Probing Top Quark - Electron Interactions at Future Colliders

In collaboration with Luigi Bellafronte, Sally Dawson, Pier Paolo Giardino
arXiv: 2507.02039

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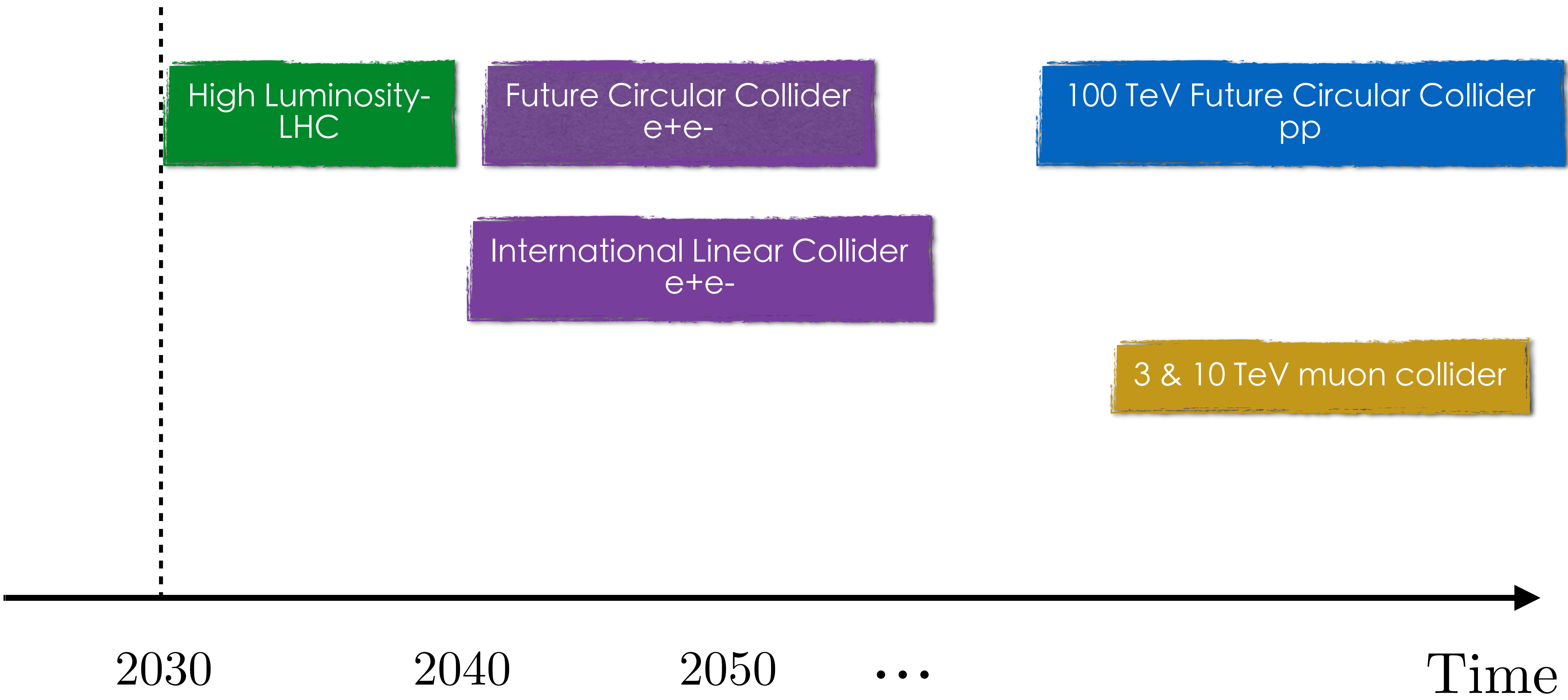
The future colliders



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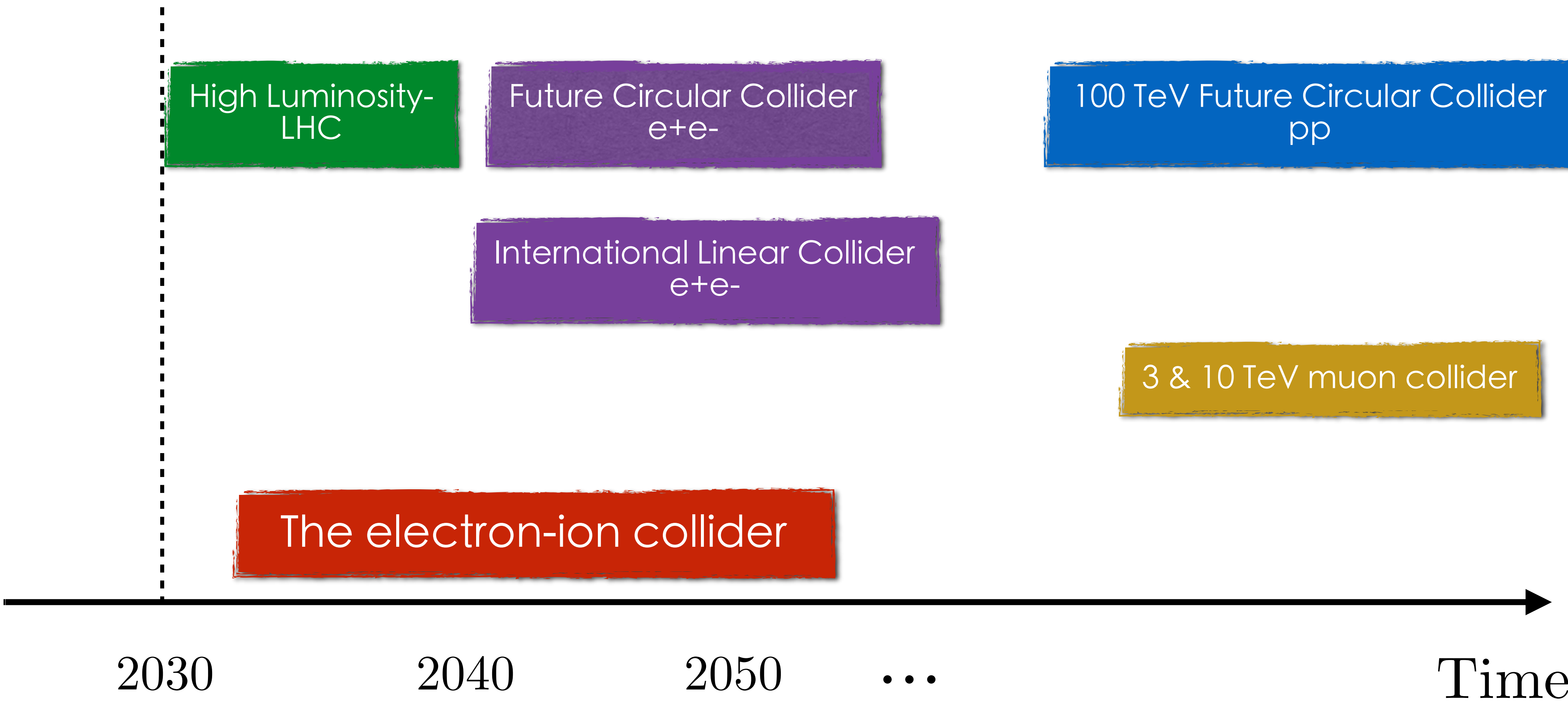
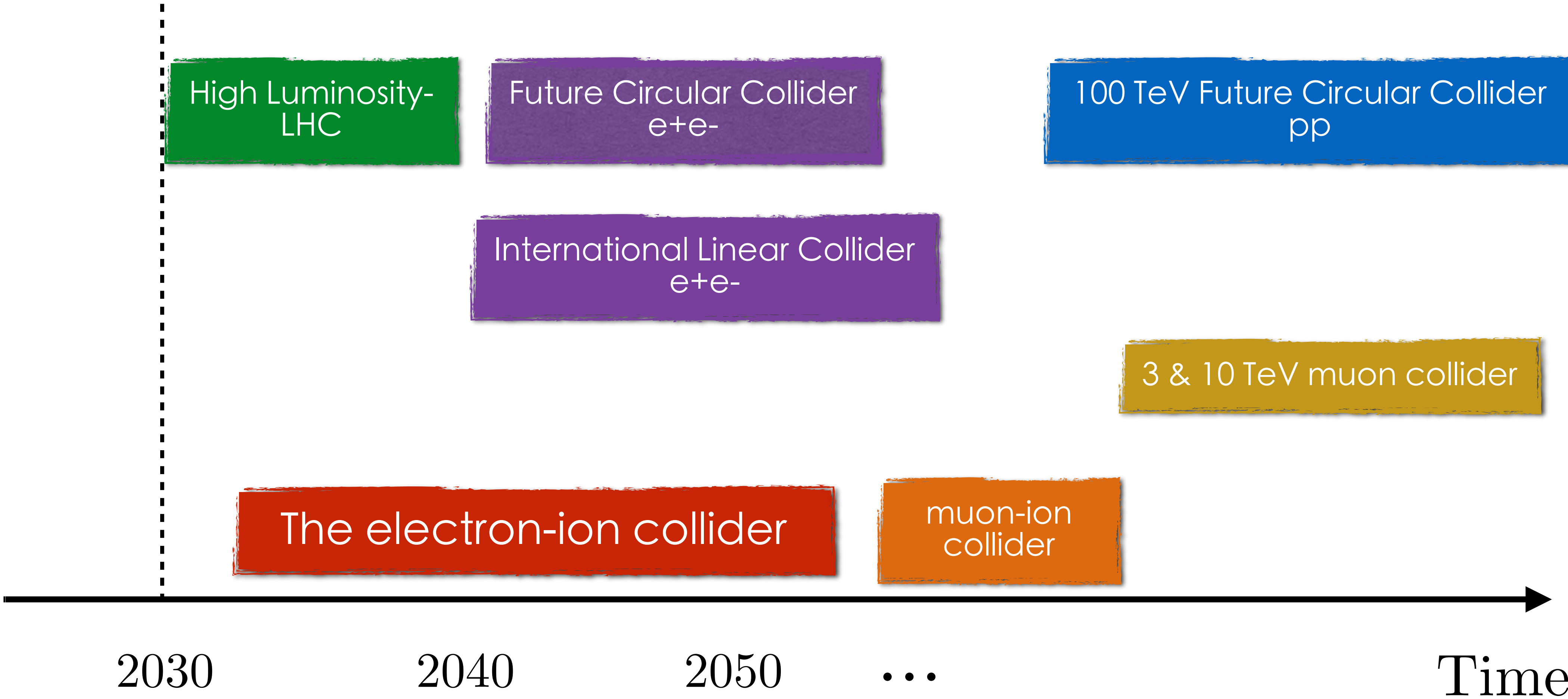
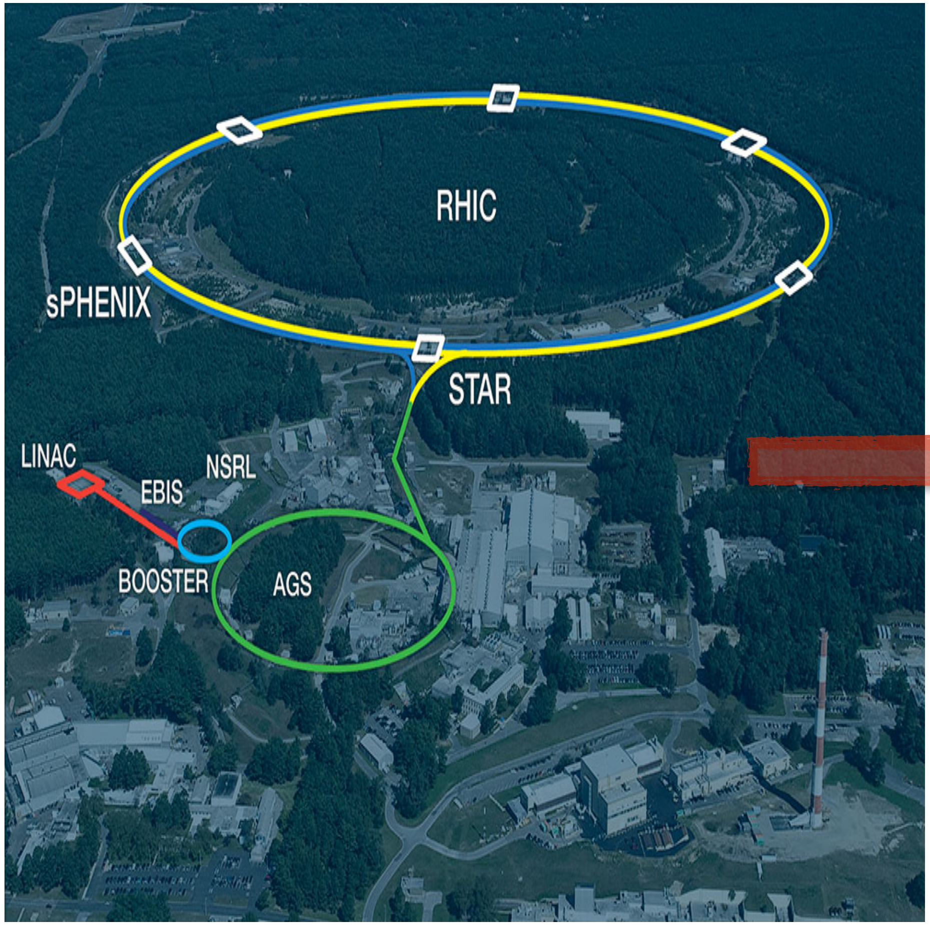


