η/s Corrections from Near-extremal, Near-horizon Quantum Fluctuations

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[Goutéraux, Ramirez, CS (2512.19642v2)]

In this talk

Interacting theories at $T > 0$

AdS/CFT – large N, strong coupling

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$$\begin{array}{c}\text{Near-Extremal} \\ \text{Black holes} \\ \Big\downarrow \text{near-horizon} \\ \mathrm{AdS}_2 \times \mathbb{M}^d\end{array}$$

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Black holes

$$\Big\downarrow \text{near-horizon}$$

$$\mathrm{AdS}_2 \times \mathbb{M}^d$$

$$\Big\downarrow \text{KK reduction}$$

JT gravity

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Thermal states $\xleftrightarrow{\text{dual}}$ Black Hole in AdS

$$\begin{tikzcd}
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 & \mathrm{AdS}_2 \times \mathbb{M}^d \arrow[d, "\text{KK reduction}"] \\
 & \text{JT gravity} \arrow[dl, "\text{low energy}"'] \\
\boxed{\text{Schwarzian}} &
\end{tikzcd}$$

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\text{SYK} \arrow[rr, leftrightarrow, "\text{dual}"] \arrow[dr] & & \text{JT gravity} \arrow[dl] \\
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\end{tikzcd}$$

low energy

Recent developments in NEBH [Iliesiu & Turiaci '20]

Deviation from extremality with correction from Schwarzian

$$\langle S\rangle = \underbrace{S_0 + 4\pi^2 CT}_{\text{semi-classical}} + \frac{3}{2}\log CT + \ldots$$

$$\underset{\text{extremal BH entropy}}{S_0 = \frac{\mathrm{Vol}(\mathbb{T}^2)}{4\ell_{\mathrm{P}}^2}\frac{L^2}{r_{\mathrm{e}}^2}}, \qquad \underset{\text{Schwarzian coupling}}{C = \frac{\mathrm{Vol}(\mathbb{T}^2)}{24\pi\ell_{\mathrm{P}}^2}\frac{L^2}{r_{\mathrm{e}}}}$$

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Natural question:

- What about out-of-equilibrium properties?

$\hookrightarrow$ hydrodynamics

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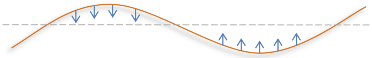
Fluid/Gravity correspondence [Rangamani 0905.4352]

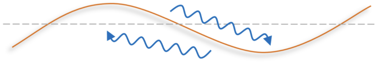
Low-energy excitations of an aAdSBH = those of hydrodynamics

Hydrodynamics

Late-time, long-distance effective description at $T \neq 0$



(a)



(b)

Credit: [Glorioso & Liu '18]

(a) non-protected operators $\Rightarrow$ decay locally

(b) conserved operators $\Rightarrow$ relax through transport

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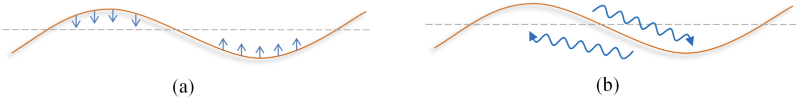

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Enhanced regime of validity of hydro for NEBH

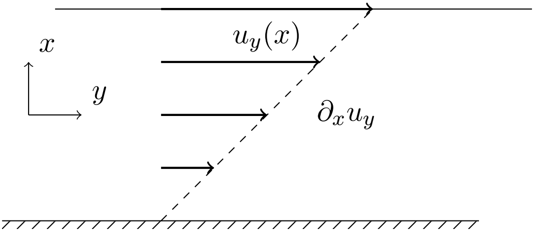
$$\omega \ll T\,, \qquad k \ll \sqrt{T/r_{\rm e}} \qquad \text{(instead of } \omega, k \ll T\text{)}$$

[Arean et al. 2011.12301, Goutéraux et al. 2506.11974]

