

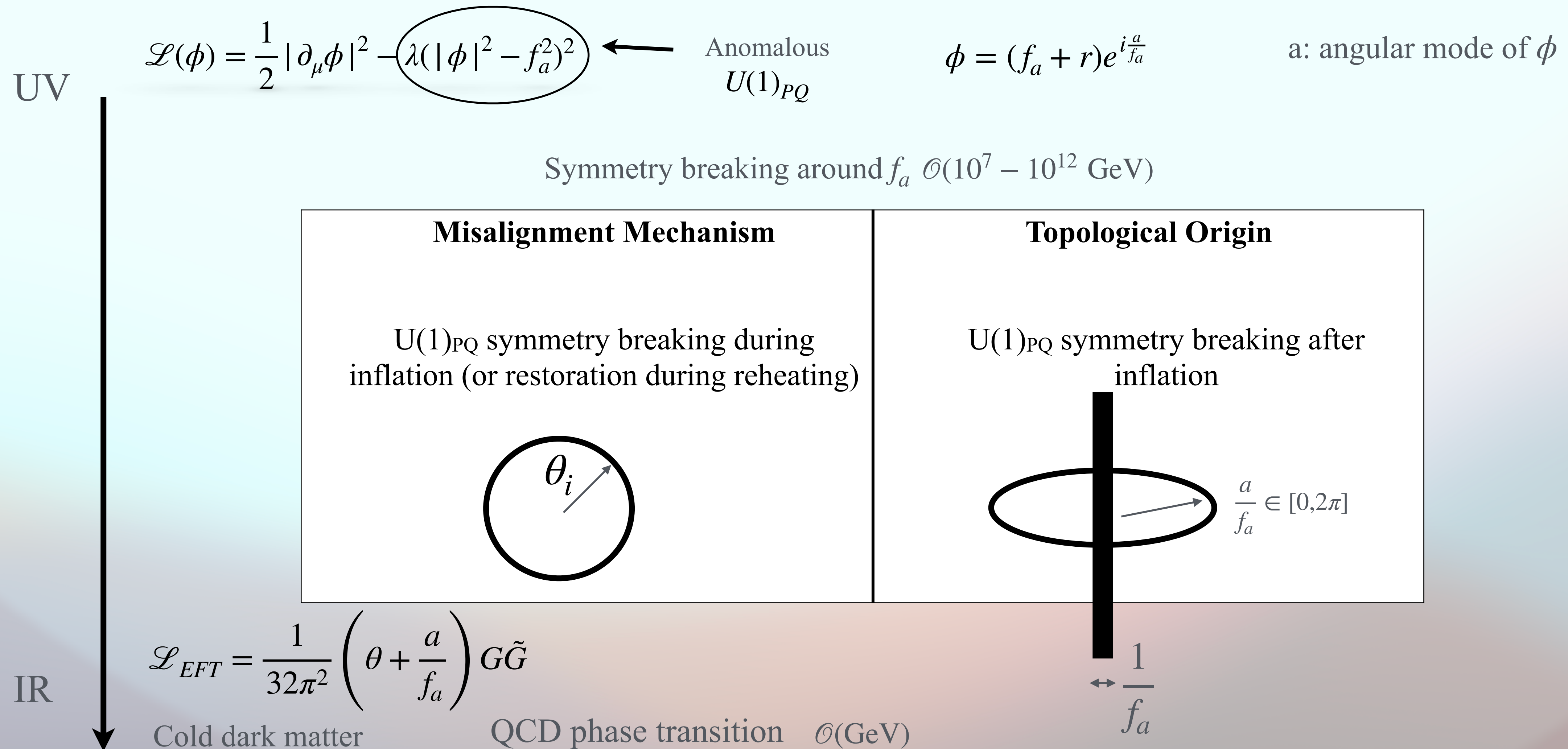


what happens when axion strings interact with the standard model plasma?

(Clue: *axions gain weight*)

