In-in EFTs for the cosmological collider

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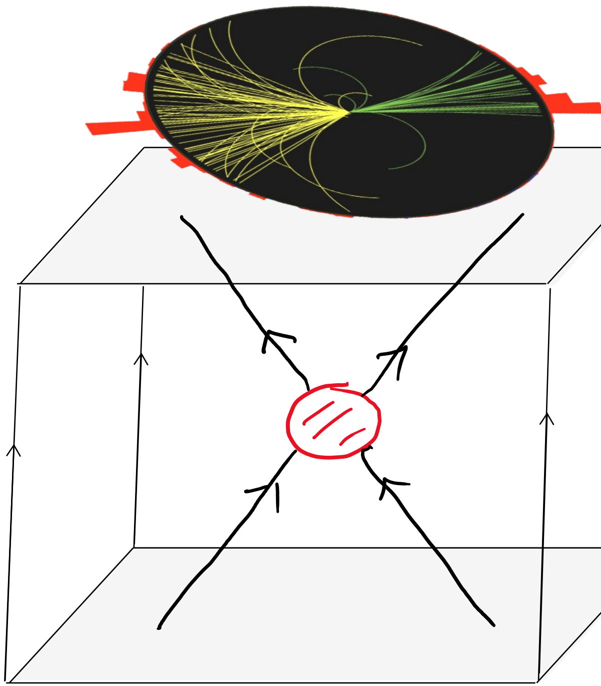
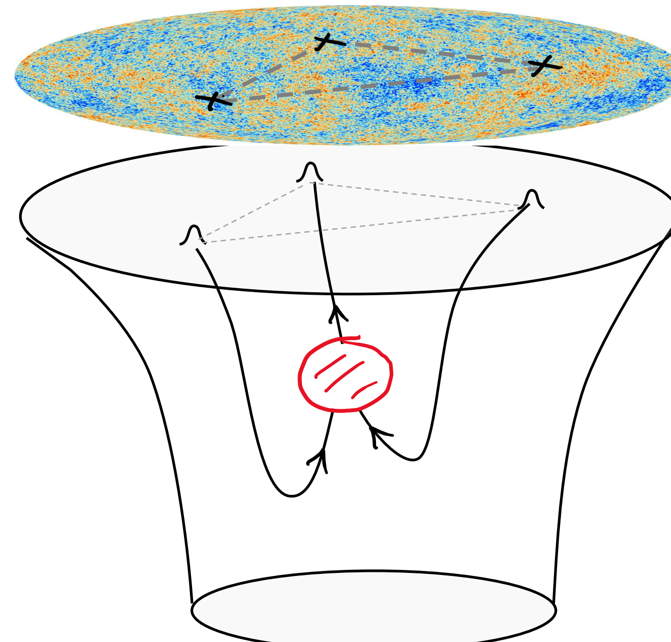
Collider

Measure cross sections to reconstruct fundamental particle physics

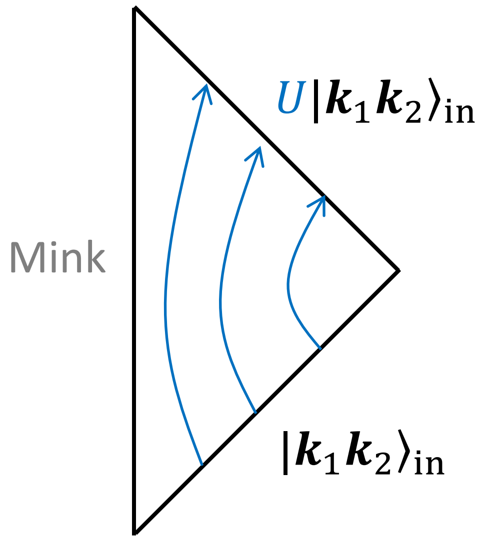


Cosmological Collider

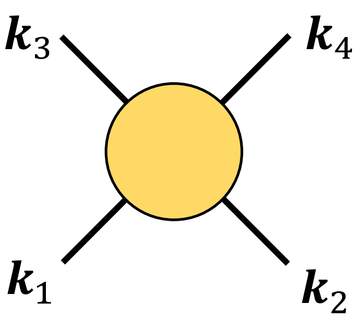
Measure correlations to reconstruct fundamental inflationary physics



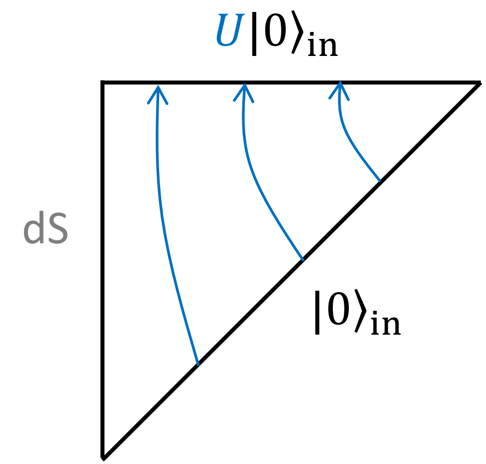
Amplitudes



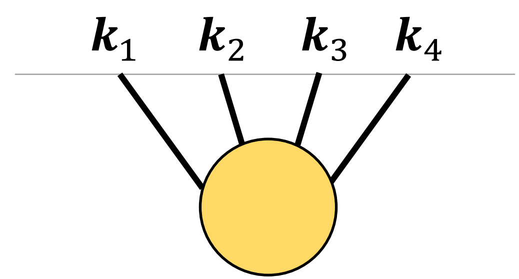
"Measure" ${}_{\text{out}}\langle \boldsymbol{k}_3\boldsymbol{k}_4|U|\boldsymbol{k}_1\boldsymbol{k}_2\rangle_{\text{in}}$



Correlators



"Measure" ${}_{\text{in}}\langle 0|U^\dagger\, \zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}}\zeta_{\boldsymbol{k}}\, U|0\rangle_{\text{in}}$



[see hep-th/0506236 for review]

For **QFT** on dS: Schwinger-Keldysh may be useful,
but not essential*

*since in-in = | in-out |^2 , i.e.

$$ {}_{\rm in}\langle \mathcal{O} \rangle_{\rm in} = \sum_n \underbrace{{}_{\rm in}\langle 0 | U^\dagger | n \rangle_{\rm out}}_{A^*_{0\to n}} \underbrace{\langle n | \mathcal{O} U | 0 \rangle_{\rm in}}_{A_{0\to \mathcal{O} n}} $$

Wavefunction coefficients are ``in-out amplitudes” with field eigenstates

[see 1406.5490 and 1709.02813, 1811.02515, 1812.07571, 2009.02898, 2009.07874, …]

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[see 1406.5490 and 1709.02813, 1811.02515, 1812.07571, 2009.02898, 2009.07874, ...]

