EFT and hydrodynamics far from equilibrium

Michal P. Heller

Funded by the European Union

European Research Council
Established by the European Commission

many works reviewed e.g. in **2005.12299** with Berges, Mazeliauskas & Venugopalan

2502.01622 with De Lescluze and **2504.18754** with Berges, Denicol and Preis

Introduction

The main motivating question

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what are the limits of applicability of hydrodynamic EFT?

In other words,
how much far from equilibrium hydrodynamics applies?

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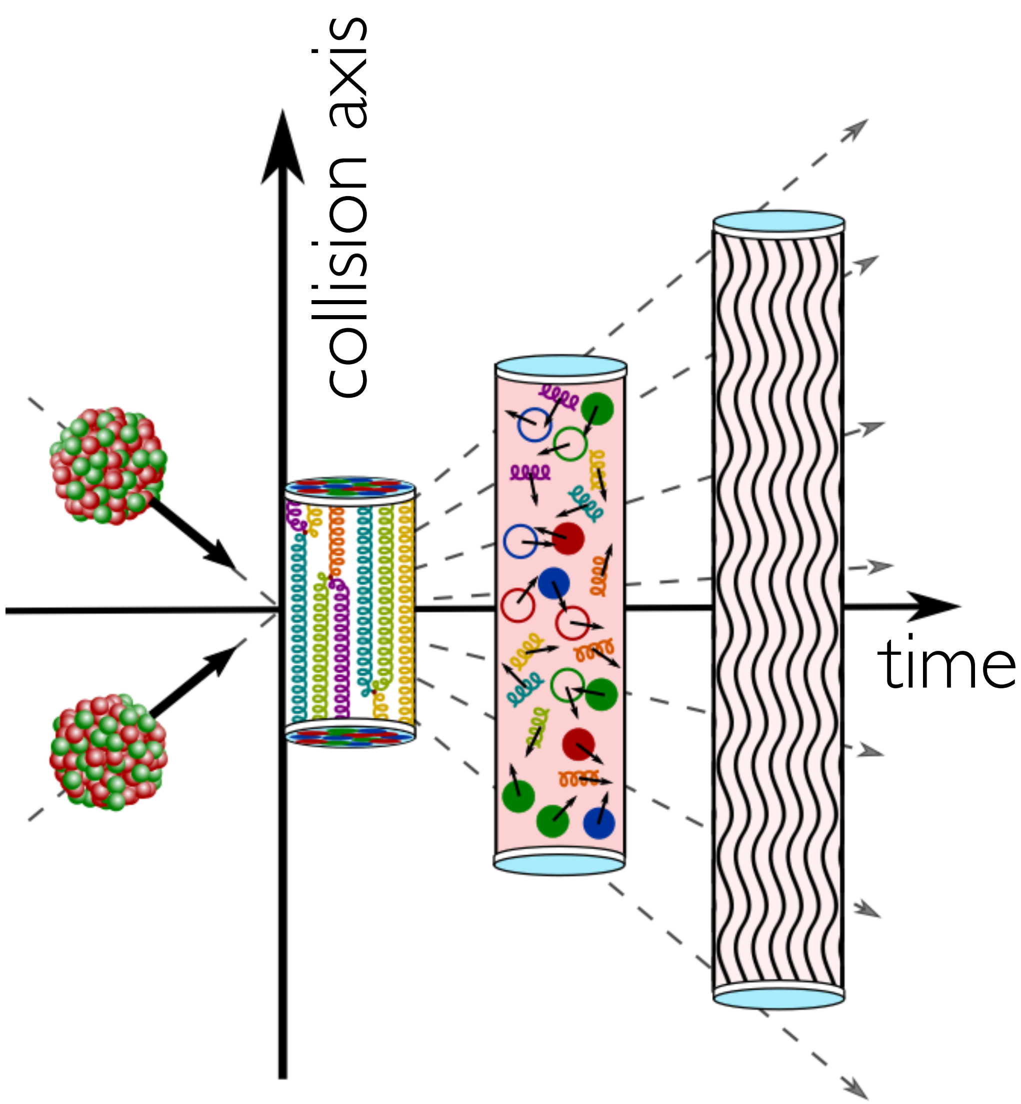
Why this question?

Relativistic hydro and ab initio simulations

heavy-ion collisions
at RHIC and LHC



2005.12299
with Berges, Mazeliauskas & Venugopalan

behaviour in
of theoretical models
(here: holographic boost-invariant dynamics)

1103.3452 with Janik & Witaszczyk

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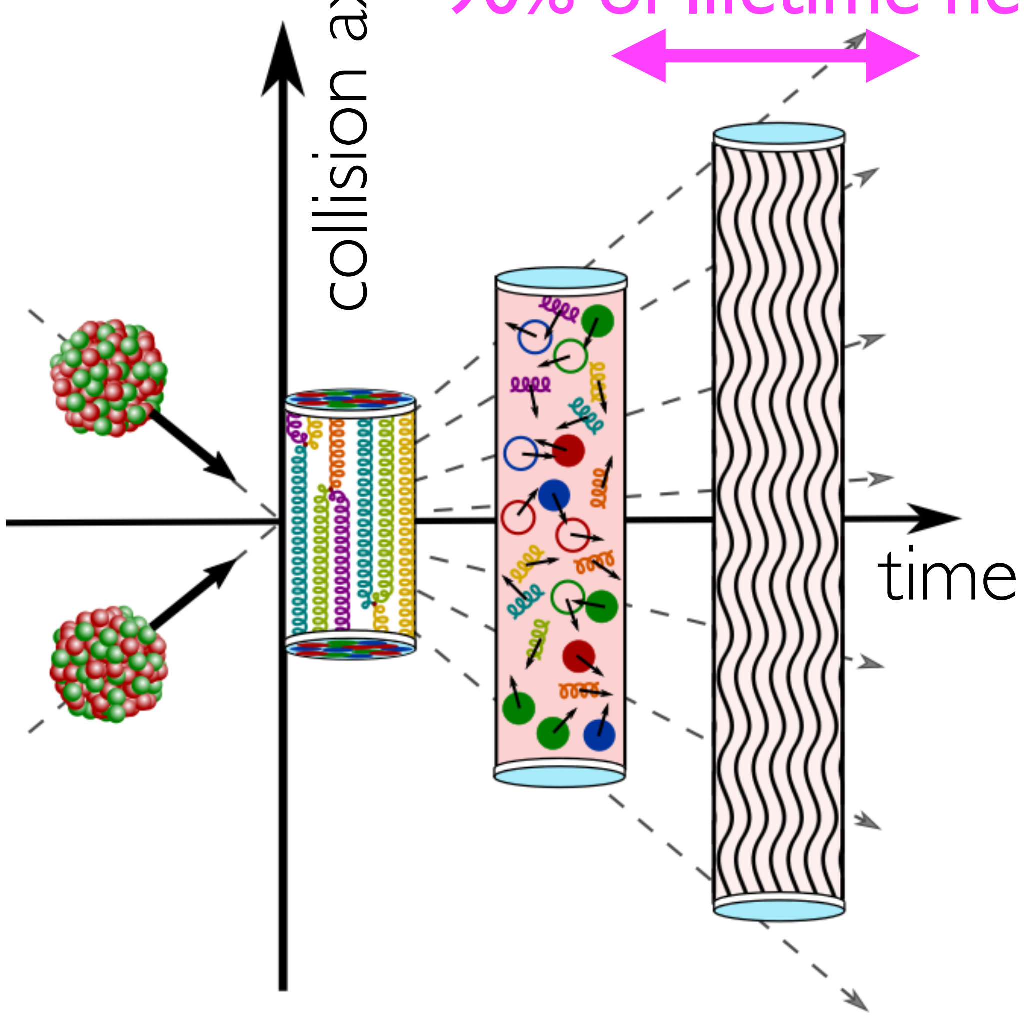
pheno:
90% of lifetime here



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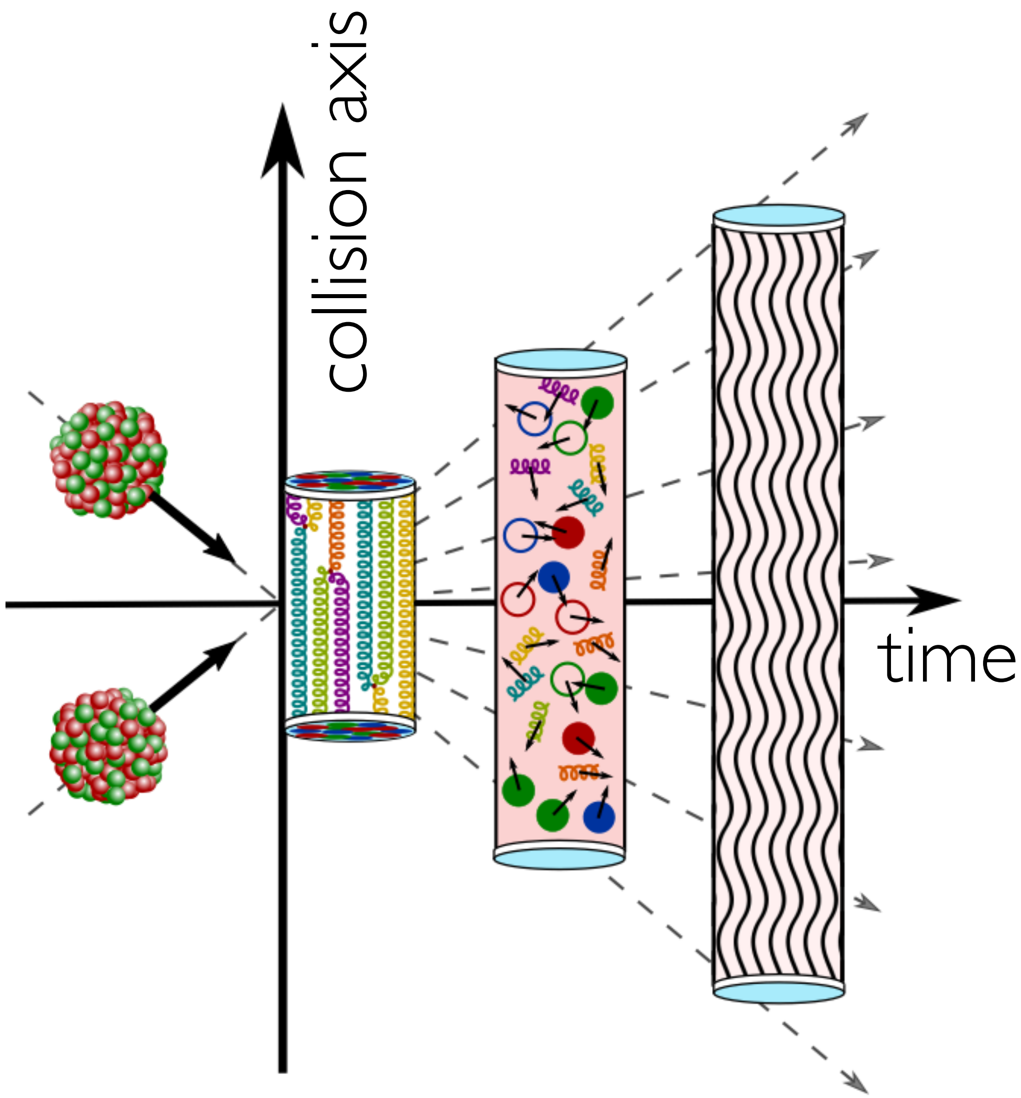

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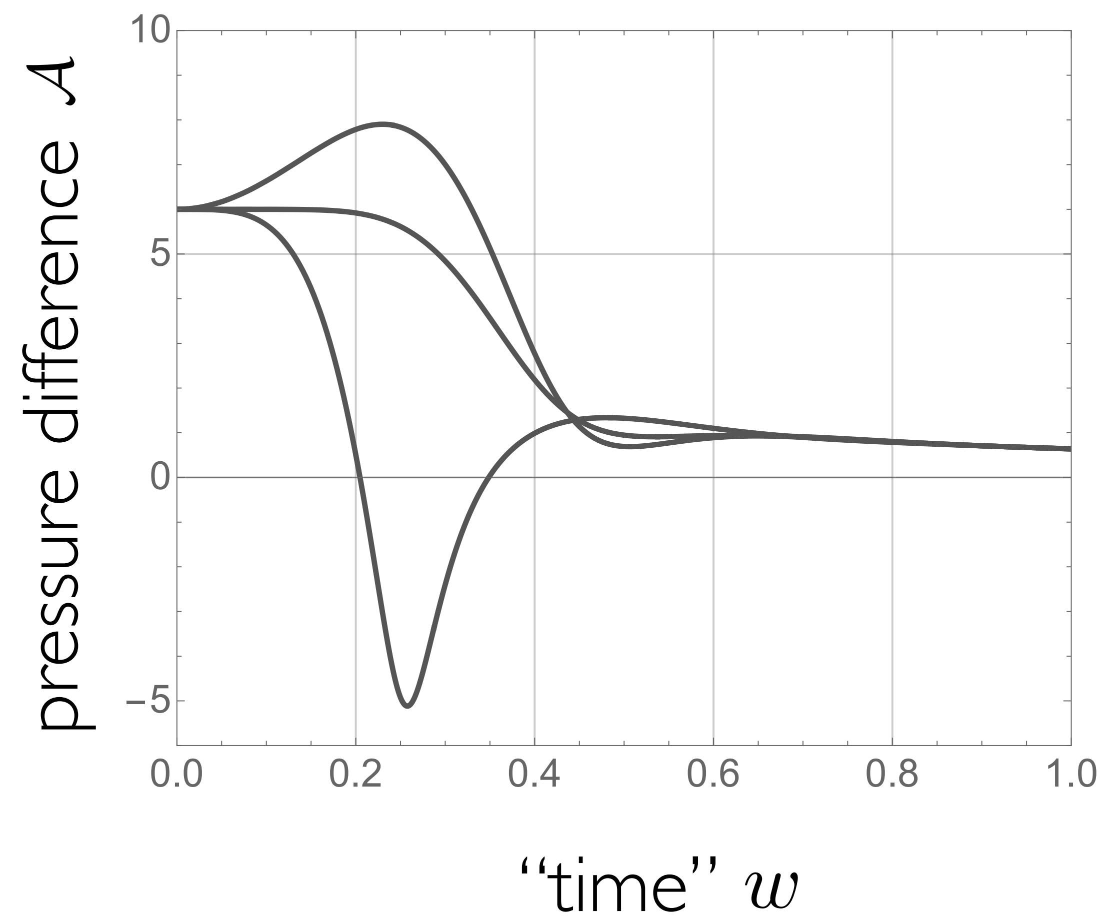

