Dynamics for theories with higher-order interference

Nabin Bhatta
(Work in progress with Djordje Minic and Tatsu Takeuchi)

Virginia Tech
Department of Physics

Pheno 2026
May 12, 2026

Motivation

Experimental observation of quantum interference and Bell inequality violations falsified classical mechanics as a fundamental theory. What could be an experimental signature that falsifies quantum mechanics?

Experimental observation of quantum interference and Bell inequality violations falsified classical mechanics as a fundamental theory. What could be an experimental signature that falsifies quantum mechanics?

Principles of GR like the equivalence principle are constantly scrutinized experimentally[1].

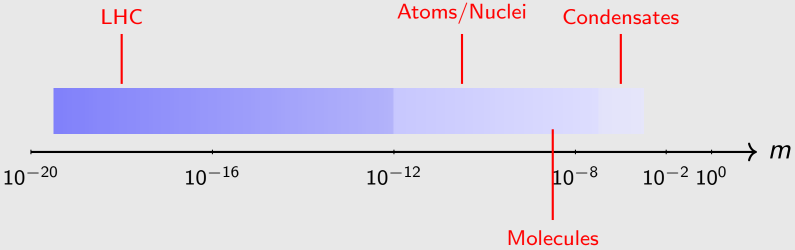
[1] Phys. Rev. Lett. 119 (2017), 231101

Experimental observation of quantum interference and Bell inequality violations falsified classical mechanics as a fundamental theory. What could be an experimental signature that falsifies quantum mechanics?

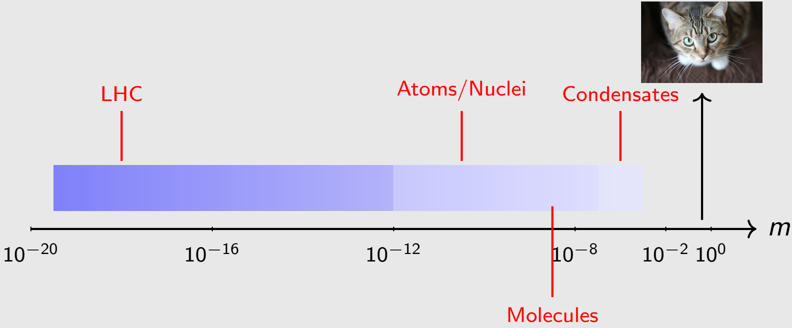
Principles of GR like the equivalence principle are constantly scrutinized experimentally[1]. Arguably, the same should be done for QM.

[1] Phys. Rev. Lett. 119 (2017), 231101

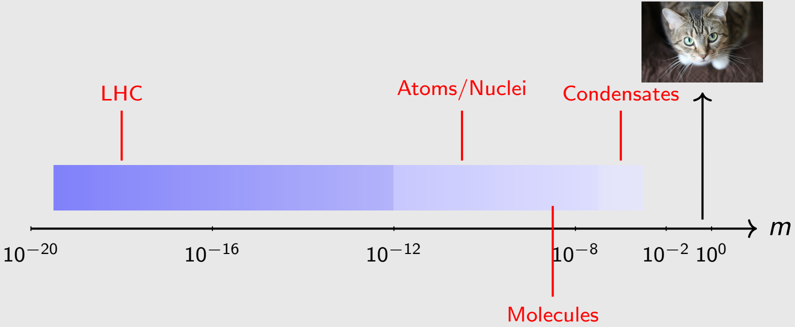
But wait...QM *has* been tested



But wait...QM *has* been tested



But wait...QM *has* been tested

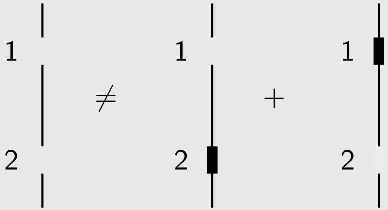


Could anything show up in low-energy experiments?

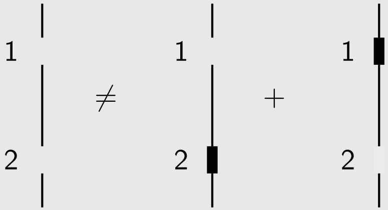
Sorkin's hierarchy of interference

Sorkin's hierarchy of interference

Sorkin's hierarchy of interference

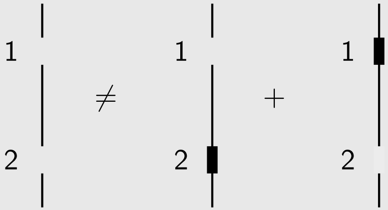


Sorkin's hierarchy of interference



$$\underbrace{P_{12}}_{\text{Both slits open}} \neq \underbrace{(P_1 + P_2)}_{\text{One slit open}}$$

Sorkin's hierarchy of interference



$$\underbrace{P_{12}}_{\text{Both slits open}} \neq \underbrace{(P_1 + P_2)}_{\text{One slit open}}$$

$$I_{12} := P_{12} - P_1 - P_2$$

Sorkin (1994)[2]: Does QM predict something more exotic if we add one more slit?

