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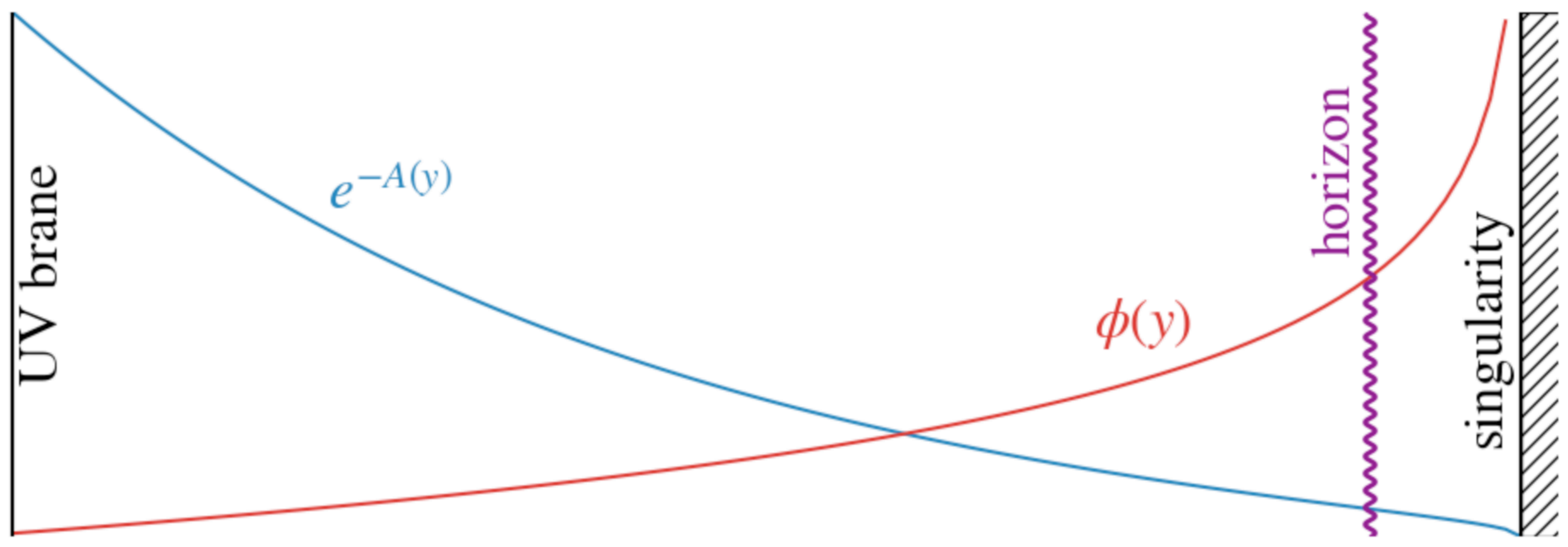
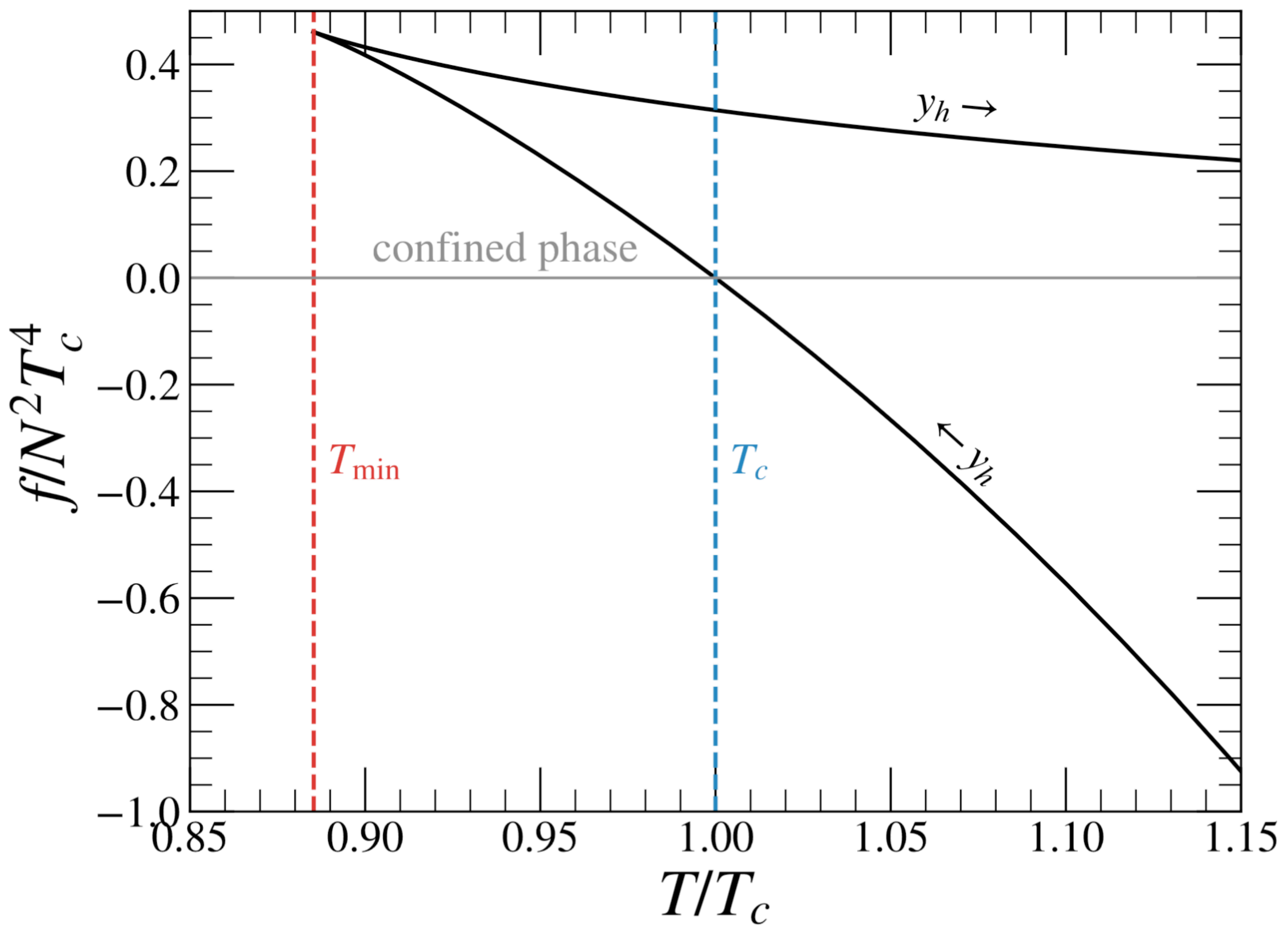
Uncool soft-wall transitions