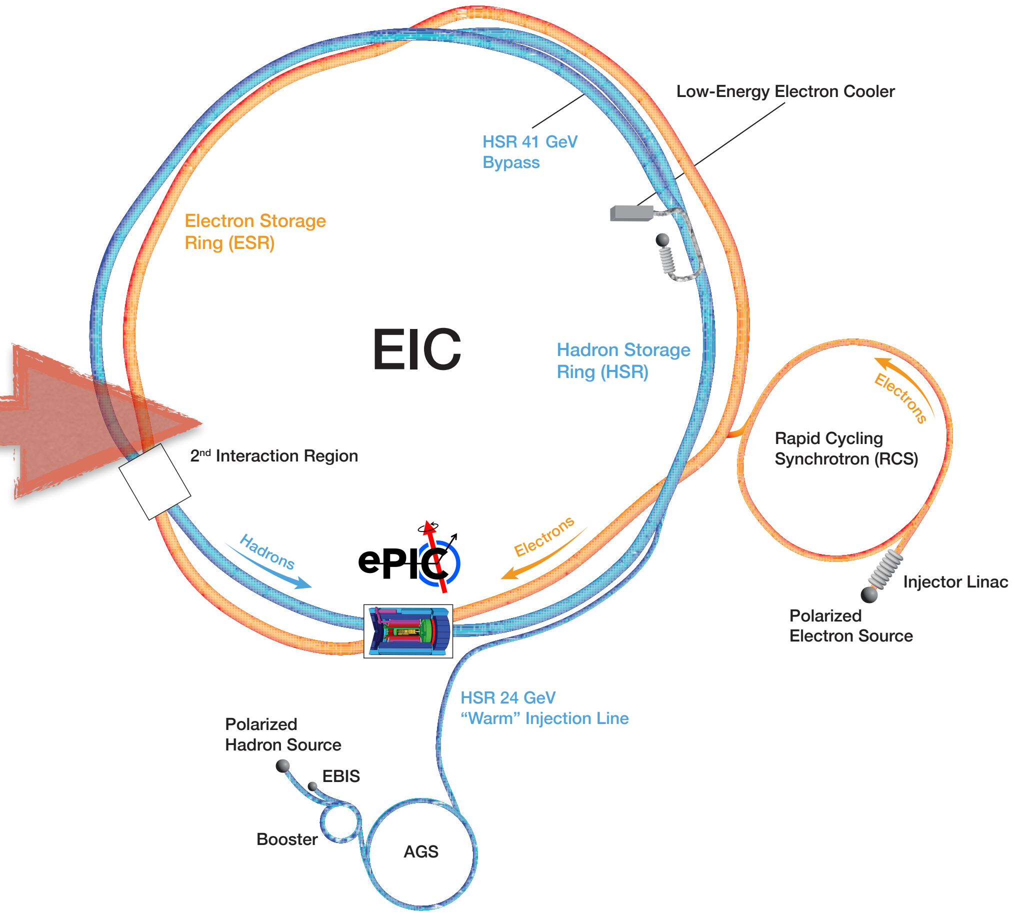
The future colliders



EIC Overview

The electron-ion collider, to be built over the coming years at the Brookhaven National Laboratory, aims to study QCD physics.



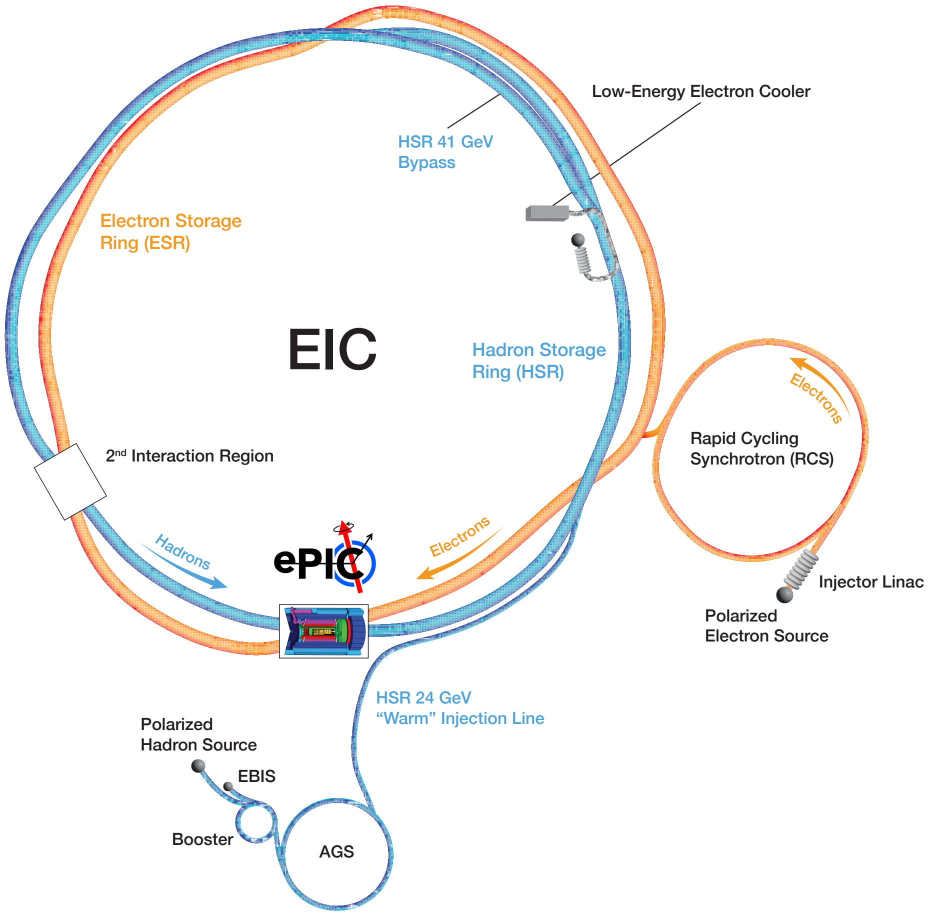


EIC Overview

The electron-ion collider, to be built over the coming years at the Brookhaven National Laboratory, aims to study QCD physics.

It has some nice features that can make it as a BSM machine as well

- Electron energy 5 - 18 GeV
- Proton energy 40-275 GeV; Ion energy ~100 GeV/nucleon
- Highly polarized (70% ~ 80%) e and p beams
- Ion beam from D to U
- High luminosity: $10^{33-34}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$ (100~1000 times HERA)



SMEFT with top-quark

Top quark, being the heaviest particle in the SM, often acts as a window to new physics.

- Two-fermion top-quark operators related to the vertices $t\bar{t}V,\ t\bar{b}W, t\bar{t}H$ are highly constrained
- Four-fermion top-quark operators $t\bar{t}q\bar{q}$ are highly constrained as well
- We focus on four-fermion top-quark operators $t\bar{t}e^-e^+$, which can be naturally probed at colliders involve leptons (lepton or lepton-hadron colliders)

If $\sqrt{s} < 2m_t$, some of the operators can only contribute via loop effects

Expansion on squared amplitude

Observables are directly related to the squared amplitude

$$|\mathcal{A}|^2 \sim |\mathcal{A}_{\rm SM}^{\rm LO}|^2 + \frac{1}{16\pi^2} 2{\rm Re}\left(A_{\rm SM}^{*\rm LO} A_{\rm SM}^{\rm NLO}\right) + \frac{1}{\Lambda^2} 2{\rm Re}\left(\sum_i C_i^6 A_{\rm SM}^{\rm LO} A_{i,\rm LO}^{*6}\right) + \frac{1}{16\pi^2\Lambda^2} 2{\rm Re}\left(\sum_i C_i^6 A_{\rm SM}^{\rm LO} A_{i,\rm NLO}^{*6}\right)$$

We also include NLO electroweak contributions

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$$+ \frac{1}{\Lambda^4} |\sum_i C_i^6 A_{i,\rm LO}^6|^2 + 2{\rm Re}\left(\sum_i \frac{C_i^8}{\Lambda^4} A_{\rm SM}^{\rm LO} A_{i,\rm LO}^{*8}\right) + \ldots$$

