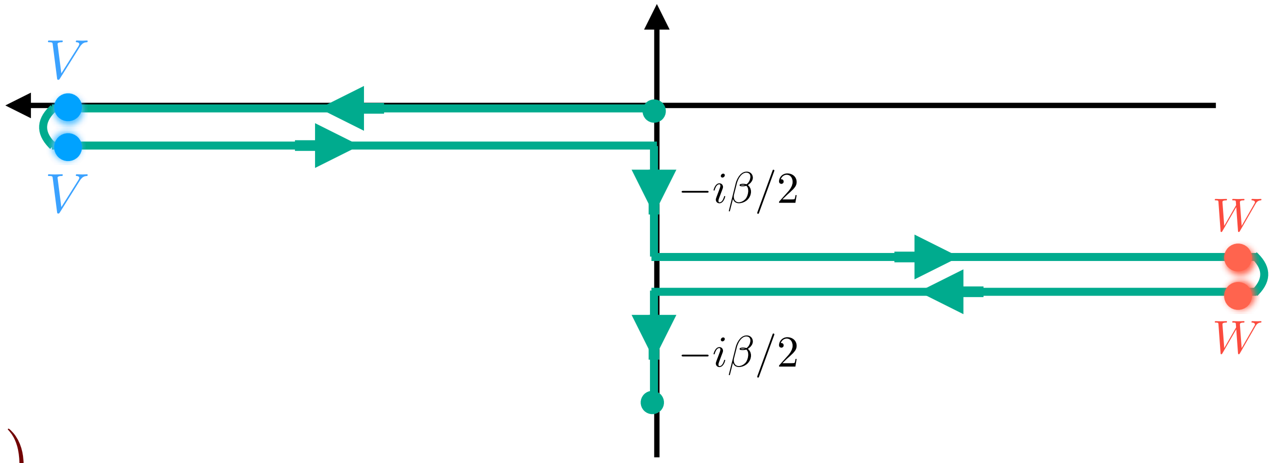

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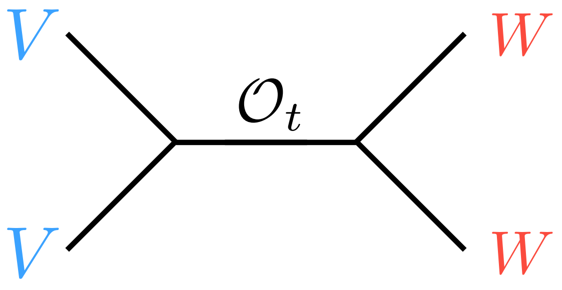

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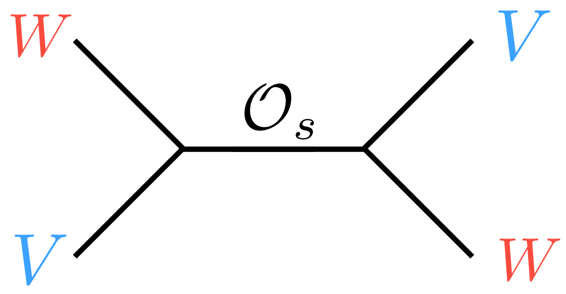

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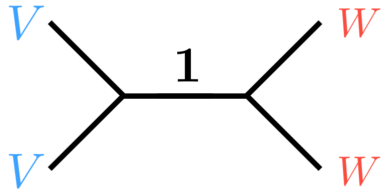
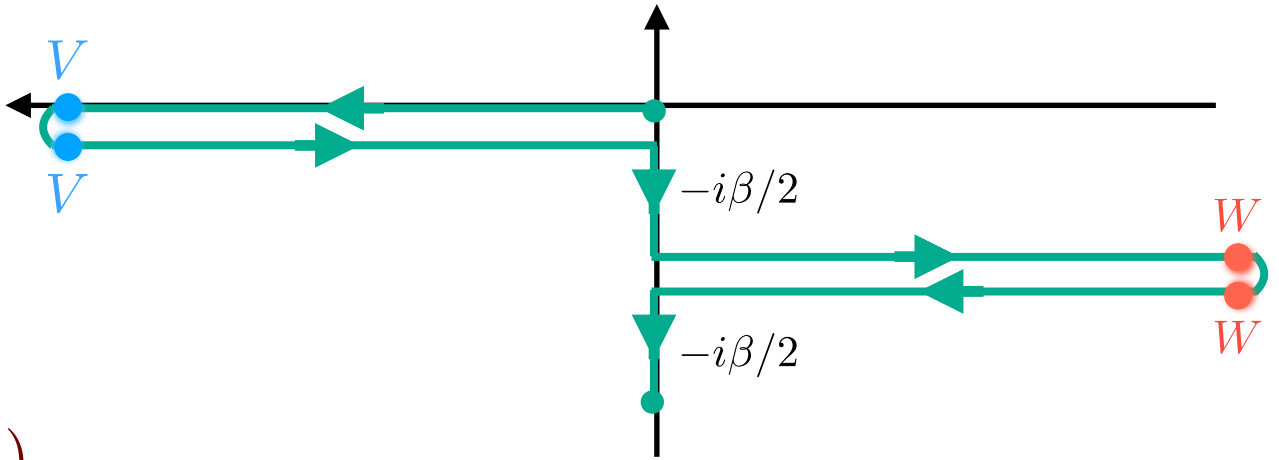


