EFT for electrodynamics in general media
and
Chiral anomalous MHD

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Questions to address

- electromagnetism interacting strongly with a non-equilibrium medium

How to describe the behavior of EM fields at macroscopic scales?

Insulators: dielectrics   conducting: magnetohydrodynamics (MHD)

- Suppose matter is chiral, with Adler-Bell-Jackiw anomaly

macroscopic manifestations, effects on transports ?

Well-understood: anomaly for a global symmetry
(chiral magnetic, chiral vortical effects ….)

Here: dynamical electromagnetic fields

Will use effective field theory (EFT) to approach these questions.

EFT has been a powerful paradigm for studying equilibrium dynamics in many areas of physics.

But much less used in non-equilibrium systems

Many far-from-equilibrium systems still possess local equilibrium

Will be interested in EFTs for such systems

Plan

1. Non-equilibrium EFT formalism

2. Effective field theory for dynamical electromagnetic fields in a medium

3. Chiral anomalous magnetohydrodynamics (MHD)

4. Future perspectives

Based on work with Michael Landry:

arXiv:2212.09757 and to appear
(also with Isaac Zhu)

Non-equilibrium EFT formalism is based on a series of earlier papers with Michael Crossley, Paolo Glorioso (2016-2018).

A review: 1805.09331 “Lectures on non-equilibrium effective field theories and fluctuating hydrodynamics”

with Paolo Glorioso

Challenges for non-equilibrium EFTs

1. What observables to focus

2. Nature of dynamical variables very different (slow variables)

Often those identified in phenomenological approaches not suitable for formulating an action principle

3. Symmetries

e.g. what symmetries define a fluid?

what symmetry gives local equilibrium?

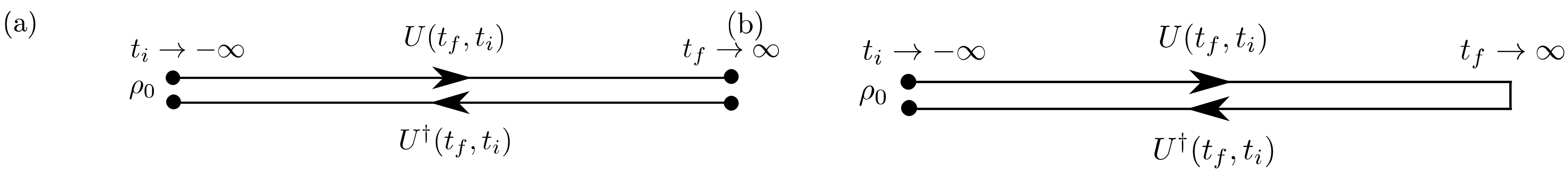
4. How to incorporate retardation and dissipation from first principle

Standard lore: Dissipative systems don't have an action formulation

$$m\ddot{x} + \nu\dot{x} = 0$$

Observables

A large class of non-equilibrium observables can be obtained from generating functionals on a closed time path (CTP) or Schwinger-Keldysh contour



$$\rho(t_f) = U(t_f, t_i)\rho_0 U^\dagger(t_f, t_i) \qquad \mathrm{Tr}\left(\rho_0 \cdots\right)$$

$$e^{W[\phi_{1i},\phi_{2i}]} = \mathrm{Tr}\left[\rho_0 \mathcal{P} e^{i\int dt\,(\mathcal{O}_{1i}(t)\phi_{1i}(t) - \mathcal{O}_{2i}(t)\phi_{2i}(t))}\right]$$

Captures all observables which do not need to run your lab backward in time.

