# Recent Developments in Calculating Bubble Wall Velocity

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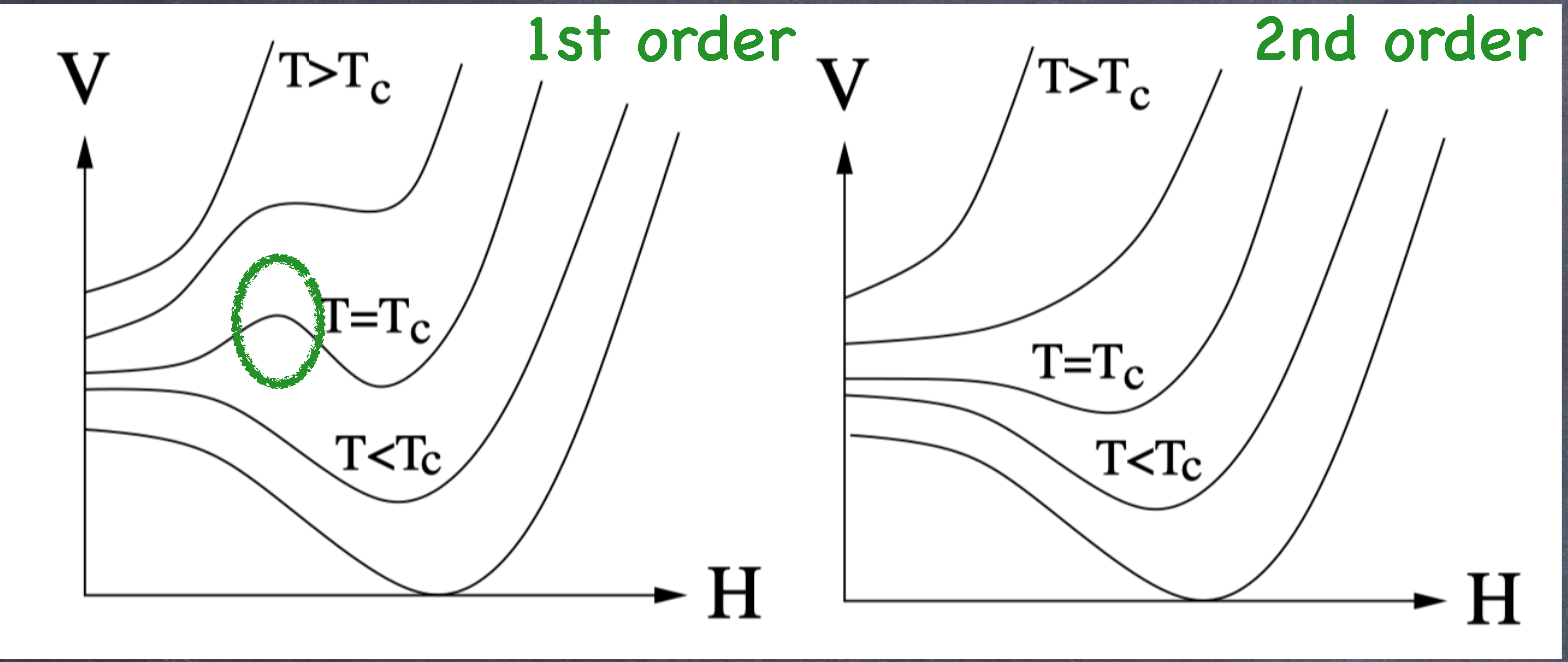
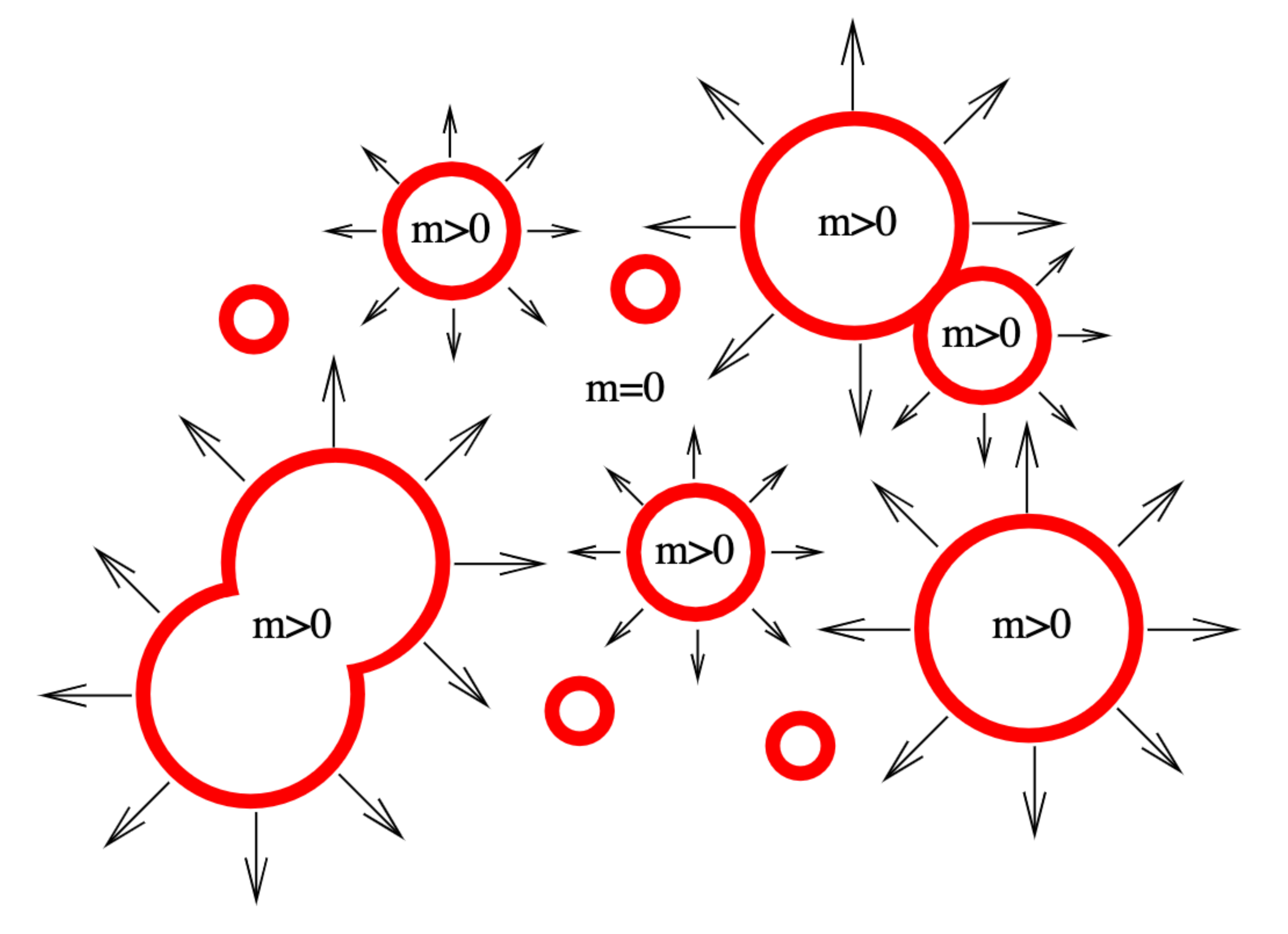
# Phase Transition: 1st vs 2nd Order



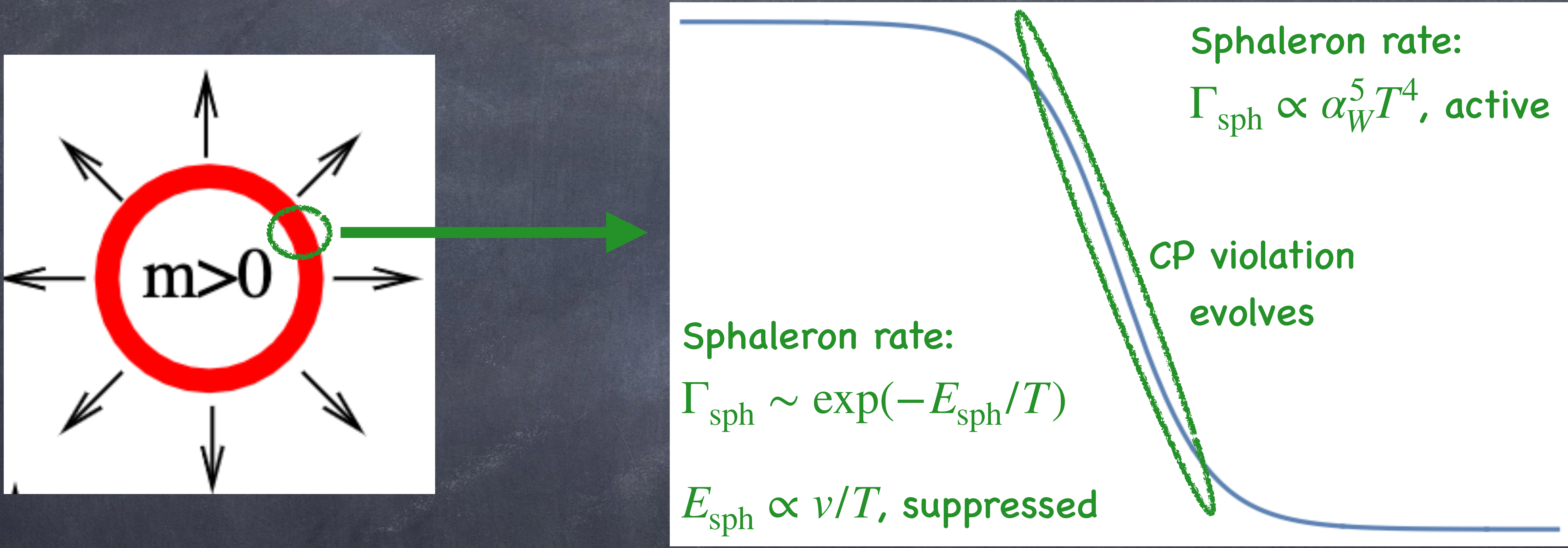

SM: 2nd order.

