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An RG approach for stochastic inflation

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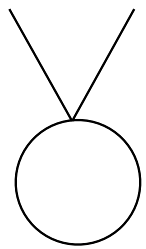
Introduction

- Application of SK EFT to old problem in cosmology
- Correlation function of a light scalar field in de Sitter have an IR problem

$$\langle \phi^2 \rangle_{1-\text{loop}} =$$



$$\sim \lambda H^8 / m^6$$

- Signals a breakdown of perturbation theory, linked to the secular problem for massless fields

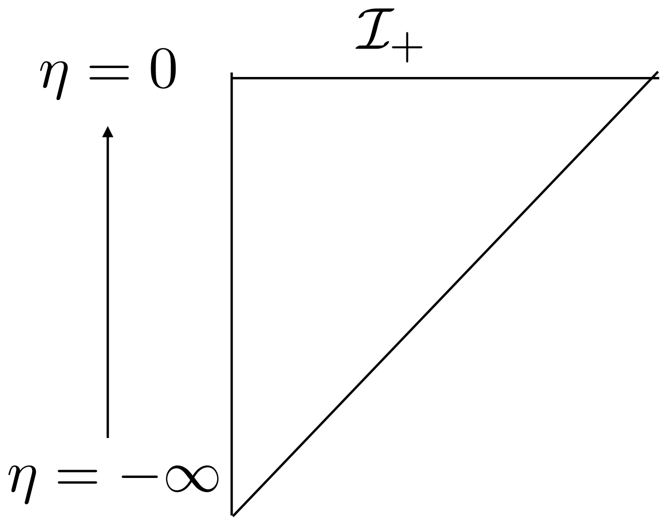
de Sitter spacetime

- Inflationary spacetimes can be approximated by de Sitter ones

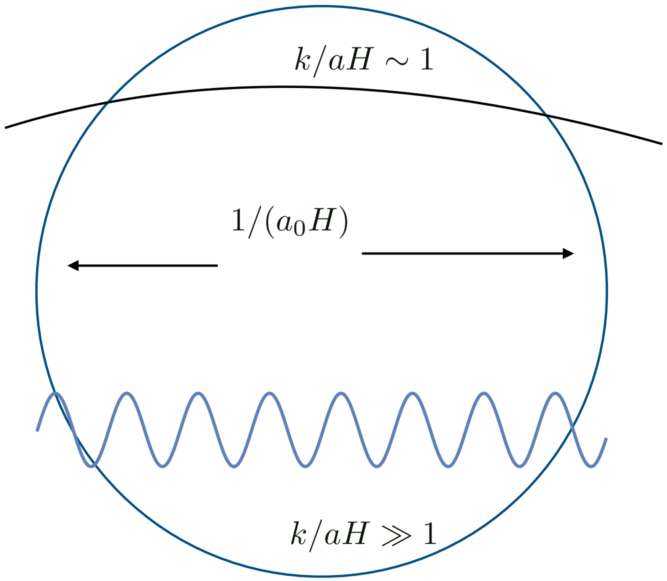
$$ds^2 = \frac{1}{H^2\eta^2}\left(d\eta^2 - d\vec{x}^2\right)$$

- Mode functions changes with time

$$\phi_k(\eta) = \frac{H}{\sqrt{2k^3}}\,(1 + ik\eta)\,e^{-ik\eta}$$



Fluctuations during inflation



- Inflation continuously separates fluctuations into long and short wavelengths
- Super-horizon modes freeze and define the relevant large-scale observables
- Short modes remain quantum and continuously interact with the long modes

Super-horizon perturbations behave as **classical stochastic variables**

$$\dot{\phi}_\ell = -\frac{V_{,\phi_\ell}}{3H} + \xi(t)$$

Quantum

Classical

Starobinsky '86
Starobinsky and Yokoyama '94

Stochastic inflation

Starobinsky '85
Starobinsky and Yokoyama '94

- Split the field between long and short wavelength modes

$$\phi = \phi_{\rm long} + \phi_{\rm short}$$

- Use a time dependent coarse graining scale $\Lambda = \epsilon/(aH)$

- Long modes obey $\dot{\phi}_{\rm long} = -\frac{1}{3H} V_{,\phi} + \eta_\phi$ $\langle \eta(t)\eta(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$