Non-equilibrium EFT

Microscopic path integrals defined on a CTP for $W[\phi_1, \phi_2]$

Integrate out the rest

Identify χ : relevant IR degrees of freedom, slow variables

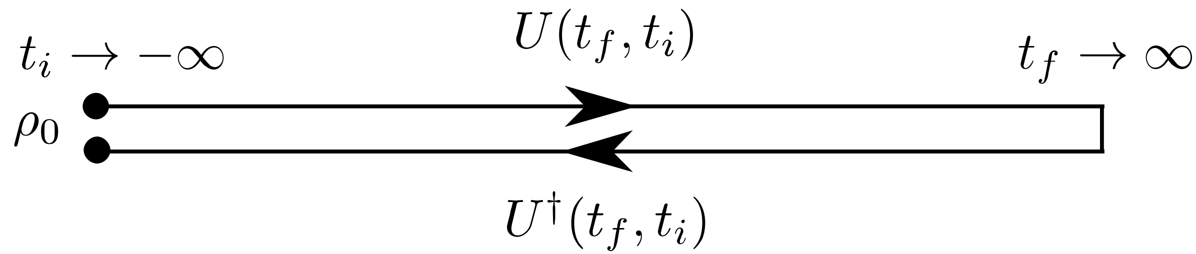
$$e^{W[\phi_1,\phi_2]} = \int D\chi_1 D\chi_2 \, e^{iS_{\rm EFT}[\chi_1,\phi_1;\chi_2,\phi_2;\rho_0]}$$

The effective action automatically incorporates retardation and dissipative effects.

Constraints from unitarity time evolution

Michael Crossley, Paolo Glorioso, HL, 2016

$$e^{iS_{\mathrm{EFT}}} =$$



1. $S^*_{\mathrm{EFT}}[\chi_1, \chi_2] = -S_{\mathrm{EFT}}[\chi_2, \chi_1]$

2. $\mathrm{Im}\, S_{\mathrm{EFT}} \geq 0$

3. $S_{\mathrm{EFT}}[\chi_1 = \chi, \chi_2 = \chi] = 0$

Not visible in Euclidean EFT

These constraints survive in the classical limit

Two significant challenges:

How to impose in the action principle the condition of local equilibrium?

How to impose microscopic time-reversal invariance in a macroscopic EFT which is not time-reversal symmetric (with dissipation)?

Dynamical KMS symmetry

Michael Crossley, Paolo Glorioso, HL, 2016, 2017
Sieberer, Buchhold, and Diehl, 2015

It turns out local equilibrium and microscopic time reversal can be imposed together by imposing an antilinear Z_2 symmetry, called dynamical KMS symmetry.

Example (non-dynamical temperature) :

$$\tilde{\chi}_1(x) = \Theta\chi_1(t - i\theta, \vec{x}), \quad \tilde{\chi}_2(x) = \Theta\chi_2(t + i(\beta_0 - \theta), \vec{x}) \ .$$

Θ : any discrete operation involving time reversal

So far, very general:

Unitarity constraints + dynamics KMS

Emergent entropy and the second law

Glorioso, HL. 2016

Combination of unitarity constraints and dynamical KMS symmetry leads to a remarkable consequence:

One can construct a local current s^μ, the "charge" of which never decreases.

This provides a field theoretical definition of entropy

In derivative expansion, the divergence of the current is everywhere non-negative.

Also gives a universal expression for entropy production.

A class of examples:

Hydrodynamics (the universal theory of conserved quantities):

For any conserved current, the hydrodynamical variables are the Stueckelberg variables for the corresponding symmetry.

EFT for a U(1) conserved current

Michael Crossley, Paolo Glorioso, HL, 2016

The sources for a conserved U(1) current are: $A_\mu, A_{a\mu}$

Hydrodynamic variables: $A_\mu + \partial_\mu \phi, A_{a\mu} + \partial_\mu \phi_a$

and in the presence of sources ($A_\mu, A_{a\mu}$), the action has the form

$$I_{\rm hydro} = I_{\rm hydro}[A_\mu + \partial_\mu \phi, A_{a\mu} + \partial_\mu \phi_a]$$

- Impose unitarity constraints and dynamical KMS
- Choice of Θ $\qquad \mathcal{T}, \mathcal{PT}, \mathcal{CT}, \mathcal{CPT}$
- Other continuous and discrete global symmetries
- invariance under (U(1) not spontaneously broken)

$$\phi \to \phi + \chi(\vec{x}), \qquad \phi_a \to \phi_a$$

A universal theory of diffusion (nonlinear)

EFT for dynamical EM field in a general medium

Consider electromagnetic field strongly couped to microscopic matter

at some finite temperature

Partial Higgs approach

Consider first the U(1) symmetry global, then we have

$$I_{\text{hydro}} = I_{\text{hydro}}[A_\mu + \partial_\mu \phi, A_{a\mu} + \partial_\mu \phi_a]$$

Now make external fields dynamical (integrate over them)

The hydro fields are now eaten,

$$I_{\text{EFT}}[A_\mu, A_{a\mu}] = I_{\text{hydro}}[A_\mu, A_{a\mu}]$$

invariance under $\phi \to \phi + \chi(\vec{x}), \quad \phi_a \to \phi_a$

Gauge symmetries of EM fields reduce to

$$A_i \to A_i' = A_i + \partial_i \chi(\vec{x})$$

We call this partial Higgs mechanism.