1st order widely exist in BSM theories



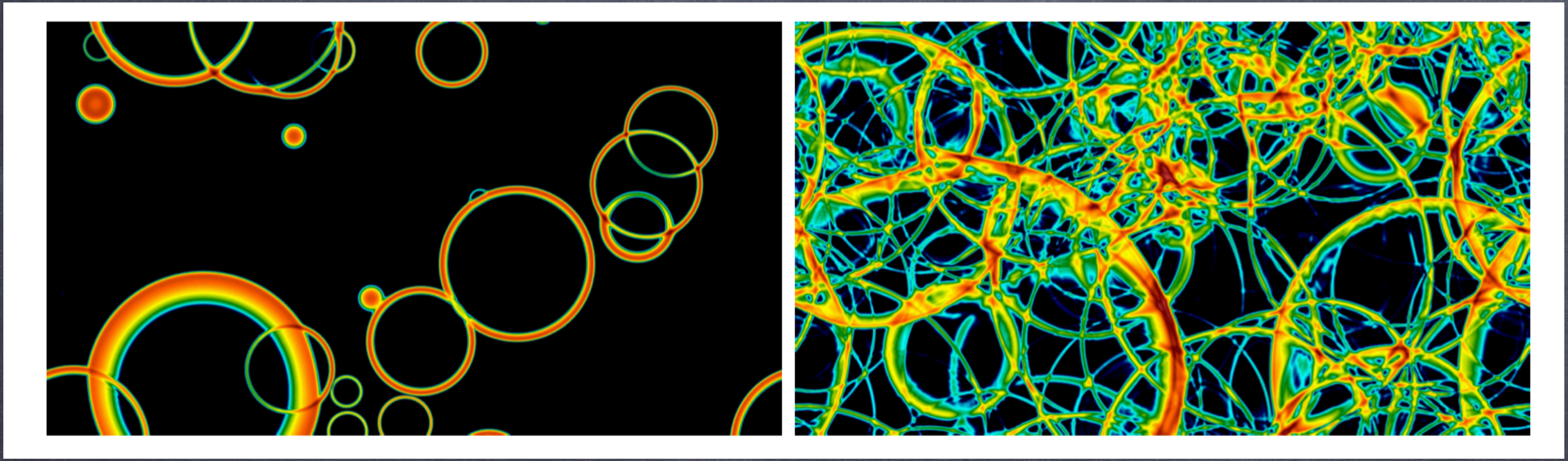

- 1st: well-defined barrier, bubble nucleation
- 2nd: smooth crossover

# What can Bubble Walls Do?




Electroweak Baryogenesis!

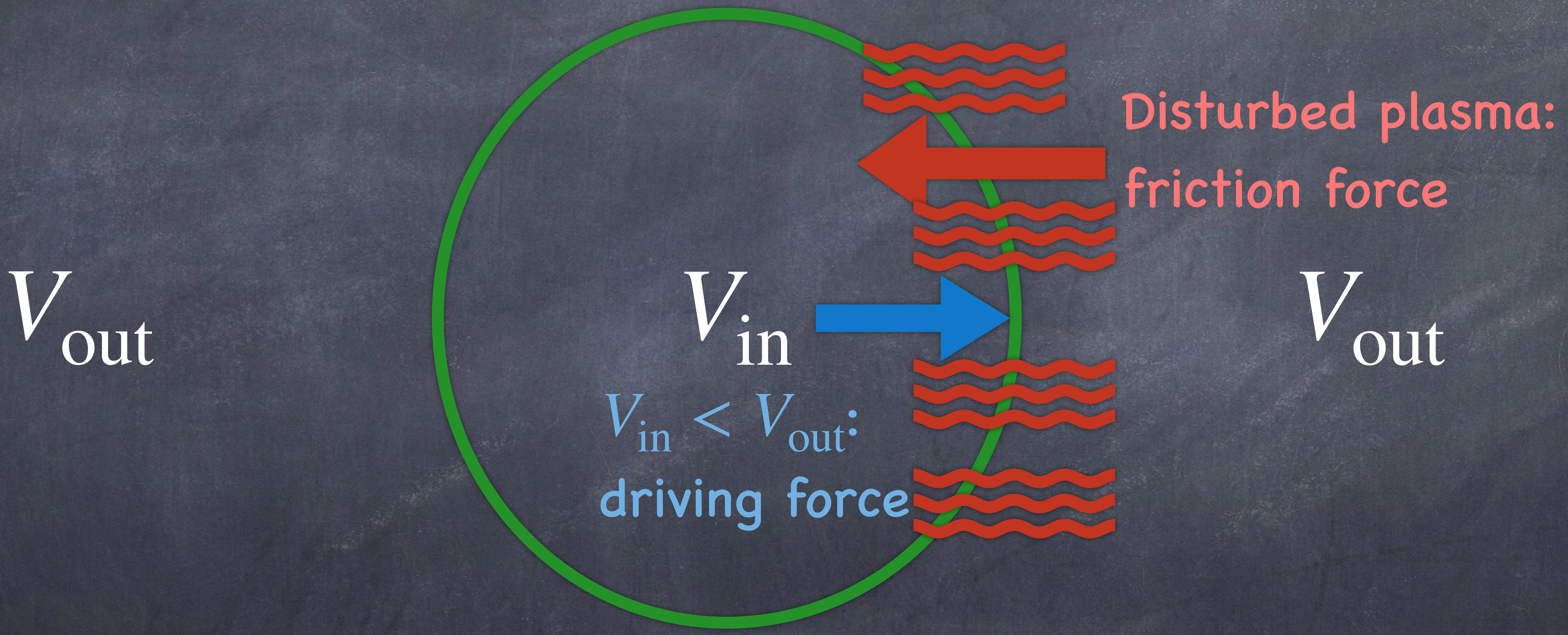
# What can Bubble Walls Do?