- Coarse-graining at the horizon generates stochastic dynamics

Fokker-Planck equation

- Langevin equation can be written as a Fokker-Planck equation

$$\frac{d}{dt}P(\phi,t) = \frac{1}{3H}\frac{\partial}{\partial\phi}(V'(\phi)P(\phi,t)) + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$

- Equilibrium solution at late times

$$\lim_{t\to\infty} P(\phi) \sim \exp\left(-\frac{8\pi^2 V(\phi)}{3H^4}\right)$$

- This allows to compute correlation functions at late times

$$\langle\phi^2\rangle_{\rm eq} = \frac{m^2}{4\lambda}\left[\frac{K_{3/4}\left(\frac{8\pi^2 m^4}{3\lambda H^4}\right)}{K_{1/4}\left(\frac{8\pi^2 m^4}{3\lambda H^4}\right)} - 1\right].$$

- This results in finite correlation functions

Starobinsky and Yokoyama '94

Outlook

- The Stochastic approach has many current applications in cosmology

- It has remarkable properties, as computing loops from a classic approach

- Still there is an active discussion in how it is derived, what it computes and what are the corrections to the Fokker-Planck equation

- This is naturally an open system so one should expect to apply those tools to this problem.

An RG approach

Burgess et al. 09-18
Gorbenko and Senatore '19
SC, Davis and Wang '24
Green and Gupta '25
SC and Colas '26

- A first problem is to identify which is computed. Take the FP equation

$$\frac{d}{dt}P(\phi,t) = \frac{1}{3H}\frac{\partial}{\partial\phi}(V'(\phi)P(\phi,t)) + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial\phi^2}P(\phi,t)$$

- One is really computing a coarse grained probability.

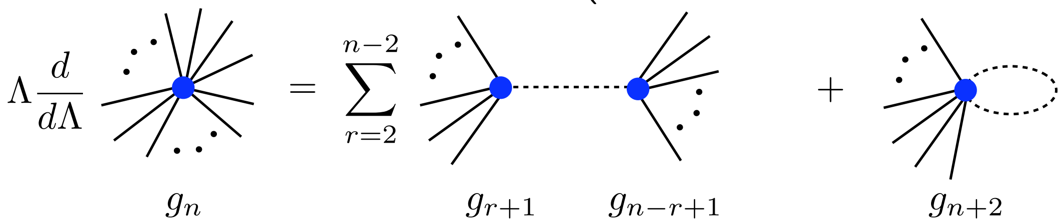
$$P_\Lambda[\phi^\ell(k,t_0)] = \int^{\phi^\ell}\mathcal{D}\varphi_r\int^0\mathcal{D}\varphi_a e^{iS^\Lambda_{\rm eff}[\varphi_r,\varphi_a]}$$

- Time evolution is given by $$\frac{dP_\Lambda}{dt} = \left.\frac{\partial P_\Lambda}{\partial t}\right|_\Lambda + \frac{d\log\Lambda}{dt}\left.\frac{\partial P_\Lambda}{\partial\log\Lambda}\right|_t$$

Wilsonian RG: integrating out short modes

- The effective description of long modes evolves as short modes are integrated out
- Physics should not depend on the arbitrary choice of cut-off
- This induces an exact, non-perturbative flow of interactions

$$\Lambda \frac{d}{d\Lambda} S_{\text{int}}^{\Lambda} = -\frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{d\Omega_\Lambda}{d \ln \Lambda} G_k \left(\left(\frac{\delta}{\delta \phi_k} S_{\text{int}}^{\Lambda} \right)^2 + \frac{\delta^2 S_{\text{int}}^{\Lambda}}{\delta \varphi_{\mathbf{k}} \delta \varphi_{-\mathbf{k}}} \right)$$



Polchinski '84

RG in the SK contour

- Given a theory with a cut-off $$\frac{d}{d\log\Lambda} Z_\Lambda[J_r, J_a] = 0$$