1103.3452 with Janik & Witaszczyk

Hydro works ≡ its constitutive relations hold

General $T^{\mu\nu}$ has 10 functions of 4 variables freedom subject to $\nabla_\mu T^{\mu\nu} = 0$

Relativistic hydro: $T^{\mu\nu}$ is systematically approx. in terms of 4 functions only

$$T^{\mu\nu} = \mathcal{E}(T) u^\mu u^\nu + \mathcal{P}(T) \left(g^{\mu\nu} + u^\mu u^\nu \right) + \pi^{\mu\nu}$$

@ conformality = 0

$$\pi^{\mu\nu} = -\eta(T)\, \nabla^{\langle\mu} u^{\nu\rangle} - \zeta(T)(g^{\mu\nu} + u^\mu u^\nu) \nabla_\alpha u^\alpha + \mathcal{O}(\nabla^2)$$

shear term | bulk term

@ conformality:
2 order: 5 terms
0712.2451 by Baier et al.
3 order: ~20 terms
1507.02461 by Grozdanov & Kaplis

The derivative expansion is an on-shell manifestation of the underlying EFT

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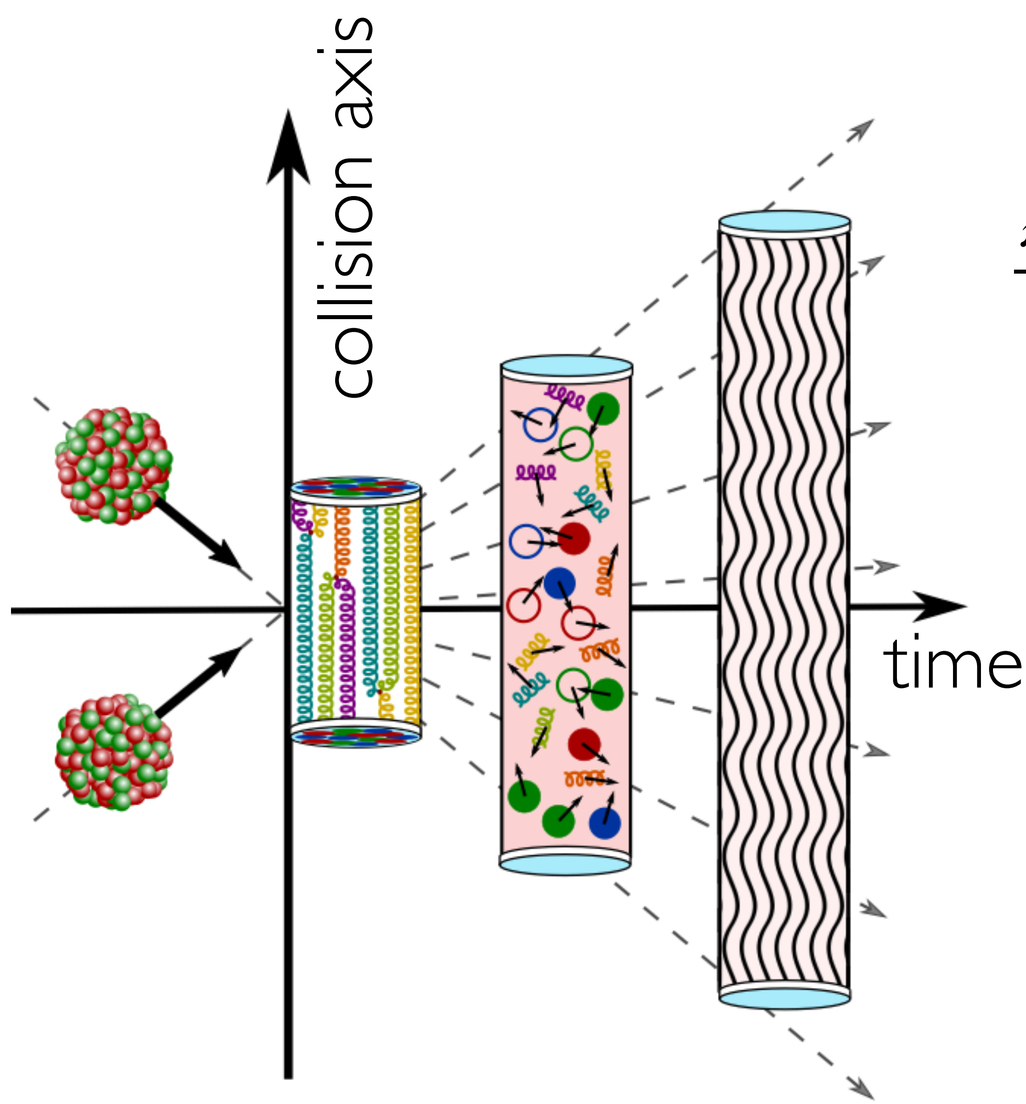

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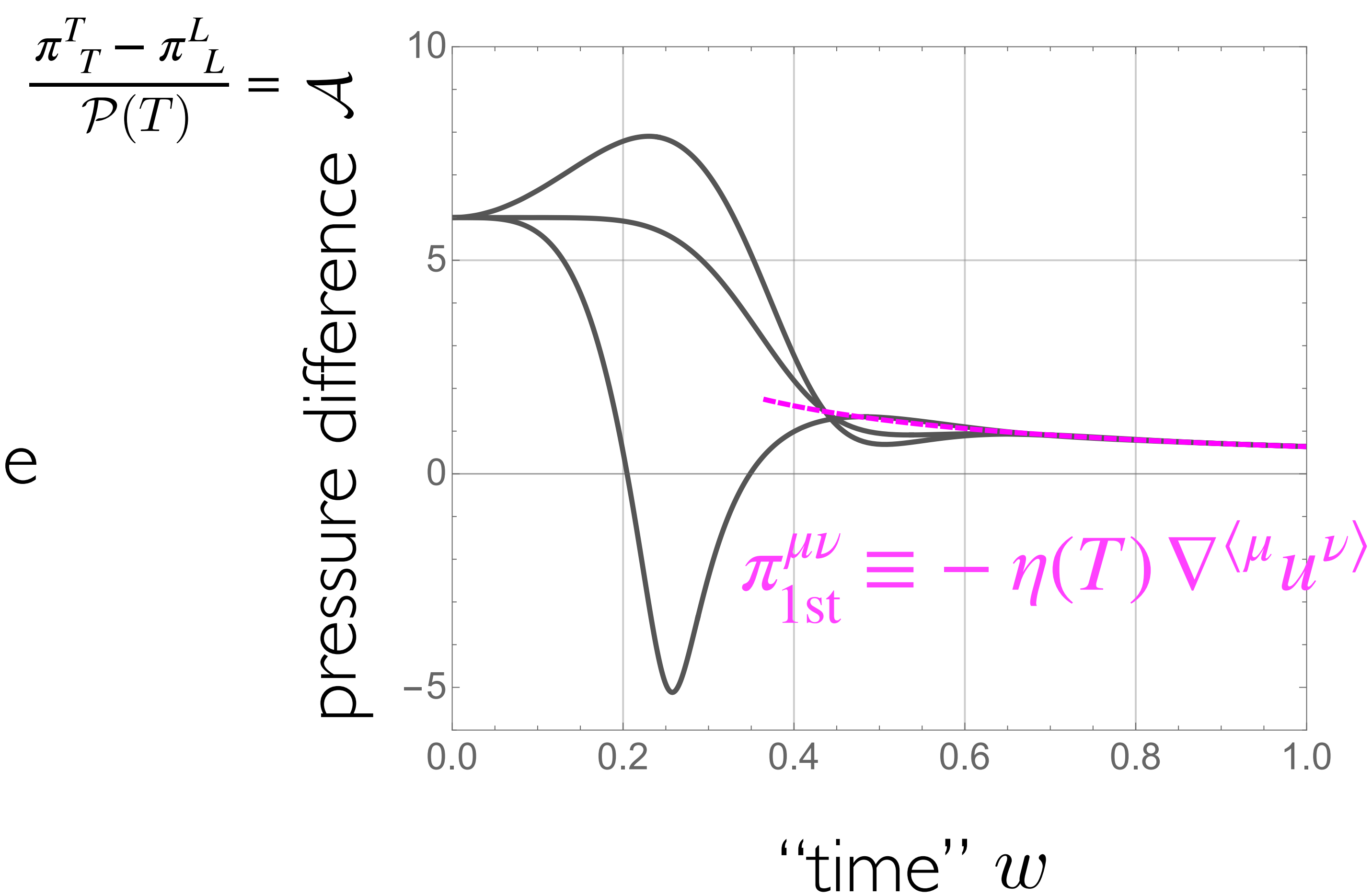

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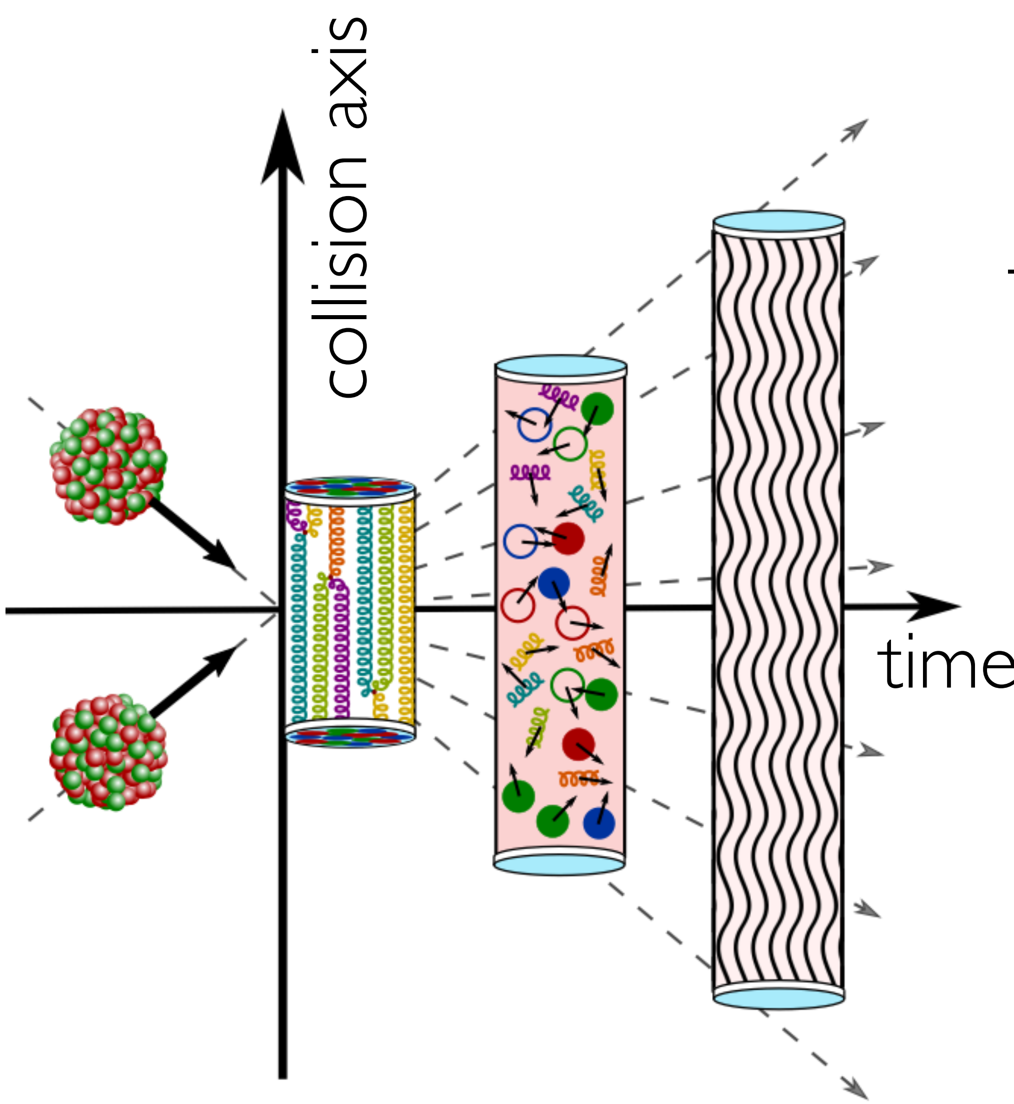

