Approaches to Schwinger-Keldysh

An Overview

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Motivation: beyond amplitudes

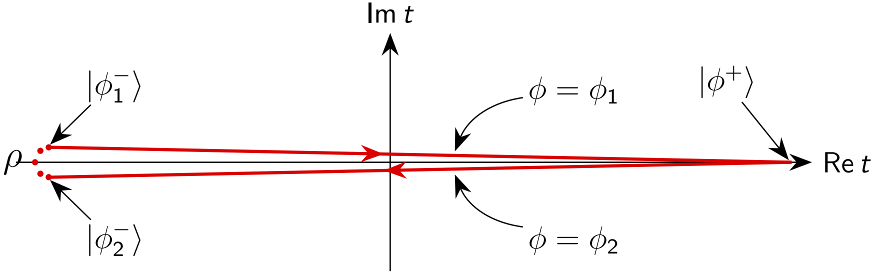
- High energy physics traditionally focused on scattering amplitudes
- We often care about expectation values:
  - Hydrodynamics
  - Gravitational Waves
  - Cosmology
  - Conformal Field Theories
- Need Schwinger-Keldysh (SK) formalism (aka "in-in" formalism)

Counterprogramming: “Beyond Amplitudes 2026”

The Schwinger-Keldysh Formalism

Interested in these expectation values:

$$\mathrm{Tr}\left[\rho \bar{T}(\dots)\, T(\dots)\right] = \int d\phi_1^- d\phi_2^- d\phi^+ \left\langle \phi_1^- \middle| \rho \middle| \phi_2^- \right\rangle \left\langle \phi_2^- \middle| \bar{T}(\dots) \middle| \phi^+ \right\rangle \left\langle \phi^+ \middle| T(\dots) \middle| \phi_1^- \right\rangle$$



Closed vs Open Systems

- **Closed systems:** von Neumann equation

$$i\partial_t \rho = [H, \rho]$$

- **Open systems:** master equation $\rightarrow$ Lindblad equation

$$i\partial_t \rho = [H, \rho] + i \sum_a \gamma_a \left(L_a \rho L_a^\dagger - \tfrac{1}{2}\{L_a^\dagger L_a, \rho\} \right), \qquad \rho = \mathrm{Tr}_{\mathrm{env}} \left(\rho_{\mathrm{closed}} \right)$$

The Schwinger-Keldysh Formalism

Interested in these expectation values:

$$\mathrm{Tr}\left[\rho\,\bar{T}(\dots)\,T(\dots)\right] = \int d\phi_1^- d\phi_2^- d\phi^+ \langle\phi_1^-|\rho|\phi_2^-\rangle\,\langle\phi_2^-|\,\bar{T}(\dots)|\phi^+\rangle\,\langle\phi^+|\,T(\dots)|\phi_1^-\rangle$$

Generated systematically by:

$$Z[J_1, J_2] = \int' D\phi_1 D\phi_2\, e^{iS[\phi_1]-iS[\phi_2]+i\Delta S_{\mathrm{open}}[\phi_1,\phi_2]+i\int J_1\phi_1 - i\int J_2\phi_2}$$

Feynman-Vernon Approach to Open Systems

Path integral first, integrate out after:

$$Z[J_1, J_2] = \int' D\phi_1 D\phi_2 D\chi_1 D\chi_2 e^{iS[\phi_1,\chi_1] - iS[\phi_2,\chi_2] + i\int J_1\phi_1 - i\int J_2\phi_2}$$

$$= \int' D\phi_1 D\phi_2 e^{iS[\phi_1] - iS[\phi_2] + i\Delta S_{\mathrm{open}}[\phi_1,\phi_2] + i\int J_1\phi_1 - i\int J_2\phi_2}$$

[Feynman-Vernon, *Annals Phys.* 24, 1963]

Special case: thermal state

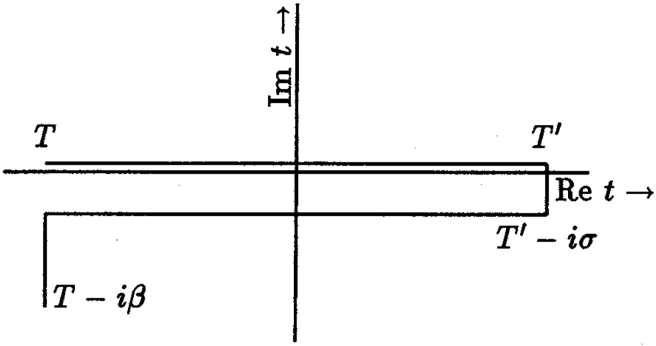


Figure: from Das, *Finite Temperature Field Theory*

Generalization: Out-of-Time-Order Correlators (OTOCs)