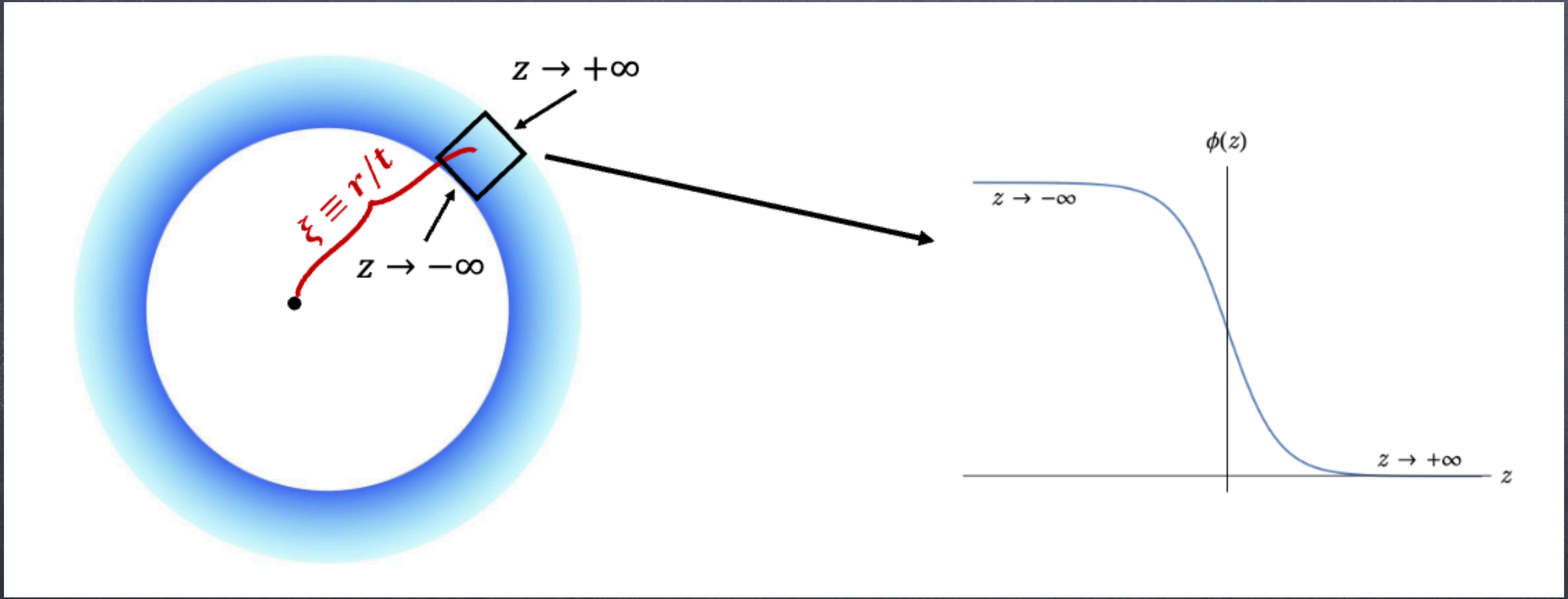
D. Weir, 1705.01783

Gravitational waves: bubble collision, sound wave, MHD turbulence, etc

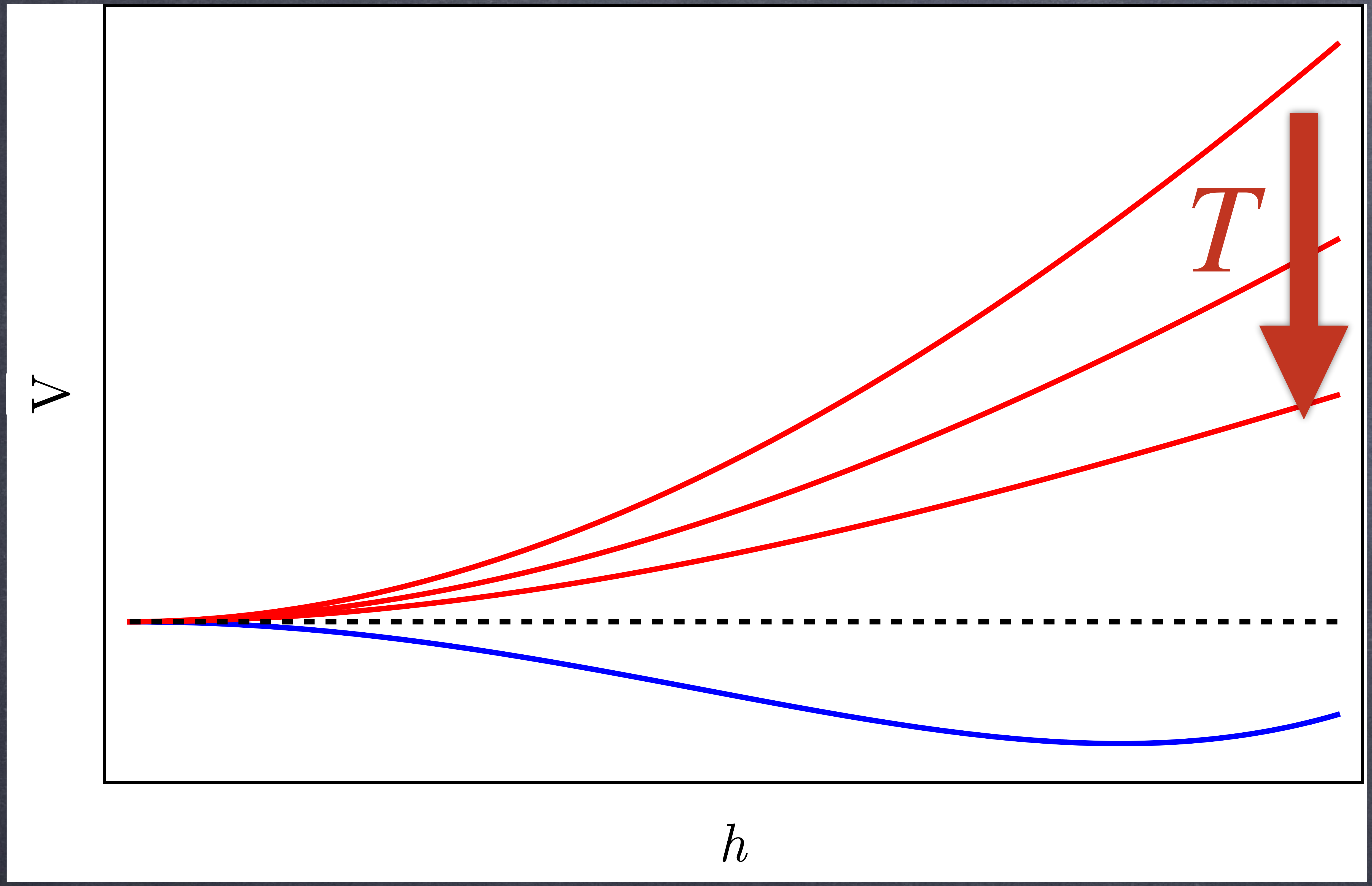
# Bubble Wall Dynamics: Intuition



# Coordinate



# Forces, from Effective Potential




$V_{\mathrm{FT}}$: finite-$T$ correction, always pushing inward (friction). Stronger for higher $T$

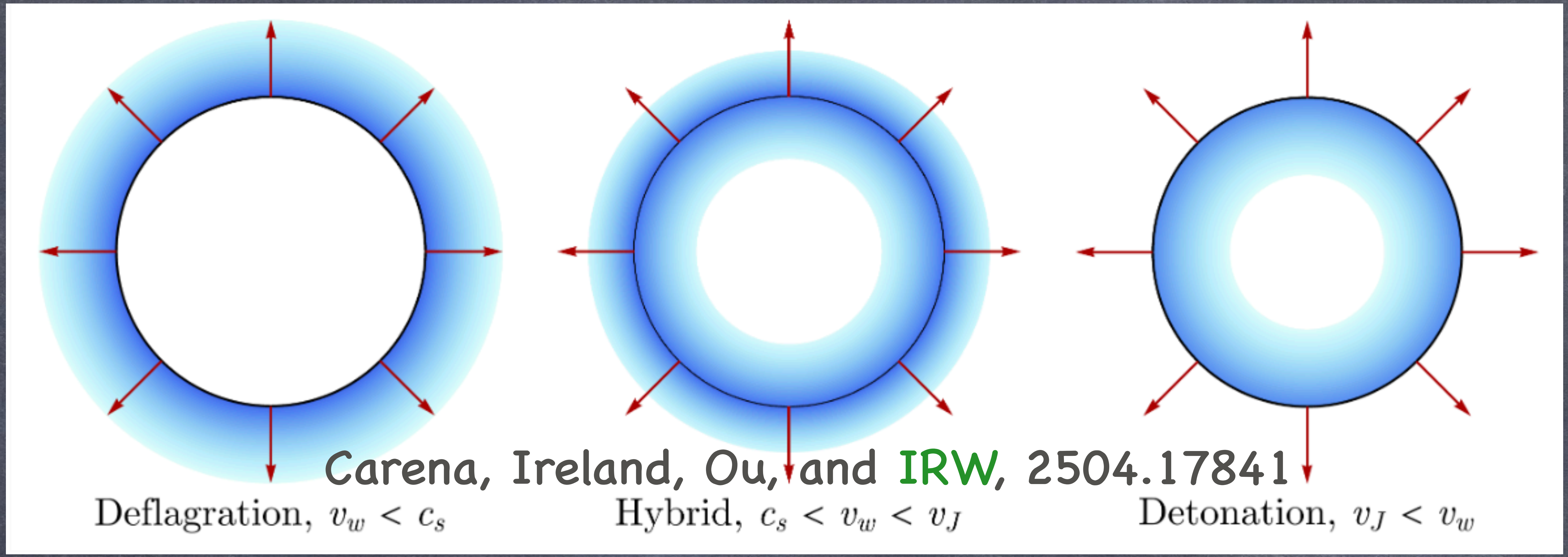
$V_0$: not changing with time, always pushing expansion

Vacuum energy release ($\alpha$) pushes acceleration, heated plasma provides friction

# Distributed Plasma: Source of Friction!

Plasma reflected and heated up in the front.

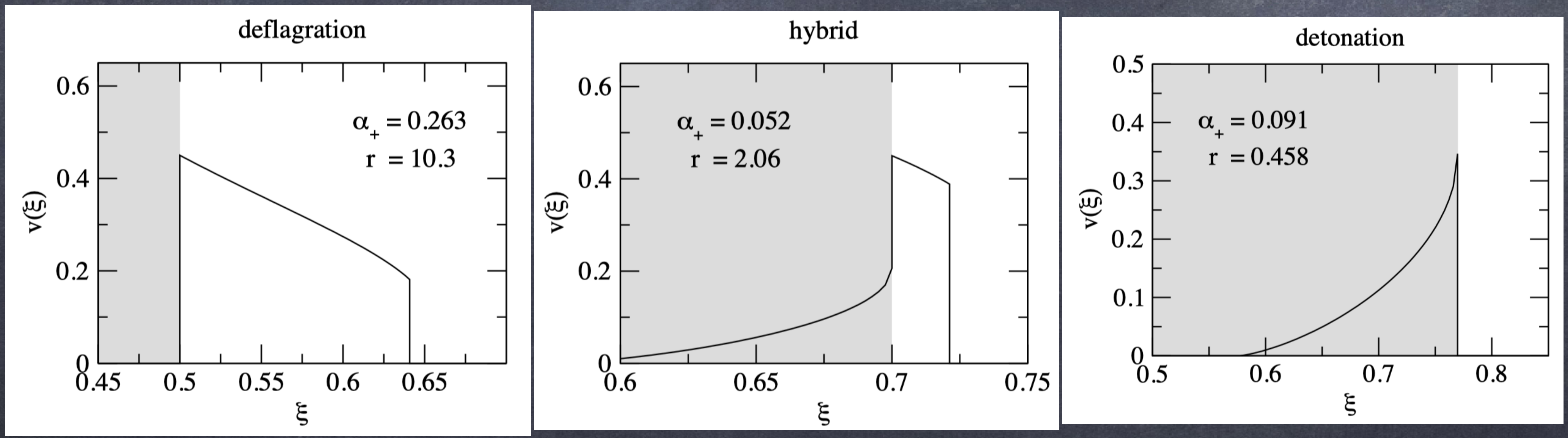
Strong friction.