and their gravitational wave signatures

arXiv: 2604.06306
w/ L.-T. Wang





Ameen Ismail Pheno 2026 11 May 2026

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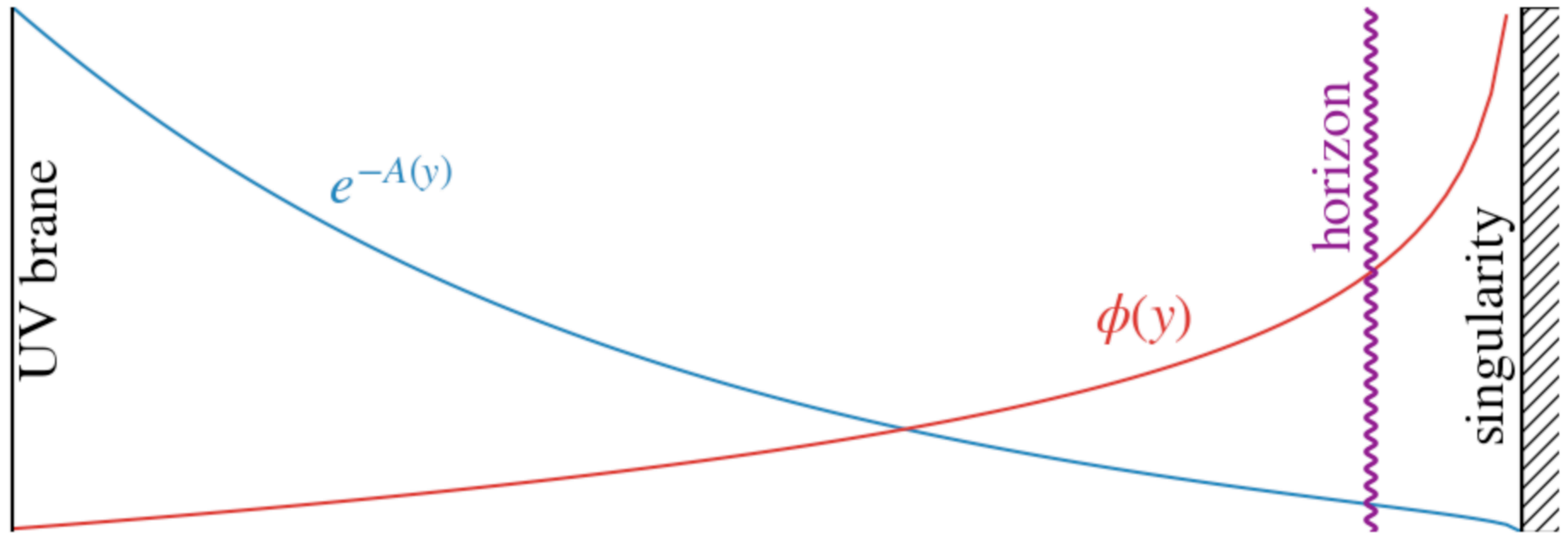
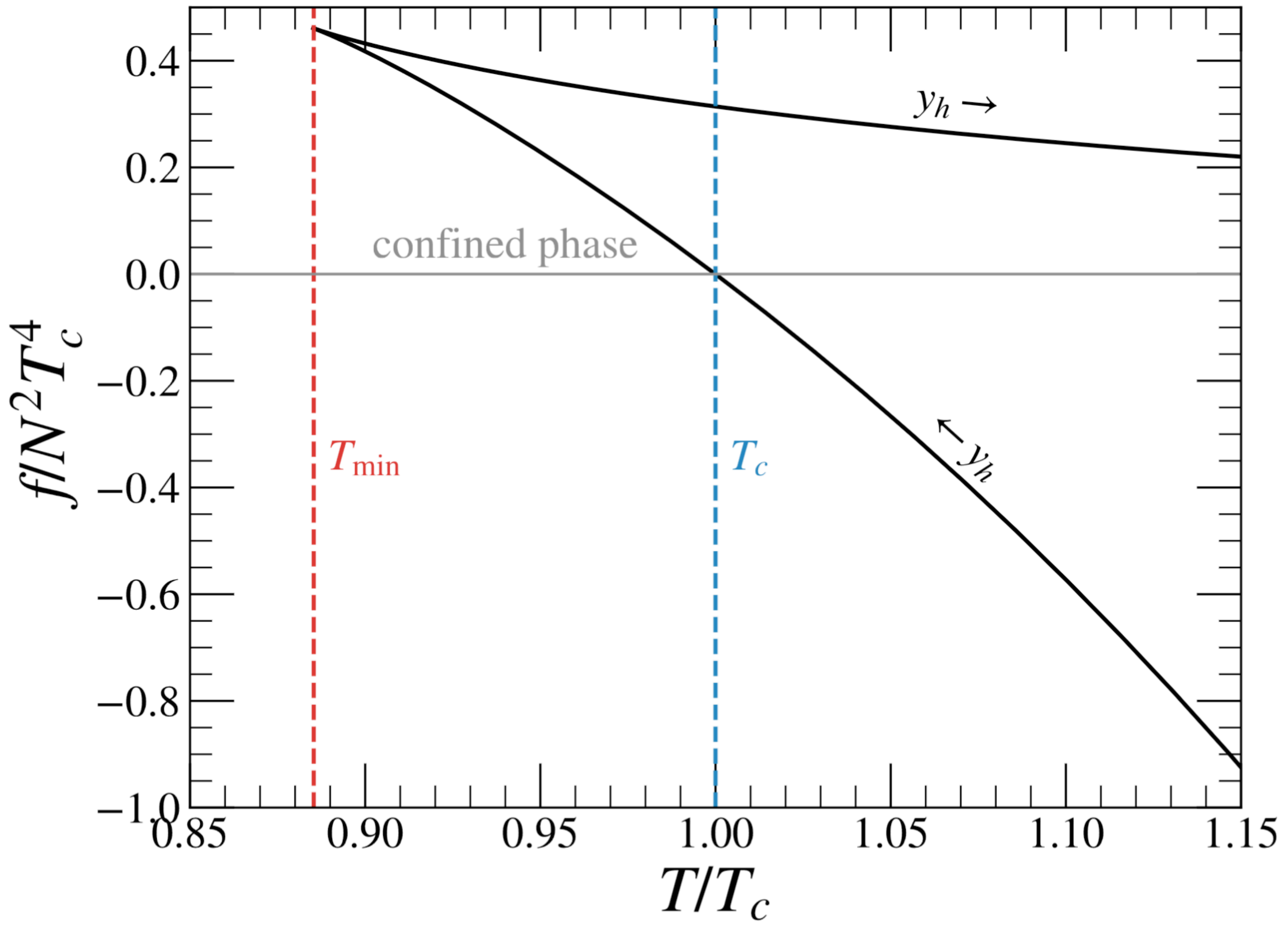
Uncool soft-wall transitions





