(Beyond) Amplitudes for Hawking radiation

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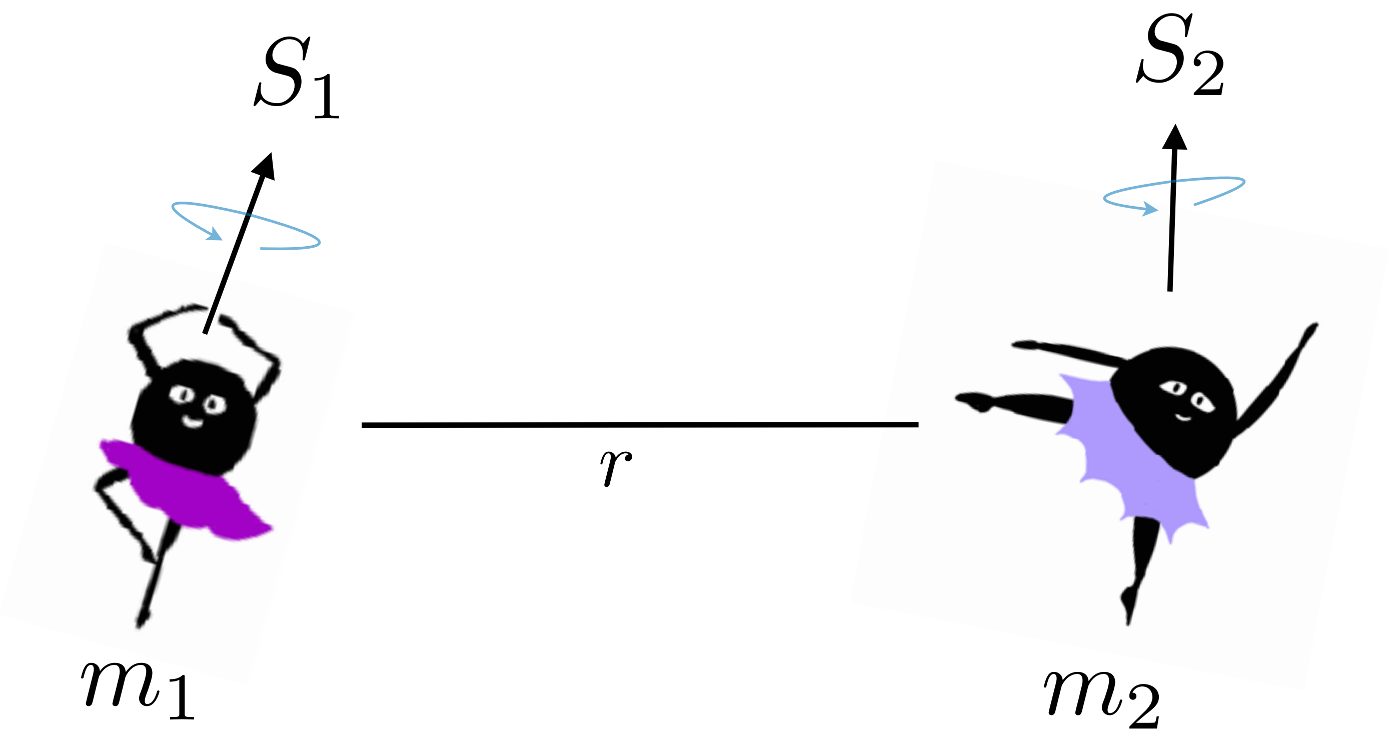
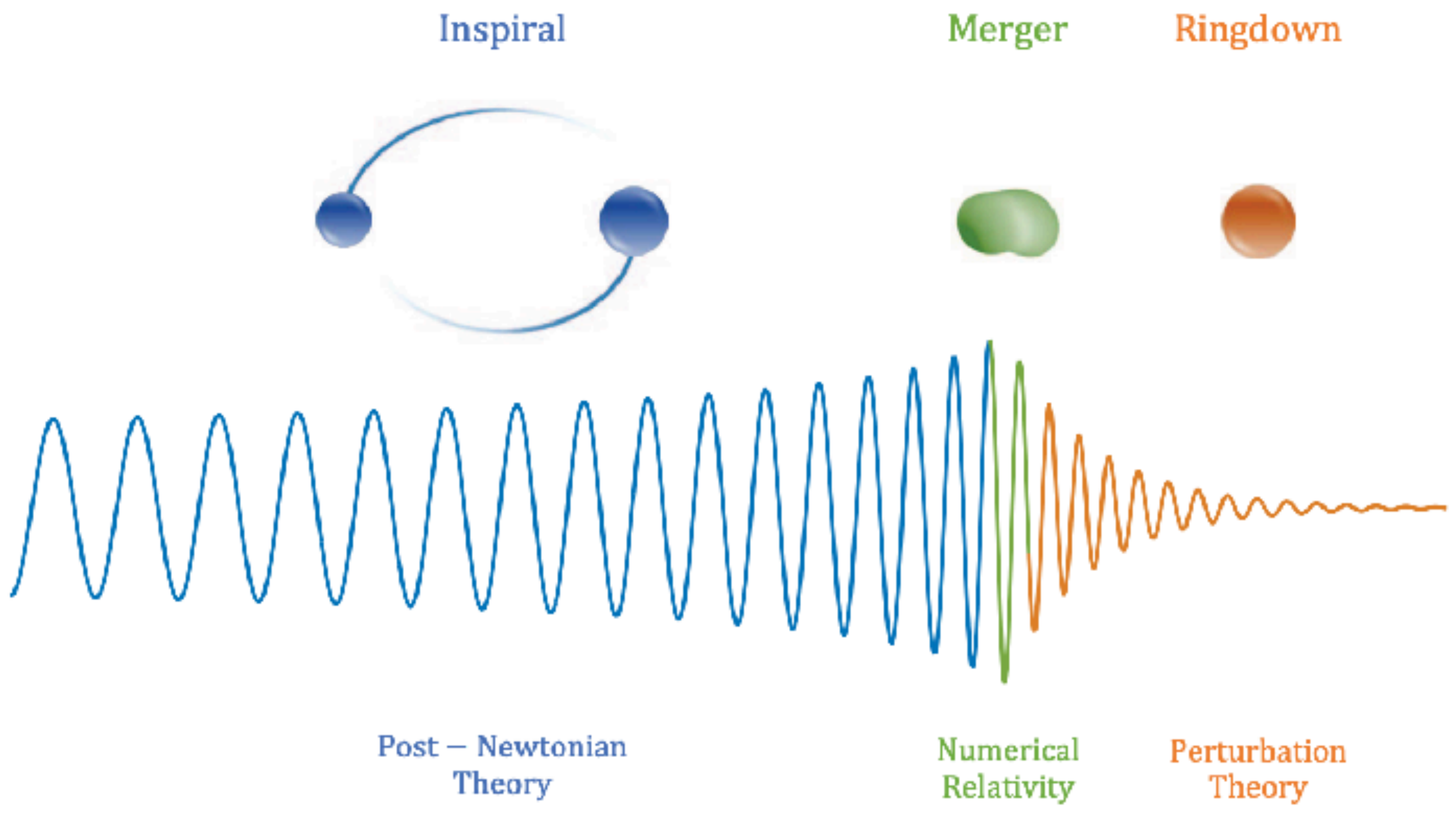
Motivation

QFT methods have successfully described many effects in classical BH dynamics.

Two-body problem:

- BH as a point particle [Goldberger, Rohstein '04]
- Large separation
- Perturbative expansion in G: Post-Minkowskian (PM) expansion





[Fig. from Antelis and Moreno, 1610.03567]

Post-Minkowskian (PM):

$$1 \gg \frac{Gm}{r}, \qquad v^2 \sim 1$$

Weak gravity

Motivation

QFT methods have successfully described many effects in classical BH dynamics.

State-of-the-art: 5PM for non spinning; 2PM (3PM) for spinning BHs, tidal, absorption ....

Success inherits knowledge from many years of QCD loop comps.
(IBPs, diff eqs, ...)

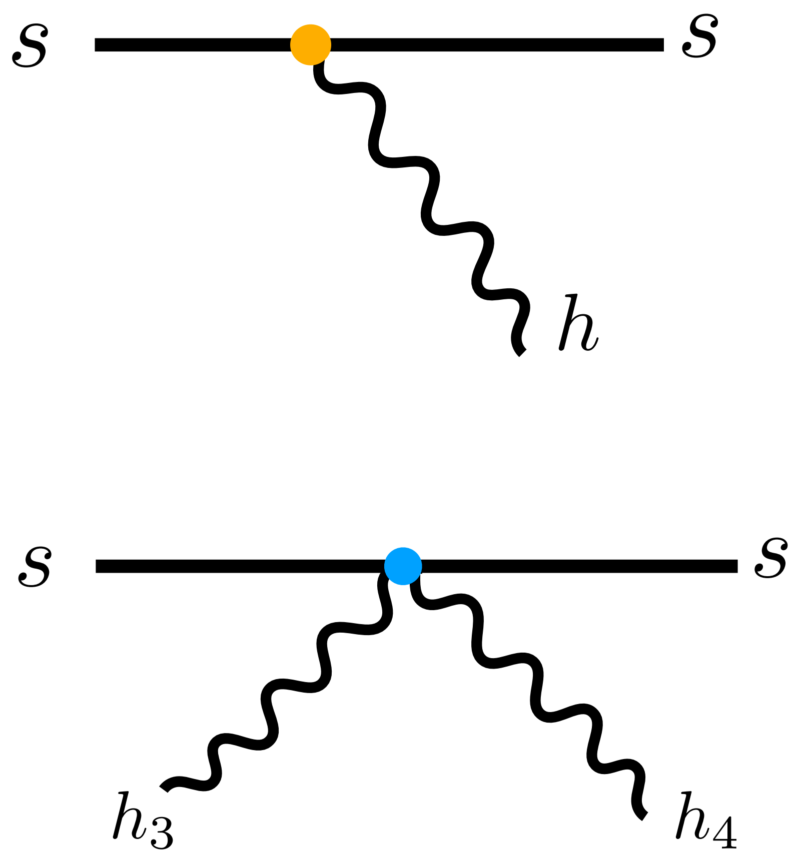
Usual pipeline: Use QFT methods for Classical BH

But amplitudes are quantum objects → We can use it to compute semiclassical observables

Hawking radiation as a scattering process

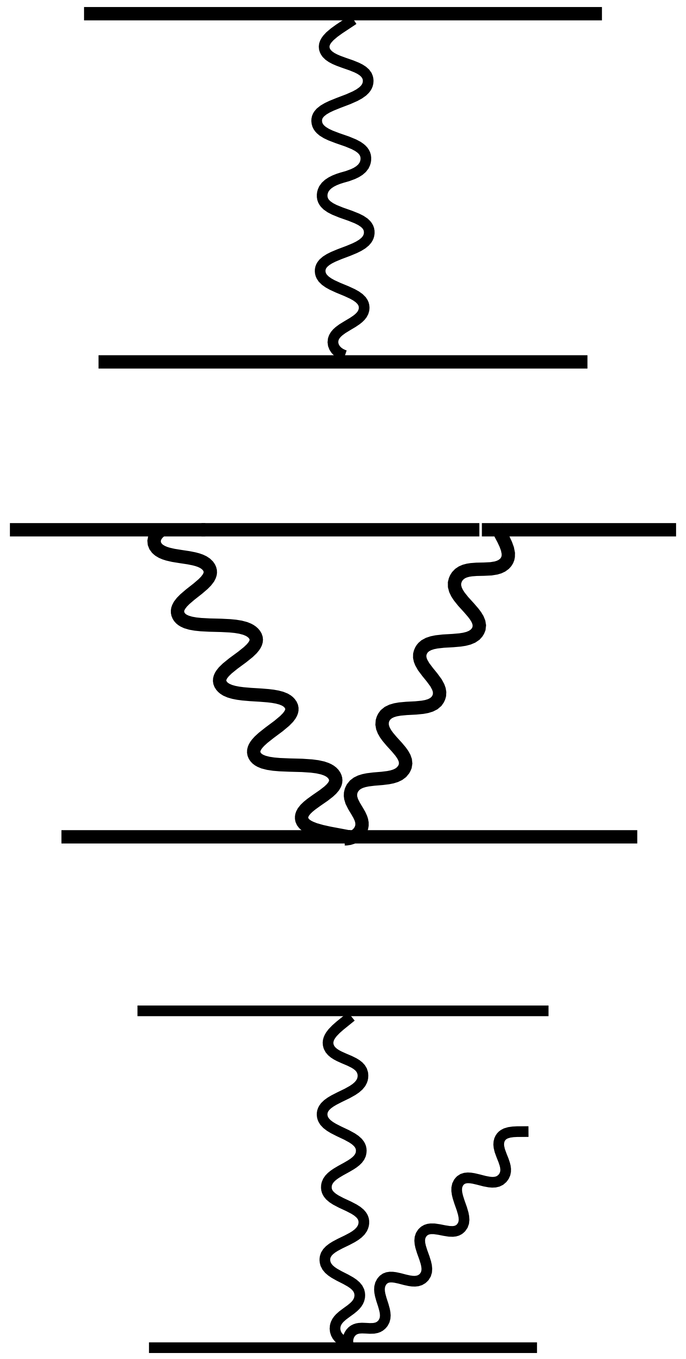
Pipeline from QFT to Classical Observables

