Decoherence or Recoherence in the Radiative Decay of the Z Boson

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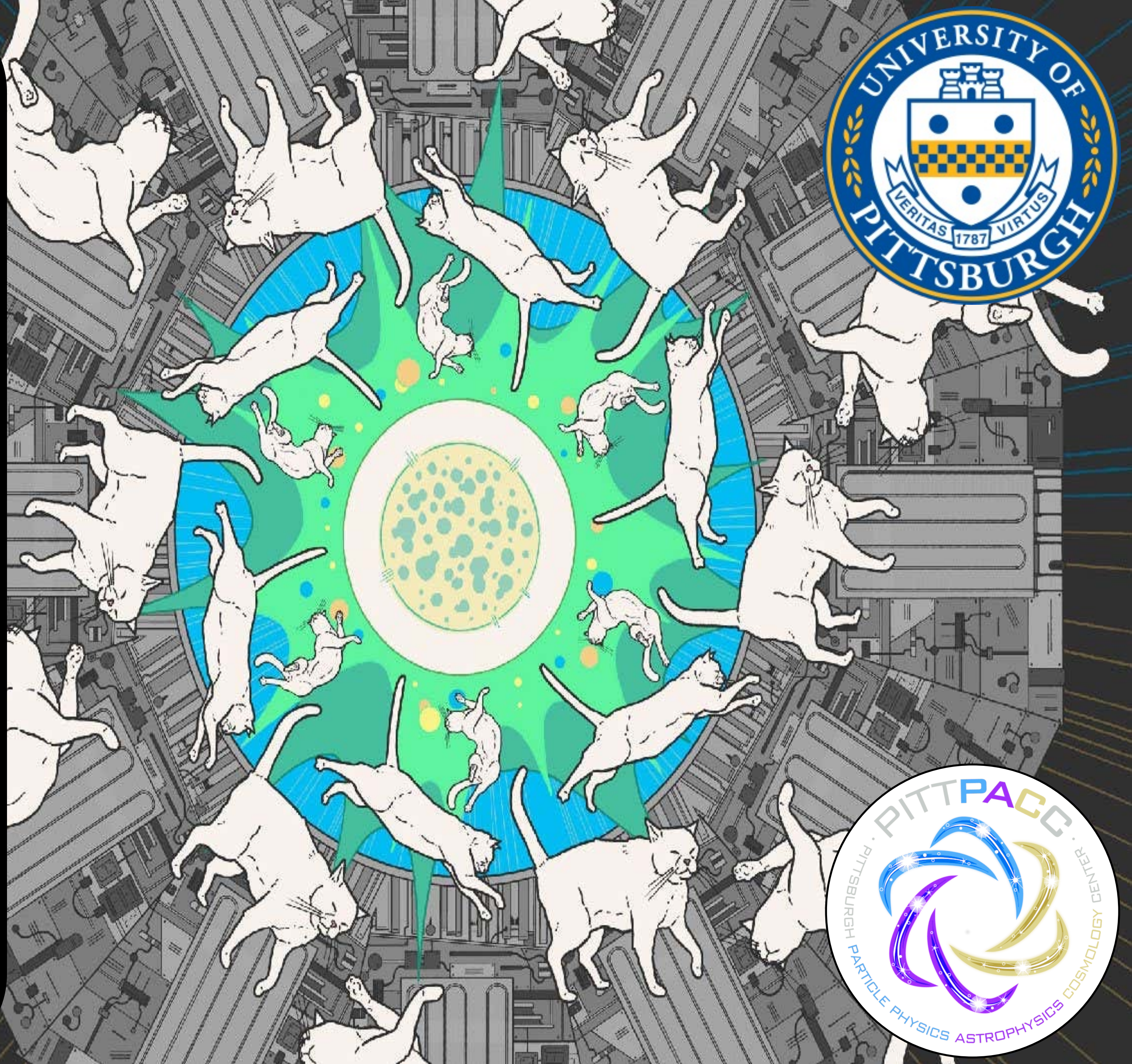

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Motivations for Decoherence

- Quantum information for collider process @ LO is mature
  - Entangled tops @ ATLAS/CMS [2311.07288, 2406.03976]
- Higher order processes like FSR are critical phenomena at colliders
- Interested in bipartite $f\bar{f}$ system but FSR is expected to cause decoherence
- Why?: Unobserved d.o.f. of radiation act as "environment" for system
  - Decoherence: interactions with environment cause loss of coherence and information
- Focus on bipartite system quantum state: Sum production polarization (σ) & trace FSR polarization (λ)

$$\rho_{a\bar{a}b\bar{b}} = \frac{\sum \left(\mathcal{M}^{\sigma}_{\lambda b\bar{b}}\right)^{*} \mathcal{M}^{\sigma}_{\lambda a\bar{a}} \, |a\bar{a}\rangle\langle b\bar{b}|}{\sum \left|\mathcal{M}^{\sigma}_{\lambda a\bar{a}}\right|^{2}}$$

Quantifying Decoherence

1) Evolution of entanglement: <u>**Concurrence**</u>

- Eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, $\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$ in descending order:

$$\mathcal{C} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \qquad 0 \leq \mathcal{C} \leq 1$$

2) Measure mixedness of state: <u>**Purity**</u>

- Tracing over FSR polarization mixes up states quantify by $\mathrm{tr}(\rho^2)$
- For bipartite: $\frac{1}{4} \leq \mathrm{tr}(\rho^2) \leq 1$ but small fermion mass: $\frac{1}{2} \lesssim \mathrm{tr}(\rho^2) \leq 1$

3) Track changing state: <u>**Magic**</u>

- Distance of state to stabilizer states (Efficient on classical computers: Bell+separable states)

$$\mathcal{M}_2 = -\log_2\left(\frac{1 + \sum_i (B_i^-)^4 + \sum_j \left(B_j^+\right)^4 + \sum_{ij}\left(C_{ij}\right)^4}{1 + \sum_i \left(B_i^-\right)^2 + \sum_j \left(B_j^+\right)^2 + \sum_{ij}\left(C_{ij}\right)^2}\right), \qquad \rho = \frac{\left(\mathbb{I}_4 + B_i^- \sigma_i \otimes \mathbb{I}_2 + B_j^+ \mathbb{I}_2 \otimes \sigma_j + C_{ij}\sigma_i \otimes \sigma_j\right)}{4}$$

Radiative Z decay as an Example

- Decoherence via radiative decay of soft/collinear radiation studied by others [2510.13951, 2504.07030]
- Higgs decay kinematically simple but dominated by NLO [1704.00790, 2403.18023]
- Focus on bipartite system from radiative Z decay to τ pair + γ

$$\mathcal{M}^{\sigma}_{\lambda a\bar{a}} = \frac{e^2}{s_W c_W}\, \epsilon^{\sigma}_{Z,\mu} \bar{u}_a \left(\not{\epsilon}^*_{\lambda} \frac{\not{p}_- + \not{k} + m_\tau}{(p_- + k)^2 - m_\tau^2} \gamma^\mu \left(g^\tau_L P_L + g^\tau_R P_R\right) - \gamma^\mu \left(g^\tau_L P_L + g^\tau_R P_R\right) \frac{\not{p}_+ + \not{k} - m_\tau}{(p_+ + k)^2 - m_\tau^2} \not{\epsilon}^*_{\lambda} \right) v_{\bar{a}}.$$