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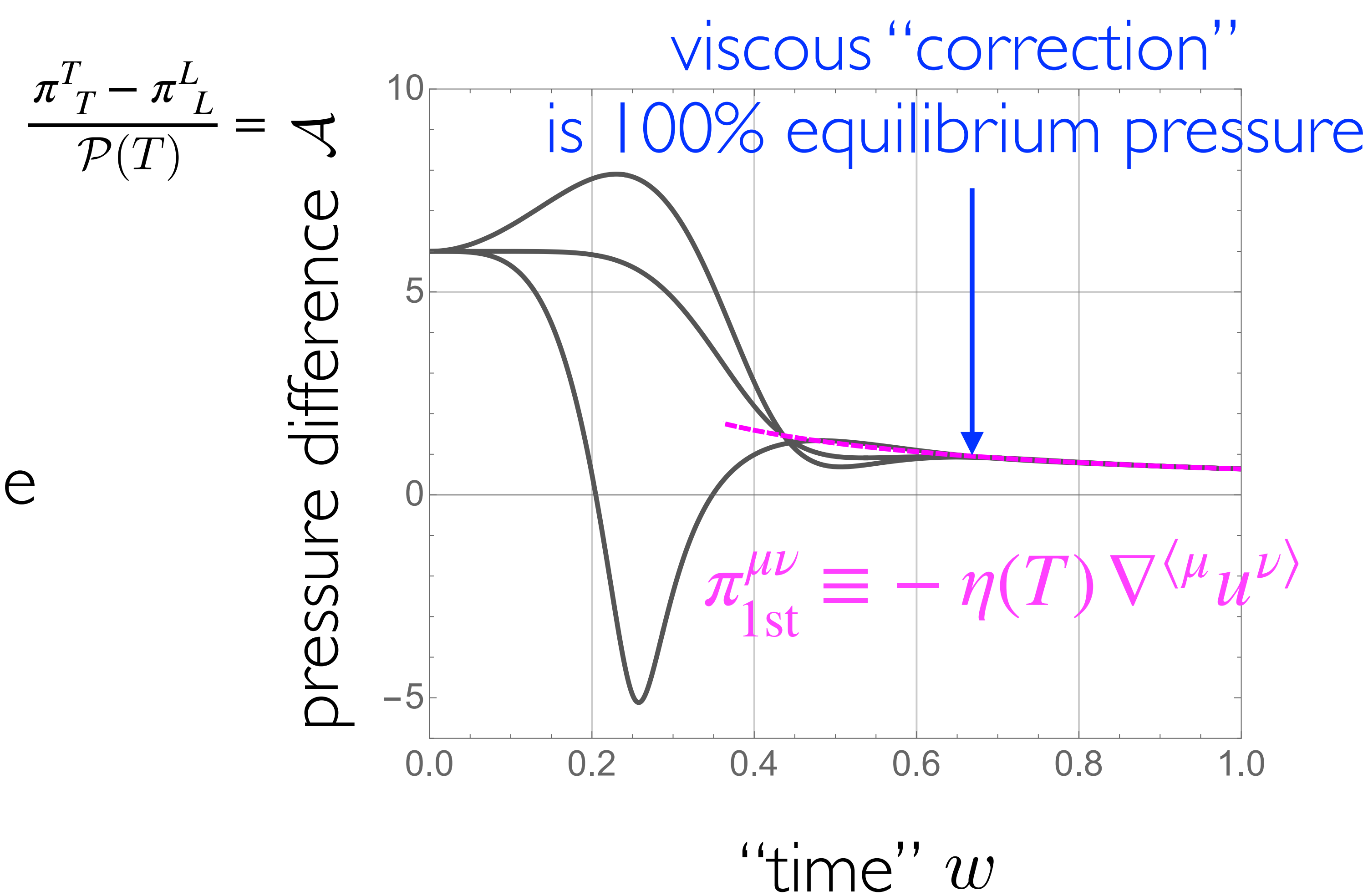

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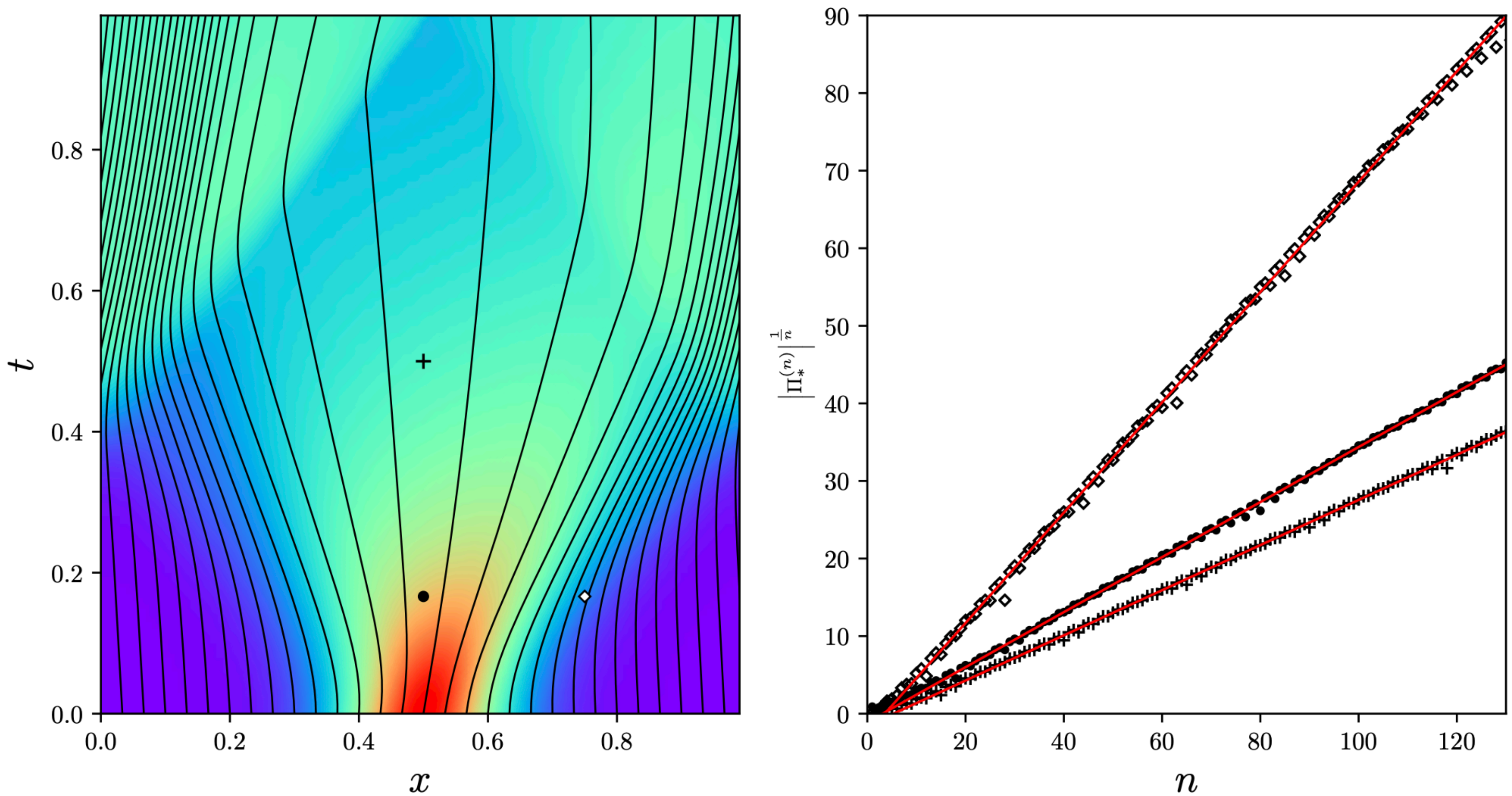
2011 ⟶ 2025



Hydro constitutive relations generally diverge factorially on-shell

1302.0697 with Janik, Witaszczyk; **1503.07514** with Spaliński;
2110.07621 with Serantes, Spaliński, Svensson, Withers

Optimal or near-optimal truncation can happen to have large derivative terms

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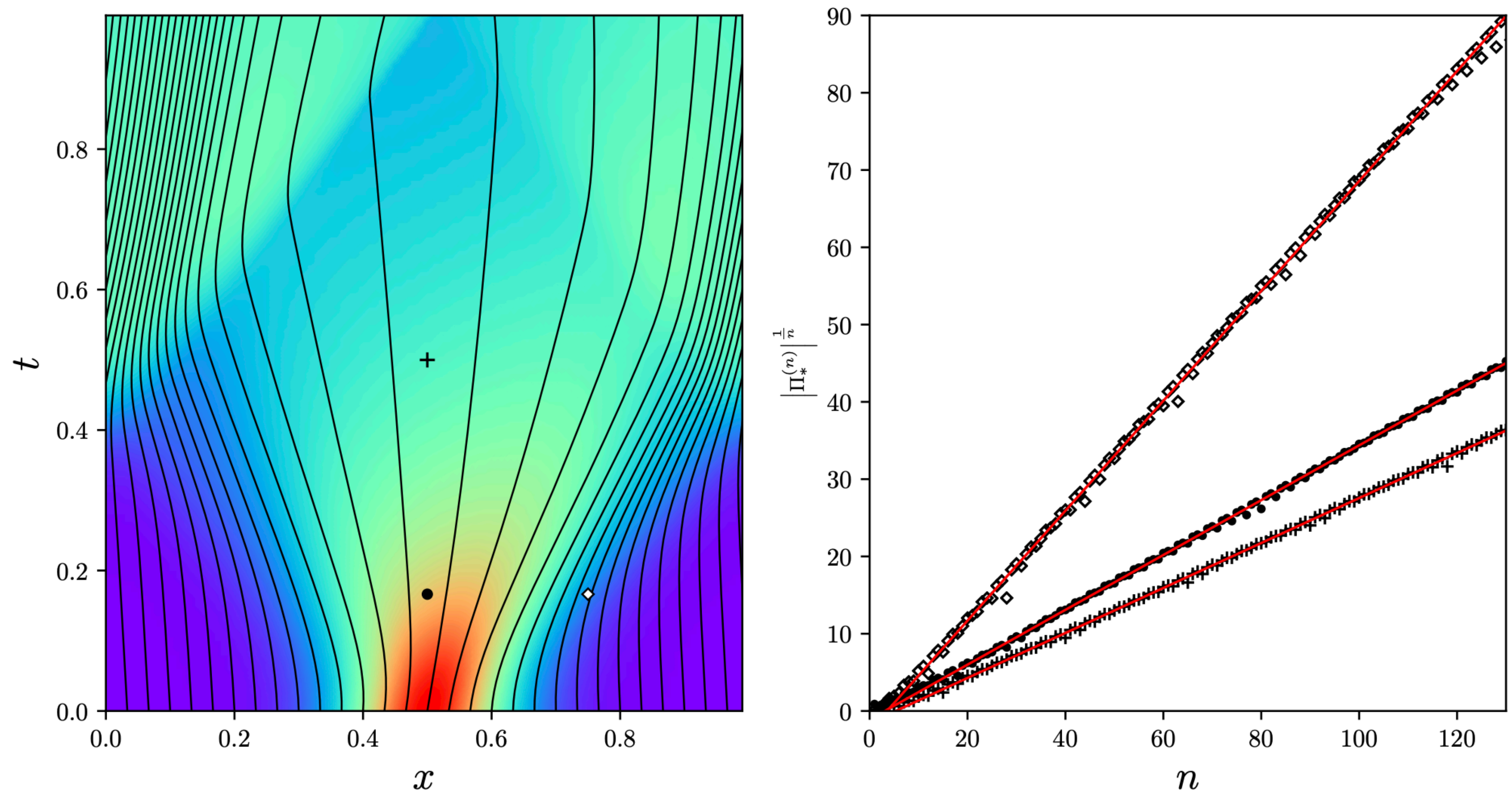

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2025-

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Ingredient I:
nonthermal fixed points

Isotropic nonthermal fixed points

e.g. **1810.08143** by Schmied, Mikheev and Gasenzer
or **2005.12299** with Berges, Mazeliauskas, Venugopalan

Conditions:

weak coupling + overoccupied initial states $f(t=0,p) \gg f_{eq}(p)\Big|_{\text{same energy density}}$

Outcome:

simulations + cold atom experiments show prolonged self-similar time evolution

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \text{ with } A(t) = \left(\frac{t-t_*}{t_0}\right)^{\alpha} \text{ and } B(t) = \left(\frac{t-t_*}{t_0}\right)^{\beta}$$

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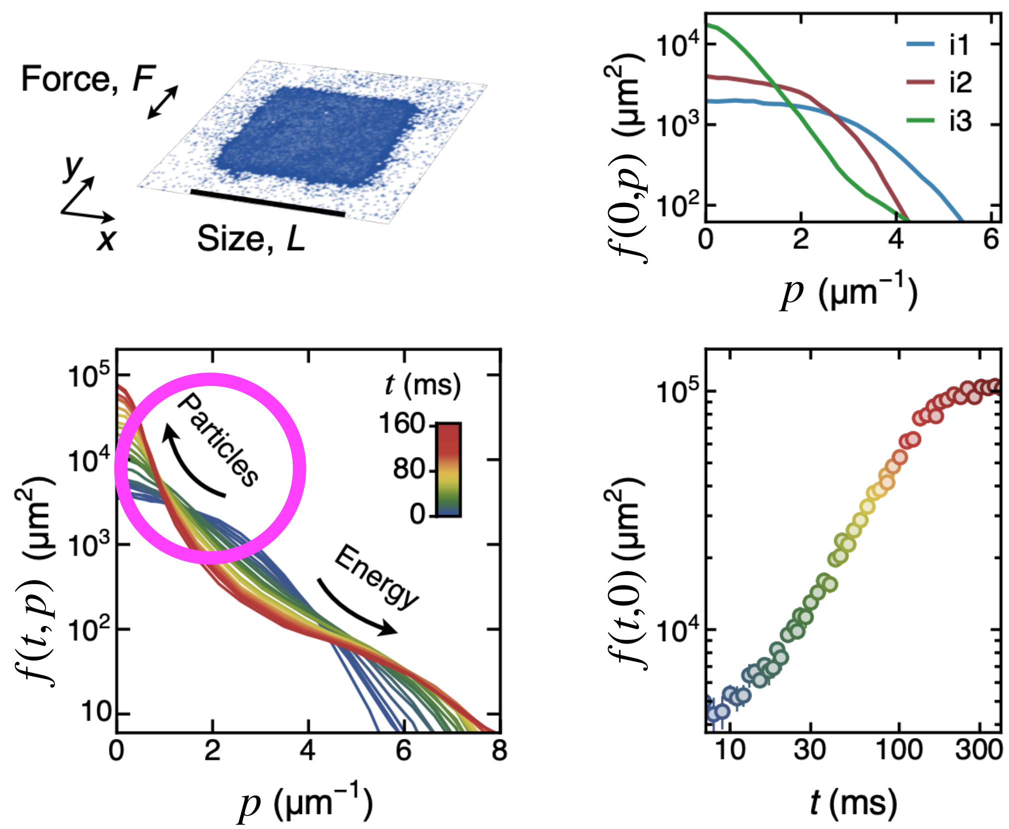
The flagship cold atom experiment

