Phases and Symmetry Breaking in the Dissipative SYK Model

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In collaboration with Lucas Sá and Benjamin Béri

Motivation

1. Is the strong-to-weak symmetry breaking in open systems **explicit** or **spontaneous**? $G_1 \times G_2 \longrightarrow G_V$

2. What about breaking a **discrete** symmetry? $\mathbb{Z}_2 \times \mathbb{Z}_2 \longrightarrow \mathbb{Z}_2$

3. Can we probe strong-to-weak symmetry breaking via an **order parameter** and an expectation value? $\langle \mathcal{O} \rangle \neq 0$

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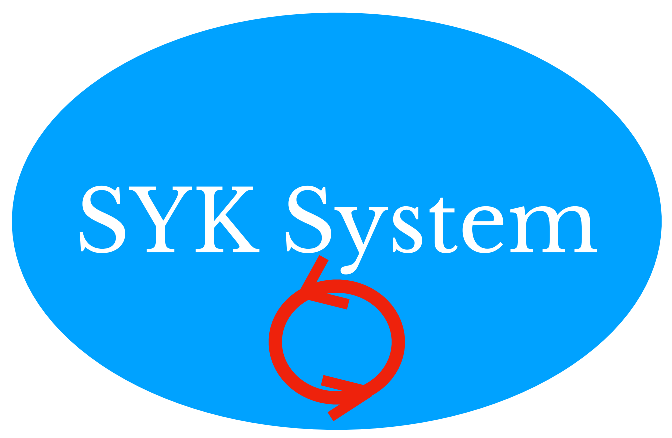