$+ \, \mathcal{O}\big(e^{-t\Delta_{\text{gap}}}\big)$

$\approx \quad 1$

Self-overlap

- Compute self-overlap of $|\Psi_{W_L V_R}\rangle$:



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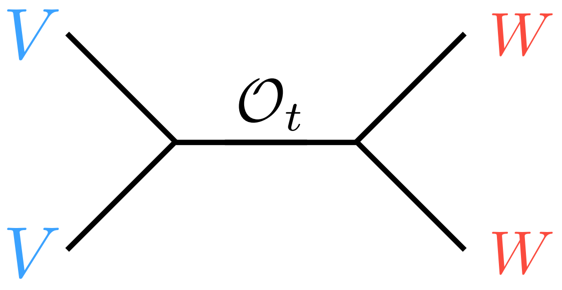
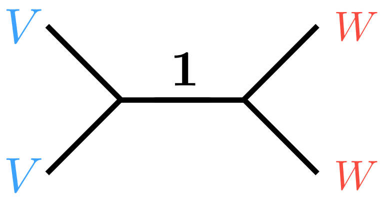


$$=$$



$$+\, \mathcal{O}\big(e^{-t\Delta_{\mathrm{gap}}}\big)$$

$$\approx\ 1$$

“s-channel”

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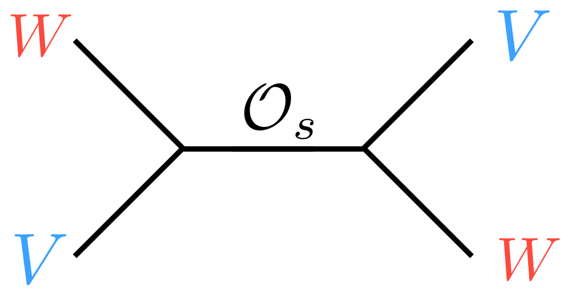

$$= \sum_{\mathcal{O}_s} C^2_{wvs}\ \underbrace{\langle \mathcal{B}_{WV\mathcal{O}_s}|\mathcal{B}_{WV\mathcal{O}_s}\rangle}_{\text{conformal blocks } \mathcal{V}_s(z,\bar{z})}$$

$$1 = \sum_{\mathcal{O}_s} C^2_{wvs} \underbrace{\langle \mathcal{B}_{WV\mathcal{O}_s} | \mathcal{B}_{WV\mathcal{O}_s} \rangle}_{\text{conformal blocks } \mathcal{V}_s(z,\bar{z})}$$

- How does the decomposition of 1 into s-channel conformal blocks change as a function of $t \equiv -t_v - t_w$?

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- How does the decomposition of 1 into s-channel conformal blocks change as a function of $t \equiv -t_v - t_w$?