We also include NLO electroweak contributions

We truncate our results at $\mathcal{O}\left(\frac{1}{16\pi^2\Lambda^2}\right)$

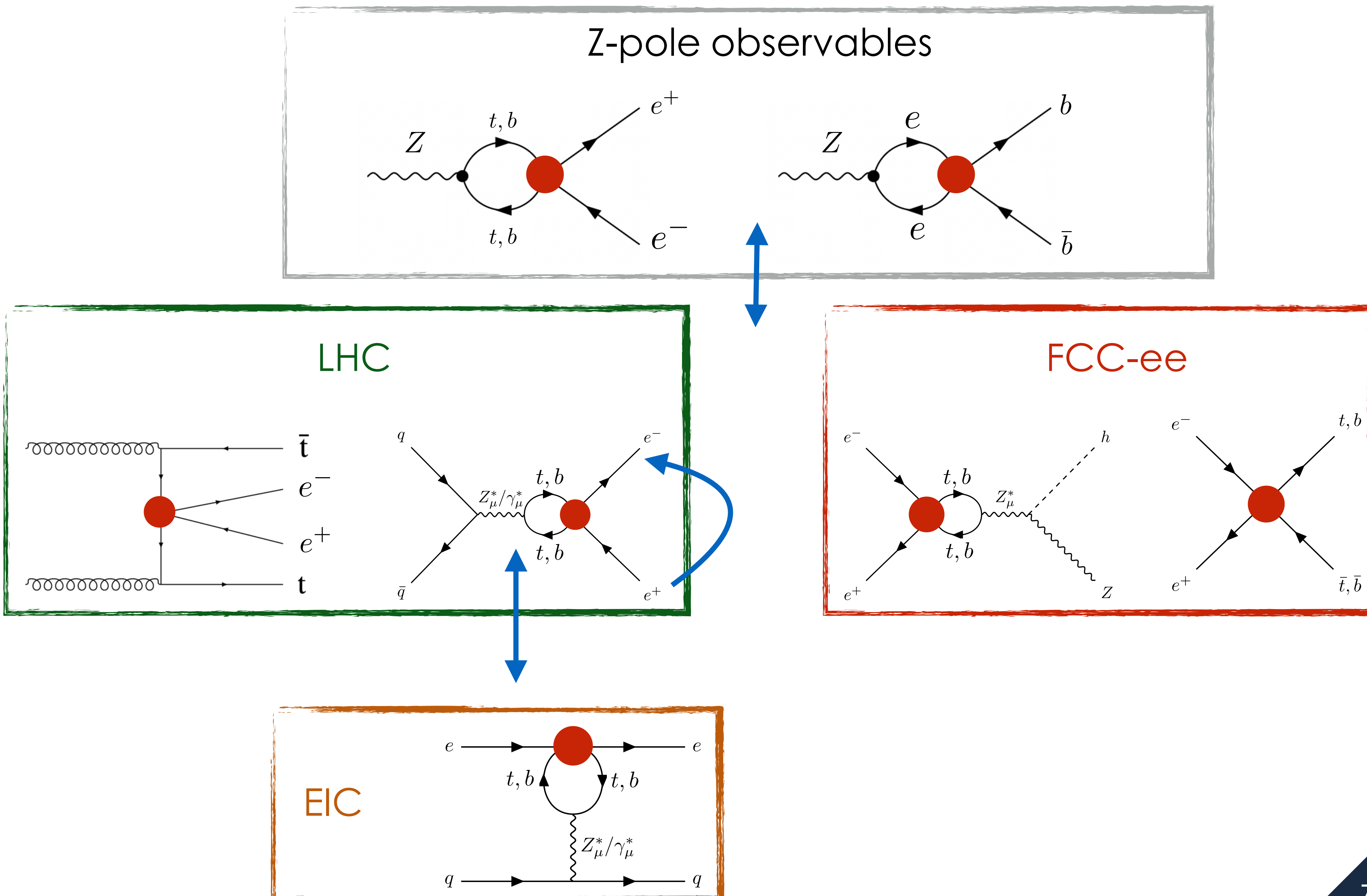
SMEFT operators

- There are five chirality conserving two-lepton-two-top operators

$\mathcal{O}_{\ell q}^{(3),1133} = (\bar{\ell}_L \gamma_\mu \tau^I \ell_L)(\bar{Q}_L \gamma^\mu \tau^I Q_L)$	
$\mathcal{O}_{\ell q}^{(1),1133} = (\bar{\ell}_L \gamma_\mu \ell_L)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{\ell u}^{1133} = (\bar{\ell}_L \gamma_\mu \ell_L)(\bar{t}_R \gamma^\mu t_R)$
$\mathcal{O}_{qe}^{3311} = (\bar{e}_R \gamma_\mu e_R)(\bar{Q}_L \gamma^\mu Q_L)$	$\mathcal{O}_{eu}^{1133} = (\bar{e}_R \gamma_\mu e_R)(\bar{t}_R \gamma^\mu t_R)$

- The contributions from chirality flipped ones are suppressed by $m_\ell/\sqrt{s}$

The observables



Z-pole observables

QED coupling: $1/\alpha_{\text{QED}}$

Total decay width: $\Gamma_Z, \quad \Gamma_W$

The R ratios:

$$R_\ell \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \ell\bar{\ell})} \qquad R_q \equiv \frac{\Gamma(Z \to q\bar{q})}{\Gamma(Z \to \text{hadrons})}$$

Hadronic cross sections:

$$\sigma_h \equiv \sigma(e^+e^- \to Z \to \text{hadrons})$$

The asymmetries:

$$A_f \equiv \frac{\sigma_{f,L} - \sigma_{f,R}}{\sigma_{f,L} + \sigma_{f,R}}$$

$$A_{f,\text{FB}} \equiv \frac{\sigma_{f,F} - \sigma_{f,B}}{\sigma_{f,F} + \sigma_{f,B}}$$

Observables	Experimental values	current (future) theoretical values
$1/\alpha_{\text{QED}}$	128.952 ± 0.014	128.316 ± 0.009
Γ_W(GeV)	2.085 ± 0.042	2.0903 ± 0.0003[55]
Γ_Z(GeV)	2.4955 ± 0.0023	2.4943 ± 0.0004 (8.0×10^{-5})[56–58]
R_e	20.804 ± 0.05	20.732 ± 0.006 (1.2×10^{-3})[56–58]
R_μ	20.784 ± 0.034	20.732 ± 0.006 (1.2×10^{-3})[56–58]
R_τ	20.764 ± 0.045	20.779 ± 0.006 (1.2×10^{-3})[56–58]
R_b	0.21629 ± 0.00066	0.2159 ± 0.0001 (2×10^{-5})[56–58]
R_c	0.1721 ± 0.0030	0.1722 ± 0.00005 (1×10^{-5})[56–58]
σ_h(nb)	41.481 ± 0.033	41.492 ± 0.006 (5×10^{-4})[56–58]
A_e(from A_{LR} had)	0.15138 ± 0.00216	0.1469 ± 0.0004[58, 59]
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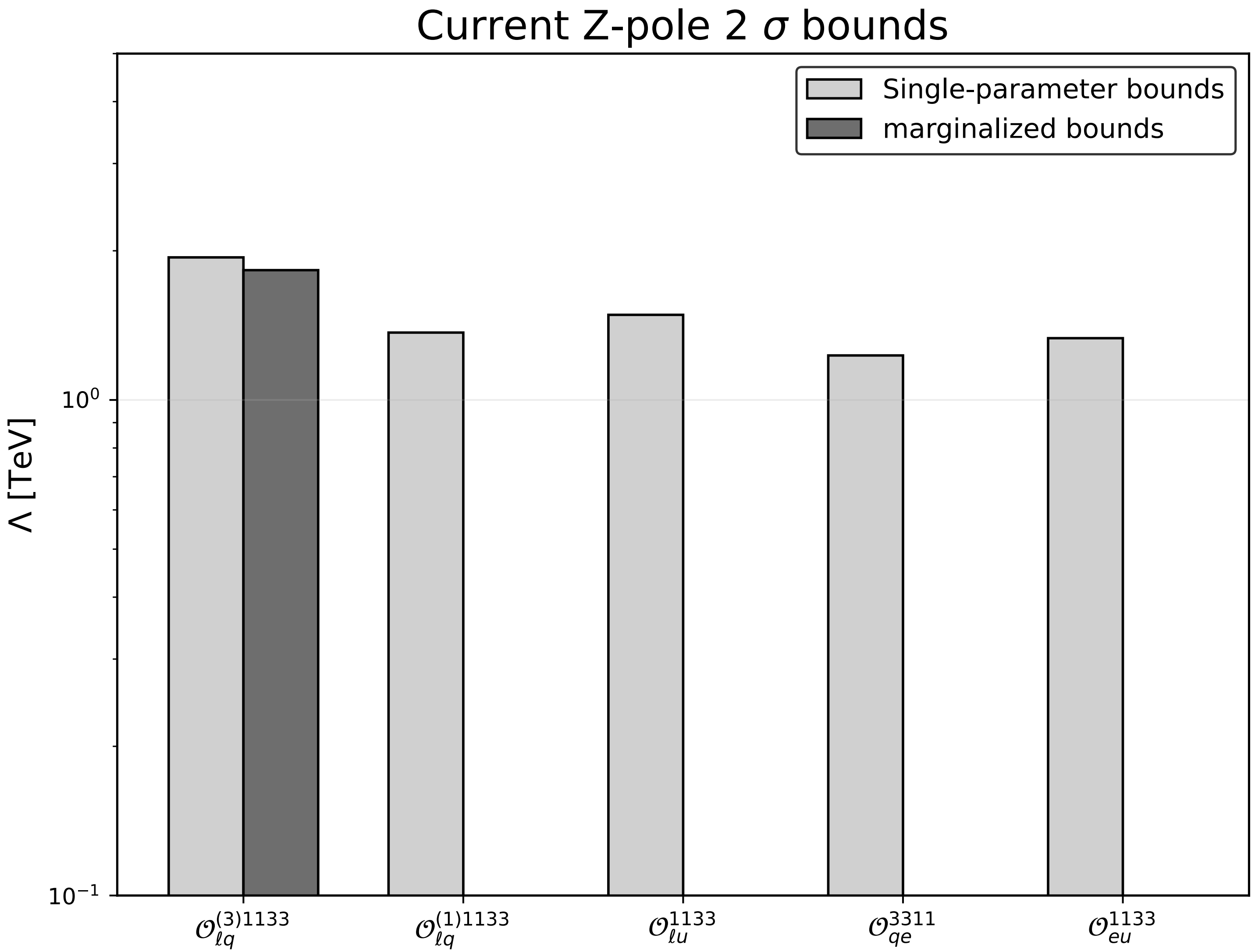
A_e(from Bhabba pol)	0.1498 ± 0.0049	0.1469 ± 0.0004[58, 59]
A_μ	0.142 ± 0.015	0.1469 ± 0.0004[58, 59]
A_τ(from SLD)	0.136 ± 0.015	0.1469 ± 0.0004[58, 59]
A_τ(τ pol)	0.1439 ± 0.0043	0.1469 ± 0.0004[58, 59]
A_c	0.670 ± 0.027	0.66773 ± 0.0002[58, 59]
A_b	0.923 ± 0.020	0.92694 ± 0.00006[56–58]
A_s	0.895 ± 0.091	0.93563 ± 0.00004[58, 59]
$A_{e,FB}$	0.0145 ± 0.0025	0.0162 ± 0.0001[58, 59]
$A_{\mu,FB}$	0.0169 ± 0.0013	0.0162 ± 0.0001[58, 59]
$A_{\tau,FB}$	0.0188 ± 0.0017	0.0162 ± 0.0001[58, 59]
$A_{b,FB}$	0.0996 ± 0.0016	0.1021 ± 0.0003[56–58]
$A_{c,FB}$	0.0707 ± 0.0035	0.0736 ± 0.0003[58, 59]
$A_{s,FB}$	0.0976 ± 0.0114	0.10308 ± 0.0003[58, 59]

