Mini Kinetic Misalignment and QCD Axion Domain Wall from Supercooling FOPT

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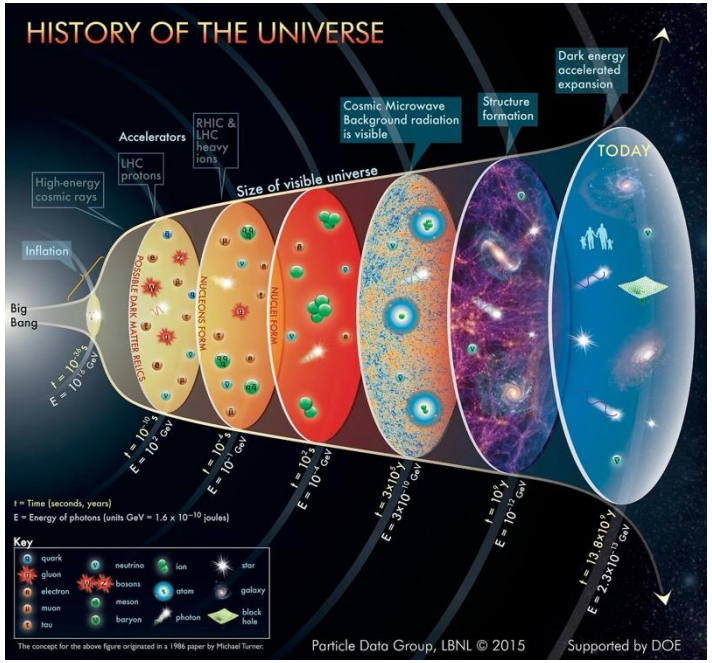

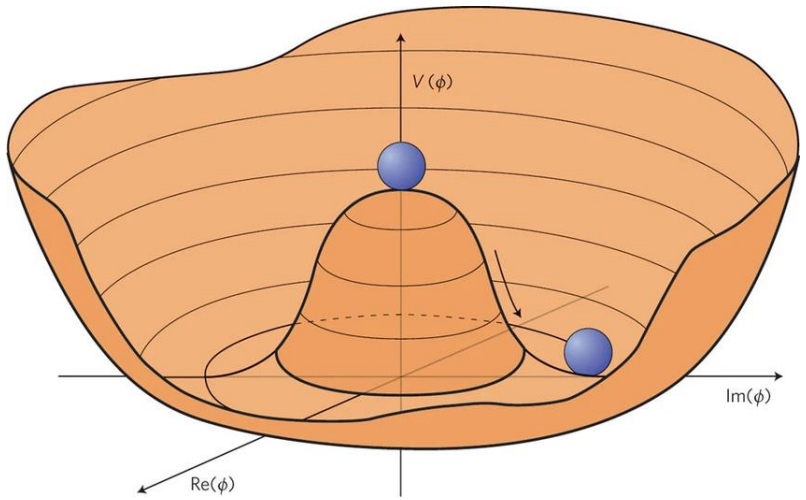

Based on 2506.19918

In collaboration with Yue Zhao

Phenomenology 2026 Symposium,

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QCD Axion

- Definition: solve strong CP problem
- Minimal Realization:

  Goldstone boson of U(1) PQ symmetry

$$(\bar{\theta} + \frac{a}{f_a}) G\tilde{G} \quad \rightarrow \theta_a$$

$$m_a = 5.70\,\mu\text{eV}\left(\frac{10^{12}\,\text{GeV}}{f_a}\right)$$

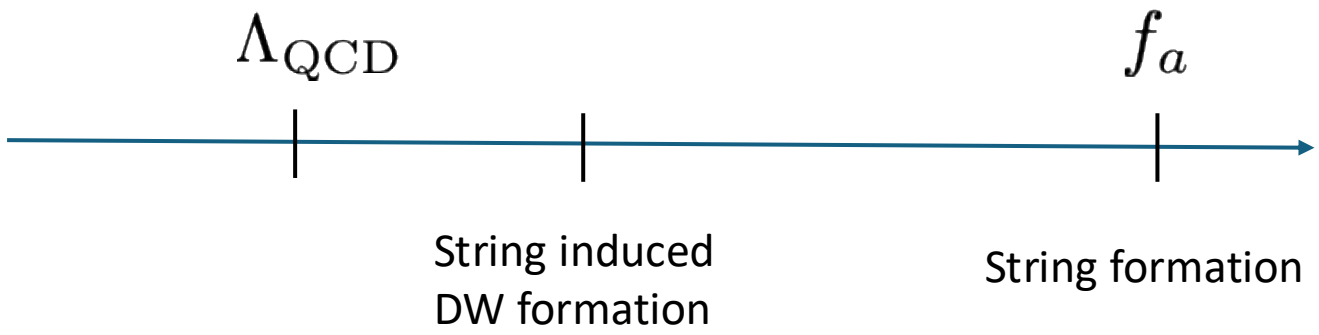
Roberto Peccei and Helen Quinn

- Dark Matter: Misalignment
- Pre-inflation case: axion is frozen until

$$m_a(T) \sim 3H(T)$$

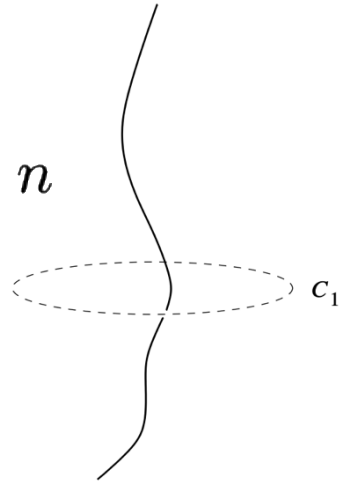
$$\Omega_a h^2 \sim 0.12\left(\frac{f_a}{10^{12}\,\text{GeV}}\right)^{7/6}\langle\theta_i^2\rangle$$