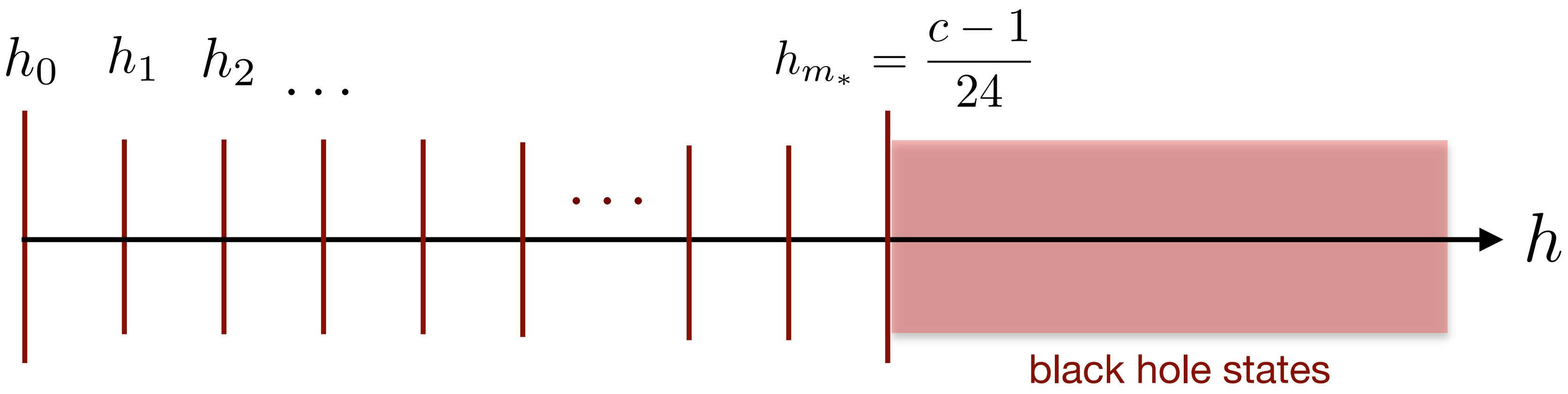
- Conformal bootstrap tells us the approximate spectrum of exchanges $\mathcal{O}_s$

  [Ponsot/Teschner] … [Caron-Huot] … [Collier/Gobeil/Maxfield/Perlmutter] [Das/Datta/Pal] …

  - Labelled by **s-channel conformal weights** $(h_s, \bar{h}_s)$ or $(E_s, J_s) = \left(\frac{h_s+\bar{h}_s}{2}, \bar{h}_s - h_s\right)$:

Discrete light "double-twist" operators $\mathcal{O}_s^{(m,\bar{m})}$: $h_m = h_v + h_w + m \qquad (m = 1, 2, \ldots, m_*)$

Continuum of heavy "mean-field" operators: $h_s > \frac{c-1}{24}$



- Higher boost t excites intermediate states $\mathcal{O}_s$ with higher energy E_s

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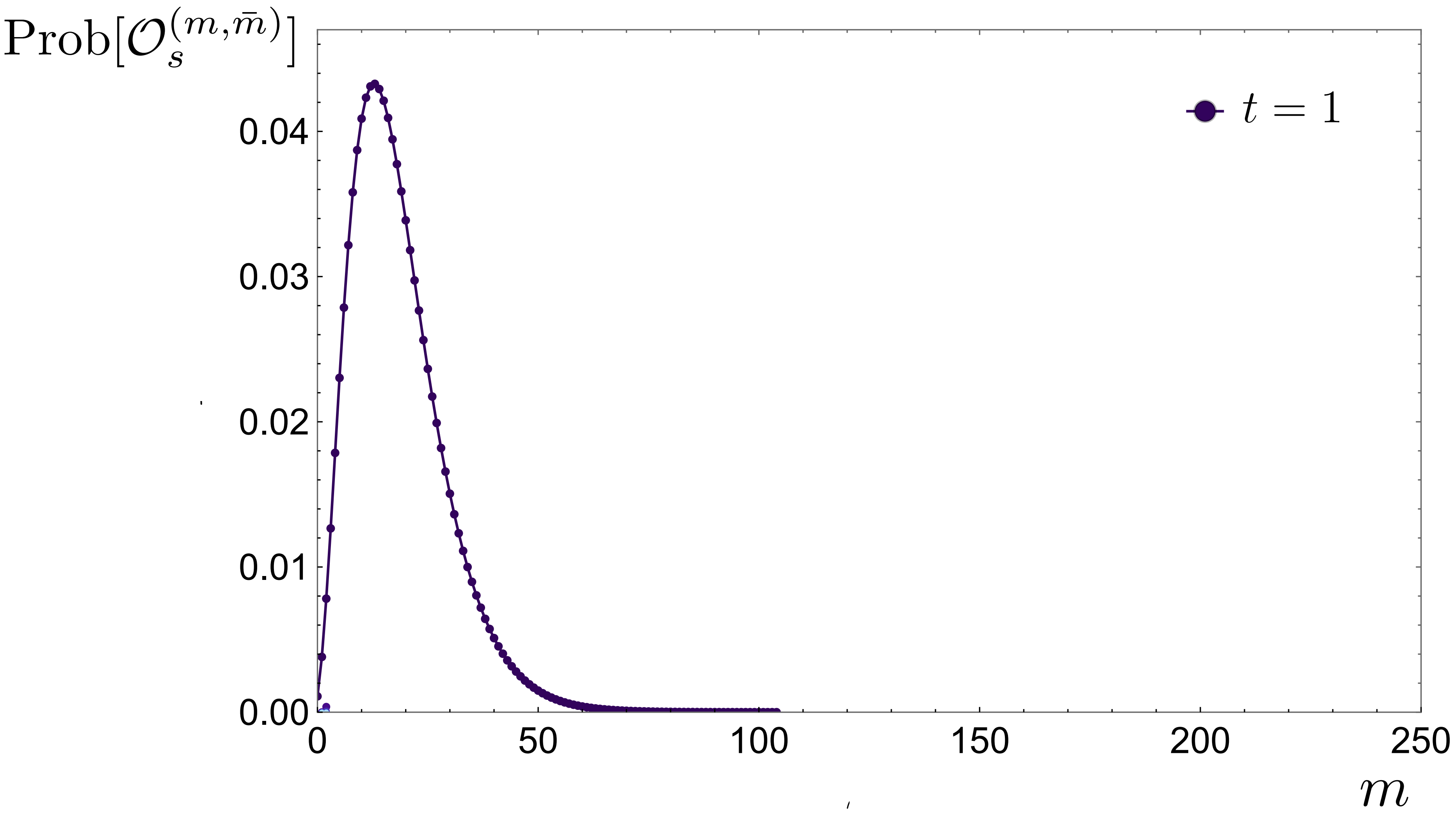