A transport coefficient: shear viscosity η

Shear viscosity = resistance to flow

Couette flow



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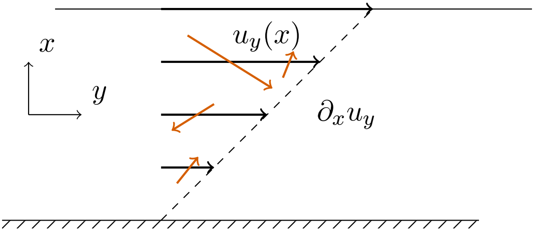

momentum transfer between the layers

$\Rightarrow \quad u_y(x)$ tends to become uniform

Shear Diffusion

$$\delta T^{xy} = -\eta \partial_x u_y$$

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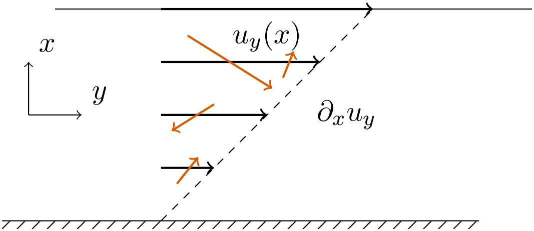

Continuity eq.:

shear stress

$$\partial_t T^{ty} + \partial_x T^{xy} = 0$$

momentum density

momentum transfer between the layers

$\Rightarrow \quad u_y(x)$ tends to become uniform

Shear Diffusion

$$\delta T^{xy} = -\eta \partial_x u_y = -\underbrace{\frac{\eta}{\varepsilon + p}}_{D_\perp} \partial_x \delta T^{ty} \quad \Rightarrow \quad \left[\partial_t - D_\perp \partial_x^2\right] \delta T^{xy} = 0$$

Hydrodynamic Green's function [Kovtun 1205.5040]

▷ **Transverse sector** natural object of interest

$$G^{\mathrm{R}}_{xy,xy}(t-t',\boldsymbol{x}-\boldsymbol{x}') = -\mathrm{i}\theta(t-t')\left\langle [T_{xy}(t,\boldsymbol{x}),T_{xy}(t',\boldsymbol{x}')]\right\rangle$$

Solution (at leading order in ∂) to

$$\left[\partial_t - D_\perp \boldsymbol{\nabla}^2\right] G^{\mathrm{R}}_{xy,xy}(t-t',\boldsymbol{x}-\boldsymbol{x}') = \delta(t-t')\delta^{(3)}(\boldsymbol{x}-\boldsymbol{x}')$$

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Fourier space: $\omega, k \ll T$, $\boldsymbol{k} = k\boldsymbol{e}_x$ (in 2+1)

$$\boxed{G^{\mathrm{R}}_{xy,xy}(\omega, k) = \frac{\eta\,\omega^2}{\mathrm{i}\omega - D_\perp k^2}}$$

Shear diffusion governed by a hydro mode: $\omega_\perp = -\mathrm{i}D_\perp k^2 + \dots$

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Shear viscosity

Kubo formula: $\quad \eta = -\lim_{\omega\to 0}\frac{1}{\omega}\lim_{k\to 0}\mathfrak{Im}\,G^{\mathrm{R}}_{xy,xy}(\omega,k)$

A minimal viscosity

- To compute η: need a microscopic theory

Semi-classical holography $\overset{\text{dual}}{\longleftrightarrow}$ large N, strongly coupled QFT

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_{\mathrm{B}}}}$$

"Universal result"

For large class of translation and rotation invariant two-derivative gravity theories

[Kovtun et al. '03, Cremonini '11]

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Kovtun-Son-Starinet bound [Kovtun et al. '03]

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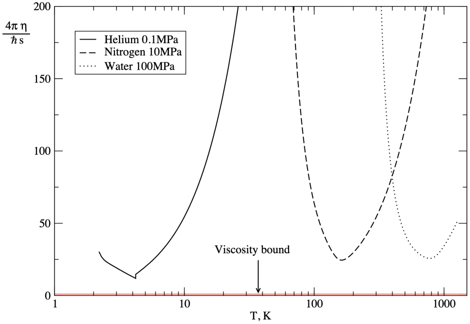
Kovtun-Son-Starinet bound [Kovtun et al. '03]

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_{\mathrm{B}}} \approx 6.08 \times 10^{-13}\,\mathrm{K}\cdot\mathrm{s}$$



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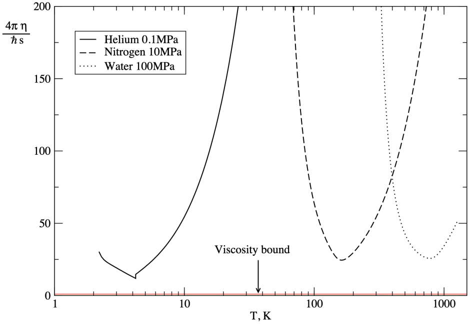

Credit: [Kovtun et al. '05]

