Naturalness of vanishing black-hole tides

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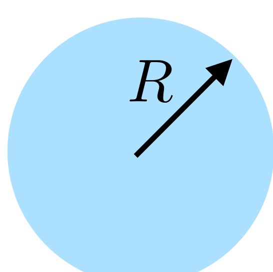
Motivations

- Tidal response and *Love numbers.*

Deformability of a body under external tidal fields. Gravitational analog of polarizability.

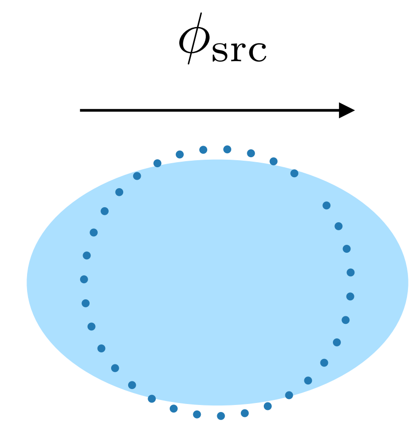
- Consider Newtonian gravity to begin with:

$$\nabla^2 \phi = 0 \qquad \text{(exterior)}$$



+

$$\phi_{\rm src} = \sum_\ell a_\ell r^\ell$$



$$\phi_{\rm tail} = \sum_\ell a_\ell r^\ell \left(1 + k_\ell \left(\frac{R}{r}\right)^{2\ell+1}\right)$$

$k_\ell \sim \mathcal{O}(1)$ Love number

Motivations

- A *naturalness puzzle* in classical physics.

Do black holes acquire induced multiples under external tidal fields ?

- Yes if the tidal fields are time-dependent.
- No if the tidal fields are static, but only in D=4 → Known for linear perturbations + some nonlinear ones

Fang, Lovelace ; Damour, Nagar ; Bennington, Poisson ; Kol, Smolkin ; Le Tiec, Casals; Chia ; Hui, Joyce, Penco, Santoni, Solomon; Riva, Vernizzi, Savic + [...]

- Valid only for black holes (cfr. neutron stars).

- This is puzzling! Seems to violate naturalness.

The problem becomes sharp in the worldline EFT formulation. Goldberger, Rothstein

Motivations

- *Why* does this happen?

  Some great work has been done to address this puzzle in BH perturbation theory:

A timeline for <u>non-rotating black holes</u>

linear LN in 4D vanish	KK decomposition, Z2 and higher D	Linear symmetries	quadratic LN in 4D vanish	Axisymmetric vanishing
2009	2011	2021	2023	2024
Damour, Nagar Bennington, Poisson	Kol, Smolkin	Charalambous+ Hui+	Riva+	Kehagias, Riotto Combaluzier+
				subclass of “electric” tides

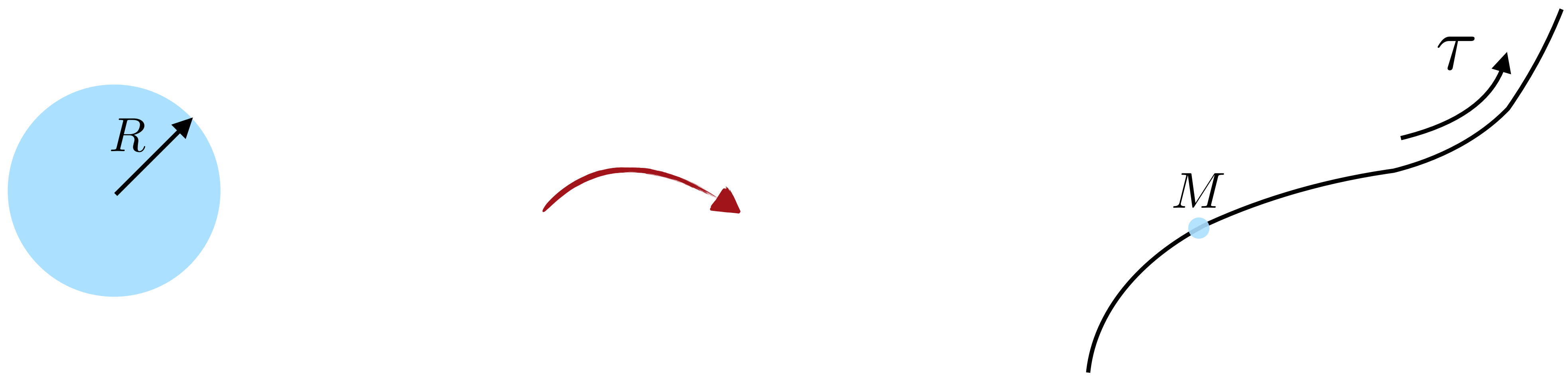
- Still many questions.