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- **How does the decomposition of 1 into s-channel conformal blocks change as a function of $t \equiv -t_v - t_w$?**
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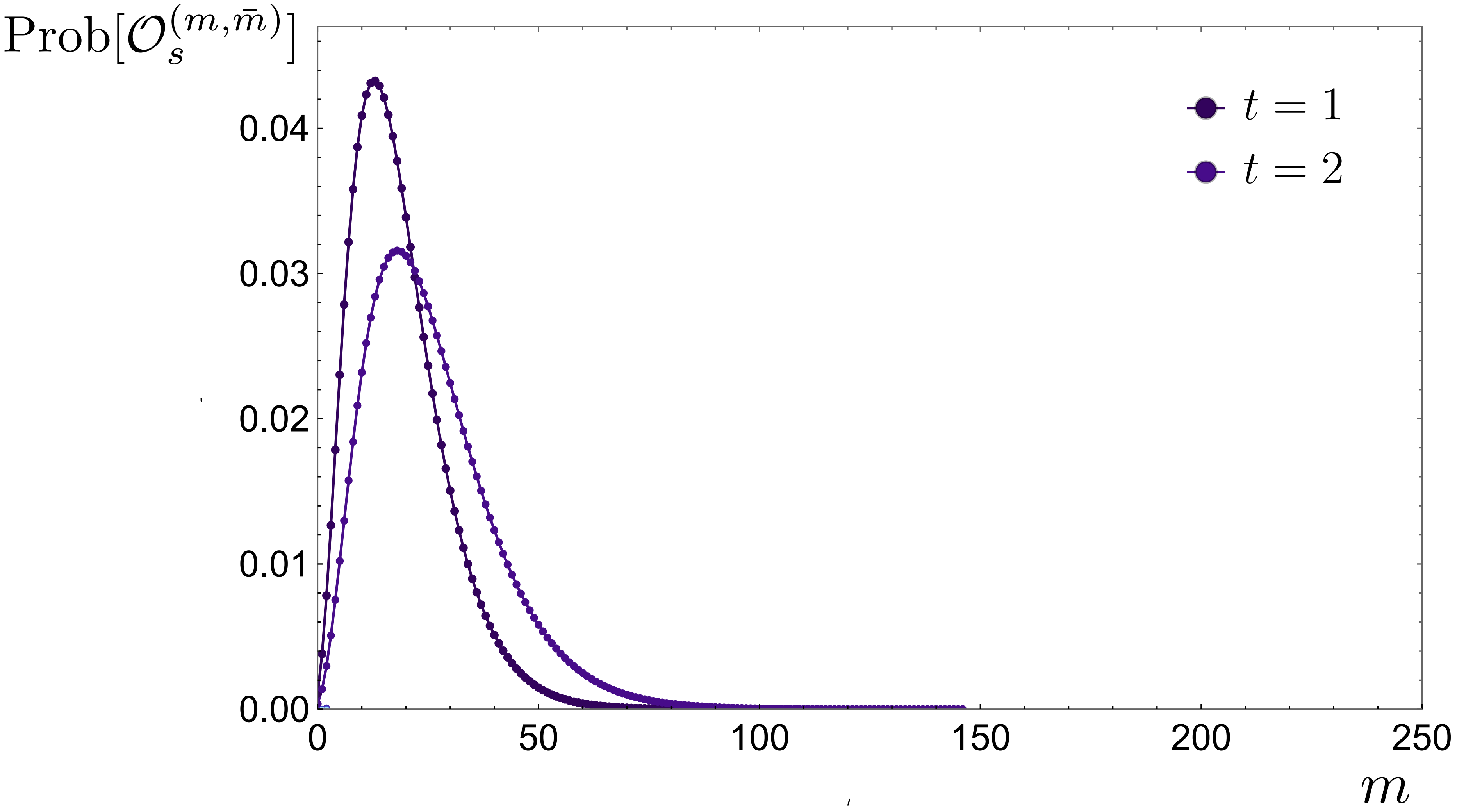

