Bubble wall dynamics from nonequilibrium quantum field theory

Bridging two regimes

with W. Ai, B. Garbrecht, C. Tamarit and M. Vanvlasselaer

Matthias Carosi

ICTP-AP
UCAS

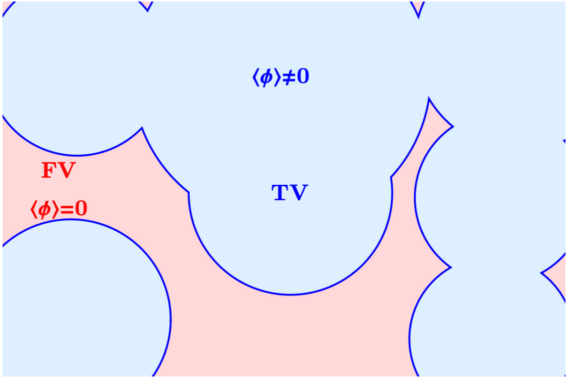
2/07/2026
Contours 2026

Outline

First-order phase transitions

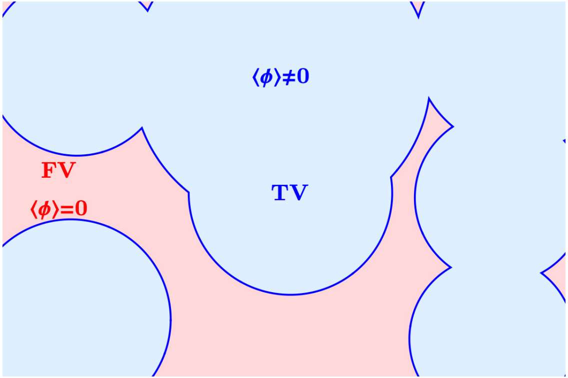
First-order phase transitions

- The early universe can undergo **first-order phase transitions**.



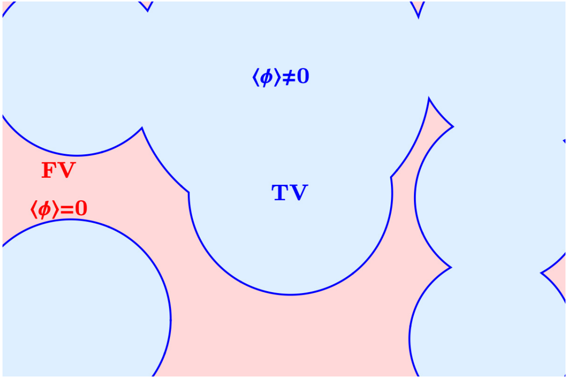
First-order phase transitions

- The early universe can undergo **first-order phase transitions**.
- They proceed via the **nucleation and expansion of bubbles** in the hot plasma.



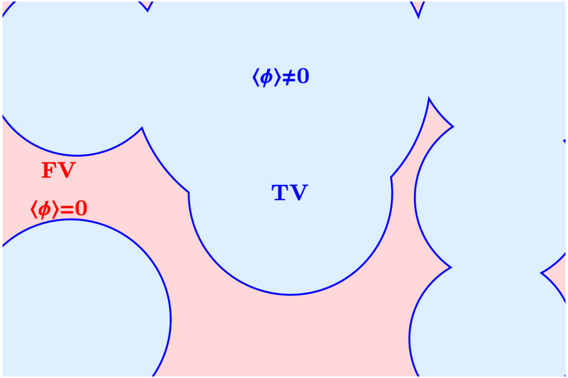
First-order phase transitions

- The early universe can undergo **first-order phase transitions**.
- They proceed via the **nucleation and expansion of bubbles** in the hot plasma.
- Bubble collisions and the stirred plasma source a background of **gravitational waves**.



First-order phase transitions

- The early universe can undergo **first-order phase transitions**.
- They proceed via the **nucleation and expansion of bubbles** in the hot plasma.
- Bubble collisions and the stirred plasma source a background of **gravitational waves**.
- Its spectrum depends on the **wall velocity** v_w $\Rightarrow$ we study the **real-time evolution of a single bubble**.



Predicting the GW spectrum

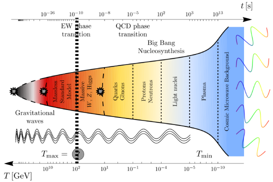
Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group ’22]

Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group ’22]

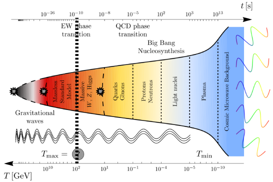
- Transition temperature



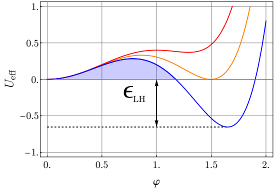
Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group '22]

- Transition temperature



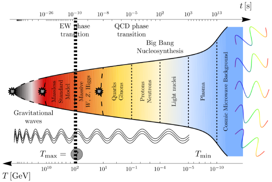
- Transition strength



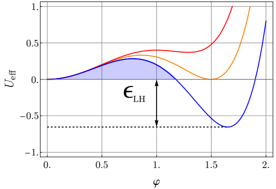
Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group ’22]