QFTs scattering amplitudes



$\Rightarrow$

Higher-loops
Higher-Spin
Emissions

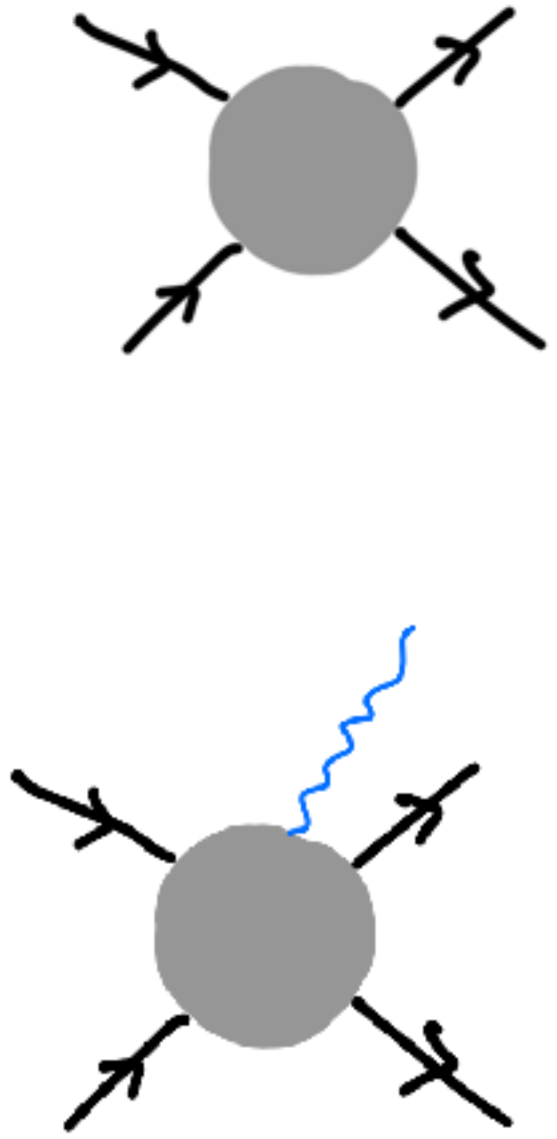


classical limit

$\Rightarrow$

$\hbar \to 0$

Classical Amps



$\Rightarrow$

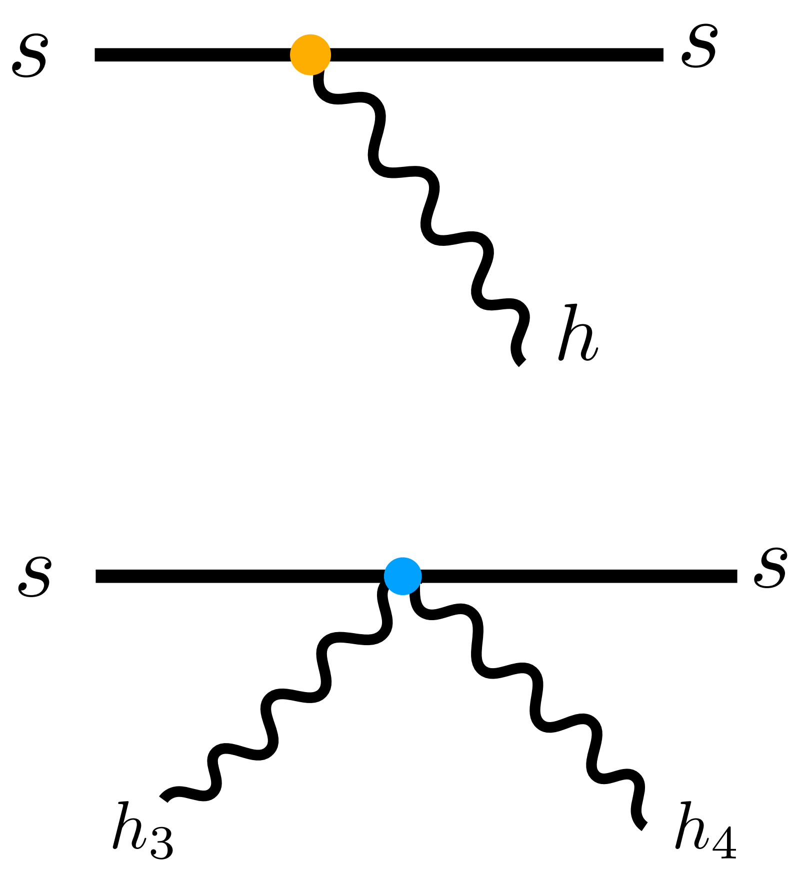
Scattering Observables

ΔO

V

Pipeline from QFT to Classical Observables

QFTs scattering amplitudes



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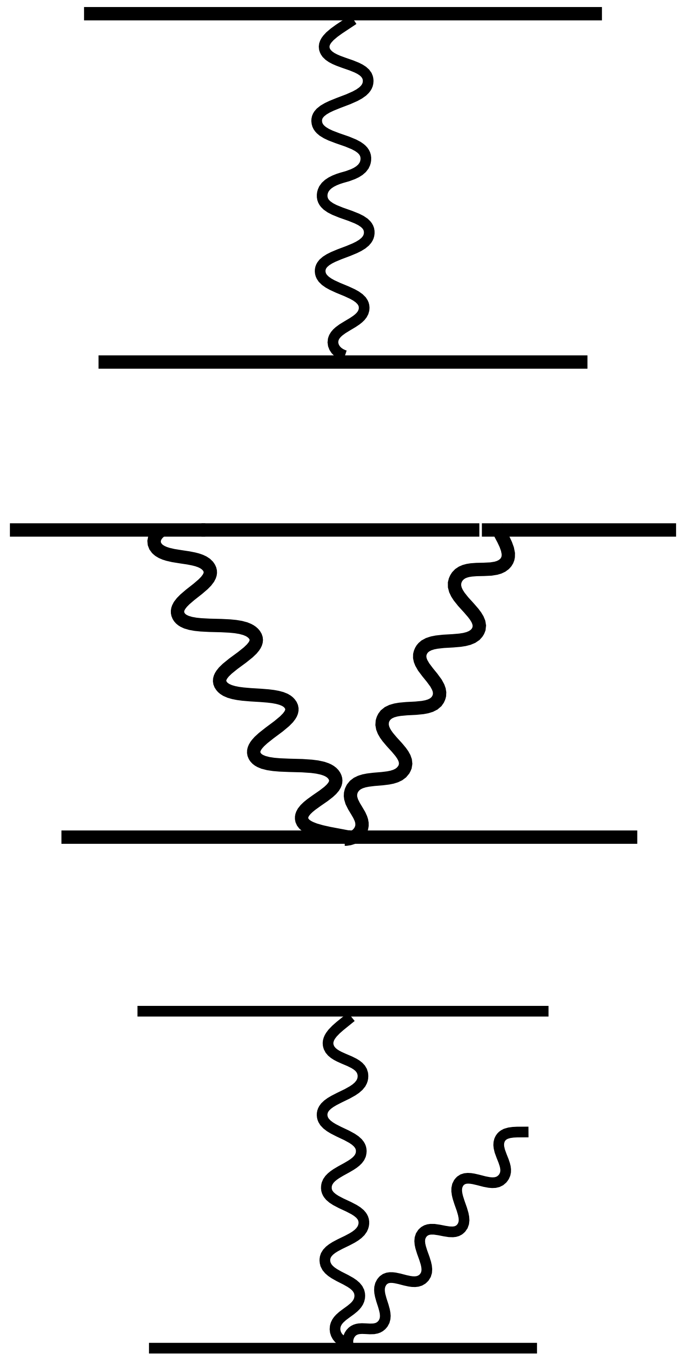

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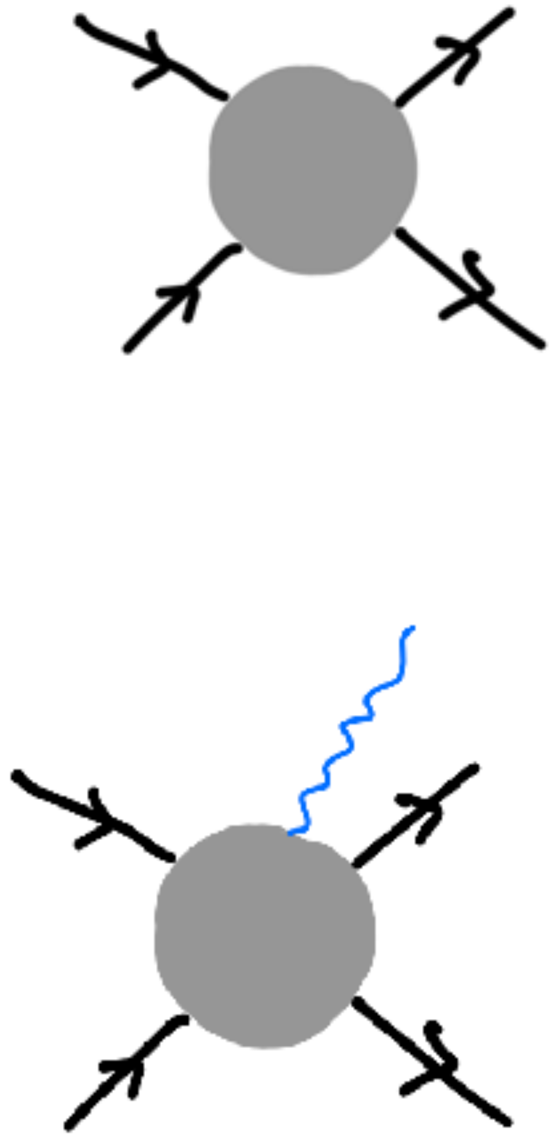