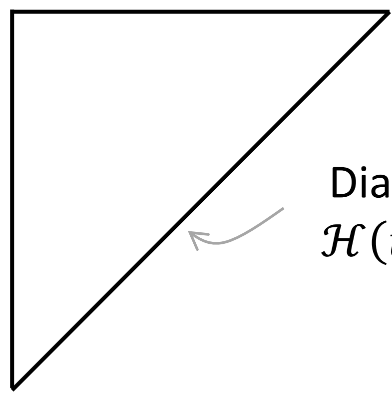
For **EFT** on dS: Schwinger-Keldysh may be essential

- The need for in-in EFTs on de Sitter

- An in-in EFT for the cosmological collider

- Constraints from causality/unitarity/locality

The need for SK: Particle production



Diagonalise $\mathcal{H}(t \to -\infty)$ $\Rightarrow$ Vaccum $a|0\rangle = 0$

(Bunch-Davies)

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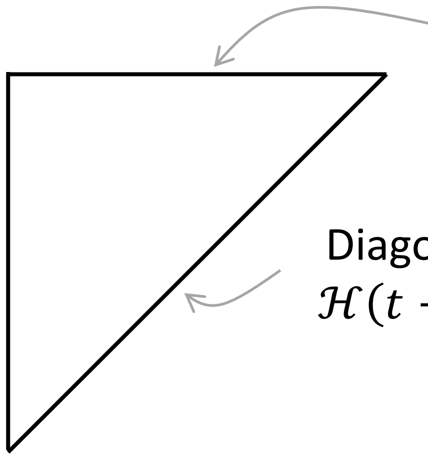

Diagonalise $\mathcal{H}(t \to +\infty)$ $\Rightarrow$ Vaccum $\tilde{a}|\tilde{0}\rangle = 0$

(Unruh-deWitt)

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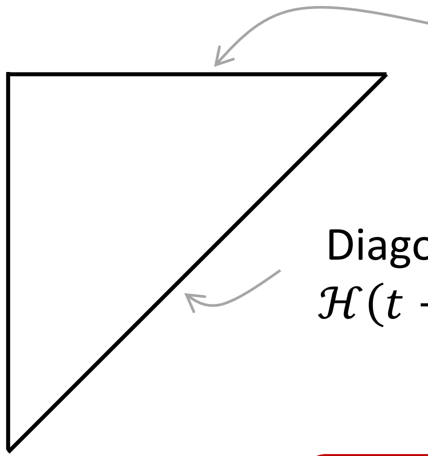

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(Bunch-Davies)

Unlike Minkowski, $|0\rangle$ does not evolve into $|\tilde{0}\rangle$

Particles are produced: $$\alpha_\mu = {}_{\text{in}}\langle 0|U^\dagger \, \tilde{a}^\dagger \tilde{a} \, U|0\rangle_{\text{in}} \sim e^{-2\pi\mu/H}$$

where $\mu = \sqrt{m^2 - d^2H^2/4}$ is effective mass

and $2\pi T = H$ is effective “temperature”

The need for SK: Particle production

in-in EFT = sum of | in-out EFTs $|^2$

$${}_{\rm in}\langle\phi^n\rangle_{\rm in} = {}_{\rm in}\langle 0|\tilde{0}\rangle_{\rm out}\langle\tilde{0}|\phi^n|0\rangle_{\rm in} + {}_{\rm in}\langle 0|\tilde{2}\rangle_{\rm out}\langle\tilde{2}|\phi^n|0\rangle_{\rm in} + \dots$$

$$\int_0^{\tilde{0}} D\chi\, e^{iS[\phi,\chi]} = e^{iS^0_{\rm EFT}[\phi]}$$

$= 0$ on Mink

$\neq 0$ on dS

$$\int_0^{\tilde{1}} D\chi\, e^{iS[\phi,\chi]} = e^{iS^1_{\rm EFT}[\phi]}$$

Particles are produced: $\alpha_\mu = {}_{\rm in}\langle 0|U^\dagger\, \tilde{a}^\dagger \tilde{a}\, U|0\rangle_{\rm in} \sim e^{-2\pi\mu/H}$

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The need for SK: Composite operators are gapless

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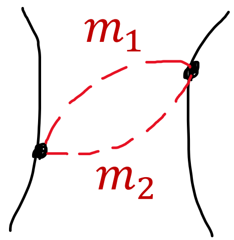
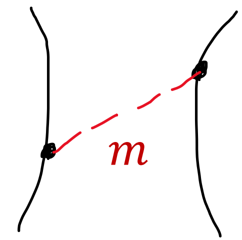


$$= \int_{m_{\min}}^{\infty} dm \; \rho(m)$$



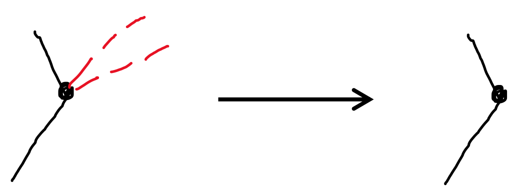
On Mink, $m_{\min} = m_1 + m_2$

$\Rightarrow$ Integrating out heavy fields ($E > m_1$ or m_2) also removes composites

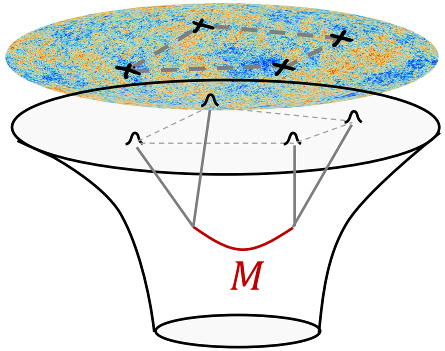
On dS, $m_{\min} = 0$ ("No mass gap")

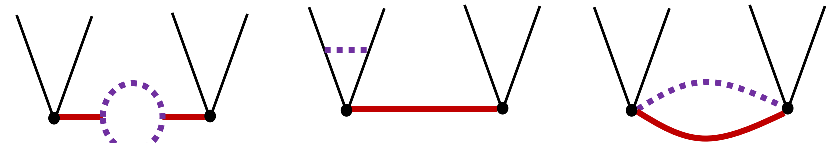
$\Rightarrow$ Two heavy fields can combine into mode of arbitrarily low energy

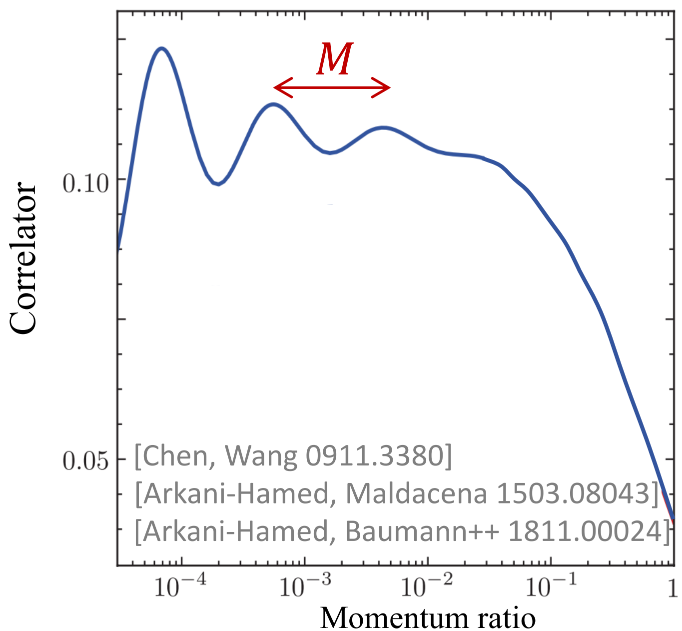
Integrating out heavy fields will produce a *non-unitary* in-out EFT