Post-inflation: Topological Defect Cosmic String and Domain Wall



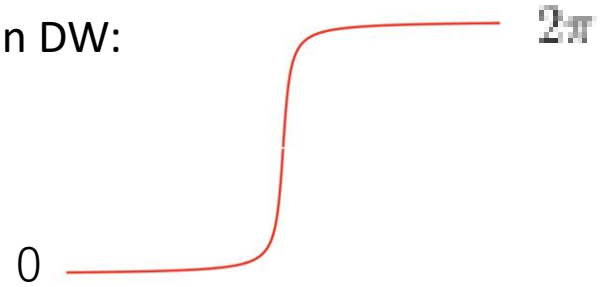
Axion string:

$\delta\theta_a = 2\pi \times n$



- If $N_{\rm DW} > 1$, the string-domain wall network will dominate universe.
- For $N_{\rm DW} = 1$, the string-domain wall network would be quickly collapsed and decay to axion radiation.

Axion DW:



Axion DW

- Domain wall is the 2D topological defect connecting the two degenerate vacuums.
- For $N_{\rm DW} = 1$, the DW profile satisfies the EOM

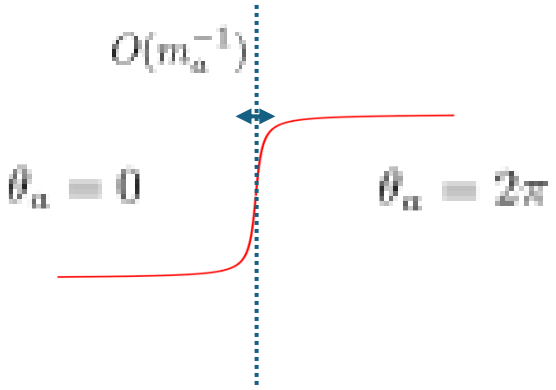
$$\frac{d^2a}{dz^2} = \frac{dV}{da} = \frac{\Lambda^4}{f_a}\sin\left(\frac{a}{f_a}\right) \qquad V(a) = \Lambda^4\left(1-\cos\left(\frac{a}{f_a}\right)\right)$$

- Sine-Gordan soliton with solution

$$\theta_a(z) = 4\arctan\left[\exp(m_a z)\right]$$

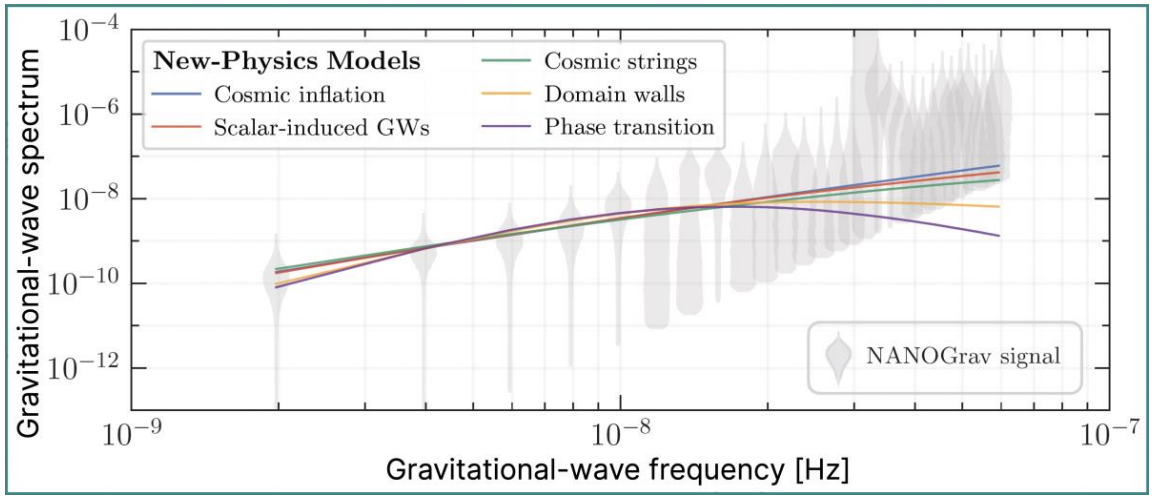
- The wall tension is given by

$$\sigma = \int_{-\infty}^{+\infty}\left[\frac{1}{2}\left(\frac{d\phi}{dz}\right)^2 + V(\phi)\right]dz = \int_{-\infty}^{+\infty} 2V(\phi)\,dz = 8m_a f_a^2$$