Carena, Ireland, Ou, and IRW, 2504.17841

Heated plasma disappears in the front.

Friction drops.




Espinosa et al, 1004.4187

# Bubble Wall Dynamics: Maths Coming!

Two equations are governing:

- Energy momentum (EMT) conservation: $\partial_\mu T^{\mu\nu} = \partial_\mu \left( T^{\mu\nu}_\phi + T^{\mu\nu}_f \right) = 0.$

Governing both the bubble wall and the disturbed plasma

- Equations of Motion: $\Box \phi_i + \frac{\partial V_{\rm eff}}{\partial \phi_i} + \frac{\partial m^2}{\partial \phi_i} \sum_j \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E_k} f_j^{\rm out}(k, z) = 0$

$$f_i = f_i^{\rm eq} + f_i^{\rm out}, f_i^{\rm eq} = \frac{1}{1 \pm \exp(E/T)}$$

# Part 1: EMT Conservation

Densities of thermodynamic quantities: enthalpy, entropy, energy, pressure

$$w_f = T\frac{\partial p_f}{\partial T}, \quad s_f = \frac{\partial p_f}{\partial T}, \quad e_f = T\frac{\partial p_f}{\partial T} - p_f, \quad p_f = -V_{\rm eff}$$

Drop the time dependencies (seeking for steady-state solutions)

At both sides of the wall:

$$w_+\gamma_+^2 v_+ = w_-\gamma_-^2 v_-, \quad w_+\gamma_+^2 v_+^2 + p_+ = w_-\gamma_-^2 v_-^2 + p_-$$

# Equation of State

The above equations cannot be analytically solved.

Approximation: Bag equation of state

$$p^f_\pm = \frac{1}{3} a_\pm T^4_\pm, \quad e^f_\pm = a_\pm T^4_\pm \,.$$

$a_\pm$: relativistic dof.

Treating everything as pure radiation, or completely decoupled.

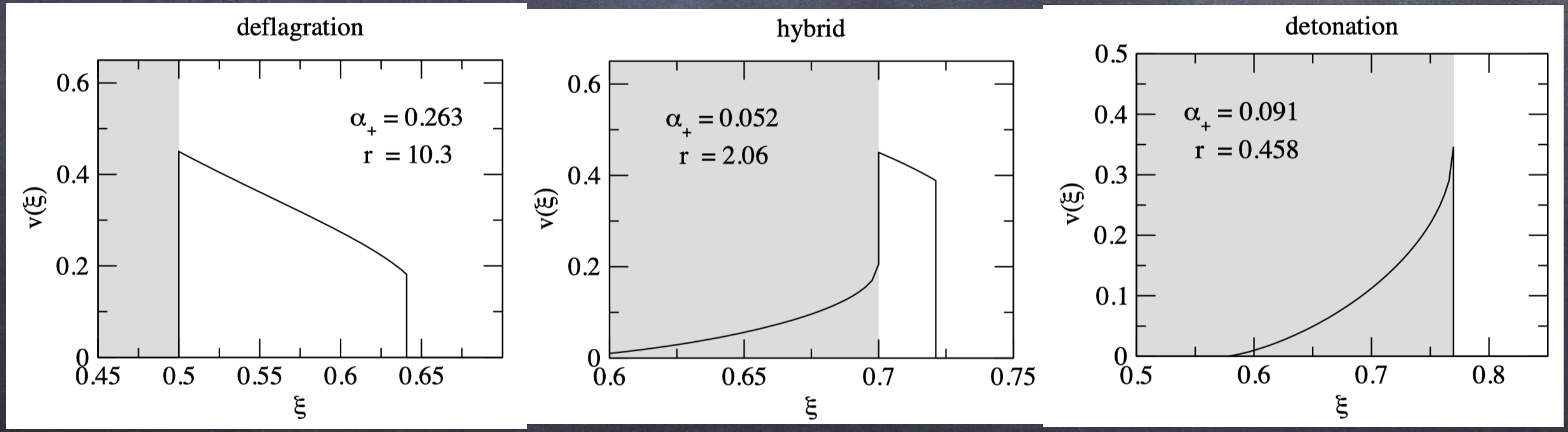
Helps giving intuitions. Not justified for EWPT!

# EMT Conservation for Fluid

Away from the wall: disturbed plasma.

Drop the scalar contribution and time dependency:

$$2\frac{v_{\rm cf}(\xi)}{\xi} = \gamma_{\rm cf}^2 (1 - v_{\rm cf}(\xi)\,\xi)\left(\frac{\mathfrak{v}(\xi, v_{\rm cf}(\xi))^2}{c_s^2(\xi)} - 1\right)\frac{\partial v_{\rm cf}(\xi)}{\partial \xi}\,,$$



# Part 2: Equations of Motion

Move terms according to our intuition: vacuum for push, plasma for friction.

$$\Box\phi_i + \frac{\partial V_0}{\partial \phi_i} + \frac{\partial V_{\rm FT}}{\partial \phi_i} + \frac{\partial m^2}{\partial \phi_i}\sum_j \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_k} f_j^{\rm out}(k,z) = 0$$

$V_{\rm FT}$ given by $f_j^{\rm eq}$

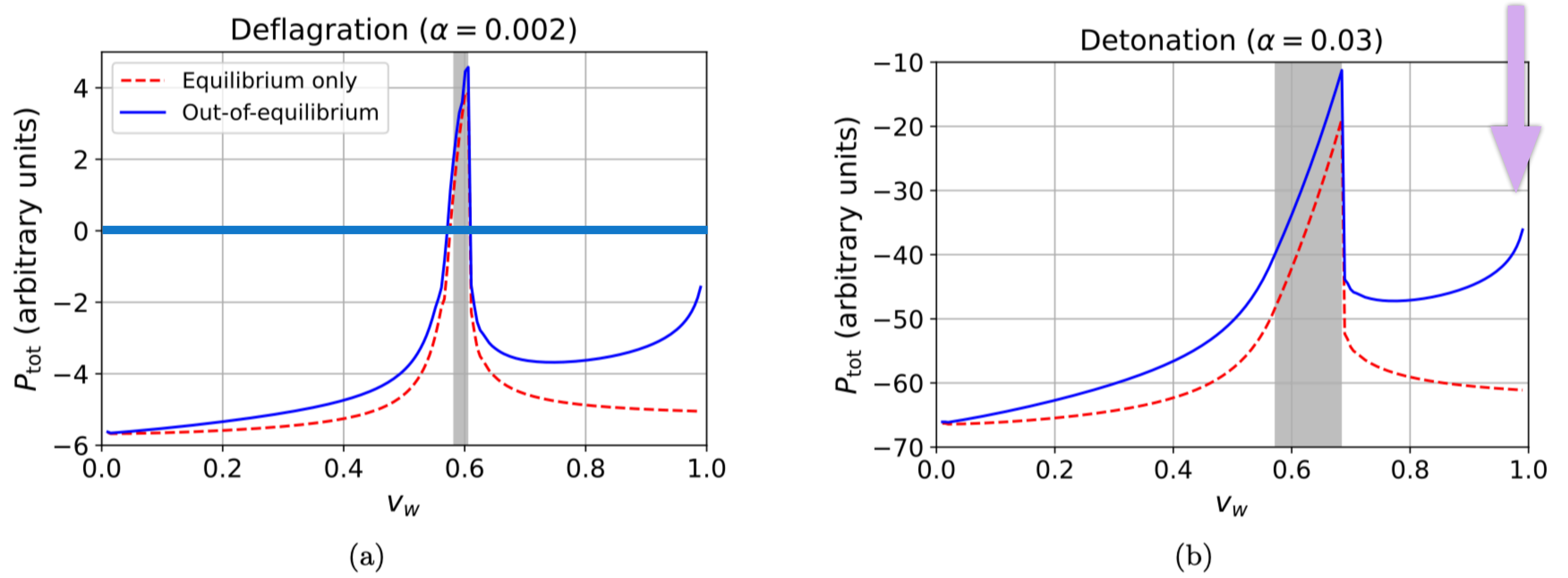
$$\ddot{\phi}_i - (\partial_z^2\phi) + \frac{\partial V_0}{\partial \phi_i} + \frac{\partial m^2}{\partial \phi_i}\sum_j \int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_k} f_j(k,z) = 0$$

Perform integration along the wall

**Pressure** $\mathscr{P}_{\rm net} \equiv \int dz(\partial_z\phi)\ddot{\phi} = -\int dz(\partial_z\phi)\left(\frac{\partial V_0}{\partial \phi} + \sum_i n_i \frac{dm_i^2}{d\phi}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2E_i} f_i(k,x)\right)$

# Pressure

When $v_w \to 1$:
$\mathscr{P} \propto \gamma_w$ or $\gamma_w^2$
Gouttenorie et al, 2021,
Hoeche et al, 2022




Cline and Laurent, 2204.13120

Terminal $v_w$: $\mathscr{P} = 0$. Out-of-equilibrium is negligible unless $v_w \to 1$