- Defining the action as $$S^\Lambda_{\rm eff}[\varphi^L_r, \varphi^L_a] = S^\Lambda_0[\varphi^L_r, \varphi^L_a] + S^\Lambda_{\rm int}[\varphi^L_r, \varphi^L_a]$$

With $$S^\Lambda_0[\phi^L_r, \phi^L_a] = -\frac{1}{2}\int_{\vec k}\int d\eta\int d\tilde\eta\,\left(\phi^L_r(\vec k,\eta),\phi^L_a(\vec k,\eta)\right)\begin{pmatrix}0 & \widehat D^A_\Lambda\\ \widehat D^R_\Lambda & -2i\widehat D^K_\Lambda\end{pmatrix}\begin{pmatrix}\phi^L_r(-\vec k,\tilde\eta)\\ \phi^L_a(-\vec k,\tilde\eta)\end{pmatrix}.$$

where $$\widehat D_{\Lambda\,\alpha\gamma}\circ G^{\gamma\beta}_\Lambda = \delta_\alpha^{\ \beta}\,\delta(\eta-\tilde\eta)$$

- Filter function on each part of the contour $$\bar G^{\alpha\beta}_\Lambda(k,\eta,\tilde\eta) = \bar\Omega_{\Lambda(\eta)}(k)\,\bar\Omega_{\Lambda(\tilde\eta)}(k)\,G^{\alpha\beta}(k,\eta,\tilde\eta)$$

SC and Colas 2026

Exact RG equation

- RG equation for the interactive part

$$\Lambda \frac{\partial}{\partial \Lambda} e^{iS^{\Lambda}_{\rm int}[\phi^L_r, \phi^L_a]} = \frac{i}{2} \int_{\vec{k}} \int d\eta \, d\tilde{\eta} \, \frac{\partial G^{\alpha\beta}_{\Lambda}(k;\eta,\tilde{\eta})}{\partial \log \Lambda} \frac{\delta^2 e^{iS^{\Lambda}_{\rm int}[\phi^L_r, \phi^L_a]}}{\delta \phi^L_\beta(\vec{k},\tilde{\eta}) \, \delta \phi^L_\alpha(-\vec{k},\eta)}.$$

Where $$G_{\alpha\beta}(k;\eta,\tilde{\eta}) = \begin{pmatrix} G_K(k;\eta,\tilde{\eta}) & G_R(k;\eta,\tilde{\eta}) \\ G_A(k;\eta,\tilde{\eta}) & 0 \end{pmatrix}.$$

- The RG flow implies that the system become open

Close interactions odd in ϕ_a

$$\dot{G}^K \left(\frac{\delta S_{\rm int}}{\delta \phi_r} \right)^2 \implies \text{Terms even in } \phi_a$$

$$S_{\rm cl}[\phi_r, \phi_a] = S[\phi_r + \phi_a/2] - S[\phi_r - \phi_a/2]$$

Green and Sun 2024
Frangi and Grozdanov 2025
SC and Colas 2026

EFT from RG

- As the cutoff is lowered the action generates diffusion and dissipation

$$\frac{\partial e^{iS^\Lambda_{\rm eff}[\phi_r,\phi_a]}}{\partial \log\Lambda} = 2i\frac{\partial G^{\alpha\beta}_\Lambda}{\partial\log\Lambda}\circ\left[-\frac{\delta^2 e^{iS^\Lambda_{\rm eff}[\phi_r,\phi_a]}}{\delta\varphi_\beta\delta\varphi_\alpha} + 2i\frac{\delta}{\delta\varphi_\alpha}\left(\frac{\delta S_0^\Lambda[\phi_r,\phi_a]}{\delta\varphi_\beta}e^{iS^\Lambda_{\rm eff}[\phi_r,\phi_a]}\right)\right]$$

Diffusion equation for the density matrix

- These terms are in principal non local

$$\frac{\partial S^\Lambda_{\rm eff}}{\partial\log\Lambda}\Big|_{aa} = -\varphi^L_a\circ\widehat{D}^A_\Lambda\circ\frac{\partial G^K_\Lambda}{\partial\log\Lambda}\circ\widehat{D}^R_\Lambda\circ\varphi^L_a$$ Diffusion

$$\frac{\partial S^\Lambda_{\rm eff}}{\partial\log\Lambda}\Big|_{ar} = -\varphi^L_a\circ\frac{\partial\widehat{G}^R_\Lambda}{\partial\log\Lambda}\circ\varphi^L_r\,.$$ Dissipation