Non-standard Cosmic Evolution

- NANOGrav Observation: GW signal at QCD scale



Supercooling FOPT

$$\frac{\Delta\rho_{\rm vac}}{\rho_{\rm rad}} \gg 1$$

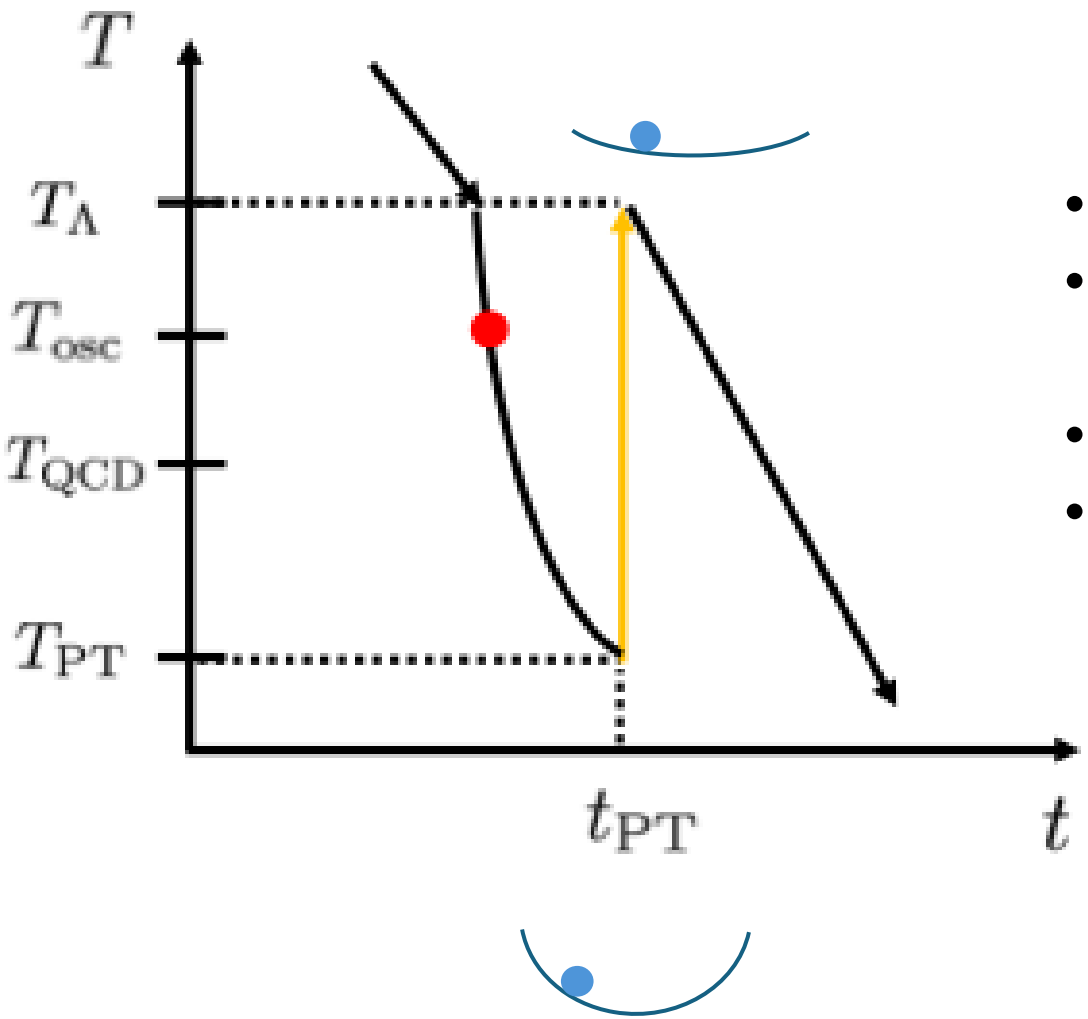
2306.16219, NANOGrav Collaboration

- Strong First Order Phase Transition (SFOPT) is one of the candidate to explain the data.
- If it is supercooling FOPT in the dark sector, the latent heat can reheat the the SM thermal bath.

Altering the QCD axion evolution

2506.19918, **KFL**, YZ

Minimal QCD Axion $N_{\rm DW} = 1$



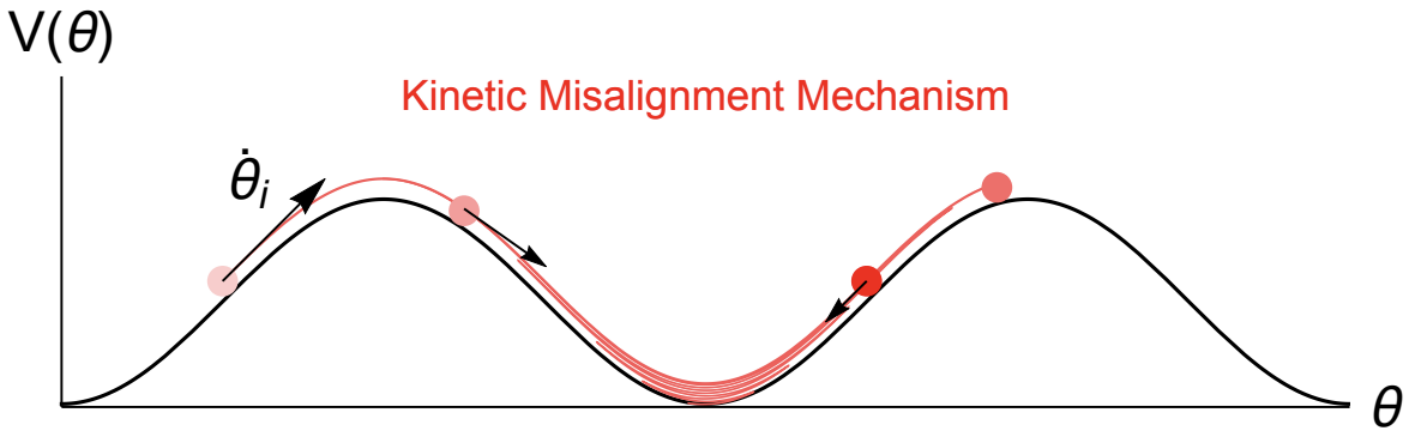
- At $T = T_{\rm osc}$, the axion field starts to oscillate.
- At $T = T_{\rm PT}$, the SM thermal bath is reheated to high temperature.
- The axion velocity is kept unchanged.
- The kinetic energy and the barrier height potential are defined to be