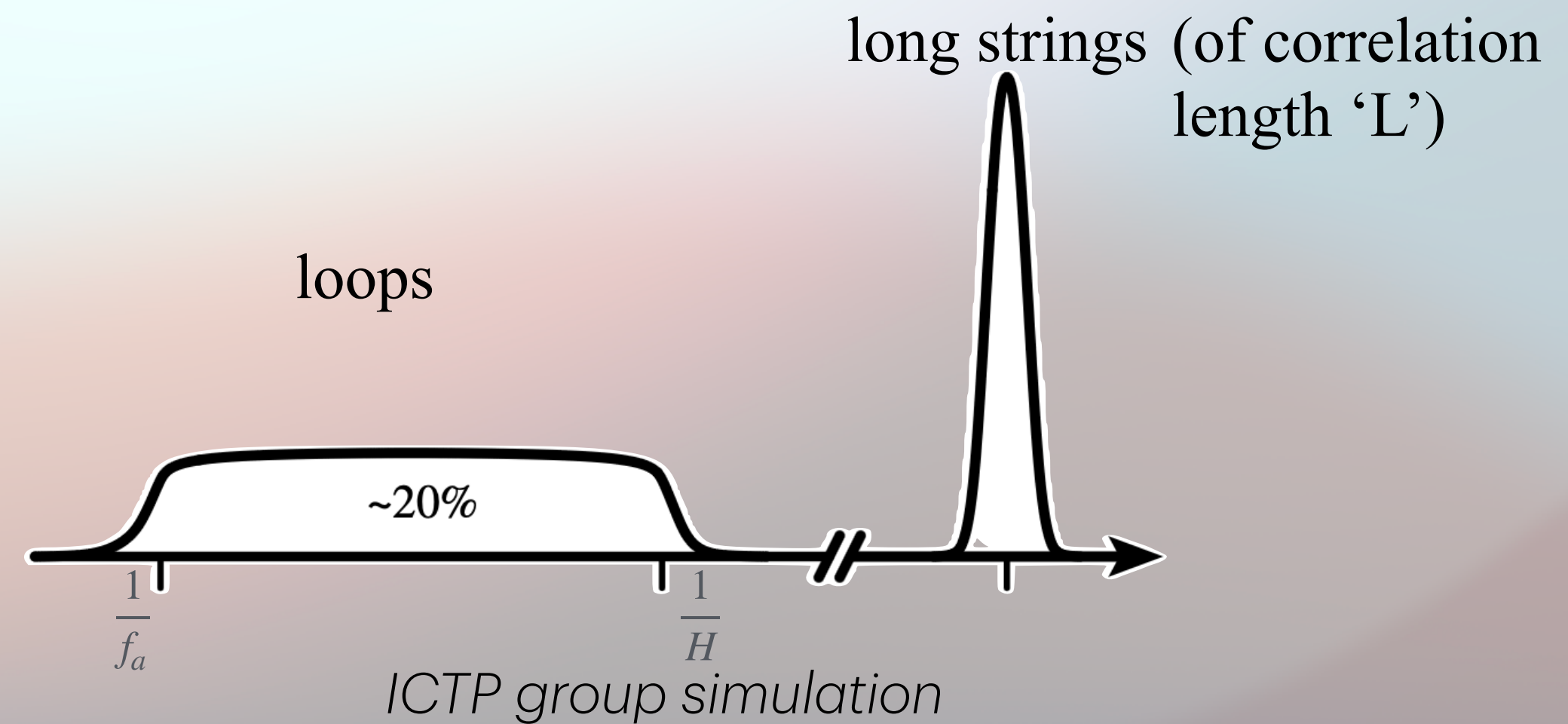
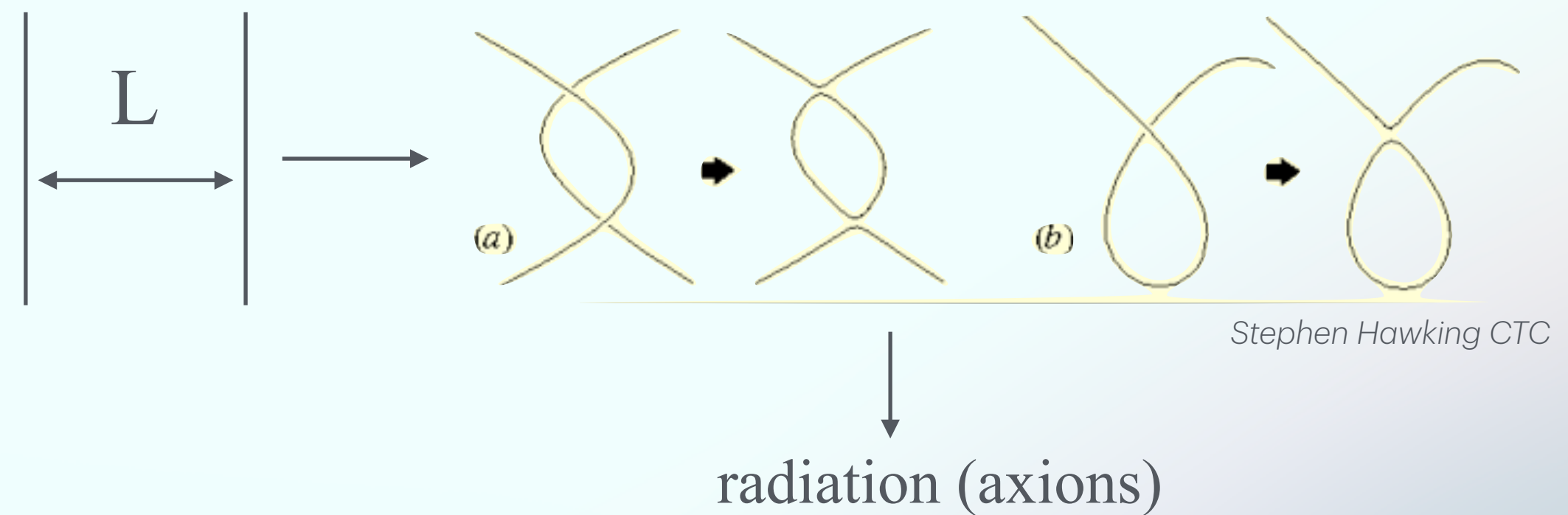
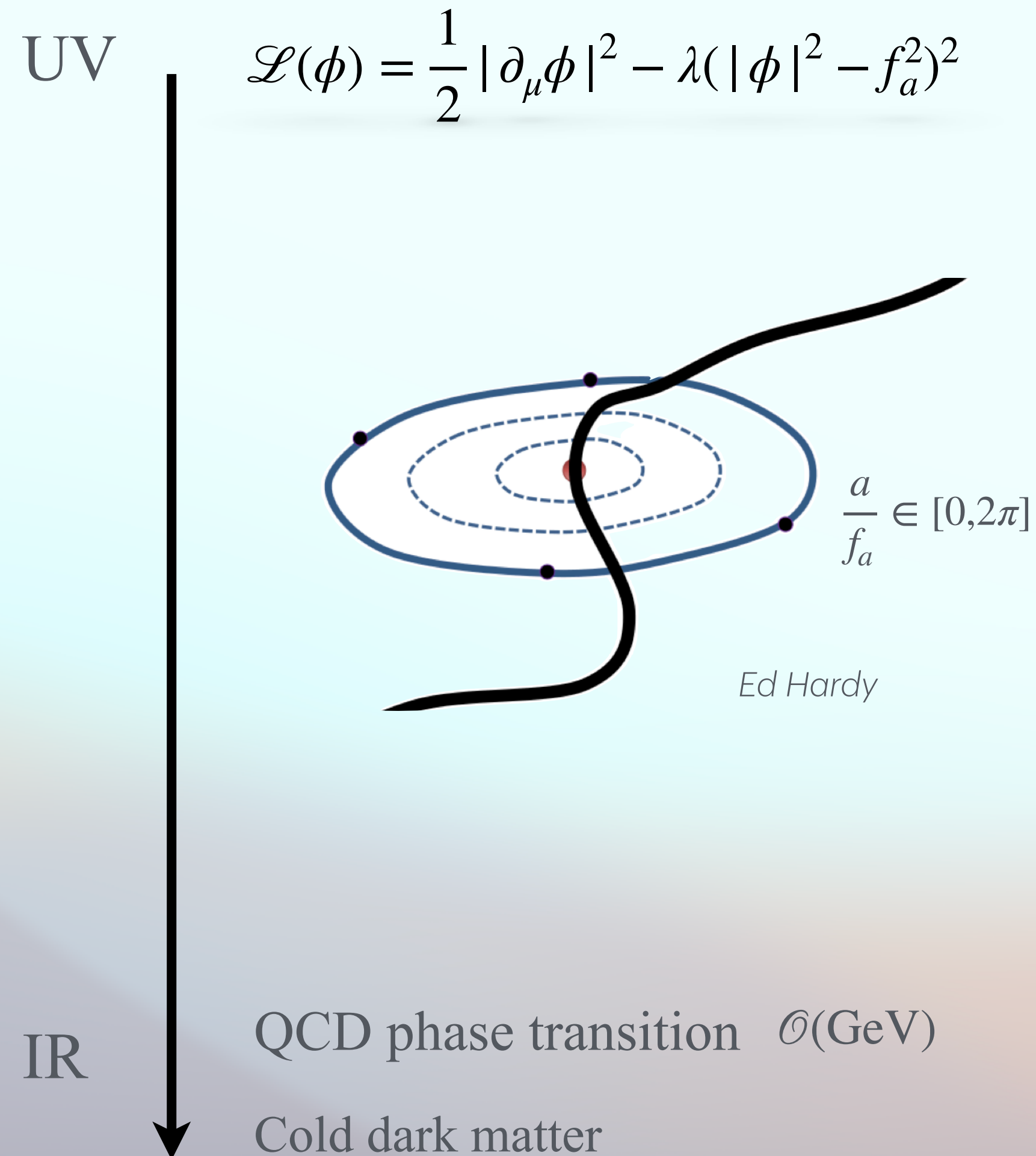
1. Introduction to Axion Strings and Friction
2. Finding n_a : A Semi-Analytic Model with Simulation Inputs
3. Conclusion

giving birth to axions



universe of (QCD) axion strings (and loops)

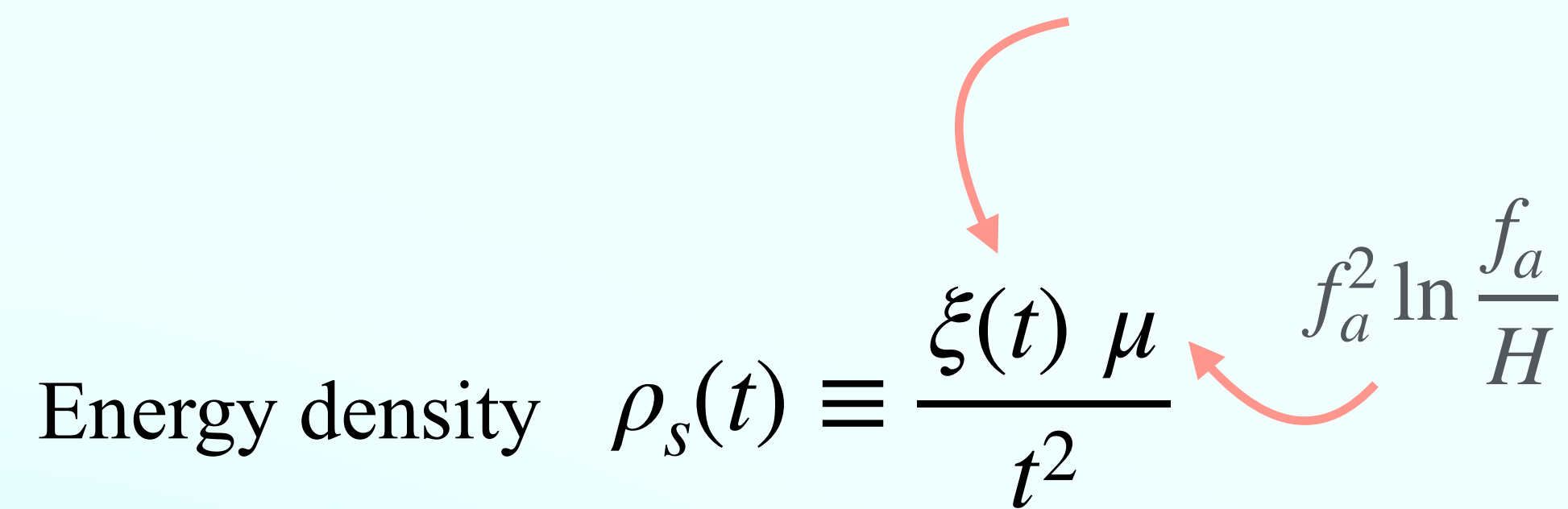
Symmetry breaking AFTER inflation



why is friction (with SM particles) important?

number of strings per
Hubble volume

Energy density $\rho_s(t) \equiv \frac{\xi(t) \mu}{t^2}$



After formation, *intuitively*,

$$\xi(t) \equiv \left(\frac{t}{L}\right)^2 \propto t$$

From simulations,

$$\xi(t) = 0.2 \ln\left(\frac{f_a}{H}\right) \propto \ln t$$

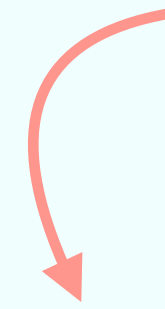
Hence, intercommutation ($\propto \xi(t)$) is unexpected

Higher string tension μ ($\propto f_a^2$) slows down loop formation for higher f_a values.