EFT for EM in general media

$$I_{\rm EFT}[A_\mu, A_{a\mu}] = I_{\rm hydro}[A_\mu, A_{a\mu}]$$

- Impose unitarity constraints and dynamical KMS
- Choice of Θ $\quad\quad \mathcal{T}, \mathcal{PT}, \mathcal{CT}, \mathcal{CPT}$
- invariance under (U(1) not spontaneously broken)

$$A_i \to A_i' = A_i + \partial_i \chi(\vec{x})$$

- Other continuous and discrete global symmetries

EFT of EM fields in general media

This leads to an EFT for EM fields in general media:

$$I_{\rm EFT}[A_\mu, A_{a\mu}] = I_{\rm hydro}[A_\mu, A_{a\mu}]$$

To perform derivative expansion, still needs a power counting scheme, which reflects different physics:

1. $[A_\mu] = 0, \quad [\partial_\mu] = 1, \quad [E, B] = 1$

   dielectric, EM field in an insulator

   Landry, HL, Zhu, to appear

2. $[A_0] = 0,\ [A_i] = -1,\ [\partial_i] = 1,\ [\partial_0] = 2, [B] = 0,\ [E] = 1$

   magnetic field: slow variable — MHD in a conducting medium

Magnetohydrodynamics (MHD)

MHD: a universal theory for conducting fluids in the presence of dynamical EM field.

Dynamical variables: hydro variables for conserved quantities and magnetic field $\vec{B}$

Difficulty: lack of a general principle to write down constitutive relations at strong field.

The theory just described gives a systematic formulation of MHD in the regime where fluid motions and temperature variations can be neglected.

can be extended to general situations

Landry and HL, to appear

An alternative approach: one-form symmetry

A new insight: $\vec{B}$ are charge densities of a conserved current associated with a generalized global symmetry (1-form symmetry)

Grozdanov, Hofman, Iqbal, arXiv:1610.07392

$$J^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho} \qquad \partial_\mu J^{\mu\nu} = 0$$

$$B_i = J^{0i} \qquad E_i = -\frac{1}{2}\epsilon_{ijk}J_{jk}$$

Combining the non-equilibrium EFT formalism, and the one-form symmetry realization,

→ action principle for MHD

Vardhan, Grozdanov, Leutheusser, HL

arXiv:2207.01636,
arXiv: 2408.12868

The two approaches to MHD can be shown to be equivalent.

In fact, one can be derived from the other via a dualization procedure.

Landry and HL, to appear

Note that the one-form approach:

Misses some “topological terms” and difficult to incorporate ABJ anomalies

See however, Luca Delacretaz 2606.02391

technically slightly more complicated

makes power counting more straightforward

Strong field MHD

Strong field MHD

Vardhan, Grozdanov, Leutheusser, HL, 2022
Landry, HL, 2022

In the formulation we still need to choose Θ

For neutron star physics: $\mathcal{T}$

missed by standard MHD

$$\mathbf{E} = c_\eta \, \mathbf{j}_B + c_a \, (\mathbf{B} \cdot \mathbf{j}) \, \mathbf{B} + c_H \, \mathbf{j}_B \times \mathbf{B} + \nabla f,$$

$$\mathbf{j}_B \equiv \mathbf{j} - \nabla \ln \mu \times \mathbf{B} \qquad \mathbf{j} = \nabla \times \mathbf{B}$$

missed by standard MHD and G-R

c_a, c_H captured by Goldreich-Reisenegger (1992)

μ : magnetic permeability. (B-dependent)

Magnetic diffusion

When linearized around a constant magnetic field, we get one more parameter than Goldreich-Reisenegger:

from first derivative w.r.t. B of μ

leads to some additional non-isotropic effects in magnetic diffusion.

