Questions on Fuzzy Dark Matter

Author: Sunhaeng Hur

Department of Physics
sunhh@vt.edu

The Core-Cusp Problem

The standard ΛCDM model is incredibly successful at large scales.

The Discrepancy

There is a significant tension between Cold Dark Matter (CDM) simulations and observations of dwarf galaxies at small scales ($<$ a few kpc).

Simulations Predict:

- NFW Profile: $\rho(r) \propto r^{-1}$ (As $r \to 0$)
- Divergent density profile at center (**Cusp**)
- $v \propto \sqrt{r}$

Observations Show:

- Flat density profiles: $\rho \approx$ const (As $r \to 0$)
- finite density (**Core**)
- $v \propto r$

Observational Evidence 1: Density Profiles

Density profiles of dwarf galaxies from the THINGS survey indicate that these galaxies favor a cored dark matter distribution.

- **CDM Prediction:** Divergent density at center
- **Data:** Flat density profile at center

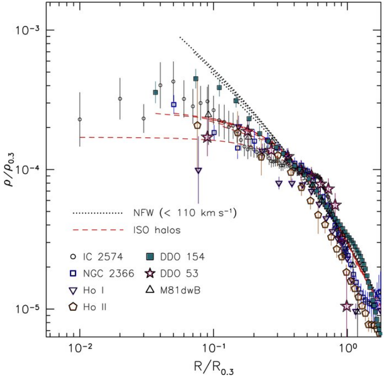


Figure: NFW Predictions vs. Observations(Oh et al. 2011)

Observational Evidence 2: Velocity Curves

This discrepancy manifests in the rotation curves as well.

	The Cusp	The Core
Density	$\rho \propto r^{-1}$	$\rho \approx \text{const}$
Mass	$\int r^2(r^{-1})\,dr \propto r^2$	$\int r^2(\text{c})\,dr \propto r^3$
Velocity$(\sqrt{GM/r})$	$\sqrt{\frac{r^2}{r}} = \sqrt{r}$	$\sqrt{\frac{r^3}{r}} = r$

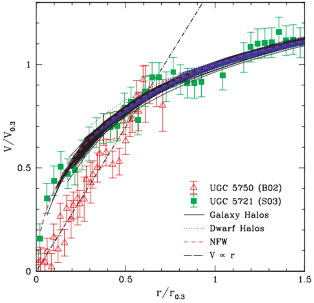


Figure: While some rotation curves scale as the square root of r, which is consistent with the NFW cusp, others show a linear rise indicating a core.(Hayashi and Navarro 2006)

Attempts to Resolve within CDM

Various solutions have been proposed to resolve this discrepancy, typically involving either Baryonic Feedback or alternative dark matter models like WDM and SIDM.

Baryonic Feedback

Supernovae bursts "blow out" gas, fluctuating the potential and smoothing the DM core.
Issue: Requires fine-tuning; hard to verify.

Warm Dark Matter

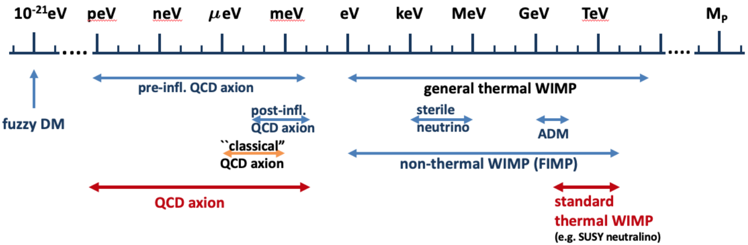
Particles with higher thermal velocities wash out small structures.
Issue: Solves "Satellites" but not "Cusps" effectively.

Self-Interacting DM

SIDM scatter off each other, transferring heat and puffing up the core.
Issue: If the self-interaction is too strong,it can recreate the cusp.

Fuzzy dark matter

Fuzzy Dark Matter is an ultralight boson with mass $m \approx 10^{-22}$ eV
At this extreme low mass, quantum effects (fuzziness) manifest on galactic scales.