2 Quantum mechanics as quantum measure theory, Mod. Phys. Lett. A9, 3119, 1994

Sorkin (1994)[2]: Does QM predict something more exotic if we add one more slit?

$$\underbrace{P_{123}}_{\text{All slits open}}$$

[2] Quantum mechanics as quantum measure theory, Mod. Phys. Lett. A9, 3119, 1994

Sorkin (1994)[2]: Does QM predict something more exotic if we add one more slit?

$$\underbrace{P_{123}}_{\text{All slits open}} = \underbrace{(P_{12} + P_{23} + P_{13})}_{\text{Two slits open}}$$

[2] Quantum mechanics as quantum measure theory, Mod. Phys. Lett. A9, 3119, 1994

Sorkin (1994)[2]: Does QM predict something more exotic if we add one more slit?

$$\underbrace{P_{123}}_{\text{All slits open}} = \underbrace{(P_{12} + P_{23} + P_{13})}_{\text{Two slits open}} - \underbrace{(P_1 + P_2 + P_3)}_{\text{Single slit open}}$$

[2] Quantum mechanics as quantum measure theory, Mod. Phys. Lett. A9, 3119, 1994

Sorkin (1994)[2]: Does QM predict something more exotic if we add one more slit?

$$\underbrace{P_{123}}_{\text{All slits open}} = \underbrace{(P_{12} + P_{23} + P_{13})}_{\text{Two slits open}} - \underbrace{(P_1 + P_2 + P_3)}_{\text{Single slit open}}$$

$$I_{123} := P_{123} - P_{12} - P_{23} - P_{13} + P_1 + P_2 + P_3 = 0$$

[2] Quantum mechanics as quantum measure theory, Mod. Phys. Lett. A9, 3119, 1994

One of the clearest signals of new physics where experiments can be done in low-energy systems[3]

[3] Sinha et. al., Science 329,418-421(2010); Kauten et. al., New J. Phys. 19 (2017) 033017

One of the clearest signals of new physics where experiments can be done in low-energy systems[3]

Our work: A theoretical model describing I_{123}

[3] Sinha et. al., Science 329,418-421(2010); Kauten et. al., New J. Phys. 19 (2017) 033017

Density matrices

In QM, three-level system $\sim 3 \times 3$ Density matrices

In QM, three-level system $\sim 3 \times 3$ Density matrices

$$\begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}$$

Three open slits

In QM, three-level system $\sim 3 \times 3$ Density matrices

$$\underbrace{\begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}}_{\text{Three open slits}} = \underbrace{\begin{bmatrix} \bullet & & \\ & & \\ & & \end{bmatrix}}_{\text{Slit 1}} + \underbrace{\begin{bmatrix} & & \\ & \bullet & \\ & & \end{bmatrix}}_{\text{Slit 2}} + \underbrace{\begin{bmatrix} & & \\ & & \\ & & \bullet \end{bmatrix}}_{\text{Slit 3}}$$

In QM, three-level system $\sim 3 \times 3$ Density matrices

$$\underbrace{\begin{bmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{bmatrix}}_{\text{Three open slits}} = \underbrace{\begin{bmatrix} \bullet & & \\ & & \\ & & \end{bmatrix}}_{\text{Slit 1}} + \underbrace{\begin{bmatrix} & & \\ & \bullet & \\ & & \end{bmatrix}}_{\text{Slit 2}} + \underbrace{\begin{bmatrix} & & \\ & & \\ & & \bullet \end{bmatrix}}_{\text{Slit 3}}$$

$$+ \underbrace{\begin{bmatrix} & \bullet & \\ \bullet & & \\ & & \end{bmatrix}}_{I_{12}} + \underbrace{\begin{bmatrix} & & \bullet \\ & & \\ \bullet & & \end{bmatrix}}_{I_{13}} + \underbrace{\begin{bmatrix} & & \\ & & \bullet \\ & \bullet & \end{bmatrix}}_{I_{23}}$$

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12}^* & \rho_{22} & \rho_{23} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} \end{bmatrix}$$