Goal: develop in-in EFT for cosmological collider signal







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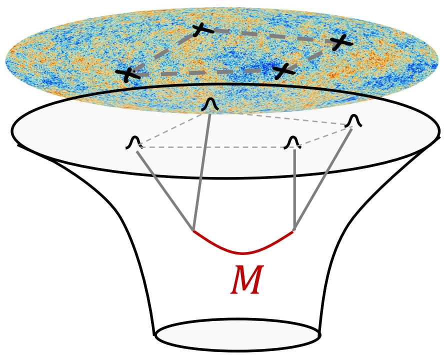
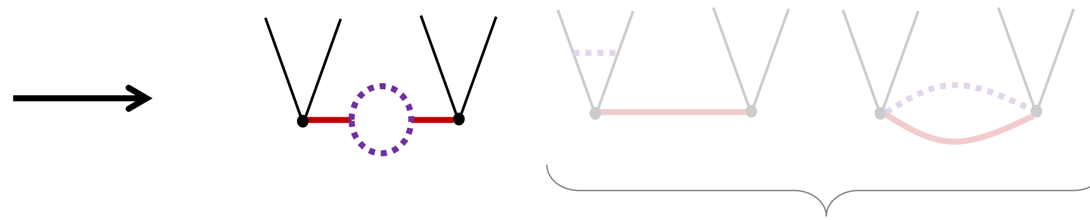
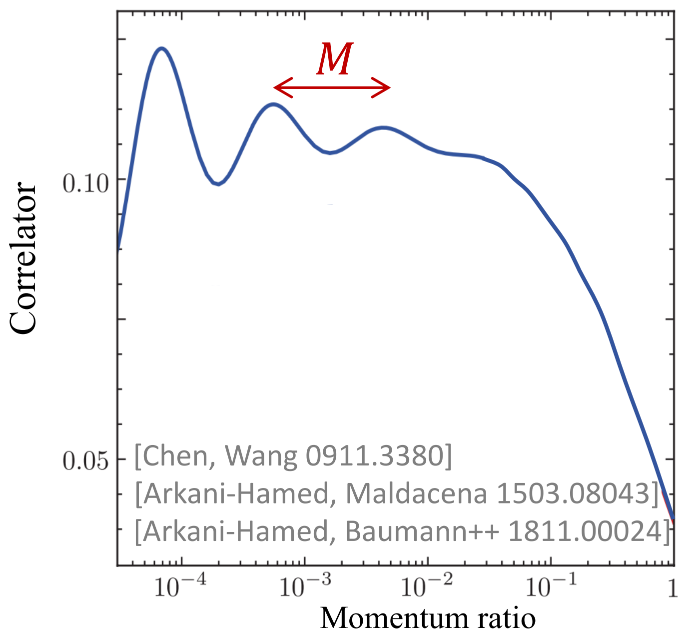
Shape of primordial non-Gaussianity

Goal: develop in-in EFT for cosmological collider signal

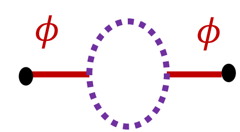






$\Downarrow$

Need EFT for propagation of massive scalar ϕ on de Sitter



$S_{\text{EFT}}[\phi]$

On Mink, analogous in-out problem: $${}_{\text{out}}\langle\Omega|T\phi_1\phi_2|\Omega\rangle_{\text{in}} = \int D\phi D\chi\; e^{iS[\phi,\chi]}\,\phi_1\phi_2$$

UV

Unitary, local (LI) system

$$S[\phi,\chi]$$

$\int D\chi$

IR

Unitary, local (LI) EFT

$$S_{\text{EFT}}[\phi] = \sum_n c_n\,\tfrac{1}{2}\phi\Box^n\phi$$

(Used for Higgs propagation in colliders)

e.g. 1903.07725

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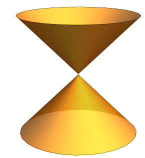
Unitary, local (LI) system

$$S[\phi, \chi]$$

+ causality for χ



$\int D\chi$

$\Downarrow$

IR

Unitary, local (LI) EFT

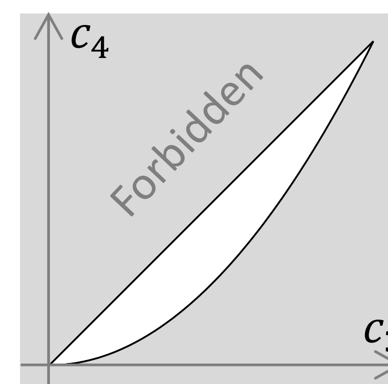
$$S_{\text{EFT}}[\phi] = \sum_n c_n \, \tfrac{1}{2} \phi \Box^n \phi$$

+ positivity bounds on c_n

$$0 < c_{n+1} < c_n / \Lambda^2$$

$$c_{n+1} c_{n-1} > c_n^2$$

c_0, c_1 are mass, wvfn normalisation



"EFThedron"

(Used for Higgs propagation in colliders)

e.g. 1903.07725

On dS: $${}_{\text{in}}\langle\Omega|T\phi_1\phi_2|\Omega\rangle_{\text{in}} = \int D\phi^{\pm}D\chi^{\pm}\, e^{+iS[\phi^+,\chi^+]}e^{-iS[\phi^-,\chi^-]}\,\phi_1^+\phi_2^+$$

UV

Closed, unitary, local (dSI) system

$$S[\phi,\chi]$$

$\int D\chi$

IR

Open, unitary, local (dSI) EFT

$$S_{\text{EFT}}[\phi^{\pm}] = \sum_n c_n\,\phi_a\Box^n\phi_r + i\gamma_n\,\tfrac{1}{2}\phi_a\Box^n\phi_a$$

Can be used for cosmo collider pheno

[Lee, SM 2512.20706]

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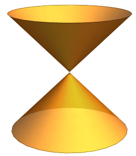

$\int D\chi$

$\Downarrow$

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$$S_{\rm EFT}[\phi^\pm] = \sum_n c_n\,\phi_a\Box^n\phi_r + i\gamma_n\,\tfrac{1}{2}\phi_a\Box^n\phi_a$$

+ positivity bounds on $g_n = c_n + f_n(\gamma)$

$$0 < g_{n+1} < g_n/\Lambda^2$$

$$g_{n+1}g_{n-1} > g_n^2$$

Can be used for cosmo collider pheno

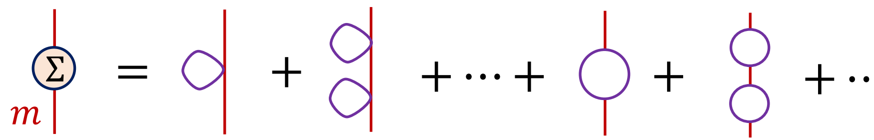
[Lee, SM 2512.20706]

$$c_2 > \begin{cases} 0 + \mathcal{O}\left(e^{-\pi\Lambda/H}\right) & \text{when } \Lambda \gg H \\ -\frac{2}{3}\gamma_0 + \mathcal{O}\left(\Lambda^2/H^2\right) & \text{when } \Lambda \ll H \end{cases}$$

Proof of positivity bounds

Proof of positivity bounds

On Mink: $${}_{\rm out}\langle T\phi(x_1)\phi(x_2)\rangle_{\rm in} = \int_p \frac{f_p^+(x_1) f_p^-(x_2)}{m^2 - p^2 - \Sigma(p)}$$ $$f_p^\pm(x) = e^{\pm i p\cdot x}$$



Causality $\Rightarrow$ $$\Sigma(p) = \int_{m_{\rm min}}^\infty dm \, \frac{\rho(m)}{m^2 - p^2 - i\epsilon}$$

Unitarity $\Rightarrow$ $$\rho = \mathrm{Im}\,\Sigma \geq 0$$

Local interactions $\Rightarrow$ $$\Sigma(p) = c_2 p^4 + c_3 p^6 + \ldots$$ at low p^2

Altogether, these imply $$c_n = \int_\Lambda^\infty \frac{dm \;\; \rho(m)}{m^{2n+2}}$$ are positive moments ($\Rightarrow$ bounded)

Proof of positivity bounds

On dS: $${}_{\rm in}\langle T\phi(x_1)\phi(x_2)\rangle_{\rm in} = \int_{\nu\mathbf{k}} f^+_{\nu\mathbf{k}}(x_1) f^+_{\nu\mathbf{k}}(x_2)\, G(\nu) \qquad f^+_{\nu\mathbf{k}} \propto e^{i\mathbf{k}\cdot\mathbf{x}} H^{(2)}_{i\nu}(-kx^0)$$

$$G(\nu) = \frac{1-\alpha_\nu}{\mu^2 - \nu^2 - \Sigma(\nu)} + \frac{\alpha_\nu}{\mu^2 - \nu^2 - \Sigma^*(\nu)}$$

Particle production

Causality ⇒ $$\Sigma(\nu) = \int_0^\infty d\mu \frac{\rho(\mu)}{\mu^2 - \nu^2 - i\epsilon}$$ [Vaziri et al, 2107.13871, 2306.00090]

Unitarity ⇒ $$\rho = \mathrm{Im}\,\Sigma \geq 0$$

Local interactions ⇒ $$\Sigma(\nu) = c_2\nu^4 + c_3\nu^6 + \ldots + i\gamma_0\nu + i\gamma_1\nu^3 + \ldots$$ at low ν

Altogether, $$g_n = \int_\Lambda^\infty \frac{d\nu\, \rho(\nu)}{\nu^{2n+2}} = c_n - \int_0^\Lambda \frac{d\nu\, \rho(\nu)}{(\nu^2 - i\epsilon)^{2n+2}}$$ are positive moments (⇒ bounded)

When does EFT break down?