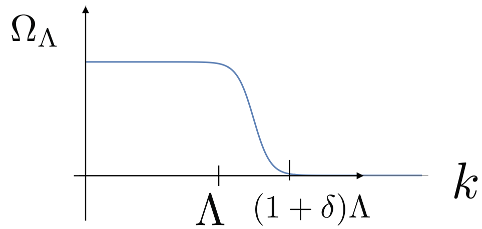
SC and Colas 2026

Open EFT for Stochastic Inflation

$$\Lambda = \epsilon a H$$

- Now lets consider the case of fields in de Sitter

$$\varphi^L(\vec{k},\eta) = \Omega_\Lambda(k)\varphi(\vec{k},\eta), \qquad \varphi^S(\vec{k},\eta) = \bar{\Omega}_\Lambda(k)\varphi(\vec{k},\eta),$$



- Thin shell of momenta implies the system is Markovian

- UV/IR mixing leads to Gaussian effective open vertices

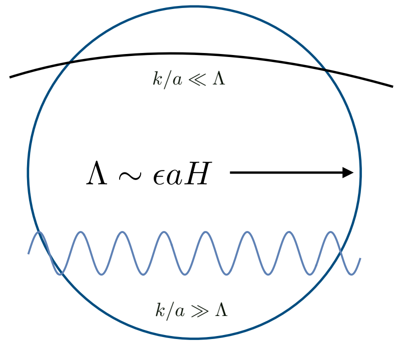
$$S_{\rm IF}^{\rm diff} = \int_{\vec{k}} \int d\eta\, a^2(\eta)\bar{\Omega}'_\Lambda(k) \int d\tilde{\eta}\, \bar{\Omega}'_{\tilde{\Lambda}}(k)\, a^2(\tilde{\eta}) \left\{ \varphi_a'^L(\vec{k},\eta)\varphi_a'^L(-\vec{k},\tilde{\eta})\, G_S^K(k,\eta,\tilde{\eta}) + \dots \right\}.$$

$$S_{\rm IF}^{\rm diss/drift} = \int_{\vec{k}} \int d\eta\, a^2(\eta)\bar{\Omega}'_\Lambda(k) \int d\tilde{\eta}\, \bar{\Omega}'_{\tilde{\Lambda}}(k)\, a^2(\tilde{\eta}) \left\{ \varphi_a'^L(\eta)\varphi_r'^L(\tilde{\eta}) G_S^{A/R}(k,\eta,\tilde{\eta}) + \dots \right\}$$

SC and Colas 2026
Li, 2025, 2026

Local EFT and stochastic limit

- For the specific time dependent filter function the expansion is organised in powers of $k/aH \sim \epsilon \ll 1$



- Filter functions make the action local in time

$$\bar{\Omega}'_\Lambda(k,\eta)\,\bar{\Omega}'_\Lambda(k,\eta') \longrightarrow \Lambda a(\eta) H\, \delta(k-\Lambda)\, \delta(\eta-\eta')$$

- Gradient expansion locality in space

Diffusion

$$S_{\rm IF}[\varphi_a^L, \varphi_r^L] \supset \quad S_{\rm IF}^{\rm diff} \simeq i \int {\rm d}^3\mathbf{x} \int {\rm d}\eta\; a^3(\eta)\; \left[\varphi_a'^L(\mathbf{x},\eta)\right]^2$$

$$S_{\rm IF}^{\rm drift/diss} = \epsilon^3 \int {\rm d}\eta\, a^3(\eta) \int {\rm d}^3\mathbf{x}\; \left[\varphi_r'^L \varphi_a^L \pm \varphi_r^L \varphi_a'^L\right] + \cdots\,,$$