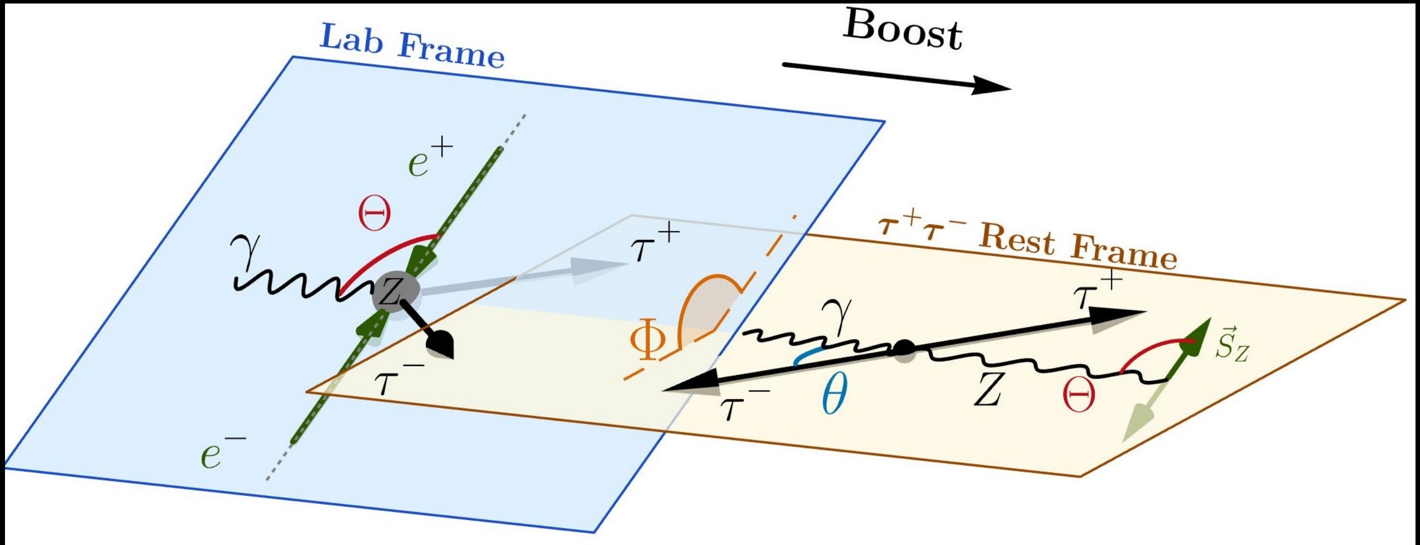
- Spinors for τ quantized along τ^- direction in $\tau\tau$ c.m. frame:

$$\tau^- \Longrightarrow u_a(p_-), \quad \tau^+ \Longrightarrow v_{\bar{a}}(p_+)$$

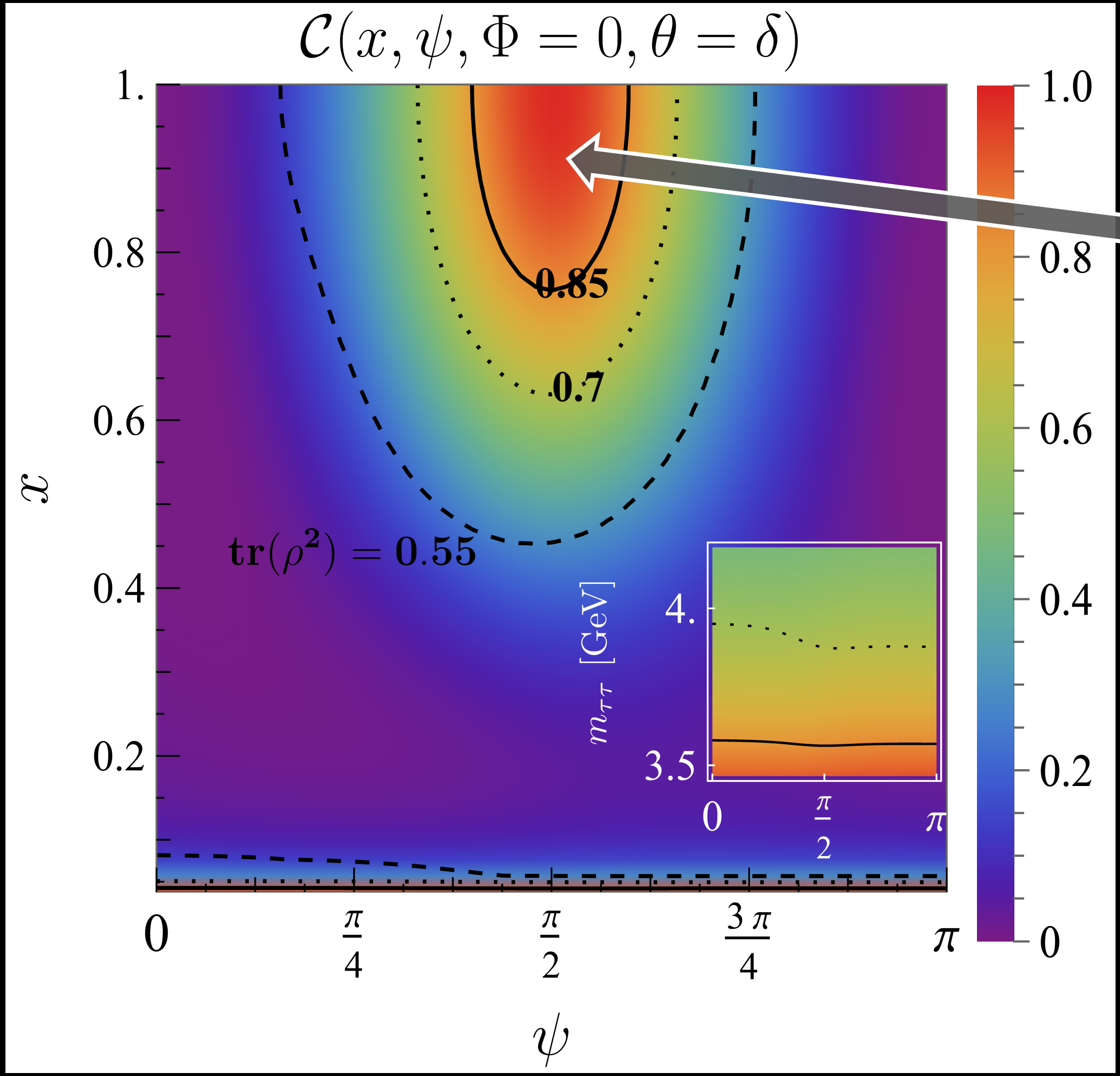
- Z polarization: $\epsilon^{\sigma}_Z(p_Z)$ can be replaced by incoming current producing it on-shell

Kinematical Set up

- Z produced on-shell by e^-e^+: only transverse polarizations
- $x = \frac{m_{\tau\tau}}{m_Z}$ – τ pair invariant mass energy fraction: $\frac{2\,m_\tau}{m_Z} \leq x \leq 1$
- Θ – Scattering angle of γ and e^- direction: $0 \leq \Theta \leq \pi$
- Φ – Azimuthal angle between planes: $0 \leq \Phi \leq 2\pi$: Primarily coplanar set $\Phi = 0$
- θ – Polar angle between τ^- and γ: $\delta \leq \Theta \leq \pi - \delta$
- Recover $2 \longrightarrow 2$ angle for soft radiation for comparing to case of no FSR: $\psi = \Theta - \theta$



Close-to-Collinear Radiation $\theta = \delta$



- Density: Concurrence & Contours: Purity
- Soft radiation ($x \to 1$) and central scattering region $\left(\psi \to \frac{\pi}{2}\right)$: Return to $Z \to \tau^-\tau^+$ known result $\mathcal{C} \sim \frac{\sin^2 \psi}{1+\cos^2 \psi}$ [2410.08303]
- Decreasing $x \longrightarrow$ decreasing entanglement: expected decoherence!
    - More energetic photon taking more info.
- Hard radiation $\left(x \to \frac{2\, m_\tau}{m_Z}\right)$: mass effect prevalent → entangled everywhere
    - γ spin align with Z & kicks $\tau\tau$ into $|1, 0\rangle$ state

