The discovery of hot water

the Schwinger-Keldysh coset construction for dissipative fluids and superfluids at finite temperature

Paolo Arcangeli - TU Munich

Master's thesis project at **EPFL**

Supervisor: **Riccardo Rattazzi**

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Contours 2026

July 3, 2026

UNIVERSITY OF CAMBRIDGE

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(+ WORK IN PROGRESS)

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Introduction and motivations

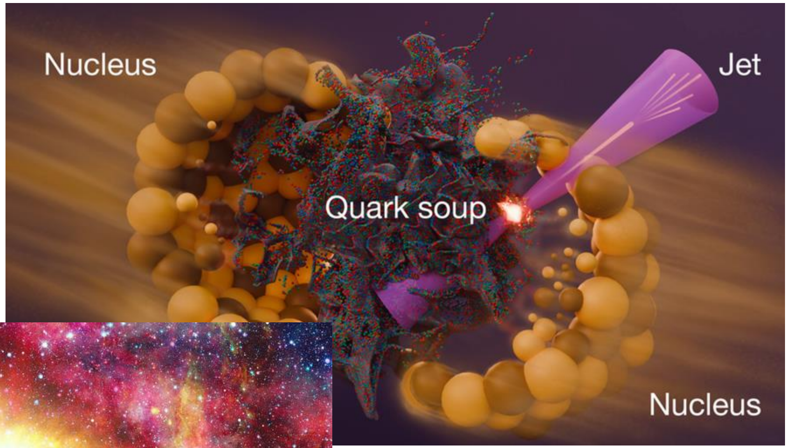
The finite temperature coset construction for SK EFTS

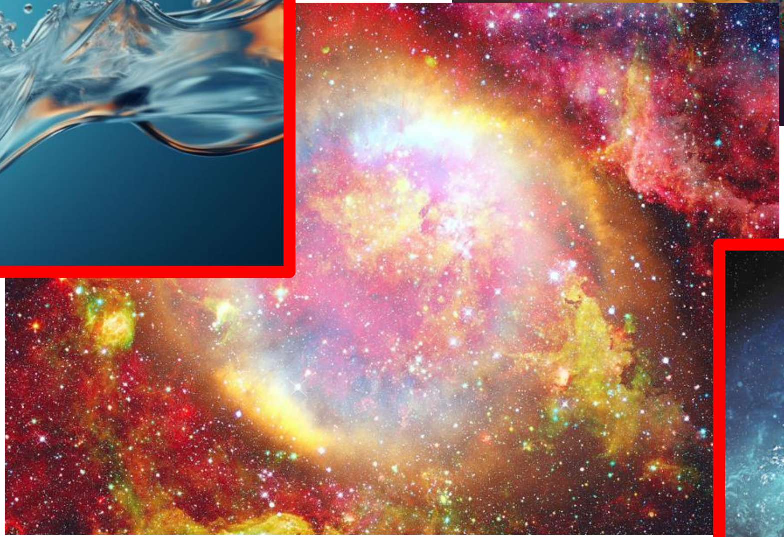
Fluid phenomenology and applications to axion physics

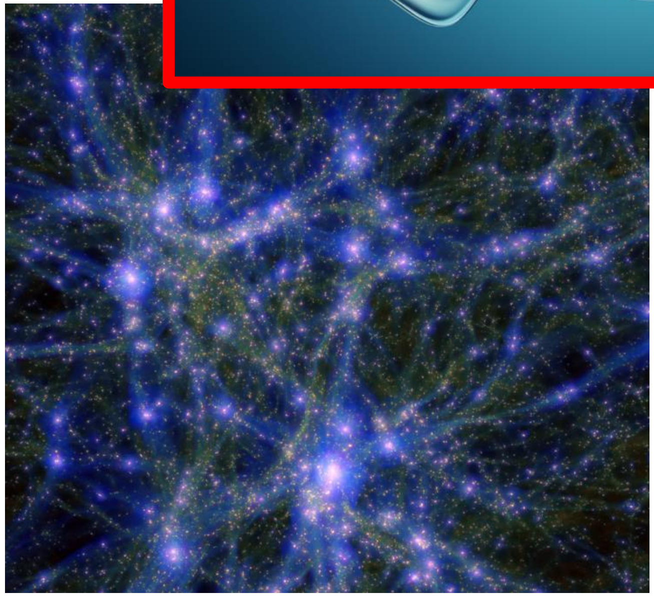
Why fluids? Why SK? Why coset?

Why fluids? Why SK? Why coset?

- Fluids are everywhere!

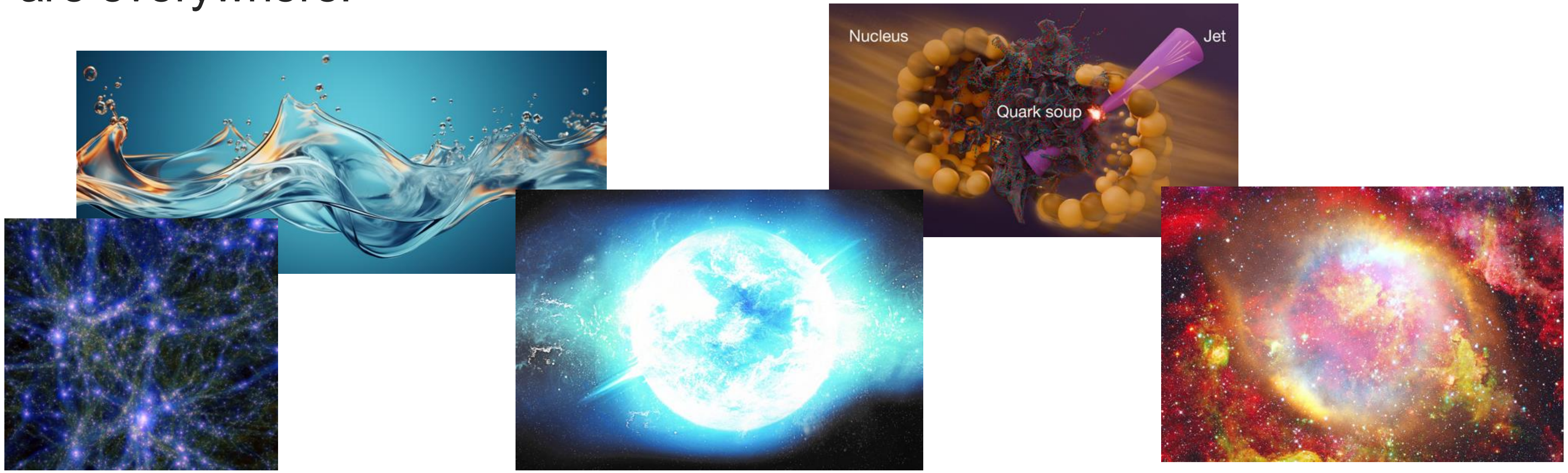






Why fluids? Why SK? Why coset?

- **Fluids** are everywhere!



- All studied through a **universal IR description** (EFT):

Hydrodynamics
Long-distance field theory for conserved quantities

Thermodynamics
Equilibrium part of hydrodynamics

Why fluids? Why SK? Why coset?

- **Symmetries** are, as usual, the good organizational principle.
    - SSB of spacetime symmetries: ***Poincaré → Rotations + Translations***
    - ***Internal symmetry*** structure can be complicated!

Why fluids? **Why SK?** Why coset?

- **Symmetries** are, as usual, the good organizational principle.
- To make a wise use of the symmetries, don't start from the conservation equations, but use a **QFT approach**:

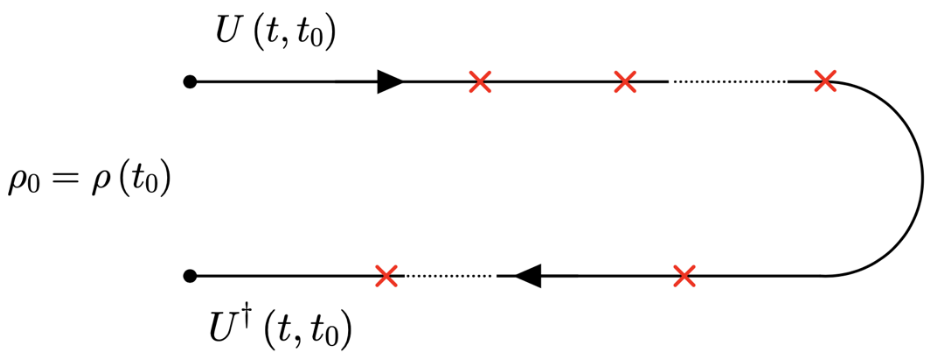
Vacuum QFT
no dissipation effects

Why fluids? **Why SK?** Why coset?