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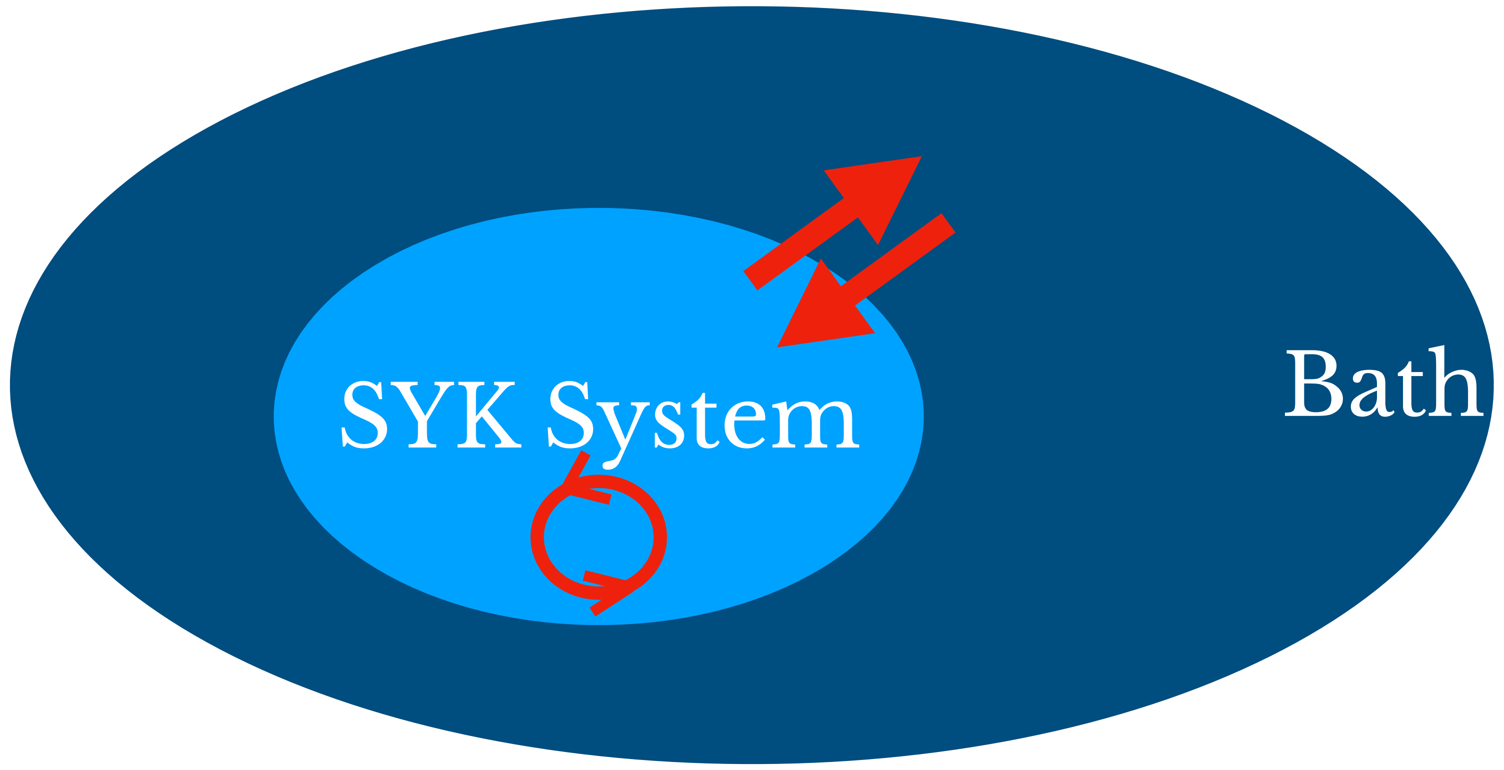
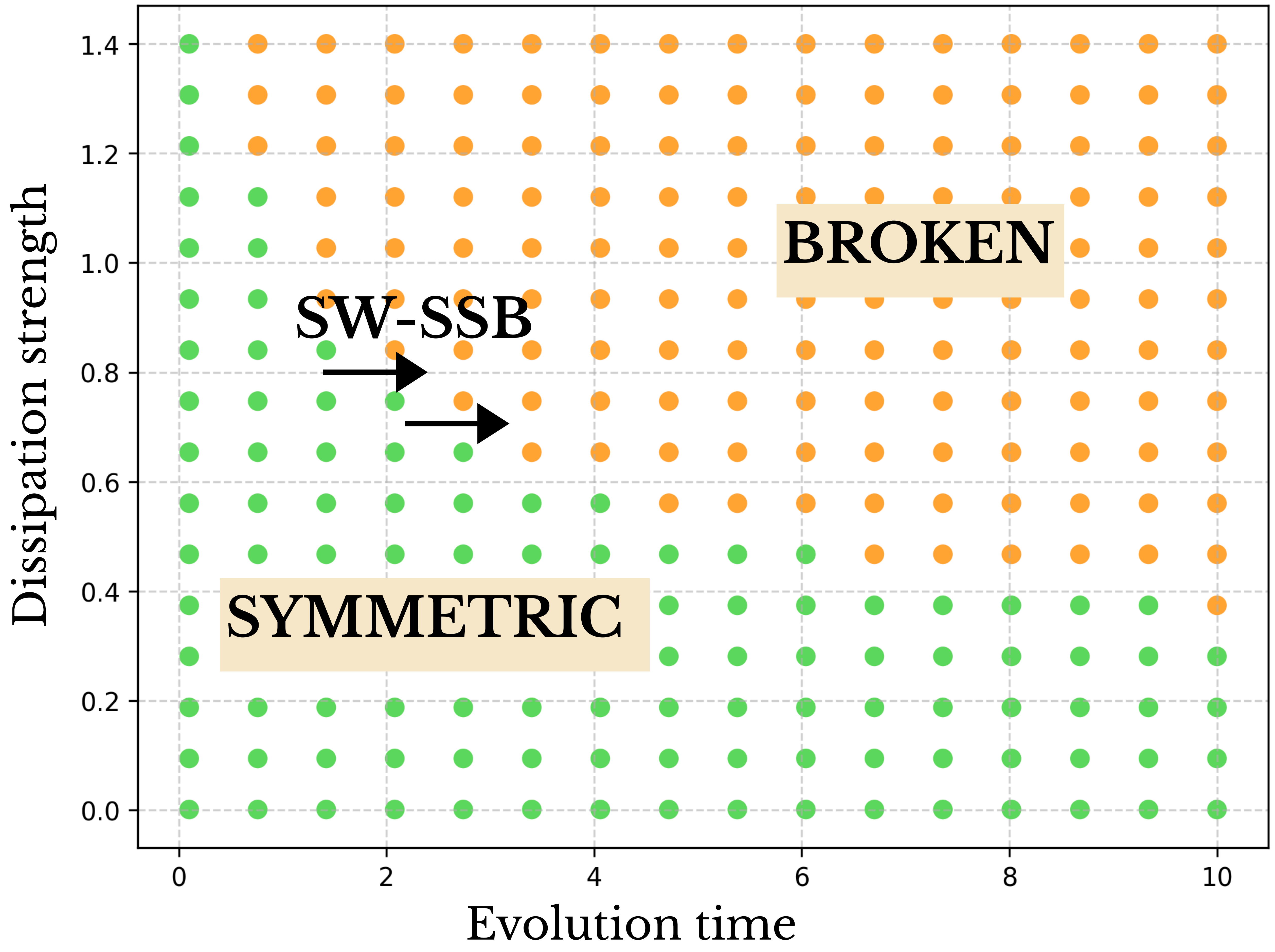


Take Away



$$\lim_{h\to 0}\lim_{N\to\infty}\frac{\langle\langle \rho\,|\,e^{T_L\mathscr{L}_h^\dagger}\mathcal{G}_{+-}e^{T_L\mathscr{L}_h}\,|\,\rho\rangle\rangle}{\langle\langle \rho\,|\,e^{T_L\mathscr{L}_h^\dagger}e^{T_L\mathscr{L}_h}\,|\,\rho\rangle\rangle}\neq 0$$