$\Rightarrow$

Scattering Observables

ΔO

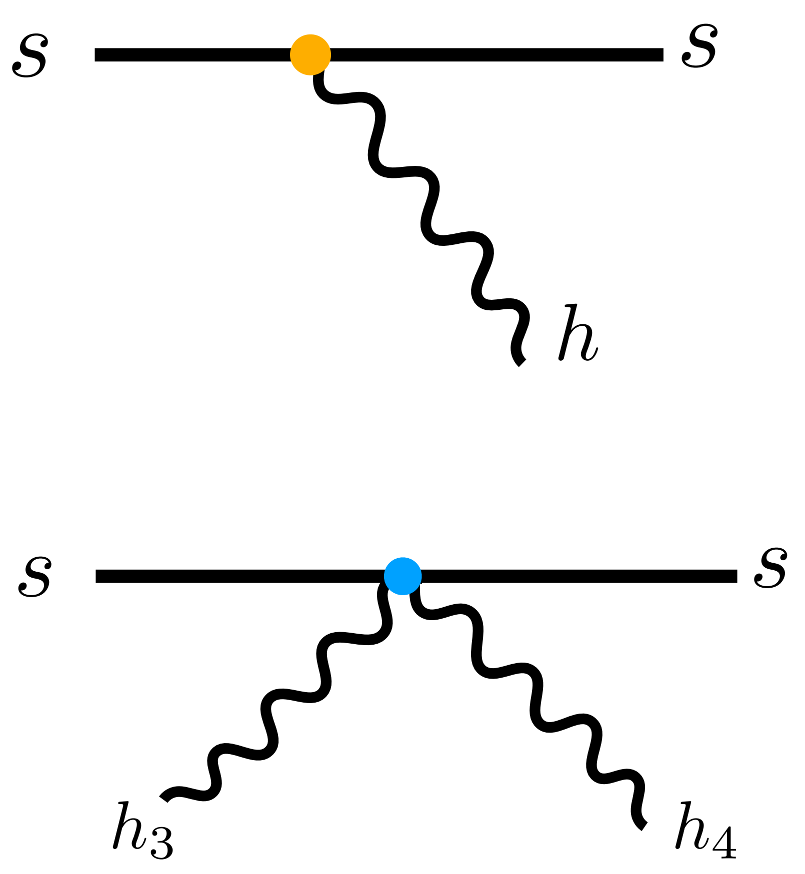
V

$\updownarrow$

Bounded Observables

Pipeline from QFT to Hawking

QFTs scattering amplitudes



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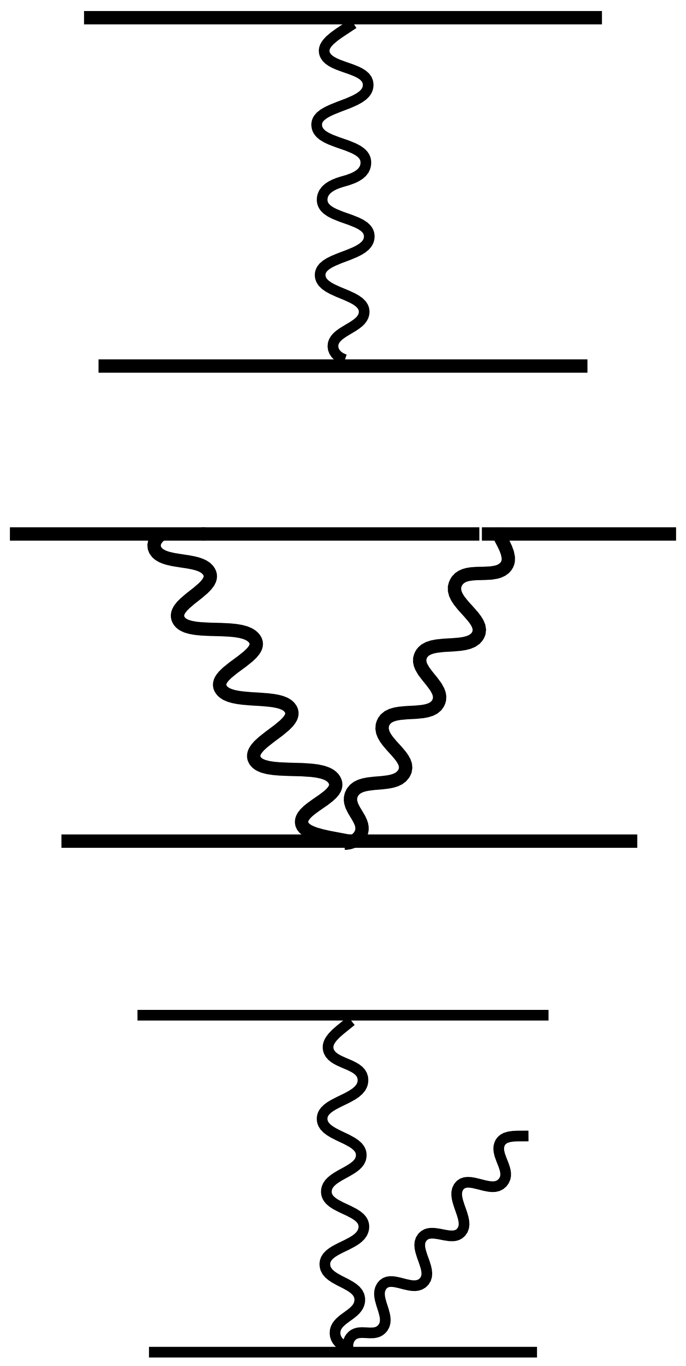
Higher-loops
Higher-Spin
Emissions



~~classical limit~~

~~$\Rightarrow$~~

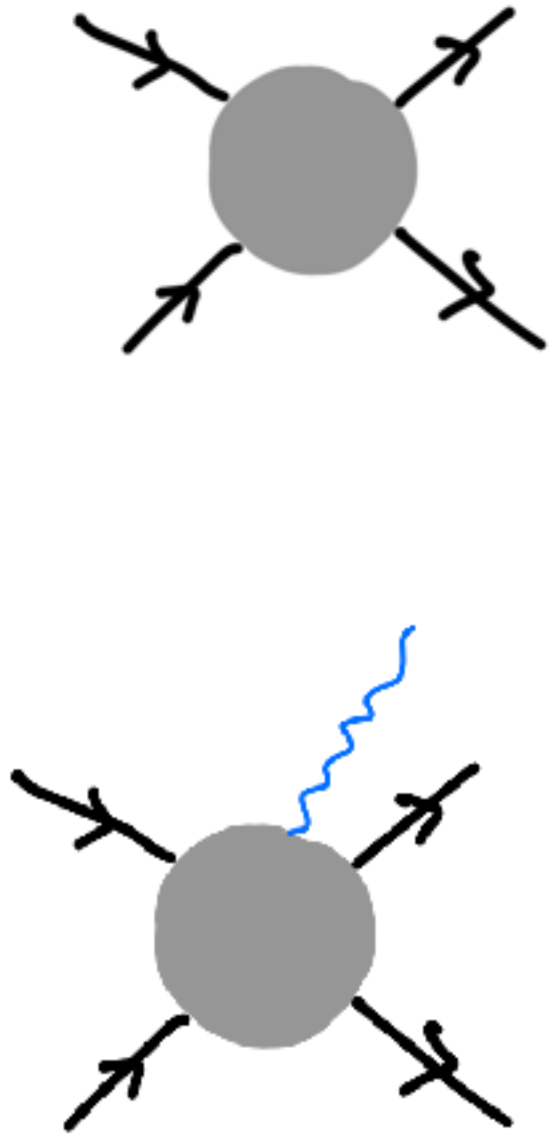
~~$\hbar \to 0$~~

geometric-optics limit

$\eta \to 0$

geometric-optics amps?

~~Classical Amps~~



$\Rightarrow$

Scattering Observables

ΔO

V

In-In vs. In-out

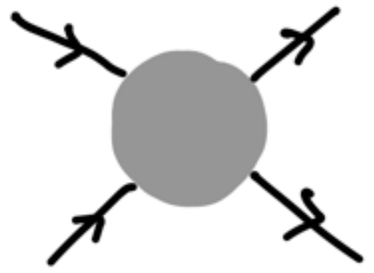
We typically formulate as a 'scattering amplitude' (in-out)

But, we are interested in observables (in-in) ⟶ Penco's Talk

KMOC framework $\Delta O = \langle \text{out}|O|\text{out}\rangle - \langle \text{in}|O|\text{in}\rangle = \langle \text{in}|S^\dagger O S|\text{in}\rangle - \langle \text{in}|O|\text{in}\rangle$.

[Kosower, Maybee, OConnell '18]

e.g Impulse $\Delta P^\mu = \int \mathrm{d}^4 q\, \delta(p_1 \cdot q)\delta(p_2 \cdot q) e^{iq\cdot b} q^\mu$



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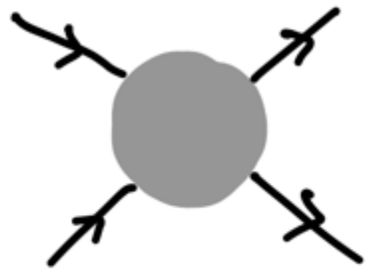
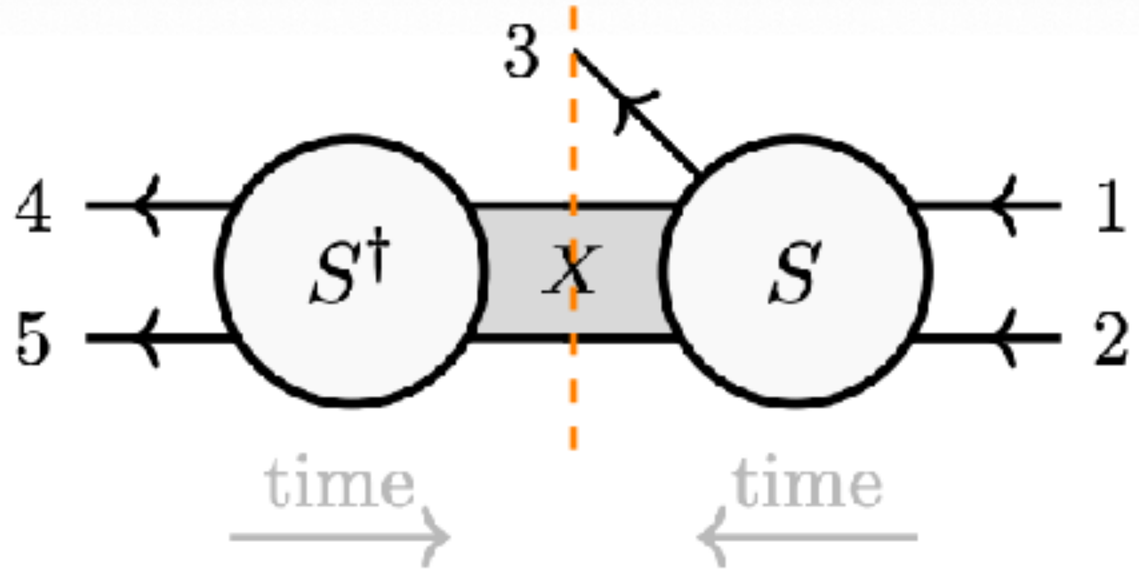


e.g Waveform [Caron-Huot, Giroux, Hannesdottir, Mizera ’23]

$$\text{Exp}_k \equiv {}_{\text{in}}\langle 2'1'|S^\dagger a_k S|12\rangle_{\text{in}} = \langle 0|a_{2'}a_{1'}\, b_k\, a_2^\dagger a_1^\dagger|0\rangle$$



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[Kosower, Maybee, OConnell ’18]

[See also: Golderberg Rothstein ’19, ’20, ’20,
Kim, Shim ’20, RA Ochirov, ’23, Aoki, Cristofoli,
Jeong, Sergola, Yoshimura ’25], ...

Collapsing background

[RA, O'Connell, Sergola, '24]

Scattering through a **time-dependent** background metric

Vaidya solution $$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM(t+r)}{r}\mathsf{k}^{\mu}\mathsf{k}^{\nu}$$

Collapsing background

[RA, O'Connell, Sergola, '24]

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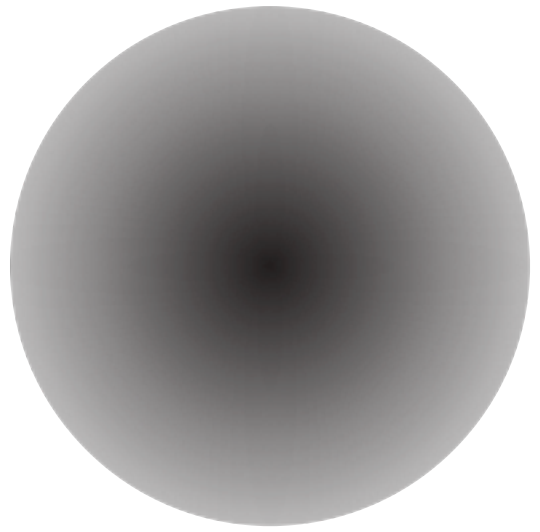
Vaidya solution
$$g^{\mu\nu} = \eta^{\mu\nu} + \frac{2GM(t+r)}{r}\mathsf{k}^\mu \mathsf{k}^\nu$$

$$M(t+r) = M\Theta(t+r)$$

collapsing bkg.