Bounds from current Z-pole observables

No meaningful marginalized bounds due to the degeneracy



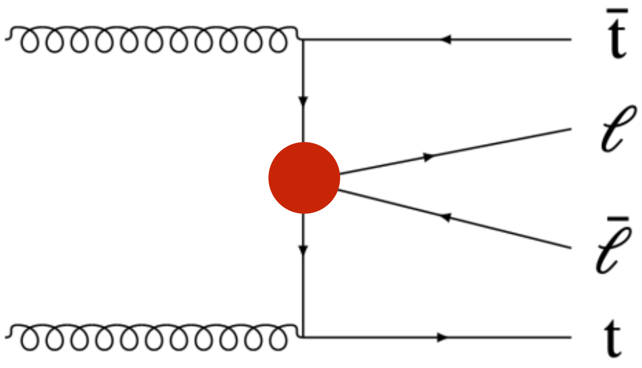
Z-pole at NLO in SMEFT:
Dawson, Giardino, 1909.02000
Bellafronte, Dawson, Giardino, 2304.00029
Biekotter, Pecjak, 2503.07724

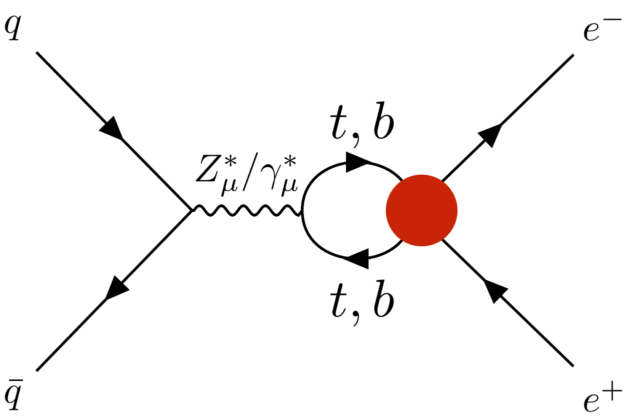
Observables at the LHC/HL-LHC

[Greljo, etc., 2104.02723]

[Camacho, Master's thesis, U. Valencia]

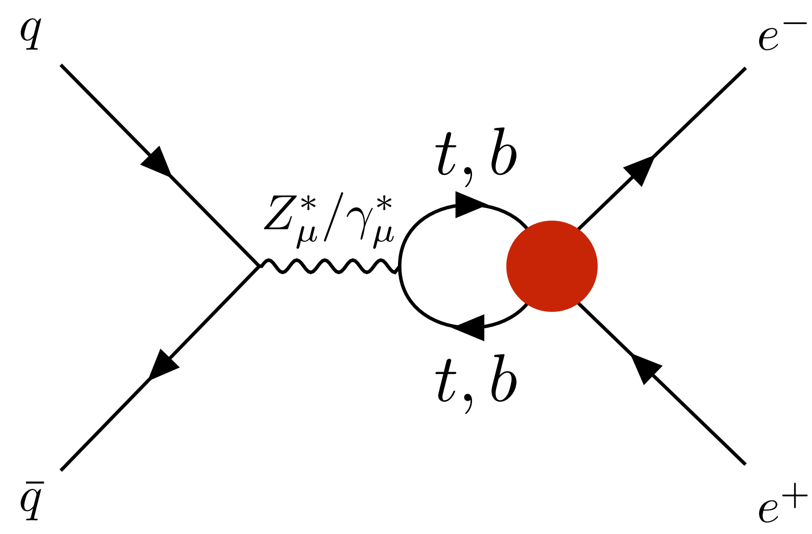
We only use bins with $m_{ee} < 800\ \mathrm{GeV}$

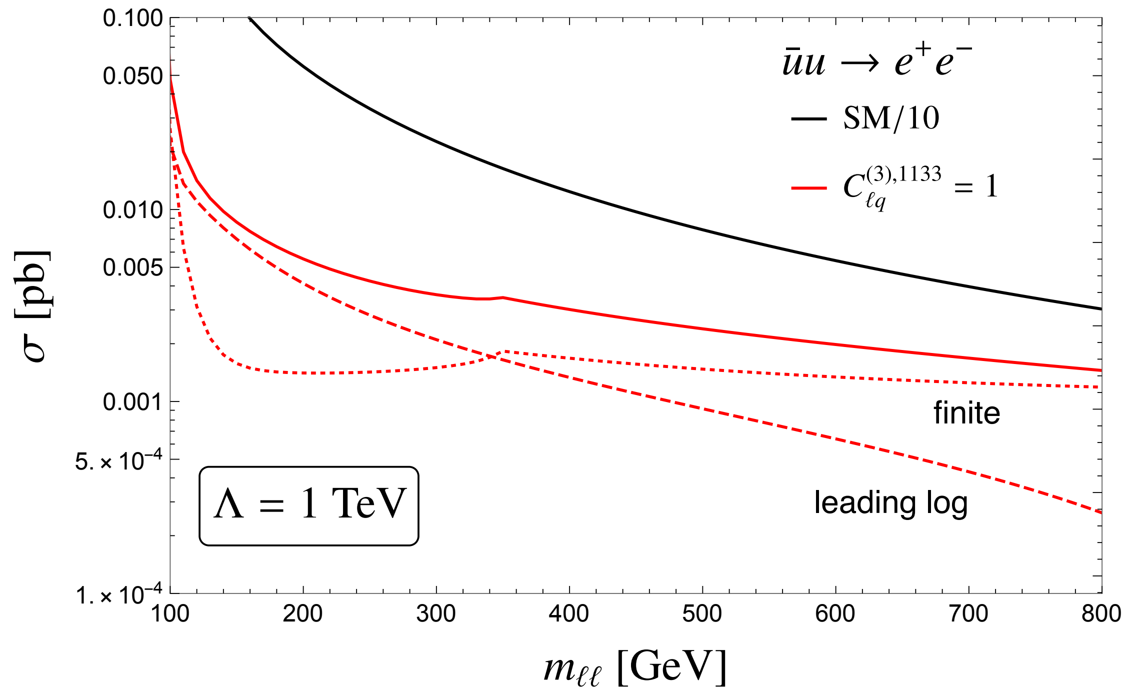




Phase space suppression ~ Loop suppression $\sim \dfrac{1}{16\pi^2}$

Drell-Yan at NLO



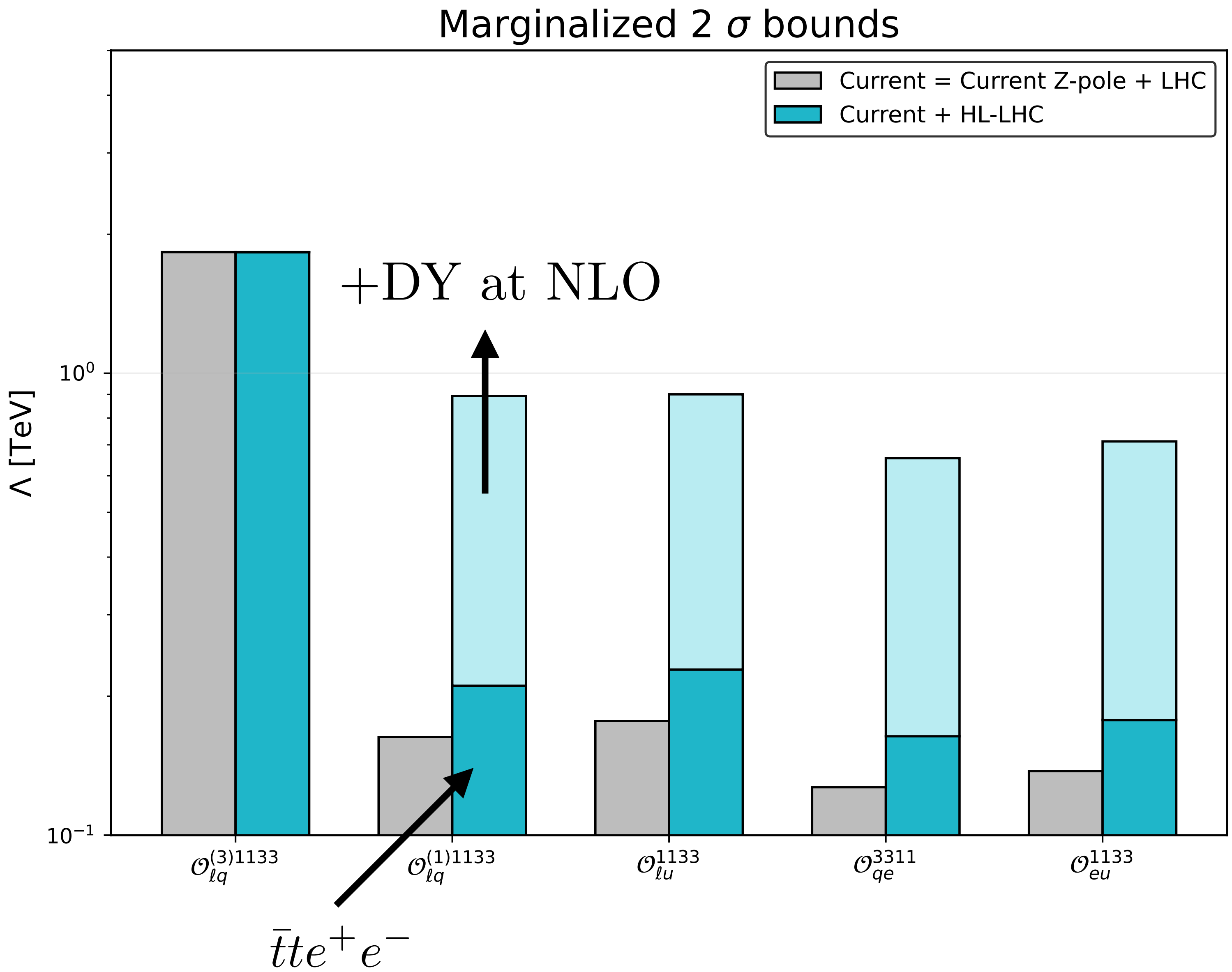


$$\frac{\hat{\sigma}_{\rm DY}^{\rm EFT}}{\hat{\sigma}_{\rm DY}^{\rm SM}} = 1 + \sum_i C_i(\mu)\{\Delta_i + \bar{\Delta}_i \log\frac{\mu^2}{s}\}$$

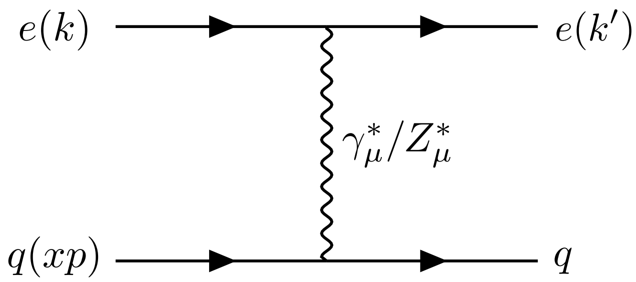
$$\mu = \Lambda = 1 \text{ TeV}, \sqrt{\hat{s}} = 600 \text{ GeV}$$