Probe for dynamical SW-SSB of fermion parity

Outline

I) Model and Path Integral

II) Symmetries

III) Phase Diagram

I) Model and Path Integral

$$\dot{\rho} = \mathcal{L}(\rho) = -i[H_{SYK}, \rho] + \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i L_i^\dagger, \rho\} \right)$$

SYK Hamiltonian

[Maldacena, Douglas, PRD (2016)]

$$H_{SYK} = -\frac{1}{4!} \sum_{i,j,k,l=1}^{N} J_{ijkl} \psi^i \psi^j \psi^k \psi^l$$

- N Majoranas
- $\{\psi^i, \psi^j\} = \delta_{ij}$
- $(\psi^i)^\dagger = \psi^i$

- All to all random couplings
- $\langle J_{ijkl} \rangle = 0$
- $\langle J_{ijkl}^2 \rangle = \frac{J^2 3!}{N^3}$

Jump Operators

[Sá, Ribeiro, Prosen, PRR (2022)]
[Kulkarni, Numasawa, Ryu, PRB (2022)]
[Kawabata et al, PRB (2023)]

1. Linear jump operators $L_i = h\psi^i$

2. Quadratic jump operators $L_{ij} = \frac{K}{\sqrt{N}} \psi^i \psi^j$

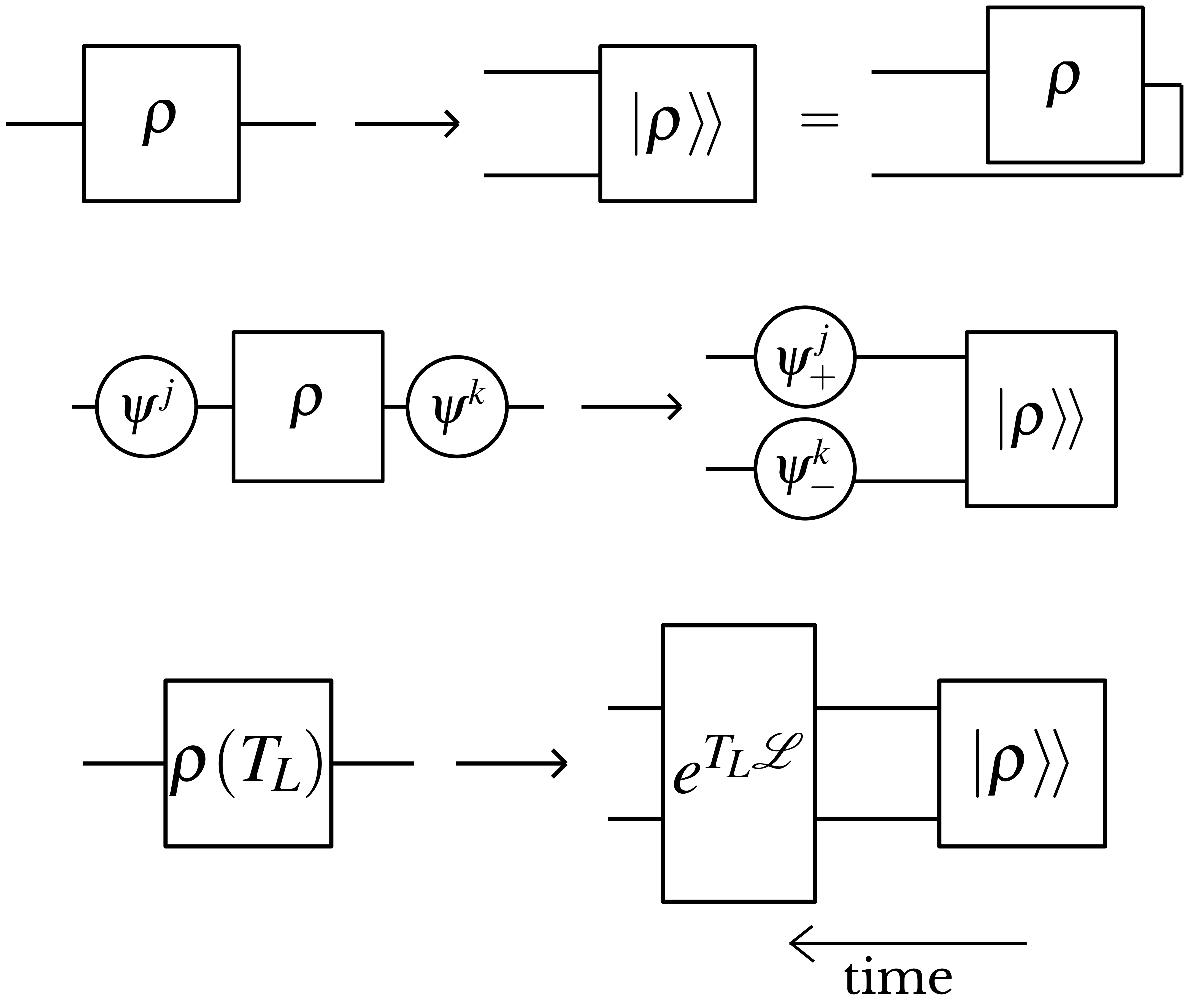
I) Model and Path Integral

$$\dot{\rho} = \mathscr{L}(\rho) = -i[H_{SYK}, \rho] + \sum_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\{L_i L_i^\dagger, \rho\} \right)$$