2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ^{39}K in the lowest hyperfine state, confined in a square box trap of size $L = 50\,\mu\mathrm{m}$ [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions ($a \to 0$) and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1–i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \to 30\,a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k)\,\mathrm{d}k$, where $\varepsilon = 2\pi\hbar^2 k^3 n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_\mathrm{B} = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].

$$t_\mathrm{uni} \equiv t - t_*$$

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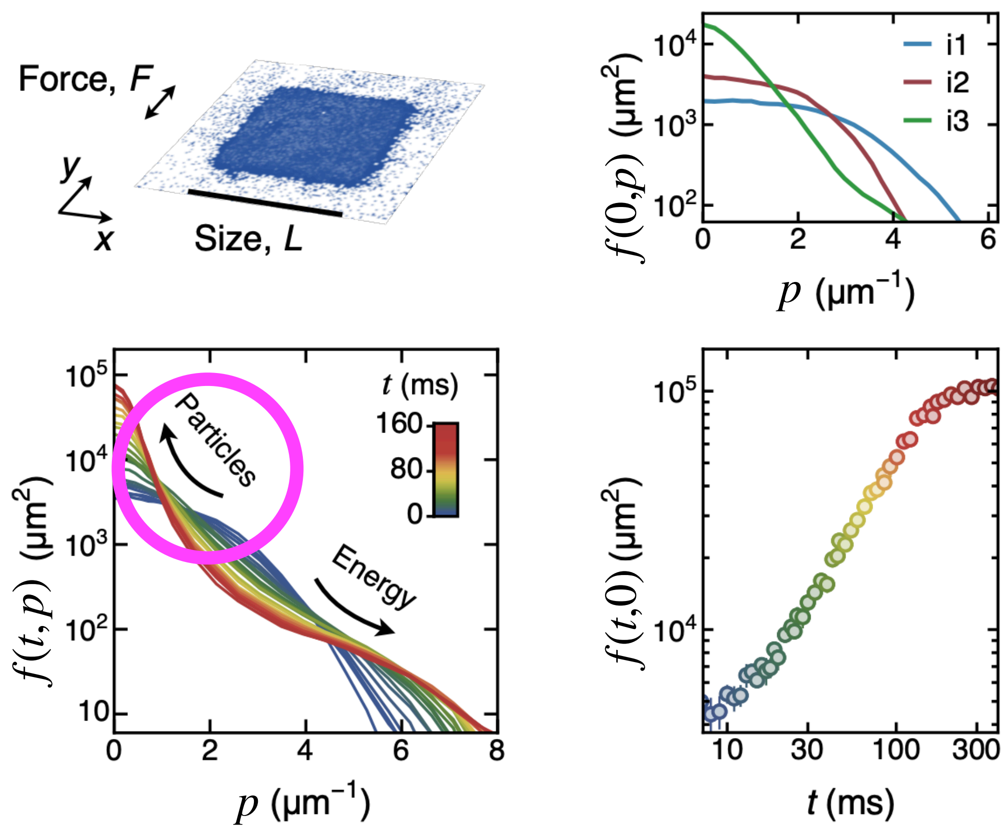

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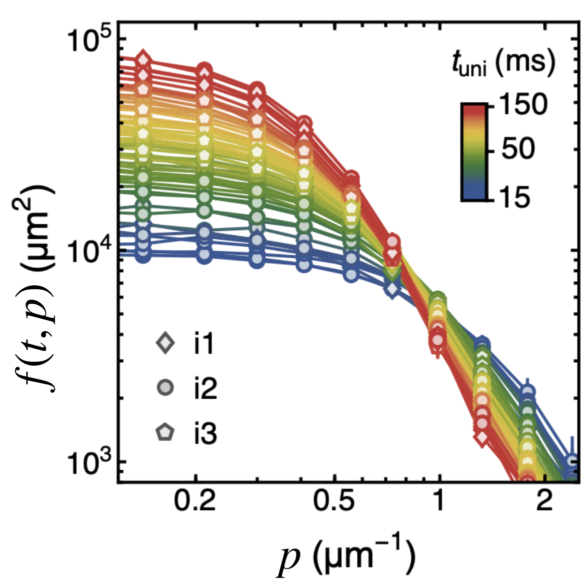

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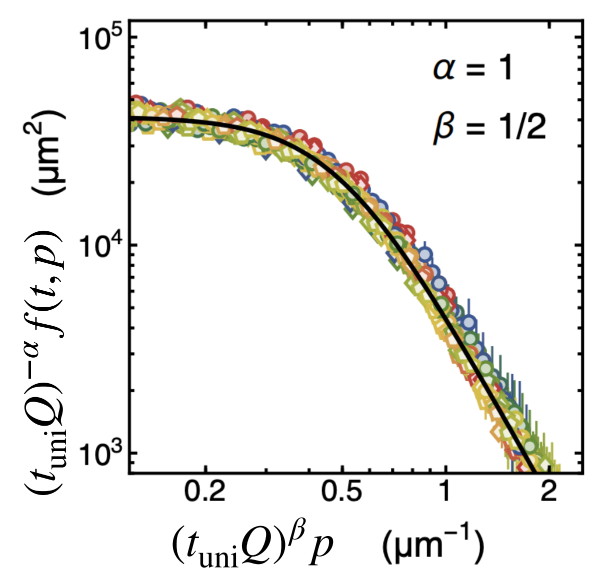

Comments on nonthermal fixed points

Nonthermal fixed points are intrinsically far from equilibrium

$$\left((t-t_*)\,Q\right)^{\alpha} \times f_s\left(\left((t-t_*)\,Q\right)^{\beta} p\right) \neq f_{eq} + \delta f$$

They are of broad th relevance: + cosmology + ultrarelativistic nuclear collisions*

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Ingredient II: quasinormal modes

Normal modes

Wave equation in a cavity $\begin{aligned} \Box_{\text{cavity}}\, \phi &= 0 \\ \phi\big|_{\text{bdries}} &= 0 \end{aligned}$ $\longrightarrow$ spectrum of normal modes e.g.

Dirichlet: $\phi(t, L) = 0$

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$$\Big\downarrow\; -\partial_t^2 \phi + \partial_x^2 \phi = 0$$

$\phi_{\text{NM}} \sim e^{-i\omega_{\text{NM}} t} \sin\left(n\pi x/L\right)$ with $\omega_{\text{NM}} = \pm\, n\pi/L$

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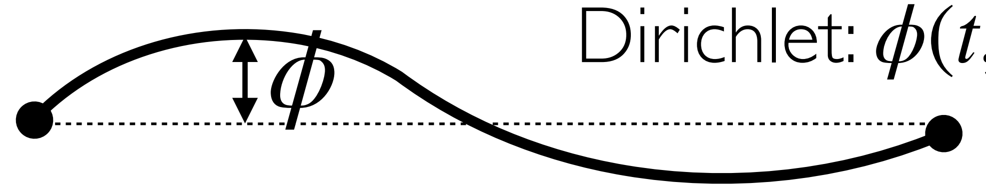

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Quasinormal modes e.g. **0905.2975** by Berti, Cardoso and Starinets

Option I: the equation breaks time symmetry explicitly, e.g.

$$\partial_t \phi - D\partial_x^2 \phi = 0 \quad \text{on a line} \quad \phi_{\mathrm{QNM}} \sim e^{-i\omega_{\mathrm{QNM}} t} e^{ikx} \quad \text{with} \quad \omega_{\mathrm{QNM}} = -iDk^2 \in \mathbb{C}$$

Option II: the boundary condition makes the problem non-Hermitian

ϕ purely ingoing at the horizon

black hole classically only absorbs : $\Box_{\text{black hole}}\, \phi = 0$ +

$\longrightarrow$ a tower of $\omega_{\mathrm{QNM}} \in \mathbb{C}$

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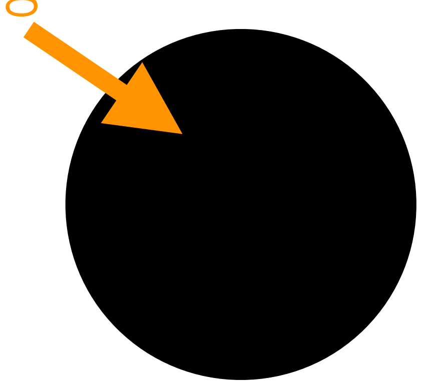

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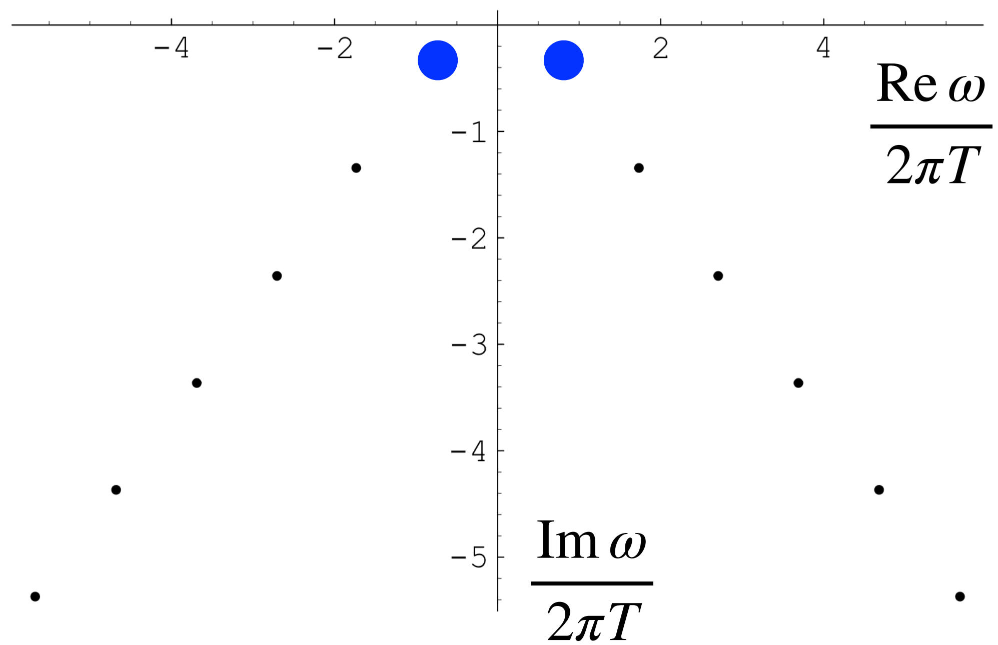
Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\, t + i\,\vec{q}\cdot\vec{x}}$

plot for $q = 2\pi T$



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→ lots of short-lived excitations

→ a few long-lived hydrodynamic modes

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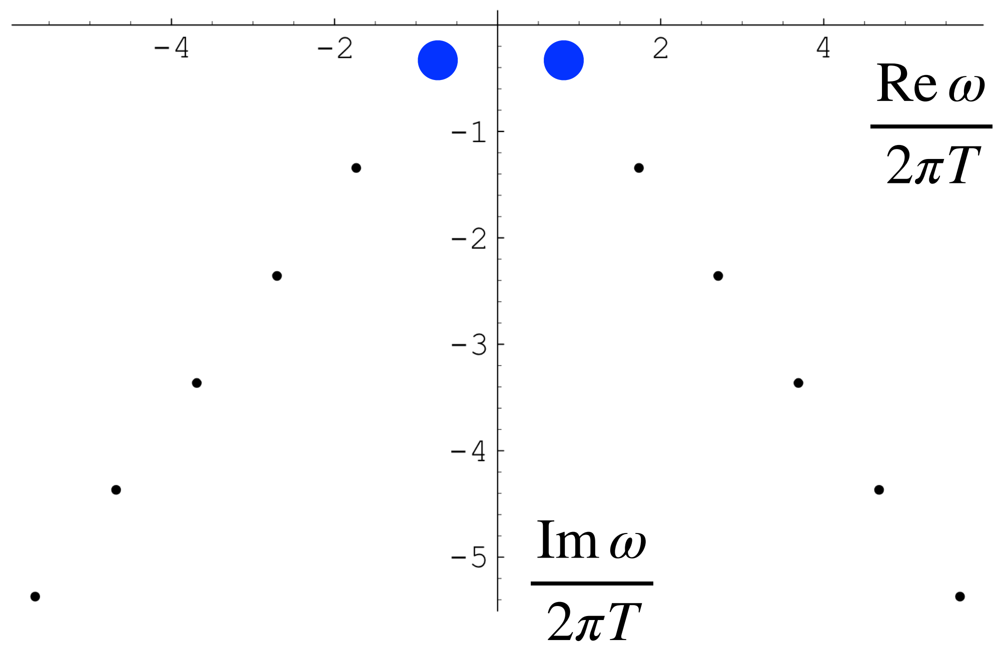