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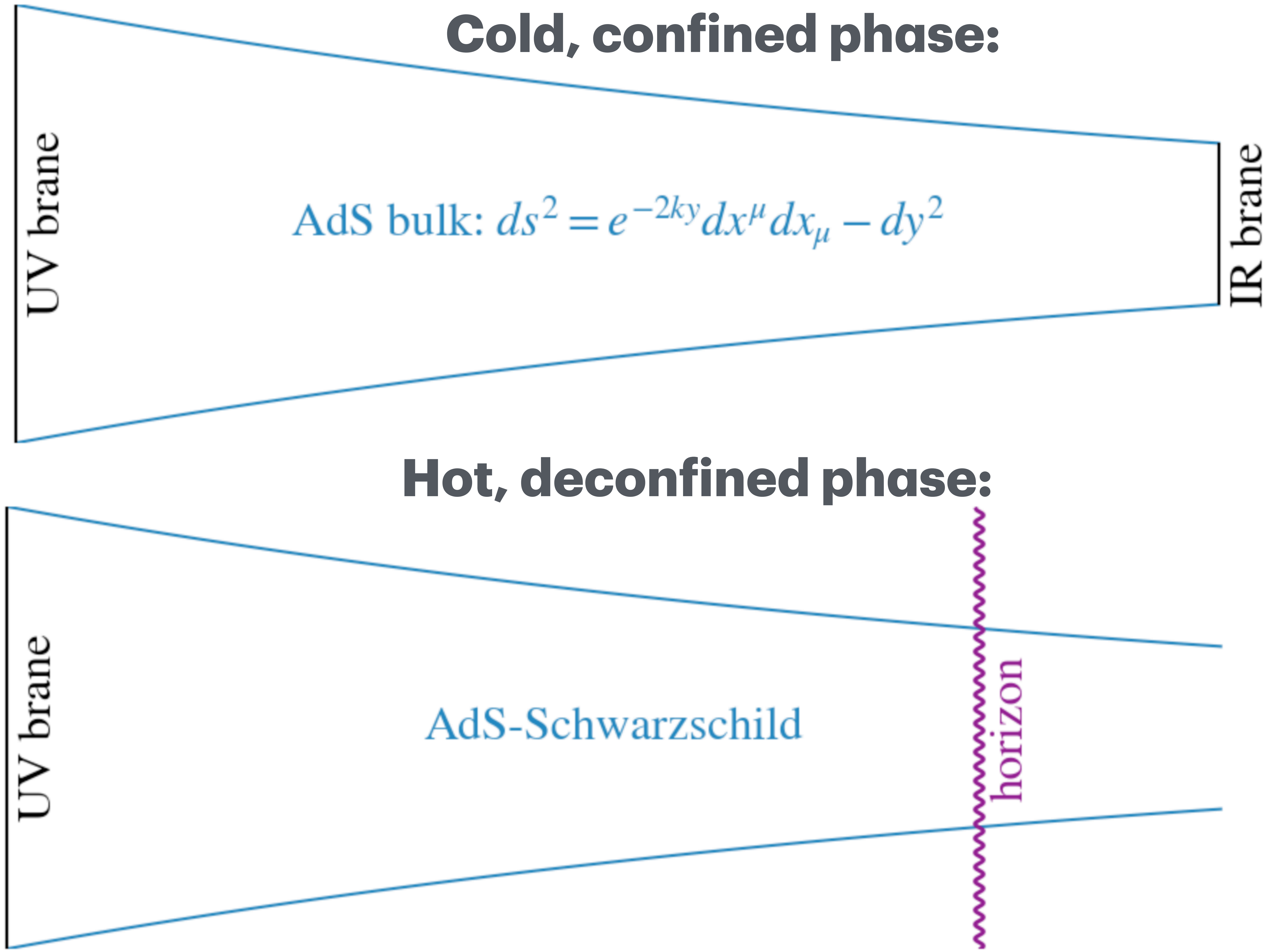
AI, L.-T. Wang