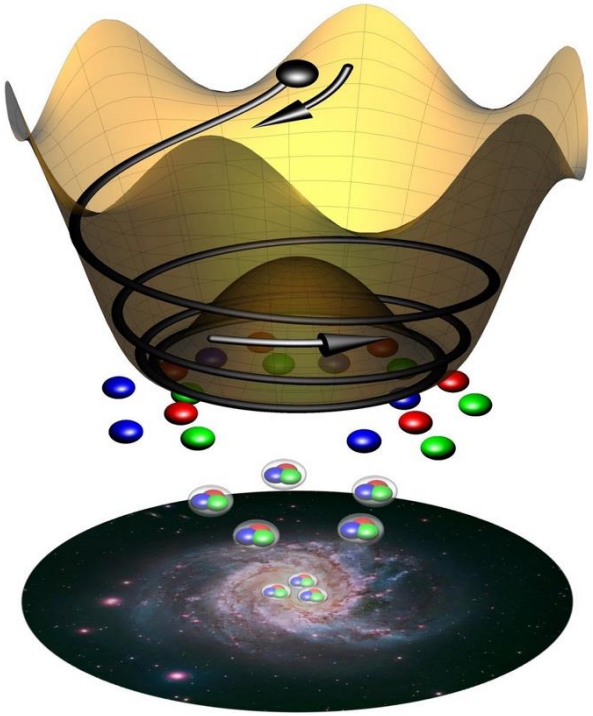
$$\theta_a = a/f_a$$

$$K = \frac{1}{2} f_a^2 \dot{\theta}_a^2 \qquad V_{\rm max} = 2 m_a (T_\Lambda)^2 f_a^2$$

Kinetic Misalignment

- If $K \gg V_{\max}$, the axion is rolling like kination



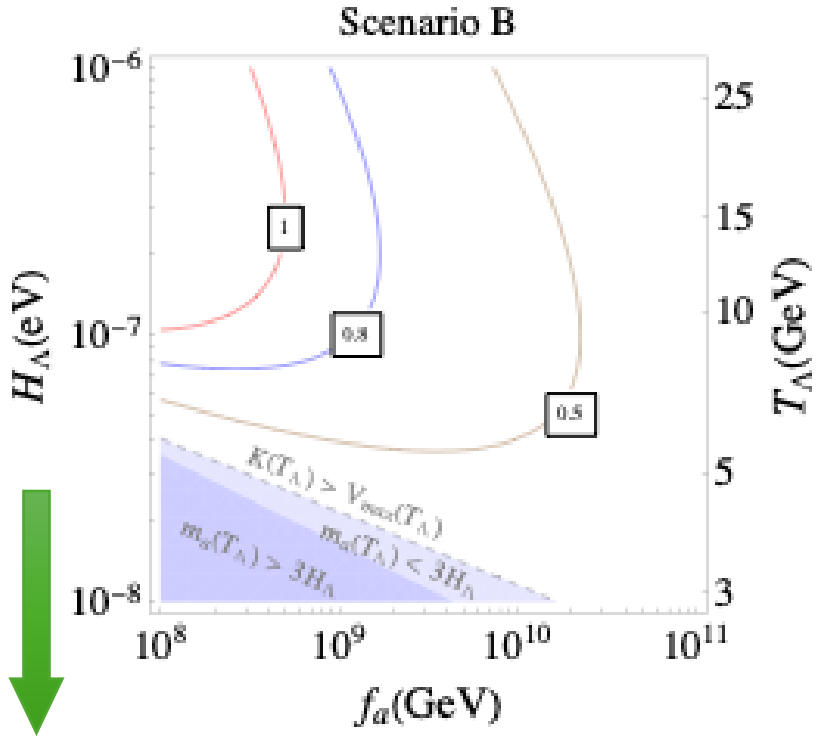


Co et al, 1910.14152

Mini Kinetic Misalignment

- The reheating time scale $t_{\rm reh}$ can be model dependent. First we consider the case $t_{\rm reh} > \Delta t_{\rm PT}$. The universe is reheated uniformly.
- There is parametric space such that $K \gg V_{\rm max}$ but not lasting long.
- The theta angle would be "reset" after being trapped again.
- The axion relic abundance can be modified.

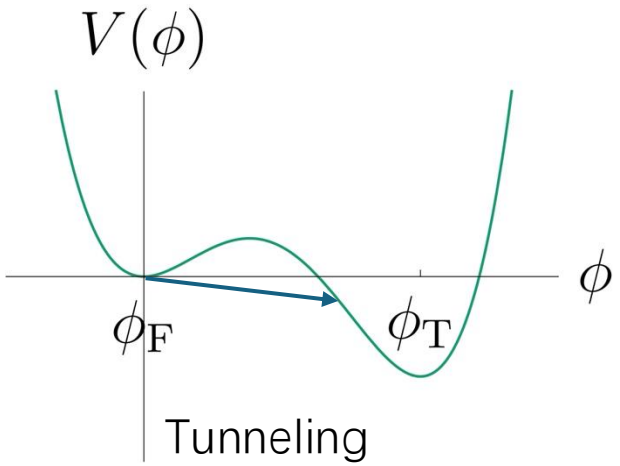
The reset field angle assuming the initial field angle is $\theta_i = 0.2$