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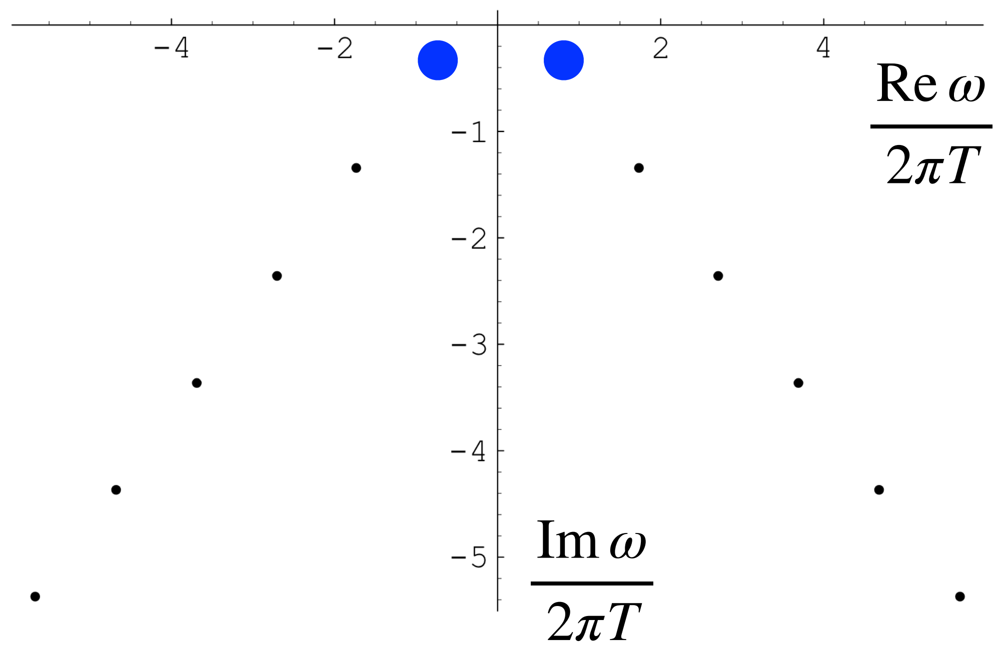

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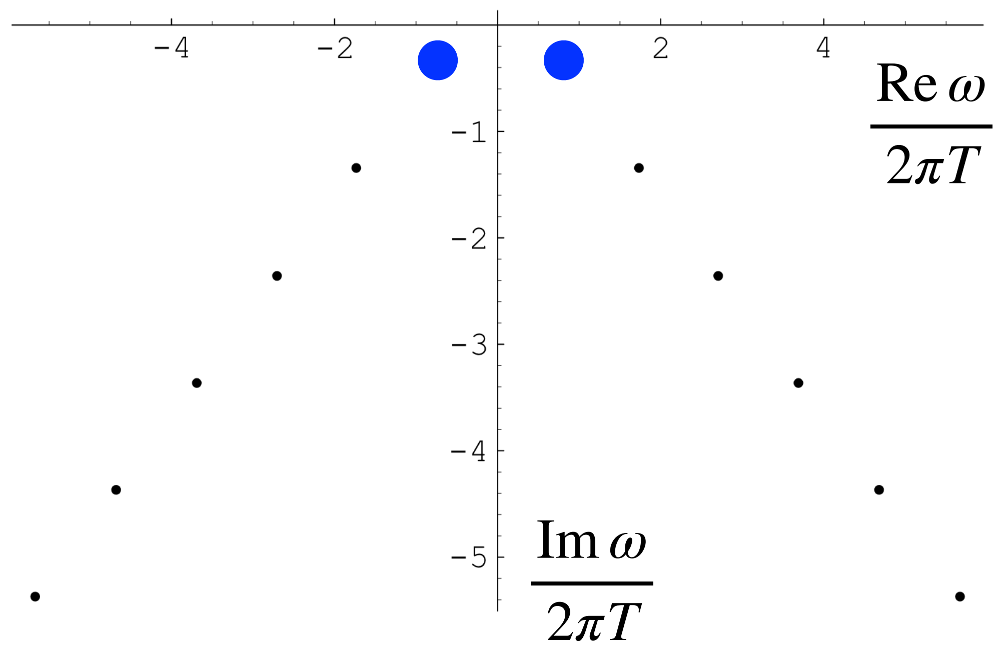

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Relativistic hydrodynamics

e.g. Florkowski, Heller & Spaliński 1707.02282

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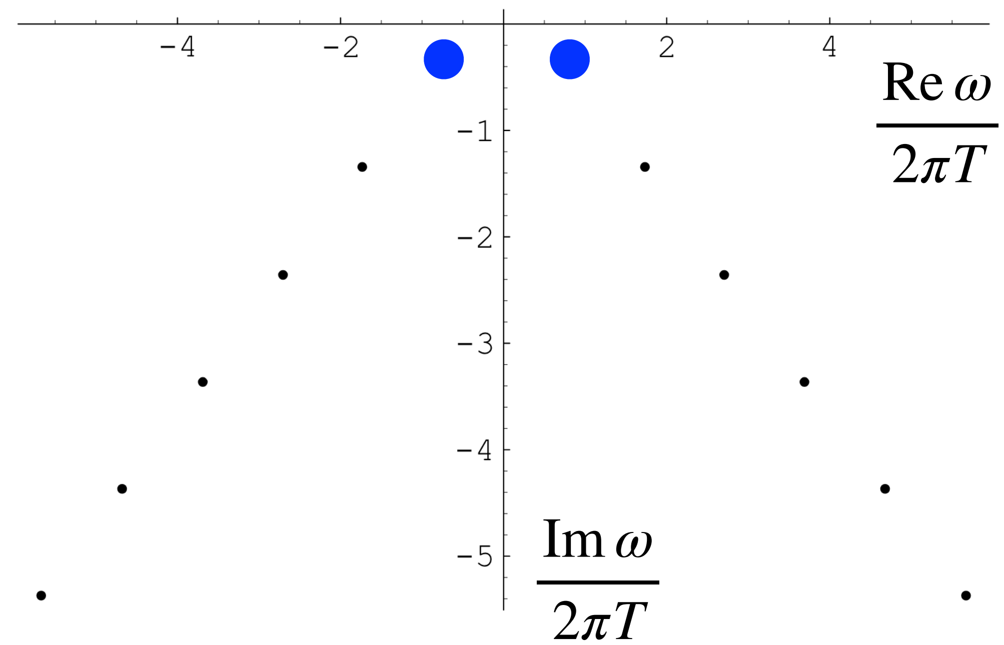

Relativistic Navier-Stokes equations dictate the properties of sound propagation:

$$\omega = \pm\, c_s\, q - i \frac{4}{3T}\frac{\eta}{s} q^2 + \ldots$$

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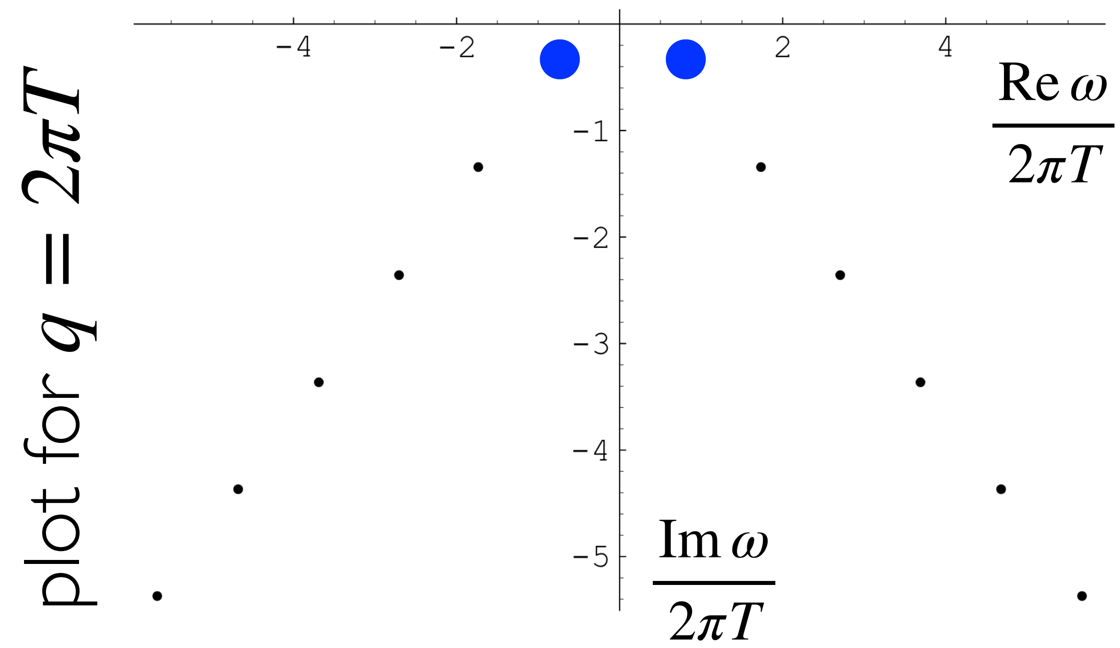

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Shear viscosity across systems

The KSS bound conjecture

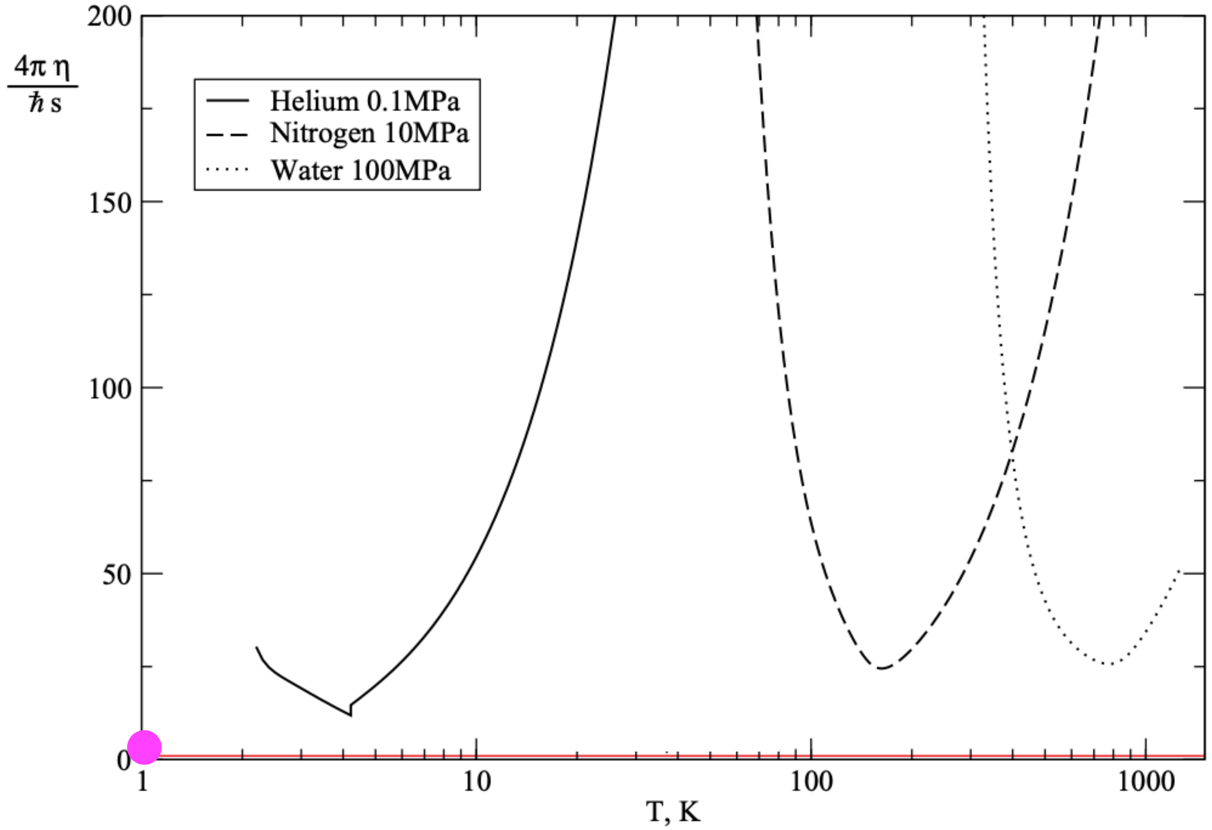
$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

hep-th/0405231 by Kovtun, Son, Starinets

$$\frac{\eta}{s} \overset{?}{\geq} \mathcal{O}\left(\frac{1}{4\pi}\right)$$

0812.2521 by Buchel, Myers, Sinha

:



Result I:
transient QNMs of nonthermal fixed points

validity: overall picture is general, implementation small angle 2-2 gluons

Key idea behind 2502.01622 with De Lescluze

There is sense in which nonthermal fixed points are static: $f_s(\bar{p})$

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Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

On a nonthermal fixed point:

$$f(t,p) \approx (t_{\rm uni}Q)^{\alpha} f_{\rm scaling}\left[(t_{\rm uni}Q)^{\beta}\, p/Q\right]$$

with $t_{\rm uni} \equiv t - t_*$

directly before that:

$$(t_{\rm uni}Q)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[(t_{\rm uni}Q)^{\beta}\, p/Q\right] + \sum_{\Omega} (t_{\rm uni}Q)^{-i\Omega}\, \delta f_{\Omega}\left[(t_{\rm uni}Q)^{\beta}\, p/Q\right]$$

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$$f(t,p) \approx (t_{\rm uni}Q)^{\alpha} f_{\rm scaling}\left[(t_{\rm uni}Q)^{\beta} p/Q\right]$$