- water at STP: 380 times larger
- liquid He at 4K: 9 times larger
- Cold atoms and QGP satisfy the bound

Near-extremal hydrodynamics

Quantify the effect of the Schwarzian fluctuations:

- A type of $1/N$ corrections to transport coefficients
- Change in the spectrum of low-energy excitations

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Focus [Goutéraux, Ramirez, CS (2512.19642)]

How is the KSS bound affected?

$$\frac{\langle \eta \rangle}{\langle s \rangle} \overset{?}{\geq} \frac{1}{4\pi}$$

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How is the KSS bound affected?

$$\frac{\langle \eta \rangle}{\langle s \rangle} \overset{?}{\geq} \frac{1}{4\pi}$$

KSS bound preserved by quantum corrections

$$\frac{\langle \eta \rangle}{s_0} > \frac{1}{4\pi}$$

as expected by [Emparan 2501.17470]

Holographic set-up: AdS–Reissner–Nordström

{Relativistic fluid charged under a global U(1) symmetry}

Holographic $\updownarrow$ dual

4D Einstein–Maxwell theory

$$\mathcal{S}_{\mathrm{EM}} = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R + 6/L^2}{2\kappa^2} - \frac{F^2}{4g_F^2} \right]$$

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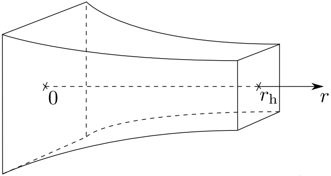
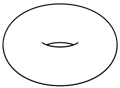


$$0 < r \leq r_\mathrm{h} < r_\mathrm{e} \sim \mu^{-1}$$



Background solution:

$$\bar{g}_{ab}\mathrm{d}x^a\mathrm{d}x^b = \frac{L^2}{r^2}\left[-f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \overbrace{\mathrm{d}x^2 + \mathrm{d}y^2}\right],\ \bar{A}_a\mathrm{d}x^a = \bar{A}_t(r)\mathrm{d}t$$

$$f(r) = 1 - \frac{r^3}{r_\mathrm{h}^3} - \frac{3r^3}{r_\mathrm{h}r_\mathrm{e}^2}\left(1 - \frac{r}{r_\mathrm{h}}\right), \quad \bar{A}_t(r) = \mu\left(1 - \frac{r}{r_\mathrm{h}}\right)$$

Holographic set-up: perturbations

What we want

$$G^{\mathrm{R}}_{xy,xy} \quad \Rightarrow \quad \text{need } g_{xy} \leftrightarrow T_{xy}$$

$$g_{ab} = \bar{g}_{ab} + \delta g_{ab}(r)\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}kx}, \quad A_a = \bar{A}_a + \delta A_a(r)\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}kx}$$

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Transverse sector:

$$(\delta g_{ty}, \delta g_{xy}, \delta A_y) \quad \xrightarrow[\text{combinations}]{\text{gauge invariant}} \quad \overset{\text{master fields}}{\boxed{(\Phi_+, \Phi_-)}}$$

[Edalati et al., '10]

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Two decoupled equations

$$\mathcal{D}_\pm \Phi_\pm = 0$$

Matching procedure [Davison, Parnachev '13]

- Need to solve: $\mathcal{D}_\pm \Phi_\pm = 0$

Scaling for quadratic mode

$$\epsilon \sim k^2 r_{\rm e}^2 \sim \omega r_{\rm e} \sim T r_{\rm e} \ll 1$$

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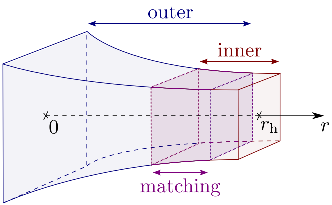

◁ $\dfrac{\omega^2 r}{f f'}, \dfrac{k^2 r}{f'} \lesssim 1$: two independent modes

▷ $r_\mathrm{h} - r \ll r_\mathrm{h} \lesssim r_\mathrm{e}$: ingoing boundary condition

⋈ $(\omega r_\mathrm{e})^{2/3}, (k r_\mathrm{e})^2 \ll \dfrac{r_\mathrm{e} - r}{r_\mathrm{e}} \ll 1$: fixes one mode in terms of the other

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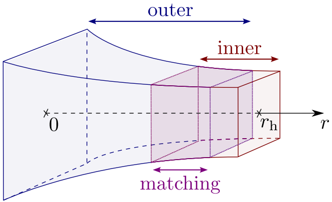