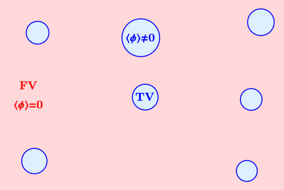
- Transition temperature



- Transition strength



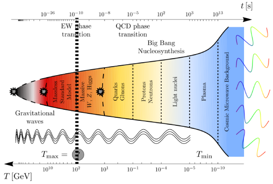
- Nucleation rate



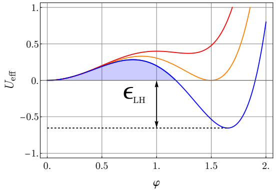
Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group ’22]

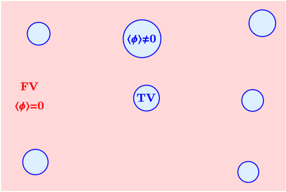
- Transition temperature



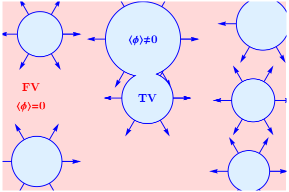
- Transition strength



- Nucleation rate



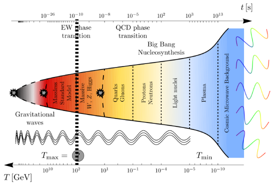
- Bubble expansion velocity



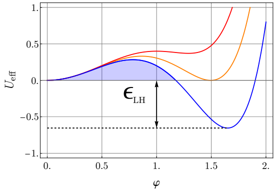
Predicting the GW spectrum

The GW spectrum mainly relies on four parameters [Lisa Cosmology working group '22]

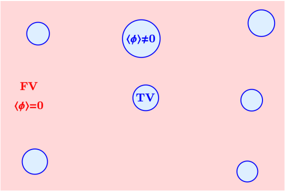
- Transition temperature



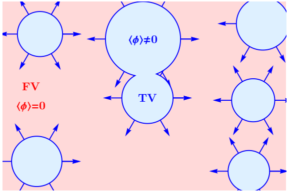
- Transition strength



- Nucleation rate

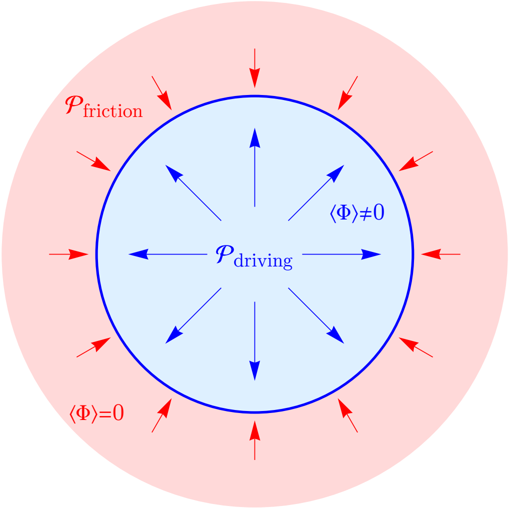


- **Bubble expansion velocity**

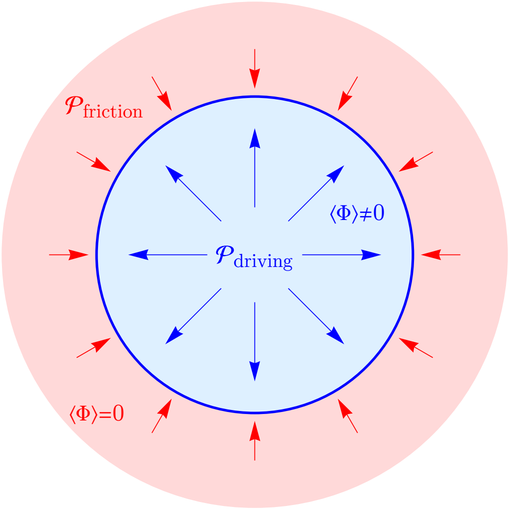


A balance of pressure

A balance of pressure



A balance of pressure



When the two pressures balance

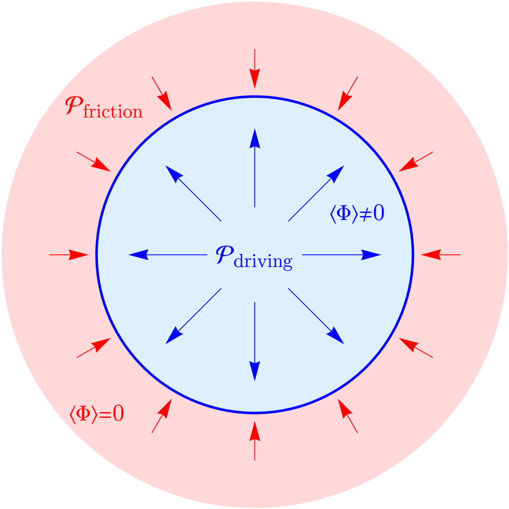
$$\mathcal{P}_{\rm friction} = \mathcal{P}_{\rm driving}$$

the system reaches a *steady state*

$\Longrightarrow$ terminal wall velocity

$\equiv v_w$

A balance of pressure



When the two pressures balance

$$\mathcal{P}_{\text{friction}} = \mathcal{P}_{\text{driving}}$$

the system reaches a *steady state*

$\Longrightarrow$terminal wall velocity

$\equiv \; v_w$

Goal: does a stable steady state exist? If so, what is v_w?