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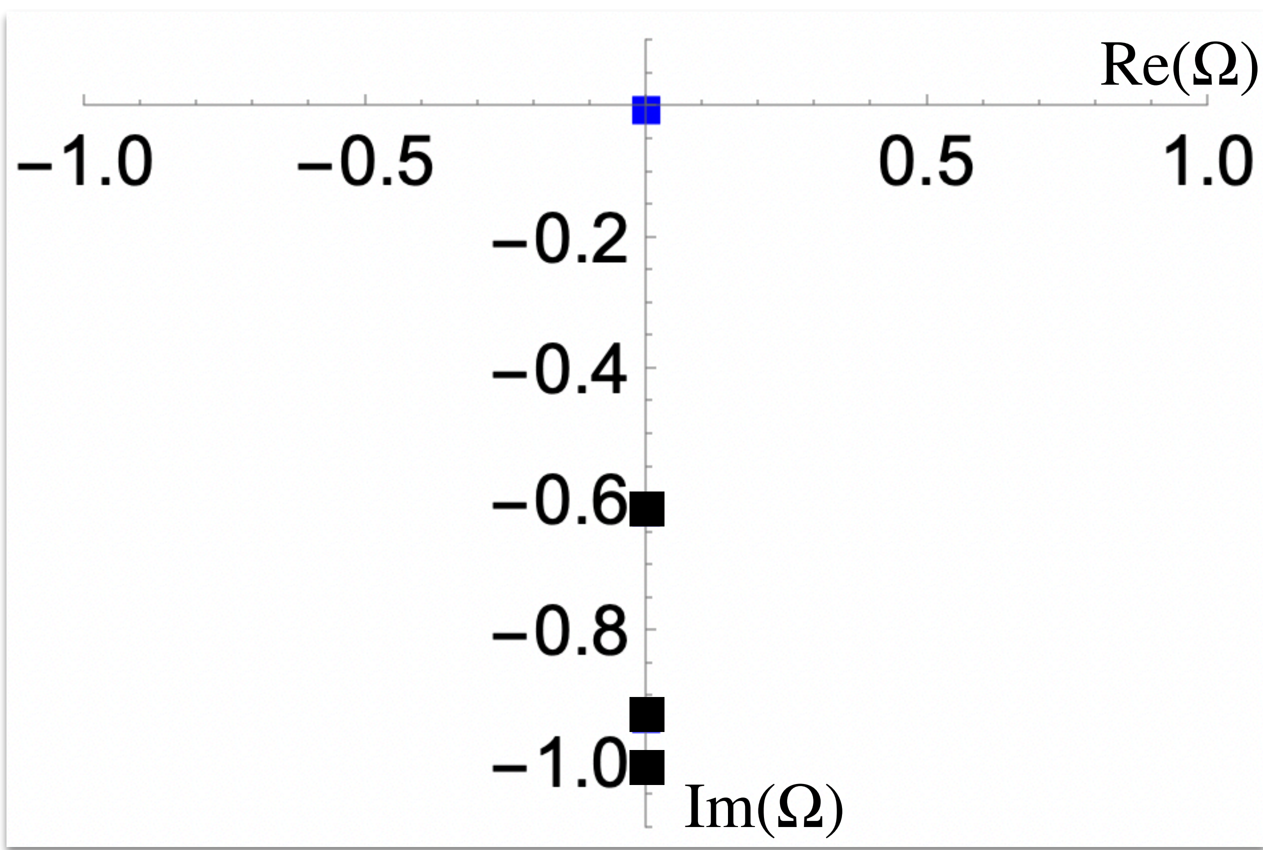
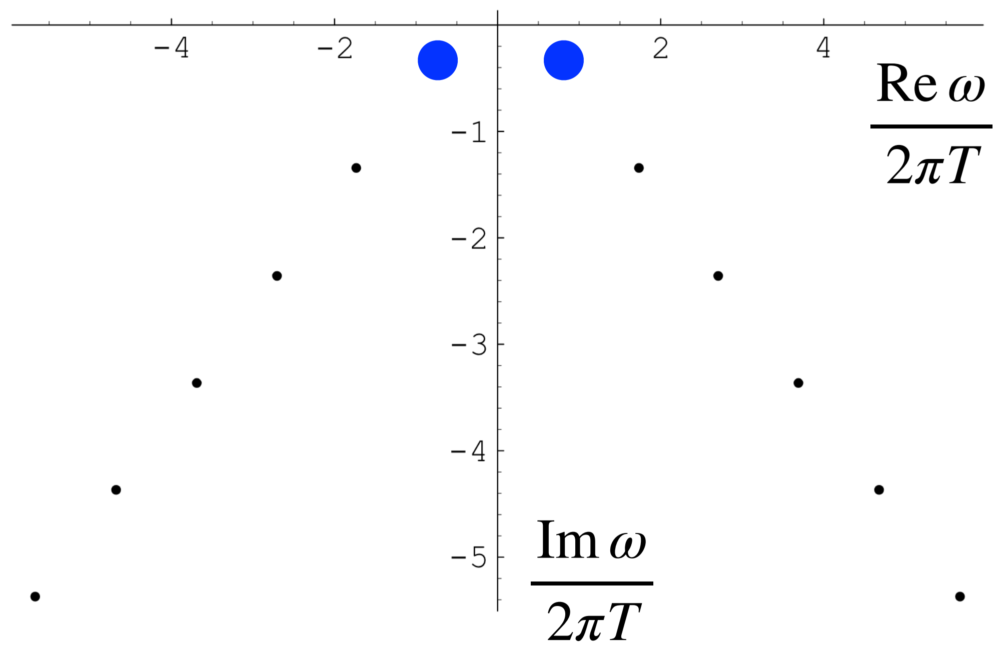


Holographic quasinormal modes (QNMs)

Horowitz and Hubeny **hep-th/9909056**; Kovtun and Starinets **hep-th/0506184**

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\,\omega\,t + i\,\vec{q}\cdot\vec{x}}$

plot for $q = 2\pi T$



Consequences for thermalization → lots of short-lived excitations

→ a few long-lived hydrodynamic modes

Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

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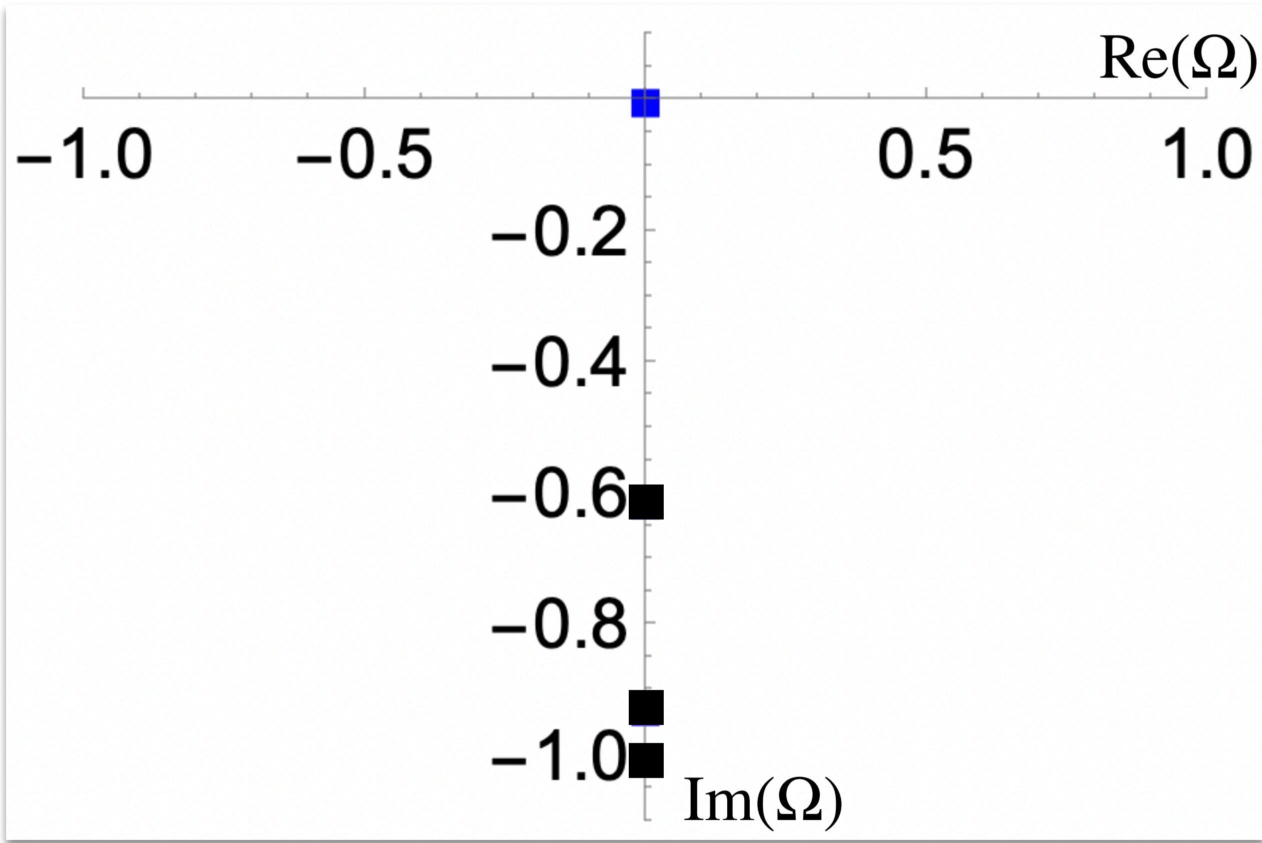

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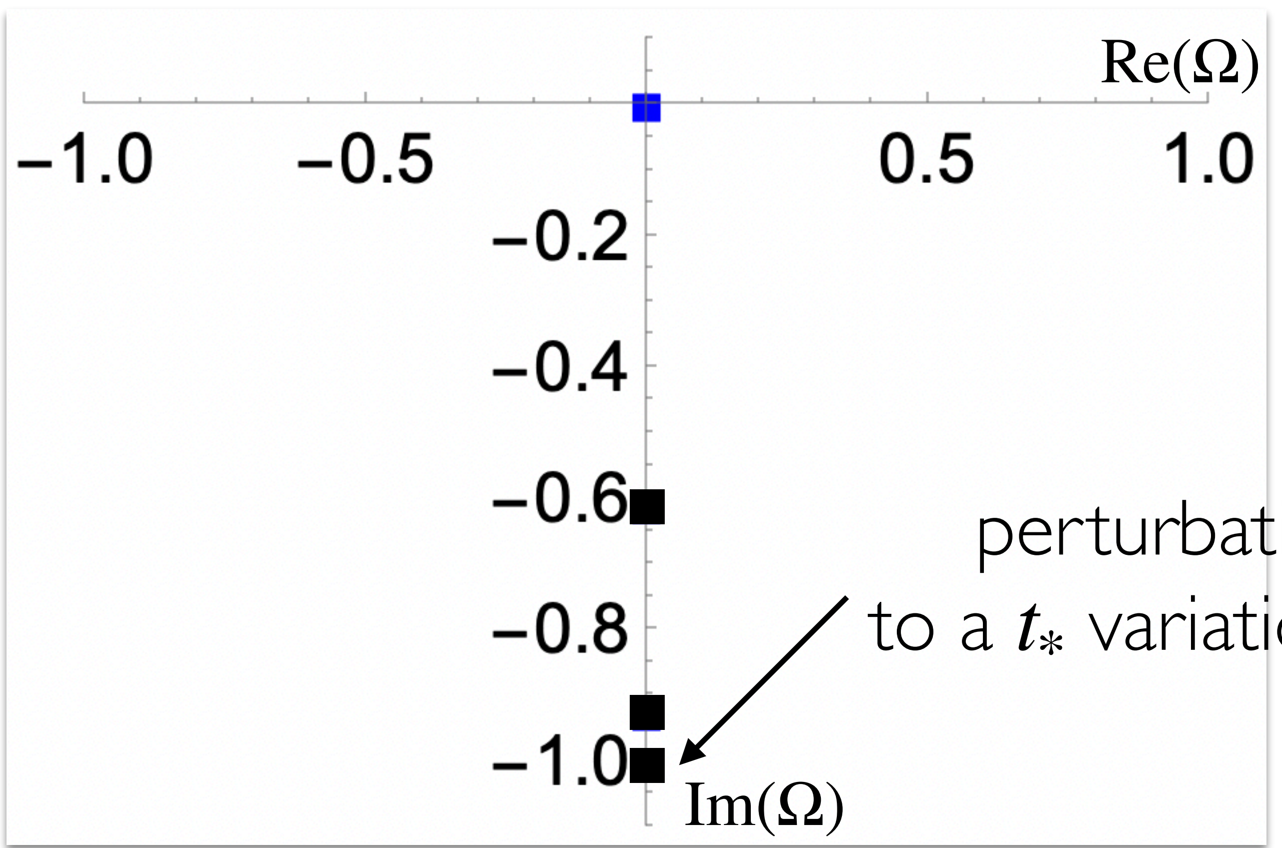
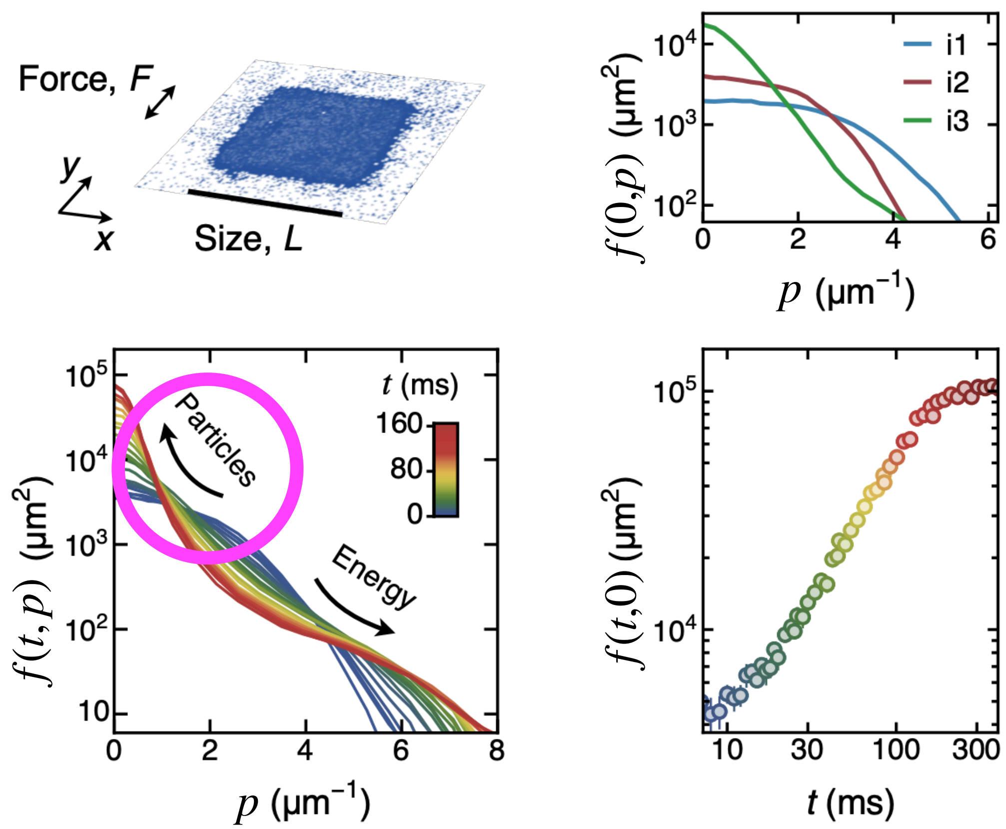


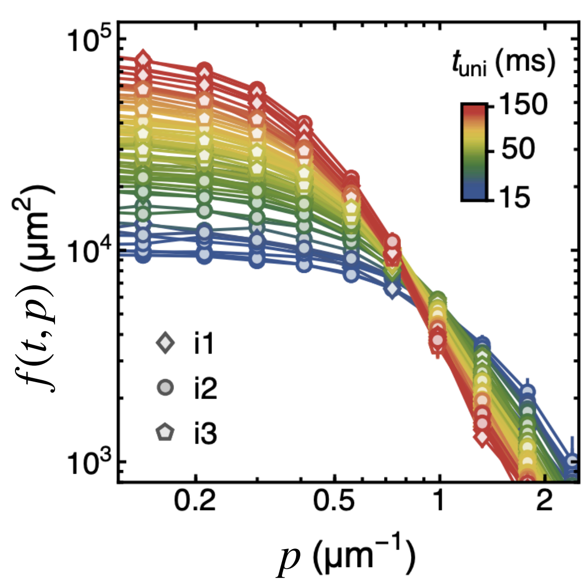
perturbation corresponding to a t_* variation of the background

The flagship cold atom experiment

2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic



We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ^{39}K in the lowest hyperfine state, confined in a square box trap of size $L = 50\,\mu$m [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions ($a \to 0$) and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1–i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \to 30\,a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k)\,\mathrm{d}k$, where $\varepsilon = 2\pi\hbar^2 k^3 n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_B = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].