When does EFT break down?

On Minkowski, the hierarchy p/Λ controls both convergence and truncation error:

$$\sum_n^{\infty} c_n p^{2n} = \sum_n^{N-1} c_n p^{2n} + \mathscr{O}\left(\frac{p^{2N}}{\Lambda^{2N}}\right)$$

converges for $|p| < \Lambda$

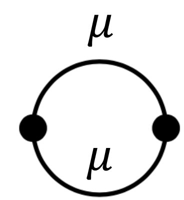
higher order terms $\sim p/\Lambda$

On dS, two hierarchies: ν/Λ (controls convergence) and ν/H (controls truncation)

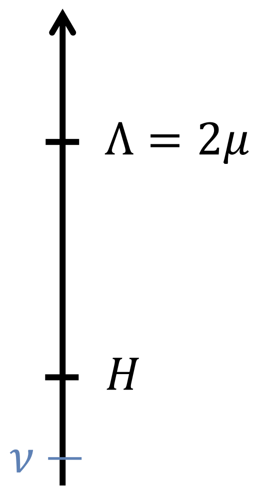
e.g. $$\frac{e^{\pi\nu/H}}{\Lambda - \nu} = \sum_n^{\infty} c_n \nu^{2n}$$ converges for any $|\nu| < \Lambda$

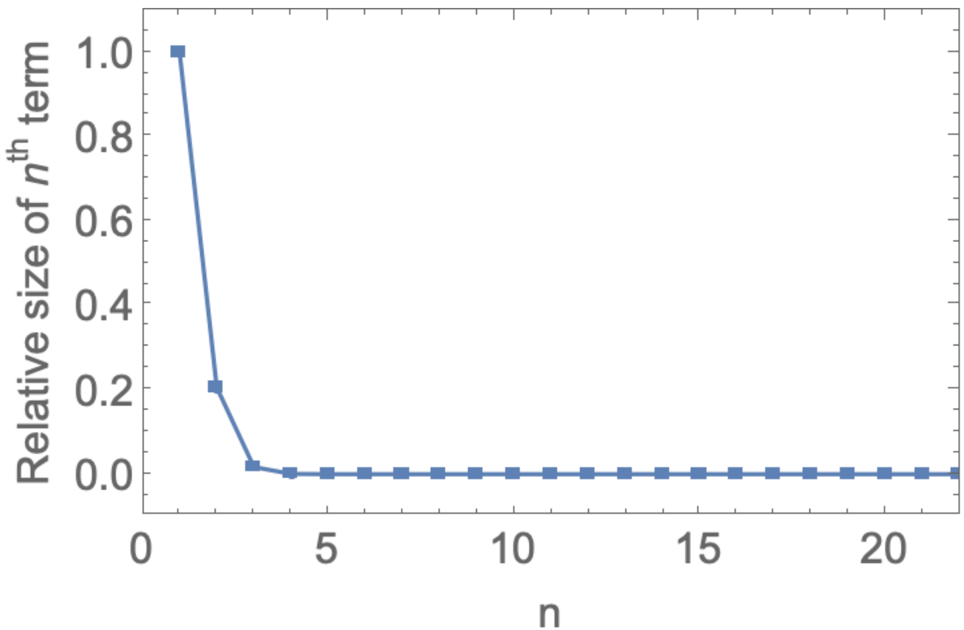
but $$= \sum_n^{N-1} c_n \nu^{2n} + \mathscr{O}\left(\frac{\nu^{2N}}{\Lambda^{2N}}\right)$$ only when $N \gtrsim |\pi\,\nu/H|$

When does EFT break down?

Example: $\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$ from 

in $d = 2$





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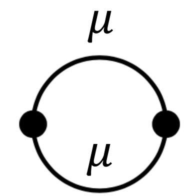
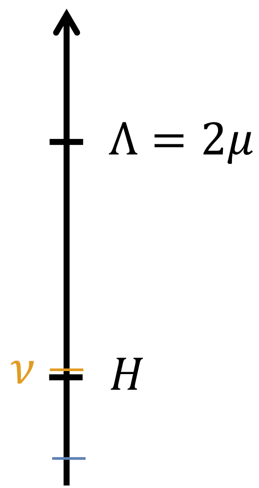
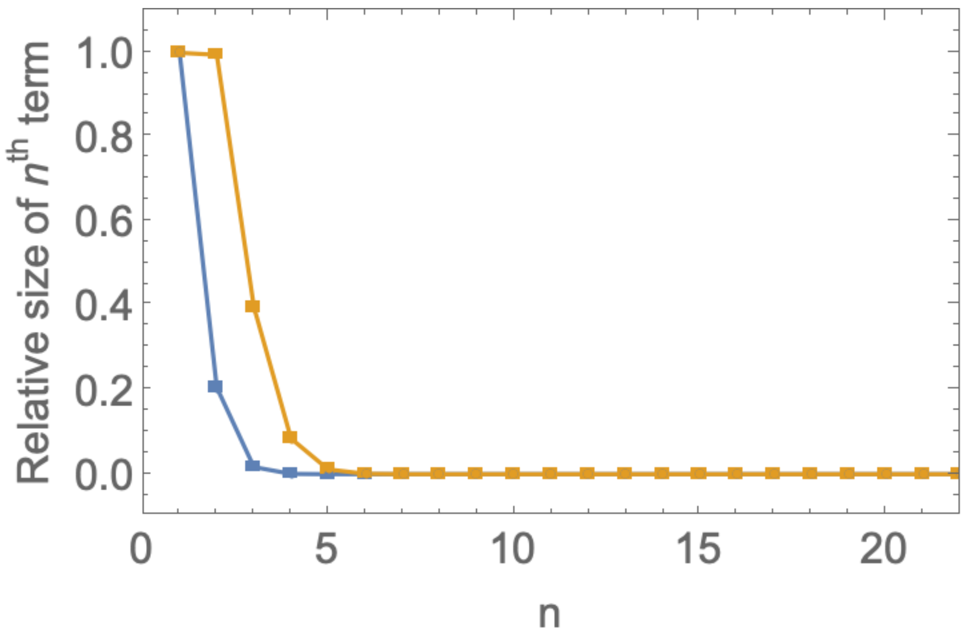
Example: $\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$ from  in $d = 2$