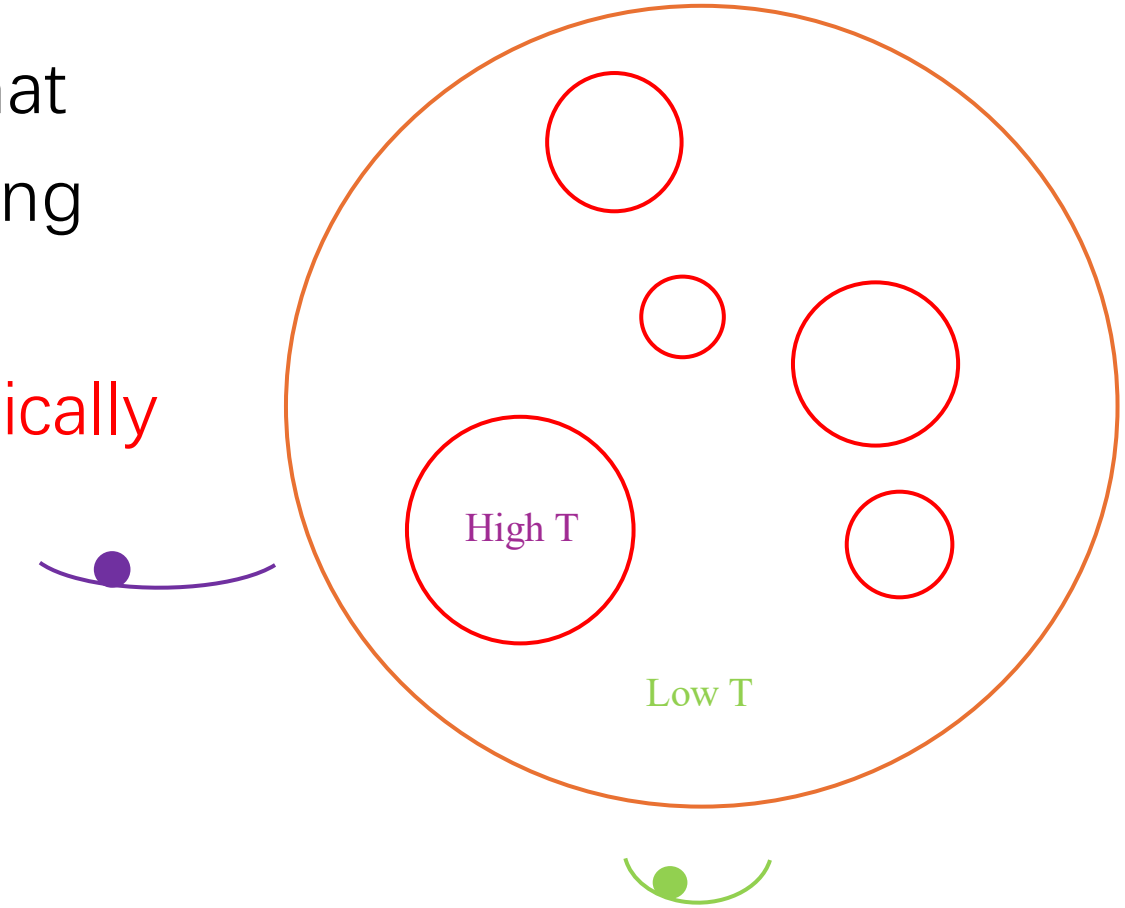


H_Λ : Effective Hubble value converted from T_Λ

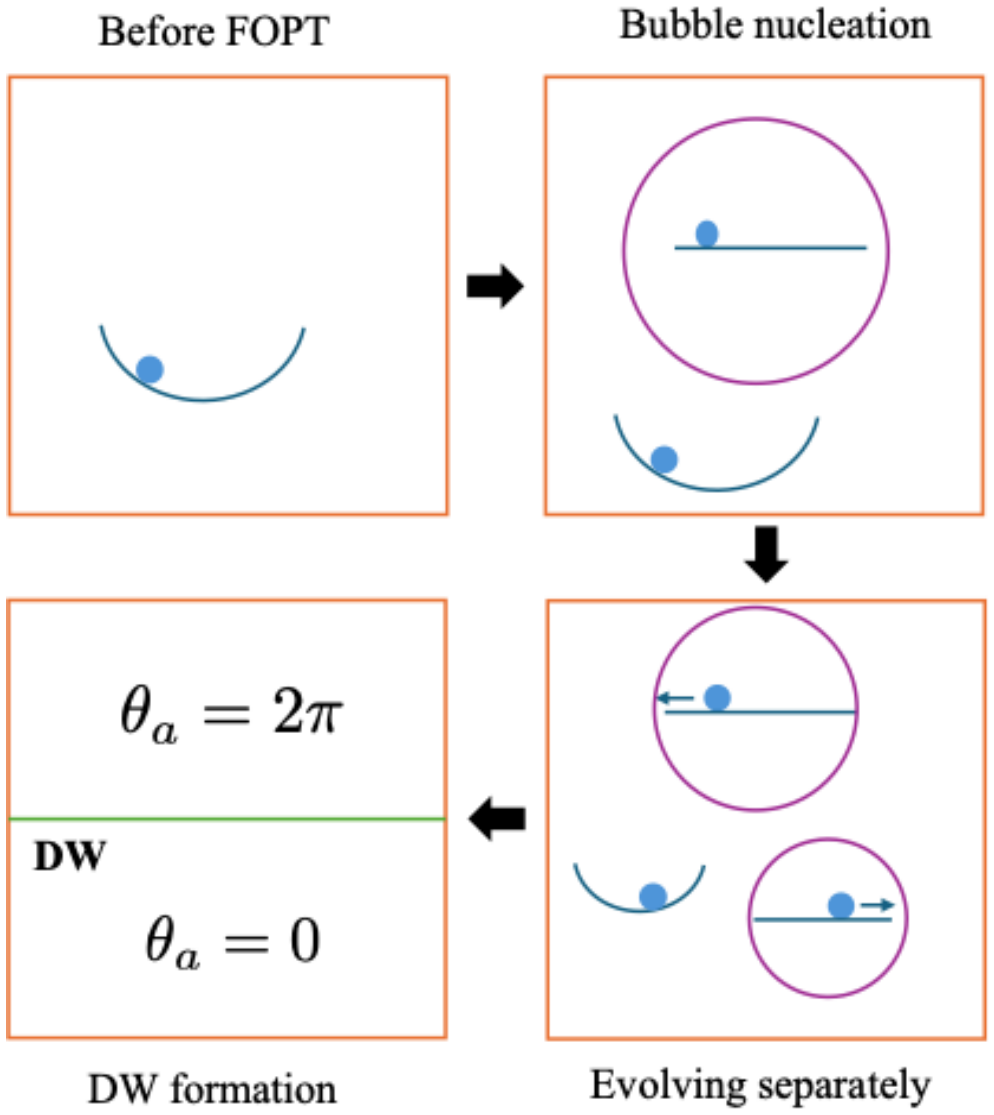
Inhomogeneity from Bubble Emergence

- If the reheating happens quickly that $t_{\rm reh} \ll \Delta t_{\rm PT} \sim O(\beta^{-1})$, more interesting phenomena.
- Bubbles can be nucleated stochastically temporally and spatially.





Schematic Illustration