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$$\boxed{G^\mathrm{R}_{xy,xy} = -\frac{L^2}{2\kappa^2}\omega^2 G^\mathrm{R}_- + O(\epsilon^3) = \frac{L^2}{2\kappa^2 r_\mathrm{e}^2}\,\frac{\omega^2 + O(\epsilon^3)}{\mathrm{i}\omega - \frac{r_\mathrm{e}}{12}k^2 + O(\epsilon^2)}}$$

Matching and $G^{\mathrm{R}}_{xy,xy}$

- **Interpret $\mathrm{i}\omega$ as an inner Green's function**

Matching and $G^{\mathrm{R}}_{xy,xy}$

▷ **Interpret iω as an inner Green's function**

Inner $\mathcal{D}_-\Phi_- = 0$: massless neutral scalar in $\mathrm{AdS}_2 + O(\epsilon)$

$$\Downarrow$$

both falloffs are normalizable

$$G^{\mathrm{R}}_{xy,xy} = \frac{L^2}{2\kappa^2 r_{\mathrm{e}}^2}\frac{\omega^2}{\mathcal{G}^{\mathrm{R}}_{\Delta=1} - \frac{r_{\mathrm{e}}}{12}k^2} \quad \text{or} \quad \frac{L^2}{2\kappa^2 r_{\mathrm{e}}^2}\frac{\omega^2}{(\mathcal{G}^{\mathrm{R}}_{\Delta=0})^{-1} - \frac{r_{\mathrm{e}}}{12}k^2}$$

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Inner Green's function

$$\mathcal{G}^{\mathrm{R}}_{\Delta=1} = \mathrm{i}\omega, \quad \mathcal{G}^{\mathrm{R}}_{\Delta=0} = \frac{1}{\mathrm{i}\omega}$$

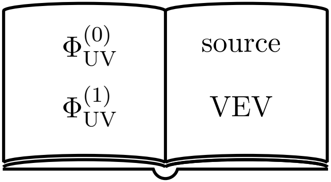
Matching and IR Green's function

Falloffs of UV and IR fields:

$$\Phi_{\mathrm{UV}} \underset{r\to 0}{=} \Phi_{\mathrm{UV}}^{(0)} + \Phi_{\mathrm{UV}}^{(1)} r + \ldots$$

$$\Phi_{\mathrm{IR}} \underset{\zeta\to 0}{=} \Phi_{\mathrm{IR}}^{(0)} + \Phi_{\mathrm{IR}}^{(1)} \zeta + \ldots$$

Dirichlet b.c.



From the matching

$$\Phi_{\mathrm{UV}}^{(0)} = \Phi_{\mathrm{IR}}^{(0)} \frac{(k r_{\mathrm{e}})^2}{12} - \Phi_{\mathrm{IR}}^{(1)}$$

$$\Phi_{\mathrm{UV}}^{(1)} = \Phi_{\mathrm{IR}}^{(0)} r_{\mathrm{e}}^{-1}$$

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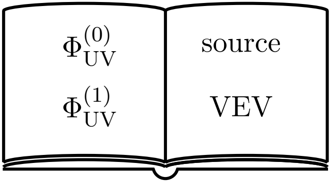

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- Mixed boundary conditions in the IR
- VEV in the IR is the **leading** mode

$$\Rightarrow \quad \boxed{\mathcal{G}^{\mathrm{R}}_{\Delta=0} = \frac{1}{\mathrm{i}\omega}}$$

Regime of validity of the calculation

- Generating functional in Schwarzian theory [Mertens & Turiaci '23]

$$\mathcal{Z}[\Phi_0] = \int [\mathcal{D}u]\ \mathrm{e}^{S_0 + C\,\mathrm{Schw}[u] + \frac{1}{2}\int \mathrm{d}\tau_1^{\mathrm{E}} \mathrm{d}\tau_2^{\mathrm{E}} \Phi_0(\tau_1^{\mathrm{E}}) \mathcal{G}_\Delta^{\mathrm{E}}(u(\tau_1^{\mathrm{E}}), u(\tau_2^{\mathrm{E}})) \Phi_0(\tau_2^{\mathrm{E}})}$$