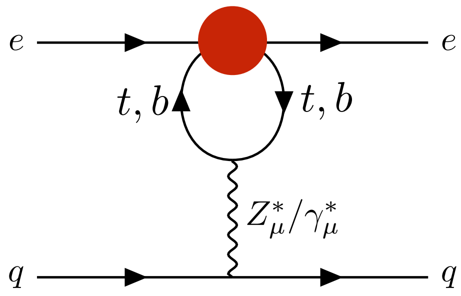
Operators		$\mathcal{O}_{\ell q}^{(3),1133}$	$\mathcal{O}_{\ell q}^{(1),1133}$	$\mathcal{O}_{\ell u}^{1133}$	$\mathcal{O}_{qe}^{3311}$	$\mathcal{O}_{eu}^{1133}$
$\bar{u}u \to e^+e^-$	Δ_i	2.5×10^{-2}	-2.1×10^{-3}	-6.1×10^{-3}	-4.0×10^{-3}	-2.8×10^{-3}
	$\bar{\Delta}_i$	1.2×10^{-2}	2.5×10^{-4}	-2.5×10^{-3}	-1.8×10^{-3}	-1.1×10^{-3}
$\bar{d}d \to e^+e^-$	Δ_i	3.3×10^{-2}	1.1×10^{-3}	-4.0×10^{-3}	-7.9×10^{-4}	-1.6×10^{-3}
	$\bar{\Delta}_i$	1.6×10^{-2}	2.4×10^{-3}	-1.5×10^{-3}	-4.3×10^{-4}	-1.2×10^{-3}

Bounds from current + HL-LHC



DIS at the EIC





$$|M_{\rm SM}|^2 \sim \mathcal{C}_{\gamma\gamma}\frac{\hat{s}^2}{Q^4} + \mathcal{C}_{\gamma Z}\frac{\hat{s}^2}{Q^2 M_W^2}$$

$$2{\rm Re}[M_{\rm SM} M^*_{\rm SMEFT}] \sim C_{\rm EFT}\frac{\hat{s}^2}{Q^2\Lambda^2}{\rm Log}\left(\frac{m_t^2}{Q^2}\right)$$

SMEFT/SM cross ratio increases with Q^2

$$\frac{\sigma_{\rm SMEFT}(ep \to ej)}{\sigma_{\rm SM}(ep \to ej)} \sim \frac{Q^2}{\Lambda^2}{\rm Log}\left(\frac{m_t^2}{Q^2}\right)$$

The EIC with polarized electron beam

- The EIC will be able to deliver 70% polarized electron beam with 1% uncertainties
- To cancel the large SM photon contributions, we compute the left-right asymmetry

$$A_{\mathrm{PV}} \equiv p_e \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

$$|M_{\mathrm{SM}}|^2 \sim \mathcal{C}_{\gamma\gamma} \frac{\hat{s}^2}{Q^4} + \mathcal{C}_{\gamma Z} \frac{\hat{s}^2}{Q^2 M_W^2}$$

$$2\mathrm{Re}[M_{\mathrm{SM}} M^*_{\mathrm{SMEFT}}] \sim C_{\mathrm{EFT}} \frac{\hat{s}^2}{Q^2 \Lambda^2} \mathrm{Log}\left(\frac{m_t^2}{Q^2}\right)$$

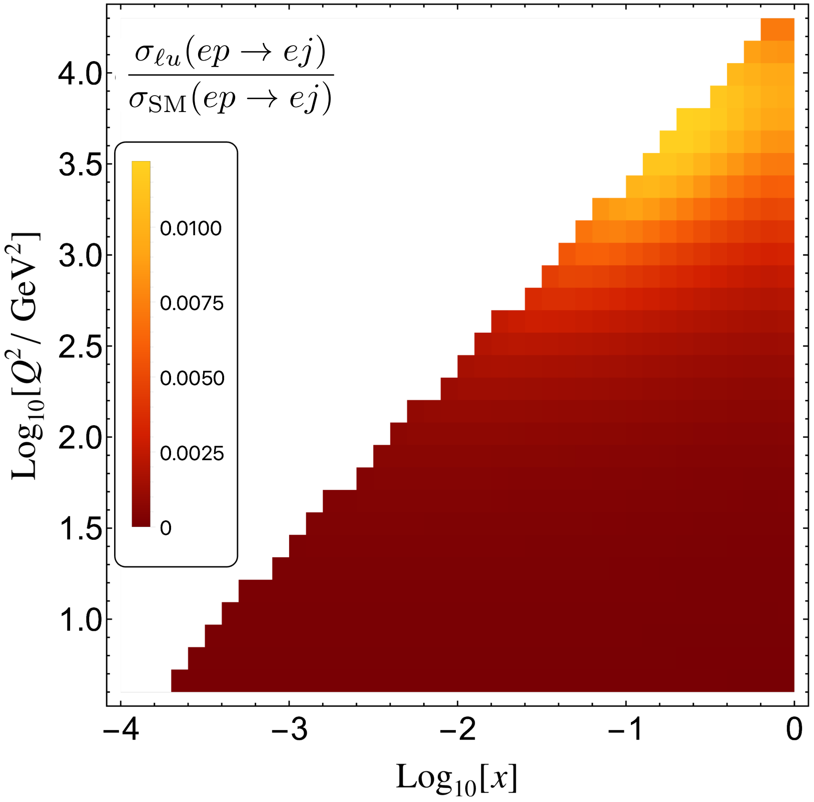
$$A_{\mathrm{PV}}^{\mathrm{SM}} \sim \frac{Q^2}{M_W^2}$$

$$A_{\mathrm{PV}}^{\mathrm{EFT}} \sim \frac{Q^2}{\Lambda^2} \mathrm{Log}\left(\frac{m_t^2}{Q^2}\right)$$

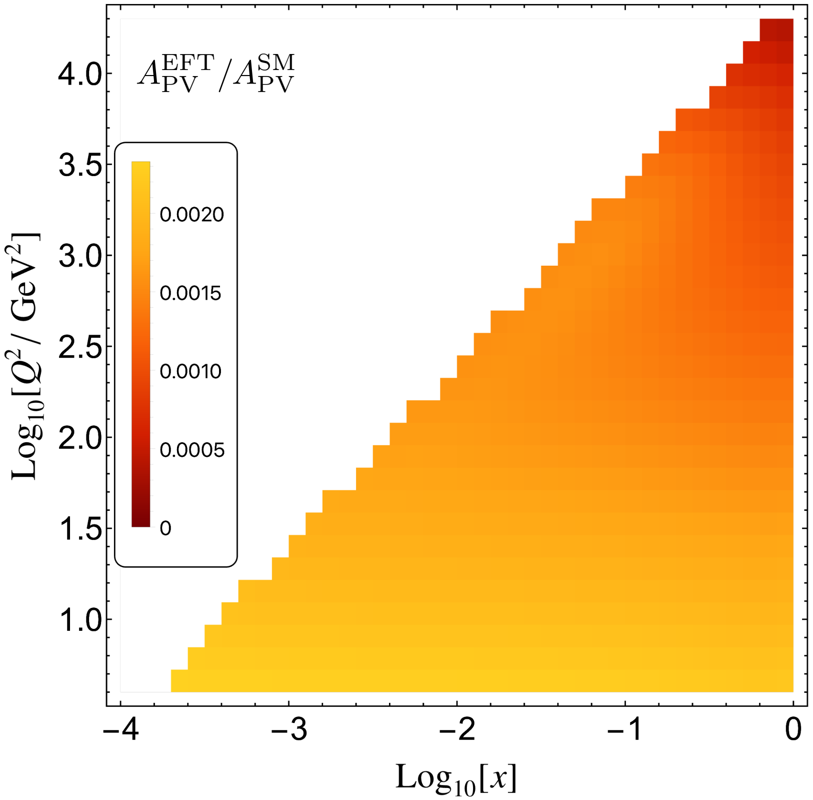
$$\frac{A_{\mathrm{PV}}^{\mathrm{EFT}}}{A_{\mathrm{PV}}^{\mathrm{SM}}} \sim \frac{M_W^2}{\Lambda^2} \mathrm{Log}\left(\frac{m_t^2}{Q^2}\right)$$

DIS at the EIC

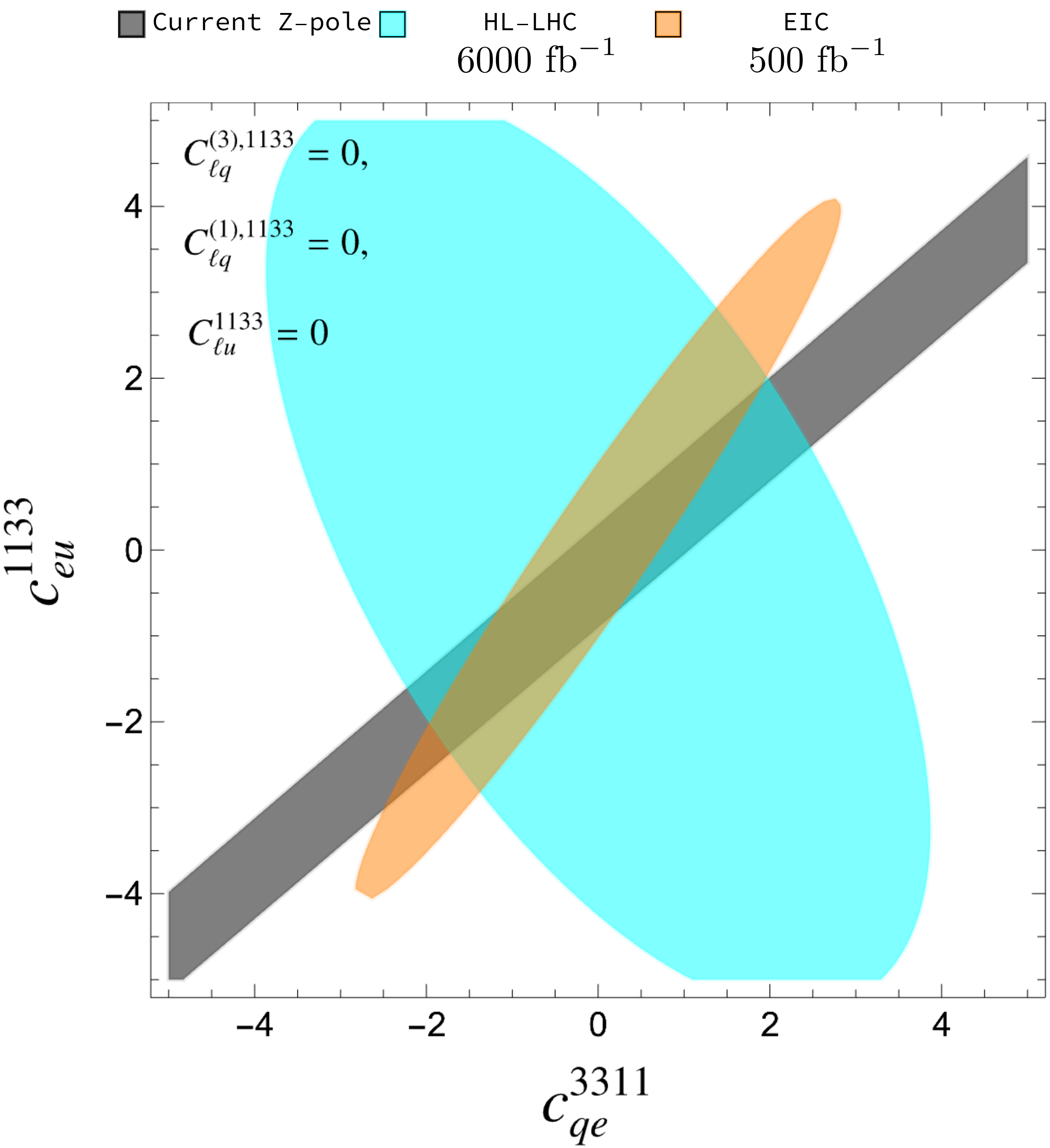
Without polarization



With polarization



Correlations



FCC-ee runs

[Prospects in Electroweak, Higgs and Top physics at FCC]

	FCC-ee			
$\sqrt{s}$	88-94 GeV	157-163 GeV	240 GeV	340-365 GeV
Run duration (years)	4	2	3	5
Integrated luminosity (ab^{-1})	205	19.2	10.8	3.12
	$N_Z \sim 10^{12}$	$N_{WW} \sim 10^8$	$N_{Zh} \sim 10^6$	$N_{t\bar{t}} \sim 10^6$

Tera Z produced at Z-pole can produce high accuracy

Above Z-pole, there are three collider energies. Energy enhanced effects are important.