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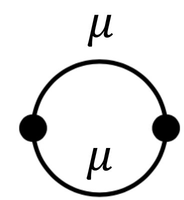
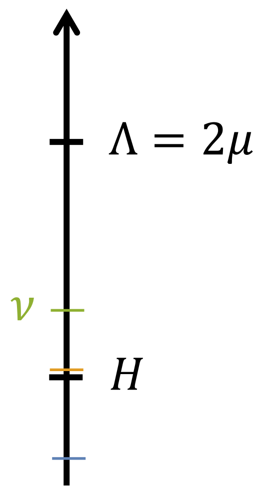
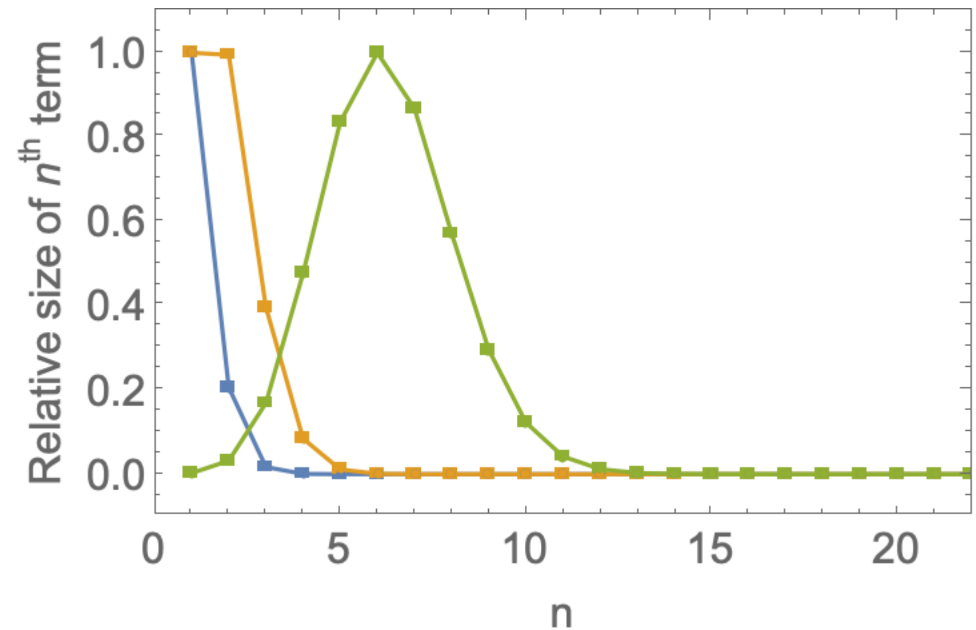
Example: $\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$ from  in $d = 2$





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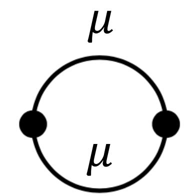
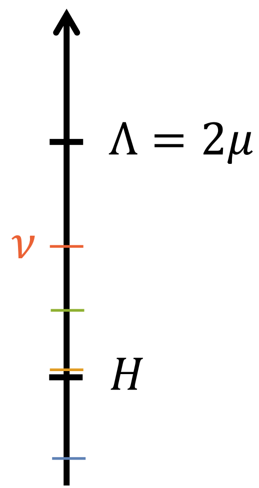
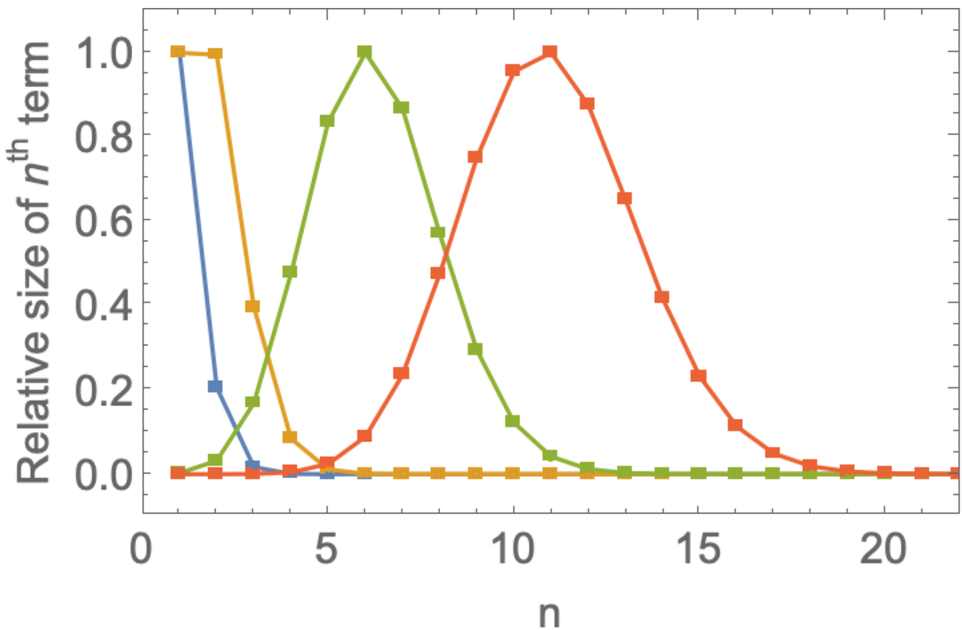
Example: $\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$ from  in $d = 2$





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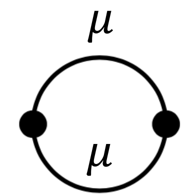
Example: $\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$ from 

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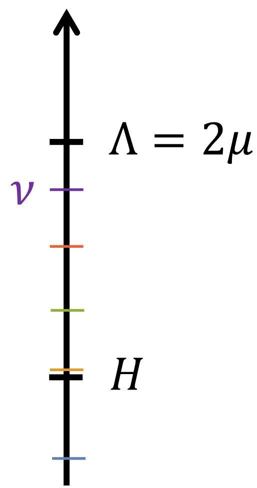

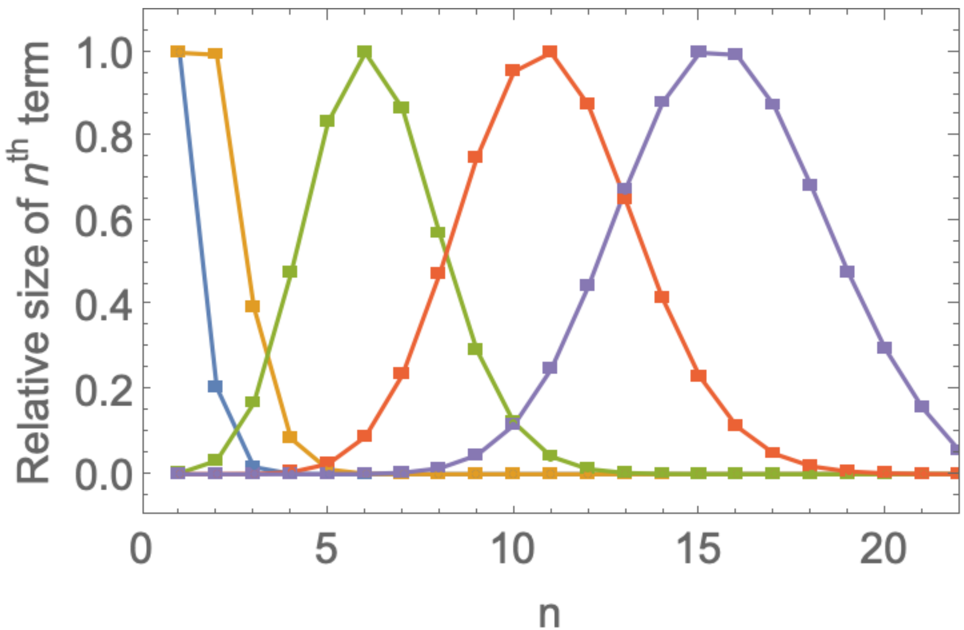

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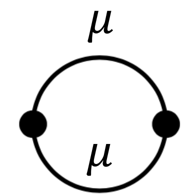
Example: $$\mathrm{Im}\,\Sigma = \sum_n \gamma_n \nu^{2n+1}$$ from 

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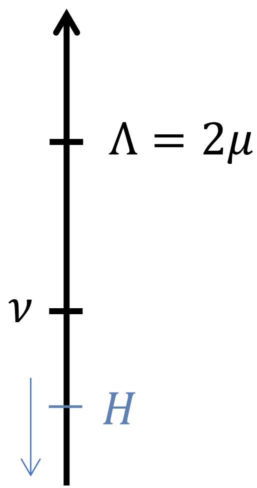