Vectorisation

- $\rho = \sum_{ij} p_{ij} |i\rangle\langle j| \to |\rho\rangle\rangle = \sum_{ij} p_{ij} |i\rangle_+ |j\rangle_-$
- $\psi^j \rho \psi^k \to \psi_+^j (-i\psi_-^k) |\rho\rangle\rangle$
- Time evolution $|\rho(T_L)\rangle\rangle = e^{T_L \mathscr{L}} |\rho\rangle\rangle$

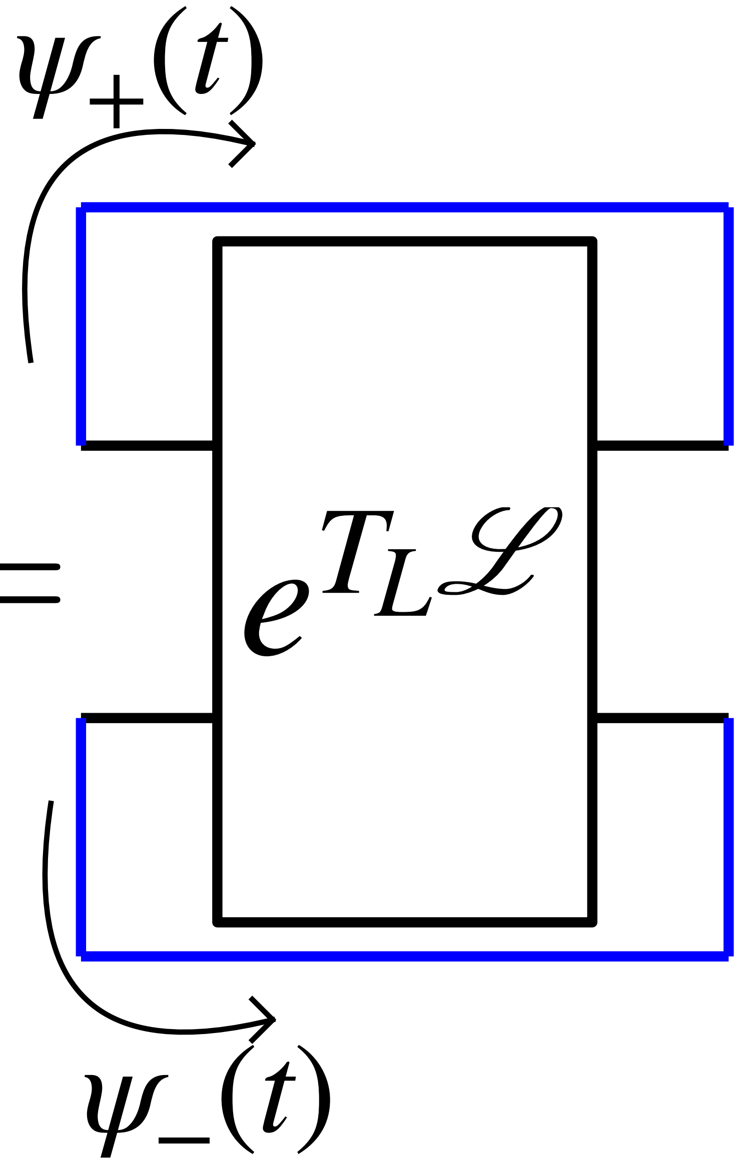
Diagrams



I) Model and Path Integral

Keldysh double contour path integral

$$\mathrm{Tr}\left(e^{T_L\mathscr{L}}\right) =$$



$$= \int \mathscr{D}\psi_+\mathscr{D}\psi_- e^{iS[\psi_+,\psi_-]} \overset{\langle\dots\rangle_J}{=} \int \mathscr{D}G\mathscr{D}\Sigma\, e^{NiS[G,\Sigma]}$$