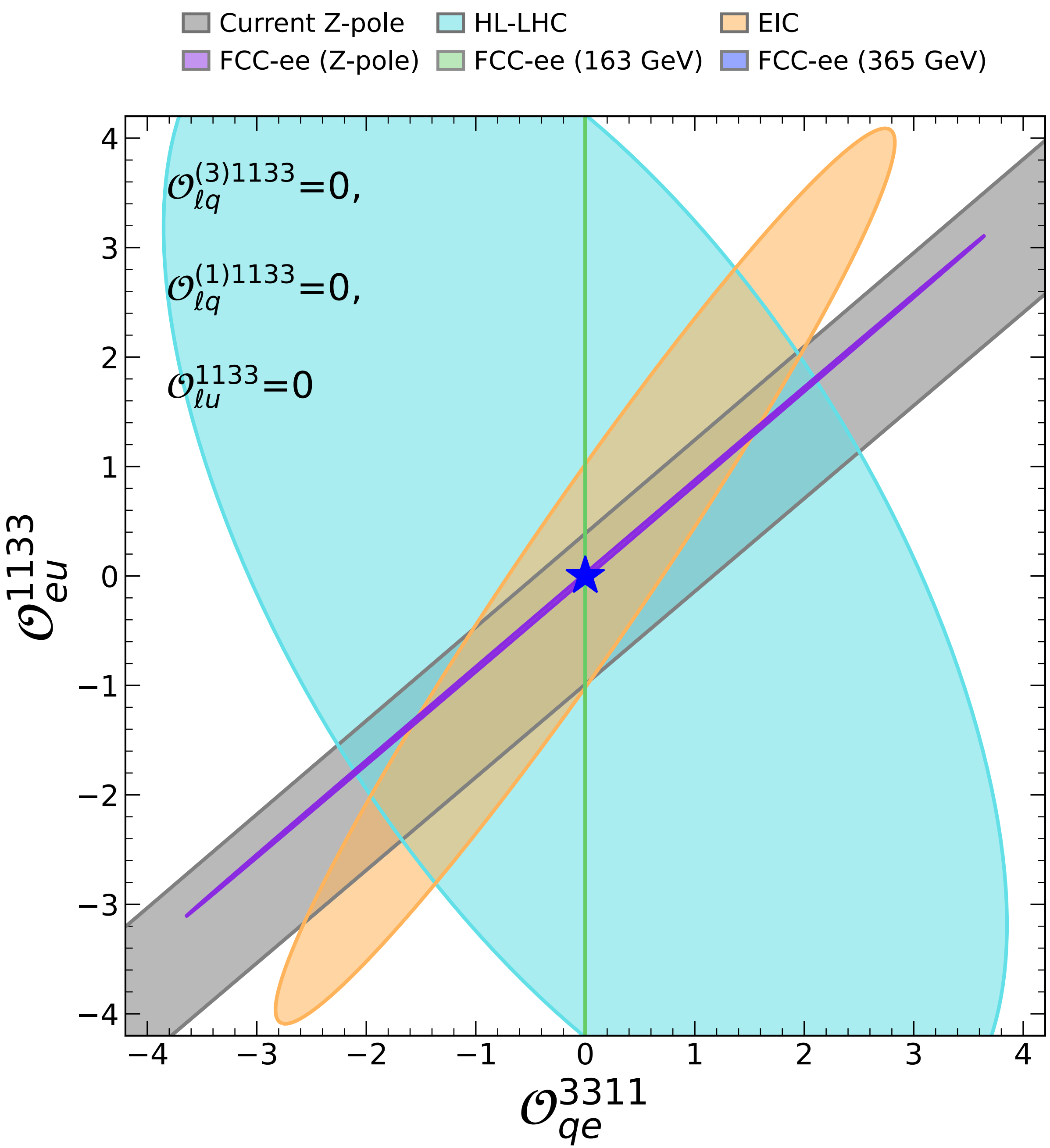
$$\Delta_{\mathrm{NLO/LO}} = \frac{1}{16\pi^2} \frac{M_Z^2}{M_{\bar{t}t}^2} \sqrt{\frac{N_Z}{N_{\bar{t}t}}} \simeq 0.3$$

FCC-ee: above the Z-pole

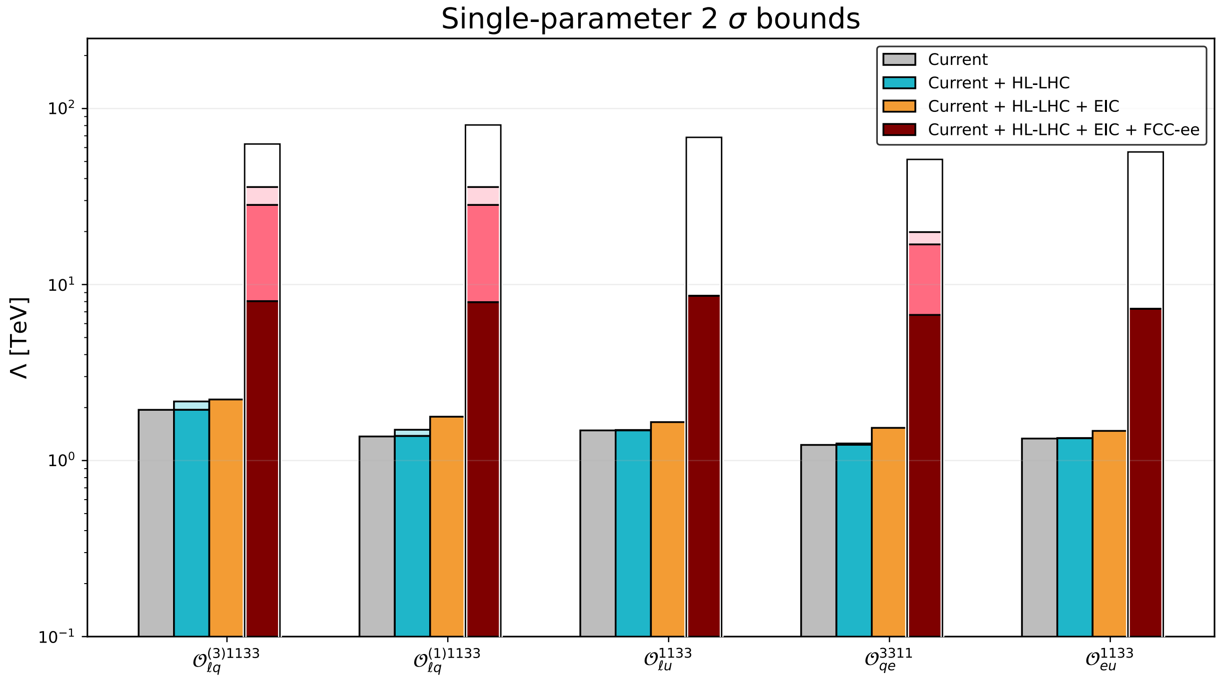
Rel. error (10^{-3})	163 GeV	240 GeV	365 GeV
R_b	0.17	0.36	0.96
σ_{Zh}	-	3.1	5.2
R_t	-	-	1.2
$A_{t,\mathrm{FB}}$	-	-	8.9

[Greljo, Tiblom, Valenti, 2411.02485; Li, etc., 2512.21290]

Correlations



Results



Single-parameter 2 σ bounds
Current
Current + HL-LHC
Current + HL-LHC + EIC
Current + HL-LHC + EIC + FCC-ee
Λ [TeV]
10^{2}
10^{1}
10^{0}
10^{-1}
$\mathcal{O}_{\ell q}^{(3)1133}$
$\mathcal{O}_{\ell q}^{(1)1133}$
$\mathcal{O}_{\ell u}^{1133}$
$\mathcal{O}_{qe}^{3311}$
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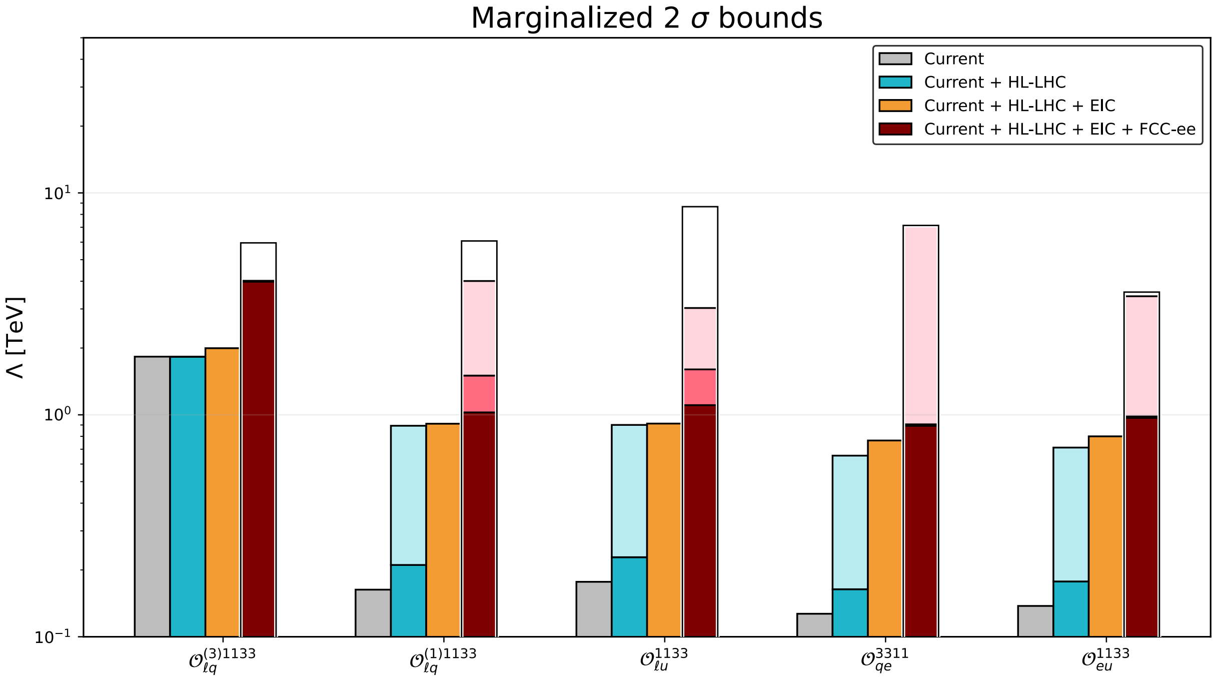


Marginalized 2 σ bounds
Current
Current + HL-LHC
Current + HL-LHC + EIC
Current + HL-LHC + EIC + FCC-ee
Λ [TeV]
10^{1}
10^{0}
10^{-1}
$\mathcal{O}_{\ell q}^{(3)1133}$
$\mathcal{O}_{\ell q}^{(1)1133}$
$\mathcal{O}_{\ell u}^{1133}$
$\mathcal{O}_{qe}^{3311}$
$\mathcal{O}_{eu}^{1133}$

Summary

★ Top quark-electron four fermion operators can be probed at the DY/DIS process via loop effects. We compute the corresponding NLO electroweak corrections.