Observed dark matter abundance for $f_a \sim 10^{12}$ GeV

why is friction (with SM particles) important?

number of strings per
Hubble volume



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Higher string tension ($\propto f_a^2$) slows down loop formation for higher f_a values.

For lower f_a values, something similar to string tension is required, which decreases interaction

Friction does the job! DM relic abundance satisfied with $f_a \sim 10^8$ GeV

(leading) sources of friction

Spin 0

$$\mathcal{L}_{int} = -\lambda \delta^2(\vec{x}) \phi^\dagger \phi$$

$$\frac{d\sigma}{dz} \approx \frac{\lambda^2}{8\pi q}$$

Everett, Vilenkin

Spin $\frac{1}{2}$

$$\mathcal{L}_{int} = c_{af} \frac{\partial_\mu a}{2f_a} \bar{\psi} \gamma^\mu \gamma_5 \psi$$

$$\left(i\gamma^\mu \partial_\mu - \frac{\partial_\mu a \gamma^\mu \gamma_5}{2f_a} \right) \psi = 0$$

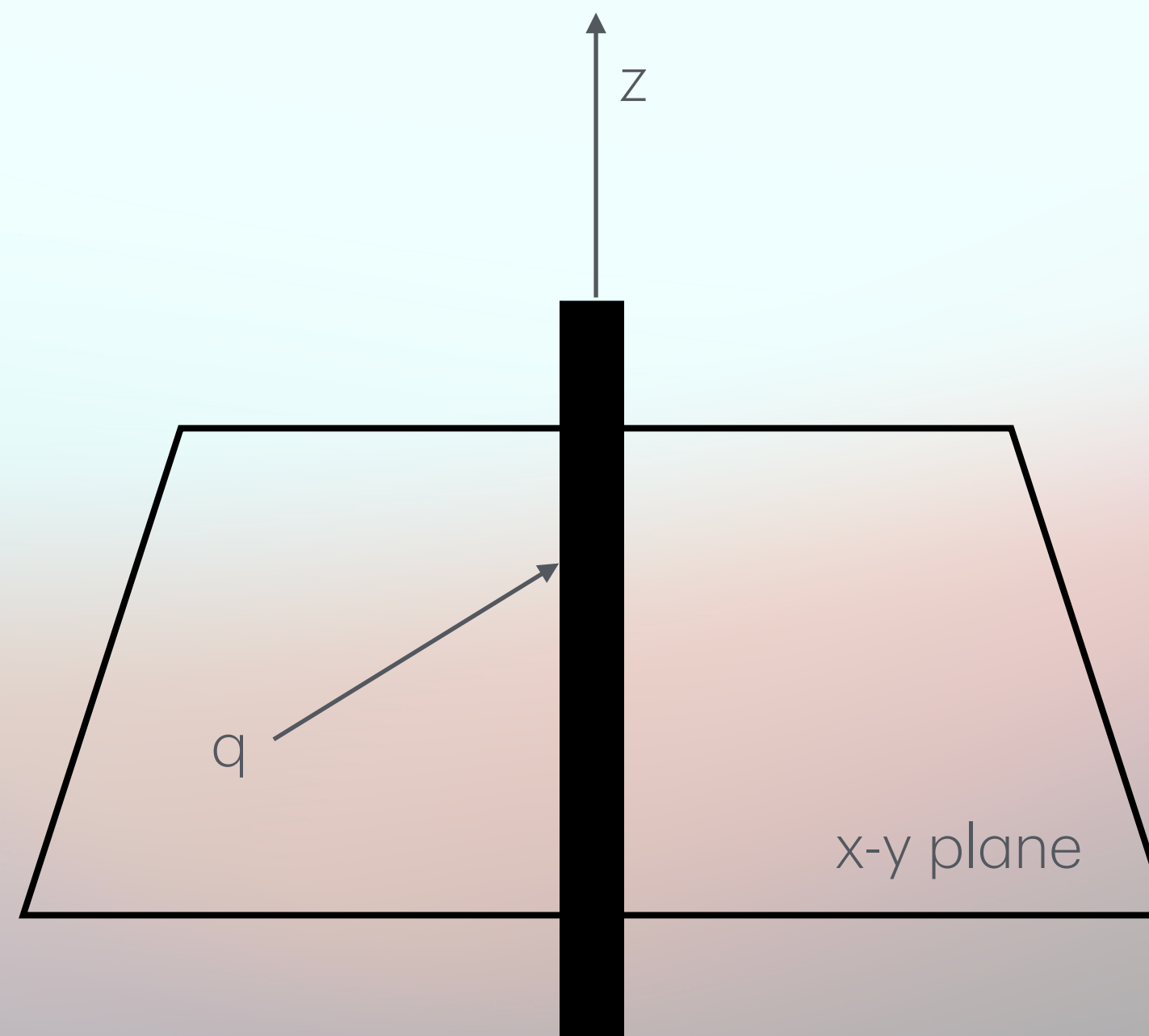
$$\frac{d\sigma}{dz} = \frac{2 \sin^2\left(\frac{\pi c_{af}}{2}\right)}{q}$$