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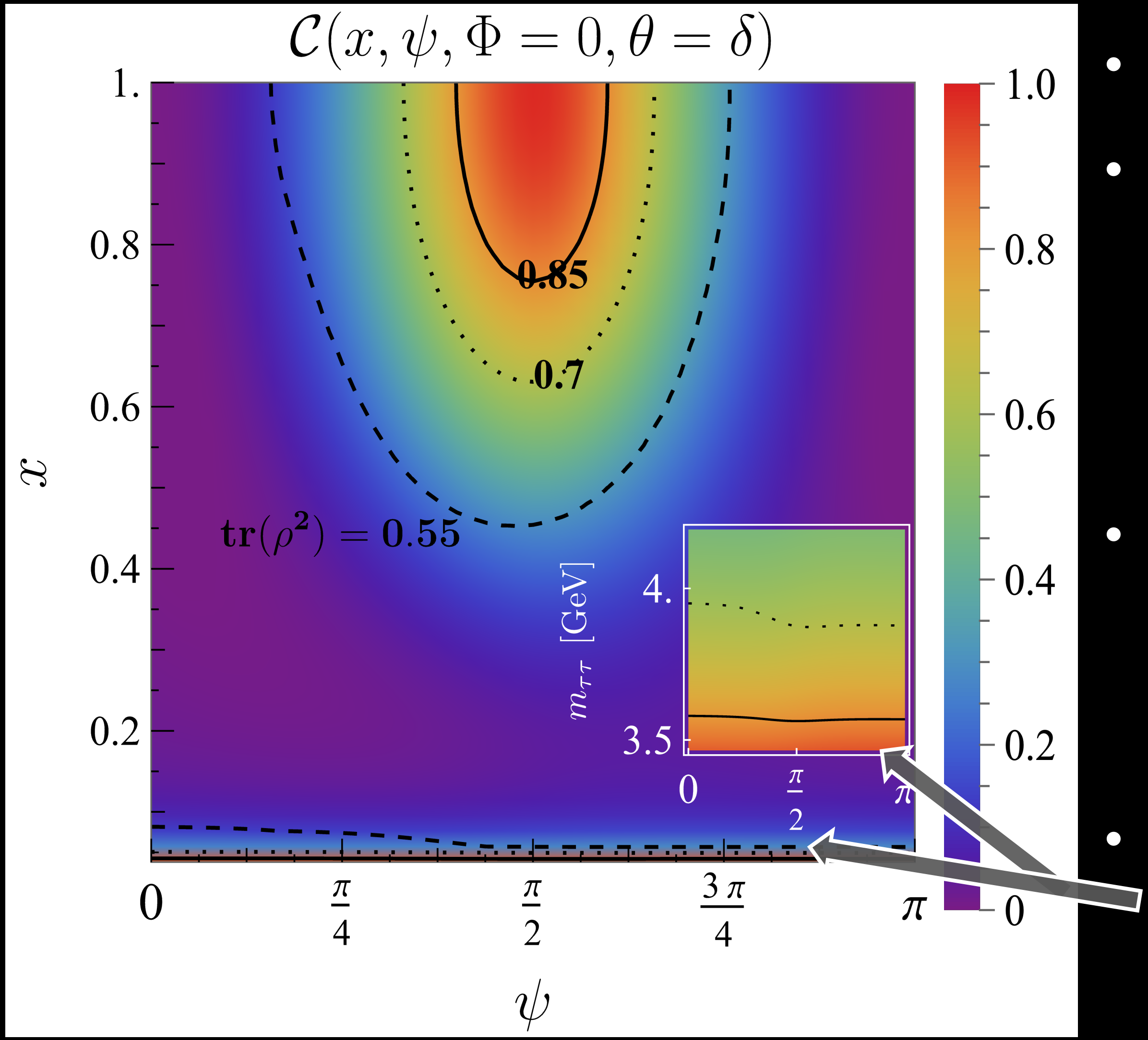

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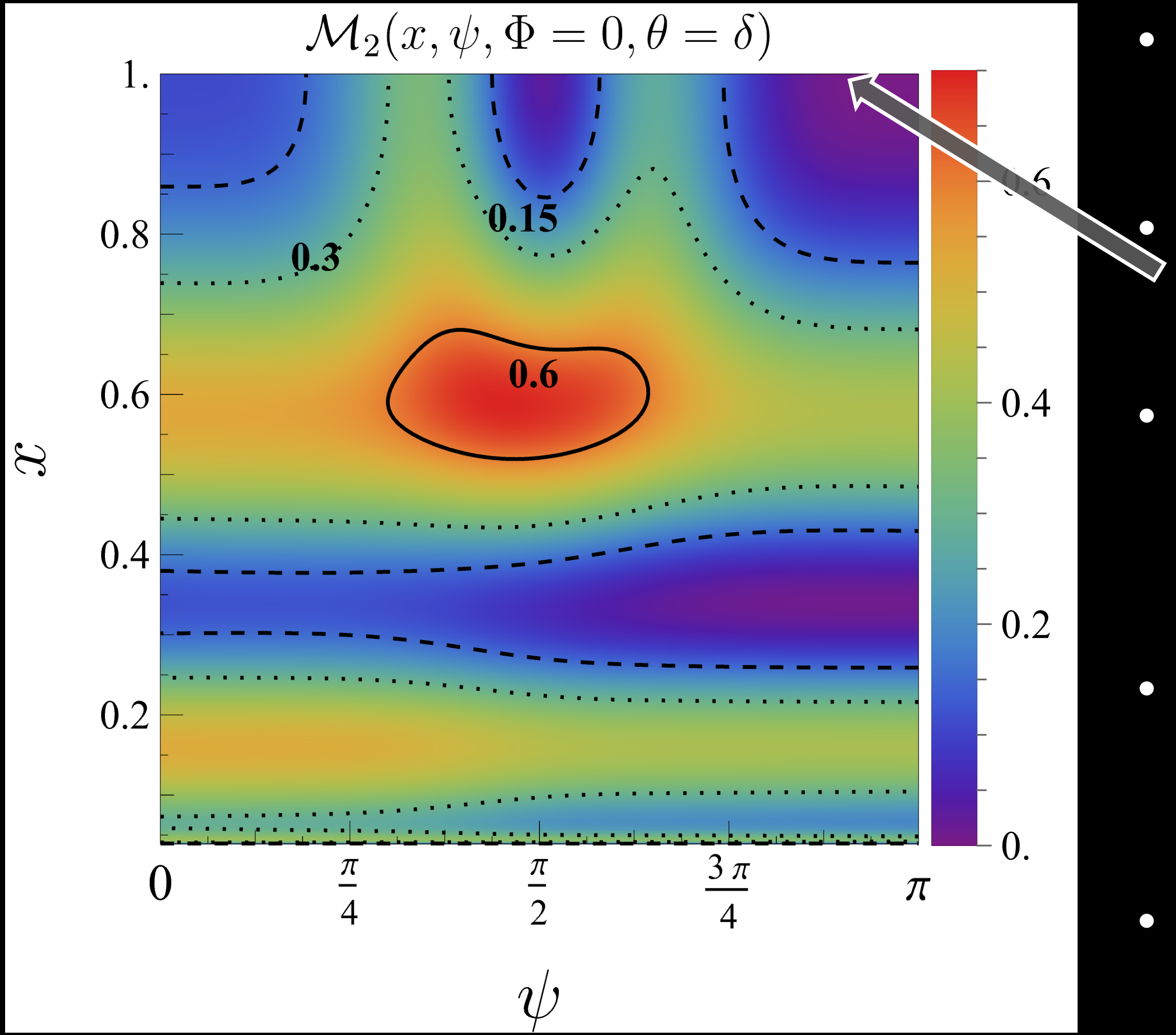

- Track changing state: contour & density = Magic
- Soft radiation: $m_\tau \approx 0$ dominated by helicity aligned states ↑↑, ↓↓ (~ Stabilizer state)
- Hard radiation: $m_\tau \neq 0$ dominated by helicity flipped states ↑↓, ↓↑ (~ Stabilizer state)
- Helicity flipped vs helicity aligned: same order when $x\theta \sim \frac{2m_\tau}{m_Z}$
- Asymmetry of forward ($\psi \to 0$) & backward ($\psi \to \pi$) due to chiral couplings...