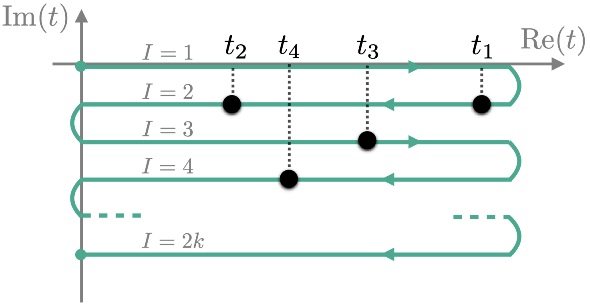


Figure: from **Haehl**-Rangamani, 2410.10602

e.g. Quantum Chaos: $\langle [A(t), B(0)]^2 \rangle$

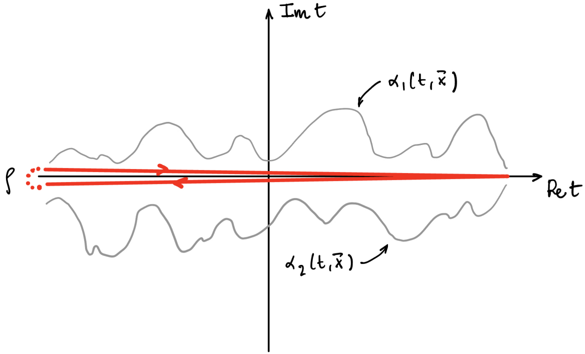
Schwinger-Keldysh Effective Field Theories

$$Z[J_1, J_2] = \int D\phi_1 D\phi_2 \, e^{iS_{EFT}[\phi_1,\phi_2]+i\int J_1\phi_1 - i\int J_2\phi_2}$$

- state + boundary conditions encoded in S_{EFT}
- non-trivial states always introduce new scales: $\beta, \mu, 1/\ell, \cdots \ll \Lambda$
- local effective description only for $\beta E, E/\mu, \ell p, \cdots \ll 1$
- state also determines symmetries and their breaking ($\rightarrow$ Nambu-Goldstone modes)

Gauge Symmetries

2 copies of **gauge** symmetries:



Global Symmetries

- **Closed:** $G_1 \times G_2 \to G_V$ (Strong to Weak SSB)
- **Open:** $G_1 \times G_2 \Rightarrow G_V$ + deformed G_A (e.g. $\phi \to \phi + e^{\Gamma t}$)
- **Emergent:** internal shifts (homogeneity), KMS (thermal), ...

[Open: **Agüí Salcedo-Colas-Pajer**, 2412.12299; **Christodoulidis**, 2509.13284]
[KMS: Glorioso-Crossley-**Liu**, 1701.07817; Sieberer *et al*, 1505.00912]

Unitarity constraints

Keldysh basis:

$$\phi_r = \frac{1}{2}(\phi_1 + \phi_2) + \dots \ , \qquad \phi_a = \phi_1 - \phi_2 + \dots$$

S_{EFT} can be complex, but must satisfy unitarity constraints:

- $\mathrm{Re}(S_{EFT})$ odd in ϕ_a
- $\mathrm{Im}(S_{EFT}) \geqslant 0$, even in ϕ_a
- $\mathrm{Im}(S_{EFT}) = 0$ when $\phi_a = 0$

Hydrodynamics

Quantities of interest: $\langle J^\mu J^\nu \rangle, \langle T^{\mu\nu} T^{\lambda\sigma} \rangle$, ...