- Microscopic action: $iS[\psi_+,\psi_-] = \int_0^{T_L} dt - \frac{1}{2}\psi_+(t)\partial_t\psi_+(t) - \frac{1}{2}\psi_-(t)\partial_t\psi_-(t) + \mathscr{L}$
- Collective fields: $G_{ab}(t,t') = -\frac{i}{N}\sum_j \psi_a^j(t)\psi_b^j(t')$
- Large N equations of motion for $G_{++}, G_{--}, G_{+-}, G_{-+}$

$$G_{ab}(t,t') = \left[\delta_{ab}\delta(t-t')\, i\partial_{t'} - \Sigma_{ab}(t,t')\right]^{-1}$$

$$\Sigma_{ab}(t,t') = -J^2 s_{ab} G_{ab}(t,t')^3 + 2K^2 G_{ab}(t,t)\delta(t-t')$$

[Sá, Ribeiro, Prosen, PRR (2022)]
[Kawabata et al, PRB (2023)]

II) Symmetries

$$\mathscr{L} = -iH_+^{SYK} + iH_-^{SYK} - \frac{K^2}{N}\sum_{i,j}\psi_+^i\psi_-^i\psi_+^j\psi_-^j - \frac{K^2N}{4}$$

Name	Symmetry Operator	Transformation
Complex Conjugation	$\mathscr{K}$	Complex Conjugation
Swap	Q	$\psi_\pm \rightarrow \mp\psi_\mp$
+ Sector Fermion Parity	P_+	$\psi_\pm \rightarrow \mp\psi_\pm$
Total Fermion Parity	P	$\psi_\pm \rightarrow -\psi_\pm$

Disorder average → Collective fields

Name	Symmetry Operator	Symmetry of the Action	Symmetry of Saddle Solutions
Modular Conjugation	$Q \circ \mathscr{K}$	Yes	Imposed
Swap	Q	Yes	Possible SSB
+ Sector Fermion Parity	P_+	Yes	Possible SSB

[Sá, Ribeiro, Prosen, PRX (2023)]

[Kawabata et al, PRX Quantum (2023)]

II) Symmetries

$$\mathscr{L} = -\, iH_+^{SYK} + iH_-^{SYK} - \frac{K^2}{N}\sum_{i,j} \psi_+^i \psi_-^i \psi_+^j \psi_-^j - \frac{K^2 N}{4}$$

Name	Symmetry Operator	Symmetry of the Action	Symmetry of Saddle Solutions
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Weak and strong symmetry

- $U\rho U^\dagger = \rho$, weak.
- $U\rho = e^{i\phi}\rho$, strong.

Weak and strong fermion parity

- $P\rho P = \rho \implies P_+ P_- |\rho\rangle\rangle = |\rho\rangle\rangle$, weak.
- $P\rho = \pm\rho \implies P_+ |\rho\rangle\rangle = \pm |\rho\rangle\rangle$, strong.

II) Symmetries

$$\mathscr{L}_h = -iH_+^{SYK} + iH_-^{SYK} - \frac{K^2}{N}\sum_{i,j}\psi_+^i\psi_-^i\psi_+^j\psi_-^j - \frac{K^2N}{4} - ih^2\sum_j \psi_+^j\psi_-^j - \frac{h^2N}{2}$$

Strong-to-weak spontaneous symmetry breaking (SW-SSB) [Lessa et al, PRX Quantum (2025)] [Ma, Turzillo, PRX Quantum (2025)] [Wang 2606.02555]

- Breaking $\mathbb{Z}_2 \times \mathbb{Z}_2 \longrightarrow \mathrm{diag}(\mathbb{Z}_2 \times \mathbb{Z}_2)$

  $P_+ |\rho\rangle\rangle = \pm |\rho\rangle\rangle$
  $P_+P_- |\rho\rangle\rangle = |\rho\rangle\rangle$ $\longrightarrow$ $P_+P_- |\rho\rangle\rangle = |\rho\rangle\rangle$

- Order parameter $\mathscr{G}_{+-} = -\frac{i}{N}\sum_j \psi_+^j\psi_-^j$

- Charged $P_+\mathscr{G}_{+-}P_+ = -\mathscr{G}_{+-}$

- Neutral $P_+P_-\mathscr{G}_{+-}P_-P_+ = \mathscr{G}_{+-}$

$$\lim_{h\to 0}\lim_{N\to\infty} \frac{\langle\langle\rho|e^{T_L\mathscr{L}_h^\dagger}\mathscr{G}_{+-}e^{T_L\mathscr{L}_h}|\rho\rangle\rangle}{\langle\langle\rho|e^{T_L\mathscr{L}_h^\dagger}e^{T_L\mathscr{L}_h}|\rho\rangle\rangle} \neq 0$$

Probe for dynamical SW-SSB of fermion parity

III) Phase Diagram

$$\lim_{h\to 0}\lim_{N\to\infty}\frac{\langle\langle\rho|e^{T_L\mathcal{L}_h^\dagger}\mathcal{G}_{+-}e^{T_L\mathcal{L}_h}|\rho\rangle\rangle}{\langle\langle\rho|e^{T_L\mathcal{L}_h^\dagger}e^{T_L\mathcal{L}_h}|\rho\rangle\rangle}\neq 0$$