Kerr-Schild vector

$$\mathsf{k}_\mu \mathrm{d}x^\mu = \mathrm{d}t + \mathrm{d}r \qquad \mathsf{k}_\mu(x) = \left(1, \frac{\boldsymbol{x}}{r}\right)$$

$$\mathsf{k}^2 = 0$$

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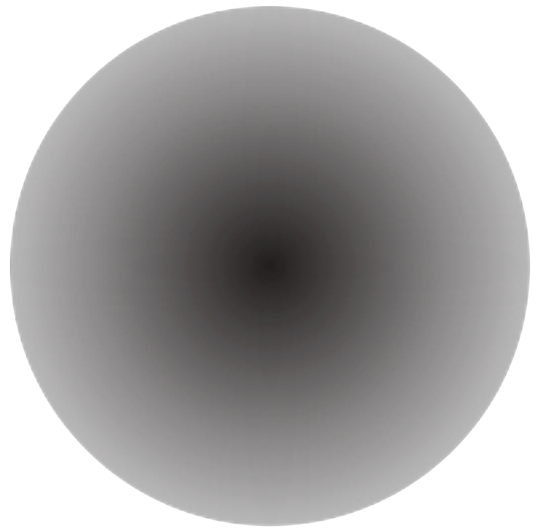

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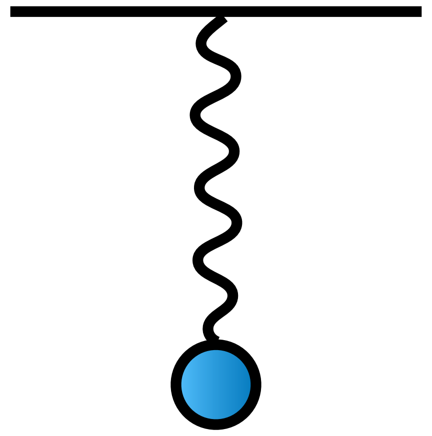
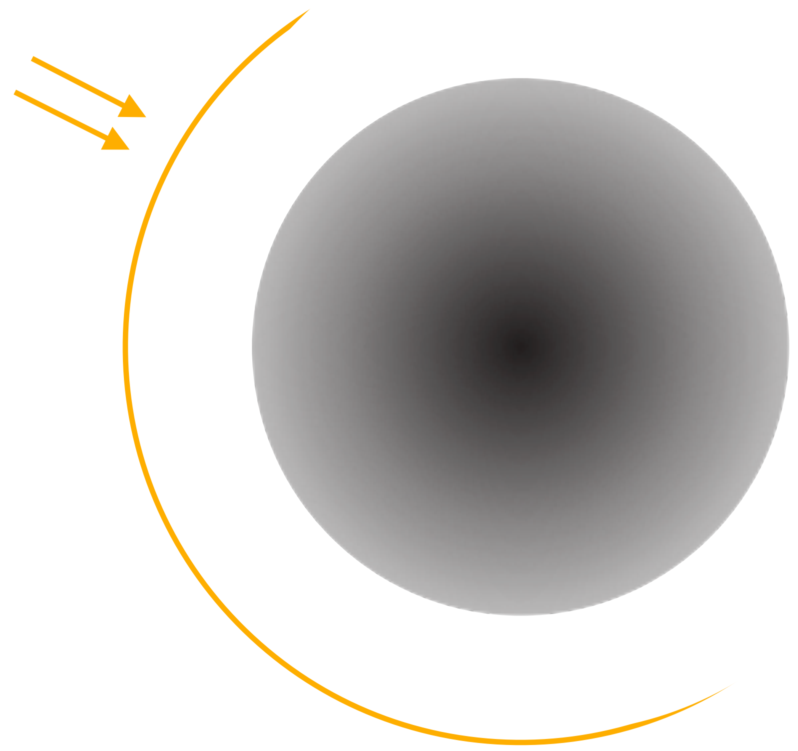
Interaction: $$\mathcal{L}_{\text{int}} = \frac{1}{2}h^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$



Initial state

[RA, O'Connell, Sergola, '24]

We scatter a massless spherical state through the Vaidya background



$$|\varphi\rangle = \int \mathrm{d}\Phi(p)\varphi(p)|p\rangle = \int \mathrm{d}\Phi(p) \int \mathrm{d}v\varphi(v)e^{iEv}|p\rangle$$

massless

spherical symmetry

$v = t + r$

Initial state

[RA, O'Connell, Sergola, '24]

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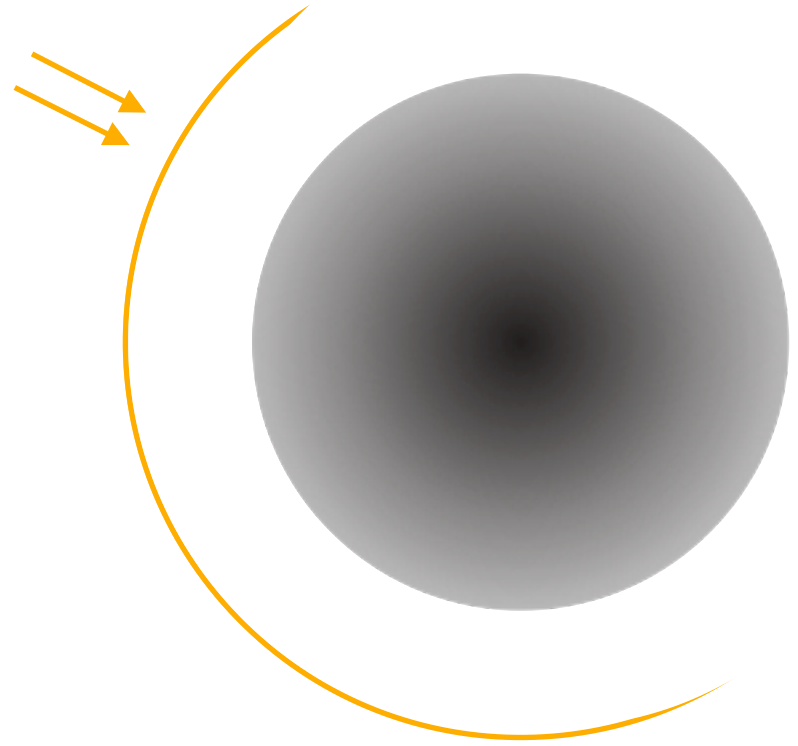


$$|\varphi\rangle = \int \mathrm{d}\Phi(p)\varphi(p)|p\rangle = \int \mathrm{d}\Phi(p) \int \mathrm{d}v \varphi(v) e^{iEv}|p\rangle$$

$$= \int \mathrm{d}\Phi(p) \int \mathrm{d}v\, \varphi(v) e^{ip\cdot b(v)}|p\rangle$$

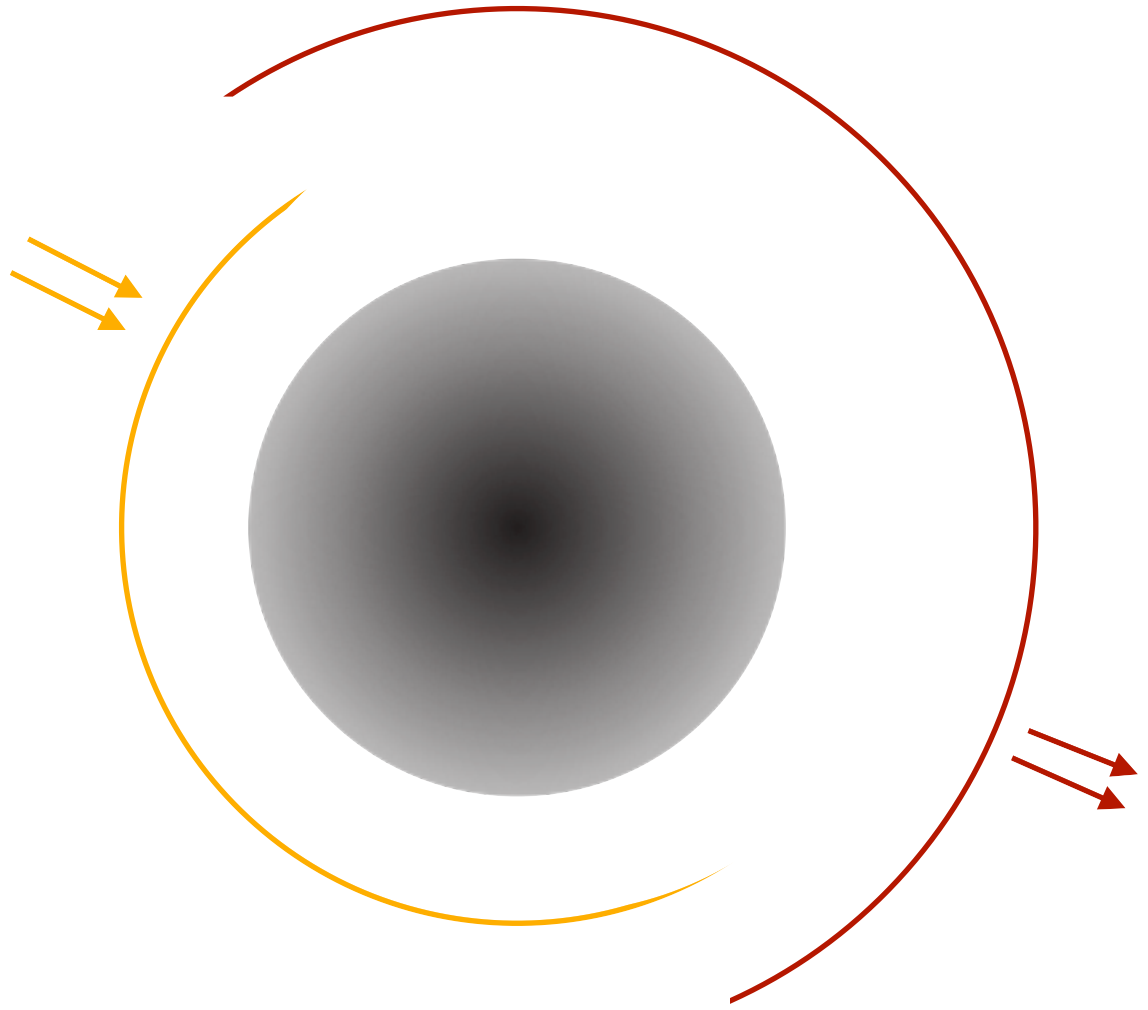
Evolve with the S-matrix

[RA, O'Connell, Sergola, '24]

Final state $$S|\varphi\rangle = \int \mathrm{d}\Phi(p')|p'\rangle\langle p'|S|\varphi\rangle + \cdots$$

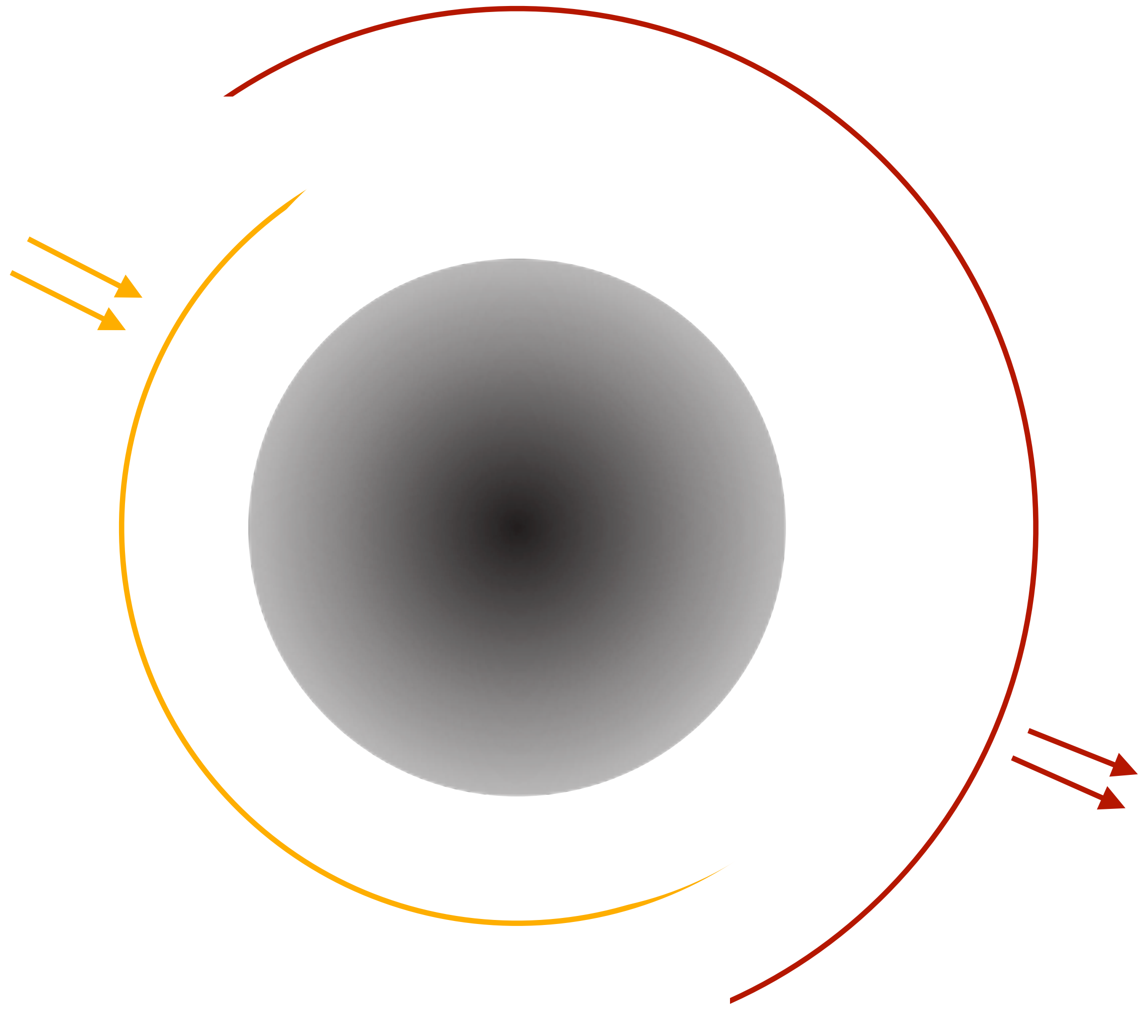
$\downarrow$

Multi particle states



Evolve with the S-matrix

[RA, O'Connell, Sergola, '24]