  - Is the property valid to full nonlinear order, including for “magnetic” tides?

  - Why is there no running from the EFT point of view?

  - How is the symmetry realized in the worldline formalism?

The Worldline EFT (non-spinning objects)



- Massive point particle

$$S_{pp} = -M \int d\tau \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} \qquad \dot{x}^{\mu} = \frac{dx^{\mu}}{d\tau}$$

- Bulk action: gravity in 4 dimension

$$S_{EH} = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R$$

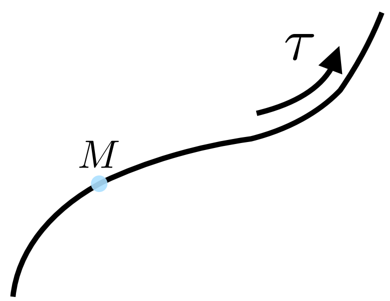
Goldberger, Rothstein 2006

The Worldline EFT (non-spinning objects)

- Ricci-flat background: non-minimal couplings defined in terms of Weyl tensor

$$E_{\mu\nu} = C_{\mu\rho\nu\sigma}\dot{x}^\rho \dot{x}^\sigma \qquad B_{\mu\nu} = \tilde{C}_{\mu\rho\nu\sigma}\dot{x}^\rho \dot{x}^\sigma$$

electric magnetic



$$\dot{x}^\mu = \delta^\mu_0$$

rest frame

$$(\partial E)_\ell \equiv \partial_{\langle i_1} \dots \partial_{i_{\ell-2}} E_{i_{\ell-1} i_\ell \rangle}$$

$$(\partial B)_\ell \equiv \partial_{\langle i_1} \dots \partial_{i_{\ell-2}} B_{i_{\ell-1} i_\ell \rangle}$$

angular momentum $\ell \geq 2$

$\langle \dots \rangle$ symmetric and traceless combination

- Finite size terms: intrinsic multipoles

$$S_{\text{mult}} = \int d\tau \, (Q^{ij} E_{ij} + \dots)$$

⟶ these are absent for black holes

The Worldline EFT (non-spinning objects)

- Finite size terms: tidal effects

Static response

$$S^{(1)}_{\rm Love} = \int d\tau \sum_{\ell} \left[\lambda^{(E)}_{\ell} (\partial E)^2_{\ell} + \lambda^{(B)}_{\ell} (\partial B)^2_{\ell}\right]$$ → linear Love numbers

$$S^{(n)}_{\rm Love} = \int d\tau \sum_{\vec{\ell}} \Big[\lambda^{(E)}_{\vec{\ell},n} \Big((\partial E)_{\ell_1} \dots (\partial E)_{\ell_{n+1}}\Big) + \lambda^{(B)}_{\vec{\ell},n} \Big((\partial B)_{\ell_1} \dots (\partial B)_{\ell_{n+1}}\Big) + \text{mixed}\Big]$$ → nonlinear Love numbers

Dynamic response

$$S_{\rm diss.} = \int d\tau \sum_{\ell} \left[\lambda^{(E)}_{\ell,\omega} (\partial \dot{E}_{-})_{\ell} (\partial E_{+})_{\ell} + \cdots\right]$$ → dissipation numbers

$$S_{\rm dynamic} = \int d\tau \sum_{\ell} \left[\lambda^{(E)}_{\ell,\omega^2} (\partial \dot{E})_{\ell} (\partial \dot{E})_{\ell} + \cdots\right]$$ → dynamic Love numbers