[**Liu**-Glorioso, 1612.07705; **Delacretaz**, 2606.02391]
[**Haehl**-Loganayagam-Rangamani, 1803.11155]

Example: $U(1)$ Diffusion

- $U(1)_1 \times U(1)_2 \to U(1)_V$: 1 Goldstone mode π_a.
- Emergent KSM symmetry: $\pi_a(t) \to -\pi_a(-t) - i\beta\partial_t\mu_r(-t) + \ldots$
- Effective action:

$$S_{\mathrm{LO}} = \int \left[\chi(\mu_r)\,\mu_r\partial_t\pi_a + \sigma(\mu_r)\vec{\nabla}\pi_a \cdot \left(\frac{i}{\beta}\vec{\nabla}\pi_a - \vec{\nabla}\mu_r\right)\right] + \mathcal{O}(\pi_a^3)$$

- Diffusion:

$$\left.\frac{\delta S_{\mathrm{LO}}}{\delta\pi_a}\right|_{\pi_a=0} = 0 \quad \longrightarrow \quad \partial_t\mu_r = \frac{\sigma}{\chi}\nabla^2\mu_r + \mathcal{O}(\mu_r^2)$$

Power Counting

- Quadratic action:

$$S^{(2)}_{\mathrm{EFF}} = \int dt d^3x \left[\bar{\chi}\, \mu_r \partial_t \pi_a + \bar{\sigma}\, \vec{\nabla}\pi_a \cdot \left(\frac{i}{\beta} \vec{\nabla}\pi_a - \vec{\nabla}\mu_r \right) \right]$$

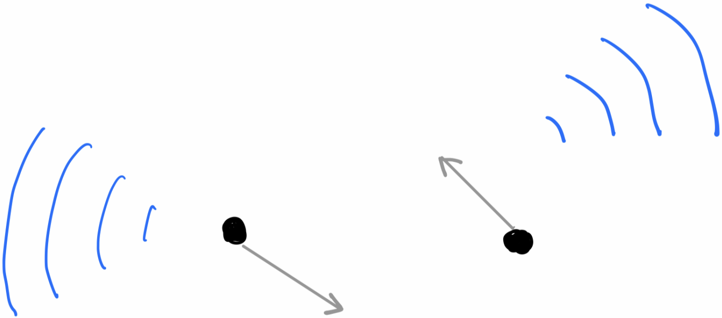
- Path integral dominated by configurations w/ $\mathrm{Re}(S_{EFF}) \sim 1$, $\mathrm{Im}(S_{EFF}) \lesssim 1$

$$\frac{\omega \pi_a}{\mu_r} \lesssim \frac{\omega}{T} \ll 1$$

- Higher derivative corrections:

$$\frac{S_{\mathrm{NLO}}}{S_{\mathrm{LO}}} \lesssim \underbrace{\omega / T}_{\text{derivative counting}} \times \underbrace{T t_{\mathrm{therm}}}_{\text{Wilson coefficients}} \sim \omega t_{\mathrm{therm}} \,,$$

Gravitational Waves



Waveform $= \langle h_{\mu\nu} \rangle$

Classical limit: $\hbar / L \ll 1$

Waveform from Schwinger-Keldysh

Effective action:

$$e^{i\Gamma[X_{1,2},J_{1,2}]} = \int Dh_1 Dh_2 e^{iS[X_1,\eta+h_1]-iS[X_2,\eta+h_2]+i\int(J_1h_1-J_2h_2)}$$

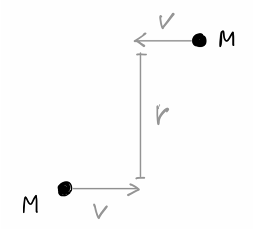
with $S = S_{\mathrm{EH}} + S_{p.p.} + S_{\mathrm{gf}} + S_{\mathrm{gh}}$.

$$\textbf{eom for } X_r^\mu \ : \ \left.\frac{\delta\Gamma}{\delta X_a^\mu}\right|_{X_a=J_1=J_2=0} = 0$$

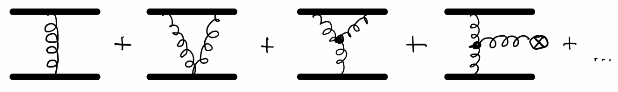
$$\textbf{waveform} \ : \ \langle h_{\mu\nu}^r\rangle = \left.\frac{\delta\Gamma}{\delta J_a^{\mu\nu}}\right|_{X_a=J_1=J_2=0} = 0$$

[Galley-Tiglio, 0903.1122]

Post-Minkowskian Expansion



Post-Minkowskian expansion of $\Gamma[X_{1,2}, J_{1,2}]$: powers of GM/r, all orders in v



[Neill-**Rothstein**, 1304.7263; Cheung-**Rothstein**-Solon, 1808.02489]

Post-Newtonian Expansion: Binary Systems

For a bound state, $GM/r \sim v^2$.