Spin 1

$$\mathcal{L}_{int} = \alpha_{EM} \delta^2(\vec{x}) A_\mu A^\mu$$

$$\frac{d\sigma}{dz} \approx \frac{\alpha_{EM}^2}{q}$$

PA, Anson Hook, JH and GMT

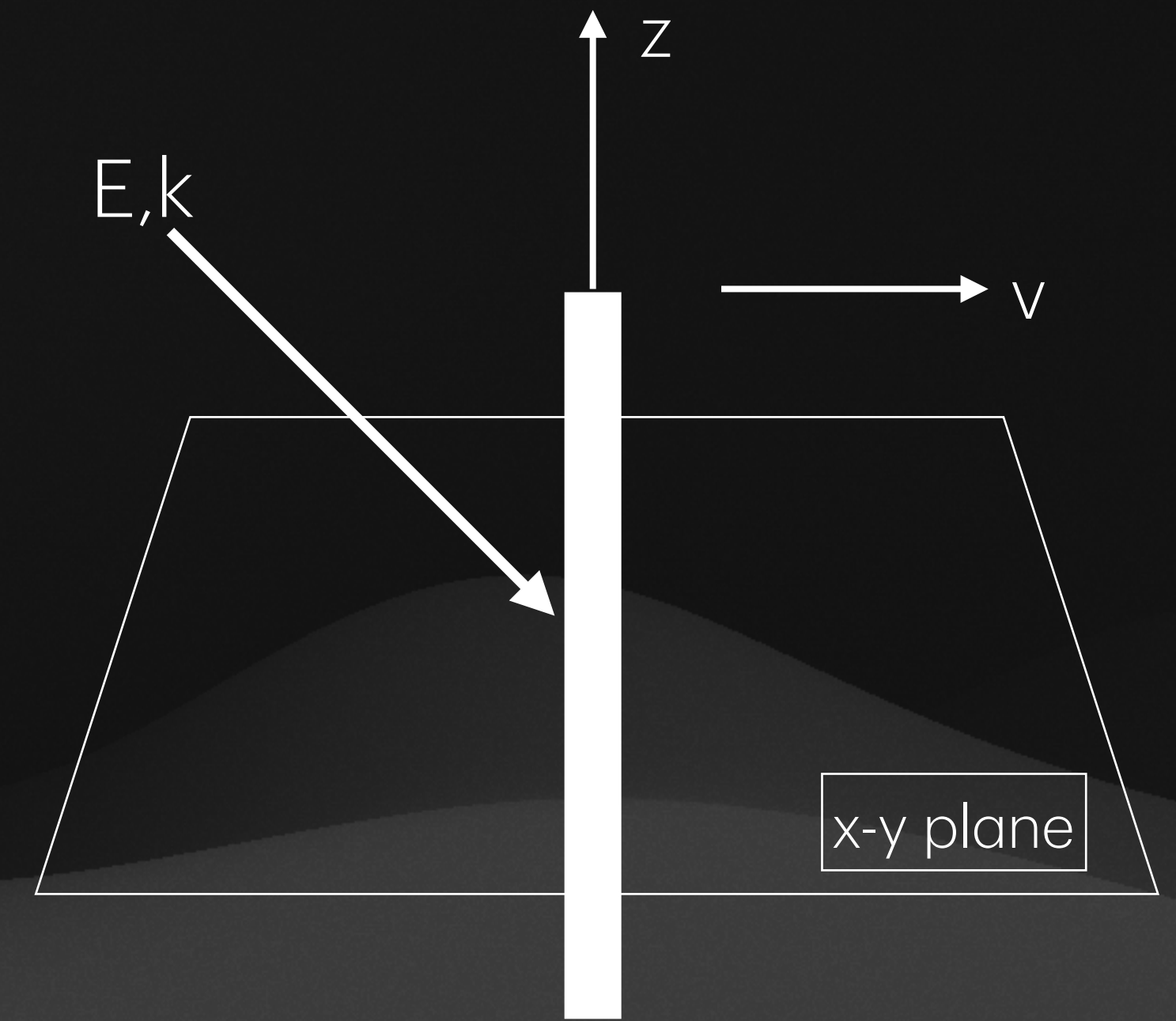


calculating friction...

$$n_k = n \left(\frac{\gamma E + \gamma \vec{v} \cdot \vec{k}}{T} \right)$$

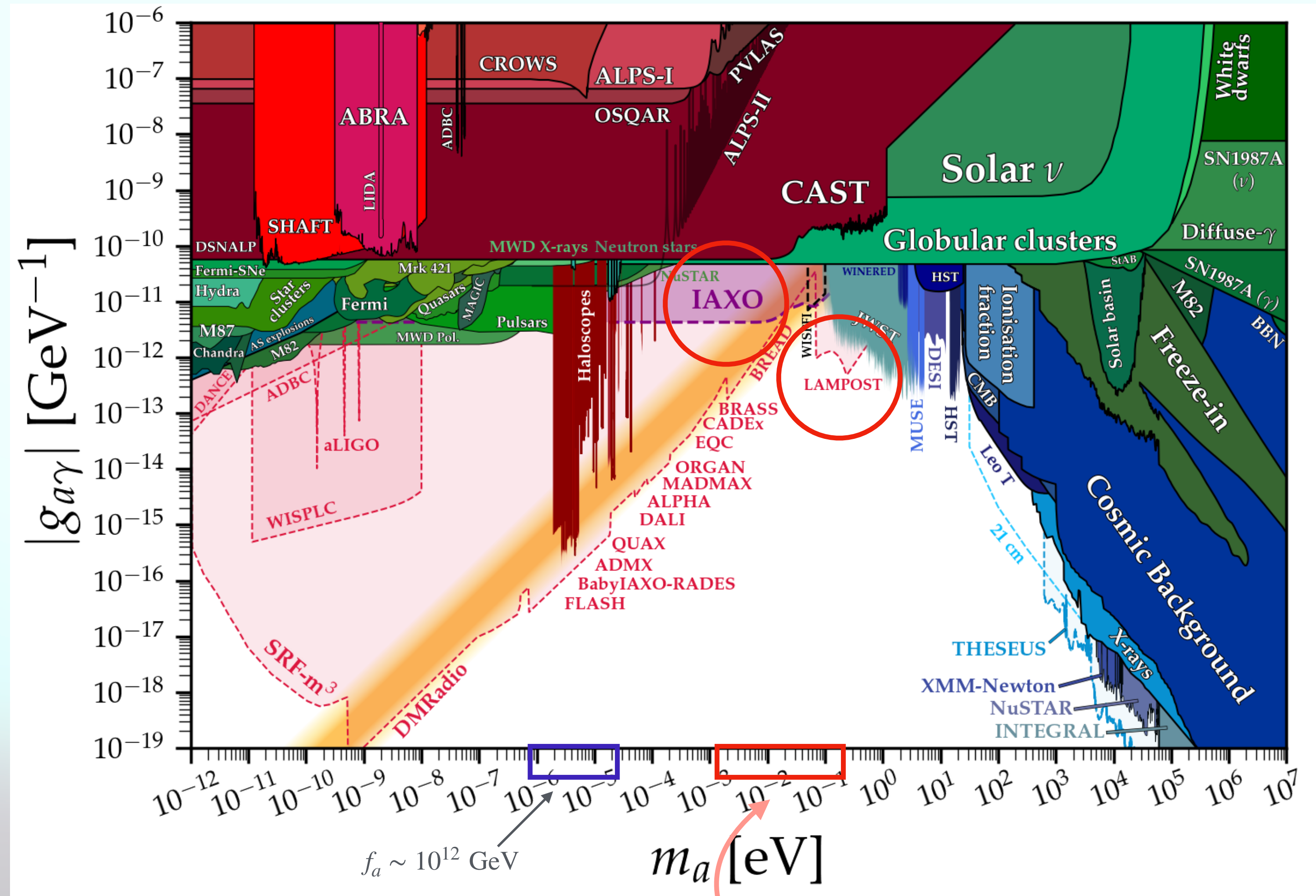
$$\frac{F}{l} = \text{Incident Flux} \times \frac{d\sigma}{d\theta dz} \times \text{momentum change}$$

$$= \int_0^\infty \frac{d^3k}{(2\pi)^3} n_k \frac{q}{|k|} \int_0^{2\pi} d\theta \frac{d\sigma}{d\theta dz} k(1 - \cos \theta)$$



$$\frac{F}{l} = \begin{cases} -\frac{\lambda^2 \zeta(3)}{2\pi^2} v \gamma T^3 & \text{(scalar)} \\ -\beta_{AB} v \gamma T^3 & \left[\beta_{AB} = \frac{3}{2\pi^2} \zeta(3) \sin^2\left(\frac{\pi c_{af}}{2}\right) \right] \text{(fermion)} \\ -2\alpha_{EM}^2 v \gamma T^3 & \text{(photon)} \end{cases}$$