$$\underbrace{S_0 = \frac{\mathrm{Vol}(\mathbb{T}^2)}{4\ell_{\mathrm{P}}^2} \frac{L^2}{r_{\mathrm{e}}^2}}_{\text{extremal BH entropy}}\,, \qquad \underbrace{C = \frac{\mathrm{Vol}(\mathbb{T}^2)}{24\pi\ell_{\mathrm{P}}^2} \frac{L^2}{r_{\mathrm{e}}}}_{\text{Schwarzian coupling}}$$

$$\mathrm{Schw}[u] = \int_0^\beta \mathrm{d}\tau^{\mathrm{E}} \{u(\tau^{\mathrm{E}}), \tau^{\mathrm{E}}\}\,, \quad \{u(\tau^{\mathrm{E}}), \tau^{\mathrm{E}}\} = \frac{u'''}{u'} - \frac{3}{2}\left(\frac{u''}{u'}\right)^2$$

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- Generating functional in Schwarzian theory [Mertens & Turiaci '23]

$$\mathcal{Z}[\Phi_0] = \int [\mathcal{D}u]\ \mathrm{e}^{S_0 + C\,\mathrm{Schw}[u] + \frac{1}{2}\int \mathrm{d}\tau_1^{\mathrm{E}}\mathrm{d}\tau_2^{\mathrm{E}}\Phi_0(\tau_1^{\mathrm{E}})\mathcal{G}_\Delta^{\mathrm{E}}(u(\tau_1^{\mathrm{E}}),u(\tau_2^{\mathrm{E}}))\Phi_0(\tau_2^{\mathrm{E}})}$$

$$\underbrace{S_0 = \frac{\mathrm{Vol}(\mathbb{T}^2)}{4\ell_{\mathrm{P}}^2}\frac{L^2}{r_{\mathrm{e}}^2}}_{\text{extremal BH entropy}}\,, \qquad \underbrace{C = \frac{\mathrm{Vol}(\mathbb{T}^2)}{24\pi\ell_{\mathrm{P}}^2}\frac{L^2}{r_{\mathrm{e}}}}_{\text{Schwarzian coupling}}$$

$$\mathrm{Schw}[u] = \int_0^\beta \mathrm{d}\tau^{\mathrm{E}}\,\{u(\tau^{\mathrm{E}}),\tau^{\mathrm{E}}\}\,, \quad \{u(\tau^{\mathrm{E}}),\tau^{\mathrm{E}}\} = \frac{u'''}{u'} - \frac{3}{2}\left(\frac{u''}{u'}\right)^2$$

$$\boxed{\begin{array}{rrcll} & \text{near-extremal} & Tr_{\mathrm{e}} \ll 1 & \leftrightarrow & T/\mu \ll 1 \\ + & \text{hydro regime} & \mathrm{Vol}(\mathbb{T}^2)^{-1} \ll T/r_{\mathrm{e}} & \leftrightarrow & k^2 \ll T\mu \\ + & \text{classical gravity} & L/\ell_{\mathrm{P}} \gg 1 & \leftrightarrow & N \gg 1 \\ & & & & \\ \Rightarrow & \text{weak quantum fluct.} & CT \gg 1 & & \end{array}}$$

Result

Plugging in near-horizon quantum fluctuations:

$$\left\langle G^{\mathrm{R}}_{xy,xy} \right\rangle = \frac{L^2}{2\kappa^2 r_{\mathrm{e}}^2} \frac{\omega^2}{\langle 1/\mathrm{i}\omega \rangle^{-1} - \frac{r_{\mathrm{e}}}{12} k^2} = \frac{\langle \eta \rangle \omega^2}{\mathrm{i}\omega - \langle D_\perp \rangle k^2}$$

$$(\langle \eta \rangle , \langle D_\perp \rangle) = (\eta^{\mathrm{cl.}}, D_\perp^{\mathrm{cl.}}) \left[1 + \frac{1}{4\pi^2 CT} + O\left(\frac{1}{(CT)^2} \right) \right]$$

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* agrees with [2510.21602, 2512.20443]

** different from [2510.16100, 2510.15411] (other approaches)

KSS bound preserved by quantum corrections

$$\frac{\langle \eta \rangle}{s_0} > \frac{1}{4\pi}$$

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$D_\perp$: Hydrodynamic mode that seemed to survive $T \to 0$ limit might get lifted by near-horizon quantum fluctuations