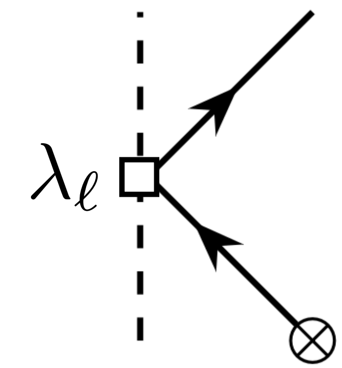
Matching and running effects in worldline EFT

- Matching UV theory and EFT

Full theory: response theory for BH perturbations

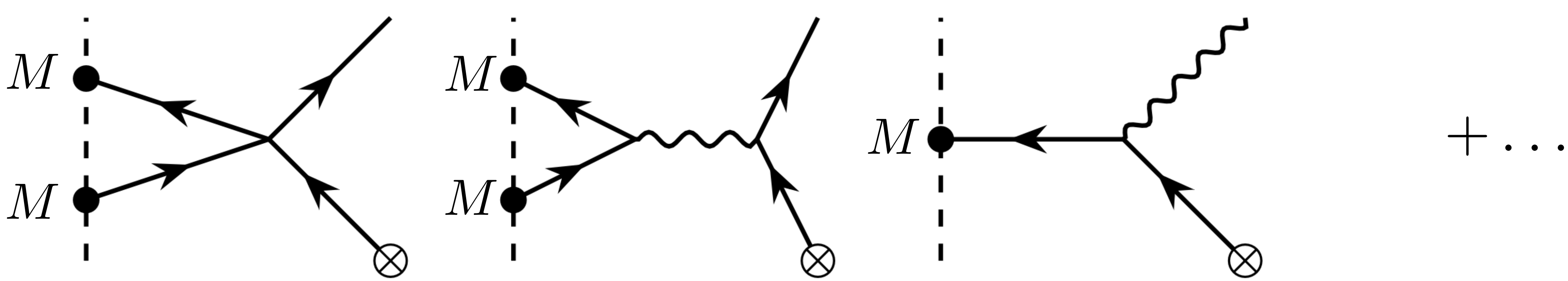
$$\mathcal{D}^2\phi_\ell(r) = 0$$

↔ matching



Kol, Smolkin ; Hui, Joyce, Penco, Santoni, Solomon; [...]

- Nonlinearities in the worldline EFT → Expect mixing and running!



see e.g. Ivanov, Zhou for probe scalar

Our proposal

- There is an accidental symmetry in the static sector of GR (at classical level)
- Some of its aspects are unique to D=4
- Broken by worldline coupling to the mass, but gives useful selection rules for BHs response
  - Implies radiative stability of Love numbers: no running (spurion analysis)
  - Analytic contributions are controlled: vanishing of Love numbers for BH in D=4
- Valid for both E, B at full nonlinear order

Time-independent metrics: Kaluza-Klein-like reduction

$$ds^2 = -e^{2\phi}(dt - A_i dx^i)^2 + e^{-2\phi}\gamma_{ij}dx^i dx^j$$

$$S_{\rm EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{\gamma}\left(-R^{(3)}[\gamma] + 2(\partial_i\phi)^2 - \frac{1}{4}e^{4\phi}F_{ij}^2\right) + \mathcal{O}(\partial_t^2)$$

3D

NB: this includes *stationary* metrics! Important to probe magnetic tides.