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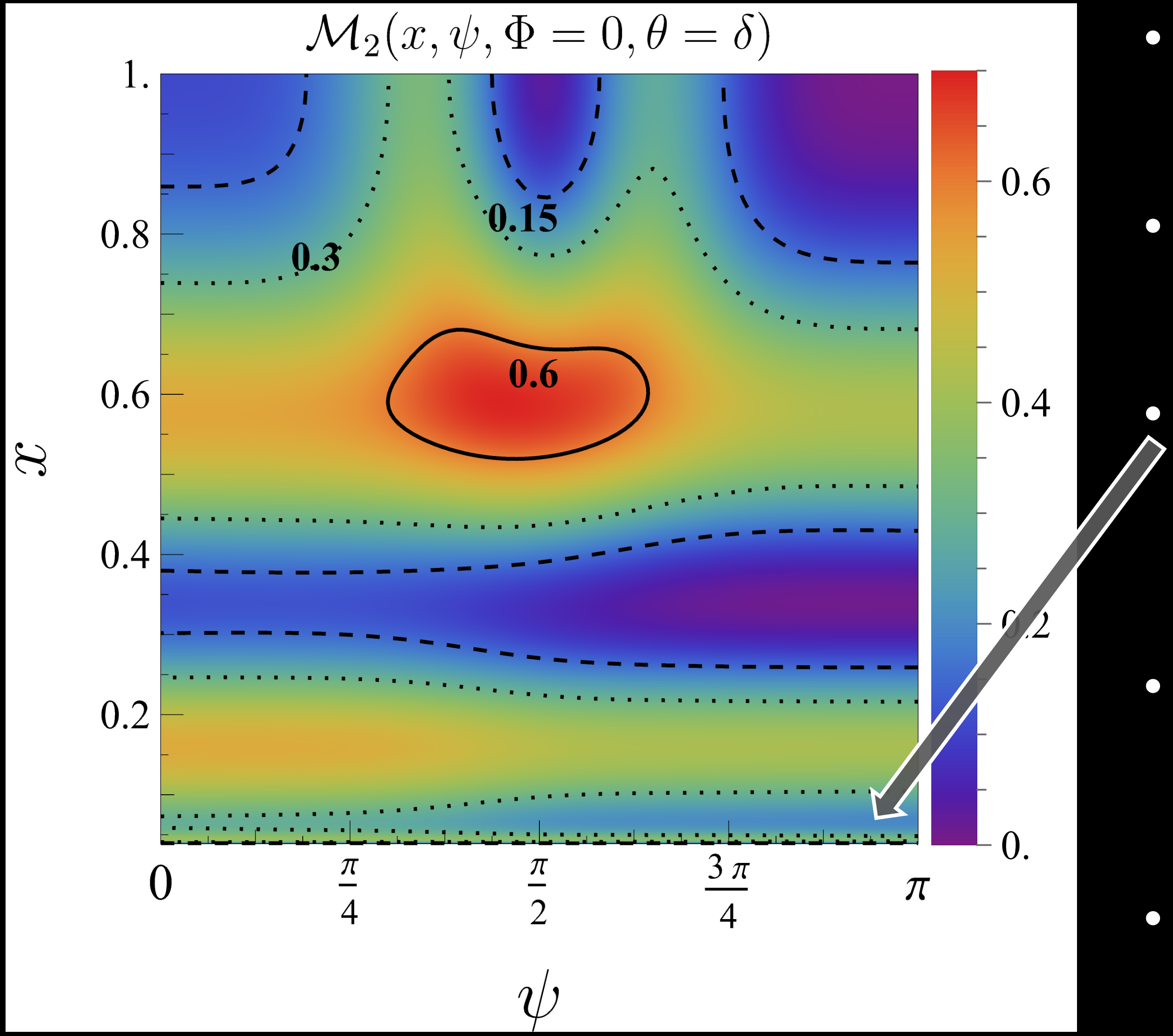

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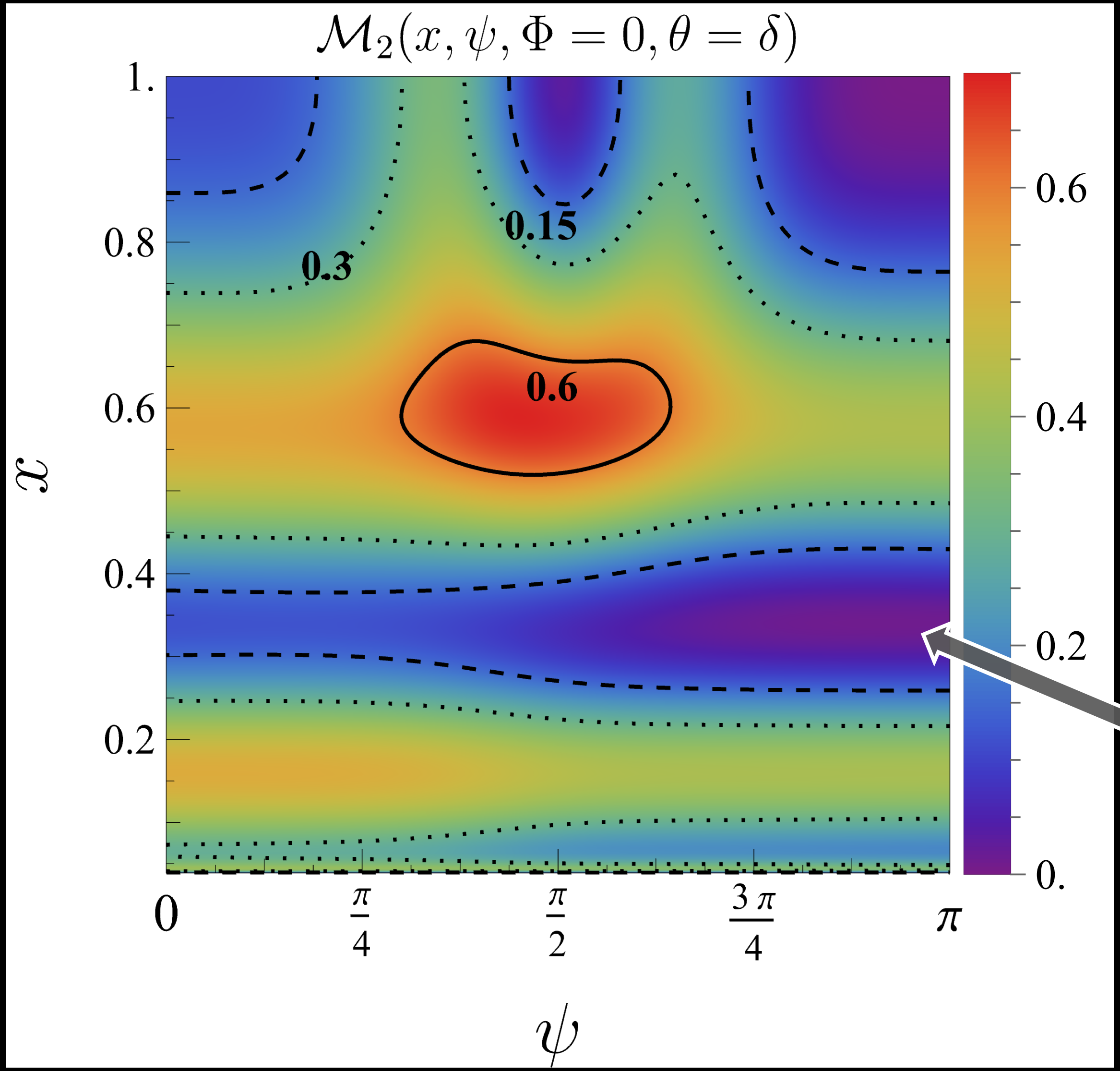