Pheno 2026

11 May 2026

First-order PTs and RS

- Introduced for Higgs hierarchy
- Dual: CFT is broken at a scale dictated by IR brane location
- High T, symmetry is restored: IR brane hidden behind black hole
- PT proceeds by nucleating bubbles of IR brane



RS PT may be strongly supercooled

- RS with the usual Goldberger-Wise stabilization* tends to exhibit very supercooled PT

see Creminelli, Nicolis, Rattazzi hep-th/0107141

- Result: large GW signal peaked around **mHz** for a TeV-scale PT

*If you're unfamiliar with Goldberger-Wise, that's fine - the details don't matter. Basically, you need some way to stabilize the size of the extra dimension, and it's a simple way to do so.

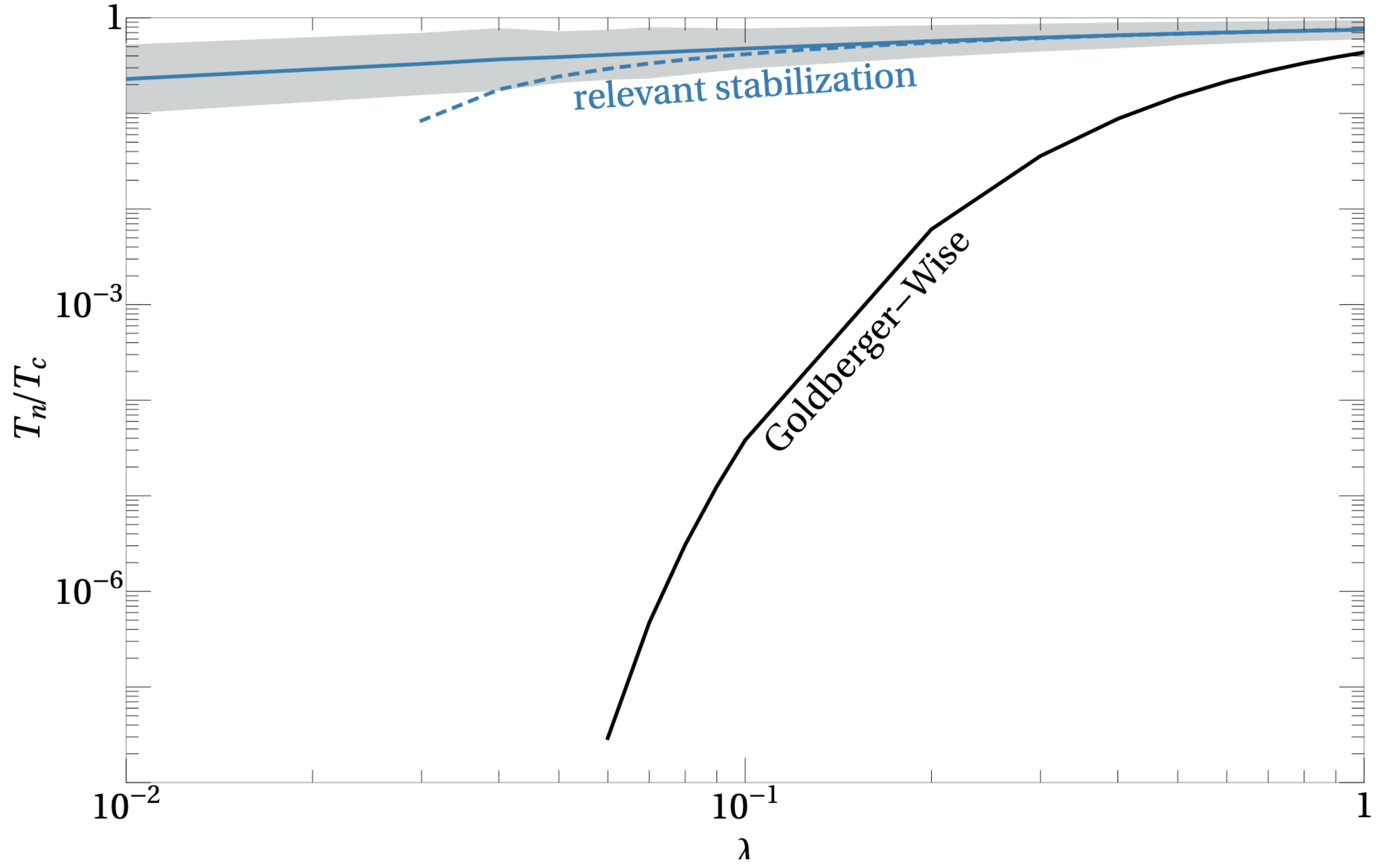
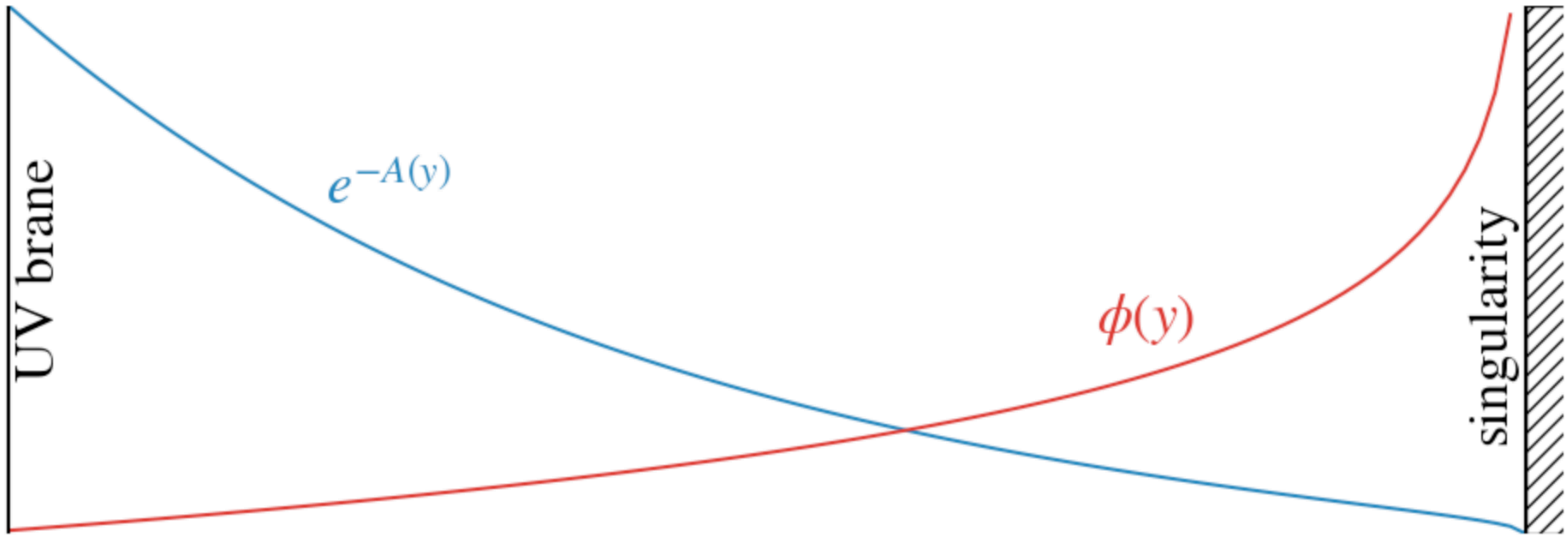