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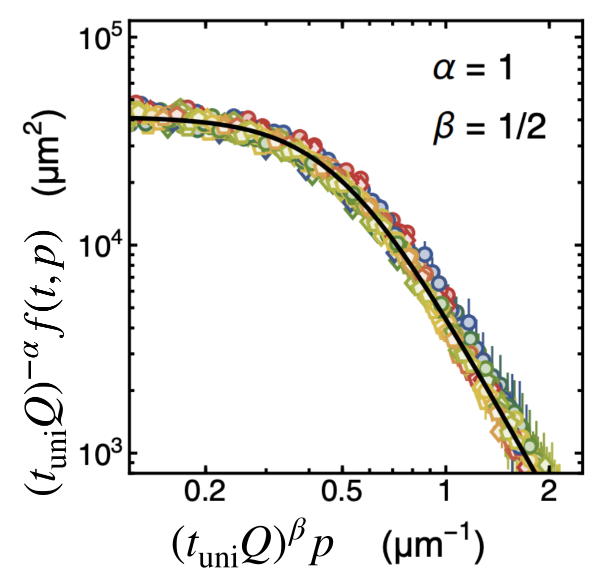

Result II:
hydrodynamics of nonthermal fixed points

validity: overall picture is general, implementation 2-2 scalars

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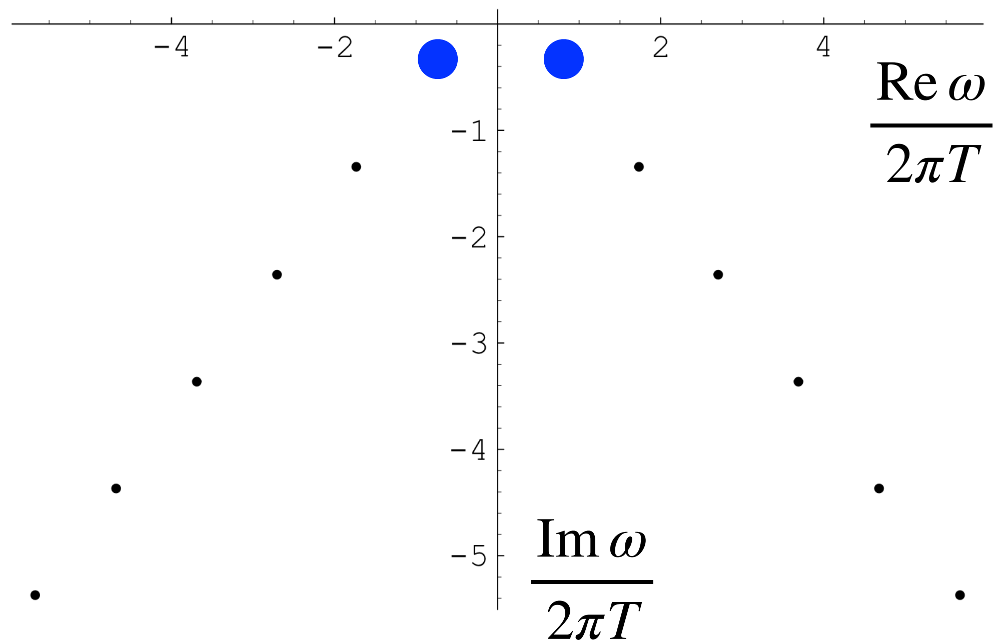

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Hydrodynamics of nonthermal fixed points

2504.18754 with Berges, Denicol and Preis

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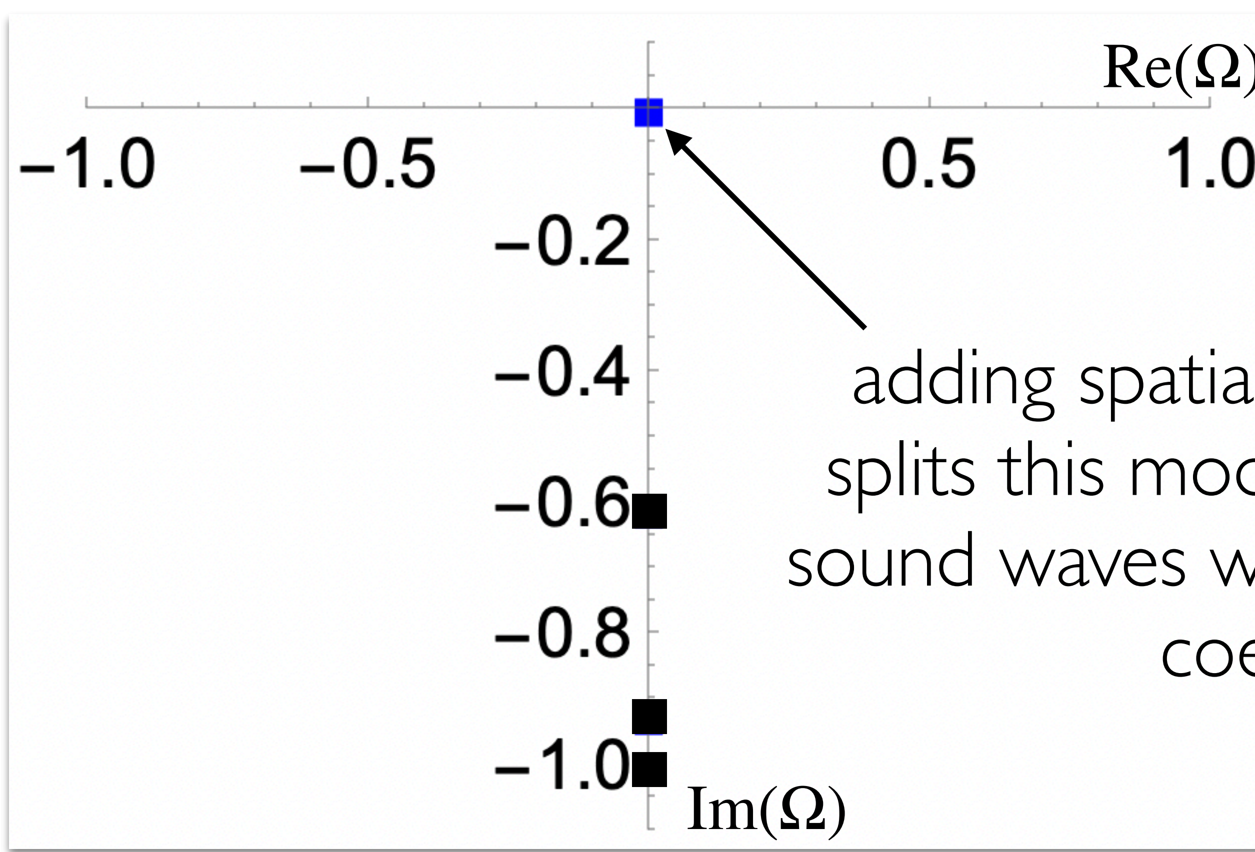


adding spatial momentum e^{iqx} splits this mode into propagating sound waves with scaling transport coefficients

What we do in practice

2504.18754 with Berges, Denicol and Preis

Input:

$$f(t,p) = \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \delta f(x^\mu, p)$$

with $\delta f(t,p) \sim p_\mu p_\nu \pi^{\mu\nu}(x^\alpha)$

Method: take truncated low moments expansion of the Boltzmann equation that was used before to derive MIS/DNMR-type equations

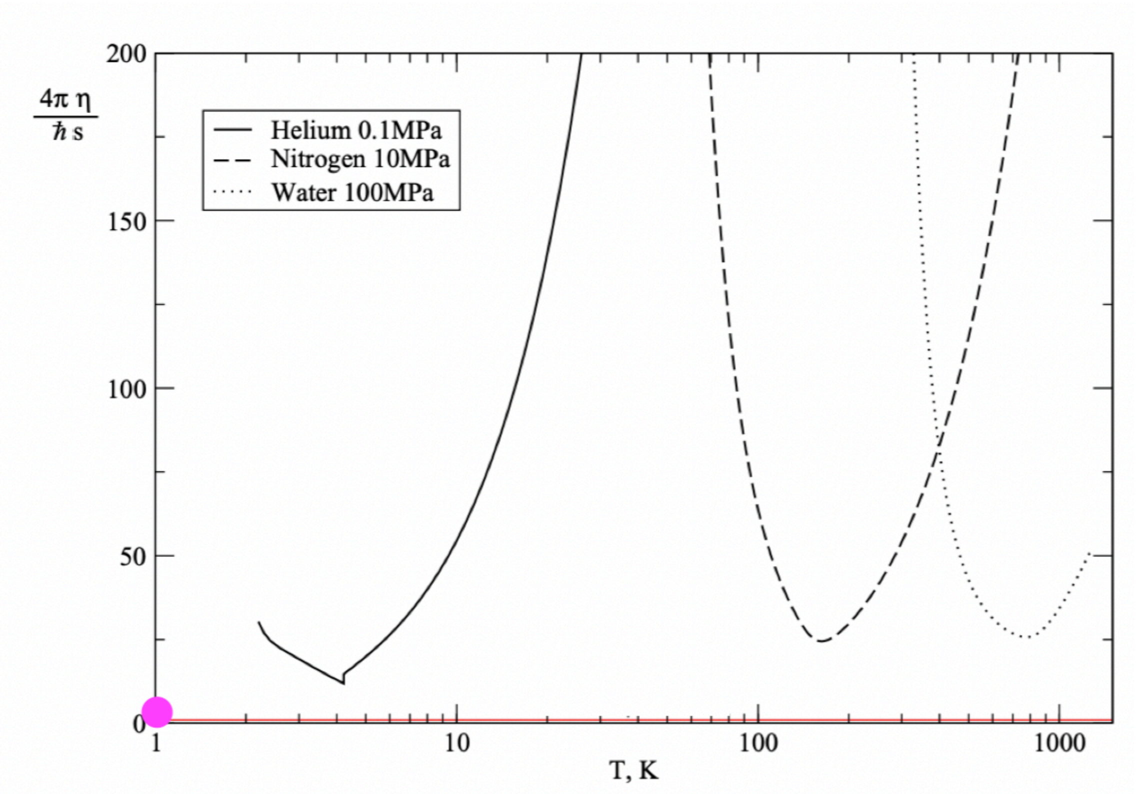
Outcome for 2-2 scattering : $\tau_\pi(t) D\pi^{\mu\nu} = -\pi^{\mu\nu} + \eta(t)\sigma^{\mu\nu} + \ldots$ with

$$\eta(t) \sim (t_{\rm uni}/t_{\rm ref})^1$$

$$\frac{\eta(t)}{\tau_\pi(t)} = \frac{4}{15} \times \text{(energy density)}$$

Beyond η/s 2504.18754 with Berges, Denicol and Preis

Comparing the nonthermal fixed point liquid with near-equilibrium liquids requires going beyond the η/s paradigm, as this ratio is now time dependent



One idea: consider $\dfrac{\eta}{2\tau_\pi \times (\text{energy density}) \times (c_s/c)^2}$