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture

[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture

[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✗ misses scattering processes

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✗ misses scattering processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\mathbf{p}} \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\, f_i(\mathbf{p})\, \Delta p_{\perp\,,i\to X}$$

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✗ misses scattering processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\mathbf{p}} \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\, f_i(\mathbf{p})\, \Delta p_{\perp\,,i\to X}$$

✔ includes all scattering processes

✗ only valid for ultrarelativistic walls

Kinetic vs. kick

Two main approaches exist for studying the dynamics of a single bubble

Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} \delta f_i(\mathbf{p}, x) = 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = -\mathcal{C}_i[f, \varphi] \end{cases}$$

✔ includes nonequilibrium effects

✗ misses scattering processes

Kick picture
[Dine et al. '92, Bodeker and Moore '09, '17]

Pressure from the flux of particles

$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\mathbf{p}} \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\, f_i(\mathbf{p})\, \Delta p_{\perp, i\to X}$$

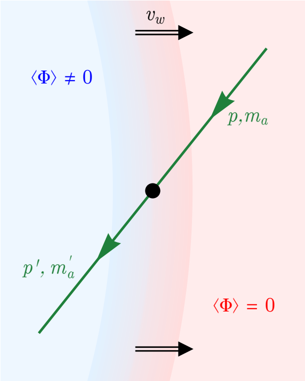
✔ includes all scattering processes

✗ only valid for ultrarelativistic walls

Why would we care about processes which are higher order in the couplings?

(some) Sources of friction in the kick picture

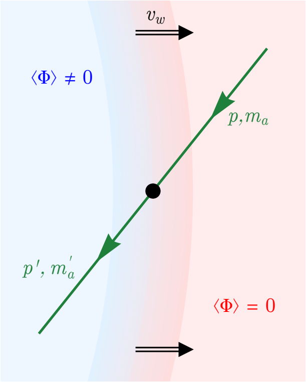
(some) Sources of friction in the kick picture



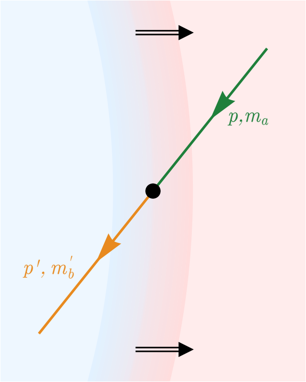
Mass gain

Captured by kinetic pic

(some) Sources of friction in the kick picture

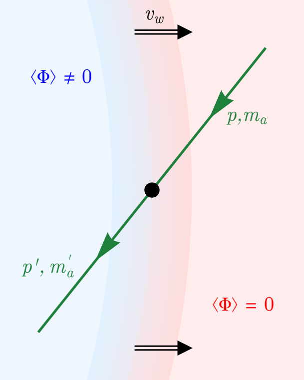


Mass gain
Captured by kinetic pic



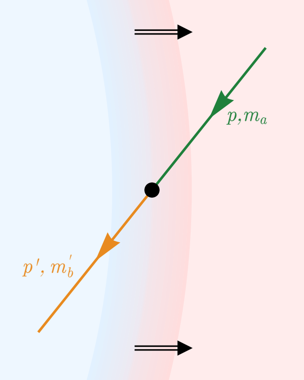
Mixing

(some) Sources of friction in the kick picture

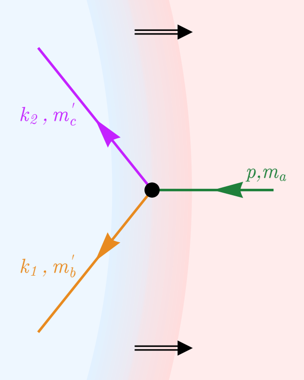


Mass gain

Captured by kinetic pic

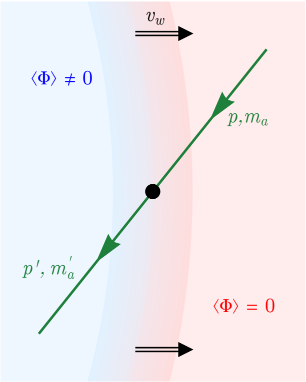


Mixing

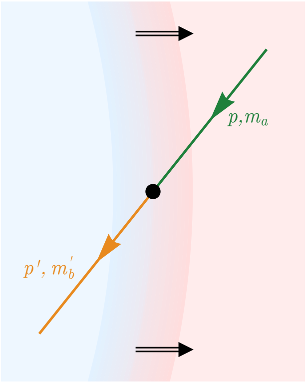


Particle production

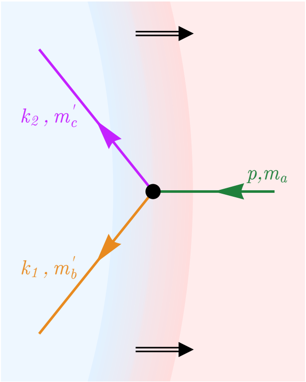
(some) Sources of friction in the kick picture



Mass gain
Captured by kinetic pic



Mixing



Particle production
$\sim \log \gamma_w$ for scalars
$\sim \gamma_w$ for gauge bosons

Outline

Bubble dynamics from first principles

Goal: develop a description of the bubble expansion from first principles using *nonequilibrium quantum field theory* and bridge the gap between kinetic and kick approaches.