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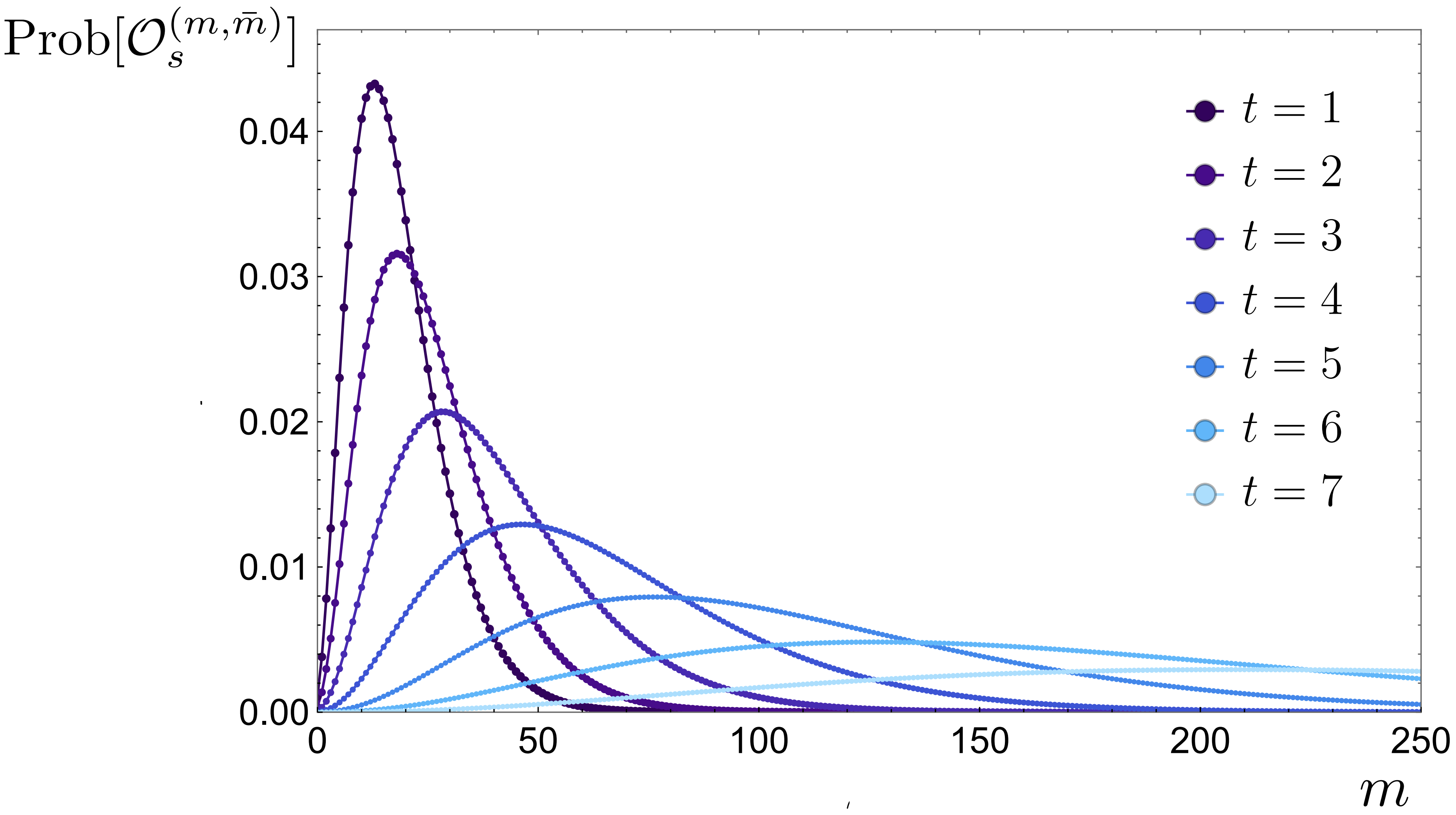