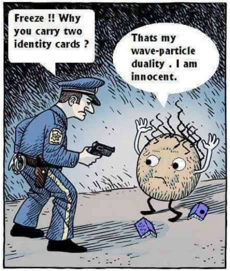


Figure: The Range of Dark Matter Model (Billard et al. 2022)

De Broglie Wavelength of FDM

Key Assumption:

Quantum Wave Scale (λ_{dB}) ≈ Galactic Core Scale (r_c)

$$\lambda_{dB} = \frac{2\pi\hbar}{mv} \approx \underbrace{\mathbf{12}\,\mathrm{kpc}}_{\text{Dwarf Galaxy Core radius}} \underbrace{\left(\frac{10^{-22}\,\mathrm{eV}}{m}\right)}_{\text{Target Mass}} \underbrace{\left(\frac{10\,\mathrm{km/s}}{v}\right)}_{\text{Dwarf Galaxy Velocity dispersion}}$$

When both the velocity dispersion and core size are constrained by observations, the boson mass is uniquely determined to be approximately 10^{-22} eV

The formation of a central cusp is naturally prevented, as particles cannot be confined within a region smaller than their de Broglie wavelength (λ_{dB}).

Missing Satellites Problem

Simulations predict that large galaxies should be orbited by an abundance of small satellite galaxies. However, observations reveal far fewer than predicted.

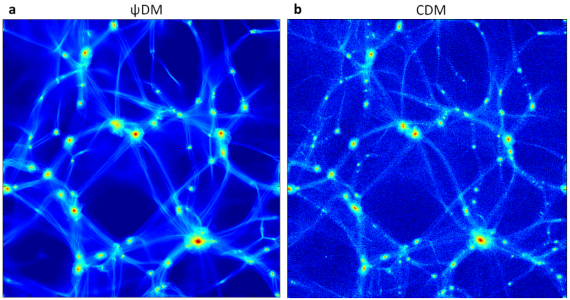


Figure: **Left**: FDM washes out the smallest scales due to wave interference. **Right**: CDM produces many small, dense substructures.(Schive, Chiueh, and Broadhurst 2014)

Motivation

- FDM solves the small-scale crisis not by adding complexity, but by adding fundamental physics.

Schrödinger-Poisson equation

FDM is modeled as a scalar field of ultralight bosons, governed by the Schrödinger-Poisson equation.

Governing Equations

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi \tag{1}$$

$$\nabla^2\Phi = 4\pi G|\psi|^2 \tag{2}$$

- The kinetic term ($-\nabla^2\psi$) manifests as a Quantum Pressure. This pressure opposes gravitational collapse (a consequence of the Uncertainty Principle).
- This balance results in a stable, cored ground state known as the **Soliton**.

Soliton solution

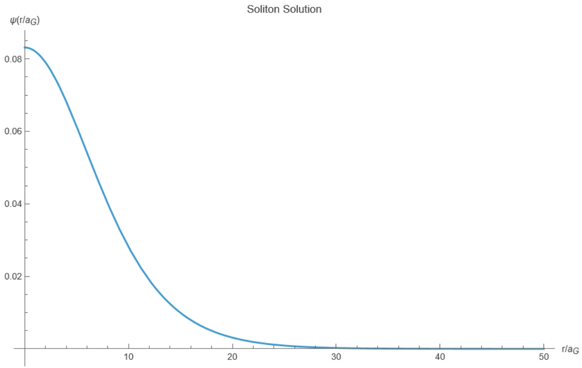


Figure: This represents the soliton ground state of the Schrödinger-Poisson equation, scaled by the gravitational Bohr radius.

The Gravitational Bohr Radius a_B

- Solving the Schrödinger-Poisson equation reveals that the size of the soliton core is fundamentally determined by the **Gravitational Bohr Radius** (a_B), analogous to the hydrogen atom:

$$a_B \equiv \frac{\hbar^2}{2GMm^2} \tag{3}$$

- This introduces a crucial relation: **the more massive the dark matter core, the smaller its radius.**

Is the Soliton Model Sufficient?

The Scaling Problem: The central issue lies in the relation between Core Density (ρ_c), Core Radius (r_c), and Core Mass (M_c).