Final state $S|\varphi\rangle = \int \mathrm{d}\Phi(p')|p'\rangle\langle p'|S|\varphi\rangle + \cdots$

Particle's wavelength $\lambda \sim \hbar/E = \hbar/|\mathbf{p}|$

Momentum transfer $q = p' - p$

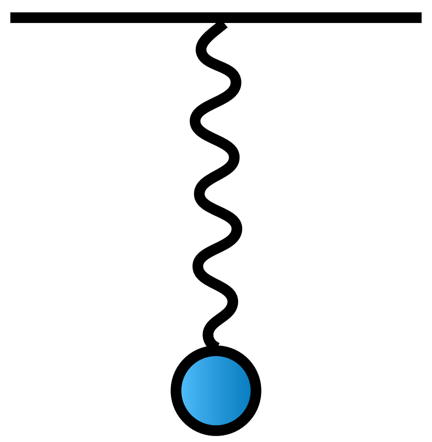
Geometric-optics limit $|\mathbf{b}| \gg \lambda, \qquad |\mathbf{q}| \ll |\mathbf{p}|$ or $\eta \equiv \frac{|\mathbf{q}|}{|\mathbf{p}|} \ll 1$

Hawking scattering

[RA, O'Connell, Sergola, '24]

Interaction: $\mathcal{L}_{\text{int}} = \frac{1}{2} h^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$ with $h^{\mu\nu} = \frac{2GM\Theta(t+r)}{r} \mathsf{k}^\mu \mathsf{k}^\nu$

Leading-order amplitude:



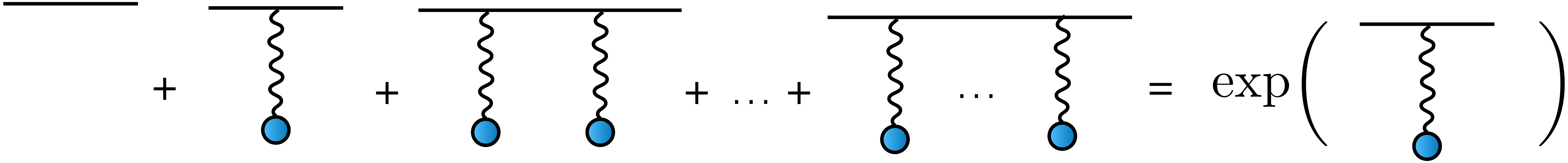
$$\langle p' | S - 1 | \varphi \rangle = \int \mathrm{d}v \, \varphi(v) e^{ip' \cdot b(v)} \left(-4GME' \log(-v/\mu) \right)$$

μ - IR phase that cancels later

Exponentiation : Resumming Feynman diagrams

Higher-loop iteration of the leading in η terms exponentiates

Similarly to the eikonal amplitude (super classical pieces)



$$\langle p'|S|\varphi\rangle = \int \mathrm{d}v\, \varphi(v) e^{ip'\cdot b(v)} \exp\left(-4iGME' \log(-v/\mu)\right)$$

Hawking Amplitude

Choosing a spherically symmetric initial state

$$|\varphi\rangle = \int \frac{\mathrm{d}\Omega_p}{4\pi}\,|E_0, E_0\hat{\boldsymbol{p}}\rangle \qquad \longleftrightarrow \qquad \varphi(v) = \frac{2\pi}{E_0}\,e^{-iE_0 v}$$

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The exponentiated amplitude

Horizon

$$\mathcal{A}(E) = \frac{2\pi}{E_0}\int_{-\infty}^{0} \mathrm{d}v\; e^{i(E-E_0)v} e^{-4iGME\,\log(-v/\mu)}$$

---

*ignores IR phase that cancels later

Hawking Amplitude

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$$= \frac{2\pi}{E_0} (i(E - E_0))^{-1+4iGME}\, \Gamma(1 - 4iGME)$$

Complete gamma function

Thermal Spectrum

The differential number spectrum in the final state is given by the integrand of $\langle\Omega|S^\dagger a^\dagger a S|\Omega\rangle$

$$\begin{aligned} \mathrm{d}n &\propto |\mathcal{A}(E)|^2 \mathrm{d}E\,\mathrm{d}E_0 \\ &\propto e^{4\pi GME}|\Gamma(1-4iGME)|^2\,\mathrm{d}E\,\mathrm{d}E_0 \end{aligned}$$

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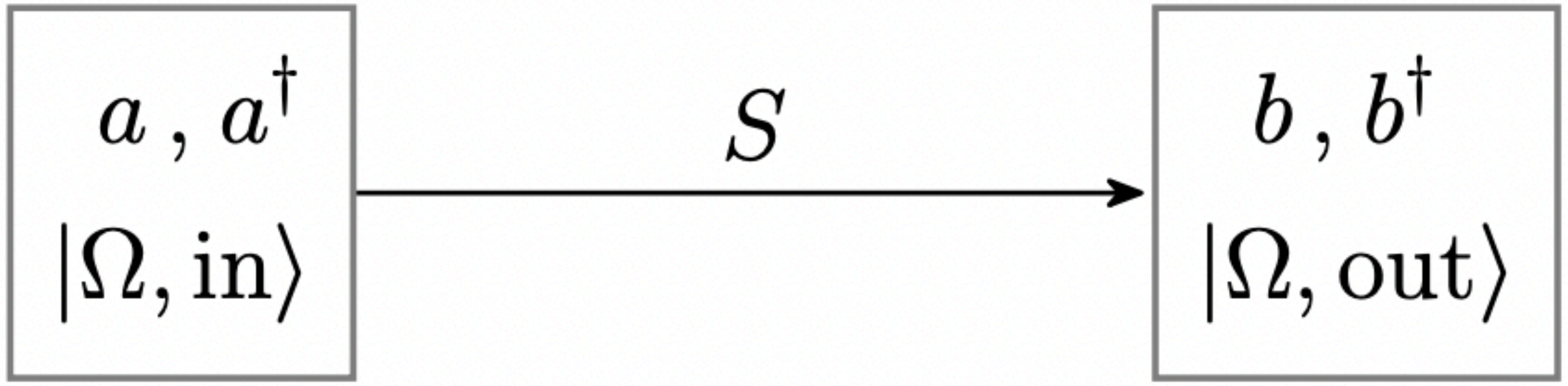
Thermal distribution

$$\mathrm{d}n \propto \mathrm{d}E\mathrm{d}E_0 \frac{1}{e^{8\pi GME}-1}$$

Temperature: $T_H = \frac{1}{8\pi GM}$

Bogoliubov

The vacuum is not the same in the initial and final state



creation and annihilation operators are related by Bogoliubov coefficients

$$b(k) = S^\dagger a(k) S = A(k,p)a(p) + B(k,p)a^\dagger(p)$$

$$a(p) = A^\dagger(p,k)b(k) - B^T(p,k)b^\dagger(k)$$

Bogoliubov and beyond amplitudes

The field can be solved in the far past or far future

$$\begin{tikzcd}
 & \int \mathrm{d}\Phi(k)\,\big(P(x,k)a(k)+\bar{P}(x,k)a^{\dagger}(k)\big) \\
\phi(x) \arrow[ur, "t\to-\infty"] \arrow[dr, "t\to+\infty"] & \\
 & \int \mathrm{d}\Phi(k)\,\big(F(x,k)b(k)+\bar{F}(x,k)b^{\dagger}(k)\big)
\end{tikzcd}$$

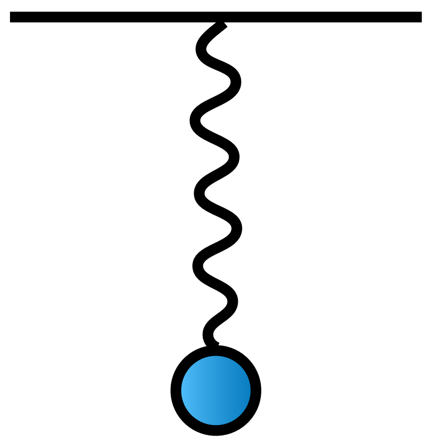
Bogoliubov $\quad A(k,p)=(F(x,k)|P(x,p))\,,\quad B(k,p)=(F(x,k)|\bar{P}(x,p))\,.$

Bogoliubov and beyond amplitudes

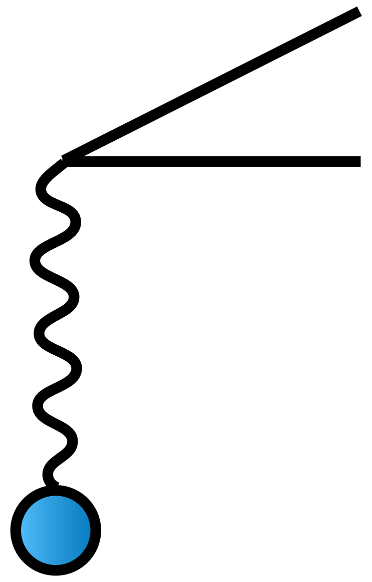
$$b(k) = S^\dagger a(k) S = \int \mathrm{d}\Phi(p) \big(A(k,p) a(p) + B(k,p) a^\dagger(p) \big) \, .$$

Bogoliubov: beyond amplitudes

$$\langle \Omega | S^\dagger \, a(k) \, S a^\dagger(p) | \Omega \rangle = A(k,p)$$



$$\langle \Omega | a(p) \, S^\dagger \, a(k) \, S | \Omega \rangle = B(k,p)$$



Beyond Amplitudes

Usual amplitudes elements are in-out objects of the form

$$\langle p_1' \cdots p_n' | S | p_1 \cdots p_m \rangle = \langle \Omega | a(p_1') \cdots a(p_n') S a^\dagger(p_1) \cdots a^\dagger(p_m) | \Omega \rangle$$

More general elements

$$\langle 0 | a(p_1') a(p_2') S^\dagger a(k) S a^\dagger(p_1) a^\dagger(p_2) | 0 \rangle \longrightarrow \text{waveform}$$

But iteration diagrams computes

$$\langle \Omega | S^\dagger \, a(k) \, S a^\dagger(p) | \Omega \rangle = A(k, p)$$

because $\langle \Omega | S^\dagger \neq \langle \Omega |$

Thermal density

Particle number
in far future

$$n = \int \mathrm{d}\Phi(p)\ \langle \Omega | S^\dagger a^\dagger(p) a(p) S | \Omega \rangle$$

where

$$\langle \Omega | S^\dagger a^\dagger(p) a(p) S | \Omega \rangle = \int \mathrm{d}\Phi(k) \bar{B}(p,k) B(p,k)$$

Thermal density

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where

$$\langle\Omega|S^\dagger a^\dagger(p)a(p)S|\Omega\rangle = \int \mathrm{d}\Phi(k)\bar{B}(p,k)B(p,k)$$

$$\boxed{\mathrm{d}n = \mathrm{d}E\,\mathrm{d}E_0\frac{2GM}{\pi E_0}\frac{1}{e^{8\pi GME}-1}}$$