An accidental symmetry in the time-independent sector of GR

In $D = 4$ we can dualize the "graviphoton" A_i to a pseudo scalar "axion" $a(x)$:

$$\frac{1}{2}\varepsilon_i^{jk} F_{jk} = e^{-4\phi}\partial_i a$$

$$2(\partial_i\phi)^2 - \frac{1}{4}e^{4\phi}F_{ij}^2 \longrightarrow 2(\partial_i\phi)^2 + \frac{1}{2}e^{-4\phi}(\partial_i a)^2$$

Introduce a complex scalar field: $z = a + ie^{2\phi}$ $(\text{Im } z > 0)$

$$S_{\text{EH}} = -\frac{1}{16\pi G}\int d^4x\sqrt{\gamma}\left(-R^{(3)}[\gamma] + \frac{1}{2}\frac{\partial_i z\,\partial^i\bar{z}}{(\text{Im } z)^2}\right) + \mathcal{O}(\partial_t^2)$$

$\text{SL}(2,\mathbb{R})/\text{SO}(2)$ Nonlinear sigma model! (Ehlers group)

accidental

Recognized as solution generating symmetry in work by Buchdahl, Ernst, Ehlers, Geroch

SL(2,R) symmetry

Symmetry action: $z \to \frac{az+b}{cz+d}$ $\qquad ad - bc = 1$

Generated by: $S: \; z \to -\frac{1}{z}, \quad T: \; z \to z + b$

In addition there is parity: $P: \; z \to -\bar{z}$

We can use a conformal map from the upper-half plane to the unit disk

$$w = \frac{z-i}{z+i} = \phi - i\frac{a}{2} + \dots \qquad S: \; w \to -w$$

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G}\int dt d^3x \sqrt{\gamma}\,\left(-R^{(3)}[\gamma] + 2\frac{\partial_i w\, \partial^i \overline{w}}{(1-\overline{w}w)^2}\right) + O(\partial_t^2)$$

This $SL(2,\mathbb{R})$ was mainly known as a generating symmetry (at the level of e.o.m.).

We see that it is an ordinary accidental symmetry of the time-independent sector

Spurionic symmetry in the "UV theory"

Minkowski vacuum:

$$z = i \quad \leftrightarrow \quad w = 0$$

Nonlinear realized symmetry

$$\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$$

$$\mathrm{SO}(2): \quad w \to e^{i\theta} w$$

Schwarzschild background in isotropic coordinates

$$ds^2 = -\left(\frac{1-X}{1+X}\right)^2 dt^2 + \left(1+X\right)^4 \delta_{ij}\, dx^i dx^j$$

$$X \equiv \frac{GM}{2r}$$

$$w_{\mathrm{Schw}} = -\frac{2X}{1+X^2}, \quad \gamma_{ij} = (1-X^2)^2 \delta_{ij}$$

The symmetry is broken.

We can restore S by assigning spurionic transformation rule

$$S: \; w \to -w$$
$$M \to -M$$

The solution corresponds to that computed diagrammatically in the worldline EFT (series in GM)

Worldline coupling and spurionic symmetry

Worldline minimal coupling:

$$S_{pp} = -M \int d\tau e^{\phi} \supset \underbrace{-M \int d\tau \phi}_{\text{only term relevant for classical response computation}} + \dots$$

We can rewrite it in terms of our field variable w

$$S_{pp} = -M \int d\tau \mathrm{Re}\, w + \dots$$

$w \to -w$ is broken but we can assign spurious transformation rule to M

$$S: \; w \to -w, \quad M \to -M$$

We can also rewrite the Weyl tensor as $E_{ij} - iB_{ij} = \partial_i \partial_j w + (\partial^2 \gamma)_{ij} + \dots$

Selection rules for static Love numbers

We can phrase all our computations in terms of w.

$$X \equiv \frac{GM}{2r}$$

$$w_{\rm tail} \sim \; w_{\rm src} \left(1 + \dots + k_\ell \, X^{2\ell+1} + \dots \right)$$ → linear Love numbers

We use the symmetry

$$S: \; w \to -w, \quad M \to -M$$

$$w_{\rm src} \left(1 + \dots + k_\ell \, X^{2\ell+1} + \dots \right) \to -w_{\rm src} \left(1 + \dots - k_\ell \, X^{2\ell+1} + \dots \right)$$