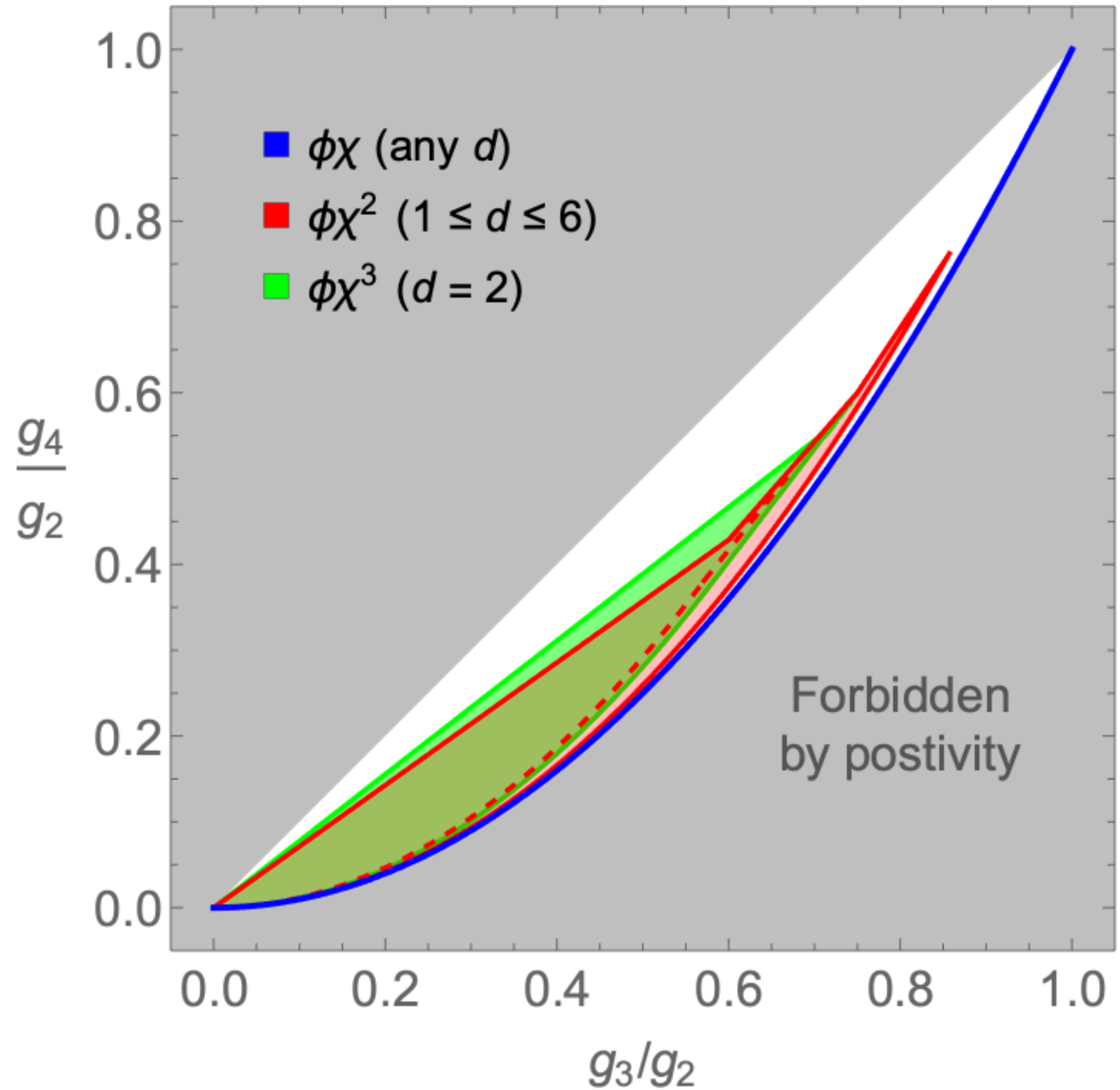
As $H \to 0$, series is dominated by terms at $n \to \infty$

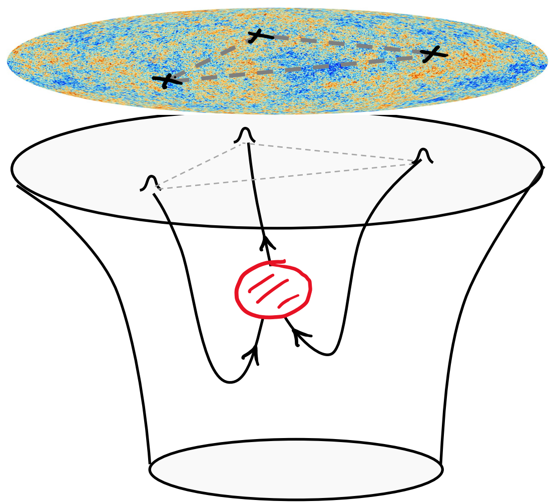
EFT series *converges* to: $$\mathrm{Im}\Sigma \sim e^{-\pi\nu/H}$$

but cannot be *truncated* at any finite order

Landscape vs Swampland

Satisfying positivity is necessary but not sufficient for a causal UV completion



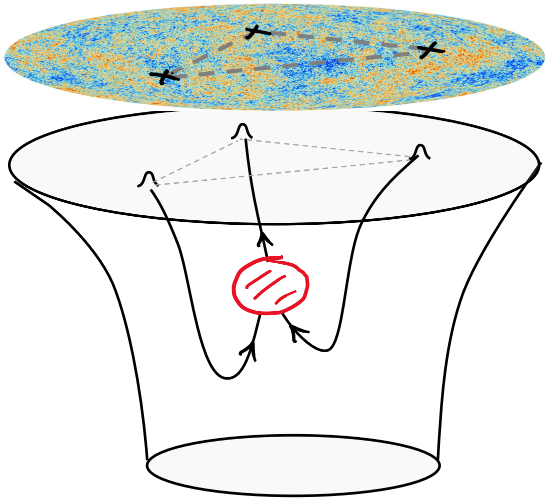


Cosmo collider signal (exchange of massive ϕ)
can be described by in-in EFT:

$$S_{\rm EFT}[\phi^{\pm}] = \sum_n c_n \, \phi_a \Box^n \phi_r + i \gamma_n \, \frac{1}{2} \phi_a \Box^n \phi_a$$

with causal (unitary/local/dSI) UV completion
only if various positivity bounds are satisfied:

e.g. $$c_2 > \begin{cases} 0 + \mathcal{O}\left(e^{-\pi \Lambda / H} \right) & \text{when } \Lambda \gg H \\ -\frac{2}{3}\gamma_0 + \mathcal{O}\left(\Lambda^2 / H^2 \right) & \text{when } \Lambda \ll H \end{cases}$$



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Questions

for non-cosmologists

(i) how would *you* describe this EFT? Is it open / unitary / local?

(ii) when is SK *necessary*? On Mink, can capture these effects with boundary terms

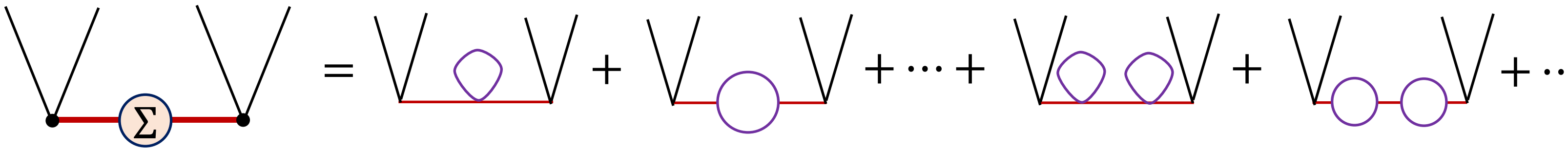
[Salcedo, Lee, SM, Pajer 2212.08009]

for cosmologists

(iii) extension to spinning ϕ?