Vardhan, Grozdanov, Leutheusser, HL

arXiv:2207.01636,
arXiv: 2408.12868

Dependence on the magnetic permeability can lead to interesting interplay with de Haas-van Alphen effect

Landry and HL, to appear

Chiral matter and ABJ anomaly

Now consider including chiral matter with ABJ anomaly:

$$\partial_\mu J_5^\mu = \frac{c}{4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

One of breakthroughs of hydrodynamics in last two decades was the realization microscopic t' Hooft anomaly can have macroscopic effects on hydrodynamics and transports.

t' Hooft anomaly: global symmetry, source not dynamical

ABJ anomaly: source dynamical

Its possible effects on hydro been an outstanding open question

$$\partial_\mu J_5^\mu = \frac{c}{4}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \qquad (*)$$

Chiral charge density not conserved, strictly speaking drops out hydrodynamics.

But if c is sufficiently small, its relaxation scale can be much slower than microscopic relaxation scales

→ Chiral anomalous MHD

Many previous works on this, but all were in weak field regime

In one-form symmetry approach (*) is hard to realize as F itself should be determined dynamically.

Chiral anomalous MHD from partial Higgs

Using partial Higgs approach, the generalization is straightforward.

φ, φ_a : hydro variables for the chiral current

$$J_5^i = (a_{50} - 2cA_0)B_i - \kappa_{ij}\partial_j \mu_5 - \lambda_{ij}^- \partial_t A_j + \epsilon_{ijk}\partial_j (mB_k)$$

$$\partial_t A_i = -r_{ij}(\epsilon_{jkl}\partial_k H_l + 2c\hat{\mu}_5 B_j + \lambda_{kj}^+ \partial_k \mu_5)$$

$$n_5 = \frac{\partial F}{\partial \mu_5}, \quad H_i = -\frac{\partial F}{\partial B_i} \qquad F(A_0, \mu_5, B^2)$$: Equilibrium free energy

$$\hat{\mu}_5 \equiv \mu_5 - \frac{a_{00}}{2c}$$

A prediction: chiral wave

$$\hat{\mu}_5 = 0, \quad B_z = B_0$$

Consider small perturbations around it with

$$\hat{\mu}_5(\vec{x}, t=0) = f_0(\vec{x})$$

$$\hat{\mu}_5(\vec{x}, t) = f_0(\vec{x}_\perp, z - v_z t) e^{-\Gamma t}$$

$$\Gamma \equiv \frac{(2cB_0)^2 r_\parallel}{\chi_5} \qquad v_z \equiv \frac{4cB_0 r_\parallel \lambda_\parallel}{\chi_5}$$

There is a similar chiral wave for B

$$\partial_t b_a = -2cB_0 r_\parallel \epsilon_{ab} \partial_b f_0(\vec{x}_\perp, z - v_z t) e^{-\Gamma t}$$

$$\hat{\mu}_5(\vec{x},t) = f_0(\vec{x}_\perp, z - v_z t)e^{-\Gamma t}$$

$$\Gamma \equiv \frac{(2cB_0)^2 r_\parallel}{\chi_5} \qquad v_z \equiv \frac{4cB_0 r_\parallel \lambda_\parallel}{\chi_5}$$

For non-dynamical external field, Kharzeev and Yee (arXiv: 1012:6026):

chiral magnetic wave $\quad \omega = \pm \frac{2cB_0}{\sqrt{\chi\chi_5}} k_z$

fluctuations in the charge density induce chiral currents and fluctuations in the chiral charge density induce electric currents via the CME, leading to the CMW.

For dynamical EM field, the electric charge density is always zero, cannot induce chiral currents. Instead, electric field fluctuations induce chiral current fluctuations, and the electric field fluctuations are in turn sourced by fluctuations in the chiral charge density (chiral chemical potential) through
the dynamical version of the CME.

Chiral instability

$$\hat{\mu}_5 = \text{const}, \quad B_i = 0$$

Helical unstable mode:

$$B_x = \mathcal{B}(t)\cos kz, \quad B_y = \mathcal{B}(t)\sin kz, \quad \mathcal{B}(t) = B_0 e^{2cr\hat{\mu}_5 kt - D_B k^2 t}$$

first pointed out by Akamatsu and Yamamoto (2013)

We presented argument that the system will evolve to

$$\hat{\mu}_5 = 0, \quad B_i = \text{const}$$

New way of generating magnetic field in astrophysics or cosmology?

Future perspectives

Various generalizations:

Effects of magnetic permeatibility on neutron stars

Nonlinear studies of chiral anomalous MHD

Explore dielectric EFT and effects of ABJ anomaly

Generalization to (2+1)-dimension

Generalization to gravity

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Thank You