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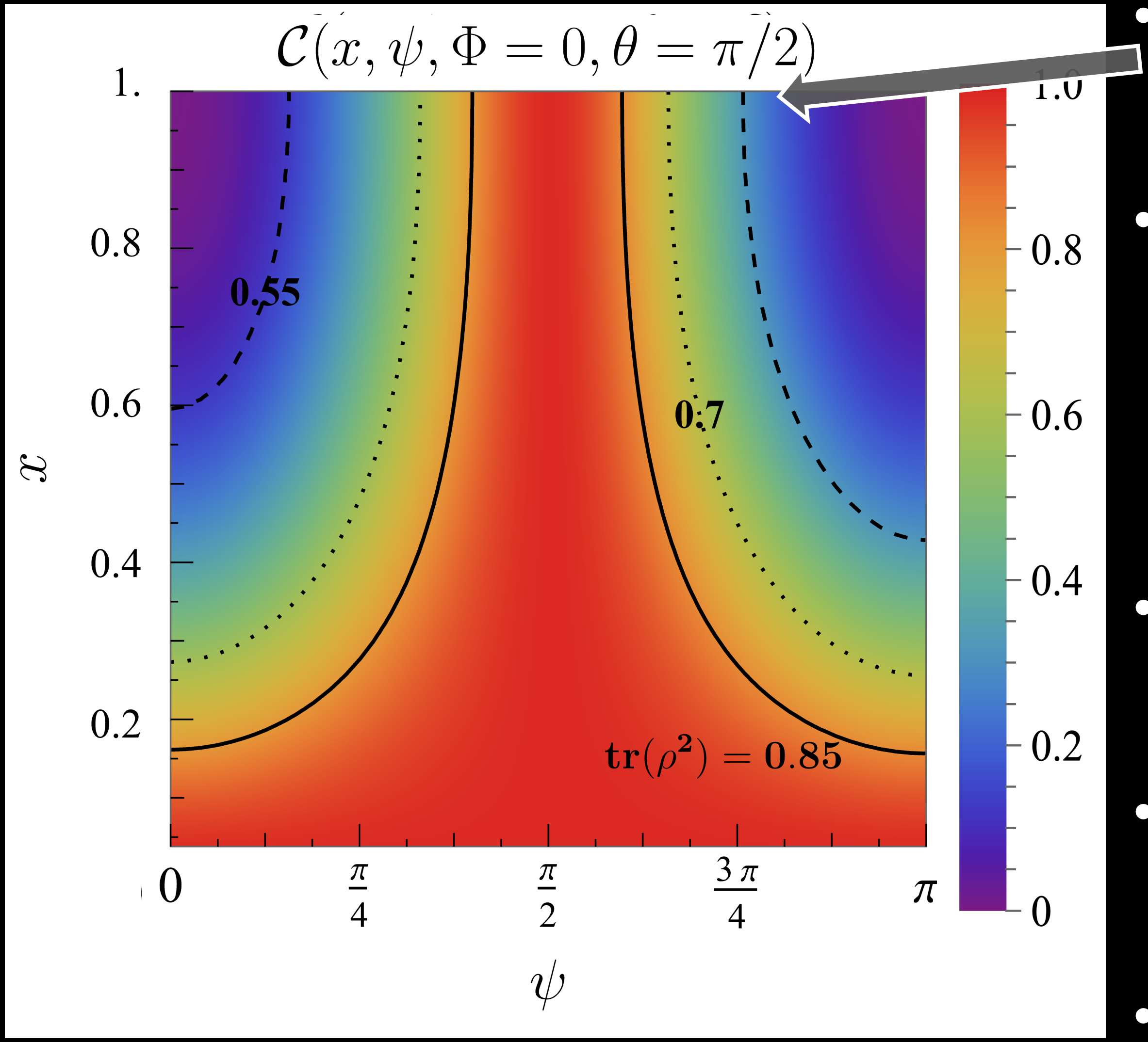

- Soft photon regime matches $\theta = \delta$: expected from Weinberg soft theorem
- Central scattering always entangled
  - Z coupling $\approx$ axial + unique kinematics $\Longrightarrow$ charge conjugation symmetry
  - Symmetry protects spin state/entanglement
- Forward & backward show increasing entanglement
- Hard photon regime entangled everywhere matching $\theta = \delta$
- No decoherence anywhere!

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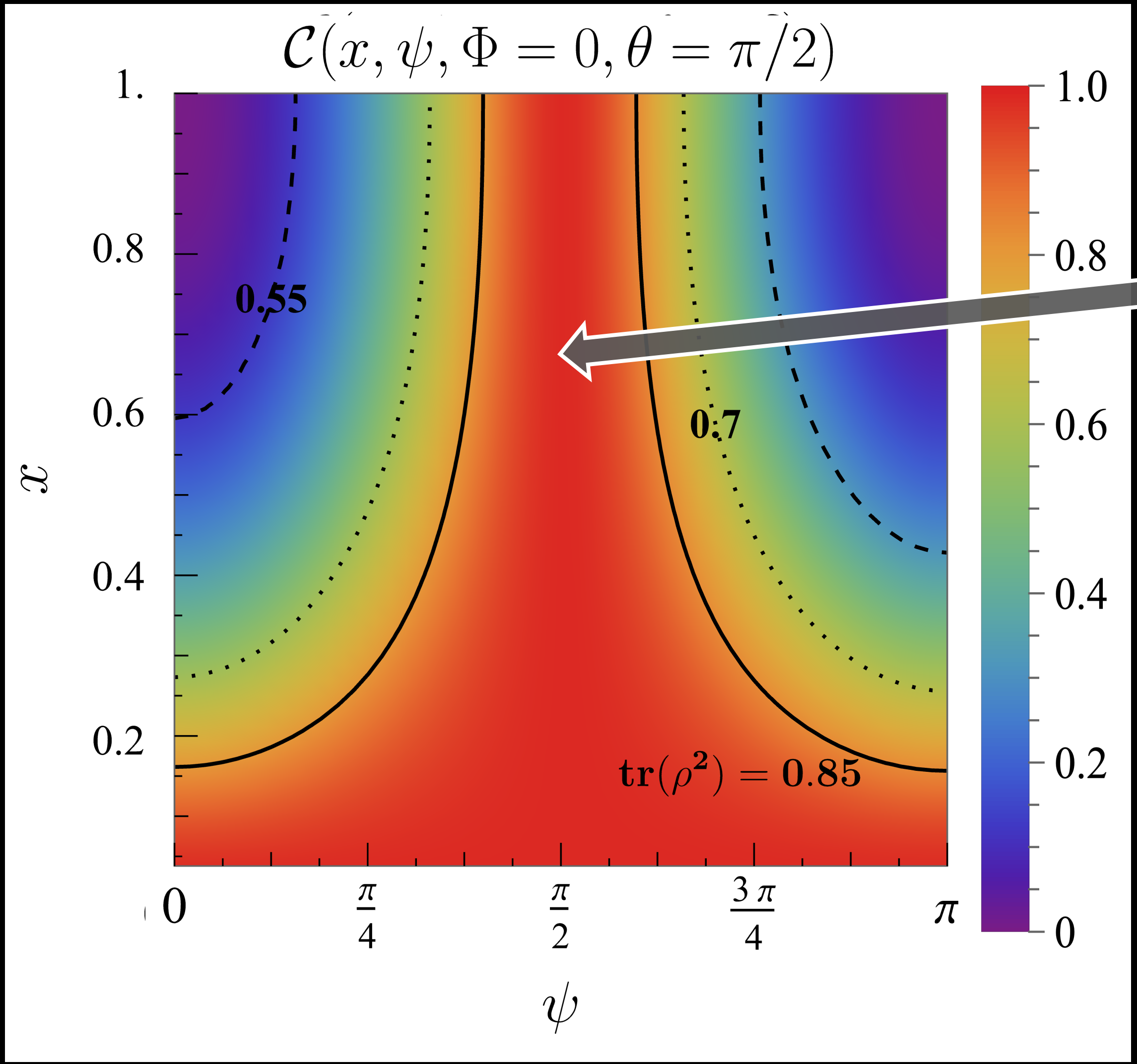