Pure state maximally entangled with a reference: $|\psi\rangle = \sum_i |i\rangle|i\rangle_R$

Density matrix: $\rho = \sum_{i,j}|i\rangle|i\rangle_R\langle j|\langle j|_R$

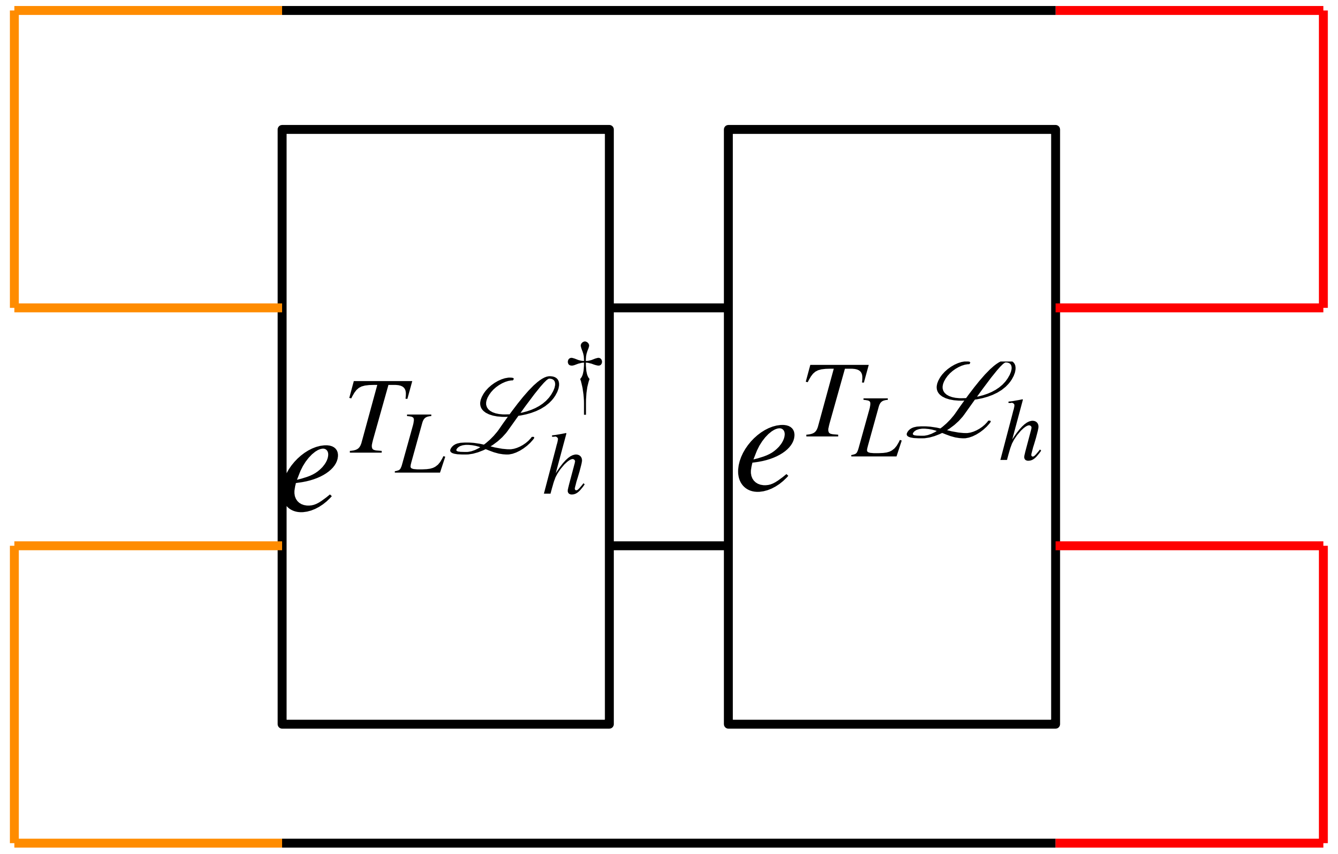
Vectorised state on double contour: $|\rho\rangle\rangle = \sum_{i,j}|i\rangle_{R+}|i\rangle_+ \otimes |j\rangle_{R-}|j\rangle_-$