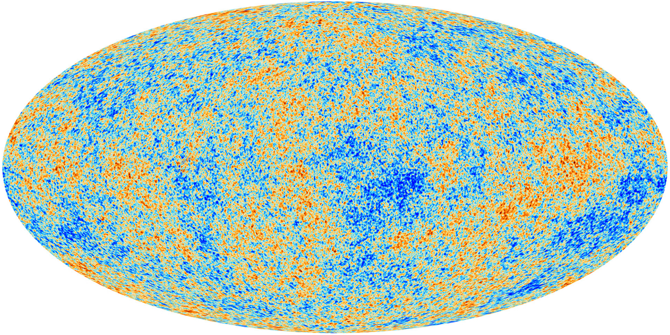
Manifest power counting inrequires method of regions: $h_{\mu\nu} = h_{\mu\nu}^{\rm pot} + h_{\mu\nu}^{\rm rad}$

$$\partial_t h^{\rm pot} \sim \partial_t h^{\rm rad} \sim v/r \;, \qquad \nabla h^{\rm pot} \sim 1/r \;, \qquad \nabla h^{\rm rad} \sim v/r$$

Effective action: $e^{i\Gamma[X_{1,2},h_{1,2}^{\rm rad}]} = \int Dh_1^{\rm pot} Dh_2^{\rm pot} e^{iS[X_1,\eta+h_1^{\rm pot}+h_1^{\rm rad}]-iS[X_2,\eta+h_2^{\rm pot}+h_2^{\rm rad}]}$

[Goldberger-**Rothstein**, hep-th/0409156 (NRGR)]

Cosmology



Observables $= \langle \delta T \delta T \rangle , \ldots$

Environment $=$ dark sector

EFT of Single-Field Inflation

Key idea: inflaton fluctuations = Goldstone of broken time translations

Effective action in unitary gauge ($\delta\phi = 0$):

$$S = \int d^4x \sqrt{-g}\, L(R_{\mu\nu,\lambda,\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t)$$

Decoupling limit:

$$S_\pi = \int d^4x\, a^3 \left[M_{\rm Pl}^2 \dot{H}\, (\partial_\mu \pi)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\, \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 + \cdots \right].$$

[Cheung-Fitzpatrick-Kaplan-Senatore-Creminelli, 0709.0293]

Open EFT of Single-Field Inflation

Open version in the decoupling limit:

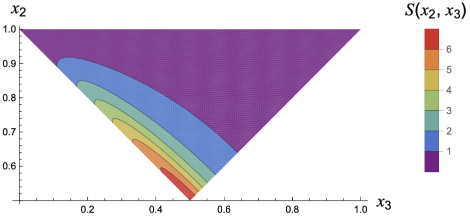
$$\mathcal{L}_{(2)} = (\alpha_0 - 2\alpha_1)\dot{\pi}_r\dot{\pi}_a - \alpha_0\partial_i\pi_r\partial^i\pi_a - 2\gamma_1\dot{\pi}_r\pi_a + i\left[\beta_1\pi_a^2 + \beta_2\dot{\pi}_a^2 + \beta_3(\partial_i\pi_a)^2\right] + \dots$$

Note: no emergent symmetry has been imposed

[**Agüí Salcedo-Colas-Pajer**, 2404.15416; + **Dufner**, 2507.03103]

Dissipation and Non-Gaussianities

Small Dissipation



Large Dissipation

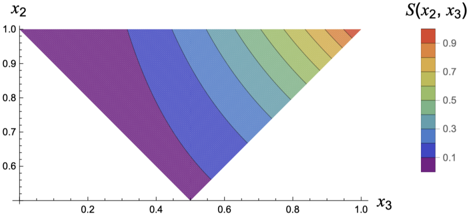
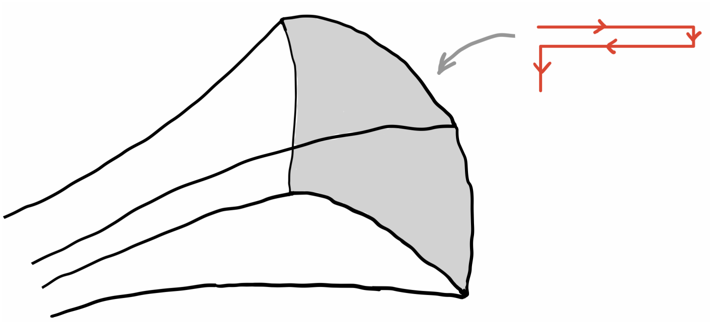