$\nearrow \approx 1$ in holography

$\rightarrow \approx 0.4$ water in room temperature

$\searrow \approx 0.4$ kinetic theory: nonthermal fixed point and equilibrium

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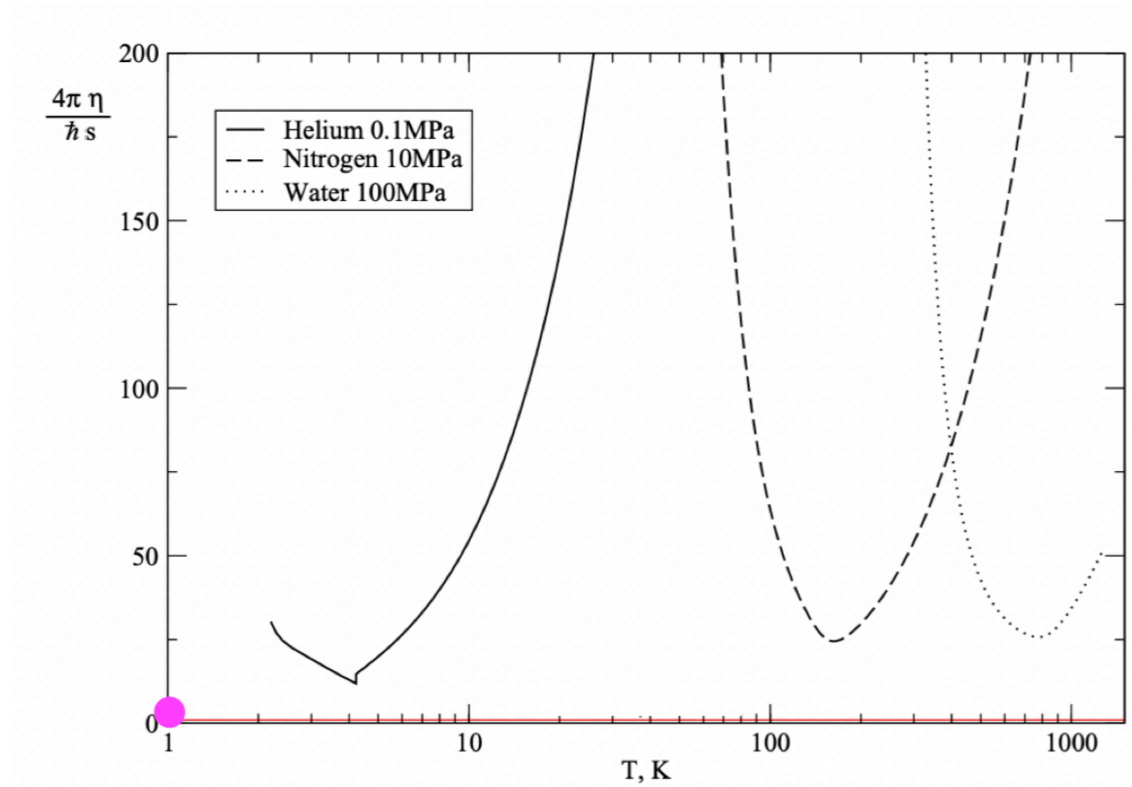

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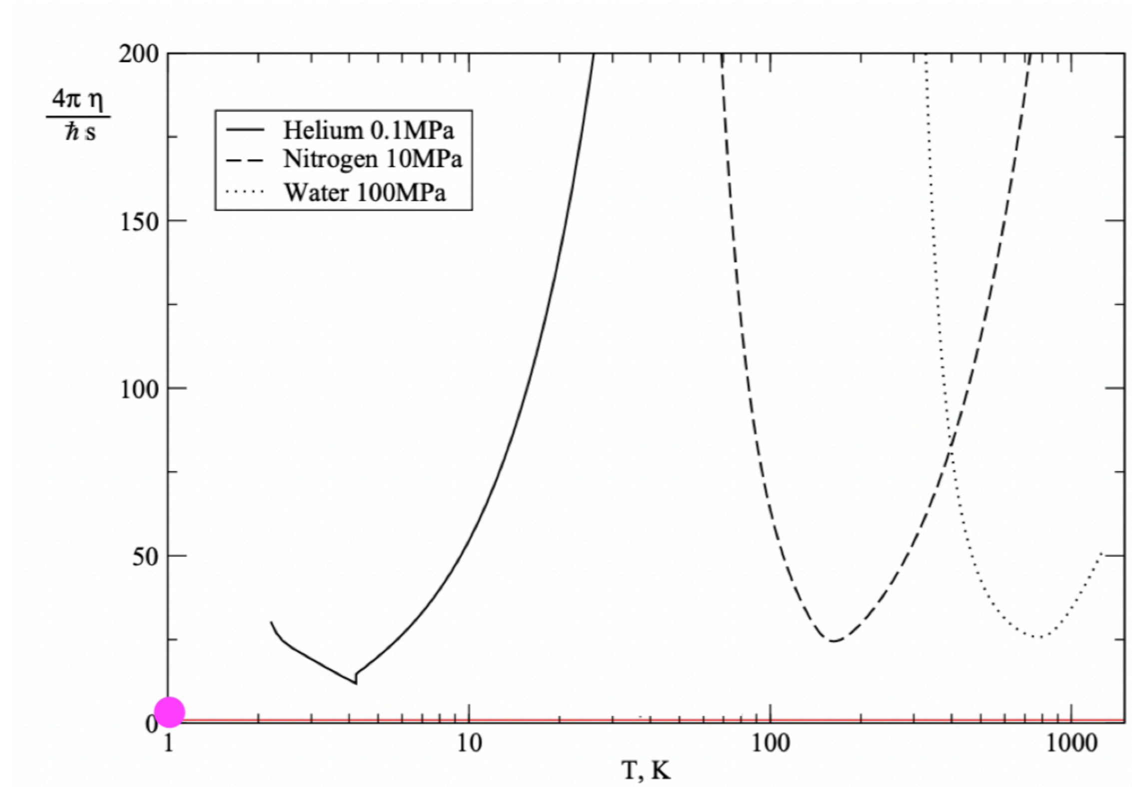

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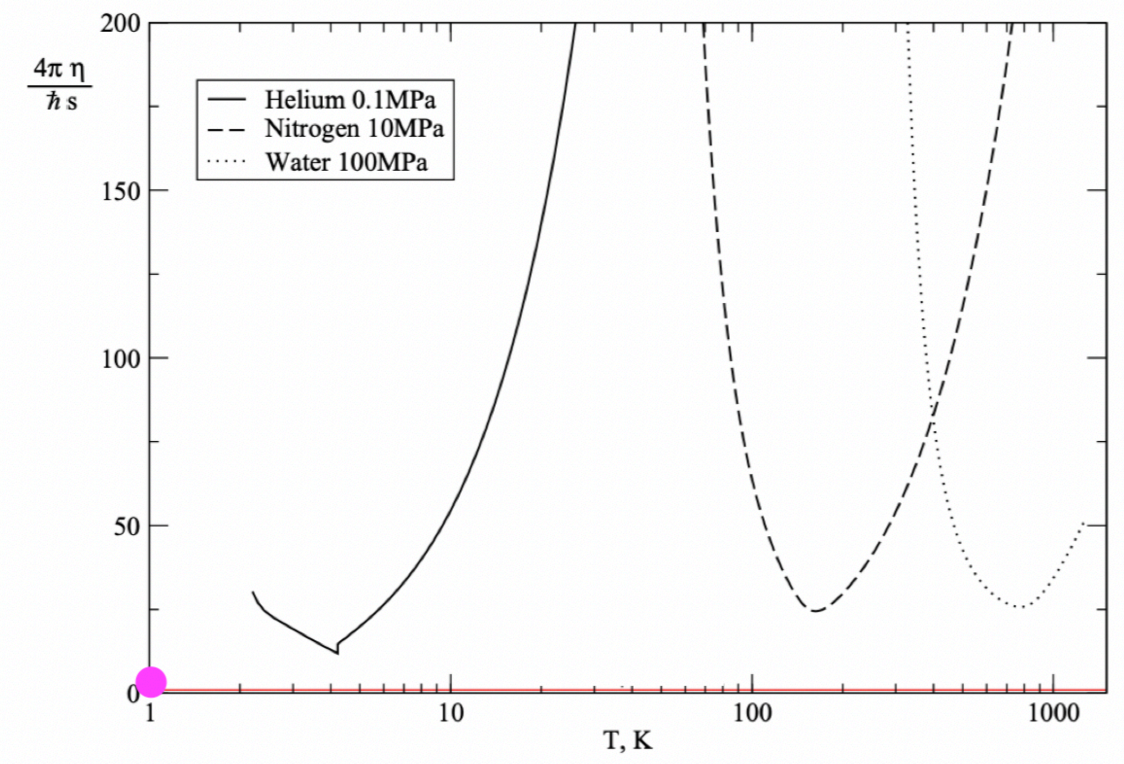

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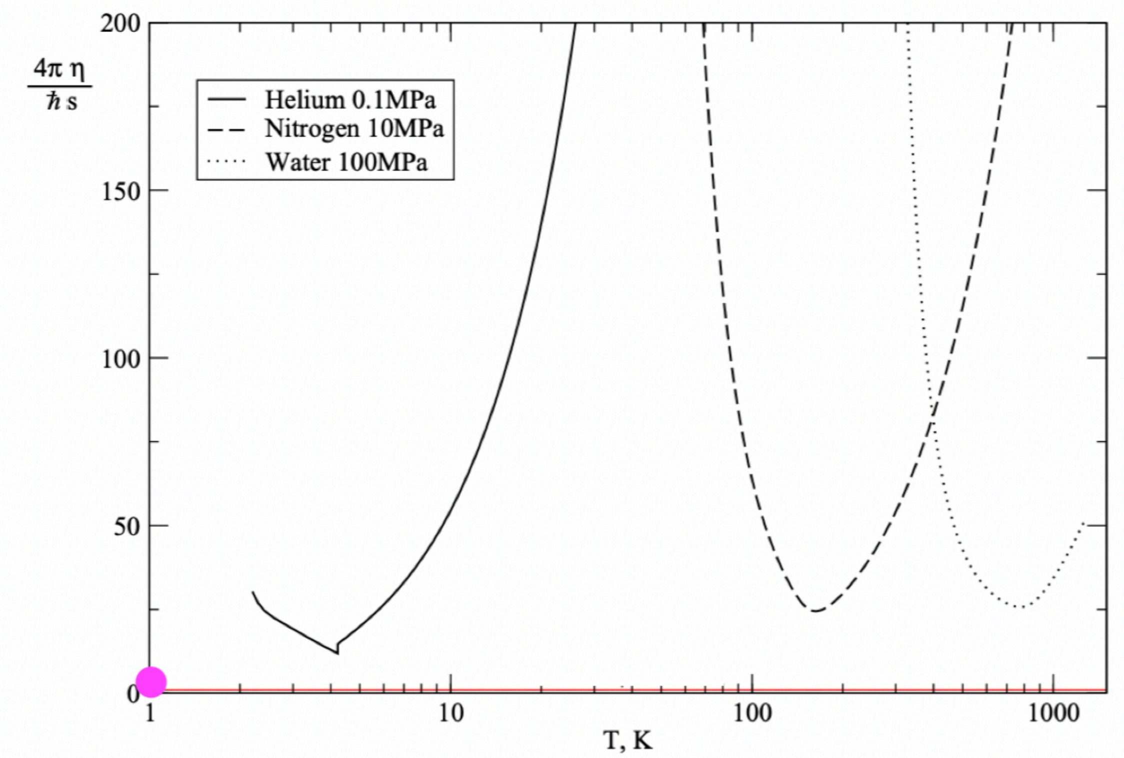

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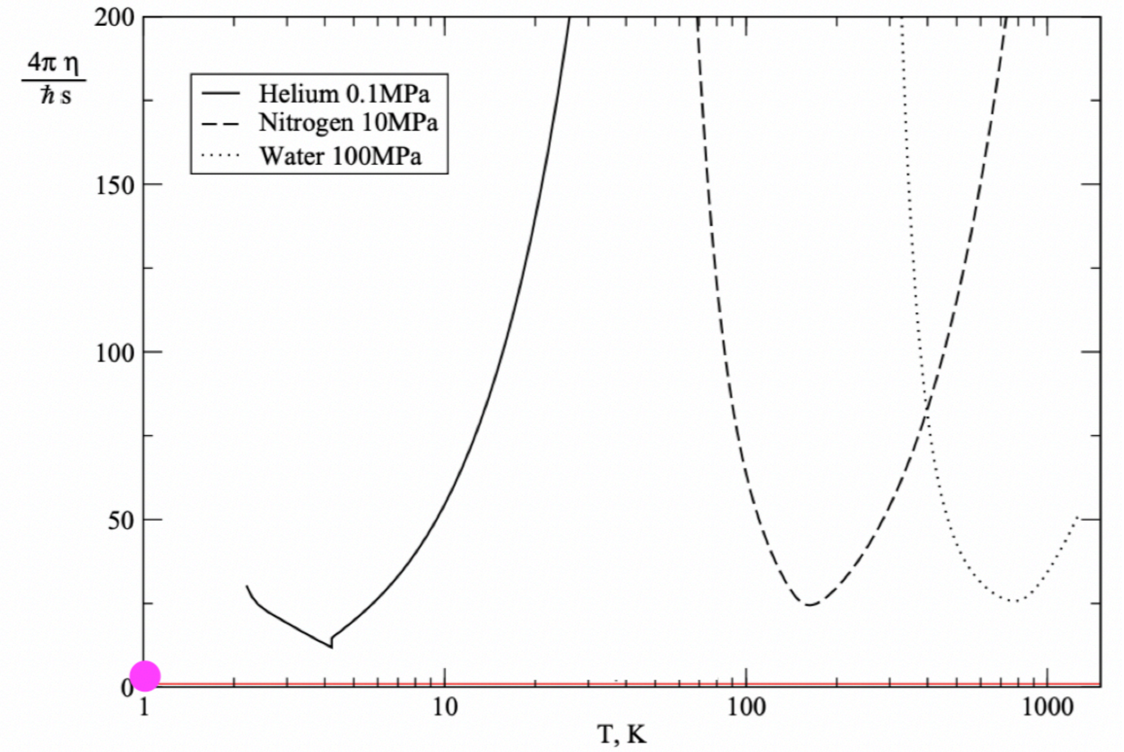

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$O(1)$

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Past 18 years or so: more subtle
- first indications in ab initio holographic sims
- vanishing radius of convergence explanation

These developments relied on the zeroth order describing local equilibrium

Past year: hydrodynamic construction around a class of far from equilibrium states

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Outlook

How the interplay between hydro and transients manifests itself in SK EFT?

2511.11555 with An, Brants and Yin

How stat. fluctuations affect the emergence of hydrodynamics?

SK EFT theory for hydrodynamics of nonthermal fixed points?

More general definition of hydrodynamics and more general SK EFTs?