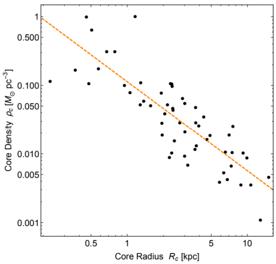
FDM Soliton Prediction

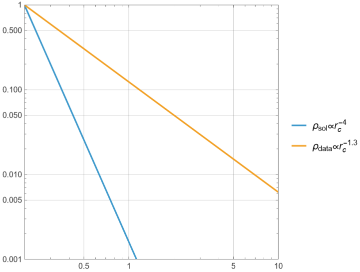
- The radius is inversely proportional to the mass: $r_c \propto M_c^{-1}$
- This implies a steep density dependence: $\rho_c \propto r_c^{-4}$
- Heavier cores must be **smaller.**

Observation

- Data suggests a shallow density profile: $\rho_c \propto r_c^{-1}$
- This implies a **positive** mass-radius correlation: $r_c \propto \sqrt{M_c}$
- Heavier cores are **larger.**

Discrepancy between FDM model and observational data





- In the left figure(Deng et al. 2018), the orange dashed curve shows that the core density is roughly inversely proportional to the core radius.($\rho_c \propto r_c^{-1.3}$)
- The right figure demonstrates the discrepancy between the theoretical prediction (blue) and the observed behavior (orange).
- Resolving this fundamental discrepancy is the primary motivation for my work.

Key Research Questions

- **Core Stability:** Is the soliton core robust against thermal perturbations?
- **Thermal Expansion:** Do finite temperature effects induce core expansion?

Eigenvalues in the Soliton Core

State (n)	ϵ_n
0	-0.0814
1	-0.0351
2	-0.0188
3	-0.0116

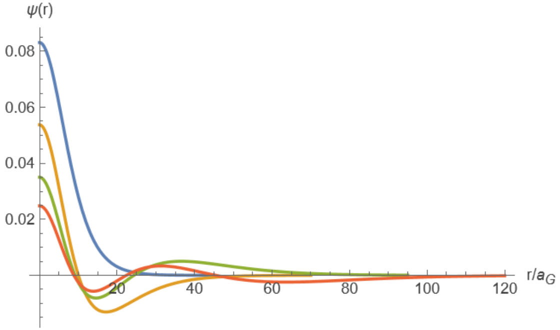


Figure: The first few s-state eigenfunctions with the potential fixed by the ground state density.

Energy Scaling Coefficient

The dimensional energy in SP equation is given by:

$$E_n = \frac{\hbar^2}{2ma_G^2}\epsilon_n \tag{4}$$

Parameters:

- FDM Mass: $m \sim 10^{-22} eV/c^2$
- $a_G \approx 1.3 \times 10^{19}$ m or $\approx 426pc$

Coefficient Calculation:

$$\frac{\hbar^2}{2ma_G^2} = \frac{(6.582 \times 10^{-16}\,\mathrm{eV}\cdot\mathrm{s})(1.05 \times 10^{-34}\,\mathrm{J}\cdot\mathrm{s})}{2 \times \left(\frac{10^{-22}\,\mathrm{eV}}{(3\times10^{8}\,\mathrm{m/s})^2}\right) \times (1.3 \times 10^{19}\,\mathrm{m})^2}$$

$$\approx 1.8 \times 10^{-49}\,\mathrm{J} \times \frac{1\,\mathrm{eV}}{1.60218 \times 10^{-19}\,\mathrm{J}}$$

$$\approx \mathbf{10^{-30}}\,\mathrm{eV}$$

Excited Energy Levels

State (n)	ϵ_n	**Result** (E_n)
0	-0.0814	$\approx -\mathbf{9} \times \mathbf{10}^{-\mathbf{30}}$ eV
1	-0.0351	$\approx -\mathbf{4} \times \mathbf{10}^{-\mathbf{30}}$ eV
2	-0.0188	$\approx -\mathbf{2} \times \mathbf{10}^{-\mathbf{30}}$ eV
3	-0.0116	$\approx -\mathbf{1.3} \times \mathbf{10}^{-\mathbf{30}}$ eV

*Because these gaps are so small, FDM can be **excited** easily.*

$\Rightarrow$ *By considering excited states, we might be able to reproduce the larger core sizes seen in observations.*

Grand Canonical Ensemble

To accurately model the system, we employ the **Grand Canonical Ensemble**, as the structure exchanges both energy and particles with the surrounding reservoir.