The tools

The tools

From *nonequilibrium quantum field theory*:

Schwinger–Keldysh

The tools

From *nonequilibrium quantum field theory*:

Schwinger–Keldysh

2PI effective action

The tools

From *nonequilibrium quantum field theory*:

Schwinger–Keldysh

2PI effective action

give coupled equations for two objects:

bubble profile $\varphi(x)$

$\updownarrow$

one-point function

The tools

From *nonequilibrium quantum field theory*:

Schwinger–Keldysh

2PI effective action

give coupled equations for two objects:

bubble profile $\varphi(x)$

$\updownarrow$

one-point function

distribution functions $f(\mathbf{k}, x)$

$\updownarrow$

two-point functions

The tools

From *nonequilibrium quantum field theory*:

Schwinger–Keldysh

2PI effective action

give coupled equations for two objects:

bubble profile $\varphi(x)$

$\updownarrow$

one-point function

distribution functions $f(\mathbf{k}, x)$

$\updownarrow$

two-point functions

the 2PI action **closes** this coupled **1-pt** $+$ **2-pt** system self-consistently

The full dynamical equations

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{Tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{Tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0$$

$$\Big\downarrow$$

bubble EoM

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{Tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0} \quad\longrightarrow\quad \boxed{\text{bubble EoM}}$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0} \quad\longrightarrow\quad \boxed{\begin{array}{c}\text{Kadanoff-Baym eqs.}\\ \text{Boltzmann eqs.}\end{array}}$$

The full dynamical equations

$$\Gamma_{2\mathrm{PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{Tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,G_0^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)} = 0} \qquad \boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

bubble EoM

Kadanoff-Baym eqs.
Boltzmann eqs.

We obtain a system of coupled equations that include all **thermal**, **quantum** and **nonequilibrium** effects at the given perturbative order.

The bubble wall equation of motion

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \underbrace{\Pi^{\mathrm{R}}(x,y)}_{\sim\frac{\delta^2\Gamma_2}{\delta\varphi(x)\delta\varphi(y)}}\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \underbrace{\frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}}_{\text{T=0 correction to } U_0'} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \underbrace{\frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}}_{\text{T=0 correction to } U_0'} + \underbrace{\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_\mathbf{k}}f(\mathbf{k},x)}_{f=f_{\mathrm{eq}}+\delta f=(T\neq 0)\text{ corr. + off-eq.}}$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

- One-loop term

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

- Packing it into a more familiar form

$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

recall kinetic approach: $\Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) = 0$

- One-loop

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

- Packing it into a more familiar form

$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i\frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

The bubble wall equation of motion

Having solved for the two-point function at leading order in the gradients (the same expansion that yields Boltzmann from Kadanoff–Baym), we have the EoM for the bubble wall

$$\Box\varphi(x) + U_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

recall kinetic approach: $\Box\varphi + U'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) = 0$

- One-loop

$$\frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) = \frac{\lambda}{2}\varphi(x)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon} + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}}f(\mathbf{k},x)$$

- Packing it into a more familiar form

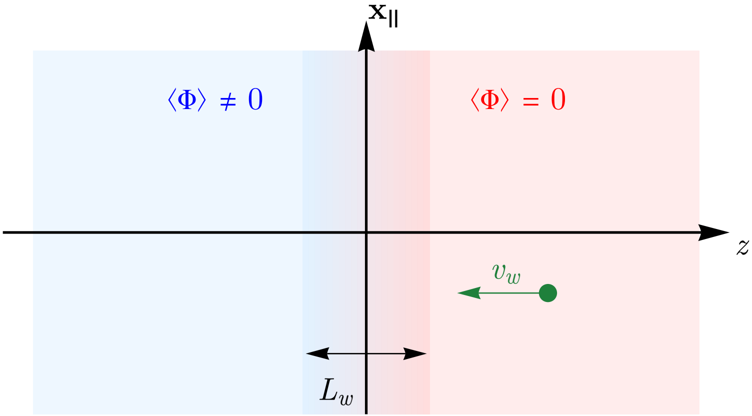
NEW TERM!

$$\Box\varphi(x) + U'(\varphi(x)) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi(x)}\int_{\mathbf{p}}\delta f_i(\mathbf{p},x) + \int_y \Pi^{\mathrm{R}}(x,y)\varphi(y) = 0$$