This spurion analysis implies that LN are not renormalized to all orders in the worldline EFT. — Naturalness

In D=4 the LN response is analytic in X (perturbative in GM), therefore static Love Numbers also have to be zero. — Vanishing

cfr. Affleck, Dine, Seiberg

Selection rules for static Love numbers

We can easily generalize the analysis to the nonlinear response.

$$S^{(n)}_{\rm Love} = \int d\tau \sum_{\vec{\ell}} \Big[\lambda^{(E)}_{\vec{\ell},n} \Big((\partial E)_{\ell_1} \dots (\partial E)_{\ell_{n+1}} \Big) + \lambda^{(B)}_{\vec{\ell},n} \Big((\partial B)_{\ell_1} \dots (\partial B)_{\ell_{n+1}} \Big) + \text{mixed} \Big]$$

$$\vec{\ell} \equiv (\ell_1, \dots, \ell_{n+1})$$

$$|\vec{\ell}| = \sum_{i=0}^{n+1} \ell_i$$

Indices are contracted with γ_{ij} $\longrightarrow$ $|\vec{\ell}|$ even

$$w_{\rm tail} \sim \left(\prod_{\alpha=1}^{n} w_\alpha \right) \left(\dots + k^w_{\vec{\ell},n} X^{|\vec{\ell}|+1} + \dots \right) \quad \xrightarrow{n \text{ odd}} \quad k^w_{\vec{\ell},n} = 0$$

$$\gamma_{\rm tail} \sim \left(\prod_{\alpha=1}^{n} w_\alpha \right) \left(\dots + k^\gamma_{\vec{\ell},n} X^{|\vec{\ell}|+1} + \dots \right) \quad \xrightarrow{n \text{ even}} \quad k^\gamma_{\vec{\ell},n} = 0$$

Static LN are not renormalized to all orders in the worldline EFT and vanish in D=4.

Accidental symmetry violations

- The symmetry is broken by time-dependent terms (with even number of derivatives).

  Dynamic Love Numbers are not zero, but leading

  $$S_{\text{dis.}} = \int d\tau \sum_{\ell} \left[\lambda_{\ell,\omega}^{(E)} (\partial \dot{E}_-)_\ell (\partial E_+)_\ell + \cdots \right]$$

  Dissipative Tides (linear and nonlinear) are not renormalized.

- Neutron stars and other compact objects can have non-vanishing LN.

  In the worldline: there are non-minimal couplings (and new dimensionful scales beyond M)

  Universal non-linearities from the leading point-particle coupling ("loops"), do not induce any running.

- The symmetry is broken by higher curvature terms, e.g. R^3, cosmological constant Λ, quantum corrections, couplings to generic matter fields (e.g. fermions)

Extensions

- In the higher dimensional case: we cannot dualize the vector to a scalar.

  There is still a $\mathbb{Z}_2$ symmetry but restricted only to electric and tensor-type operators.

Power counting is different because $$X = \frac{1}{4}\left(\frac{R_s}{r}\right)^{D-3} \propto GM$$

$$g_{\text{tail}} \sim g_{\text{src}}\left(1 + \cdots + k_\ell\, X^{2\hat{\ell}+1} + \dots\right) \qquad \hat{\ell} = \frac{\ell}{D-3}$$

$\hat{\ell} \in \mathbb{N}$ — Vanishing

$\frac{2\hat{\ell}+1}{2} \in \mathbb{N}$ — Running

$2\hat{\ell} \notin \mathbb{N}$ — Non-renormalization

$$\phi \to -\phi, \qquad M \to -M$$

Kol and Smolkin

- We can also extend the analysis to the case of a shift-symmetric scalar field coupled to gravity, in Schwarzschild background.

  Relation to no-hair theorem + vanishing static LN for nonlinear even tides.

Selection rules for shift-symmetric scalars

Consider the action of a shift-symmetric scalar with minimal coupling

$$S_\psi = -\int d^4x\sqrt{\gamma}\, e^{-2\phi}\, P\Big(e^{2\phi}\gamma^{ij}\partial_i\psi\partial_j\psi\Big)$$

$\psi \to -\psi$ is an exact symmetry.