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12}^* & \rho_{22} & \rho_{23} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} \end{bmatrix}$$

$\rho_{ij}, i \neq j \sim$ Coherence between **two states** $\implies I_{ij} \neq 0$

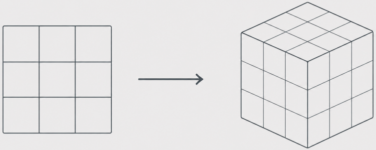
QM is a probability theory that allows second-order interference but not higher

QM is a probability theory that allows second-order interference but not higher

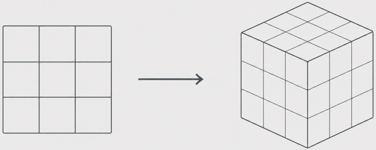
What kind of probability theories would lead to third-order interference?

Density cubes

B Dakić et. al. 2014 (New J. Phys. 16 023028)



B Dakić et. al. 2014 (New J. Phys. 16 023028)



$$\begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{12}^* & \rho_{22} & \rho_{23} \\ \rho_{13}^* & \rho_{23}^* & \rho_{33} \end{bmatrix}$$

$$\downarrow$$

$$\left\{ \begin{bmatrix} \rho_{111} & \rho_{112} & \rho_{113} \\ \rho_{112} & \rho_{122} & \rho_{123} \\ \rho_{113} & \rho_{123}^* & \rho_{133} \end{bmatrix}, \begin{bmatrix} \rho_{112} & \rho_{122} & \rho_{123}^* \\ \rho_{122} & \rho_{222} & \rho_{223} \\ \rho_{123} & \rho_{223} & \rho_{233} \end{bmatrix}, \begin{bmatrix} \rho_{113} & \rho_{123} & \rho_{133} \\ \rho_{123}^* & \rho_{223} & \rho_{233} \\ \rho_{133} & \rho_{233} & \rho_{333} \end{bmatrix} \right\}$$

$$\rho = \left\{ \begin{bmatrix} \rho_{111} & \rho_{112} & \rho_{113} \\ \rho_{112} & \rho_{122} & \rho_{123} \\ \rho_{113} & \rho_{123}^{*} & \rho_{133} \end{bmatrix}, \begin{bmatrix} \rho_{112} & \rho_{122} & \rho_{123}^{*} \\ \rho_{122} & \rho_{222} & \rho_{223} \\ \rho_{123} & \rho_{223} & \rho_{233} \end{bmatrix}, \begin{bmatrix} \rho_{113} & \rho_{123} & \rho_{133} \\ \rho_{123}^{*} & \rho_{223} & \rho_{233} \\ \rho_{133} & \rho_{233} & \rho_{333} \end{bmatrix} \right\}$$

$$\rho = \left\{ \begin{bmatrix} \rho_{111} & \rho_{112} & \rho_{113} \\ \rho_{112} & \rho_{122} & \rho_{123} \\ \rho_{113} & \rho_{123}^* & \rho_{133} \end{bmatrix}, \begin{bmatrix} \rho_{112} & \rho_{122} & \rho_{123}^* \\ \rho_{122} & \rho_{222} & \rho_{223} \\ \rho_{123} & \rho_{223} & \rho_{233} \end{bmatrix}, \begin{bmatrix} \rho_{113} & \rho_{123} & \rho_{133} \\ \rho_{123}^* & \rho_{223} & \rho_{233} \\ \rho_{133} & \rho_{233} & \rho_{333} \end{bmatrix} \right\}$$

$\rho_{123} \sim$ Coherence between three states $\Longrightarrow I_{123} \neq 0$

What about time evolution?

What about time evolution?

For density matrices,

$$U \cdot \rho(0) \to \rho(t)$$

What about time evolution?

For density matrices,

$$U \cdot \rho(0) \to \rho(t)$$

Corresponds to Heisenberg/von-Neumann equation of motion

$$\frac{d}{dt}\rho \; = \; i\big[\rho, H\big]$$

What about time evolution?

For density matrices,

$$U \cdot \rho(0) \to \rho(t)$$

Corresponds to Heisenberg/von-Neumann equation of motion

$$\frac{d}{dt}\rho \;=\; i\big[\rho, H\big]$$

Analog provided in Dakić et. al.

$$T \cdot \rho(0) \to \rho(t)$$

What about time evolution?

For density matrices,

$$U \cdot \rho(0) \to \rho(t)$$

Corresponds to Heisenberg/von-Neumann equation of motion

$$\frac{d}{dt}\rho \;=\; i\left[\rho, H\right]$$

Analog provided in Dakić et. al.

$$T \cdot \rho(0) \to \rho(t)$$

Our work: What is the analog of von-Neumann equation for density cubes?