experimental motivation

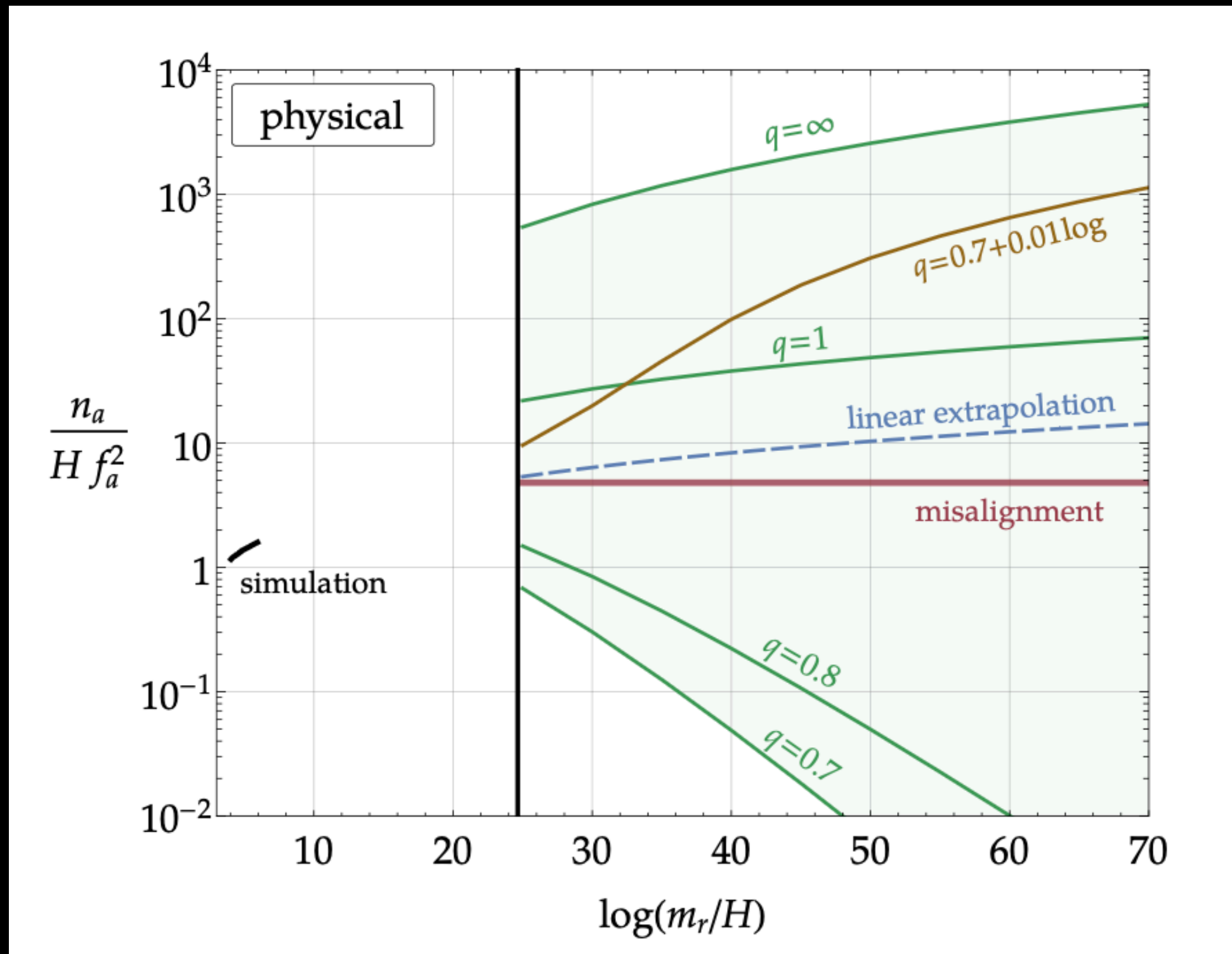


cajohare.github.io/AxionLimits/

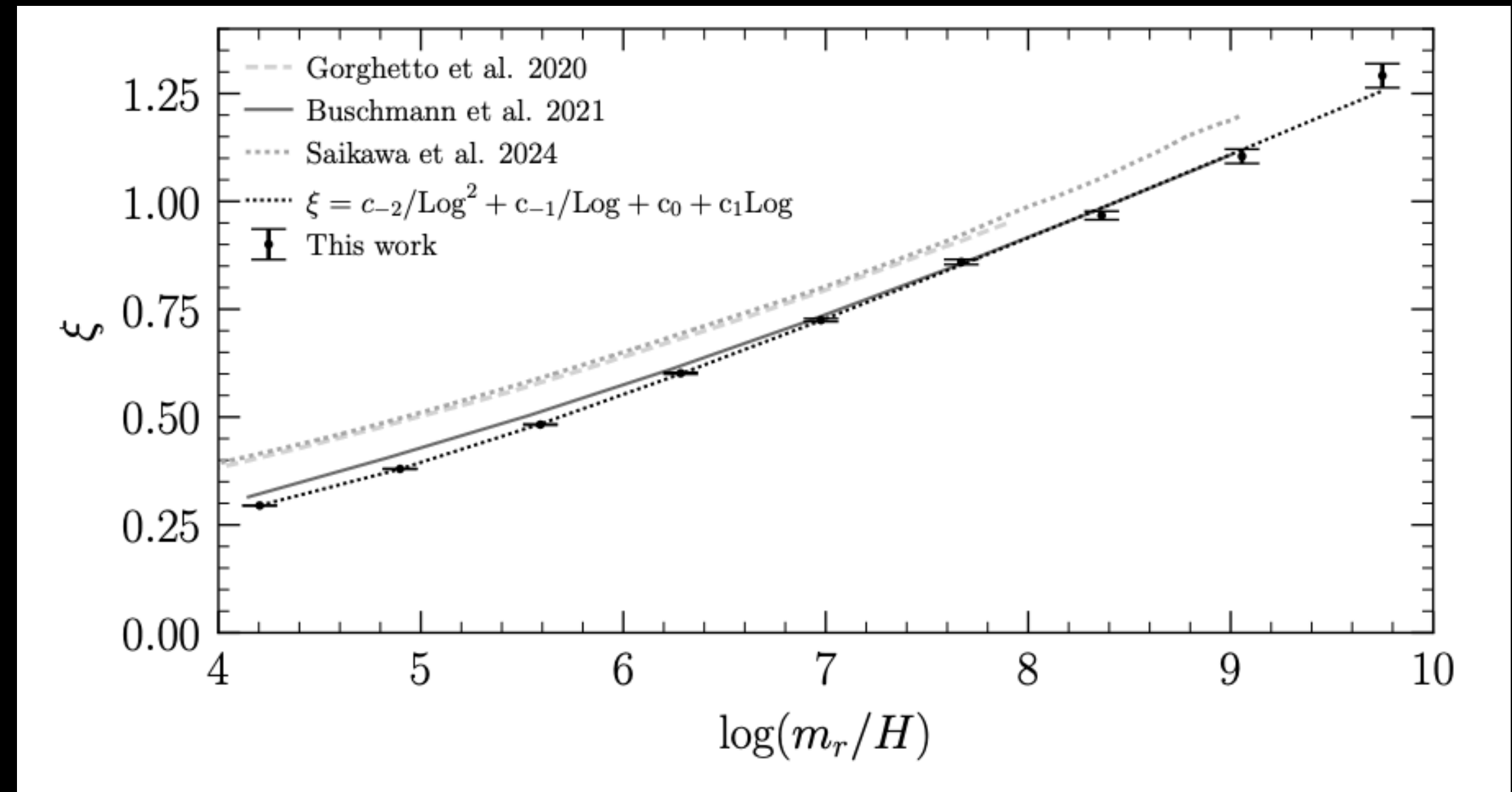
$$10^7 \text{ GeV} \leq f_a \leq 10^9 \text{ GeV}$$

but numerics is HARD!

(Previous Work)



ICTP group



Berkeley group

the Velocity One Scale (VOS) model

$$2\frac{dL}{dt} = 2HL(1 + v^2) + cv + \frac{Lv^2}{\ell_f}$$

$$\frac{dv}{dt} = (1 - v^2) \left[\frac{k}{L} - v \left(2H + \frac{1}{\ell_f} \right) \right]$$

$$\ell_f = \frac{\mu}{\beta T^3}$$

$$\theta = \frac{\beta}{\sqrt{f}} \left(\frac{t_c}{t_{pl}} \right)^{\frac{1}{2}}$$

c: loop chopping efficiency

k: momentum parameter

$$c = k = 1$$

Stretching

$$L \propto t^{\frac{1}{2}}$$

$$v = \frac{\ell_f}{L}$$

$$t_c \propto \frac{1}{f_a^2}$$

time



Kibble (friction)

$$L = \sqrt{\frac{3}{2\theta(1+c)}} \frac{t^{\frac{5}{4}}}{t_c^{\frac{1}{2}}} \sqrt{\ln N}$$

$$v = \sqrt{\frac{2(1+c)}{3\theta}} \left(\frac{t}{t_c} \right)^{\frac{1}{4}} \sqrt{\ln N}$$