Horizon effects and higher-order terms

In the standard eikonal exponentiation

[Di Vecchia, Heissenberg, Russo, Veneziano ’23]
[Cristofoli, Gonzo, Moynihan, O’Connell, Ross, Sergola, White ’21]

$$e^{i\chi(b)/\hbar}\left(1+i\hbar\Delta(b)\right)=1+i\tilde{\mathcal{A}}(b).$$

quantum remainder

Expand LHS and RHS in powers of $\hbar$ and G:

$i\tilde{\mathcal{A}}_L^{(n)}$ n-fragments
L-loops

$$i\tilde{\mathcal{A}}_0=i\chi_0$$

$$i\tilde{\mathcal{A}}_1=\frac{1}{2!}(i\chi_0)^2+i\chi_1+i\Delta_1$$

... two loops

Tree-level quantum remainder

$$i\Delta_0=i\tilde{\mathcal{A}}_0^{(1)}=0$$

Horizon effects and higher-order terms

In the geometric optics limit – GM expansion

$$(1+i\Delta)\exp\left(i\chi/\eta\right) = (1+i\Delta)\exp\left(i\frac{\chi_0+\chi_1+\cdots}{\eta}\right)$$

Expand LHS and RHS in powers of η and G:

$i\tilde{\mathcal{A}}_L^{(n)}$

n-fragments
L-loops

$$i\tilde{\mathcal{A}}_0 = i\chi_0 + i\Delta_0$$

$$i\tilde{\mathcal{A}}_1 = \frac{1}{2!}(i\chi_0)^2 + i\chi_1 + i\Delta_0 i\chi_0 + i\Delta_1$$

Tree-level remainder

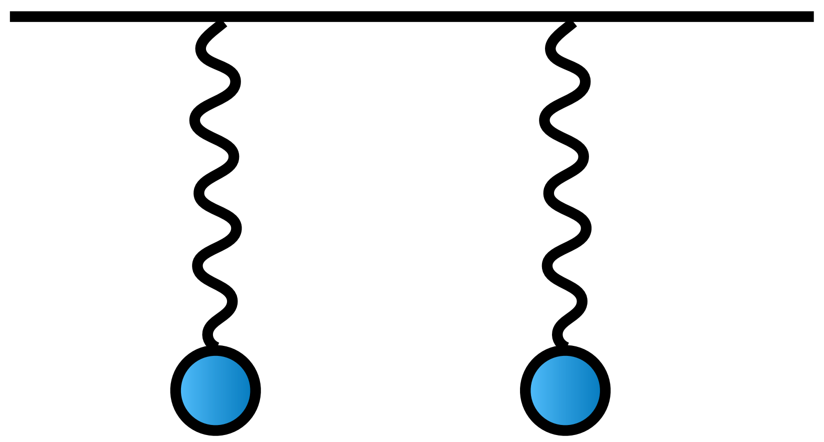
$$i\Delta_0 = i\tilde{\mathcal{A}}_0^{(1)} \neq 0$$

Horizon effects and higher-order terms

In the geometric optics limit – GM expansion

$$\exp\left(i\chi/\eta\right) = \exp\left(i\frac{\chi_0 + \chi_1 + \cdots}{\eta}\right)$$

NLO term computed perturbatively from higher orders in η



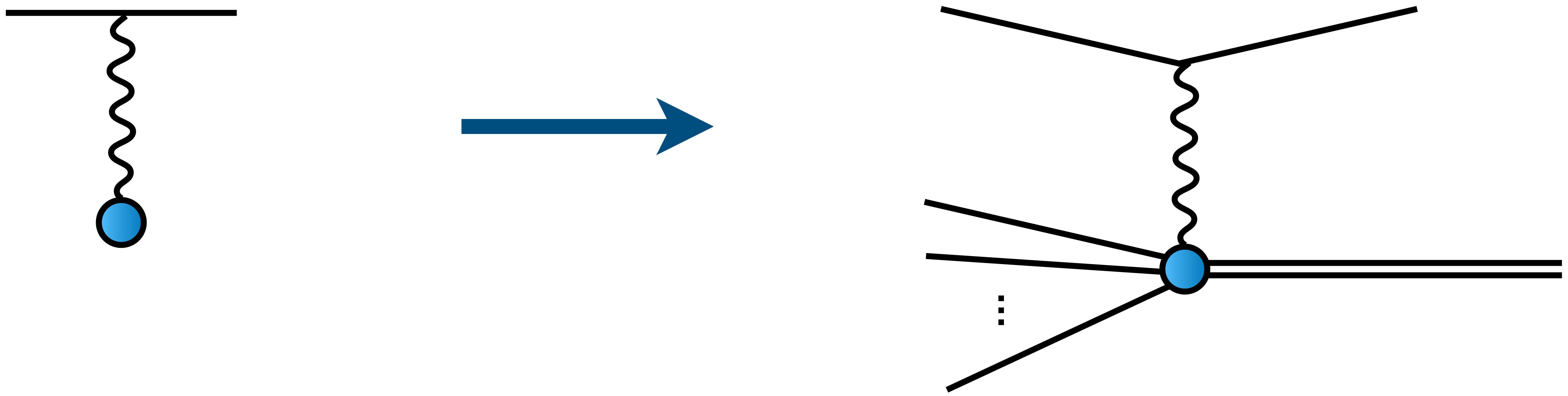
$$\chi_0 + \chi_1 = -4GME\log(-v/\mu) - \frac{16G^2M^2E}{v}$$

$$\simeq -4GME\log(-(v+4GM)/\mu)$$

→ finite radius of horizon

Beyond the background description

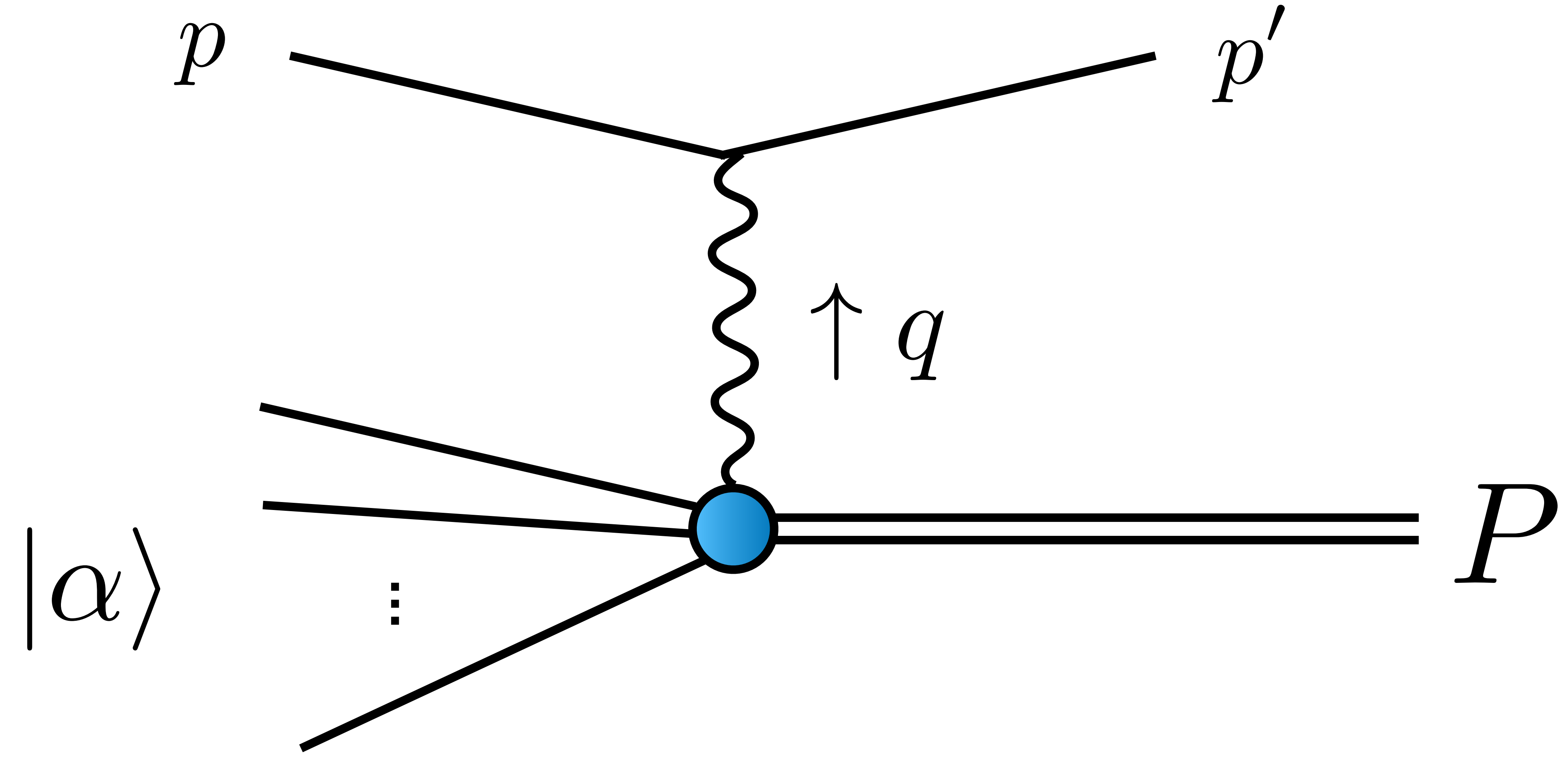
We can give dynamical d.o.f to the collapsing background



RA, O’Connell, Sergola (to appear)

Beyond the background description

Same massless probe



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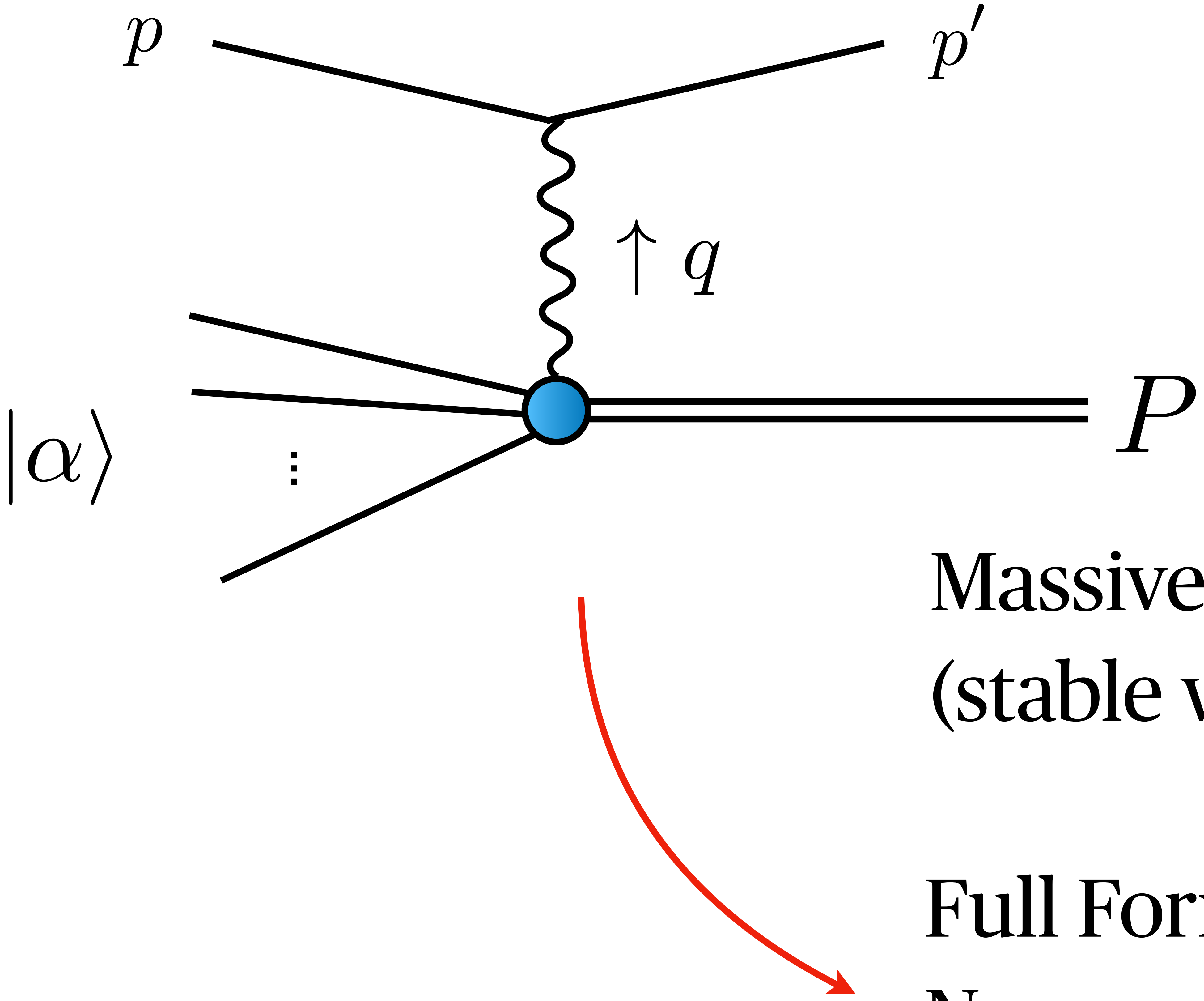