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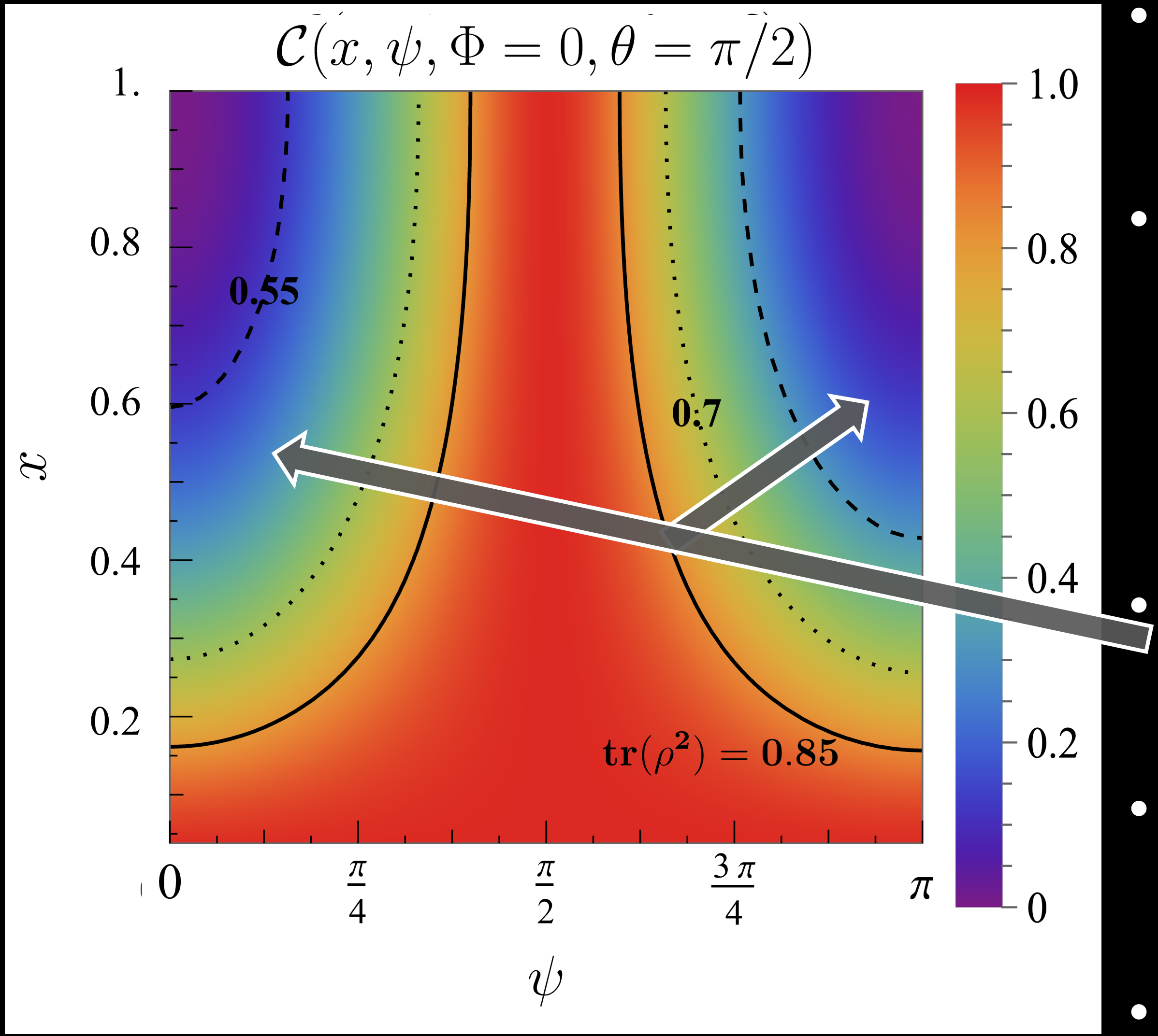

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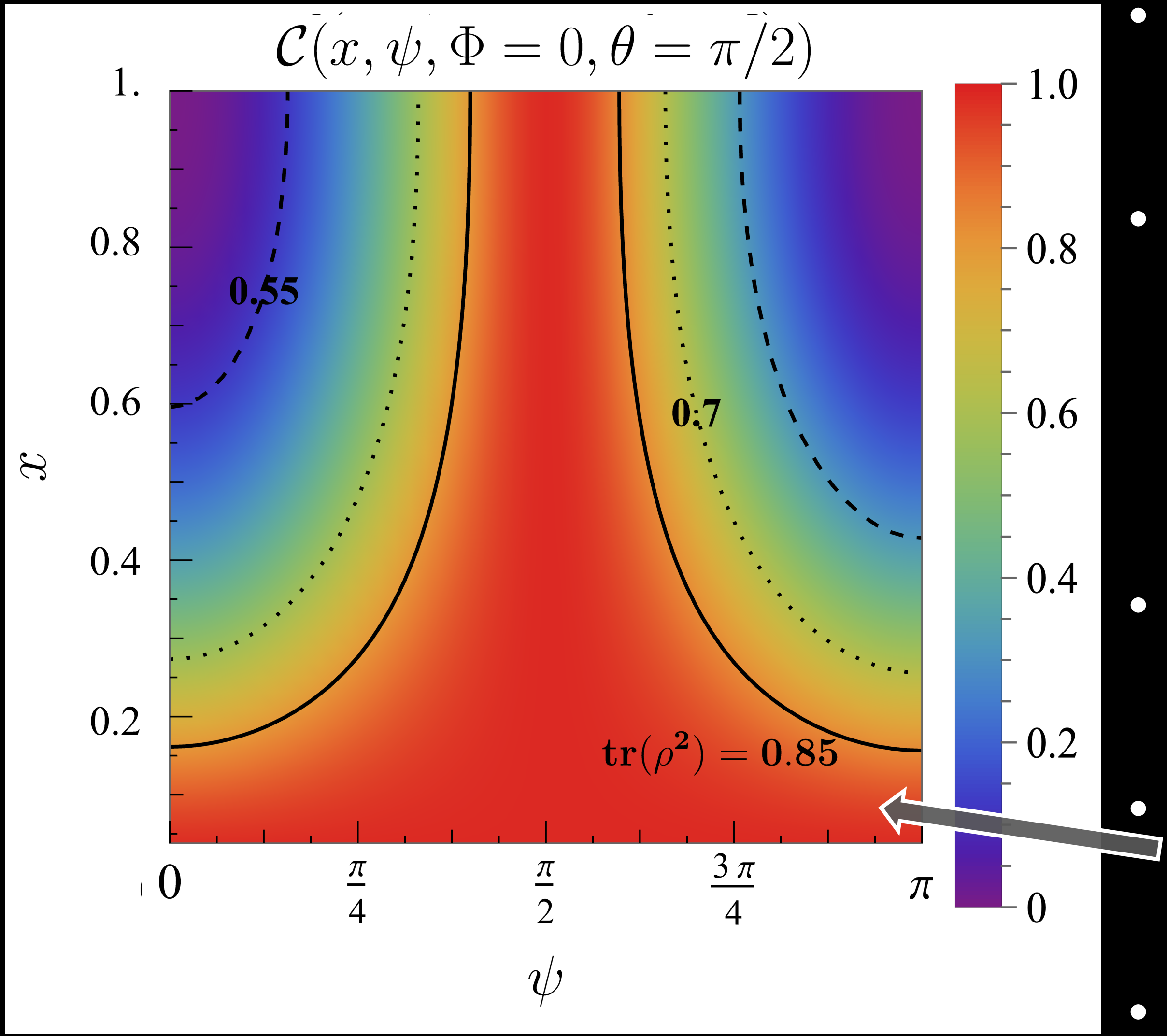