★ The current Z-pole observables impose a stringent constraint on the SU(2) triplet 4-fermion operator $\mathcal{O}_{\ell q}^{(3),1133}$. While others are unconstrained due to degeneracies.

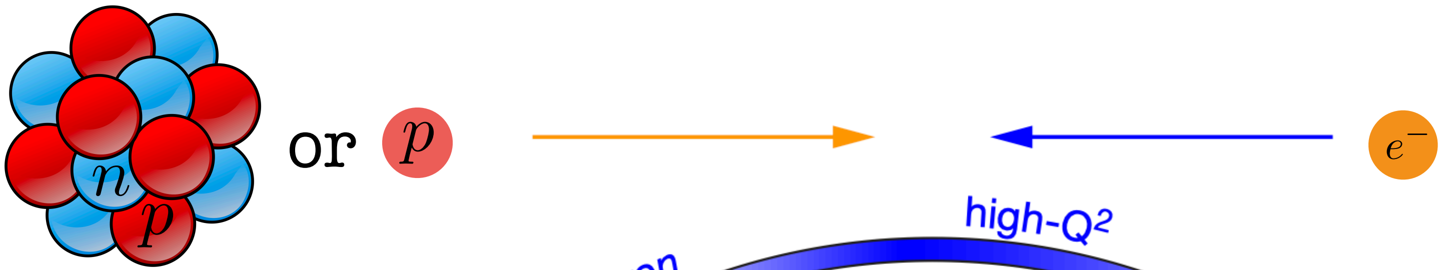
★ The EIC with polarized electrons beams is complementary to the HL-LHC and helps to break degeneracies.

★ FCC-ee can improve the bounds significantly due to its high-luminosity and the energy availability to produce top-pair.

Back-up slides

EIC Overview

The different detector systems observe different particle distributions.

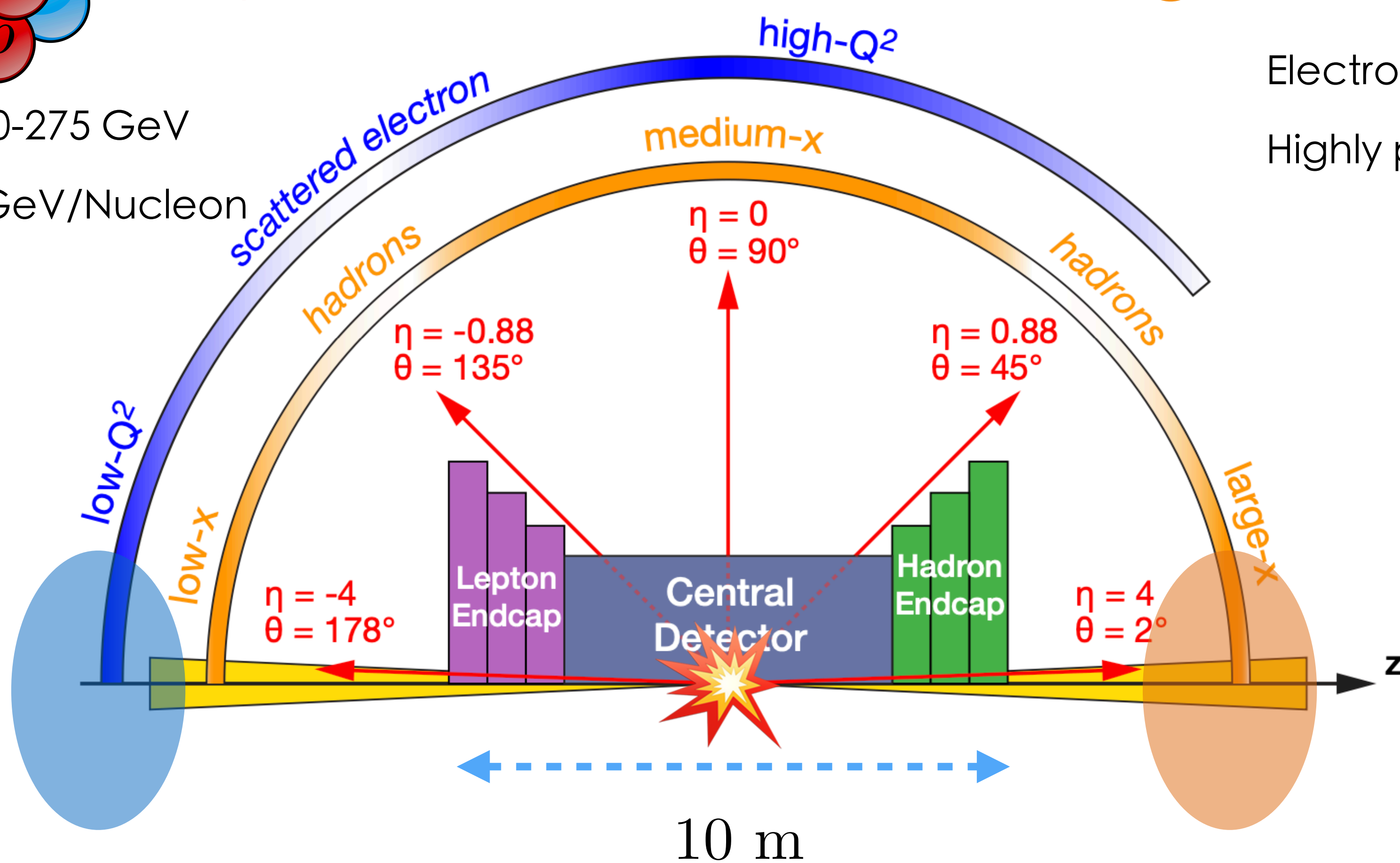


Proton: 40-275 GeV

Ion: 100 GeV/Nucleon

Electron: 5 - 18 GeV

Highly polarized (~70%)