Identifying sources of friction

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**



Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \underbrace{\pi^R(z, z')}_{\int \mathrm{d}\mathbf{x}_\parallel \mathrm{d}\mathbf{x}'_\parallel \Pi^R(\mathbf{x},\mathbf{x}')} \varphi(z') \quad = 0$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2 + U(\varphi(z), T) \right] = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial U}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + U(\varphi(z), T) \right] = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial U}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\left[\cancel{\frac{1}{2}\left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\right)^2} + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial U}{\partial T}\frac{\mathrm{d}T}{\mathrm{d}z} - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial U}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} \right. \qquad \equiv \mathcal{P}_{\text{LTE}}$$

$$- \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z)$$

$$\left. - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\mathrm{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial U}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} \right. \qquad \equiv \mathcal{P}_{\mathrm{LTE}}$$

$$- \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) \qquad \equiv \mathcal{P}_{\mathrm{dissipative}}$$

$$\left. - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the *stationary planar* wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + U'(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

$$\begin{aligned}
\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + U(\varphi(z), T) \right]}_{\Delta U \equiv \mathcal{P}_{\mathrm{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \, \Bigg[& \frac{\partial U}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} && \equiv \mathcal{P}_{\mathrm{LTE}} \\
& - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) && \equiv \mathcal{P}_{\mathrm{dissipative}} \\
& - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg] && \equiv \mathcal{P}_{\mathrm{vertex}}
\end{aligned}$$

The vertex pressure in Wigner space

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\text{vertex}} = -\int \mathrm{d}z\, \mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \, \pi^R(z, z') \, \varphi(z') \; \equiv \; -\,\mathrm{Tr}\Big[\pi^R \, \varphi \, \partial_z \varphi\Big]$$

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\text{vertex}} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \equiv -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad \Longrightarrow \quad \mathcal{P}_{\text{vertex}} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\frac{1}{2}\partial_u - ik\Big]\widetilde{\pi}^R(k,u)$$

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\text{vertex}} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \;\equiv\; -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\text{vertex}} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\cancel{\frac{1}{2}\partial_u} - ik\Big]\widetilde{\pi}^R(k,u)$$

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\text{vertex}} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \equiv -\operatorname{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\operatorname{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\text{vertex}} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\cancel{\frac{1}{2}\partial_u} - ik\Big]\widetilde{\pi}^R(k,u)$$

Split by parity in k: $\operatorname{Re}\widetilde{\pi}^R$ **even,** $\operatorname{Im}\widetilde{\pi}^R$ **odd**

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\rm vertex} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \;\equiv\; -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\rm vertex} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\cancel{\tfrac{1}{2}\partial_u} - ik\Big]\widetilde{\pi}^R(k,u)$$

Split by parity in k: $\mathrm{Re}\,\widetilde{\pi}^R$ **even**, $\mathrm{Im}\,\widetilde{\pi}^R$ **odd**

$\mathrm{Re}\,\widetilde{\pi}^R$: odd integrand $\rightsquigarrow\ 0$

enters only at **NLO** in gradients

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\mathrm{vertex}} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \;\equiv\; -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\mathrm{vertex}} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\cancel{\tfrac{1}{2}\partial_u} - ik\Big]\widetilde{\pi}^R(k,u)$$

Split by parity in k: $\mathrm{Re}\,\widetilde{\pi}^R$ **even**, $\mathrm{Im}\,\widetilde{\pi}^R$ **odd**

$\mathrm{Re}\,\widetilde{\pi}^R$: odd integrand $\rightsquigarrow$ 0

enters only at **NLO** in gradients

$\mathrm{Im}\,\widetilde{\pi}^R$: even integrand $\rightsquigarrow$ **LO**

genuine **dissipation** (fluctuation–dissipation)

The vertex pressure in Wigner space

Focus on the new term

$$\mathcal{P}_{\rm vertex} = -\int \mathrm{d}z\,\mathrm{d}z'\,\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z}\,\pi^R(z,z')\,\varphi(z') \equiv -\,\mathrm{Tr}\Big[\pi^R\,\varphi\,\partial_z\varphi\Big]$$

It is a **trace**. In **Wigner space** it stays a trace,

$$\mathrm{Tr}[A\,B] = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{A}(k,u)\,\widetilde{B}(k,u) \quad\Longrightarrow\quad \mathcal{P}_{\rm vertex} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,\widetilde{\varphi^2}(k,u)\Big[\cancel{\tfrac{1}{2}\partial_u} - ik\Big]\widetilde{\pi}^R(k,u)$$

Split by parity in k: $\mathrm{Re}\,\widetilde{\pi}^R$ **even**, $\mathrm{Im}\,\widetilde{\pi}^R$ **odd**

$\mathrm{Re}\,\widetilde{\pi}^R$: odd integrand $\rightsquigarrow$ 0

enters only at **NLO** in gradients

$\mathrm{Im}\,\widetilde{\pi}^R$: even integrand $\rightsquigarrow$ **LO**

genuine **dissipation** (fluctuation–dissipation)

$$\boxed{\mathcal{P}_{\rm vertex} = \int \frac{\mathrm{d}k}{2\pi}\mathrm{d}u\,k\,\widetilde{\varphi^2}(k,u)\;\mathrm{Im}\,\widetilde{\pi}^R(k,u)}$$

Outline

Particle production

Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2 \,, \qquad m_\chi \gg m_\phi \,, T \implies f_\chi \sim 0$$

Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\rm vertex} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\; \mathrm{Im}\,\widetilde{\pi}^{\rm R}(q_\perp)$$