(iv) extension to higher-point interactions?

$$\langle \sigma^4 \rangle = \int_0^\infty d\nu \, V^{\nu}_{k_1 k_2} \left[\frac{g^2}{M^2 - \nu^2 - \Sigma(\nu^2)} \right] V^{\nu}_{k_3 k_4}$$ captures:



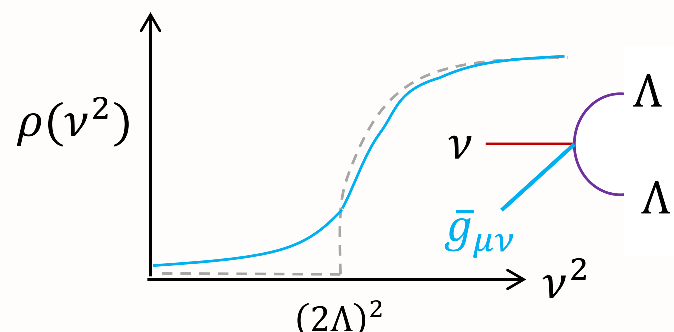
Any (causal) χ physics gives

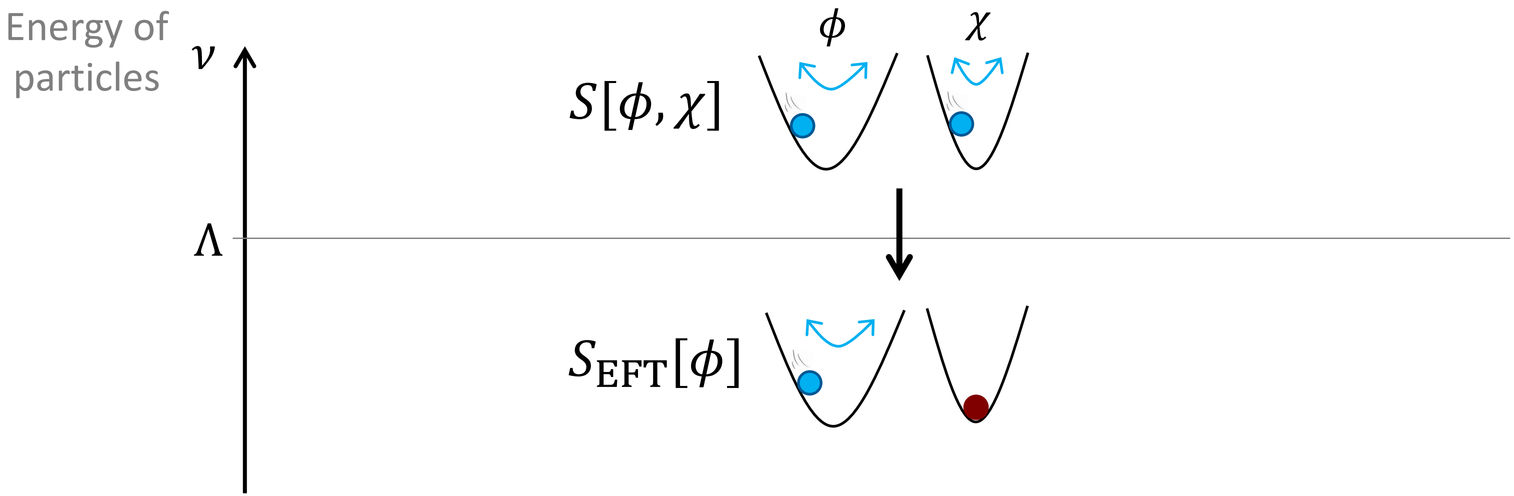
$$\Sigma(\nu^2) = \int_0^\infty dm^2 \, \frac{\rho(m^2)}{m^2 - \nu^2}$$

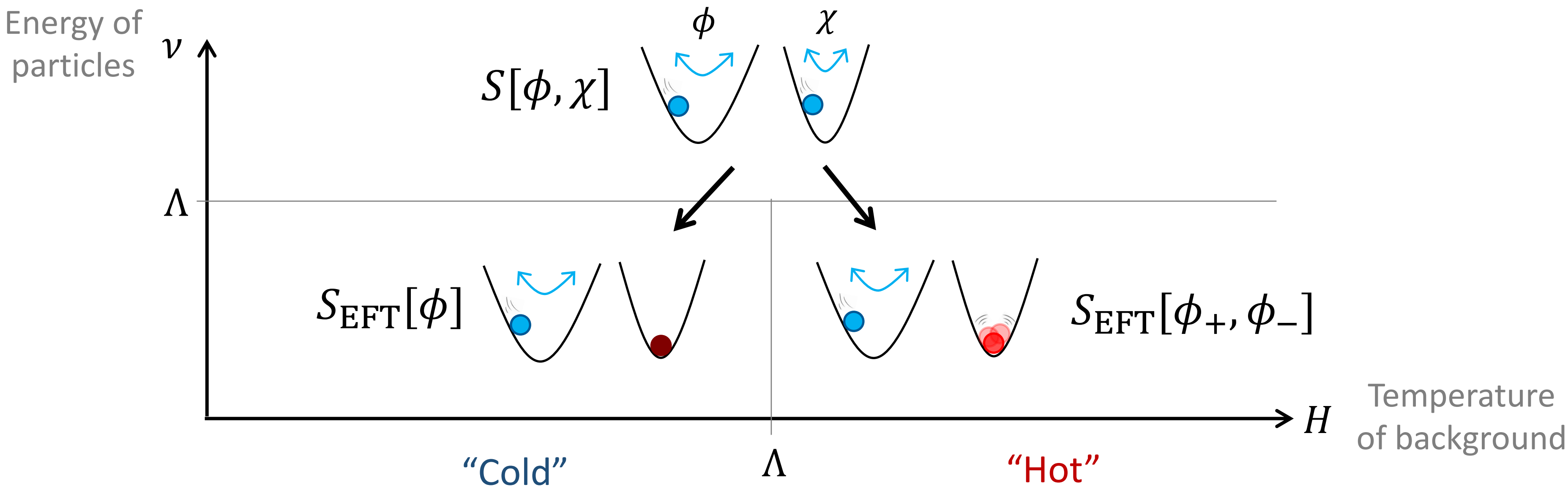
[Vaziri et al, 2107.13871, 2306.00090]

ρ is a **positive** density of states (unitarity)