- **Symmetries** are, as usual, the good organizational principle.
- To make a wise use of the symmetries, don't start from the conservation equations, but use a **QFT approach**:

Vac… QFT
no diss… effects

Schwinger-Keldysh formalism
(real time) path-integral over a closed-time path contour



$$\mathcal{Z}[J_1, J_2] = \int_{BC} \mathcal{D}\Phi_1 \mathcal{D}\Phi_2 \, e^{iS(\Phi_1) - iS(\Phi_2) + i\int J_1 \cdot \mathcal{O}_1 - i\int J_2 \cdot \cdot \mathcal{O}_2}$$

$$\phi_r \equiv \frac{1}{2}(\phi_1 + \phi_2)$$
$$\phi_a \equiv \phi_1 - \phi_2$$

Glorioso & Liu, 1805.09331

Why fluids? Why SK? Why coset?

- We can construct the **SK EFT action** that simplifies the evaluation of the correlation functions:

$$Z[J_1, J_2] = \int \mathcal{D}\phi_1 \mathcal{D}\phi_2 e^{iS_{EFT}[\phi_1, J_1; \phi_2, J_2]}$$

BCs and KMS symmetry are implicit in the EFT action

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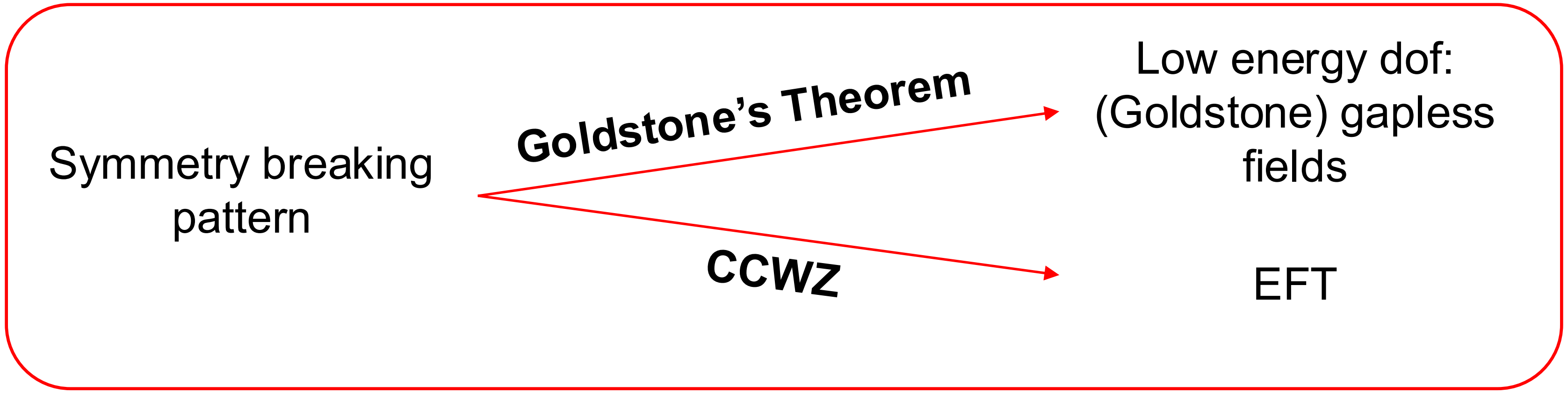
BCs and KMS symmetry are implicit in the EFT action

- We will use the **coset construction** (or **CCWZ**) to determine it:



Akyuz, Goon, Penco, 2306.17232v2

Why fluids? Why SK? **Why coset?**

- **Systematic**
  - Naturally organizes the EFT in a derivative expansion of building blocks
  (if supplemented by a power counting rule)

- **Algorithmic**
  - Can be used for generic symmetry content
  Energy and momentum modes from breaking of spacetime symmetries

- Clarifies the difference between **physical symmetries** and **redundancies**

  e.g. U(1) charged fluid: **diffeos** in fluid spacetime, **chemical shift** symmetry

What is this talk about?

1. Coset construction
For Lagrangians describing finite T, dissipative fluid with generic internal symmetries

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2. Redundancies vs physical symmetries
Show that dynamical KMS is responsible for the typical redundancy symmetries of a fluid

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3. Fluid pheno, and axion physics applications
Dynamics of an axion – star system to obtain new bounds in axion parameter space *(in progress…)*

Introduction and motivations

The finite temperature coset construction for SK EFTs

Fluid phenomenology and applications to axion physics

SK effective action for dissipative fluids?

- *Some* worked out cases:
  - SK EFTs for dissipative fluids (neutral or with conserved U(1) charge)

Crossley, Glorioso, Liu, 1511.03646v3

More general internal symmetry structure? → Coset construction

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  - SK coset construction for internal, non-Abelian symmetry breaking

    Firat, Gomes, Nardi, Penco, Rattazzi, 2508.18346

    No spacetime symmetry breaking → No fluids

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- In this work, **SK EFT** Lagrangians for **any type of dissipative fluid**

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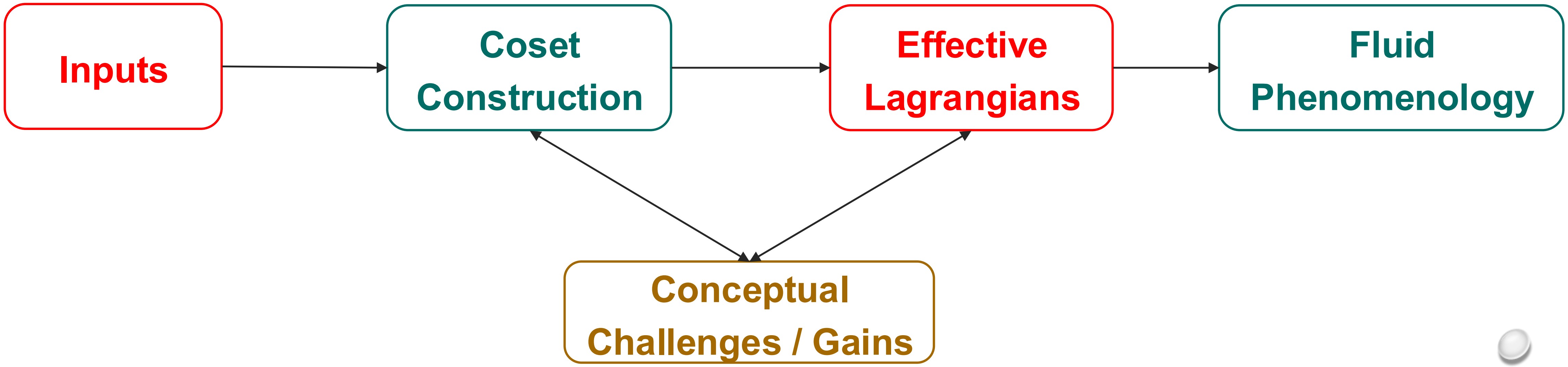

SK coset construction for dissipative fluids

Inputs

Initial state: $\rho_0 = e^{-\beta_0 H}$

DKMS symmetry

Unitarity constraints

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Initial state: $\rho_0 = e^{-\beta_0 H}$

DKMS symmetry

Unitarity constraints

"Fluid" symmetry breaking pattern: $(ISO(D-1,1)\times G_{\text{int}})_1 \times (ISO(D-1,1)\times G_{\text{int}})_2 \to (\mathbb{R}^4 \times SO(D-1)\times H_{\text{int}})_{\text{diag}}$

Poincaré

Internal symmetries

Diagonal
(1-2 basis)

SK coset construction for dissipative fluids

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Coset $\Omega \supset$ ***Broken / non linearly*** *realised generators*

- *Diagonal (1-2 basis) broken* — *e.g. superfluid*
- *Diagonal (1-2 basis) unbroken* — *e.g. U(1) charged fluid*

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Coset $\Omega \supset$ ***Broken / non linearly*** *realised generators* ⟷ ***GBs****

- *Diagonal (1-2 basis) broken* *e.g. superfluid* $\{X_r^\mu, X_a^\mu, \pi_r, \pi_a\}$
- *Diagonal (1-2 basis) unbroken* *e.g. U(1) charged fluid* $\{X_r^\mu, X_a^\mu, \varphi_a\}$

** Inverse Higgs constraints* Landry, 1912.12301

SK coset construction for dissipative fluids

Construction and challenges

- Enlarge the coset

$$\{X_r^\mu, X_a^\mu, \varphi_a\} \longrightarrow \{X_r^\mu, X_a^\mu, \varphi_a, \varphi_r\}$$

$$\Phi_r \xrightarrow{\text{DKMS}} \Phi_r$$

$$\Phi_a \xrightarrow{\text{DKMS}} \Phi_a + i\beta_0^A \partial_A \Phi_r$$

Overparametrization!