Equation of motion

Poisson bracket $\rightarrow$ Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$
$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Poisson bracket → Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$

$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Quantization

$$\frac{d}{dt}\rho = i[\rho, H]$$

Poisson bracket $\rightarrow$ Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$

$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Quantization

$$\frac{d}{dt}\rho = i[\rho, H]$$

$$[A, B] := AB - BA$$

Poisson bracket → Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$
$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Quantization

$$\frac{d}{dt}\rho = i\left[\rho, H\right]$$

$$[A, B] := AB - BA$$

Nambu (PRD 7, 2405, 1973)

$$\frac{d}{dt}\rho = \{\rho, H_1, H_2\}$$
$$= \sum_{ijk} \varepsilon_{ijk} \frac{\partial \rho}{\partial q_i} \frac{\partial H_1}{\partial q_j} \frac{\partial H_2}{\partial q_k}$$

Poisson bracket → Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$
$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Quantization

$$\frac{d}{dt}\rho = i[\rho, H]$$

$$[A, B] := AB - BA$$

Nambu (PRD 7, 2405, 1973)

$$\frac{d}{dt}\rho = \{\rho, H_1, H_2\}$$
$$= \sum_{ijk} \varepsilon_{ijk} \frac{\partial \rho}{\partial q_i} \frac{\partial H_1}{\partial q_j} \frac{\partial H_2}{\partial q_k}$$

"Quantization"

$$\frac{d}{dt}\rho = i[\rho, H_1, H_2]$$

Poisson bracket → Commutator

$$\frac{d}{dt}\rho = \{\rho, H\}$$
$$= \sum_{ij} \varepsilon_{ij} \frac{\partial \rho}{\partial q_i} \frac{\partial H}{\partial q_j}$$

Quantization

$$\frac{d}{dt}\rho = i[\rho, H]$$

$$[A, B] := AB - BA$$

Nambu (PRD 7, 2405, 1973)

$$\frac{d}{dt}\rho = \{\rho, H_1, H_2\}$$
$$= \sum_{ijk} \varepsilon_{ijk} \frac{\partial \rho}{\partial q_i} \frac{\partial H_1}{\partial q_j} \frac{\partial H_2}{\partial q_k}$$

“Quantization”

$$\frac{d}{dt}\rho = i[\rho, H_1, H_2]$$

$$[A, B, C] := ABC - BAC + BCA - CBA + CAB - ACB$$

$$[A, B, C] = ABC - BAC + BCA - CBA + CAB - ACB$$

$$[A, B, C] = ABC - BAC + BCA - CBA + CAB - ACB$$

Many inequivalent ways of defining the product[4] of three cubes

[4] Awata et. al., JHEP02 (2001) 013; Kawamura, Prog. Theor. Phys. 110, 579–587 (2003)

$$[A, B, C] = ABC - BAC + BCA - CBA + CAB - ACB$$

Many inequivalent ways of defining the product[4] of three cubes

We need to ensure

- Hermitian $\rho, H_1, H_2 \implies$ Hermitian $i[\rho, H_1, H_2]$

[4] Awata et. al., JHEP02 (2001) 013; Kawamura, Prog. Theor. Phys. 110, 579–587 (2003)

$$[A, B, C] = ABC - BAC + BCA - CBA + CAB - ACB$$

Many inequivalent ways of defining the product[4] of three cubes

We need to ensure

- Hermitian $\rho, H_1, H_2 \implies$ Hermitian $i[\rho, H_1, H_2]$
- Trace preserving evolution of ρ

[4] Awata et. al., JHEP02 (2001) 013; Kawamura, Prog. Theor. Phys. 110, 579–587 (2003)

Depends crucially on how you define the product ABC

Depends crucially on how you define the product ABC

We use

$$(ABC)_{ijk} = \sum_{\ell} A_{\ell jk} B_{i\ell k} C_{ij\ell}$$

Depends crucially on how you define the product ABC

We use

$$(ABC)_{ijk} = \sum_{\ell} A_{\ell jk} B_{i\ell k} C_{ij\ell}$$

Gives us a trace-preserving dynamics for I_{123}

Takeaway: Why care about dynamics?

An equation of motion will allow us to:

Takeaway: Why care about dynamics?

An equation of motion will allow us to:

Calculate **other consequences** of the theory

Takeaway: Why care about dynamics?

An equation of motion will allow us to:

Calculate **other consequences** of the theory and **borrow constraints** from different experiments

Thank You! Questions?

nabinb@vt.edu