High-Temperature Limit Approximation

In thermal equilibrium, assuming large occupation numbers ($\mathcal{N}_{nl} \gg 1$), the Bose-Einstein distribution simplifies to:

$$\mathcal{N}_{nl} = \frac{1}{e^{\beta(E_{nl}-\mu)} - 1} \approx \frac{1}{\beta(E_{nl} - \mu)} \tag{5}$$

Where $\beta = 1/(k_B T)$ and μ is the chemical potential.

The Hartree Approximation

The Hartree Approximation is a mean-field theory effective for many-body system. It simplifies the system by assuming each particle moves independently in an average potential $\hat{V}$ generated by the density of all particles.

$$\hat{\nabla}_r^2 \hat{R}_{n\ell}(\hat{r}) = \left[\hat{V}(\hat{r}) + \frac{\ell(\ell+1)}{\hat{r}^2} - \hat{\varepsilon}_{n\ell}\right] \hat{R}_{n\ell}(\hat{r}) \tag{6}$$

$$\hat{\nabla}_r^2 \hat{V}(\hat{r}) = \sum_{n\ell} c_{n\ell} |\hat{R}_{n\ell}(\hat{r})|^2 \tag{7}$$

where $c_{n\ell} = N_{n\ell}/N$. However, naively summing over **all states** to determine the total particle number N results in a UV divergence:

$$N = \sum_{n\ell}^{\infty} N_{n\ell} \approx \sum_{n\ell}^{\infty} \frac{1}{\beta(E_{n\ell} - \mu)} \to \infty \tag{8}$$

Chemical Potential

The Role of Chemical Potential μ(T)

To regularize this divergence, we must explicitly impose the chemical potential as a constraint on the total particle number N. We can estimate its asymptotic behavior:

- **Low T ($T \ll 1$):** Particles condense (BEC).

$$\mu \to E_0 \quad \text{(Ground State Energy)}$$

- **High T ($T \gg 1$):** The chemical potential dominates the energy ($-\mu \gg E_i$). Assuming n available states:

$$N \approx \sum_{i=1}^{n} \frac{1}{-\beta\mu} = -\frac{n}{\beta\mu} \implies \mu \approx -\frac{n}{N\beta}$$

**Exact $\mu(T)$ for intermediate temperatures requires numerical calculation.*

Parameter Reduction: The $N\beta$ Scaling

Since N and β are not independent in this regime, we treat the product $N\beta$ as a **single parameter**:

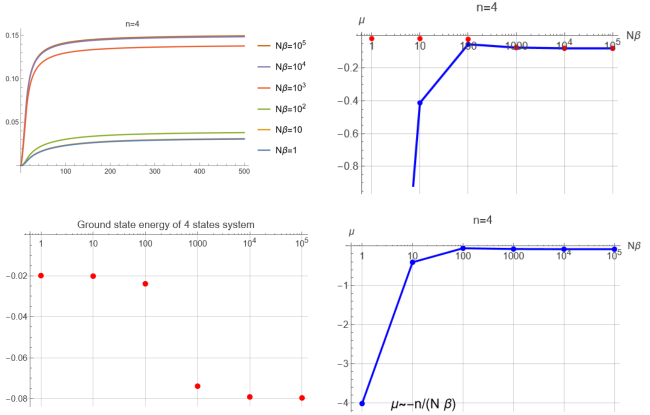
$$\boxed{N\beta = \sum_{i}^{n} \frac{1}{E_i - \mu}} \tag{9}$$

By fixing the coupled parameter $N\beta$, we can numerically invert the constraint equation to determine the chemical potential μ for a finite basis of n states. This allows us to observe the thermal evolution of the potential.