III) Phase Diagram

b. Pure state maximally entangled with a reference

$$|\rho\rangle\rangle = \sum_{i,j} |i\rangle_{R+} |i\rangle_{+} \otimes |j\rangle_{R-} |j\rangle_{-}$$

$$\langle\langle\rho | e^{T_L \mathscr{L}_h^\dagger} e^{T_L \mathscr{L}_h} | \rho\rangle\rangle =$$



$$= \mathrm{Tr}\left(e^{T_L \mathscr{L}_h^\dagger} e^{T_L \mathscr{L}_h}\right)$$

$$\lim_{h\to 0} \lim_{N\to\infty} \frac{\langle\langle\rho | e^{T_L \mathscr{L}_h^\dagger} \mathscr{G}_{+-} e^{T_L \mathscr{L}_h} | \rho\rangle\rangle}{\langle\langle\rho | e^{T_L \mathscr{L}_h^\dagger} e^{T_L \mathscr{L}_h} | \rho\rangle\rangle} = \lim_{h\to 0} \lim_{N\to\infty} \frac{\mathrm{Tr}\left(e^{T_L \mathscr{L}_h^\dagger} \mathscr{G}_{+-} e^{T_L \mathscr{L}_h}\right)}{\mathrm{Tr}\left(e^{T_L \mathscr{L}_h^\dagger} e^{T_L \mathscr{L}_h}\right)} = \lim_{h\to 0} \lim_{N\to\infty} \frac{\int \mathscr{D}G \mathscr{D}\Sigma \, G_{+-}(T_L, T_L) \, e^{NiS[G,\Sigma]}}{\int \mathscr{D}G \mathscr{D}\Sigma \, e^{NiS[G,\Sigma]}}$$

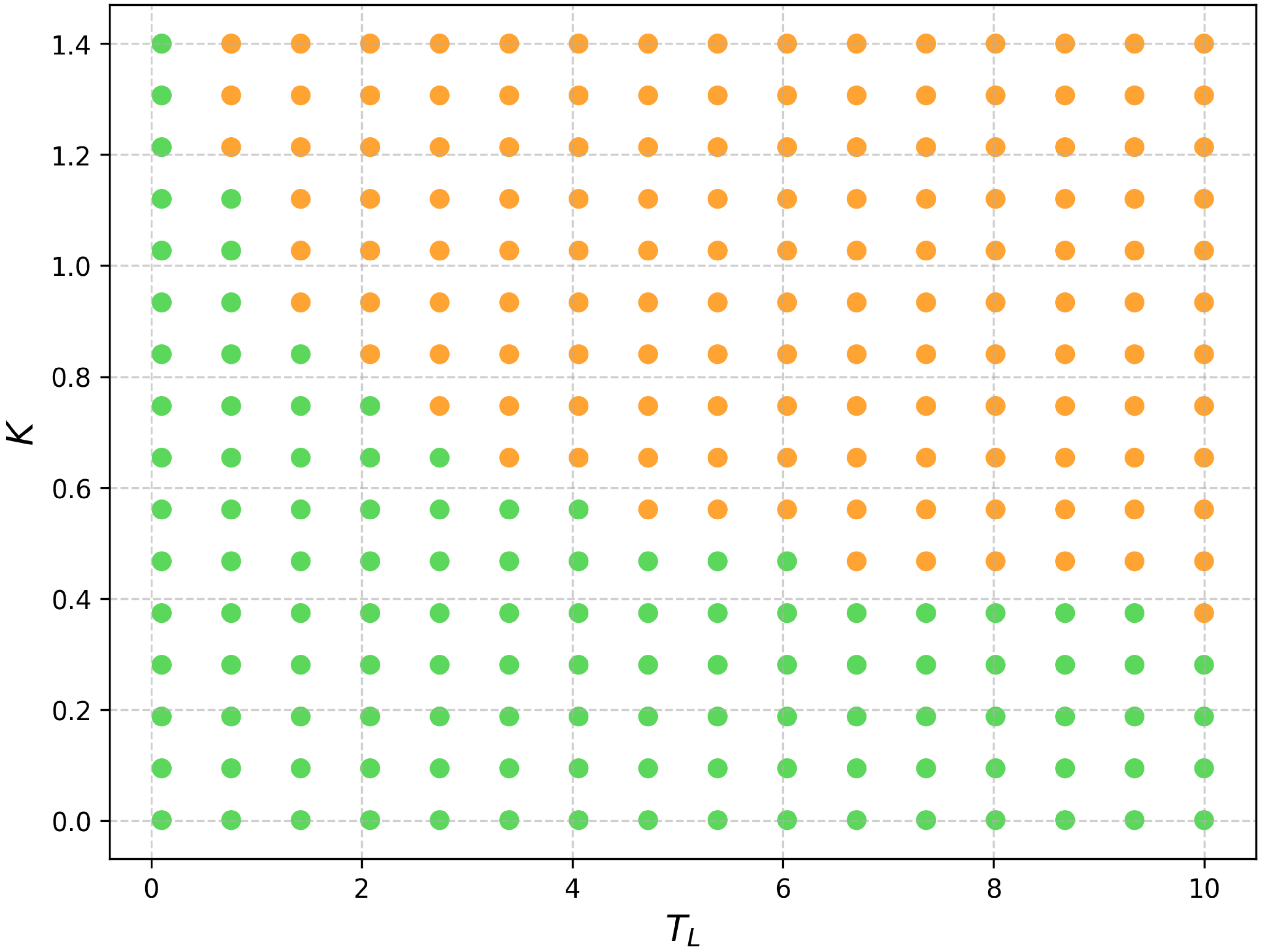
$$= G_{+-}(T_L, T_L)$$

Saddle solution for cross-contour correlator probes SW-SSB

III) Phase Diagram

b. Pure state maximally entangled with a reference

$$|\rho\rangle\rangle = \sum_{i,j} |i\rangle_{R+} |i\rangle_{+} \otimes |j\rangle_{R-} |j\rangle_{-}$$