Axion Field with Bubble

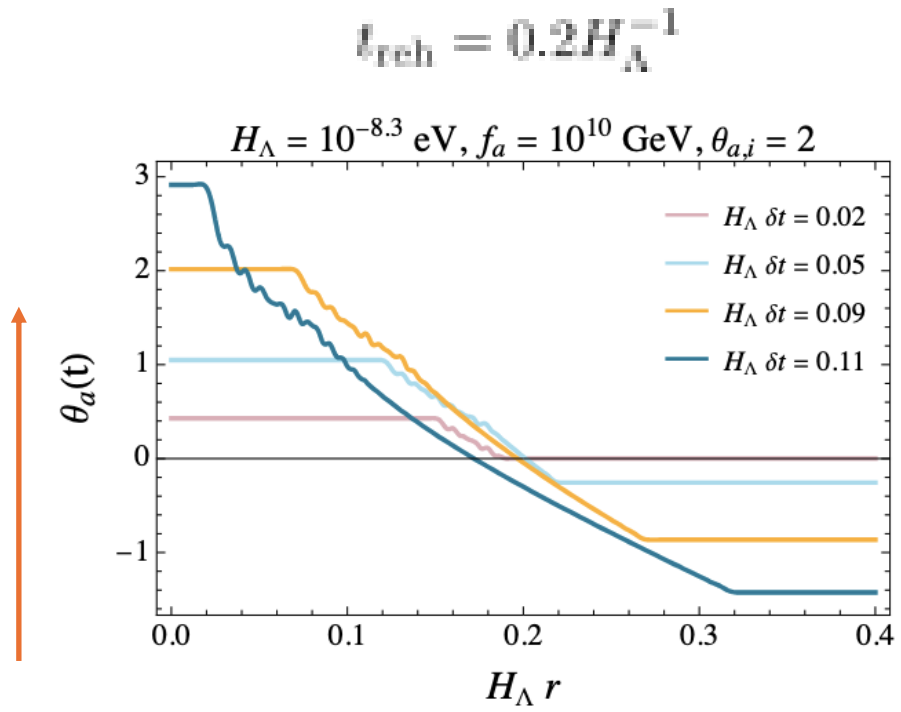
- The axion potential differs inside and outside the bubble.

- Solve the EOM of the axion with the wall boundary condition.

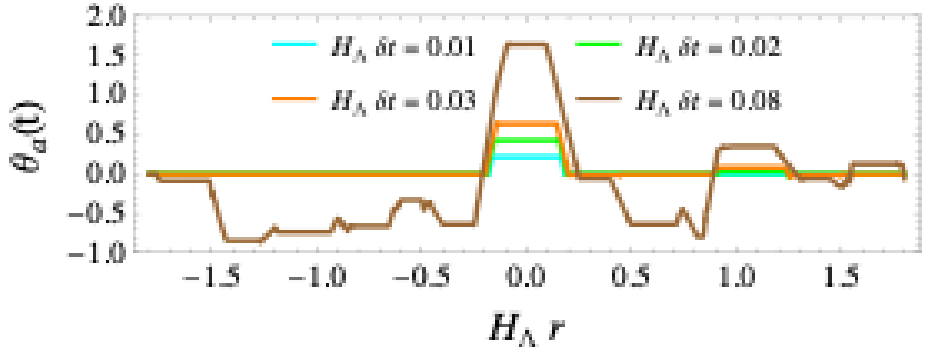
$$\ddot{\theta}_a + 3H\dot{\theta}_a - \frac{\nabla^2}{a(t)^2}\theta_a + m_a^2(T)\sin\theta_a = 0.$$

- Spatial fluctuation will be generated!