Drift/Dissipation

Moss and Rigopoulous '16
Pinol et al '20
SC and Colas '26
Liu, '25, '26

Semiclassical Limit

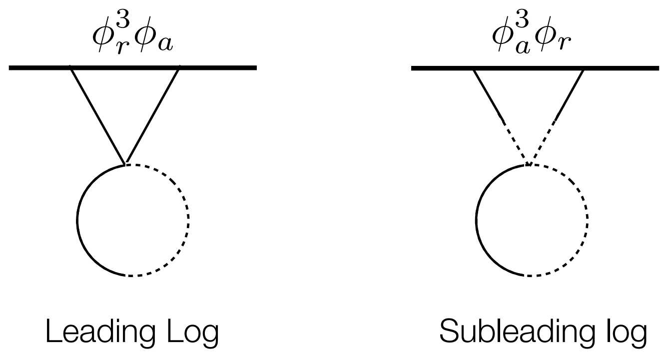
- At late times the systems is out of equlibrium $\frac{\text{dissipation}}{\text{diffusion}} \sim \epsilon^3.$

- This is more strictly formulated as a semiclassical limit

$$\phi_r \to \phi_c, \qquad \phi_a \to \epsilon^3 \hbar \phi_a$$

- Scaling matches perturbative computations. Some loop diagrams become more important

$\langle \phi^2 \rangle_{1-\text{loop}} =$



Baumgart and Sundrum 2019
SC, Davis and Wang '24
Launay et al '25
SC. Qin and Wang '26
SC and Colas '6

EFT for long modes

- Local action reproduces correlation functions at finite time

$$S = \int d^3x \int dt\, a^3(t) \Big[-\partial_\mu \phi_r^L\, \partial^\mu \phi_a^L - \frac{\lambda}{3!}(\phi_r^L)^3 \phi_a^L + iF[\phi_r](\dot{\phi}_a^L)^2 \Big].$$

- Using Hubbard-Stratonovich and integrating out the fast variables

$$S_{\rm red} = \int dt\, d^3\vec{x}\, a^3(t)\, \chi_a^L \left[\eta - \dot{\varphi}_r^L - \frac{1}{3H} V_{,\varphi}(\varphi_r^L) + \frac{1}{3H}\frac{\partial^2}{a^2}\varphi_r^L \right] + \cdots . \qquad \chi_a \sim \dot{\phi}_r$$

- This reproduces Langevin equation from stochastic inflation

$$\dot{\phi}_{\rm long} = -\frac{1}{3H} V_{,\phi} + \eta_\phi \qquad \langle \eta(t)\eta(t')\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$$

SC and Colas 2026
Li, 2025, 2026

Stochastic fixed point?

- Non relativistic theory has local equilibrium

$$S^{(2)}_{\rm red} = \int dt\, d^3\vec{x}\, a^3(t) \left[-\chi^L_a \left(\partial_t + \frac{1}{3H} V_{,\varphi\varphi} - \frac{1}{3H}\frac{\partial^2}{a^2}\right)\varphi^L_r + iD(\chi^L_a)^2\right]. \qquad \chi_a \sim \dot{\phi}_r$$

- Dynamical KMS at zero momenta

$$\varphi^L_r(t,\vec{x}) \to \varphi^L_r(-t,\vec{x}), \qquad \chi^L_a(t,\vec{x}) \to \chi^L_a(-t,\vec{x}) + iT^{-1}_{\rm eff}\dot{\varphi}^L_r(-t,\vec{x}). \qquad T_{\rm eff} = \frac{3H}{8\pi^2}.$$

- Scaling changes at late times

$$[\varphi^L_r] = \frac{d-2}{2}, \qquad [\chi^L_a] = \frac{d+2}{2}.$$

$$\begin{aligned} \widehat{\chi}^L_a(\varphi^L_r)^3 &: \quad \text{relevant for } d<4, \\ (\widehat{\chi}^L_a)^2(\varphi^L_r)^2 &: \quad \text{irrelevant for } d>2, \\ (\widehat{\chi}^L_a)^3\varphi^L_r &: \quad \text{strongly irrelevant for } d>0. \end{aligned}$$