fig. from Csáki, Geller, Heller-Algazi, **AI**, 2301.10247

But you could also have a soft wall

$$ds^2 = e^{-2A(y)}\left[dt^2 - dx^i dx_i\right] - dy^2,$$

$$S = \int d^5x\sqrt{g}\left[-\frac{1}{2\kappa^2}R + \frac{1}{2}g^{AB}\partial_A\phi\partial_B\phi - V(\phi)\right].$$



The potential needs to grow exponentially to form a soft wall.

Note: the 5D gravity EFT breaks down near the singularity, but this isn't important for our results.

Why you should care

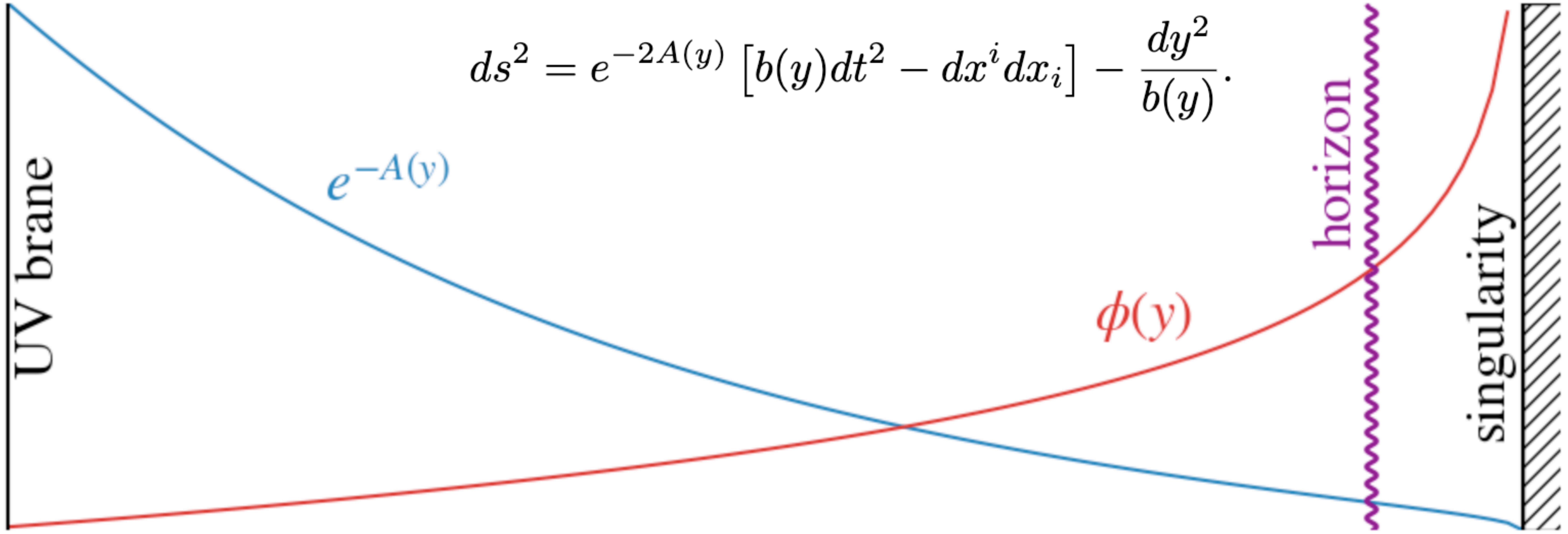
- This happens in stringy warped compactifications — see Buchel, 2103.15188, 1809.08484, 2411.15950
- Recent work suggests dramatically **different PT dynamics** — see Mishra, Randall 2401.09633
- GW signatures have not been studied much*

*Aside from some results in specific holographic QCD models and in models that do not exhibit confinement at low temperature.

What GW signals should we expect from a soft wall PT?