SK coset construction for dissipative fluids

Construction and challenges

- Enlarge the coset → Emergent **chemical shift**-like symmetry

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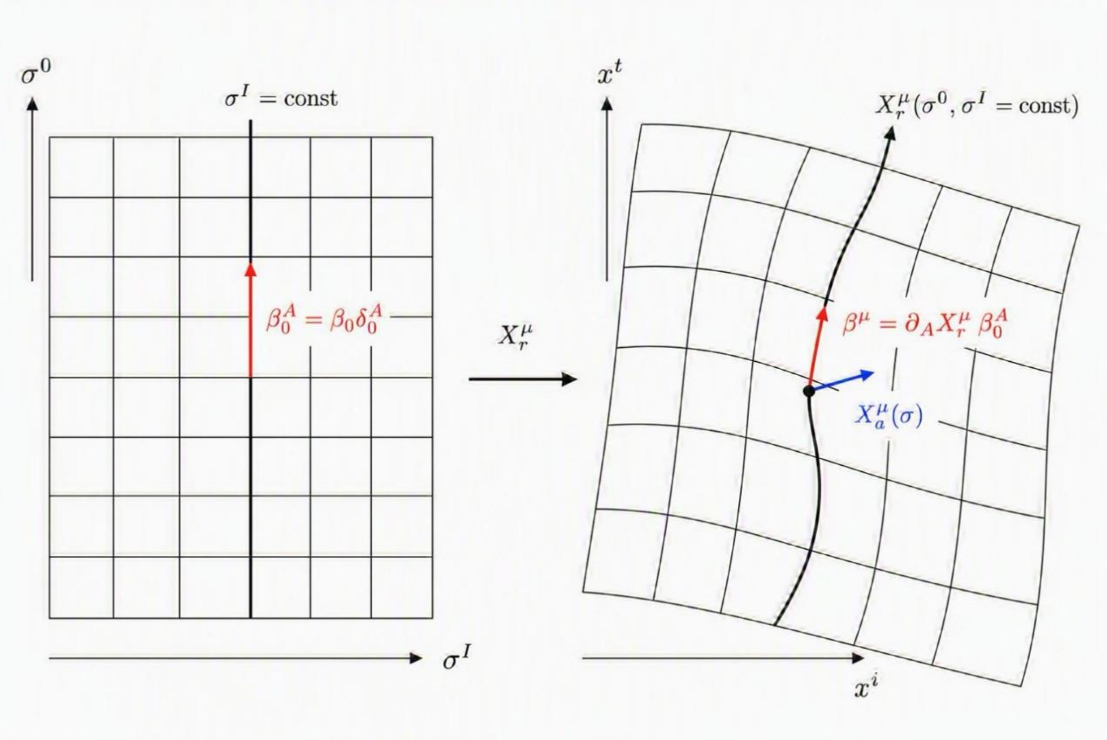
Overparametrization ⟶ $\varphi_r(\sigma^A) \xrightarrow{\text{CS}_\lambda} \varphi_r(\sigma^A) + \lambda(\sigma^A)$

SK coset construction for dissipative fluids

Construction and challenges

- Enlarge the coset → Emergent **chemical shift**-like symmetry
- Introducing a **fluid spacetime**

$$\rho_0 = e^{-\beta_0 H}$$



Glorioso, Liu, 1805.09331

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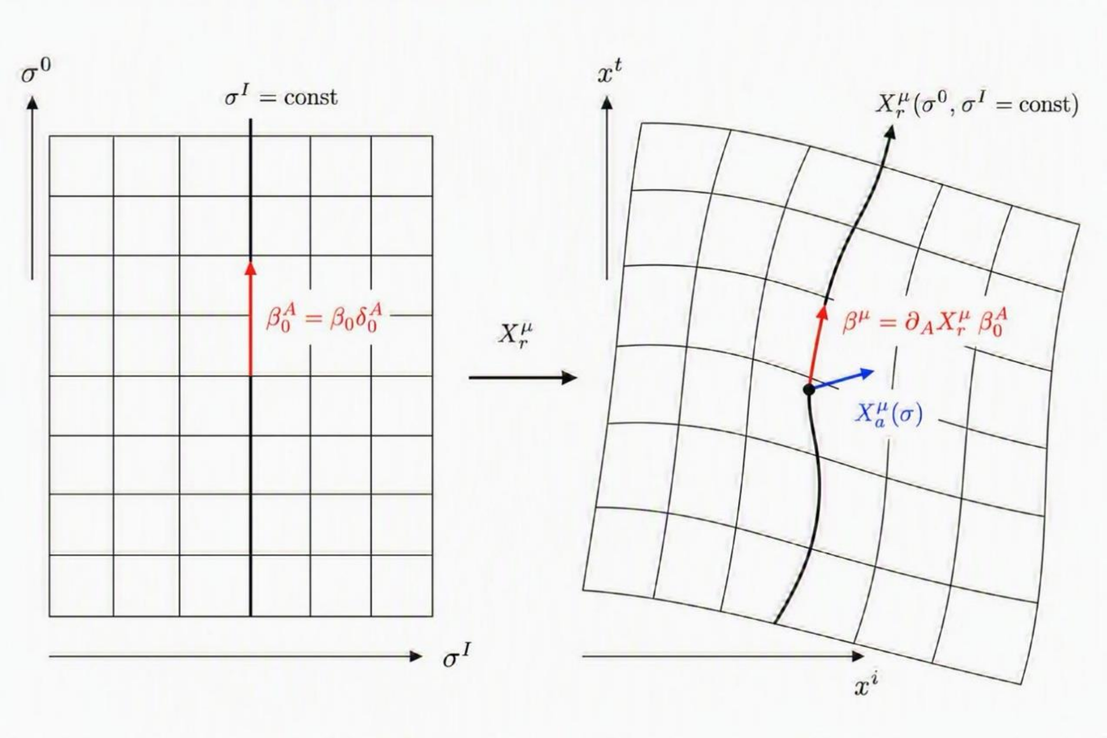

$$\sigma^A \xrightarrow{\text{Diff}_\xi} \sigma^A + \xi^A(\sigma^B)$$

Glorioso, Liu, 1805.09331

SK coset construction for dissipative fluids

Construction and challenges

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} **Redundancy symmetries**

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$$\varphi_r(\sigma^A) \xrightarrow{\text{CS}_\lambda} \varphi_r(\sigma^A) + \lambda(\sigma^A)$$

Overconstrained building blocks…
We are forgetting something

SK coset construction for dissipative fluids

Construction and challenges

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- ? → **restrictions**

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→

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SK coset construction for dissipative fluids

Construction and challenges

- Enlarge the coset → Emergent **chemical shift**-like symmetry
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} Redundancy symmetries

- **DKMS** symmetry → **restrictions**

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Physical symmetries vs redundancies

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Symmetry: **physical** ⟶ $\text{DKMS}\,(\Phi) \sim \text{DKMS}\,(\Phi')$

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Physical symmetries vs redundancies

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But…
Is it non trivial?