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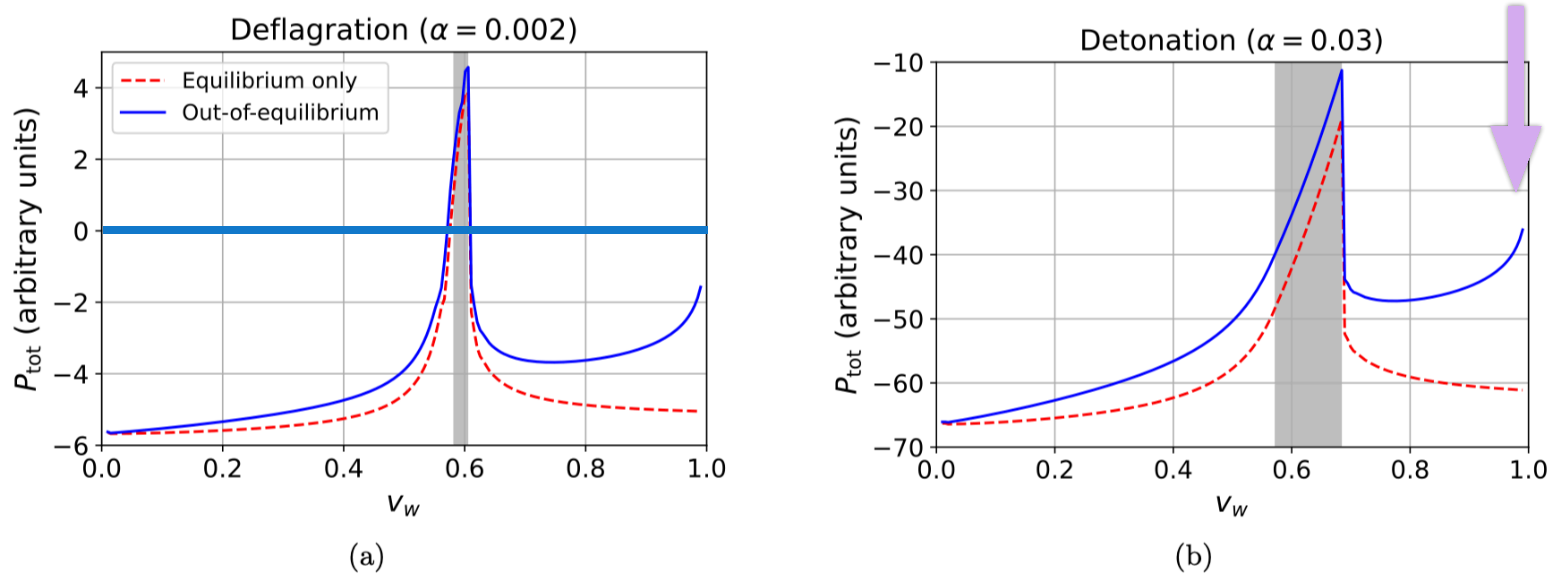



Cline and Laurent, 2204.13120

Conclusion: $v_w$ either located around 0.6, or goes up around 1.

# Local Thermal Equilibrium (LTE)

From EOM: $u_\nu \partial^\nu \phi \left( \Box \phi + \frac{\partial V_{\rm eff}}{\partial \phi} \right) + T \partial_\mu S^\mu = 0 \,,$

Entropy conservation: $s_+ \gamma_+ v_+ = s_- \gamma_- v_-$, calculation simplified!

From Bag model:

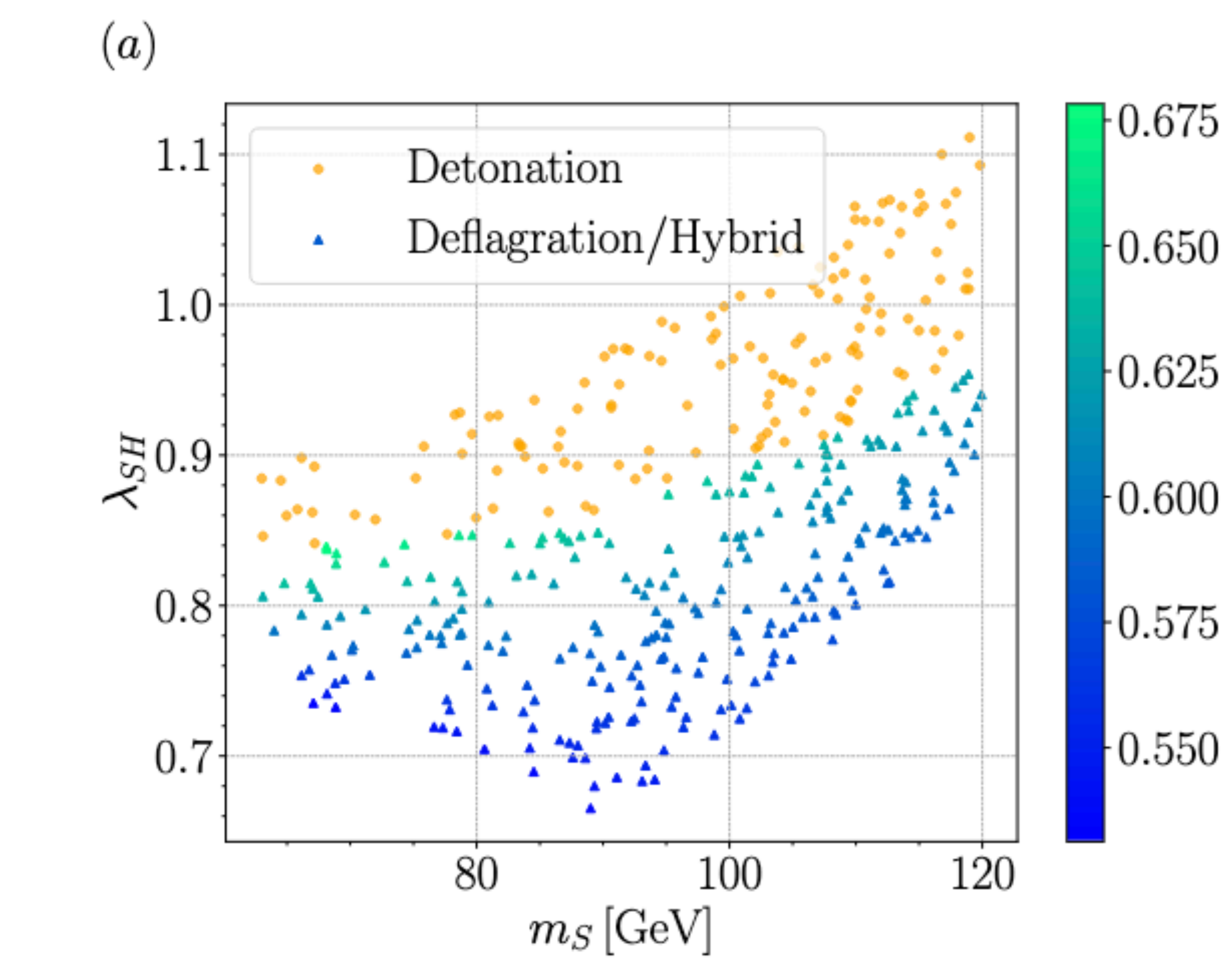
$b \equiv a_- / a_+$, just counting.

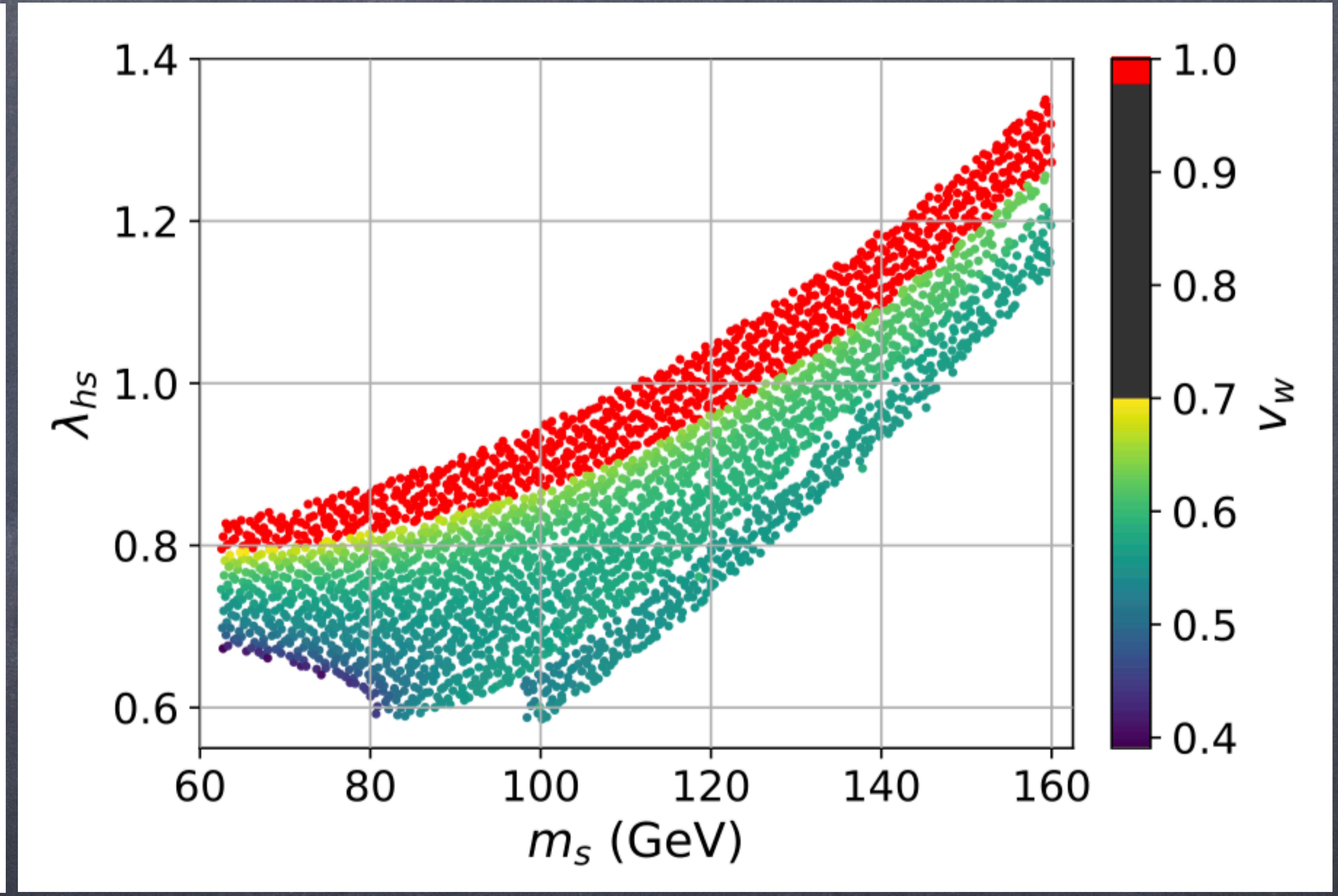
Critical $\alpha$ to prevent a runaway!