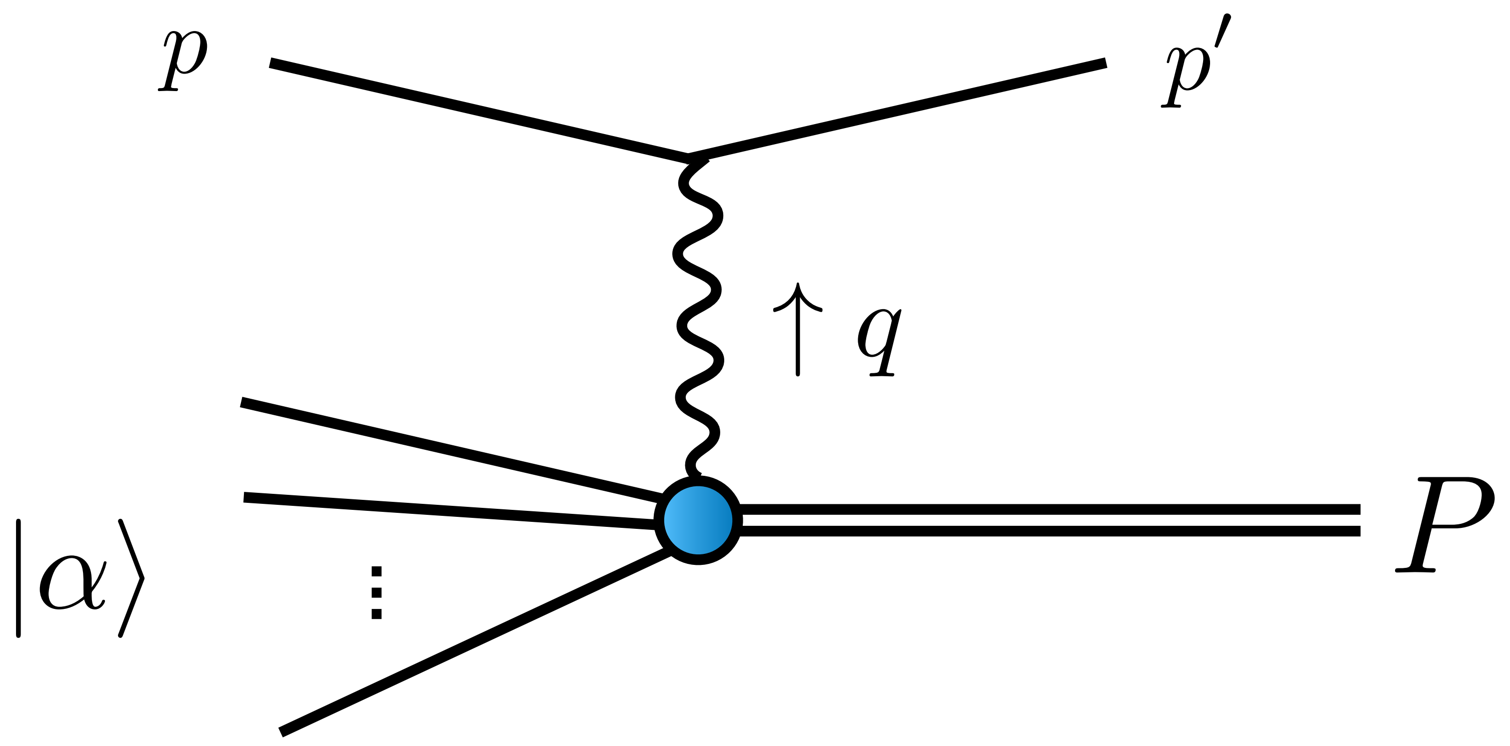
Initial Coherent state
(Properly chosen)

Massive BH
(stable within timescales)

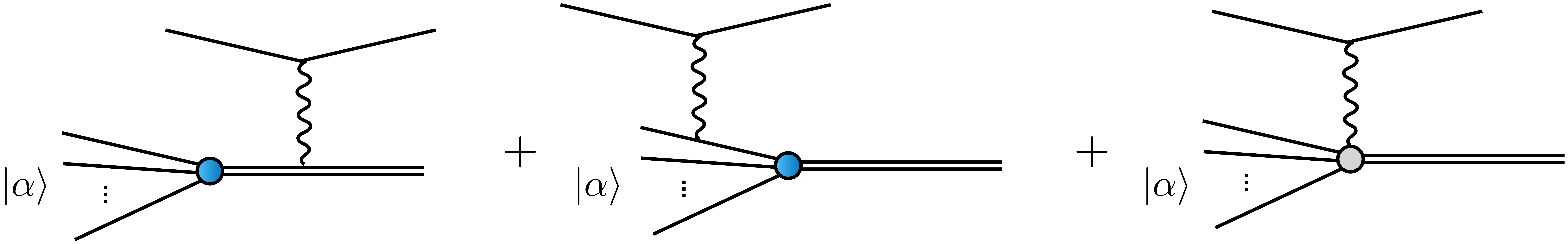
Full Form Factor
Non-pert + perturb.

Beyond the background description

RA, O'Connell, Sergola (to appear)



Three contributions:



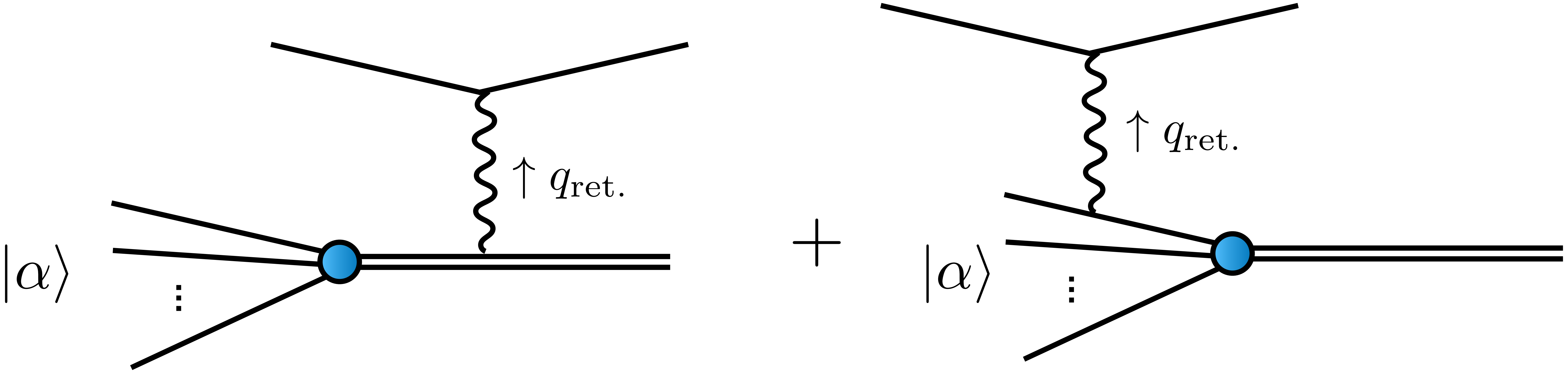
Pure contact (NZ)

Beyond the background description

Better to compute the in-in Bogoliubov coefficient

$$A(k,p) \coloneqq \langle\alpha|S^\dagger a(k) S a^\dagger(p)|\alpha\rangle$$

S-matrix decomposition: Non-pert. (hard) + pert. (soft)



$$A(p,v) = 4GME\log(-v/\mu)$$

Conclusion and future directions

- Hawking thermal spectrum can be obtained from scattering amplitudes
  - Exponentiation leads to a Hawking amplitude
  - NLO terms access the horizon

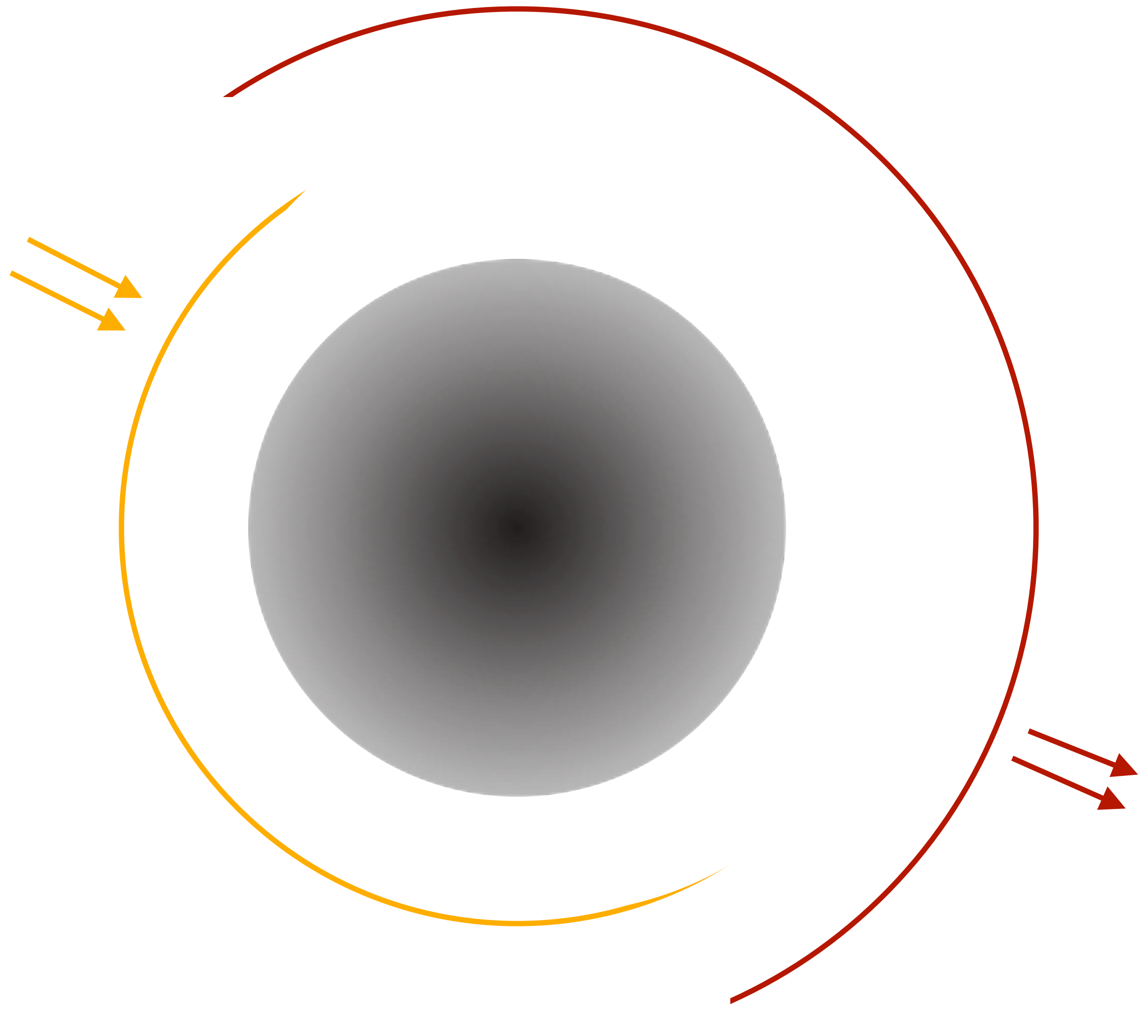
- Bogoliubov coefficients are related to in-in correlators $\langle\Omega|S^\dagger a^\dagger a S|\Omega\rangle$
  - Crossing related coeffs A and B [RA, Elkhidir, Ilderton, O'Connell, Rajeev '26]