Physical symmetries vs redundancies

$$\mathrm{DKMS}\cdot\mathcal{R}\cdot\mathrm{DKMS}^{-1}=\mathcal{R}$$

But…
Is it non trivial?

$$\sigma^A \xrightarrow{\mathrm{Diff}_\xi} \sigma^A+\xi^A(\sigma^B)$$

$$\begin{pmatrix} e^{i\varphi_1(\sigma)\cdot T} \\ e^{i\varphi_2(\sigma)\cdot T}\end{pmatrix} \xrightarrow{CS_\lambda} \begin{pmatrix} e^{i\varphi_1(\sigma)\cdot T}e^{i\lambda(\sigma)\cdot T} \\ e^{i\varphi_2(\sigma)\cdot T}e^{i\lambda(\sigma)\cdot T}\end{pmatrix}$$

Diagonal in 1-2 basis

$$\varphi_1(\sigma^A) \xrightarrow{\mathrm{DKMS}} \varphi_1(\sigma^A+i\beta_0^A/2)$$

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Non-diagonal in 1-2 basis

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Diagonal in 1-2 basis

Non-diagonal in 1-2 basis

→

$$\mathcal{L}_{\beta_0}\xi^A(\sigma)=0$$
$$\mathcal{L}_{\beta_0}\lambda(\sigma)=0$$

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$$\sigma^A \xrightarrow{\mathrm{R\,Diff}_\xi} \sigma^A+\xi^A(\sigma^I)$$
$$\tilde{\Omega} \xrightarrow{\mathrm{R\,CS}_\lambda} \tilde{\Omega}e^{i\lambda(\sigma^I)\cdot T_r}$$

SK coset construction - result

Effective Lagrangians (LO and NLO)

$$\text{Non-dissipative:}\quad \mathcal{L}_1 = \mathcal{D}_\mu \Pi_a^K \, \Pi_{r,0}^{\mu K}$$

$$\text{Dissipative:}\quad \mathcal{L}_2 = \mathcal{D}_\mu \Pi_a^K \left(\Pi_{r,1}^{\mu K} + \frac{i}{2} W_{r,0}^{\mu K,\nu J} \mathcal{D}_\nu \Pi_a^J \right)$$

a-type building blocks

$$\supset \{\partial_\mu X_a^\nu \,,\ \partial^\mu \pi_a \,,\ \partial^\mu \varphi_a \,,\ \ldots\}$$

r-type building blocks

$$\supset \{\beta^\mu \,,\ \partial^\mu \pi_r \,,\ \partial^\mu \varphi_r \,,\ \ldots\}$$

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Dissipation ⟷ *Fluctuation /noise*

DKMS

DKMS imposes **constraints** on the most general structures

SK U(1) charged fluid

$$G_{\text{int}} = U(1)\,, \;\; H_{\text{int}} = U(1)$$

Non dissipative order

First dissipative order

SK U(1) charged fluid

$$G_{\text{int}} = U(1)\,,\;\; H_{\text{int}} = U(1)$$

Non dissipative order

$$\mathcal{L}_1^{(C)} = \partial_\mu X_{a\nu}\, T_0^{(C)\mu\nu} + \partial_\mu \varphi_a\, j_0^{(C)\mu}$$

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Imposing DKMS…

$$T_0^{(C)\mu\nu} = (Y p_Y - \beta p_\beta)\, u^\mu u^\nu + p\,\eta^{\mu\nu}$$

$$j_0^{(C)\mu} = p_Y\, u^\mu$$

✓

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First dissipative order

DKMS + Landau frame

$$T_1^{(C)\mu\nu} = -\zeta \partial_\alpha u^\alpha \Delta^{\mu\nu} - \eta \Delta^{\mu\alpha} \Delta^{\nu\beta} \left(\partial_{(\alpha} u_{\beta)} - \frac{2}{\mathrm{Tr}\,\Delta} \eta_{\alpha\beta} \Delta^{\rho\sigma} \partial_\rho u_\sigma \right)$$

$$j_1^{(C)\mu} = -\frac{\sigma}{\beta} \partial_\alpha \left(\beta y\right) \Delta^{\mu\alpha}$$

Three transport coefficients:

SK U(1) charged fluid

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$$j_1^{(C)\mu} = -\frac{\sigma}{\beta}\partial_\alpha(\beta y)\,\Delta^{\mu\alpha}$$

Unitarity constraints

Three transport coefficients:

- Bulk viscosity : $\zeta \geq 0$
- Shear viscosity : $\eta \geq 0$
- Charge conductivity : $\sigma \geq 0$

SK superfluid

$$G_{\text{int}} = U(1)\,, \quad H_{\text{int}} = \emptyset\,, \quad \bar{P}^t_r = P^t_r + \mu Q_r$$

SK superfluid

$$G_{\text{int}} = U(1)\,, \quad H_{\text{int}} = \emptyset\,, \quad \bar{P}_r^t = P_r^t + \mu Q_r$$

~~Chemical shift~~ →

Additional r–type vector: $w^\mu \equiv \partial^\mu \psi_r$

Additional r–type scalar: $X = w^\mu w_\mu$

SK superfluid

$$G_{\text{int}} = U(1)\,, \quad H_{\text{int}} = \emptyset\,, \quad \bar{P}^t_r = P^t_r + \mu Q_r$$

Non dissipative order

$$T_0^{(S)\mu\nu} = (yp_y - \beta p_\beta)\, u^\mu u^\nu + p\eta^{\mu\nu} - 2p_X w^\mu w^\nu$$

$$j_0^{(S)\mu} = p_y u^\mu + 2p_X w^\mu$$

First dissipative order

SK superfluid

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Landau's 2-fluid model

finite T superfluid: (T = 0 superfluid) + (finite T neutral fluid)

Nicolis 1108.2513

SK superfluid

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First dissipative order

$T_1^{(S)\mu\nu}$, $j_1^{(S)\mu}$ — Determined by **14 transport coefficients**

(Details)

Introduction and motivations

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Dispersion relations

- Linearize conservation equations:

$$T(x) = T_0 + \delta T(x)\,, \quad \mu_{td}(x) = \mu_{td0} + \delta\mu_{td}(x)\,, \quad u^\mu(x) = \left(1,\ \mathbf{v}(x)\right)^\mu$$

Dispersion relations

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- **Charged fluid**:

one **diffusive mode**, associated to $\mathbf{v}_\perp$: $\omega = -iD^{(\mathbf{v}_\perp)}k^2$

one **diffusive mode**, associated to $\delta\mu$: $\omega = -iD^{(\delta\mu)}k^2$

one **propagating, attenuated mode**, associated to $\mathbf{v}_\parallel$ and δT: $\omega = \pm v_s k - iD^{(\delta T)}k^2$

Dispersion relations

- Linearize conservation equations:

$$T(x) = T_0 + \delta T(x)\,, \quad \mu_{td}(x) = \mu_{td0} + \delta\mu_{td}(x)\,, \quad u^\mu(x) = \begin{pmatrix} 1 & \mathbf{v}(x) \end{pmatrix}^\mu$$

- **Charged fluid**:

one **diffusive mode**, associated to $\mathbf{v}_\perp$: $\omega = -iD^{(\mathbf{v}_\perp)}k^2$

one **diffusive mode**, associated to $\delta\mu$: $\omega = -iD^{(\delta\mu)}k^2$

one **propagating, attenuated mode**, associated to $\mathbf{v}_\parallel$ and δT: $\omega = \pm v_s k - iD^{(\delta T)}k^2$

- **Superfluid**:

one **diffusive mode**, associated to $\mathbf{v}_\perp$: $\omega = -iD^{(\mathbf{v}_\perp)}k^2$

(up to) two **propagating, attenuated mode** associated to $\mathbf{v}_\parallel$, δT, and $\delta\mu$: $\omega = \pm v_{s(j)}k - iD^{(j)}k^2$

Herzog et al., 1101.3330

Axion-sourced stellar remnants

With: T. **Kutter**, K. **Bartnick**, A. **Weiler** *(pheno side)*
E. **Firat** *(theory side)*

Axion-sourced stellar remnants

- **Axion** interacting with **nucleons** in stars *(white dwarfs, neutron stars, …)*

Axion-sourced stellar remnants

WORK IN PROGRESS

- **Axion** interacting with **nucleons** in stars *(white dwarfs, neutron stars, …)*

$$V(\phi) = \sigma_N \Big(\underbrace{\bar{N}N}_{n} - \underbrace{\epsilon\frac{m_\pi^2 f_\pi^2}{\sigma_N}}_{n_c}\Big)\left(\cos\left(\frac{\phi}{f}\right) - 1\right)$$