$$\frac{b(\alpha^{\rm det}_{n,\rm max}+1)^4}{\left[2\alpha^{\rm det}_{n,\rm max}+1-\sqrt{\alpha^{\rm det}_{n,\rm max}(3\alpha^{\rm det}_{n,\rm max}+2)}\right]^2} + 3\alpha^{\rm det}_{n,\rm max}$$
$$= 1 + 2\frac{3(\alpha^{\rm det}_{n,\rm max})^2 + \alpha^{\rm det}_{n,\rm max} + (1-\alpha^{\rm det}_{n,\rm max})\sqrt{3(\alpha^{\rm det}_{n,\rm max})^2 + 2\alpha^{\rm det}_{n,\rm max}}}{2\alpha^{\rm det}_{n,\rm max}+1-\sqrt{3(\alpha^{\rm det}_{n,\rm max})^2+2\alpha^{\rm det}_{n,\rm max}}}.$$

Ai et al, 2401.05911

# LTE: Beyond the Bag EOS




LTE+entropy with general EOS
Carena, Ireland, Ou, and IRW, 2025

Full result, Cline and Laurent, 2022

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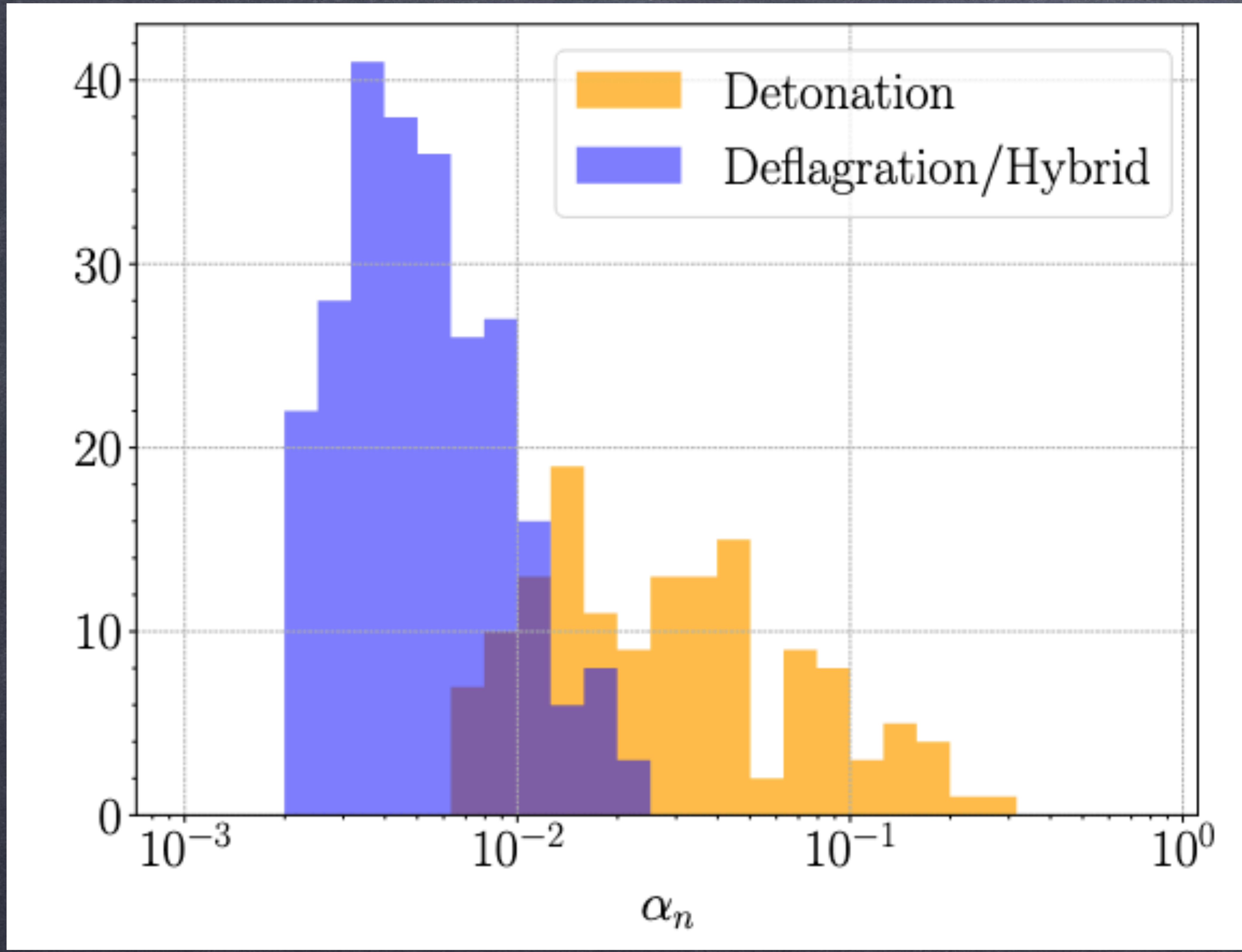


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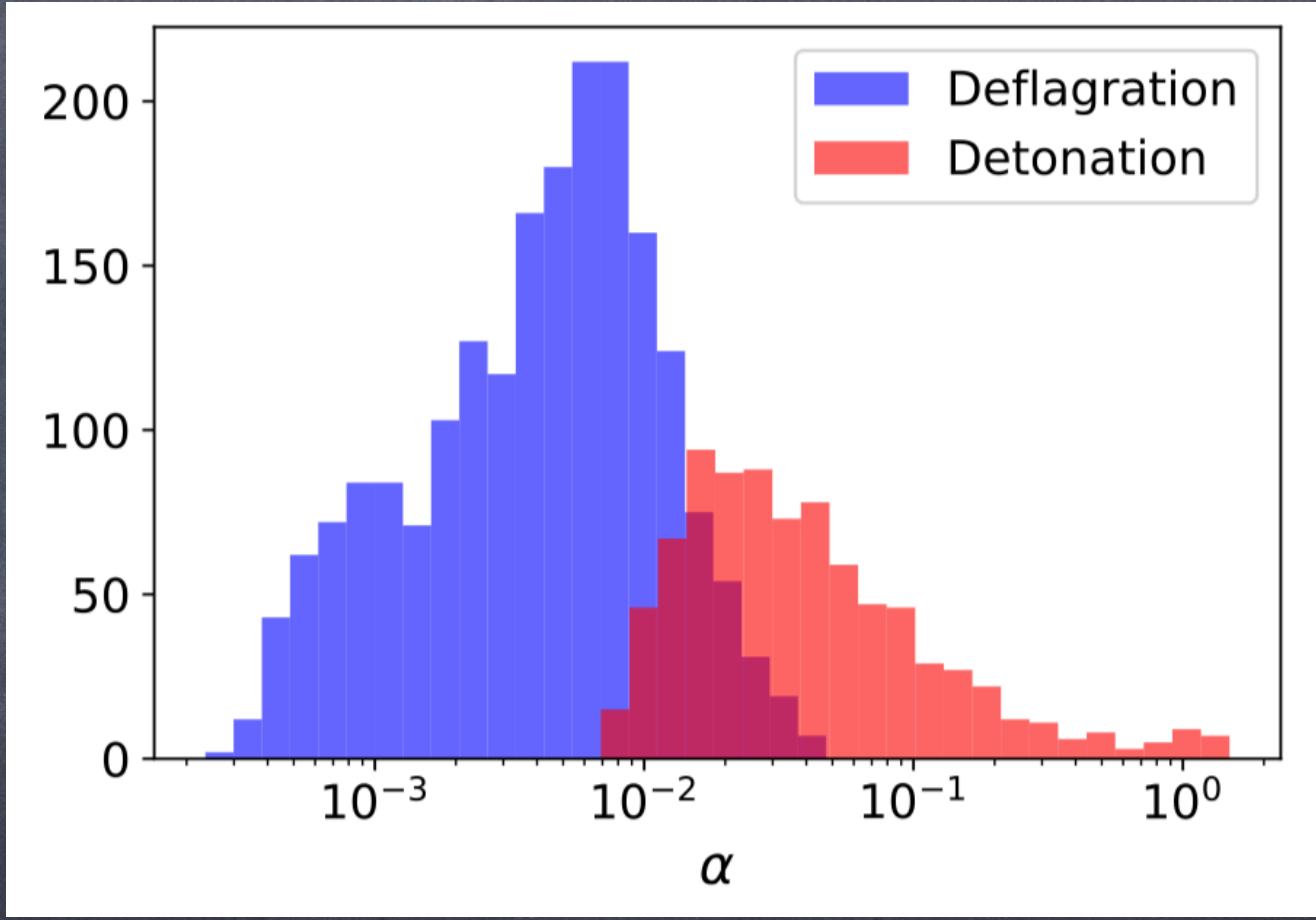