Outlook: follow-ups

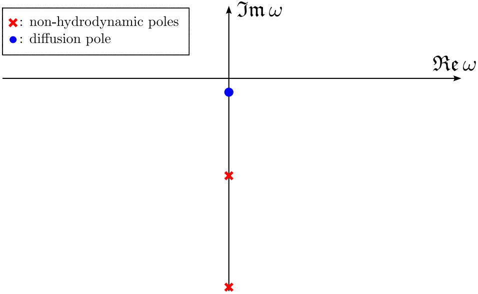
Possible follow-ups

- Explore $CT \lesssim 1$ regime, beyond hydrodynamics
- Next order in ϵ to consider $\langle s \rangle$ [Iliesiu & Turiaci '23]
- 5D calculation to connect to $\mathcal{N} = 4$ SYM

Outlook: EFT with gapped modes

Remark

Corrections to diffusion constant, similar as **stochastic corrections** to hydrodynamics [Chen-Lin, Delacrétaz & Hartnoll '19] but without branch-cut



EFT with gapped modes

Integrate over gapped modes to obtain an effective action for the gapless mode

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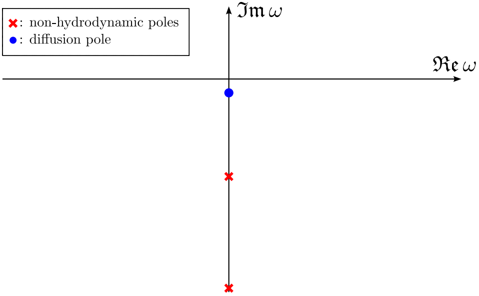


EFT with gapped modes

Integrate over gapped modes to obtain an effective action for the gapless mode

End

A minimal viscosity [Kovtun et al. '05]

- Hydrodynamics

$$\text{positivity of entropy production} \quad \Rightarrow \quad \frac{\eta}{s} \geq 0$$

- Kinetic theory and Heisenberg uncertainty principle

$$\eta \propto \varepsilon \frac{\ell_{\text{mfp}}}{\bar{u}} = \varepsilon \tau_{\text{mft}}\,, \quad s \sim n k_{\text{B}} \quad \Rightarrow \quad \frac{\eta}{s} \sim \frac{1}{k_{\text{B}}} \frac{\varepsilon}{n} \tau_{\text{mft}} \geq \frac{\hbar}{k_{\text{B}}}$$

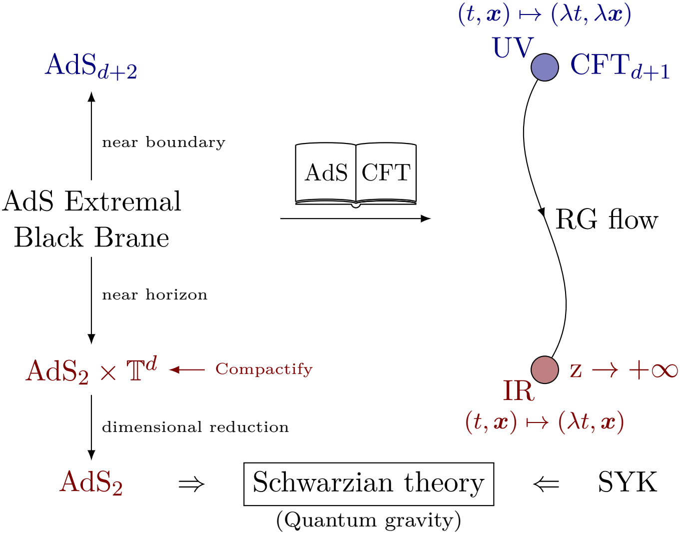
- Semi-classical holography $\xleftrightarrow{\text{dual}}$ large N, strongly coupled QFT

$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_{\text{B}}}}$$

"Universal result"

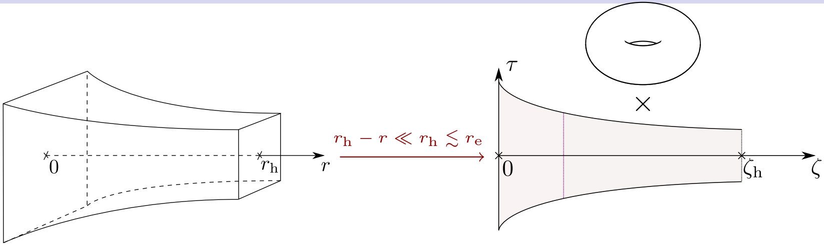
For large class of translation and rotation invariant two-derivative gravity theories