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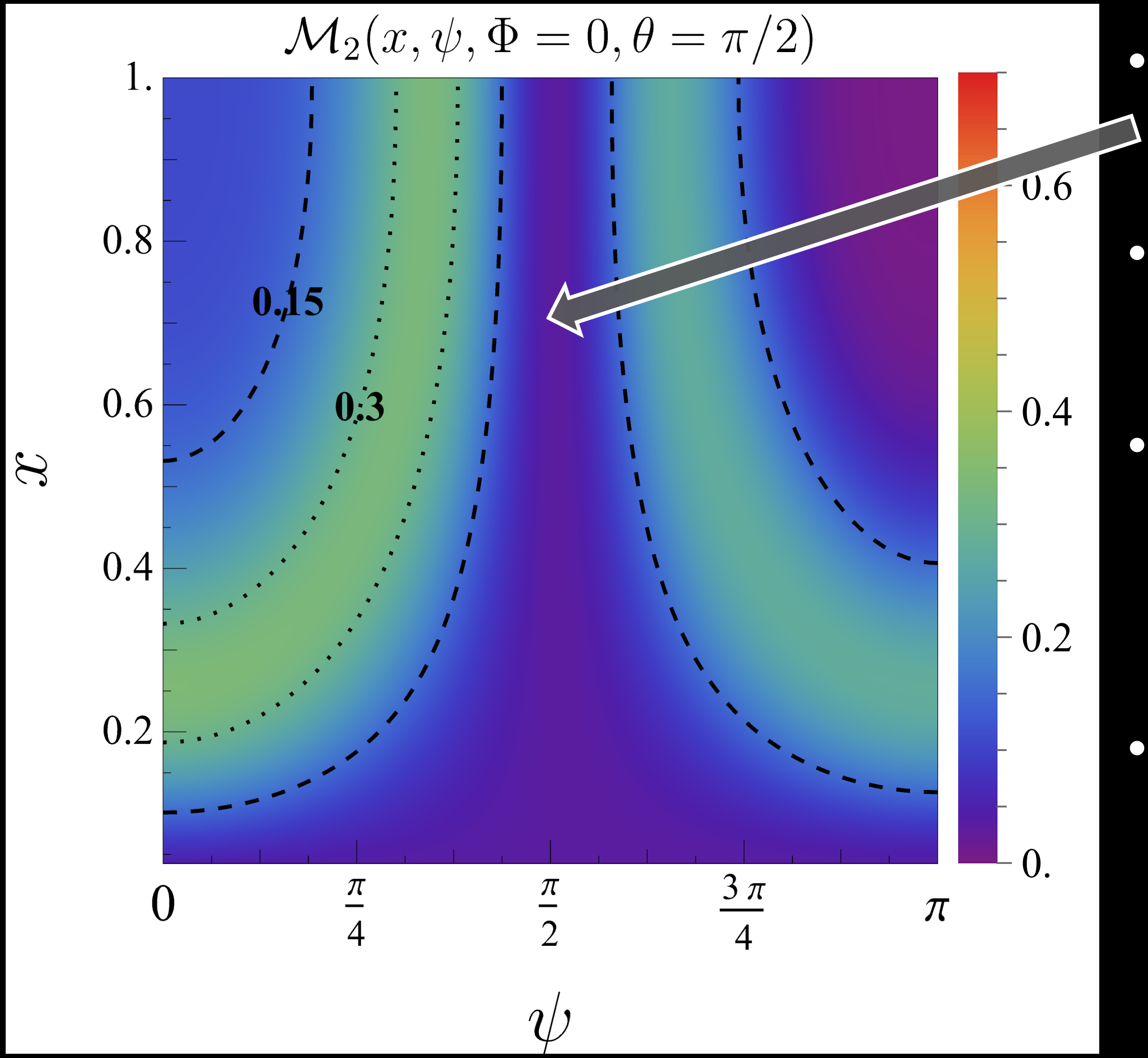

- Central scattering: No state change due to symmetry protection
- Forward & backward asymmetry from chiral coupling
- Backward: e^- & τ^- opposite helicity
  - $|\uparrow\uparrow\rangle$ & $|\downarrow\downarrow\rangle$ have same amplitude $g_L g_R$
  - Stabilizer state but separable
- Forward: e^- & τ^- same helicity
  - $|\uparrow\uparrow\rangle$ & $|\downarrow\downarrow\rangle$ slightly different amplitude $g_L^2 \sim g_R^2$
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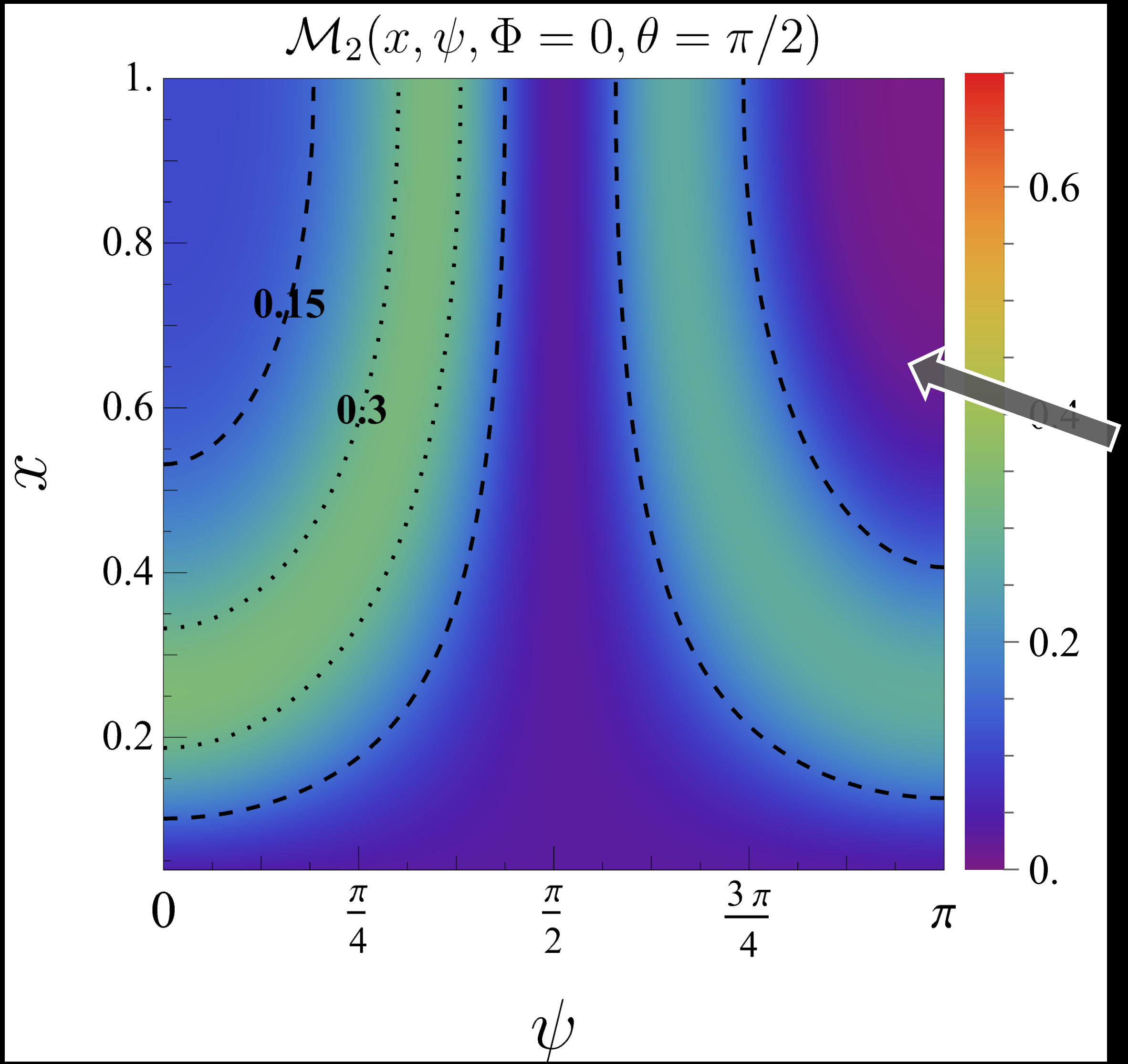