Figure: from **Agüí Salcedo-Colas-Pajer**, 2404.15416

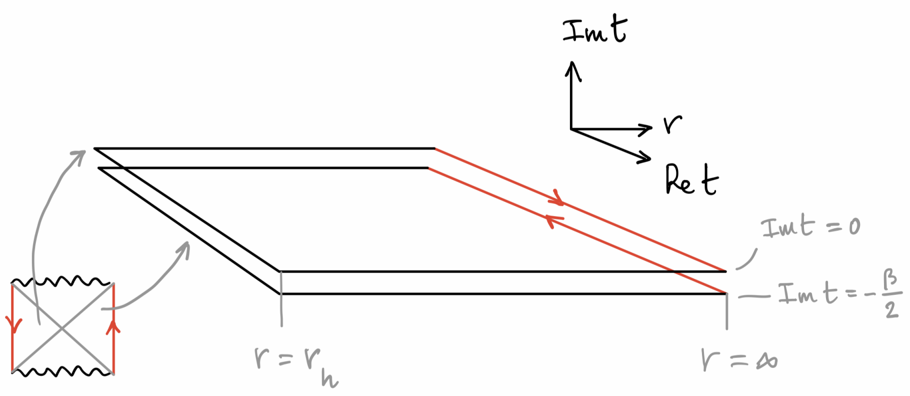
$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

Holography



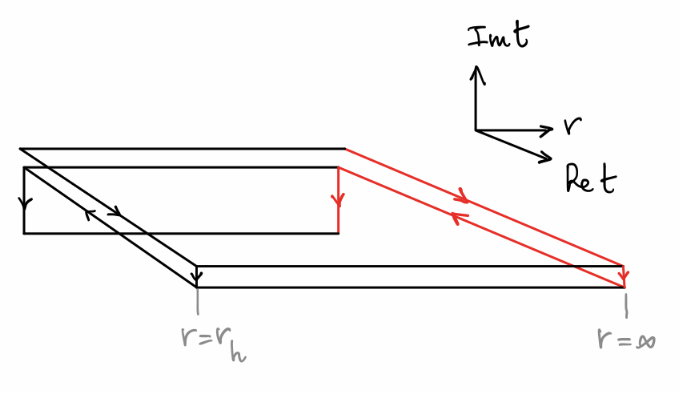
$$2 \times \text{ boundary d.o.f.} \longrightarrow 2 \times \text{ bulk spacetime}$$

Herzog-Son



Herzog-Son, hep-th/0212072
Son–Starinets, hep-th/0205051
Maldacena, hep-th/0106112

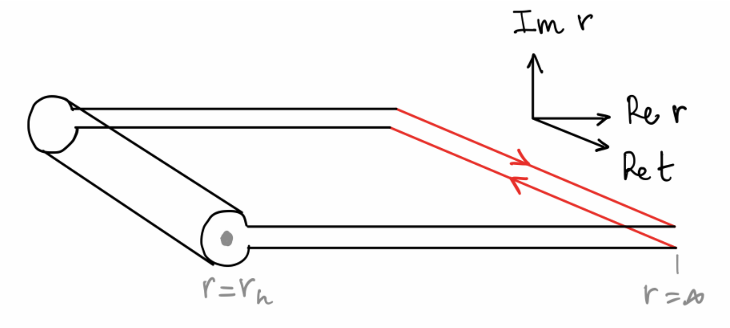
Skenderis-Van Rees



Skenderis-Van Rees, 0812.2909

de Boer-**Heller-Pinzani Fokeeva**, 1812.06093

Glorioso-Crossley-Liu



Glorioso-Crossley-**Liu**, 1812.08785

Thank you

Collaborators:

- Can Onur Akyuz, Garrett Goon (formerly CMU)
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