Near-Extremal Black Holes



Same universality class as SYK models

Holographic Set-up: IR Geometry

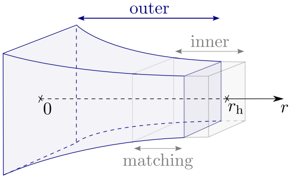


Zoom in near horizon [Faulkner et al. '09]

$$r = r_\mathrm{e} - \epsilon \frac{r_\mathrm{e}^2}{6\zeta}\,, \quad t = \epsilon^{-1}\tau\,, \quad T = \frac{\epsilon}{4\pi\zeta_\mathrm{h}} + O(\epsilon^2)$$

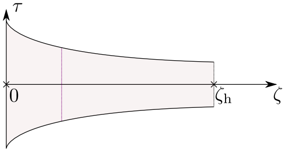
$$\mathrm{d}s^2 \underset{\epsilon\to 0}{=} \underbrace{\frac{L^2}{6\zeta^2}\left[-\left(1-\frac{\zeta}{\zeta_\mathrm{h}}\right)\mathrm{d}\tau^2 + \frac{\mathrm{d}\zeta^2}{1-\zeta/\zeta_\mathrm{h}}\right]}_{\text{Black Hole in AdS}_2} + \underbrace{\frac{1}{r_\mathrm{e}^2}\left[\mathrm{d}x^2 + \mathrm{d}y^2\right]}_{\mathbb{T}^2} + O(\epsilon)$$

Solve the EoM: Matching Procedure [Davison, Parnachev '13]



$\triangleleft\ \dfrac{\omega^2 r}{f f'}, \dfrac{k^2 r}{f'} \ll 1$

$$f\left(f\Phi_\pm'\right)' + \left[\underbrace{\omega^2 - f k^2}_{O(\omega^2, k^2)} - \frac{f f'}{r} - \frac{6 r \beta_\pm}{r_\mathrm{e}^2 r_\mathrm{h}} f\right]\Phi_\pm = 0$$



$\triangleright\ r_\mathrm{h} - r \ll r_\mathrm{h} \lesssim r_\mathrm{e}$

Zoom in near horizon [Faulkner et al. '09]

$$r = r_\mathrm{e} - \epsilon\frac{r_\mathrm{e}^2}{6\zeta}, \quad t = \epsilon^{-1}\tau, \quad T = \frac{\epsilon}{4\pi\zeta_\mathrm{h}} + O(\epsilon^2)$$

$$\mathrm{d}s^2 \underset{\epsilon\to 0}{=} \frac{L^2}{6\zeta^2}\left[-\left(1 - \frac{\zeta}{\zeta_\mathrm{h}}\right)\mathrm{d}\tau^2 + \frac{\mathrm{d}\zeta^2}{1 - \zeta/\zeta_\mathrm{h}}\right] + \frac{1}{r_\mathrm{e}^2}\left[\mathrm{d}x^2 + \mathrm{d}y^2\right] + O(\epsilon)$$

Scaling for quadratic mode

$$\epsilon \sim k^2 r_\mathrm{e}^2 \sim \omega r_\mathrm{e} \sim T r_\mathrm{e} \ll 1$$

Holographic Set-up: Perturbations

What we want

$$G^{\mathrm{R}}_{xy,xy} \quad \Rightarrow \quad \text{need } g_{xy} \leftrightarrow T_{xy}$$

$$g_{ab} = \bar{g}_{ab} + \delta g_{ab}(r)\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}kx}, \quad A_a = \bar{A}_a + \delta A_a(r)\mathrm{e}^{-\mathrm{i}\omega t + \mathrm{i}kx}$$

Transverse sector:

$$(\delta g_{ty}, \delta g_{xy}, \delta A_y) \quad \xrightarrow[\text{combinations}]{\text{gauge invariant}} \quad \overset{\text{master fields}}{\boxed{(\Phi_+, \Phi_-)}}$$

Two decoupled equations:

$$f\left(f\Phi_\pm'\right)' + \left[\omega^2 - fk^2 - \frac{ff'}{r} - \frac{6r\beta_\pm}{r_\mathrm{e}^2 r_\mathrm{h}} f\right]\Phi_\pm = 0$$

$$\beta_\pm = \frac{3}{4}\left(1 + \frac{r_\mathrm{e}^2}{3r_\mathrm{h}^2}\right)\left[1 \pm \sqrt{1 + \frac{16k^2 r_\mathrm{e}^2}{27}}\left(1 + \frac{r_\mathrm{e}^2}{3r_\mathrm{h}^2}\right)^{-2}\right]$$