$$1 = \sum_{\mathcal{O}_s} C_{wvs}^2 \underbrace{\langle \mathcal{B}_{WV\mathcal{O}_s} | \mathcal{B}_{WV\mathcal{O}_s} \rangle}_{\text{conformal blocks } \mathcal{V}_s(z,\bar{z})}$$

- How does the decomposition of 1 into s-channel conformal blocks change as a function of $t \equiv -t_v - t_w$?
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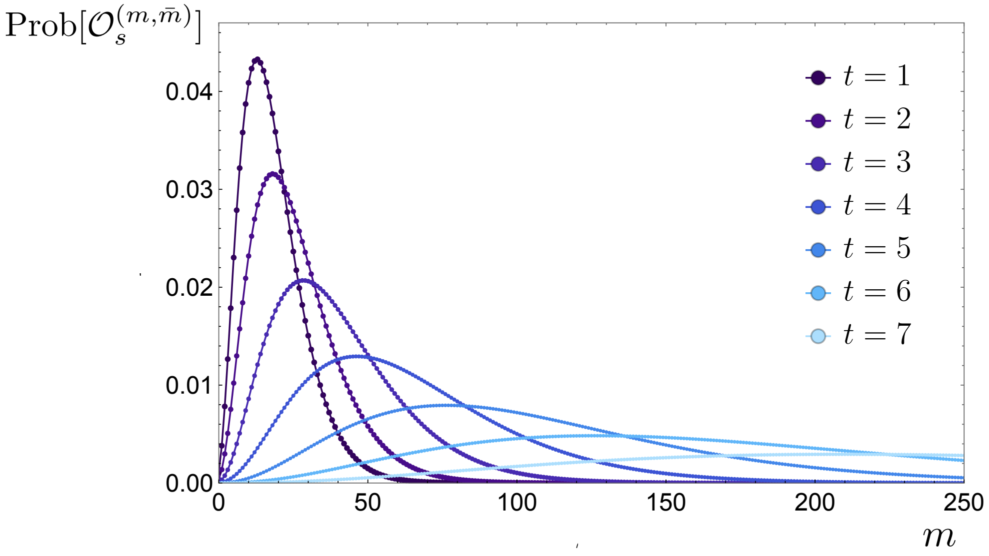
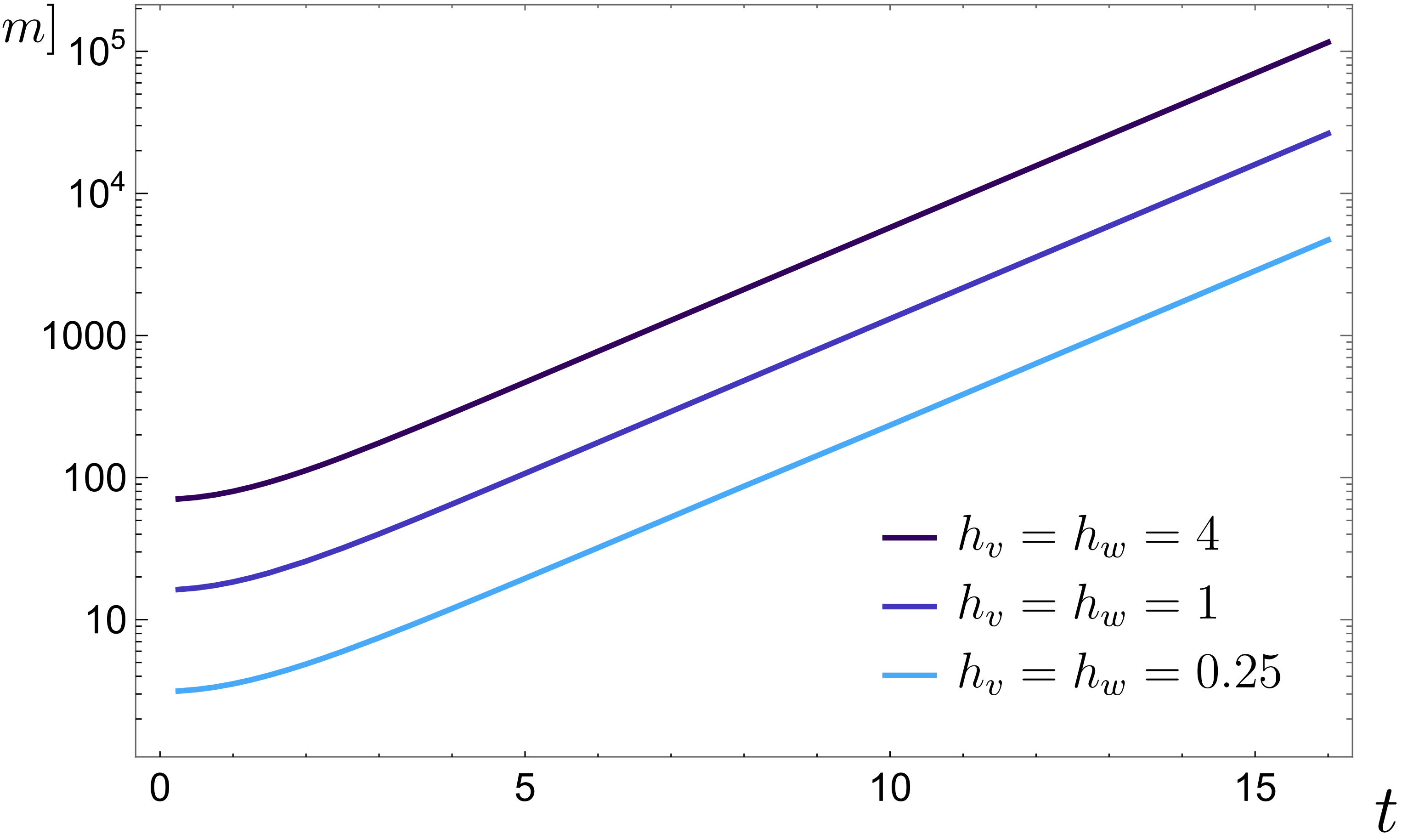
$$\mathbb{E}[m] = \sum_m m\,\mathrm{Prob}[m]$$