Hook, Huang, 1708.08464
Banerjee, Buen-Abad, Hook, 2507.02049

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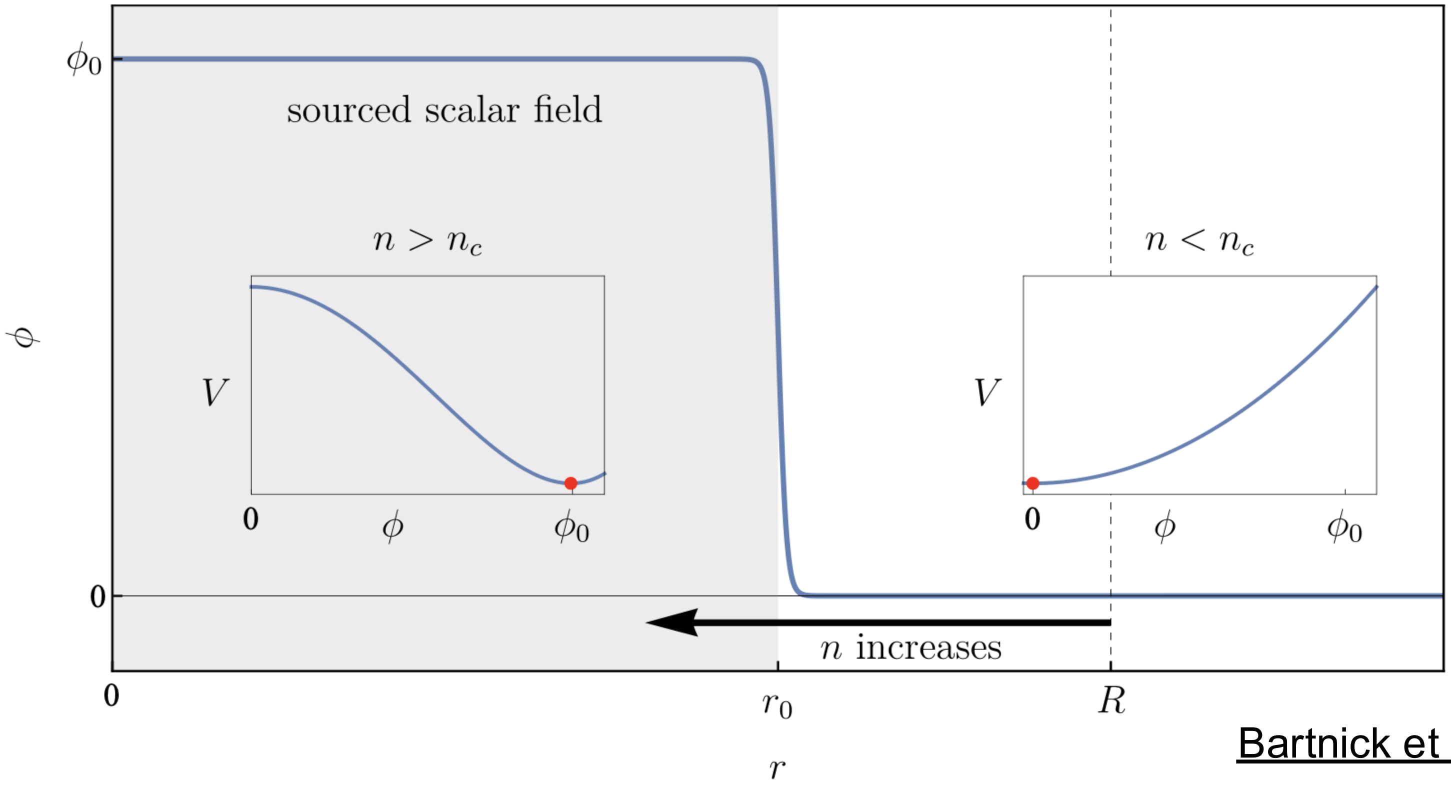

Bartnick et al. 2510.06312

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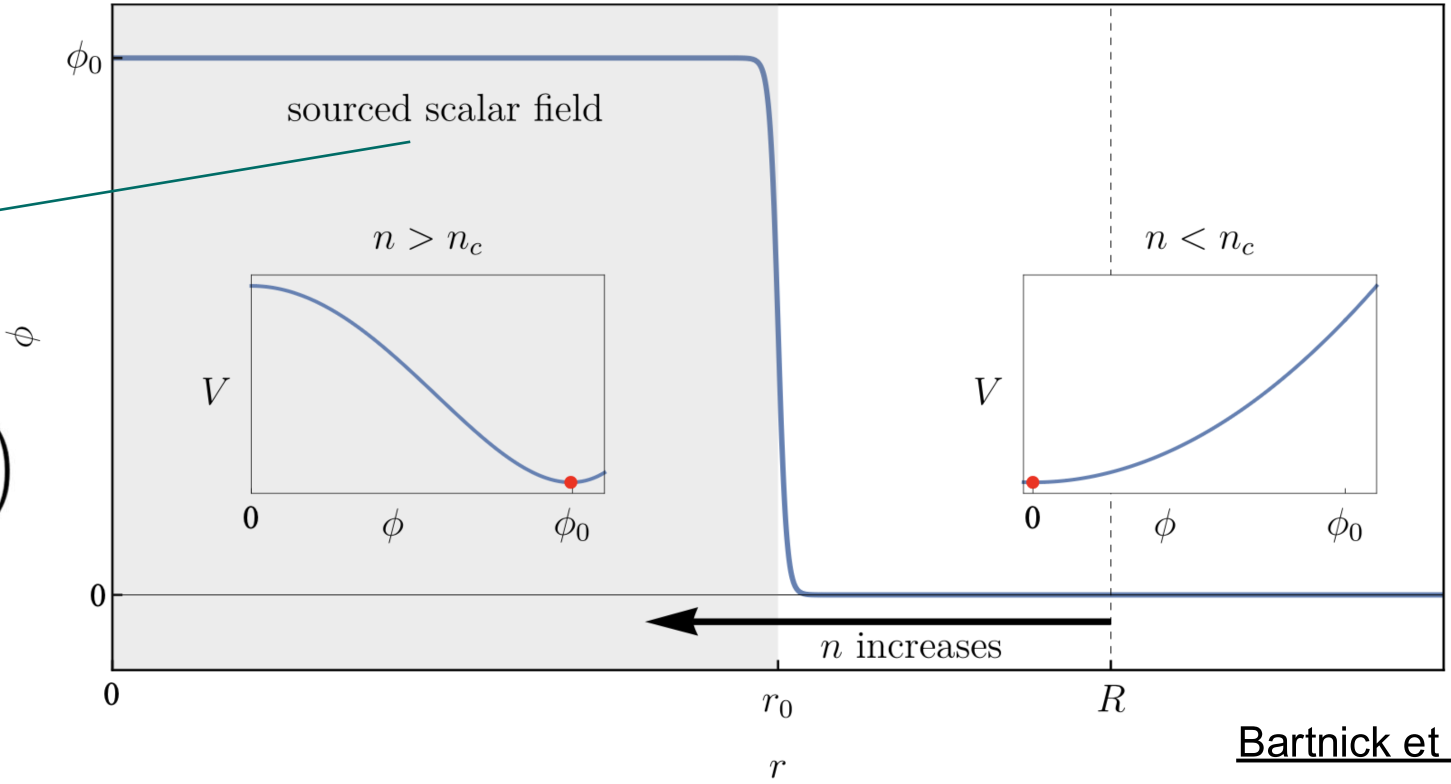
Nucleon mass **reduced**:

$$\delta m_N \simeq 32\,\text{MeV} \left(\frac{\sigma_N}{50\,\text{MeV}} \right)$$

Balkin et al. 2211.02661v2



Bartnick et al. 2510.06312

Axion-sourced stellar remnants

- Realistic star → Include gravity:
  - **Equilibrium** configuration = **Axion-modified TOV equations**