$w \to -w, \quad M \to -M$ is a symmetry for a free scalar, or for the leading quadratic terms.

$$S^{(n)}_{\psi,\mathrm{Love}} = \int d\tau \sum_{\vec{\ell}} \lambda^{(\psi)}_{\vec{\ell},n}\Big((\partial\psi)_{\ell_1}\dots(\partial\psi)_{\ell_{n+1}}\Big)$$

$$\vec{\ell} \equiv (\ell_1, \dots, \ell_{n+1})$$

$$|\vec{\ell}| = \sum_{i=0}^{n+1} \ell_i$$

$$\psi_{\mathrm{tail}} \sim \left(\prod_{\alpha=1}^{n}\psi_\alpha\right)\Big(c_n + \dots + k^{(\psi)}_{\vec{\ell},n}\, X^{|\vec{\ell}|+1} + \dots\Big)$$

The response vanishes for any even n, because of $\psi \to -\psi$ symmetry. → Includes no-hair

The LN vanish for $n = 1$ for interacting scalar, because of $w \to -w$ symmetry.

Using SL(2,R) : Lorentzian Taub-NUT

$G = 1$

Consider the Lorentzian Taub-NUT spacetime:

$$ds^2 = -f(\rho)(dt - A)^2 + \frac{1}{f(\rho)}d\rho^2 + (\rho^2 + N^2)(d\theta^2 + \sin^2\theta d\phi^2)$$

$$A = -2N\cos\theta d\phi, \qquad f(\rho) = \frac{\rho^2 - 2M\rho - N^2}{\rho^2 + N^2}$$

↓

"magnetic" monopole $\quad F_i = \frac{2N}{\rho^2}\hat{\rho}_i$

We can rewrite it in isotropic coordinates with change of variables:

$$\rho \to r + M + \frac{M^2 + N^2}{4r}.$$

Valid for $r > \frac{\sqrt{M^2+N^2}}{2}$

Outer horizon

$$ds^2 = -e^{2\phi}(dt - A)^2 + e^{-2\phi}\gamma_{ij}dx^i dx^j.$$

Using SL(2,R) : Lorentzian Taub-NUT

$G = 1$

We can re-express this in a simple form using our field basis.

Recall the complex scalar field: $z = a + ie^{2\phi}$ $\longleftrightarrow$ $w = \dfrac{z-i}{z+i}$

$$w_{\mathrm{TN}} = -\frac{2Z}{1+\bar{Z}Z} \qquad \gamma_{ij} = (1-\bar{Z}Z)^2\delta_{ij}$$

with $Z \equiv X - iY$ $\quad X \equiv \dfrac{M}{2r}$ $\quad Y \equiv \dfrac{N}{2r}$

Outer horizon:

$\bar{Z}Z = 1$

We recover Schwarzschild for $Y = 0$ $\longrightarrow$ Related by $U(1) \subset SL(2,R)$

Self-dual Taub-NUT: $N = -iM$, $Z = 0, \quad \bar{Z} \neq 0$ $\longrightarrow$ $w_{\mathrm{SD-TN}} = 0, \quad \overline{w}_{\mathrm{SD-TN}} = -2\bar{Z}$, $\gamma_{ij} = \delta_{ij}$

We can repeat our analysis for the nonlinear tidal response using full SL(2,R)

Vanishing tides!

Conclusions

- We presented an accidental symmetry of the static sector of GR
- We explained how the symmetry is broken in a black hole background but can be restored assigning spurionic transformation rules to M
- The selection rules imply non-renormalization (naturalness) and vanishing
- The results are fully nonlinear and apply to both electric and magnetic tides
- This symmetry is consistent with all previous results in D=4 and higher dim, and with all non-vanishing results

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- The construction can be extended to EM charged black holes + also NUT charge
- Can we test it in the nonlinear regime (exp or NR)? Extension to spinning BHs ?