Color	P_+
	✗
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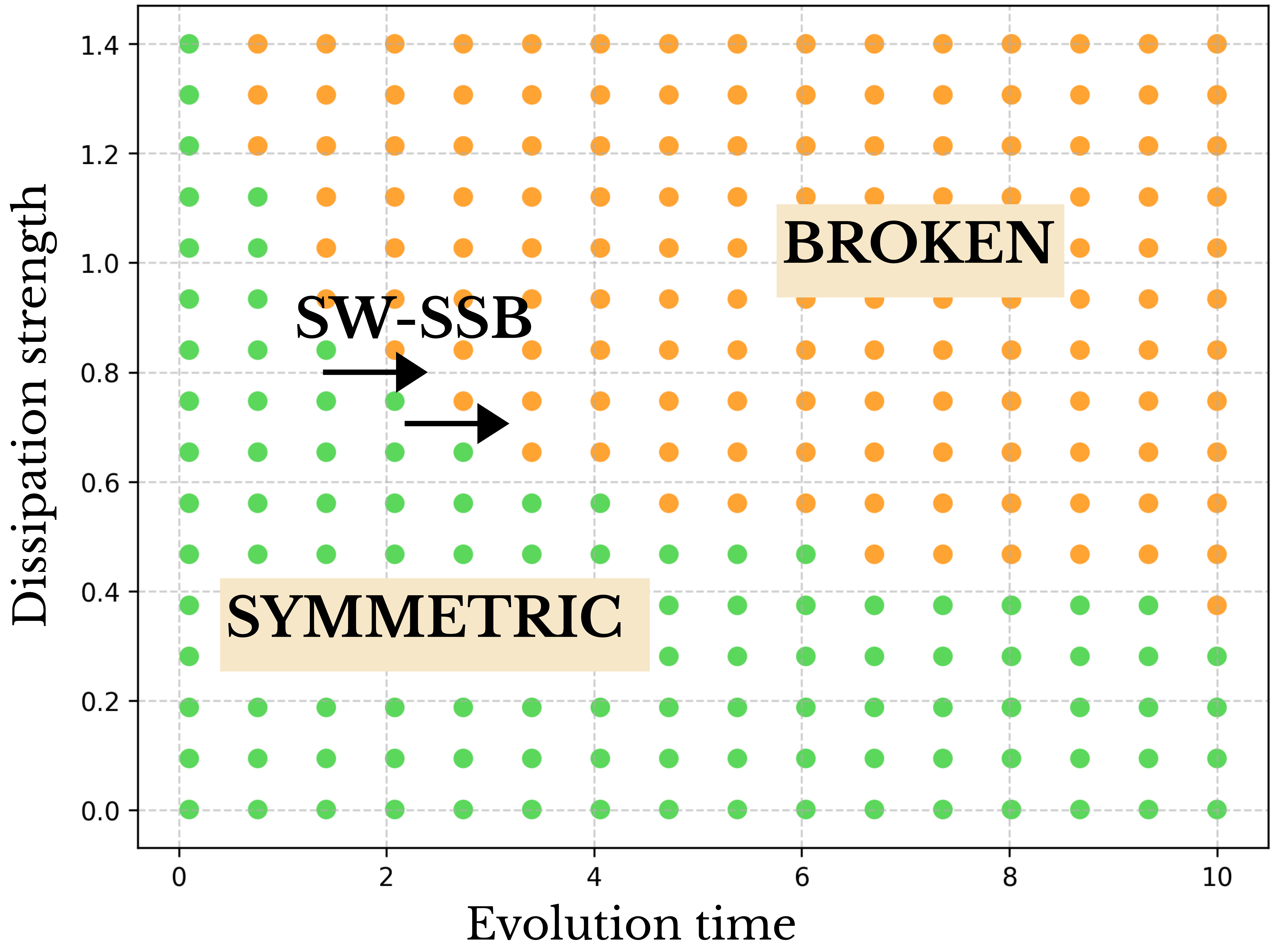
Fig.1: Phase diagram for $\mathrm{Tr}\left(e^{T_L \mathscr{L}^\dagger} e^{T_L \mathscr{L}}\right)$.
Quadratic jump operators.
($J = 1$).

Take Away



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