$$t_K \propto t_c^{\frac{4}{3}}$$

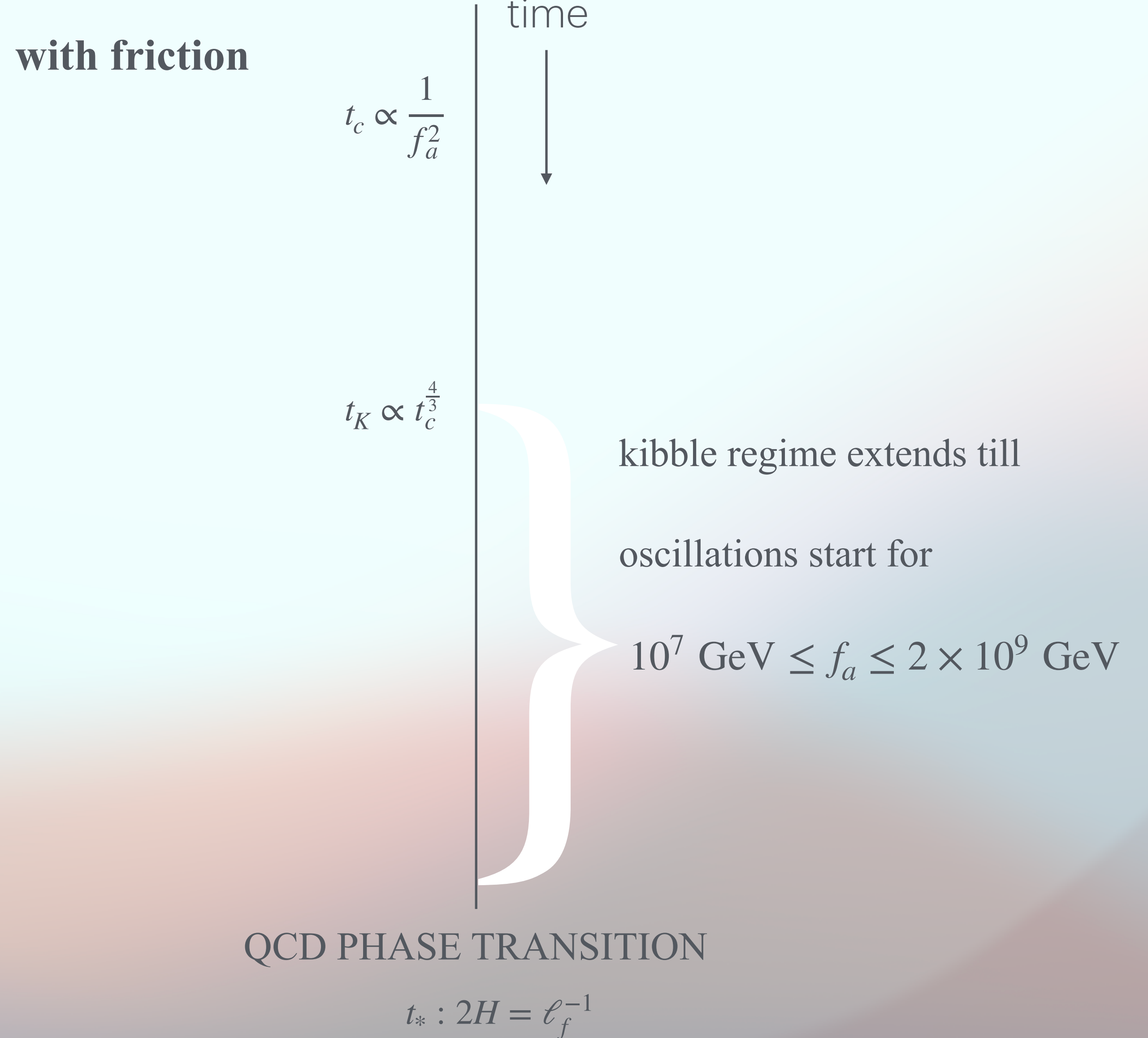
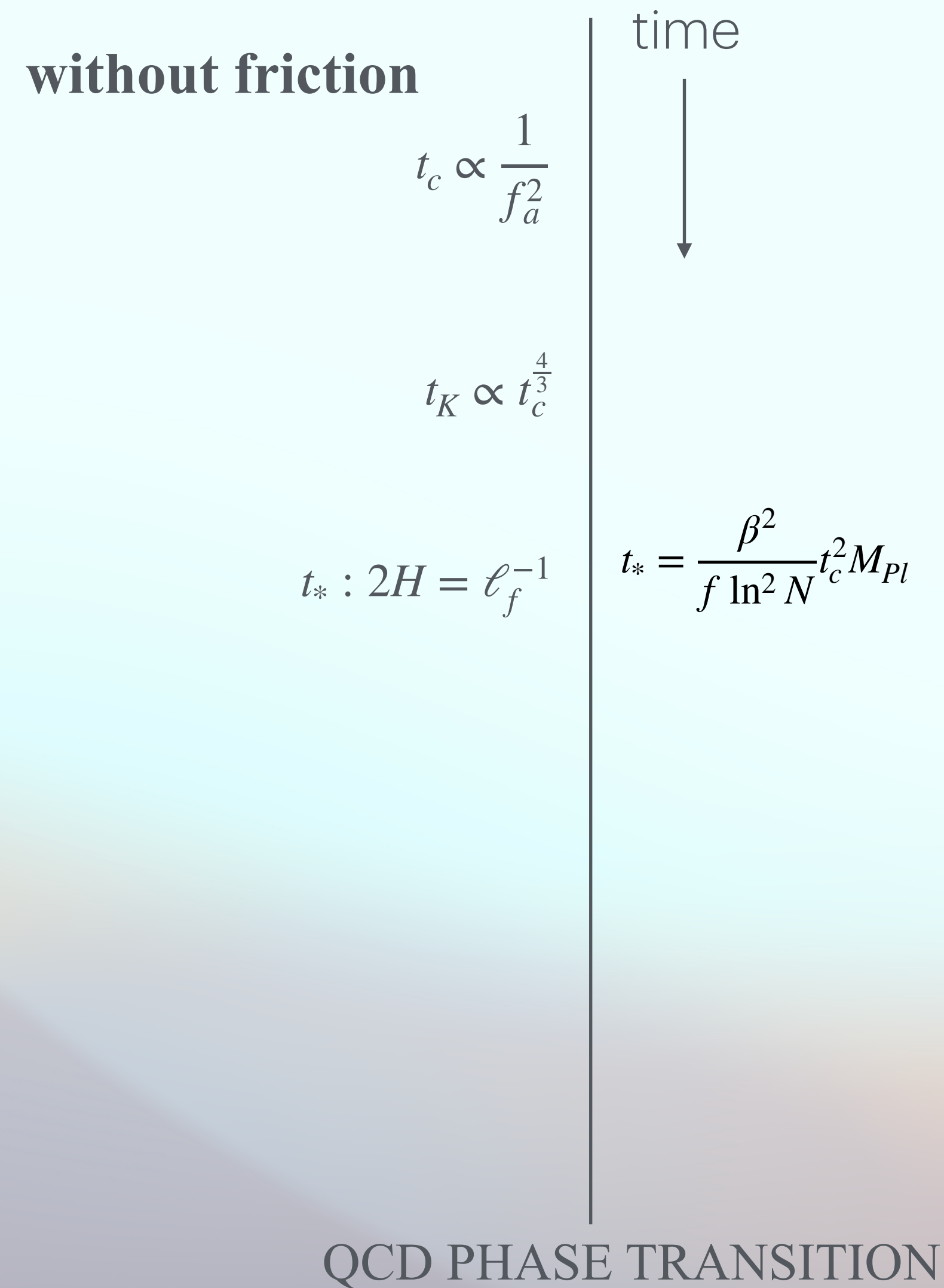
Scaling

$$L = \sqrt{k(k+c)}t$$

$$v = \sqrt{\frac{k}{k+c}}$$

$$t_* : 2H = \ell_f^{-1}$$

importance of friction



calculating number density

$$n(t_0) = \int_{t_K}^{t_0} d\tilde{t} \left(\frac{dn}{d\tilde{t}} \right) \left[\frac{a(\tilde{t})}{a(t_0)} \right]^3$$

Inputs :

Rescale ρ_s :

$$\rho_s(t) = \frac{\frac{4}{5} \rho_{\text{num}}^{\text{scaling}}(t)}{\rho_{\text{VOS}}^{\text{scaling}}(t)} \rho_{\text{VOS}}^K(t)$$

Set the length scale:

$$\ell_{\text{crit}}(t) = \min \left\{ \alpha \ell_f(t), \frac{\beta}{x_{\text{IR}}} L(t) \right\}$$

method: log-uniform distribution

simulation input

$$k_{\text{min}}(t) = \max \left\{ (\alpha \ell_f(t))^{-1}, \left(\frac{\beta}{x_{\text{IR}}} L(t) \right)^{-1} \right\}$$