Particle production

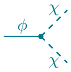
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\ \mathrm{Im}\,\widetilde{\pi}^{\mathrm{R}}(q_\perp)\Big\} \sim \mathrm{Im}$$

[

] = | 

|^2

optical theorem = cutting rule

Particle production

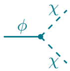
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\ \mathrm{Im}\,\widetilde{\pi}^{\mathrm{R}}(q_\perp)\Big\} \sim \mathrm{Im}\left[\vphantom{\int}\right.$$

$$\left.\vphantom{\int}\right] = \left|\vphantom{\int}\right.$$



$$\left.\vphantom{\int}\right|^2$$

$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

$$\times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

Particle production

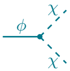
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2 \ \mathrm{Im}\,\widetilde{\pi}^{\mathrm{R}}(q_\perp)\Big\} \sim \mathrm{Im}\Big[$$

$$\Big] = \Big|$$



$$\Big|^2$$

$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

density of incoming particles

$$\times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

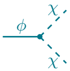
Particle production

- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\ \mathrm{Im}\,\widetilde{\pi}^{\mathrm{R}}(q_\perp)\Big\} \sim \mathrm{Im}\left[\ \cdots\ \right] = \Big|\ \cdots\ \Big|^2$$



$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

$$\times\ f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$$

density of incoming particles

momentum exchange

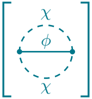
Particle production

- Introduce a heavy scalar field χ in the Lagrangian

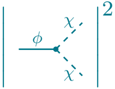
$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

$$\mathcal{P}_{\text{vertex}} \approx -\int \frac{\mathrm{d}q_\perp}{2\pi}\, q_\perp\, |\widetilde{\varphi}(q_\perp)|^2\, \underbrace{\mathrm{Im}\,\widetilde{\pi}^{\mathrm{R}}(q_\perp)} \sim \mathrm{Im}$$



$=$



$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

density of incoming particles $\quad \times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2 \longleftarrow$ Fourier tf. of the wall

momentum exchange

Particle production

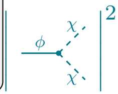
- Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\rm int} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

- The **new term** leads to a new contribution to the friction, encoding higher-order effects

recall: $\mathcal{P}_{\rm kick} = \sum_{i,X}\int_{\mathbf{p}} \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p})\, f_i(\mathbf{p})\, \Delta p_{\perp\,, i\to X}$

$$\mathcal{P}_{\rm vertex} \approx$$

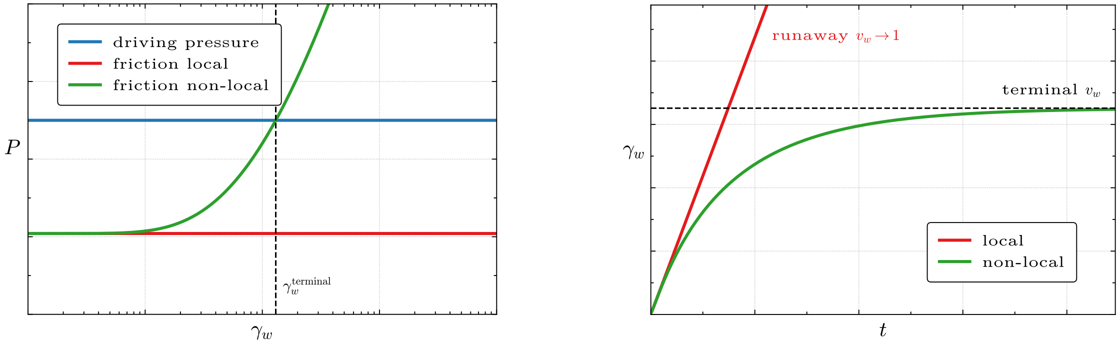


$$= \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^2\delta^{(2)}(\mathbf{p}_\parallel - \mathbf{k}_{1,\parallel} - \mathbf{k}_{2,\parallel})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\times$$

density of incoming particles $\quad \times\, f_\phi(\mathbf{p}) \quad \Delta p_\perp \quad |\tilde{\varphi}(\Delta p_\perp)|^2$ $\longleftarrow$ Fourier tf. of the wall

momentum exchange

Dramatic consequences!



Evolution of a *rigid* tanh-wall with $L = 1/T = 1$, $g = 3$, $v = 2$, $\Delta V = 0.005$ and $\Delta m_\phi^2 = 0.05$.

In a region of parameter space, neglecting the non-local self-energy (i.e. particle production) can mischaracterise whether a wall accelerates indefinitely or reaches a terminal velocity.

Entropy production in local thermal equilibrium

Entropy production in local thermal equilibrium

[Ai, Laurent, Van de Vis '23]: the dissipative friction (total minus the hydrodynamic obstruction) is proportional to the **entropy production** Δs.