Full result, Cline and Laurent, 2022

Bag result: $\alpha_{\rm crit} \simeq 0.05$

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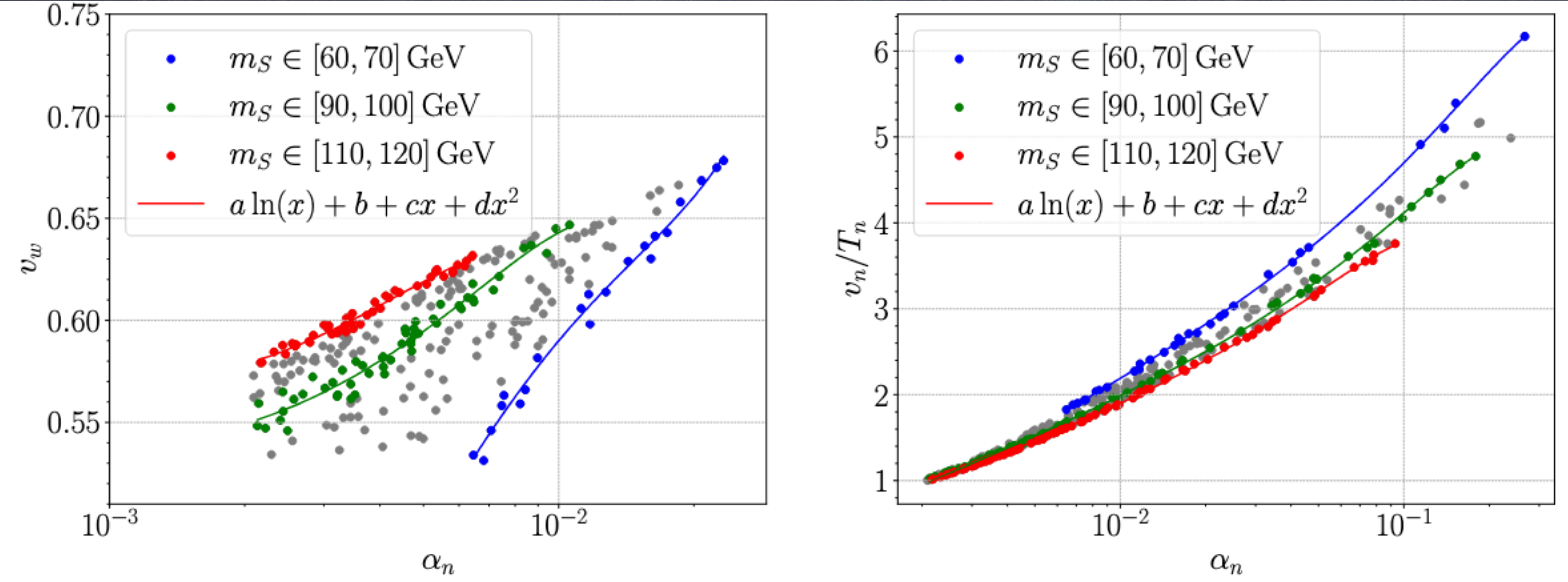
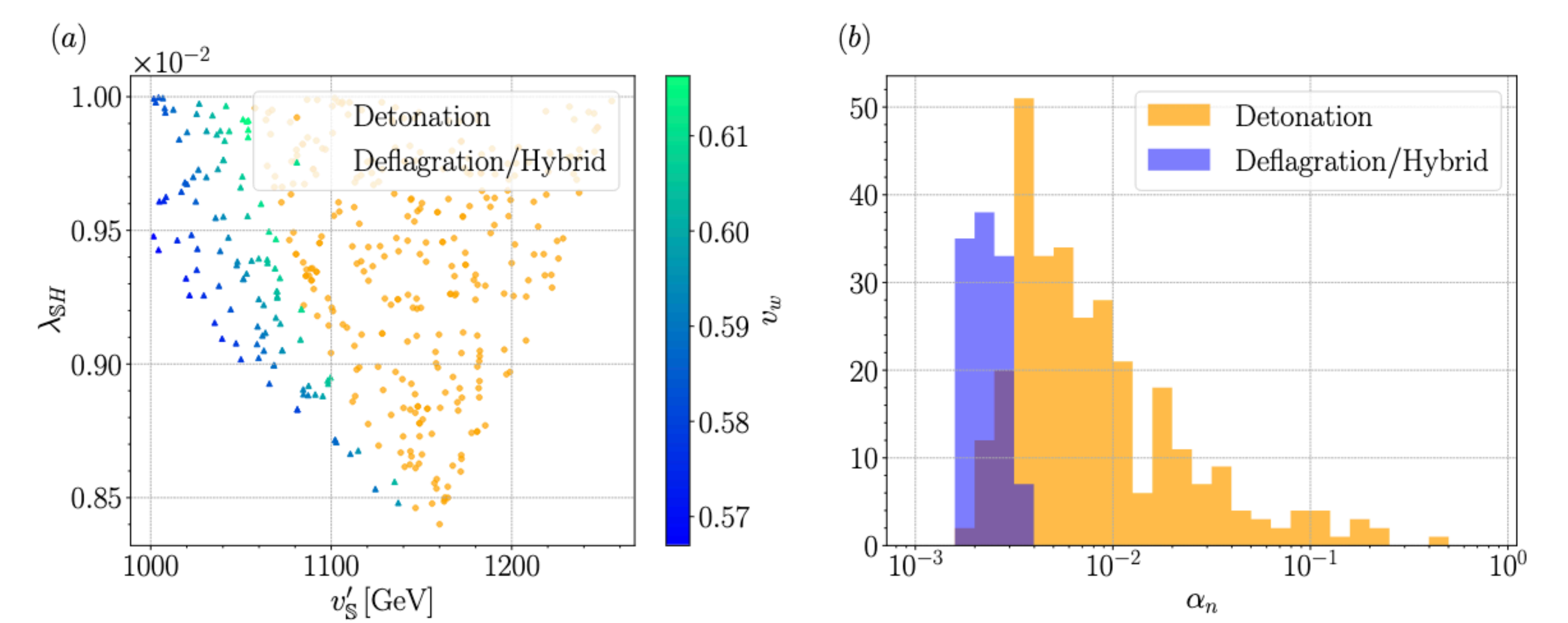



| $m_S$ range (GeV) | $(a, b, c, d)$ for $v_w = f(\alpha_n)$ | $(a, b, c, d)$ for $v_n/T_n = f(\alpha_n)$ |
|---|---|---|
| 60 − 70 | (0.27, 2.00, −22.83, 382.07) | (0.78, 5.68, 9.59, −14.60) |
| 90 − 100 | (−0.016, 0.40, 27.24, −1021.60) | (0.59, 4.60, 11.48, −26.61) |
| 110 − 120 | (−0.028, 0.34, 36.93, −2114.66) | (0.52, 4.13, 16.41, −78.95) |