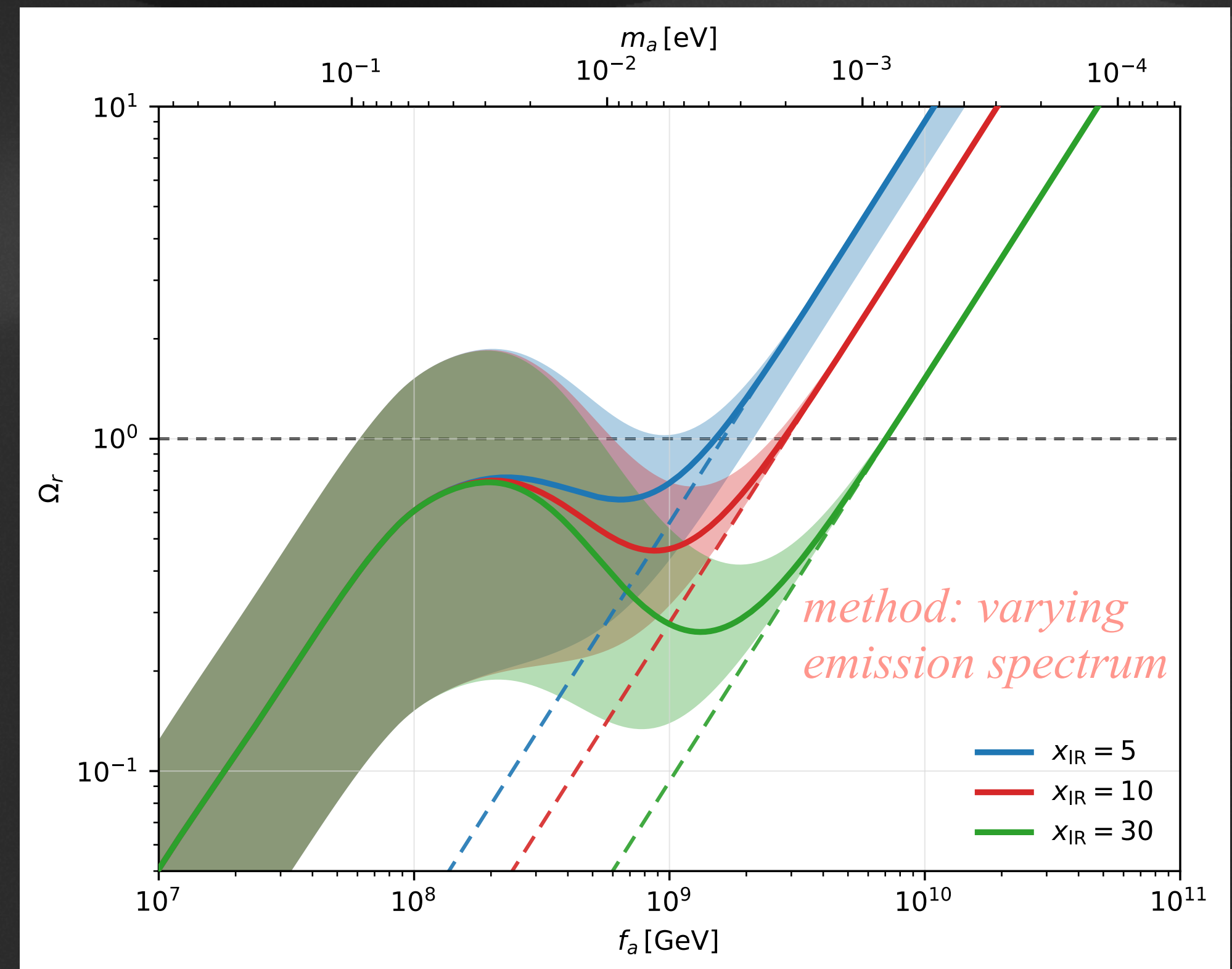
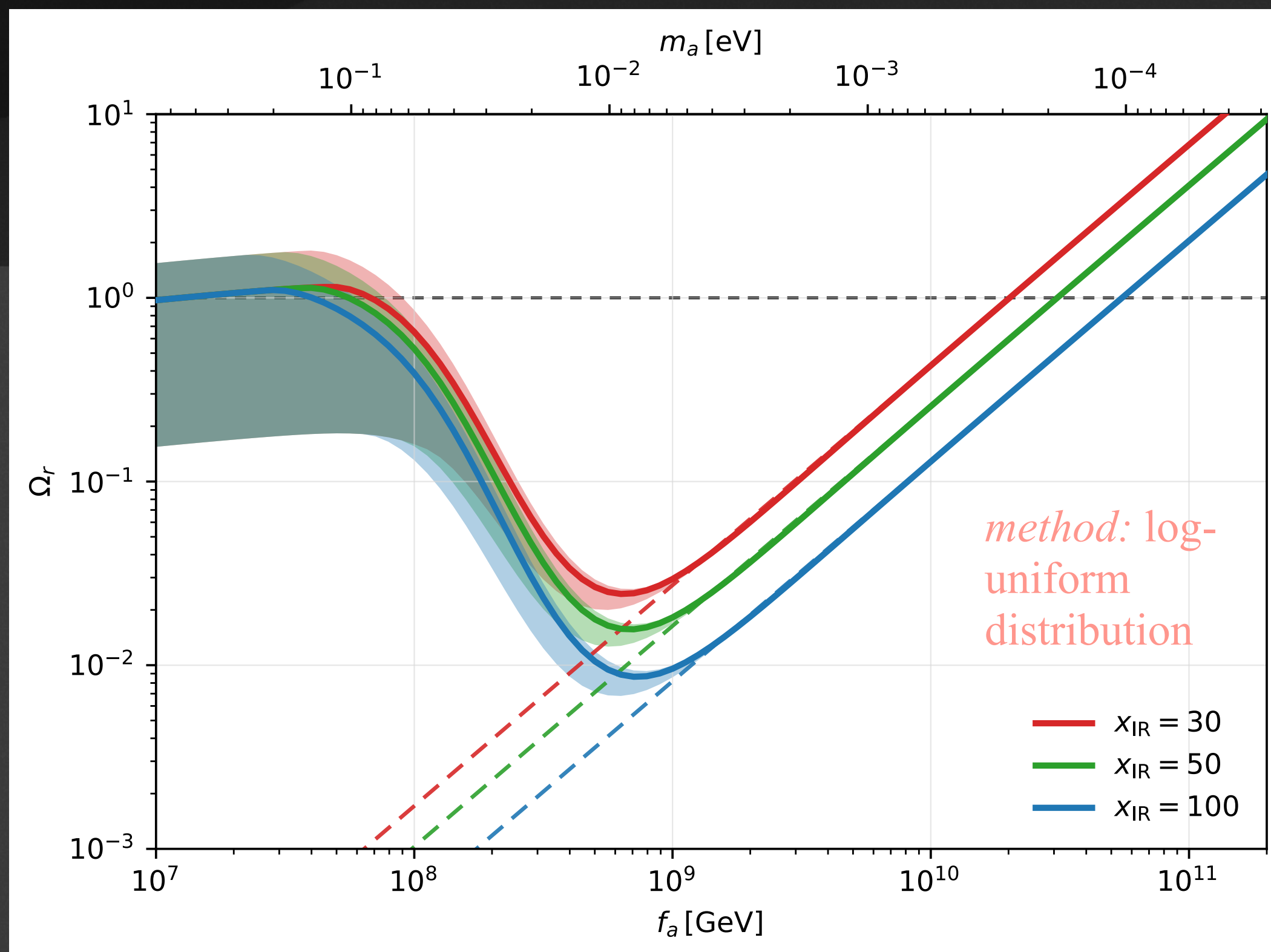
method: varying emission spectrum

calculating number density

$$\Omega_r(t_{today}) = \frac{n(t_{today})}{n_{DM}^{req}(t_{today})}$$

$$n(t_0) = 0.2 \frac{\Gamma_a f_a^2}{2\pi} \frac{1}{t_0^{3/2}} \int_{t_K}^{t_0} d\tilde{t} \frac{\xi^{3/2}(\tilde{t})}{\tilde{t}^{3/2}} l_{crit}(\tilde{t})$$

$$n_a(t_0) = \int_{t_K}^{t_0} d\tilde{t} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^3 \int_{\max[\frac{1}{HL}, \frac{1}{H\ell_f}]}^{f_a} d\left(\frac{k}{H}\right) \frac{1}{k} \frac{d\rho_a}{d \ln k}$$