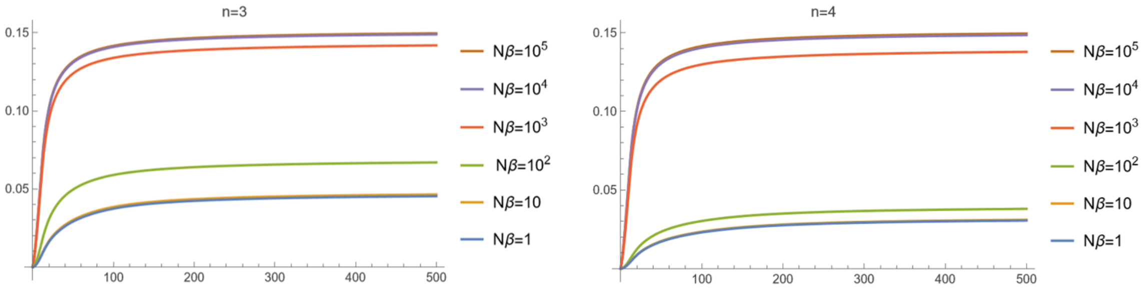
Numerical Results: 4-State System ($n = 4$)

Key Observations

1. A sharp transition in the potential profile occurs in the regime: $10^2 < N\beta < 10^3$
2. Coinciding with this transition, the energy between μ and the ground state (E_0) begins not to align.
3. As the potential becomes shallower, the ground state energy decreases.
4. The numerical results for μ are consistent with our prediction:
   - **High-T ($N\beta = 1$):** $\mu \approx -\frac{n}{N\beta} = -4$
   - **Low-T ($N\beta \gg 1$):** $\mu \to E_0$



Sharpening of the Phase Transition ($n = 3$ vs $n = 4$)



- Both systems transition within $10^2 < N\beta < 10^3$.
- However, the transition for $n = 4$ **is significantly more abrupt** than for $n = 3$.
- This trend intensifies as we include more states.

Physical Implication: Evaporation Instability

Implication

- This implies that in the limit of large state occupancy, the density drops below the background threshold ($\bar{\rho}_{\text{univ}}$) once the critical temperature is reached, triggering **evaporation**.

Conclusion

- We are currently working to define the UV cutoff required to **regularize the partition function** and calculate the **critical temperature** (T_c).
- This approach can resolve the density profile discrepancy while simultaneously explaining the suppression of low-mass halos.

Thank You!

References I

Billard, Julien et al. (Apr. 2022). "Direct detection of dark matter—APPEC committee report*". In: *Reports on Progress in Physics* 85.5, p. 056201. ISSN: 1361-6633. DOI: 10.1088/1361-6633/ac5754. URL: http://dx.doi.org/10.1088/1361-6633/ac5754.

Deng, Heling et al. (July 2018). "Can light dark matter solve the core-cusp problem?" In: *Physical Review D* 98.2. ISSN: 2470-0029. DOI: 10.1103/physrevd.98.023513. URL: http://dx.doi.org/10.1103/PhysRevD.98.023513.

References II

Hayashi, Eric and Julio F. Navarro (Nov. 2006). “Hiding cusps in cores: kinematics of disc galaxies in triaxial dark matter haloes”. In: *Monthly Notices of the Royal Astronomical Society* 373.3, pp. 1117–1124. ISSN: 0035-8711. DOI: 10.1111/j.1365-2966.2006.10927.x. eprint: https://academic.oup.com/mnras/article-pdf/373/3/1117/18665524/mnras0373-1117.pdf. URL: https://doi.org/10.1111/j.1365-2966.2006.10927.x.

Oh, Se-Heon et al. (May 2011). “DARK AND LUMINOUS MATTER IN THINGS DWARF GALAXIES”. In: *The Astronomical Journal* 141.6, p. 193. ISSN: 1538-3881. DOI: 10.1088/0004-6256/141/6/193. URL: http://dx.doi.org/10.1088/0004-6256/141/6/193.

References III

Schive, Hsi-Yu, Tzihong Chiueh, and Tom Broadhurst (June 2014). "Cosmic structure as the quantum interference of a coherent dark wave". In: *Nature Physics* 10.7, 496–499. ISSN: 1745-2481. DOI: 10.1038/nphys2996. URL: http://dx.doi.org/10.1038/nphys2996.