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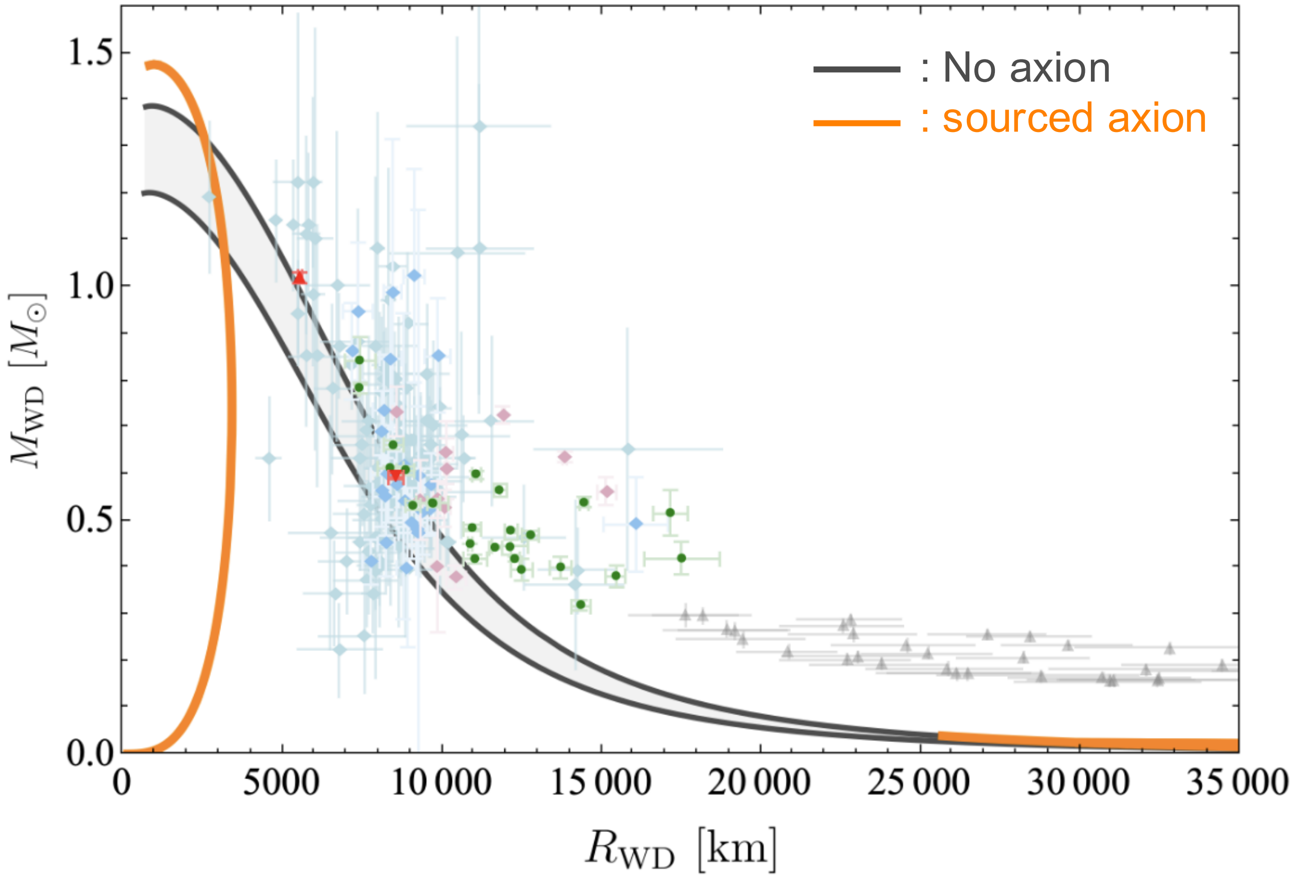
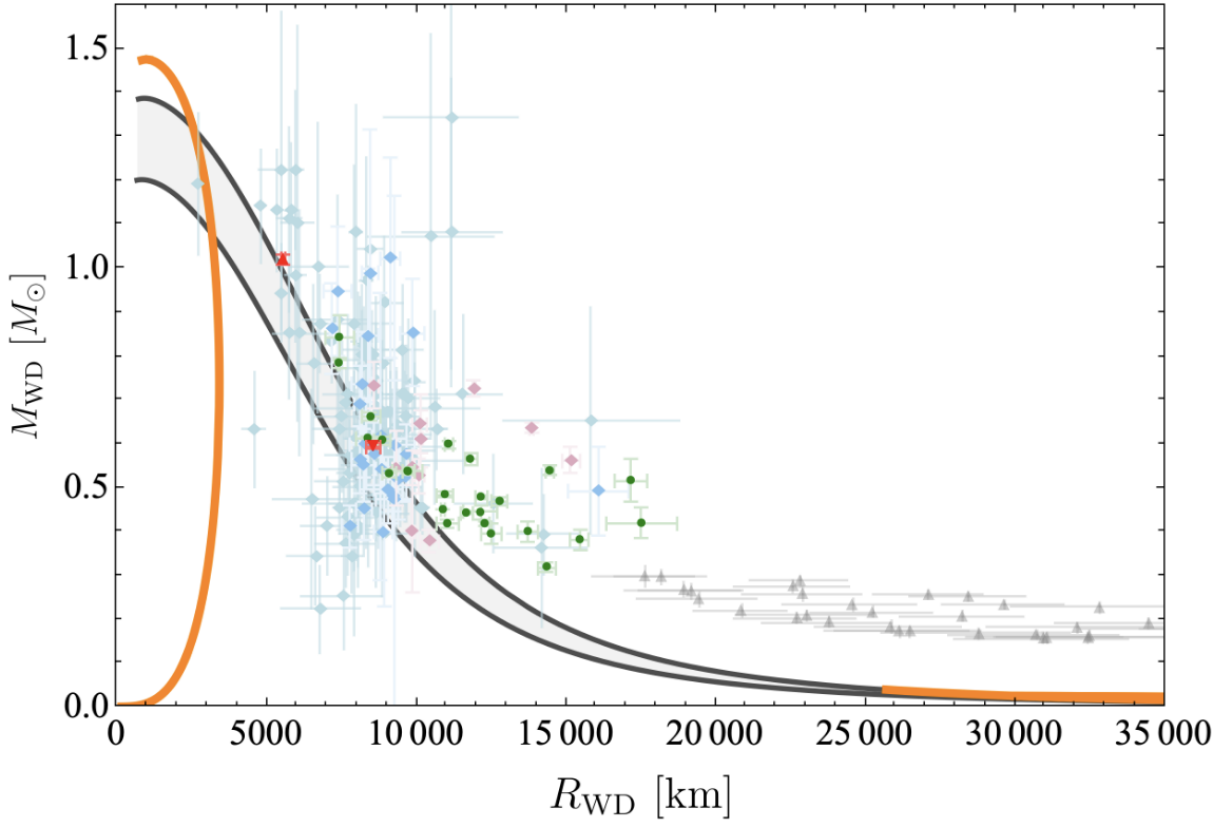


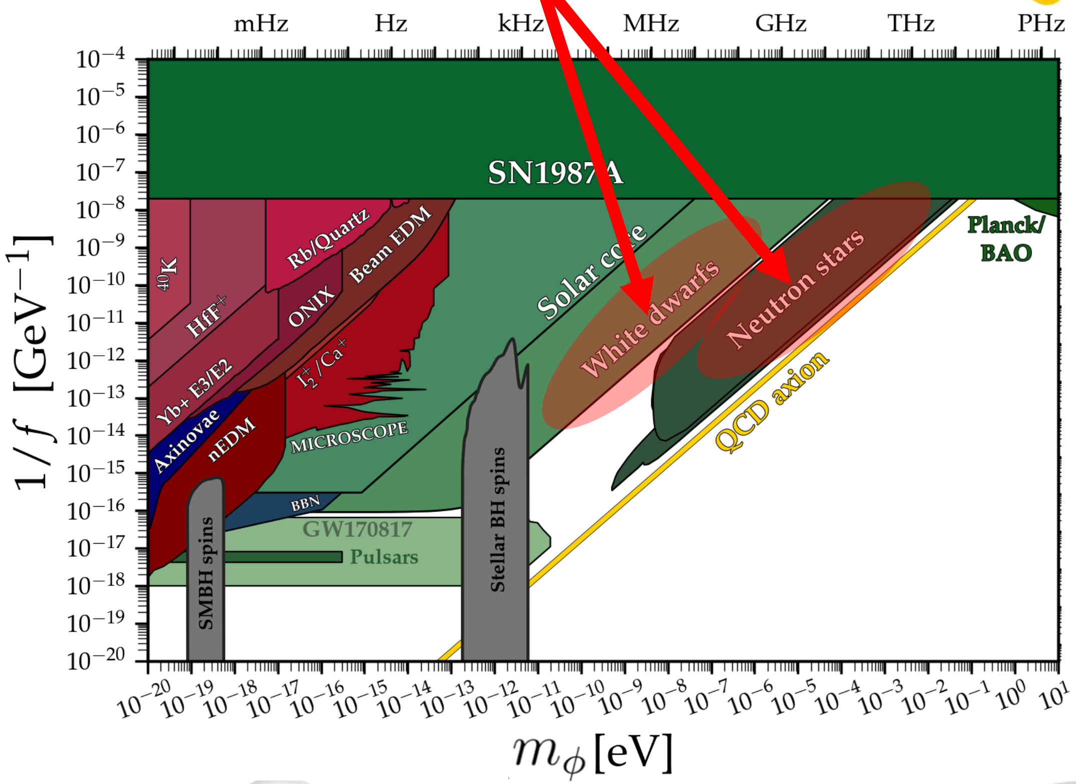
Balkin et al. 2211.02661

(Why?)