Non-Linear Effects

t_0

t_l



$$H = m_a$$

$$\rho(t_0) \gg m_a^2(t_0)f_a^2$$

(even for soft modes)

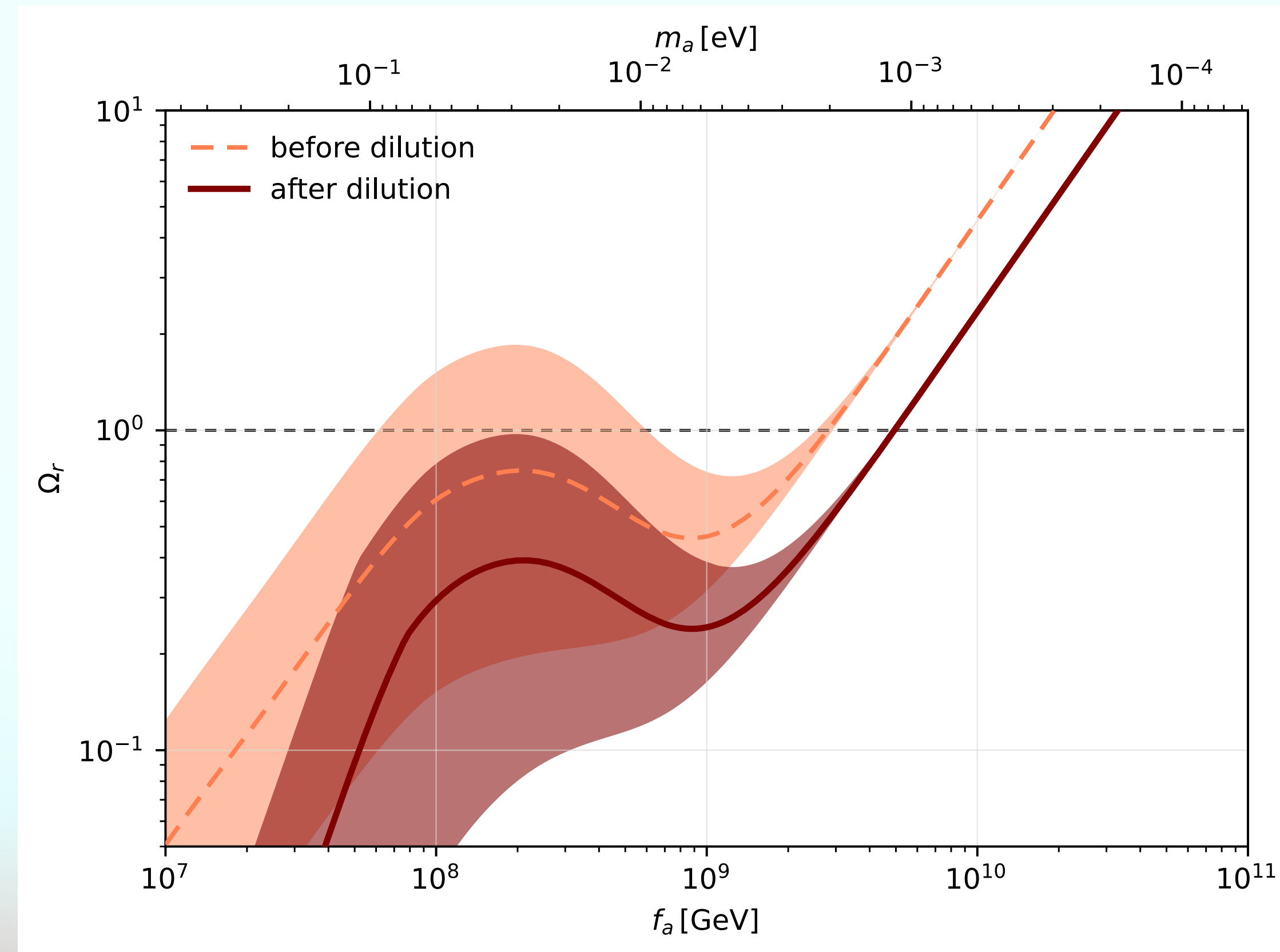
$$\frac{\partial \rho_a}{\partial k}(t, k) = \left[\frac{a(t_{\text{osc}})}{a(t)} \right]^3 \frac{\partial \rho_a}{\partial k}(t_{\text{osc}}, k_{\text{osc}})$$

$$\rho_{\text{IR}}(t_l) \simeq m_a^2(t_l)f_a^2$$

$$m_a(t) \approx m_a(t_0) \left(\frac{t}{t_0} \right)^2$$

$$\Omega_r \equiv \frac{\rho_a}{\rho_{\text{DM}}} = \frac{m_a^0 n_a(t_\ell)}{\rho_{c,0} \Omega_c} \left[\frac{a(t_\ell)}{a_0} \right]^3$$

dilution (varying emission spectrum)





[arXiv:2603.00237](https://arxiv.org/abs/2603.00237)

calculating number density

$$n(t_0) = \int_{t_K}^{t_0} d\tilde{t} \left(\frac{dn}{d\tilde{t}} \right) \left[\frac{a(\tilde{t})}{a(t_0)} \right]^3$$

$$\frac{dn(\tilde{t})}{d\tilde{t}} = \int_0^{\ell_{\text{crit}}(\tilde{t})} d\ell \frac{dn_\ell(\tilde{t})}{d\ell} \frac{dN_a}{d\tilde{t}} \Big|_\ell$$

$$\frac{dN_a}{dt} = \frac{dE_a}{k} = \Gamma_a f_a^2 \ell$$

\uparrow
 $\mathcal{O}(10^2)$

We integrate between all acceptable loop sizes

$$n(t_0) = 0.2 \frac{\Gamma_a f_a^2}{2\pi} \frac{1}{t_0^{3/2}} \int_{t_K}^{t_0} d\tilde{t} \frac{\xi^{3/2}(\tilde{t})}{\tilde{t}^{3/2}} l_{\text{crit}}(\tilde{t})$$

method:
log-uniform
distribution

calculating number density (again)

$$n_a(t_0) = \int_{t_K}^{t_0} d\tilde{t} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^3 \int d\left(\frac{k}{H}\right) \frac{1}{k} \frac{d\dot{\rho}_a}{d \ln k}$$

$$\frac{d\dot{\rho}_a}{d \ln k}(t) = \Gamma_K(t) \rho_s(t) F\left(x \equiv \frac{k}{H}, y \equiv \frac{m_r}{H}\right)$$

$$= \int_{t_K}^{t_0} d\tilde{t} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^3 \frac{\Gamma_K(\tilde{t})}{H(\tilde{t})} \rho_s(\tilde{t}) \int_{k_{\min}(\tilde{t})}^{m_r} \frac{dk}{k} F\left(\frac{k}{H(\tilde{t})}, \frac{m_r}{H(\tilde{t})}\right)$$

$F(x, y) \propto \frac{1}{x^q}$

We integrate between all applicable k modes

$$n(t_0) = 3 \frac{0.2 \ln N \mu_0 \ln N}{\xi_0 t_0^{3/2}} \frac{q-1}{q} \int_{t_K}^{t_0} d\tilde{t} \frac{\tilde{t}^{3/2}}{L_K^2(\tilde{t})} \left(\frac{x_0^q - y^q}{x_0^q y - y^q x_0} \right)$$

*method: varying
emission spectrum*

calculating number density (again)

$$\Omega_r(t_0) = \frac{n(t_0)}{n_{\text{DM}}^{\text{req}}(t_0)}$$

*method: varying
emission spectrum*

comparing the methods

$$\left(\frac{dn}{dt}\right)_{\text{M1}} \propto \mathcal{N}(t) \ell_{\text{crit}}(t) \sim \frac{\rho_s(t)}{L(t)} \ell_{\text{crit}}(t)$$

$$\begin{aligned} \left(\frac{dn}{dt}\right)_{\text{M2}} &\propto \int_{k_{\min}}^{m_r} \frac{dk}{H} \frac{1}{k} \frac{d\rho_a}{d \ln k} \\ &\sim \int_{k_{\min}}^{m_r} \frac{dk}{H} \frac{1}{k^2} \Gamma_k \rho_s \\ &\sim H \rho_s \ell_{\text{crit}} \end{aligned}$$

they differ in considering the length scale over which density changes,
while being similar for $q=1$, when friction not included