Entropy production in local thermal equilibrium

[Ai, Laurent, Van de Vis '23]: the dissipative friction (total minus the hydrodynamic obstruction) is proportional to the **entropy production** Δs.

in LTE: $\delta f = 0 \;\Rightarrow\; \Delta s = 0 \;\Rightarrow\;$ friction vanishes $\;\Rightarrow\;$ **upper bound on** v_w

Entropy production in local thermal equilibrium

[Ai, Laurent, Van de Vis '23]: the dissipative friction (total minus the hydrodynamic obstruction) is proportional to the **entropy production** Δs.

in LTE: $\delta f = 0 \;\Rightarrow\; \Delta s = 0 \;\Rightarrow\;$ friction vanishes $\;\Rightarrow\;$ **upper bound on** v_w

But this rests on the *incomplete* EoM. With the new term

$$\Delta s \;\propto\; \underbrace{\mathcal{P}_{\text{dissipative}}}_{\propto\,\delta f = 0 \text{ in LTE}} + \underbrace{\mathcal{P}_{\text{vertex}}}_{\neq 0 \text{ in LTE!}}$$

Entropy production in local thermal equilibrium

[Ai, Laurent, Van de Vis '23]: the dissipative friction (total minus the hydrodynamic obstruction) is proportional to the **entropy production** Δs.

in LTE: $\delta f = 0 \;\Rightarrow\; \Delta s = 0 \;\Rightarrow$ friction vanishes $\Rightarrow$ **upper bound on** v_w

But this rests on the *incomplete* EoM. With the new term

$$\Delta s \propto \underbrace{\mathcal{P}_{\text{dissipative}}}_{\propto \delta f = 0 \text{ in LTE}} + \underbrace{\mathcal{P}_{\text{vertex}}}_{\neq 0 \text{ in LTE!}}$$

- entropy production **does not vanish** in LTE
- the LTE upper bound on v_w must be **revisited**
- dissipative effects are captured already in LTE, at LO in the gradients

Outline

Summary and outlook

Summary and outlook

- A single first-principle EoM **bridges the kinetic and kick pictures**.

Summary and outlook

- A single first-principle EoM **bridges the kinetic and kick pictures**.
- In Wigner space the self-energy splits by parity:
  $\mathrm{Re}\,\tilde{\pi}^R \to$ dispersion (NLO), $\;\mathrm{Im}\,\tilde{\pi}^R \to$ **dissipation** (LO) $=$ vertex friction.

Summary and outlook

- A single first-principle EoM **bridges the kinetic and kick pictures**.
- In Wigner space the self-energy splits by parity:
  $\mathrm{Re}\,\tilde{\pi}^R \to$ dispersion (NLO), $\mathrm{Im}\,\tilde{\pi}^R \to$ **dissipation** (LO) $=$ vertex friction.
- **Entropy production does not vanish in LTE** $\Rightarrow$ the LTE upper bound on v_w must be revisited.

Summary and outlook

- A single first-principle EoM **bridges the kinetic and kick pictures**.
- In Wigner space the self-energy splits by parity:
  $\mathrm{Re}\,\tilde{\pi}^R \rightarrow$ dispersion (NLO), $\mathrm{Im}\,\tilde{\pi}^R \rightarrow$ **dissipation** (LO) $=$ vertex friction.
- **Entropy production does not vanish in LTE** $\Rightarrow$ the LTE upper bound on v_w must be revisited.

Future directions

- investigate out-of-equilibrium effects, such as gauge boson saturation,
- find general bounds for friction strength,
- study numerically the impact of non-local effects for intermediate wall velocities.

Summary and outlook

- A single first-principle EoM **bridges the kinetic and kick pictures**.
- In Wigner space the self-energy splits by parity:
  $\mathrm{Re}\,\tilde{\pi}^R \to$ dispersion (NLO), $\mathrm{Im}\,\tilde{\pi}^R \to$ **dissipation** (LO) $=$ vertex friction.
- **Entropy production does not vanish in LTE** $\Rightarrow$ the LTE upper bound on v_w must be revisited.

Future directions

- investigate out-of-equilibrium effects, such as gauge boson saturation,
- find general bounds for friction strength,
- study numerically the impact of non-local effects for intermediate wall velocities.

THANK YOU

arXiv:2504.13725

+ upcoming work

BACK-UP SLIDES

The ultrarelativistic regime

The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24 \times 32\pi^2} \log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

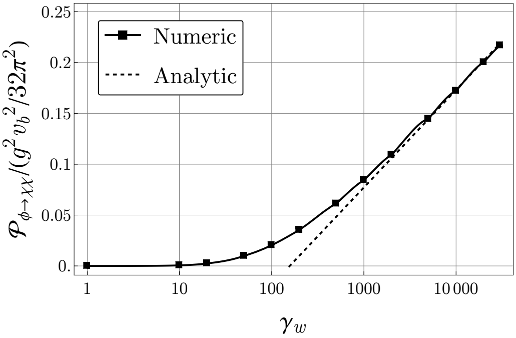
which approaches the result from the kick picture.

The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approaches the result from the kick picture.

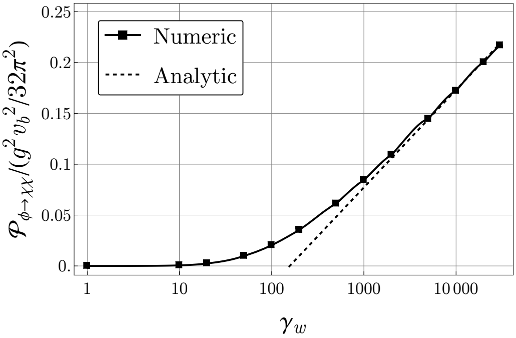


The ultrarelativistic regime

Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24 \times 32\pi^2} \log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

which approaches the result from the kick picture. Similarly, we show in our work that particle mixing and transition radiation are also captured within this framework.