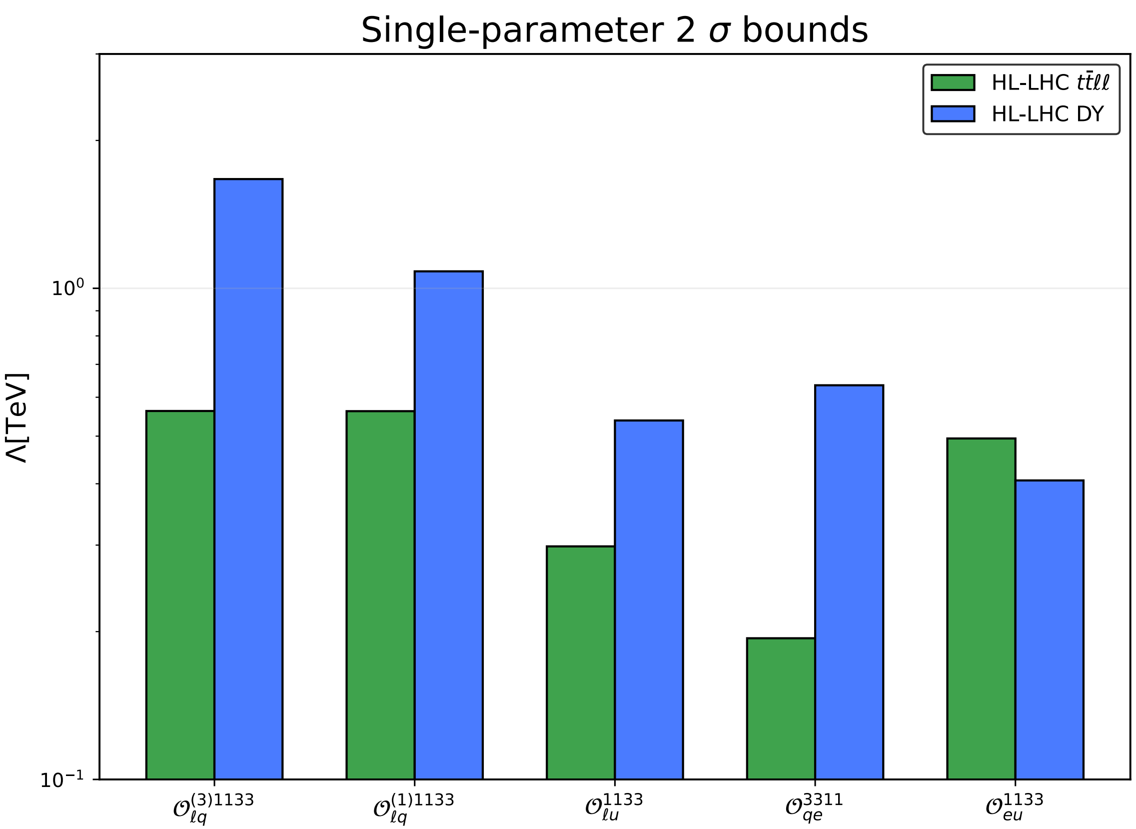
Far-backward detector

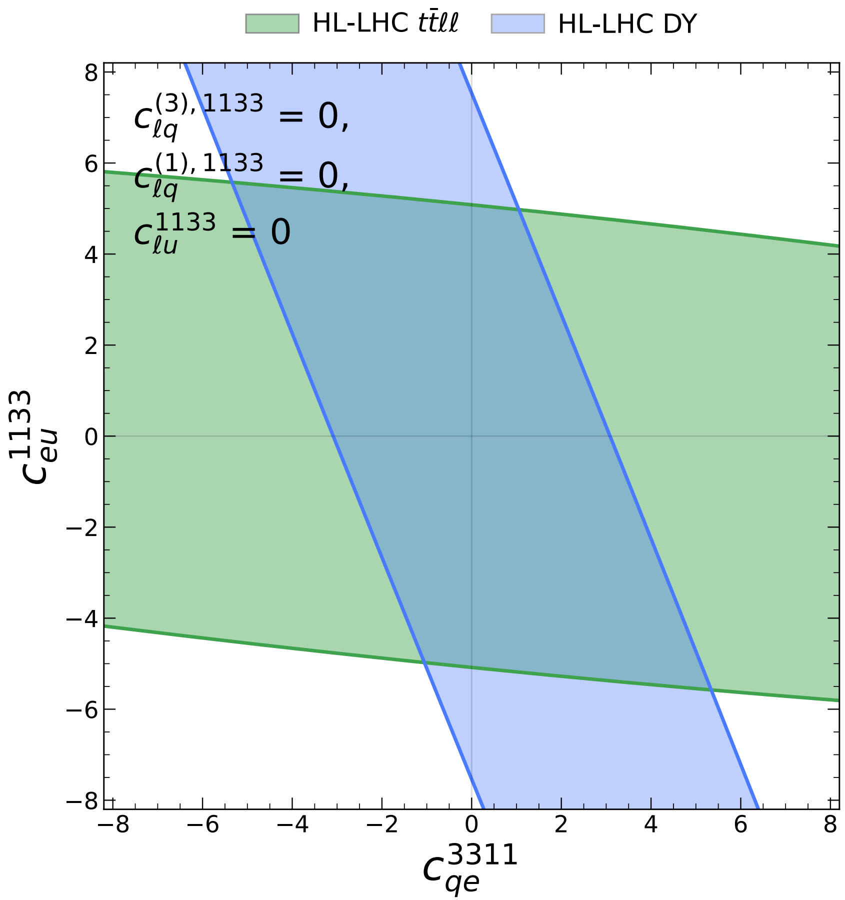
Far-forward detector

Excellent reconstruction accuracy $Q^2 > 10^{-3}\ \mathrm{GeV}^2$

Observables at the LHC/HL-LHC

6/ab integrated luminosity is assumed





FCC-ee: Z,W-pole

The future uncertainties will be dominated by the theory

Observables	Experimental values	current (future) theoretical values	FCC-ee projections
$1/\alpha_{\text{QED}}$	128.952 ± 0.014	128.316 ± 0.009	$\pm 0.8 \times 10^{-3}$
Γ_W(GeV)	2.085 ± 0.042	2.0903 ± 0.0003[55]	$\pm 2.7 \times 10^{-4} \pm 2.0 \times 10^{-4}$
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R_e	20.804 ± 0.05	20.732 ± 0.006 (1.2×10^{-3})[56–58]	$\pm 3.4 \times 10^{-6} \pm 2.3 \times 10^{-6}$
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A_s	0.895 ± 0.091	0.93563 ± 0.00004[58, 59]	$\pm 1.2 \times 10^{-4}$
$A_{e,FB}$	0.0145 ± 0.0025	0.0162 ± 0.0001[58, 59]	$\pm 3.3 \times 10^{-6} \pm 2.4 \times 10^{-6}$
$A_{\mu,FB}$	0.0169 ± 0.0013	0.0162 ± 0.0001[58, 59]	$\pm 2.3 \times 10^{-6} \pm 2.4 \times 10^{-6}$
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$A_{b,FB}$	0.0996 ± 0.0016	0.1021 ± 0.0003[56–58]	$\pm 4 \times 10^{-6} \pm 4 \times 10^{-6}$
$A_{c,FB}$	0.0707 ± 0.0035	0.0736 ± 0.0003[58, 59]	$\pm 5 \times 10^{-6} \pm 5 \times 10^{-6}$
$A_{s,FB}$	0.0976 ± 0.0114	0.10308 ± 0.0003[58, 59]	$\pm 7.4 \times 10^{-6} \pm 7.4 \times 10^{-6}$
M_W(GeV) PDG World Ave	80.377 ± 0.012	80.357 ± 0.004 (0.001)[60, 61]	$\pm 1.8 \times 10^{-4} \pm 1.6 \times 10^{-4}$

Z-pole observables

Input: $\{G_F, m_W, m_Z\}$ $\qquad C/\Lambda^2 = 1/\mathrm{TeV}^2$

QED coupling: $1/\alpha_{\rm QED}$

Total decay width: $\Gamma_Z, \quad \Gamma_W$

The R ratios:

$$R_\ell \equiv \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \ell\bar{\ell})} \qquad R_q \equiv \frac{\Gamma(Z \to q\bar{q})}{\Gamma(Z \to \text{hadrons})}$$

Hadronic cross sections:

$$\sigma_h \equiv \sigma(e^+e^- \to Z \to \text{hadrons})$$

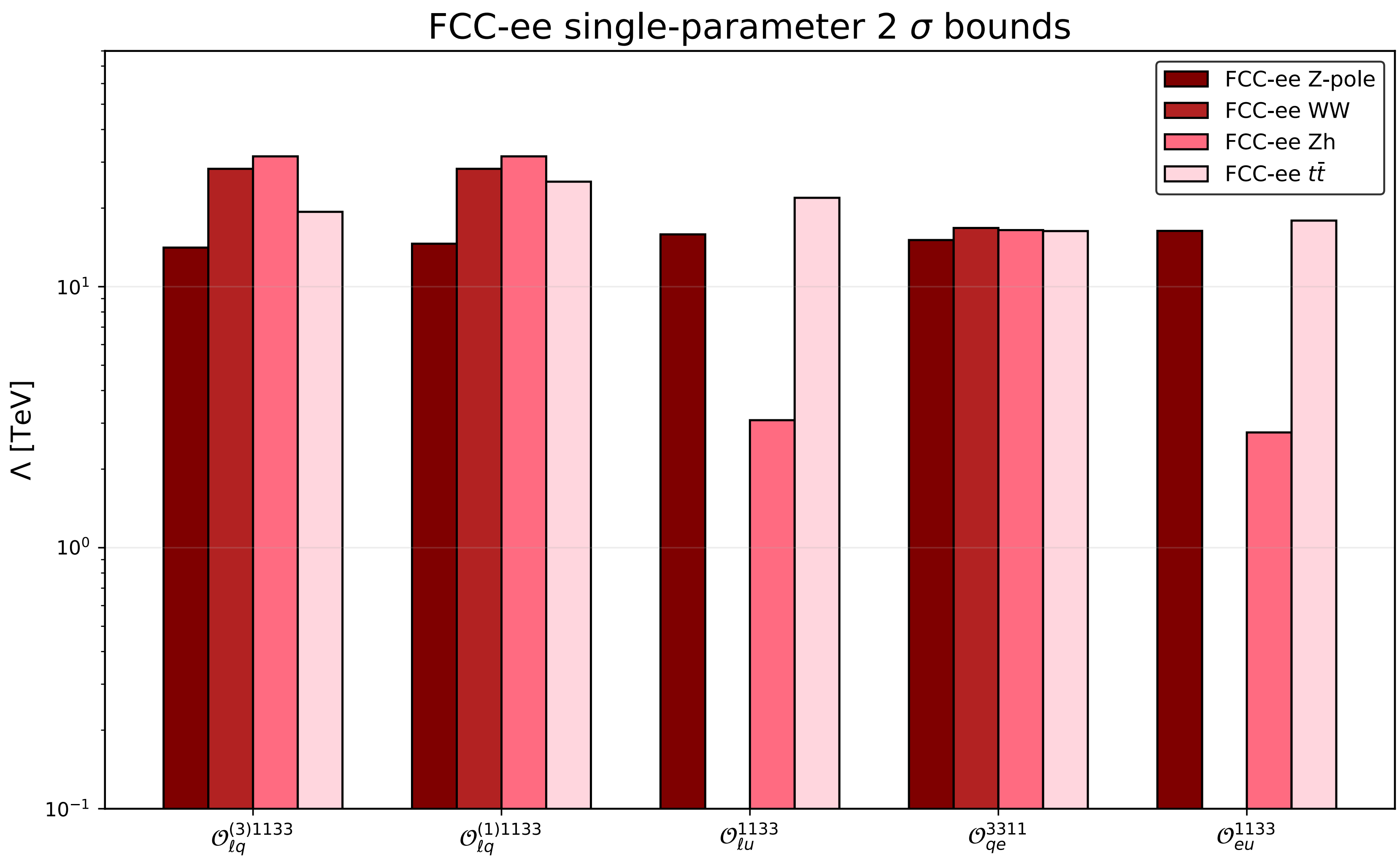
The asymmetries:

$$A_f \equiv \frac{\sigma_{f,L} - \sigma_{f,R}}{\sigma_{f,L} + \sigma_{f,R}}$$

$$A_{f,\rm FB} \equiv \frac{\sigma_{f,F} - \sigma_{f,B}}{\sigma_{f,F} + \sigma_{f,B}}$$

Observable	$\mathcal{O}_{\ell q}^{(3)1133}$	$\mathcal{O}_{\ell q}^{(1)1133}$	$\mathcal{O}_{\ell u}^{1133}$	$\mathcal{O}_{qe}^{3311}$	$\mathcal{O}_{eu}^{1133}$
$1/\alpha_{\rm QED}$	4.4×10^{1}	0	0	0	0
Γ_W	1.8×10^{-2}	0	0	0	0
Γ_Z	1.8×10^{-1}	7.1×10^{-2}	8.3×10^{-2}	7.1×10^{-2}	8.3×10^{-2}
R_e	2.04	4.09	2.01	1.69	3.71
R_μ	1.4×10^{-1}	2.3×10^{-2}	0	2.3×10^{-2}	0
R_τ	2.5×10^{-2}	3.8×10^{-3}	0	3.8×10^{-3}	0
R_c	5.9×10^{-4}	4.9×10^{-5}	0	4.9×10^{-5}	0
R_b	8.4×10^{-2}	1.8×10^{-2}	0	1.8×10^{-2}	0
σ_h	7.92	1.3×10^{1}	1.3×10^{1}	3.85	9.15
A_e	1.25	3.84	7.17	4.53	9.65
A_μ	1.0×10^{-1}	0	0	0	0
A_τ	1.05	0	0	0	0
A_s	1.5×10^{-3}	0	0	0	0
A_c	4.3×10^{-3}	0	0	0	0
A_b	2.9×10^{-3}	6.6×10^{-5}	0	6.6×10^{-5}	0
$A_{\rm FB}^e$	5.4×10^{-2}	2.6×10^{-1}	2.3×10^{-1}	3.4×10^{-1}	2.8×10^{-1}
$A_{\rm FB}^\mu$	1.2×10^{-1}	1.5×10^{-1}	2.5×10^{-1}	1.9×10^{-1}	3.3×10^{-1}
$A_{\rm FB}^\tau$	2.9×10^{-1}	3.7×10^{-1}	4.8×10^{-1}	4.7×10^{-1}	6.1×10^{-1}
$A_{\rm FB}^s$	2.6×10^{-2}	1.3×10^{-1}	1.2×10^{-1}	1.7×10^{-1}	1.4×10^{-1}
$A_{\rm FB}^c$	6.5×10^{-2}	5.3×10^{-1}	4.2×10^{-1}	7.0×10^{-1}	5.0×10^{-1}
$A_{\rm FB}^b$	5.6×10^{-1}	3.26	2.07	4.39	2.31

FCC-ee fits



FCC-ee single-parameter 2 σ bounds
FCC-ee Z-pole
FCC-ee WW
FCC-ee Zh
FCC-ee $t\bar{t}$
Λ [TeV]
10^{1}
10^{0}
10^{-1}
$\mathcal{O}_{\ell q}^{(3)1133}$
$\mathcal{O}_{\ell q}^{(1)1133}$
$\mathcal{O}_{\ell u}^{1133}$
$\mathcal{O}_{qe}^{3311}$
$\mathcal{O}_{eu}^{1133}$