SC and Colas 2026

From RG to a equation for the PDF

- Define probability as

$$P_\Lambda[\phi^L] = \int \mathcal{D}\varphi_r^L\, \mathcal{D}\varphi_a^L\, e^{iS_{\rm eff}^\Lambda[\varphi_r^L,\varphi_a^L]}\, \delta\big[\varphi_r^L(t_0) - \phi^L\big]\, \delta\big[\varphi_a^L(t_0)\big].$$

- Projecting into RG flow implies an defining a saddle point first

$$\frac{\partial P_\Lambda[\phi^L]}{\partial \log\Lambda} = i\int d^3x\, d^3y\, \frac{\partial G_\Lambda^K(|\vec{x}-\vec{y}|;t_0,t_0)}{\partial\log\Lambda}\, \frac{\delta^2 P_\Lambda[\phi^L]}{\delta\phi^L(\vec{x})\,\delta\phi^L(\vec{y})} + \int d^3x\, d^3y \int^{t_0} dt'\, \frac{\partial G_\Lambda^R(|\vec{x}-\vec{y}|;t_0,t')}{\partial\log\Lambda}\, \frac{\delta}{\delta\phi^L(\vec{x})}\Big[E^\Lambda(t',\vec{y};[\phi^L])\, P_\Lambda[\phi^L]\Big].$$

- Corrections are computed around this saddle point. Eg $\phi_a^3\phi_r$

$$\partial_{\log\Lambda} P_\Lambda \supset \lambda\int d^3x\, d^3y\, C_\Lambda(\vec{x},\vec{y})\, \phi^L(\vec{x})\, \frac{\delta^3 P_\Lambda}{\delta\phi^L(\vec{y})^3}.$$

Green and Gupta '25
SC and Colas '26,

From RG to Starobinsky

- FP equation is given by $$\frac{d}{dt}P_\Lambda[\phi^L,t] = \frac{\partial}{\partial t}P_\Lambda[\phi^L,t] + \dot{\Lambda}\frac{\partial}{\partial\Lambda}P_\Lambda[\phi,t]$$

- In this limit we can rewrite the RG equation as the Starobinsky-Yokoyama equation

$$\frac{\partial P[\phi^L]}{\partial t} = \frac{\partial}{\partial\phi^L}\left(\frac{V'(\phi^L)}{3H}P[\phi^L]\right) + \frac{H^3}{8\pi^2}\frac{\partial^2 P[\phi^L]}{\partial\phi^{L2}}$$

- This shows that the Starobinsky equation is the leading (diffusive) limit of the RG flow, with controlled ϵ -suppressed corrections.

$$\frac{\partial P[\phi^L]}{\partial t} \supset \epsilon^2\frac{\partial}{\partial\phi^L}\Big(F[\phi^L]P[\phi^L]\Big), \qquad \lambda\epsilon^6\frac{\partial^3\phi^L, P[\phi^L]}{\partial(\phi^L)^3}.$$

AC Davis and D-G Wang '23,
Green and Gupta '25
SC and T Colas '26

Stochastic inflation as a Lindbladian

- The FP equation can also be obtained from a Lindbladian operator

$$\partial_t \rho = -i[H_r, \rho] + \gamma \left(L\rho L - \frac{1}{2}\{L^2, \rho\} \right), \qquad \gamma = \frac{H_0^3}{4\pi^2}.$$

$$L = (1+\alpha)\Pi + \frac{a^3}{3H_0} v'''(\phi^c)(1+\beta)\phi,$$

$$\alpha = \frac{\lambda(\phi^c)^2}{6H_0^2} \log + \mathcal{O}(\lambda^2), \qquad \beta = \frac{\lambda(\phi^c)^2}{6H_0^2}(1 + 3\log) + \mathcal{O}(\lambda^2).$$

- This is equivalent to the RG evolution and thus the two things must be related

Goldman et al 24
Li 25,26
Christie et al 25,26

Conclusions

- Low energy EFTS of $\lambda\phi^4$ share the same features: diffusion and decoherence. Semiclassical limit arises
- Diffusion leads to an stochastic description
- We get a more general RG equation, does it have any applications?
- A lot of disagreement around non Gaussian noise
- We need to consider gravity to understand backreation