The full dynamical equations

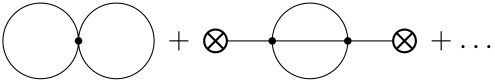
The equations of motion are now readily obtained

$$\left.\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi^+(x)}\right|_{\varphi^+=\varphi^-=\varphi} = \frac{\delta S}{\delta\varphi(x)} - \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\Delta^T(x,x) + \left.\frac{\delta\Gamma_2}{\delta\varphi^+(x)}\right|_{\varphi^+=\varphi} = 0$$

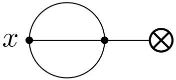
$$\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta^{ab}(x,y)} = 0 \quad \Rightarrow \quad \Delta^{ab,-1}(x,y) - G_\varphi^{ab,-1}(x,y) + 2i\frac{\delta\Gamma_2}{\delta\Delta^{ab}(x,y)} = 0$$

For a scalar theory with quartic self-interaction, we have

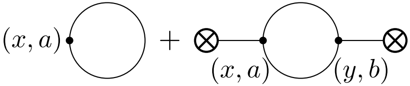
$$\mathcal{L}_{\mathrm{int}} = -\frac{\lambda}{4!}\phi^4 \longrightarrow i\Gamma_2 =$$



$$i\frac{\delta\Gamma_2}{\delta\varphi(x)} \supset$$



$$2i\frac{\delta\Gamma_2}{\delta\Delta^{ab}(x,y)} \supset$$



The self-energy

The self-energy

At leading order in the gradient expansion

$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \, |\tilde{\varphi}(q^z)|^2 \, \tilde{\pi}^R(-q^z)$$

The self-energy

At leading order in the gradient expansion

$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \left|\tilde{\varphi}(q^z)\right|^2 \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \left|\tilde{\varphi}(q^z)\right|^2 \mathrm{Im}\tilde{\pi}^R(q^z)$$

The self-energy

At leading order in the gradient expansion

$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \left|\tilde{\varphi}(q^z)\right|^2 \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \left|\tilde{\varphi}(q^z)\right|^2 \mathrm{Im}\tilde{\pi}^R(q^z)$$

Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

The self-energy

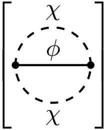
At leading order in the gradient expansion

$$\mathcal{P}_{\mathrm{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \, |\tilde{\varphi}(q^z)|^2 \, \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \, |\tilde{\varphi}(q^z)|^2 \, \mathrm{Im}\tilde{\pi}^R(q^z)$$

Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\mathrm{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$ 

The self-energy

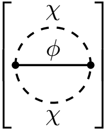
At leading order in the gradient expansion

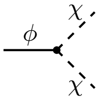
$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} i q^z \, |\tilde{\varphi}(q^z)|^2 \, \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \, |\tilde{\varphi}(q^z)|^2 \, \mathrm{Im}\tilde{\pi}^R(q^z)$$

Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}\Big[$ 

$\Big] = \Big|$ 

$\Big|^2$

The self-energy

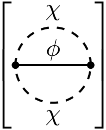
At leading order in the gradient expansion

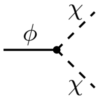
$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z'\, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} iq^z \left|\tilde{\varphi}(q^z)\right|^2 \tilde{\pi}^R(-q^z)$$

$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \left|\tilde{\varphi}(q^z)\right|^2 \mathrm{Im}\tilde{\pi}^R(q^z)$$

Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}\Big[$ 

$\Big] = \Big|$ 

$\Big|^2 \implies$ pair production!

A comment on the gradient expansion

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \;\equiv\; \text{typical momentum of a particle } \textbf{in the wall frame}$$

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$k \sim \gamma_w T \;\equiv\;$ typical momentum of a particle **in the wall frame**

$$\Longrightarrow \gamma_w T L_w \gg 1$$

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \;\equiv\; \text{typical momentum of a particle } \textbf{in the wall frame}$$

$$\implies \gamma_w T L_w \gg 1$$

The gradient expansion is valid if the wall is either **fast** or **thick**.

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \;\equiv\; \text{typical momentum of a particle } \textbf{in the wall frame}$$

$$\Longrightarrow \gamma_w T L_w \gg 1$$

The gradient expansion is valid if the wall is either **fast** or **thick**. For the numerical and analytical results, we assumed the plasma outside the bubble to be **in equilibrium**, which is once again only valid if the wall is very fast.

Mixing

Assume two mixing scalar species χ and s interacting through the background

$$\mathcal{L}_{\rm int} \supset -\kappa\varphi\chi s\,, \qquad \text{and} \quad m_\chi \gg m_s$$

Particles χ are absent in the plasma but are generated via mixing as s-particles go through the wall. In the ultrarelativistic limit

$$\mathcal{P}^{\gamma_w\to\infty}_{s\to\chi} = \frac{2\kappa^2 v_b^2}{m_\chi^2}\frac{T^2}{24}$$