Axion-sourced stellar remnants

- Realistic star → Include gravity:
  - **Equilibrium** configuration = **Axion-modified TOV equations** → **Bounds**





Balkin et al. 2211.02661

Bartnick et al. 2408.07740

Axion-sourced stellar remnants

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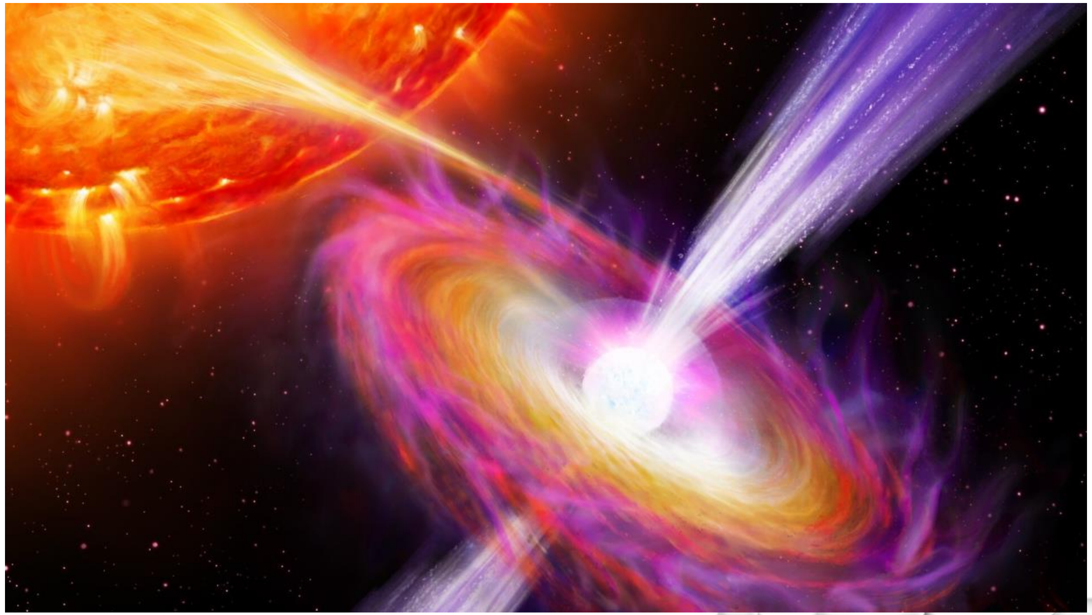
  - Near a **dynamical transition**?

$$n < n_c \Rightarrow n > n_c$$



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How the system **dissipates/fluctuates**?

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Dissipation effects →

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Dissipation effects → **SK EFT**

Conclusions and further outlooks

Coset construction in **dissipative** SK, with spacetime symmetry breaking

→ **Redundancies** vs **physical** symmetries (DKMS). *Beyond thermal state…*

→ Hydrodynamics of a **finite T**, **dissipative superfluid**

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Algorithmic procedure, "easy" to generalise

→ **Non-abelian** internal symmetries
Non-abelian superfluids, low-energy QCD …

→ Hydrodynamics of **astrophysical** objects, and **axion-induced sourcing**

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Thank you!

Backup slides

Thermodynamics and Hydrodynamics in theoretical physics

- Many-body physics (and physics in general) relies on the concept of **separation of scales**, between IR and UV variables.
  - E.g. Multipole expansion in classical EF, or Effective Field Theories (EFTs) in Quantum Field Theory (QFT).
  - UV variables are averaged out: only the IR degrees of freedom survive!

- We want to study the **hydrodynamics** and **thermodynamics** of systems at finite temperature, in a field-theoretical fashion.
  - Hydrodynamics: long-distance field theory for conserved quantities (energy, momentum, internal charges)
  - Thermodynamics: equilibrium (zero-derivative) part of hydrodynamics

Old-school hydrodynamics – more details

Kovtun 1205.5040

Inputs

- Initial state: $\rho_0 = e^{-\beta H}$
- **Conservation laws for energy-momentum and internal symmetry currents**
- Time reversal invariance

Linear response theory

Technicalities

- **Kubo formulas**: relate retarded correlation functions and transport coefficients
- **Onsager relations**: correlation functions are related → reduces the number of independent transport coefficients
- **Local II law of thermodynamics** → transport coefficients are non-negative

Fluid phenomenology

Punch line:

it's **difficult** to describe temperature fluctuations in system involving hydrodynamical modes! Things get messy soon…

Doesn't capture quantum fluctuations!

(Back)

Unitarity constraints

- Constraints derived from unitarity of time evolution operator $Te^{i\int A_s \cdot \mathcal{O}_s}, \quad s \in \{1,2\}$

$$S_{\text{EFT}}[\phi_r, \phi_a = 0, A_{\mu r}, A_{\mu a} = 0] = 0$$

$$S^*_{\text{EFT}}[\phi_r, \phi_a, A_{\mu r}, A_{\mu a}] = -S_{\text{EFT}}[\phi_r, -\phi_a, A_{\mu r}, -A_{\mu a}]$$

$$\text{Im}\, S_{\text{EFT}}[\phi_r, \phi_a, A_{\mu r}, A_{\mu a}] \geq 0$$

- They imply: presence of a-fields in each term, a-even terms being imaginary, and positivity of imaginary terms

Power counting argument

- Schematically, the SK EFT action (no spacetime symmetry breaking) can be written as:

$$\frac{S}{\hbar} = \frac{p_*}{\hbar} \int dt\, d^3\mathbf{x} \left[\frac{k_*^2}{\omega_*} \dot{\phi}_a \dot{\phi}_r - \nabla\phi_a \nabla\dot{\phi}_r + \frac{i}{\beta\hbar} (\nabla\phi_a)^2 \right]$$

- Unsuppressed field configuration satisfy:

$$\frac{\mathrm{Im}\ S}{\hbar} \lesssim 1$$

Imaginary part → oscillations

$$\frac{\mathrm{Re}\ S}{\hbar} \sim 1$$

Configurations around saddle point

- This implies: $\frac{\phi_a}{\phi_r} \lesssim \frac{\hbar\omega}{T}$

Kubo-Martin-Schwinger (KMS) relations

time-reversal invariance

$\boldsymbol{\rho_0 = e^{-\beta H}}$

$\mathcal{Z}[J_1, J_2]$ invariant under a **KMS transformation** of the sources

$$\begin{pmatrix} J_r^\Theta(x^\mu) \\ J_a^\Theta(x^\mu) \end{pmatrix} = \eta_\Theta \begin{pmatrix} \cosh\left(i\frac{\beta}{2}\partial_t\right) & \frac{1}{2}\sinh\left(i\frac{\beta}{2}\partial_t\right) \\ 2\sinh\left(i\frac{\beta}{2}\partial_t\right) & \cosh\left(i\frac{\beta}{2}\partial_t\right) \end{pmatrix} \begin{pmatrix} J_r(y^\mu) \\ J_a(y^\mu) \end{pmatrix} \Bigg|_{y=-x}$$

$$\beta = \frac{1}{T_0}$$

Usually truncated at LO in $\frac{\hbar\omega}{T_0}$

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$$\beta = \frac{1}{T_0}$$

- This symmetry implies

Fluctuation-dissipation theorem

→ Einstein relation $\nu = \frac{1}{2}\sigma\beta$

Onsager relations for transport coefficients

Superfluid – more details

$$T_1^{(S)\mu\nu} = -\left(A_1\beta\tilde{\Delta}^{\alpha\beta}\partial_\alpha u_\beta + B_2\beta b^\alpha\partial^{(b)}u_\alpha + A_1'\partial^{(u)}(\beta y) + A_5'\partial^{(b)}(\beta y)\right)\tilde{\Delta}^{\mu\nu}$$
$$- \left(B_2\beta\tilde{\Delta}^{\alpha\beta}\partial_\alpha u_\beta + A_3\beta b^\alpha\partial^{(b)}u_\alpha + A_3'\partial^{(u)}(\beta y) + A_7'\partial^{(b)}(\beta y)\right)b^\mu b^\nu$$
$$- \left(C_2 b^{(\gamma}\partial^{\alpha)}\beta_\gamma + B_2'\partial_\alpha(\beta y)\right)b^{(\mu}\tilde{\Delta}^{\nu)\alpha}$$
$$- \beta E\tilde{\Delta}^{\mu\alpha}\tilde{\Delta}^{\nu\beta}\left(\partial_{(\alpha}u_{\beta)} - \frac{2}{\mathrm{Tr}\,\tilde{\Delta}}\eta_{\alpha\beta}\tilde{\Delta}^{\rho\sigma}\partial_\rho u_\sigma\right)$$

$$j_1^{(S)\mu} = -\left(A_1'\beta\tilde{\Delta}^{\alpha\beta}\partial_\alpha u_\beta + A_3'\beta b^\alpha\partial^{(b)}u_\alpha + A_1''\partial^{(u)}(\beta y) + A_3''\partial^{(b)}(\beta y)\right)u^\mu$$
$$- \left(A_5'\beta\tilde{\Delta}^{\alpha\beta}\partial_\alpha u_\beta + A_7'\beta b^\alpha\partial^{(b)}u_\alpha + A_3''\partial^{(u)}(\beta y) + A_2''\partial^{(b)}(\beta y)\right)b^\mu$$
$$- \left(B_2' b_{(\gamma}\partial_{\alpha)}\beta^\gamma + B_1''\partial_\alpha(\beta y)\right)\tilde{\Delta}^{\mu\alpha},$$

Onsager-type relations

- If the superfluid velocity is *almost* parallel to the fluid velocity, only 5 independent coefficients:

Charge conductivity : $\sigma \equiv \beta B_1'' \geq 0\,,$ Shear viscosity : $\eta \equiv \beta E \geq 0$

Bulk viscosities : $\beta\mathbf{S} = \begin{pmatrix} \zeta \equiv \beta A_1 & \zeta' \equiv \beta A_1' \\ \zeta' \equiv \beta A_1' & \zeta'' \equiv \beta A_1'' \end{pmatrix} \geq 0$

Unitarity constraints

Coherent with **Landau's** description of **Helium-4 in He-II phase**!