- Mean value of *m* grows exponentially:

$$\mathbb{E}[m] \sim e^{\frac{t-b}{2}}$$

$$\mathbb{E}[\bar{m}] \sim e^{\frac{t+b}{2}}$$

$t \equiv -t_v - t_w$

$b \equiv x_v - x_w$

$$1 = \sum_{\mathcal{O}_s} C^2_{wvs} \underbrace{\langle \mathcal{B}_{WV\mathcal{O}_s} | \mathcal{B}_{WV\mathcal{O}_s} \rangle}_{\text{conformal blocks } \mathcal{V}_s(z,\bar{z})}$$

- How does the decomposition of 1 into s-channel conformal blocks change as a function of $t \equiv -t_v - t_w$?
- Higher boost t excites intermediate states $\mathcal{O}_s$ with higher energy E_s
- Mean value of *m* grows exponentially: $\mathbb{E}[m] \sim e^{\frac{t-b}{2}}$

$$\mathbb{E}[\bar{m}] \sim e^{\frac{t+b}{2}}$$

$t \equiv -t_v - t_w$

$b \equiv x_v - x_w$

  - Quantum butterfly effect / operator growth!
  - $W \times V \to \mathcal{O}_s^{(m,\bar{m})}$: increasingly heavy & broad superpositions of $\mathcal{O}_s^{(m,\bar{m})}$
  - Exponent = $\frac{1}{2} \times$ [maximal chaos exponent]

—> Expected for these kind of observables

—> Previously seen in 6-point OTOCs and quantum circuit models

[**FMH**/Zhao] [**FMH**/Streicher/Zhao]