Alternate Quantization

The quantization scheme is decided the matching
Imposes **alternate** quantization

$$\mathcal{G}^{\mathrm{R}}_{\Delta=0}(\omega) = \frac{1}{\mathrm{i}\omega} \quad \xrightarrow[\text{rotate}]{\text{Wick}} \quad \mathcal{G}^{\mathrm{E}}_{\Delta=0}(\omega_{\mathrm{E}}) = \frac{-1}{|\omega_{\mathrm{E}}|} \quad \xrightarrow{\mathscr{F}^{-1}} \quad \frac{\gamma}{\pi} + \frac{1}{2\pi}\boxed{\log|\tau_{\mathrm{E}}|^2}$$

To connect with Schwarzian literature: go Euclidean

$\langle \log|\tau_{\mathrm{E}}|^2 \rangle$

Logarithmic Correlator

$$\mathcal{G}^{\mathrm{R}}_{\Delta=0}(\omega) = \frac{1}{\mathrm{i}\omega} \quad \xrightarrow[\text{rotate}]{\text{Wick}} \quad \mathcal{G}^{\mathrm{E}}_{\Delta=0}(\omega_{\mathrm{E}}) = \frac{-1}{|\omega_{\mathrm{E}}|} \quad \xrightarrow{\mathscr{F}} \quad \frac{\gamma}{\pi} + \frac{1}{2\pi}\log|\tau_{\mathrm{E}}|^2$$

Need to compute $\left\langle \log|\tau_{\mathrm{E}}|^2 \right\rangle$

Extracted from expansion of [Mertens et al., '17]:

$$\left\langle \mathcal{G}^{\mathrm{E}}_{\Delta} \right\rangle = \frac{\mathrm{e}^{S_0}}{\mathcal{Z}(\beta)} \int \mathrm{d}\mu(p_1)\mathrm{d}\mu(p_2) \mathrm{e}^{-\tau_{\mathrm{E}}\frac{p_1^2}{2C}} \mathrm{e}^{-(\beta-\tau_{\mathrm{E}})\frac{p_2^2}{2C}} \frac{2\Gamma(\Delta \pm \mathrm{i}p_1 \pm \mathrm{i}p_2)}{(2C)^{2\Delta}\Gamma(2\Delta)}$$

Logarithmic Correlator

To extract logarithm, expand in Δ

$$\mathcal{G}^{\mathrm{E}}_{\Delta}(\tau_1^{\mathrm{E}}-\tau_2^{\mathrm{E}}) = \left(\pi^2 T^2 \frac{u'(\tau_1^{\mathrm{E}})u'(\tau_2^{\mathrm{E}})}{\sin^2(\pi T|u(\tau_1^{\mathrm{E}})-u(\tau_2^{\mathrm{E}})|)}\right)^{\Delta} = \left(\mathcal{G}^{\mathrm{R}}_{\Delta=1}\right)^{\Delta}$$

$$\Big\downarrow \; \Delta \to 0$$

$$1+\Delta \log \mathcal{G}^{\mathrm{E}}_{\Delta=1}$$

$$\Big\downarrow \; u \mapsto \mathrm{id}, T \to 0$$

$$1-\Delta \boxed{\log|\tau_1^{\mathrm{E}}-\tau_2^{\mathrm{E}}|^2}$$

$$\mathcal{Z}[\Phi_0] = \int [\mathcal{D}u]\; \mathrm{e}^{S_0 + C\,\mathrm{Schw[u]} + \frac{1}{2}\int \mathrm{d}\tau_1^{\mathrm{E}}\mathrm{d}\tau_2^{\mathrm{E}}\Phi_0(\tau_1^{\mathrm{E}})\mathcal{G}^{\mathrm{E}}_{\Delta}\Phi_0(\tau_2^{\mathrm{E}})}$$

$$\left\langle \mathcal{G}^{\mathrm{E}}_{\Delta}\right\rangle = \frac{1}{\mathcal{Z}[\Phi_0]}\frac{\delta^2\mathcal{Z}[\Phi_0]}{\delta\Phi_0(\tau_1^{\mathrm{E}})\delta\Phi_0(\tau_2^{\mathrm{E}})}\Bigg|_{\Phi_0=0}$$

[Mertens et al., '17]