(Back)

Dispersion relations – more details/1

- Consider small perturbations around equilibrium, and linearize conservation equations:

$$T(x) = T_0 + \delta T(x)\,, \quad \mu_{td}(x) = \mu_{td0} + \delta\mu_{td}(x)\,, \quad u^\mu(x) = \begin{pmatrix}1 & \mathbf{v}(x)\end{pmatrix}^\mu$$

$$\begin{cases} \partial_\mu j^{(C)\mu} = 0 \\ \partial_\mu T^{(C)\mu t} = 0 \\ \partial_\mu T^{(C)\mu i} = 0 \end{cases} \Rightarrow \quad \mathbf{M}\vec{a} = 0\,, \quad \vec{a} = \begin{pmatrix}\delta T & \delta\mu & \mathbf{v}\end{pmatrix}^T$$

- The roots of $\det M$:

one **diffusive mode**, associated to $\mathbf{v}_\perp$: $\omega = -iD^{(\mathbf{v}_\perp)}k^2$

one **diffusive mode**, associated to $\delta\mu$: $\omega = -iD^{(\delta\mu)}k^2$

one **propagating, attenuated mode**, associated to $\mathbf{v}_{||}$ and δT: $w = \pm v_s k - iD^{(\delta T)}k^2$

Qualitative and quantitative agreement with literature!

Dispersion relations – more details/2

- Compared to charged fluid, now also the superfluid velocity field $b^\mu = \begin{pmatrix} 0 & \mathbf{b} \end{pmatrix}^\mu$
- We have an additional constraint (by construction), which closes the system:

$$\partial^{[\mu} w^{\nu]} = 0 \xrightarrow{\mu=t\ ,\ \nu=i\ ,\ \text{Fourier}} \mathbf{b} = -\frac{\mathbf{k}}{\omega}\delta\mu + \mu_0 \mathbf{v}$$

- Roots of $\det M$:

one **diffusive mode**, associated to $\mathbf{v}_\perp$	$: \ \omega = -iD^{(\mathbf{v}_\perp)}k^2$
(up to) two **propagating, attenuated mode** associated to $\mathbf{v}_\parallel$, δT, and $\delta\mu$	$: \ \omega = \pm v_{s(j)}k - iD^{(j)}k^2$

Again, qualitative and quantitative agreement with literature!

(Back)

“Proving” the II law of thermodynamics

- From the SK EFT action, **entropy current**:

Glorioso & Liu, 1612.07705v2

$$s^\mu \equiv V_0^\mu - (T_0^{\mu\nu} + T_1^{\mu\nu})\,\beta u_\nu - (j_0^\mu + j_1^\mu)\,\beta \mu_{td} \equiv s_0^\mu + s_1^\mu$$

$$\mathcal{L}_1 \xrightarrow{\mathrm{DKMS_{cl}}} \mathcal{L}_1 + i\partial_\mu V_0^\mu$$

$$\mathcal{L}_2 \xrightarrow{\mathrm{DKMS_{cl}}} \mathcal{L}_2$$

Bhattacharya et al., 1101.3332

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Non-dissipative term

$$s_0^\mu = p_T u^\mu \equiv s u^\mu$$

$$\partial_\mu s_0^\mu = 0$$

Noether current for (local) KMS

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Glorioso & Liu, 1612.07705v2

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$$\partial_\mu s_0^\mu = 0$$

Noether current for (local) KMS

Dissipative terms

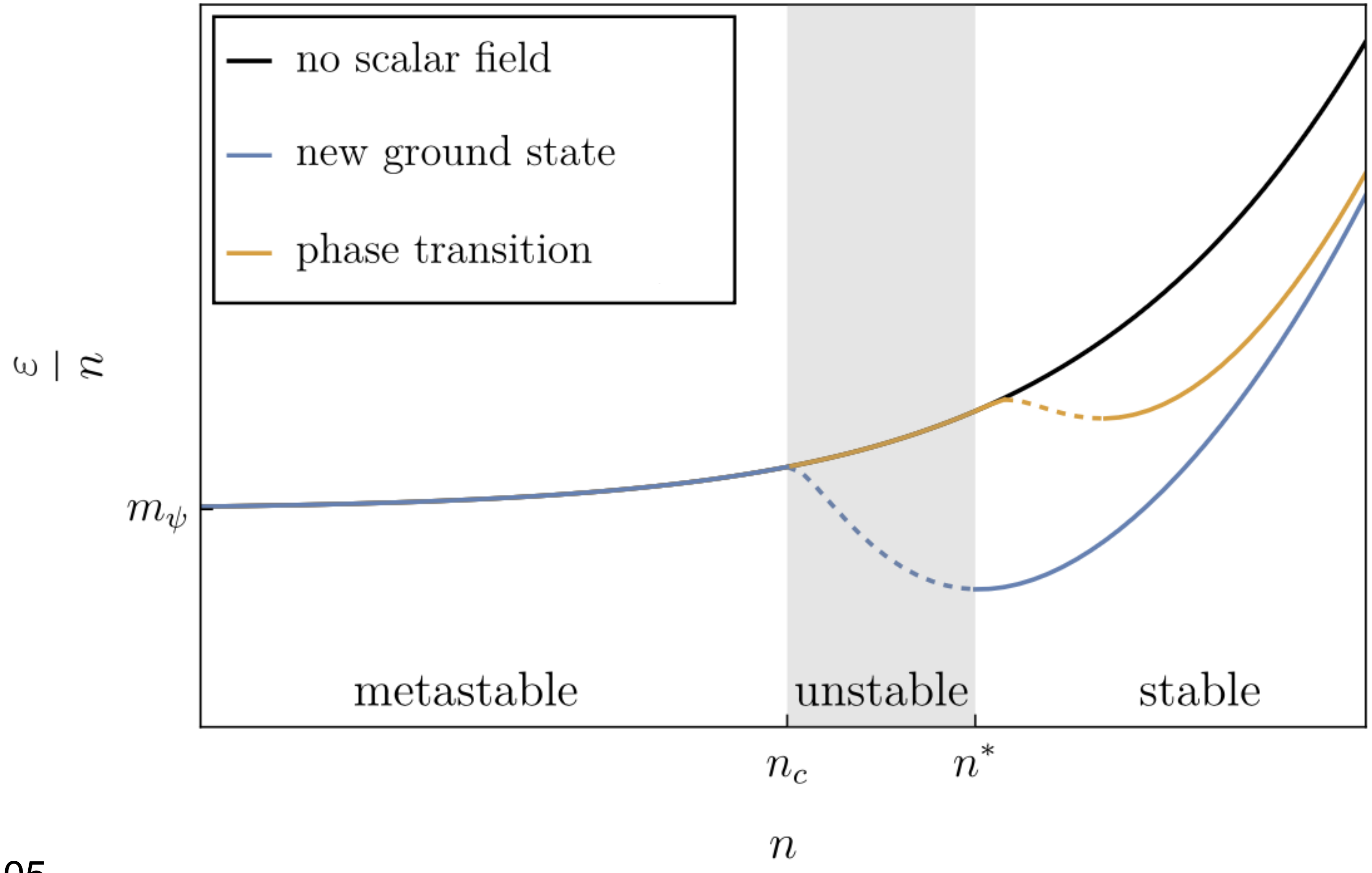
$$\partial_\mu s^\mu \,\big|_{\mathrm{EOM}} \geq 0$$

KMS + unitarity constraints

→ **local II law of thermodynamics**

Axion-sourced stellar remnants - NGS

- Energy/particle can develop a new minimum → **New ground state**



Bartnick et al. 2509.25305
Balkin et al. 2211.02661

(Back)