- Flat-space formulation with coherent states [RA, O'Connell, Sergola (to appear)]

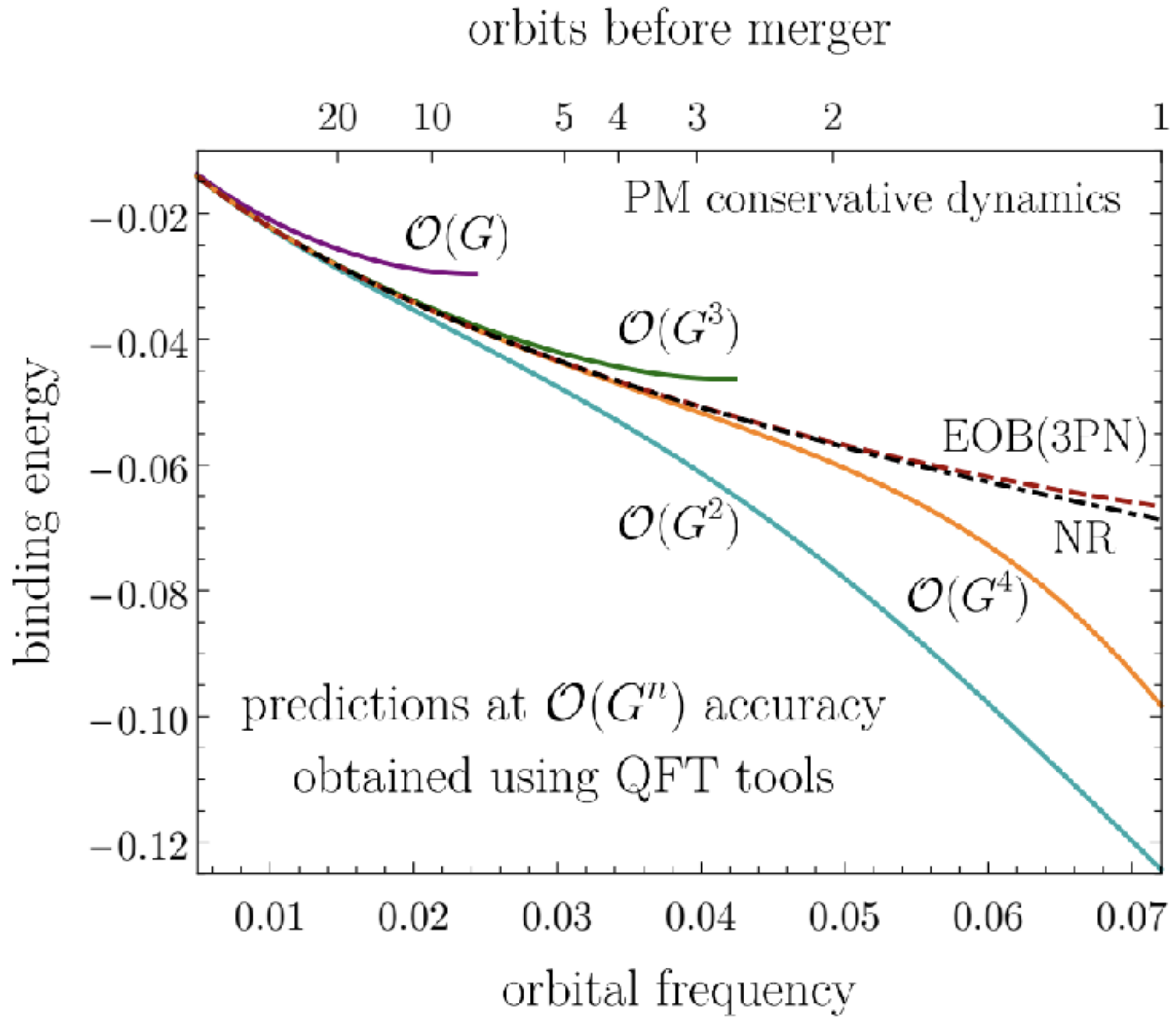
- Hawking radiation and the Double copy [RA, O'Connell, Sergola, White '25] [Ilderton, Landed, Rajeev '25] [Carrasco, Chen, '25]

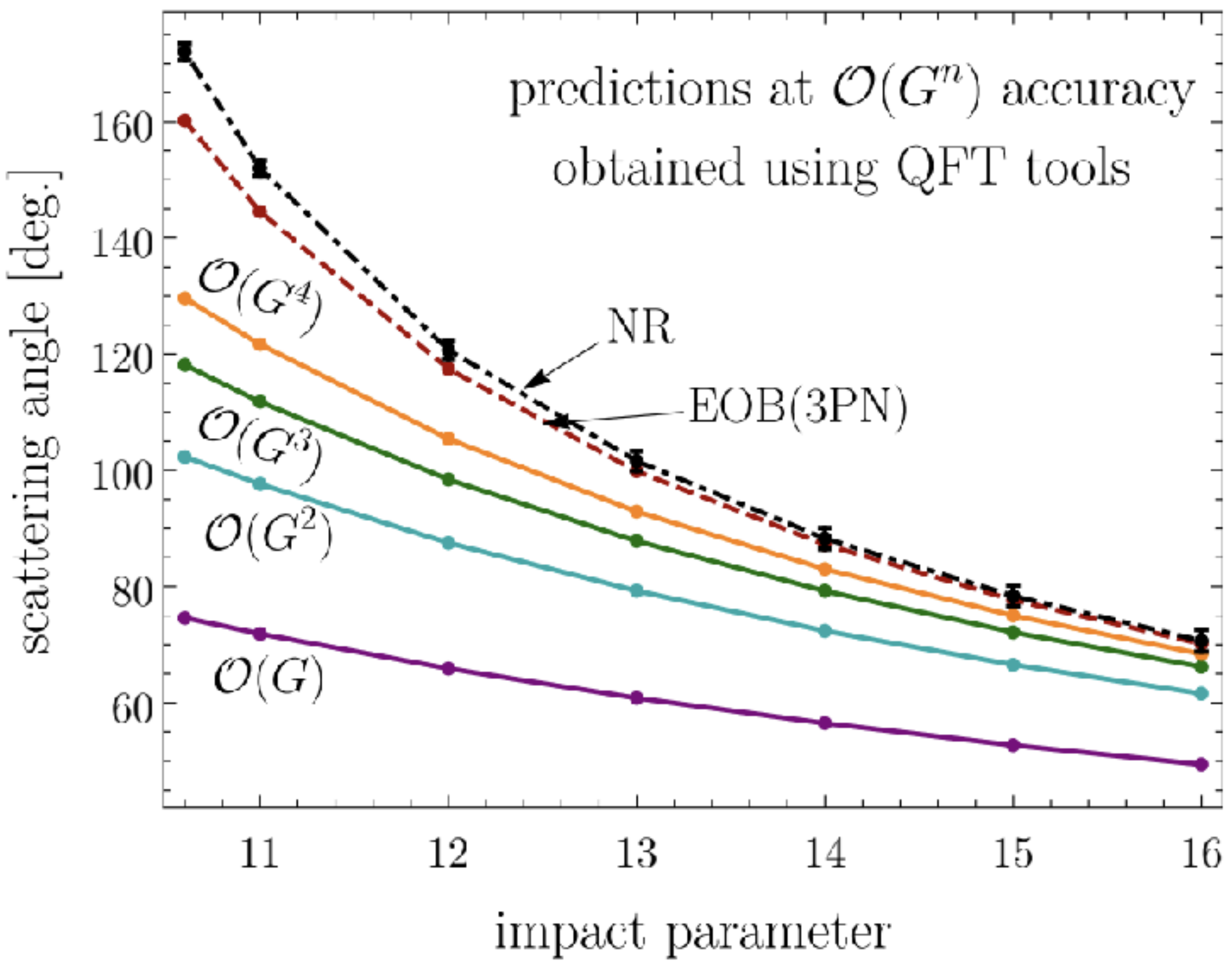
- Many directions: NNLO, spinning BHs, charged, ...

Thank you !



QFT tools progress for *spinless* dynamics





Fast progress in spinless dynamics via QFT methods

[Snowmass: Gravitational Waves and Scattering Amplitudes, adapted from M. Khalil, A. Buonanno, J. Steinhoff and J. Vines, 22]

Rôle of the theta function

Tree-level

$$\langle p|S_{\mathrm{tree}}-1|\psi\rangle = \int \mathrm{d}v\, \varphi(v) e^{ip\cdot b(v)} \int \mathrm{d}^4x\, \hat{\mathrm{d}}^4 q\, \hat{\delta}(2p\cdot q) \\ \times \frac{2iGM(t+r)}{r} E^2 (1+\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{p}})^2 e^{iq\cdot(x-b(v))}.$$

at the WL

$$\int \mathrm{d}^4x\, \hat{\mathrm{d}}^4 q\, \hat{\delta}(2p\cdot q) f(x) e^{iq\cdot(x-b)} = \int \mathrm{d}\lambda\, f(b+2\lambda p)\,.$$

lambda-integral

$$\begin{aligned}\int_{-\infty}^{\infty} \mathrm{d}\lambda \left[\frac{M(t+r)}{r}(1+\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{p}})^2\right]_{x=b+2\lambda p} &= \int_{-\infty}^{\infty} \mathrm{d}\lambda \frac{M\,\Theta(v+2E(\lambda+|\lambda|))4\Theta(\lambda)}{2E|\lambda|} \\ &= \frac{2M}{E}\int_{-\frac{v}{4E}}^{\infty} \frac{\mathrm{d}\lambda}{\lambda} \\ &= -\frac{2M}{E}\log(-v/\mu) + \frac{2M}{E}\log(4E\lambda_{\infty}/\mu),\end{aligned}$$

Relation between in and out vacua

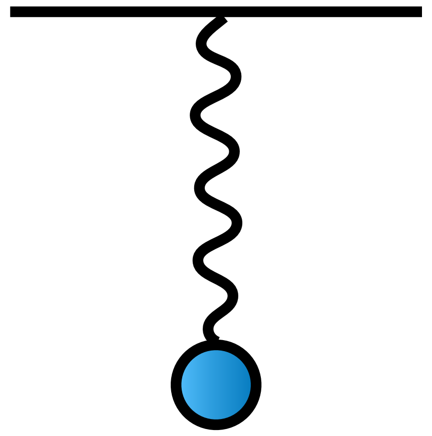
Using the Bogoliubov relation $a(p) = A^\dagger(p,k)b(k) - B^T(p,k)b^\dagger(k)$

$a(p)S\,|\Omega\rangle = \xi(p,k)a^\dagger(k)S\,|\Omega\rangle$ where $\xi(p,k) = (A^\dagger)^{-1}(p,q)B(q,k)$

out vacuum is a coherent squeeze state

$$S\,|\Omega\rangle = \mathcal{N}_\Omega \exp\left[\frac{1}{2}a^\dagger(p)\xi(p,k)a^\dagger(k)\right]|\Omega\rangle$$

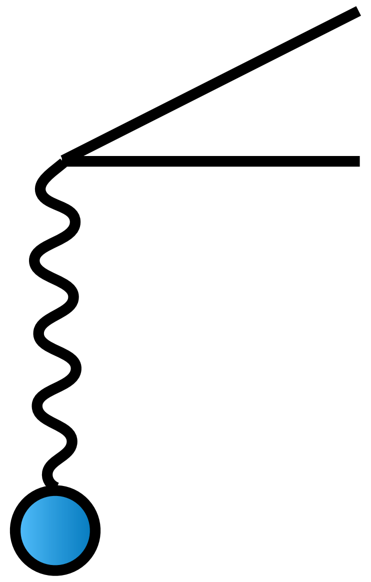
Bogoliubov and crossing



$$\mathcal{A}(E) = \mathcal{N} \int_{-\infty}^{0} \mathrm{d}v \; e^{i(E-E_0)} e^{-4iGME\log(-v/\mu)}$$

$$\Big\downarrow$$

Crossing



$$\mathcal{B}(E) = \mathcal{N} \int_{-\infty}^{0} \mathrm{d}v \; e^{i(E+E_0)} e^{-4iGME\log(-v/\mu)}$$