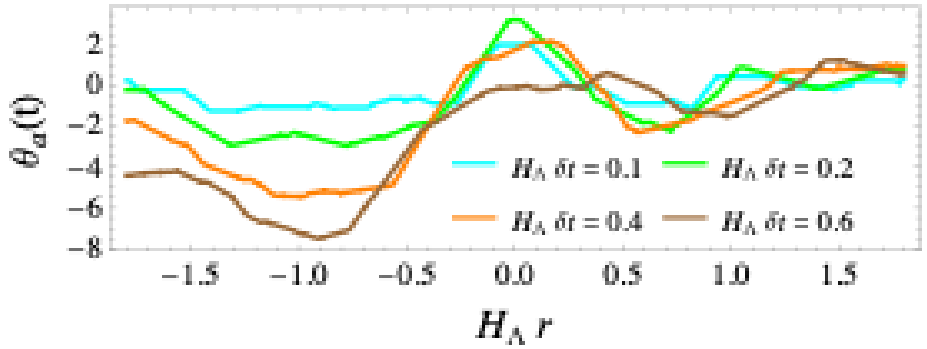
Axion field value at the center keeps rising first!



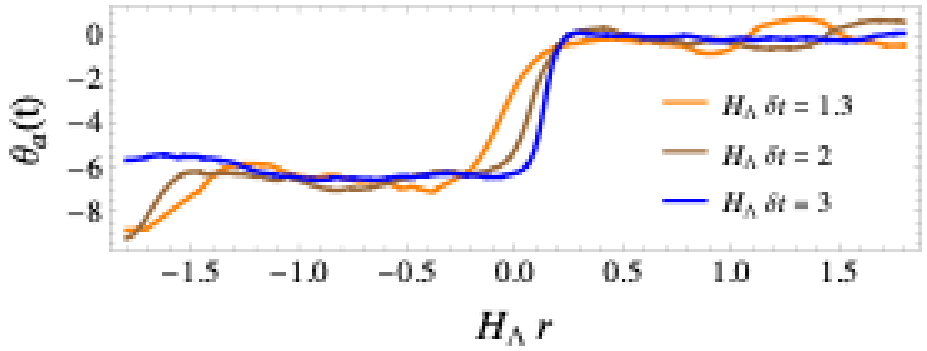
- With bubbles merging, the significant axion spatial fluctuation is developed.
- As potential barrier height increases, axion fields at different points will be trapped into distinct vacuum position.
- DW can be formed.
- 1D demonstration



$m_a \sim 0$



m_a non-negligible



Trapped DW formation

DW Formation

- The condition for DW formation

$$\frac{\dot{\theta}_{a,\mathrm{PT}}}{\beta}\left(\frac{T_{\mathrm{trap}}}{T_{\Lambda}}\right) > \pi$$

Redshift factor

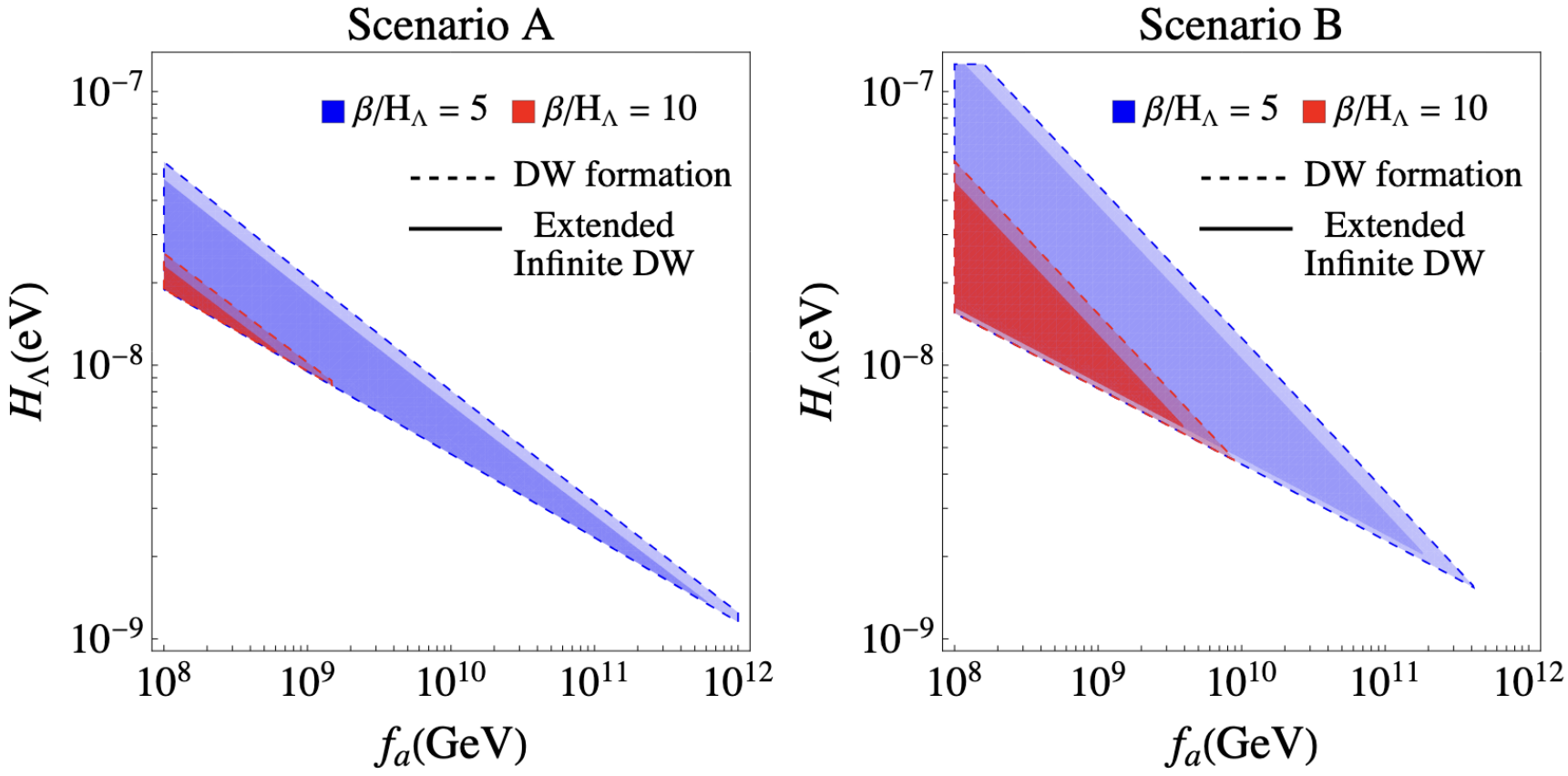
- Since there is no cosmic string, no boundary for the DW.
- Finite-sized enclosed domain walls:

  unstable, quickly shrink and collapse
- Infinite DW network:

  Determined by the percolation theorem

  Stable, will dominate the universe

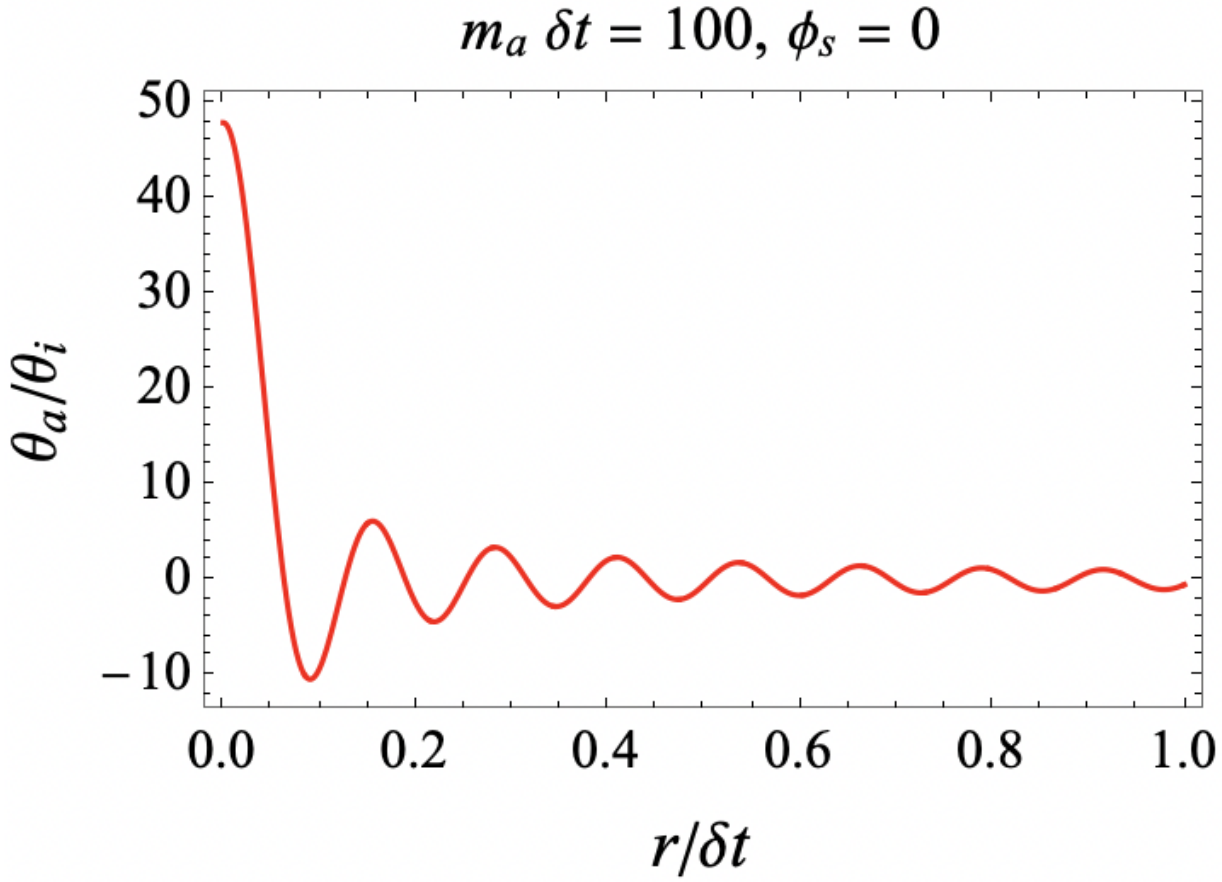
DW Formation



$$\theta_{a,i} = 2$$

Instantaneous Reheating

- For $t_{\rm reh} \ll \Delta t_{\rm PT}$



Only the center gains large field value.
Small enclosed bubbles are produced.

Conclusion:

- FOPT at QCD scale is hinted by NANOGrav.
- The existence of QCD axion is very motivated.

⇨ Combining these two aspects leads to interesting cosmology!

- Mini kinetic misalignment is a natural prediction.
- Axion relic abundance is reset.
- Domain wall formation, without cosmic strings, from the stochastic bubbles is expected.

Thank you!