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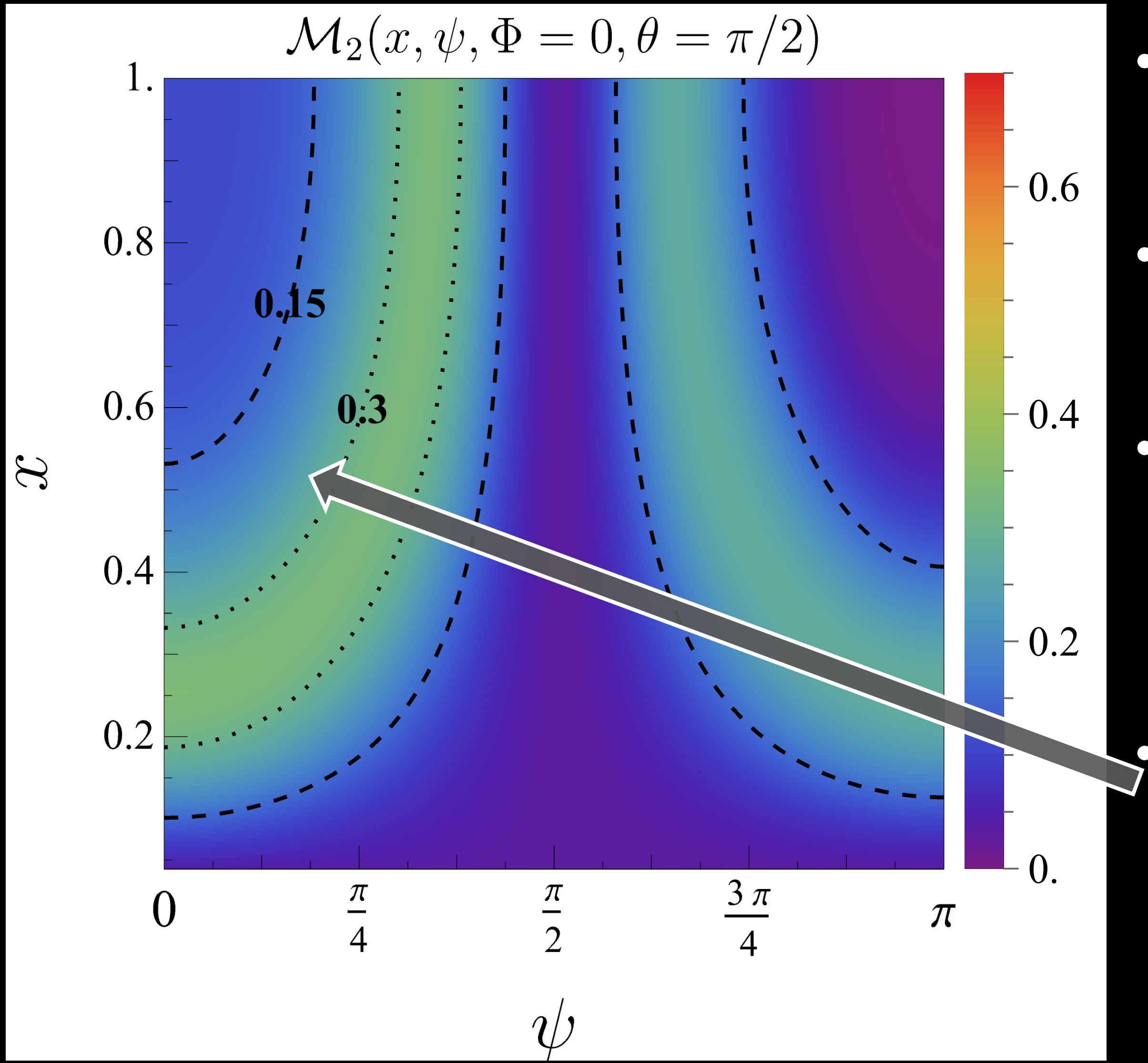

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Conclusions

- Decoherence is an expected phenomena when system interacts with environment
- However, the effect of decoherence on bipartite system is largely dependent on kinematics
  I. Close-to-collinear: Expected decoherence effect
  II. Perpendicular: Kinematic setup + axial vector coupling provides symmetry protecting spin structure and entanglement
  III. Threshold: Mass effect prevalent leading to entanglement regardless of scattering angle
- Questions?

Backup Slides

Purity

- Typically, states are pure: $\rho = |\psi\rangle\langle\psi|$
- Tracing over photon polarization mixes up states potentially giving mixed states $\rho = \sum p_i |\psi_i\rangle\langle\psi_i|$
- Define purity as $\mathrm{tr}(\rho^2)$
- Typically ranges between $\frac{1}{d} \leq \mathrm{tr}(\rho^2) \leq 1$
- Helicity flip terms are suppressed by small lepton mass realistic range is

$$\frac{1}{2} \lesssim \mathrm{tr}(\rho^2) \leq 1$$

Magic

- Define Pauli string: $\mathcal{P}_2 = \{\mathbb{I}_2 \otimes \mathbb{I}_2, \sigma_i \otimes \mathbb{I}_2, \mathbb{I}_2 \otimes \sigma_j, \sigma_i \otimes \sigma_j\}$
- Eigenstates = Stabilizer States
  - Easily simulated on classical computer: Bell and separable states
- Distance of state relative to these stabilizer states: **Magic**
- Decompose $\rho = \frac{1}{4}\left(\mathbb{I}_4 + B_i^- \sigma_i \otimes \mathbb{I}_2 + B_j^+ \mathbb{I}_2 \otimes \sigma_j + C_{ij} \sigma_i \otimes \sigma_j\right)$
- second Stabilizer Rényi entropy to find Magic

$$\mathcal{M}_2 = -\log_2\left(\frac{1 + \sum_i (B_i^-)^4 + \sum_j \left(B_j^-\right)^4 + \sum_{ij} \left(C_{ij}\right)^4}{1 + \sum_i \left(B_i^-\right)^2 + \sum_j \left(B_j^-\right)^2 + \sum_{ij} \left(C_{ij}\right)^2}\right)$$

Coplanar

- Sum helicities for squared amplitude and integrate out θ, Θ