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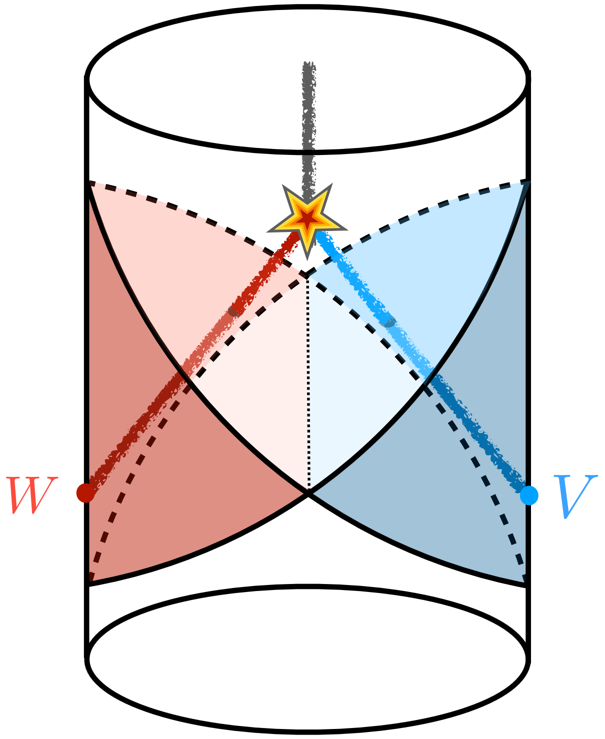
$t \equiv -t_v - t_w$

$b \equiv x_v - x_w$

  - Fusion into discretum of light states **breaks down** when:



$$\mathbb{E}[m] \sim \mathbb{E}[\bar{m}] \sim \frac{c}{24}$$

$$\Leftrightarrow \qquad t - |b| \sim 2 \times \log c$$

(2 x “scrambling time”)

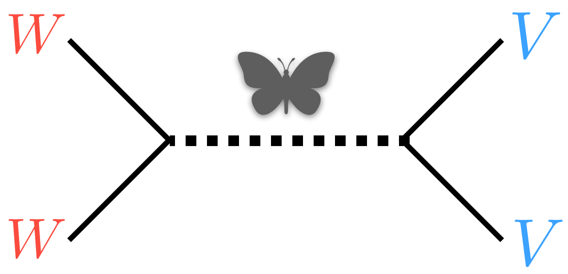
- In AdS3/CFT2, this is the threshold for **black hole formation**

- Shockwave collision creates black hole with mass $M = \mathbb{E}[\frac{m+\bar{m}}{2}] - \frac{c}{12}$ and spin $J = \mathbb{E}[m - \bar{m}]$

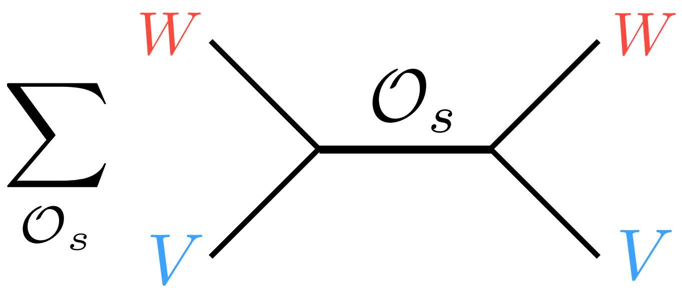
[Matschull]...[Chowdhury/**FMH**/Sanchez-Garrido/Zhao] [deBoer/Hollander/Rolph]

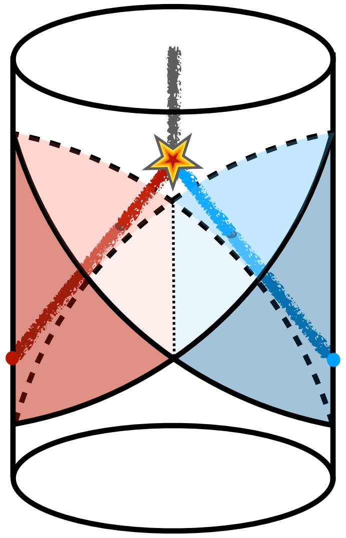
Summary

- The **‘EFT of quantum chaos’** is a powerful and useful paradigm
  - <u>Outlook:</u> develop for more general theories (e.g. CFTs) and incorporate into wider framework of quantum chaos



- **‘Two-shockwave state’** $|\Psi_{W_L V_R}\rangle = W(-t_w - i\pi, x_w)V(t_v, x_v)|0\rangle$
  - OPE decomposition depends on $e^{(t\pm b)/2}$
  - Qualitative change after (2x) scrambling time
  - New & precise notion of operator growth





- Holographic interpretation:
  black hole formation via shockwave collision
- <u>Outlook:</u> mean-field theory of ‘heavy’ black hole states