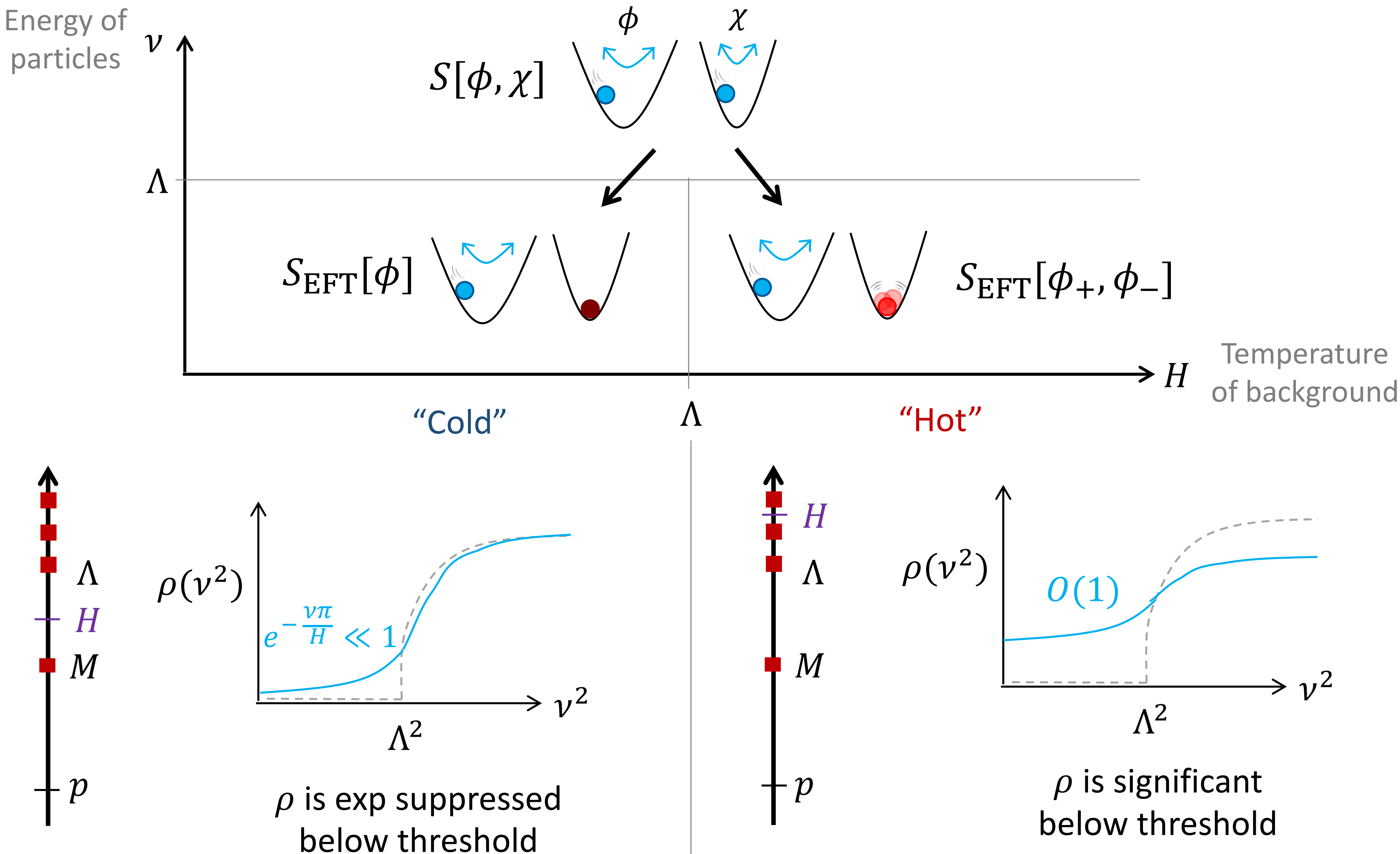
… but is no longer gapped



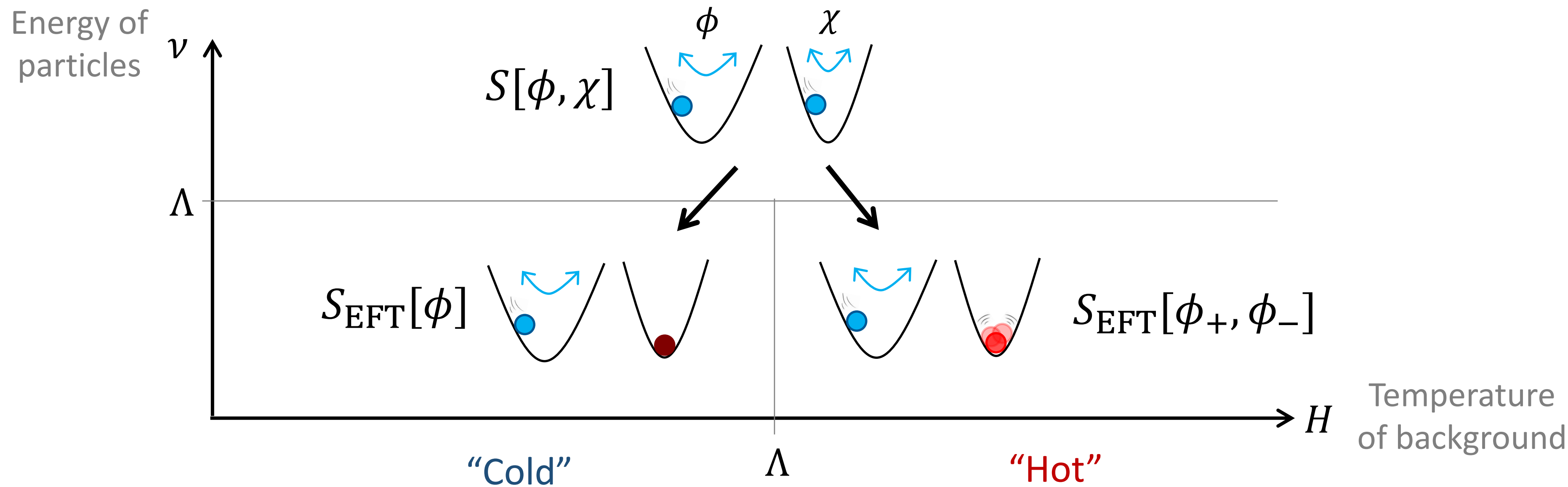

Energy of particles
ν
Λ
ϕ
χ
$S[\phi, \chi]$
$S_{\mathrm{EFT}}[\phi]$



Energy of particles
ν
S[ϕ, χ]
ϕ
χ
Λ
S_EFT[ϕ]
S_EFT[ϕ+, ϕ−]
H
Temperature of background
"Cold"
Λ
"Hot"



Energy of particles
ν
ϕ
χ
$S[\phi, \chi]$
Λ
$S_{\text{EFT}}[\phi]$
$S_{\text{EFT}}[\phi_+, \phi_-]$
H
Temperature of background
"Cold"
Λ
"Hot"
Λ
H
M
p
$\rho(\nu^2)$
$e^{-\frac{\nu\pi}{H}} \ll 1$
ν^2
Λ^2
ρ is exp suppressed below threshold
H
Λ
M
p
$\rho(\nu^2)$
$O(1)$
ν^2
Λ^2
ρ is significant below threshold



Matching is unaffected by curvature, find same EFT coefficients c_n as Mink:

$$\mathcal{L} = \sum_n \left[\; c_n \, \phi \nabla^{2n} \phi \right.$$
$$\left. + \mathcal{O}(e^{-\Lambda\pi/H}) \; \right]$$

Need additional EFT coefficients γ_n to capture curvature (dissipation) effects:

$$\mathcal{L} = \sum_n \left[\; c_n \, \phi_+ \nabla^{2n} \phi_+ \right.$$
$$\left. + i\gamma_n \, \phi_+ \nabla^{2n} (\phi_+ + \phi_-) \; + \mathrm{c.c.} \right]$$

$$\Sigma(\nu^2) = \int_0^\infty dm^2 \frac{\rho(m^2)}{m^2 - \nu^2} \quad \text{with } \rho > 0$$

If "hot", can expand density at low energies: $\rho = (\gamma_0 + \gamma_1 \nu^2 + \cdots)\nu$

$$\underbrace{\Sigma(\nu^2) - \int_0^{\Lambda^2} dm^2 \frac{\rho(m^2)}{m^2 - \nu^2}}_{\substack{(c_0 - \gamma_0 2\Lambda + \cdots) \\ +(c_1 + \cdots)\nu^2 + \cdots}} = \underbrace{\int_{\Lambda^2}^\infty dm^2 \frac{\rho(m^2)}{m^2 - \nu^2}}_{g_0 + g_1 \nu^2 + \cdots}$$

Measure these

bound these

$$g_n = \int_{\Lambda^2}^\infty \frac{d\nu^2}{\nu^{2n+2}} \rho(\nu^2)$$

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$$\Sigma(\nu^2) - \int_0^{\Lambda^2} dm^2 \, \frac{\rho(m^2)}{m^2 - \nu^2} = \int_{\Lambda^2}^\infty dm^2 \, \frac{\rho(m^2)}{m^2 - \nu^2}$$

$$(c_0 - \gamma_0 2\Lambda + \cdots) + (c_1 + \cdots)\nu^2 + \cdots \qquad g_0 + g_1 \nu^2 + \cdots$$

Measure these — bound these

$$g_n = \int_{\Lambda^2}^\infty \frac{d\nu^2}{\nu^{2n+2}} \, \rho(\nu^2)$$

g_n are bounded moments:

e.g. positivity $g_2 > 0, \; g_3 > 0, \; \ldots$

'convergence' $\Lambda^2 g_3 < g_2, \; \ldots$

(g_0, g_1 renormalize the mass and residue)