Carena, Ireland, Ou, and IRW, 2025

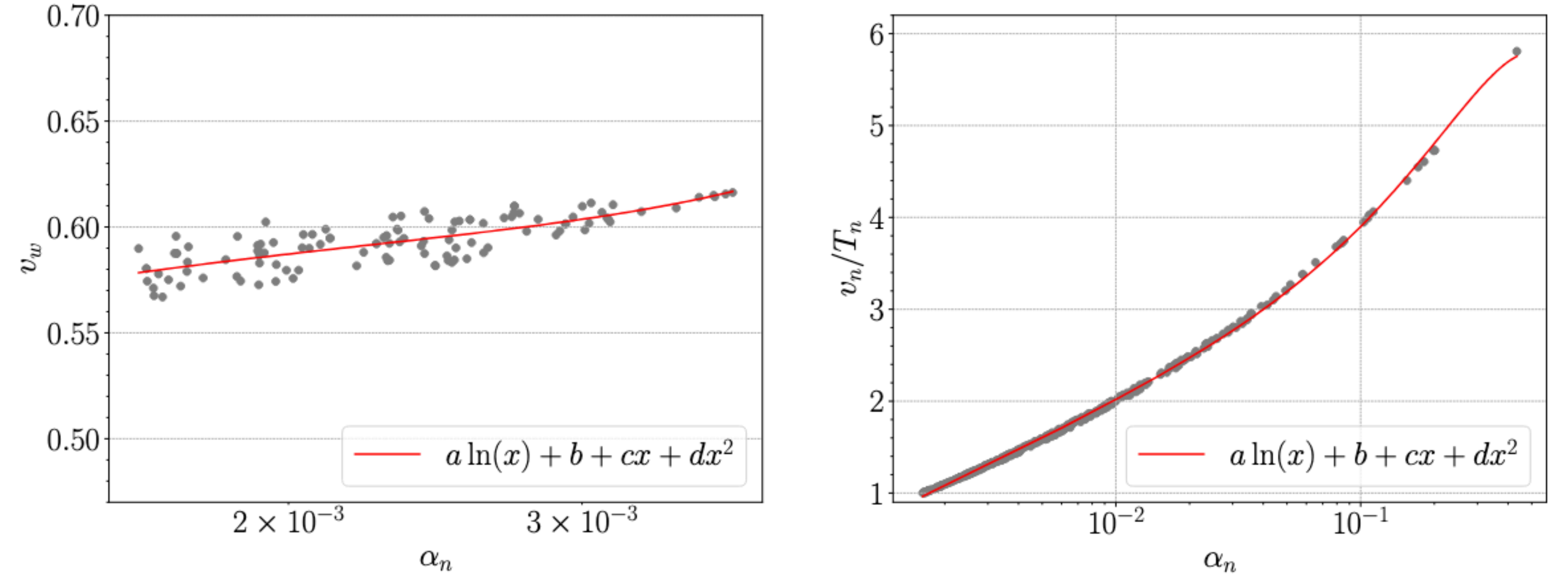
# Another BSM Model: DarkCPV




Carena, Ireland, Ou, and IRW, 2025

Bag result: $\alpha_{\text{crit}} \simeq 0.05$

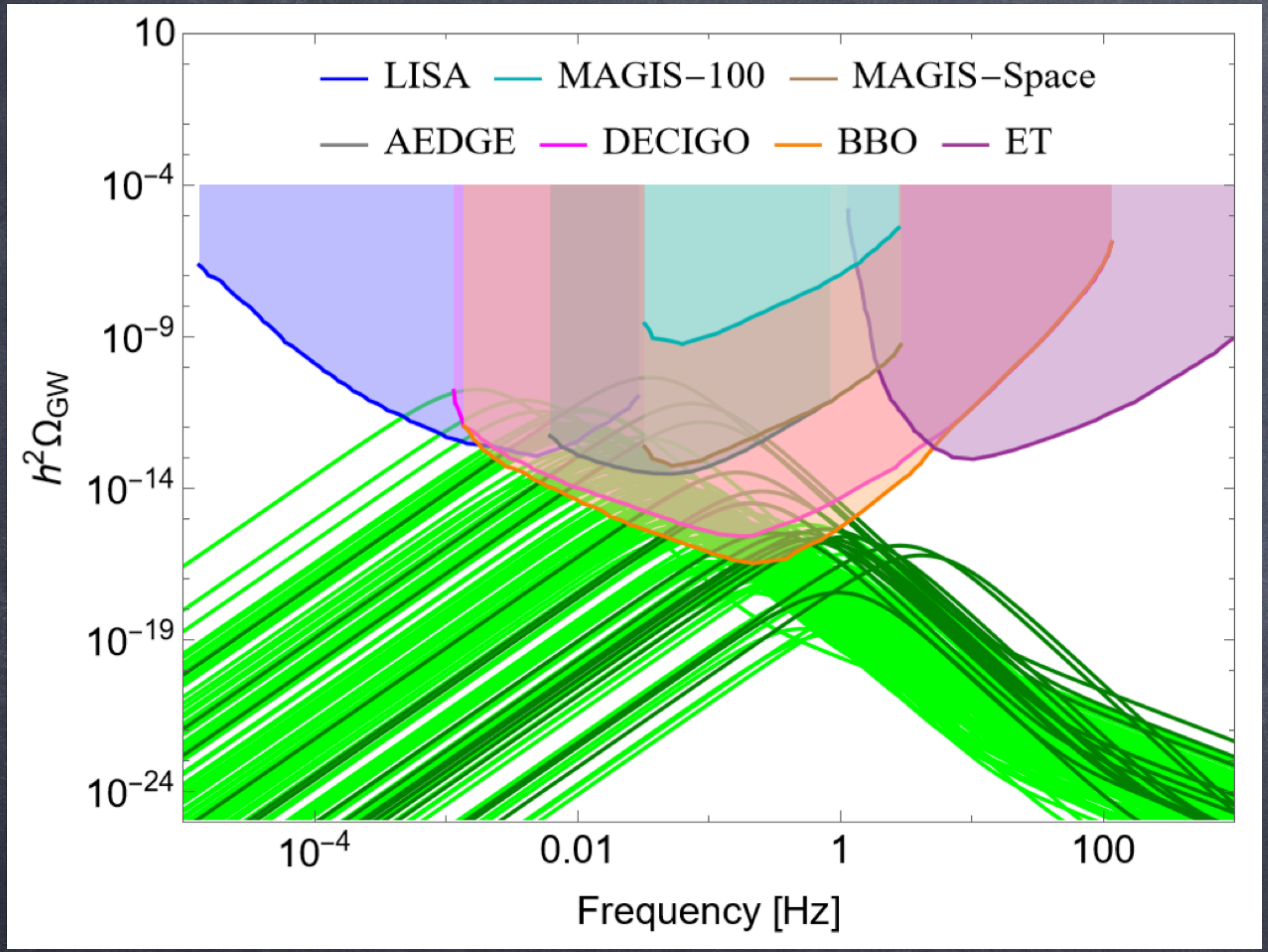
# DarkCPV: Continue




Carena, Ireland, Ou, and IRW, 2025

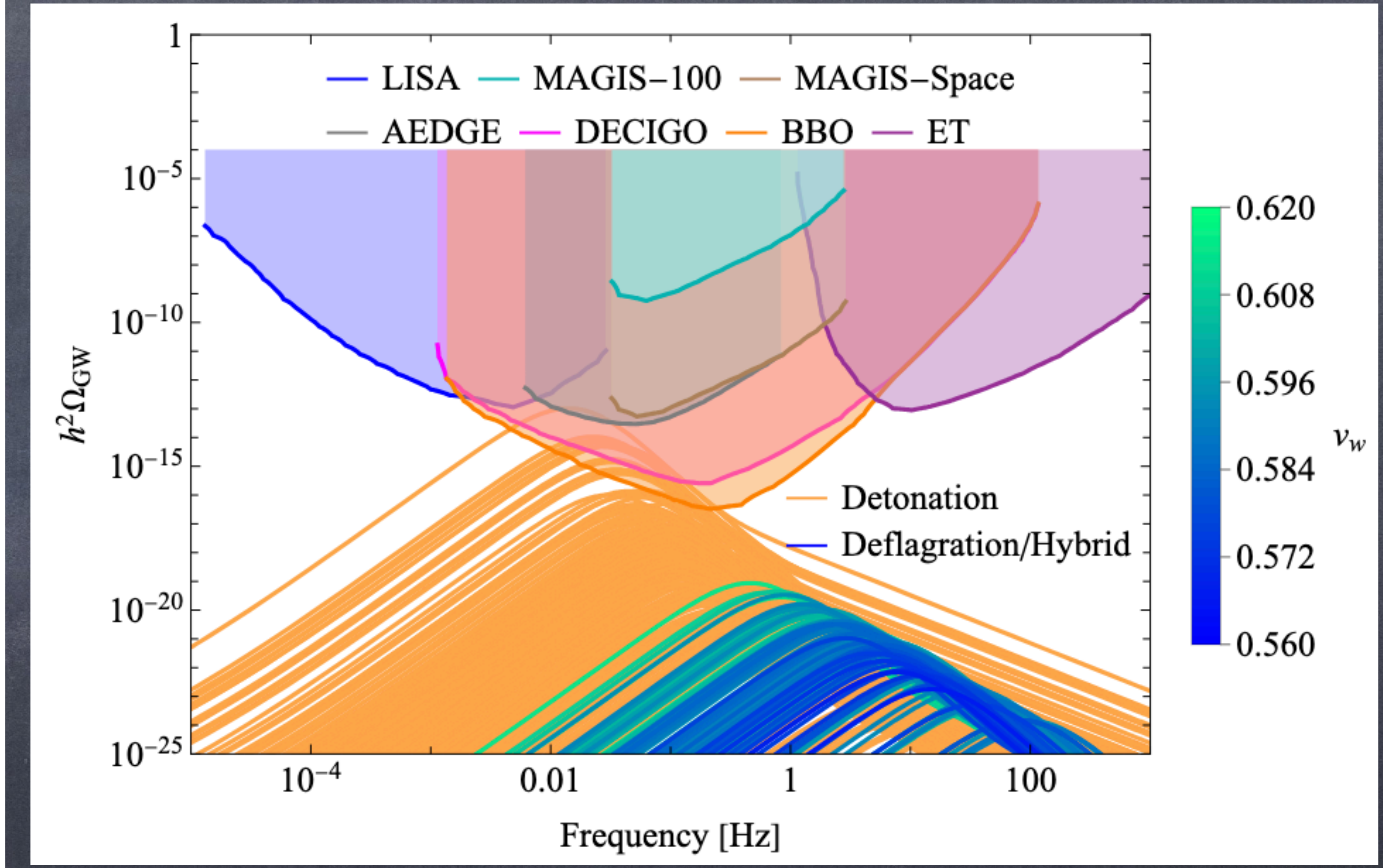
$$v_w = f(\alpha_n): \ a = 0.15,\ b = 1.64,\ c = -95.07,\ d = 1.05 \times 10^4.$$
$$\frac{v_n}{T_n} = f(\alpha_n): \ a = 0.54,\ b = 4.44,\ c = 8.01,\ d = -9.10.$$

# Implications on GW




Fix $v_w = 0.5$

Carena et al, 2022




Real $v_w$

Carena, Ireland, Ou and IRW, 2025

# Next Directions

- We assumed steady state for fluid. Is that really the case? (2411.16580)
- How well does LTE behave for more general BSM models? (2303.05846)
- Describing out-of-equilibrium with entropy production (ongoing)
- What's the behavior for $v_w \to 1$?

Thank you!