Soft walls, the hot phase

- Introduce a "blackening factor" $b(y)$
- Singularity hidden behind horizon, where $b(y_h) = 0$

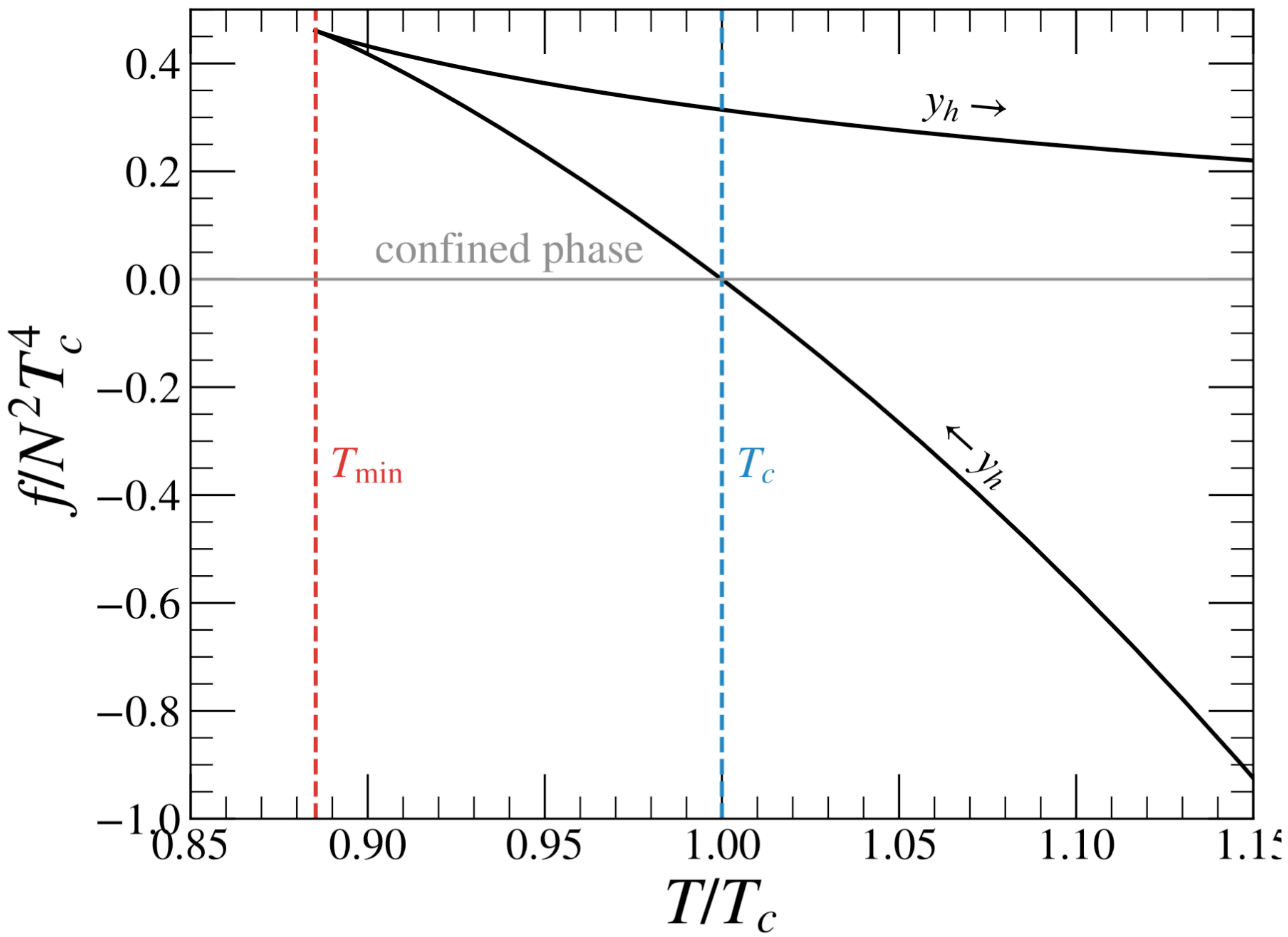


(in cold phase: $y_h \to y_s$)

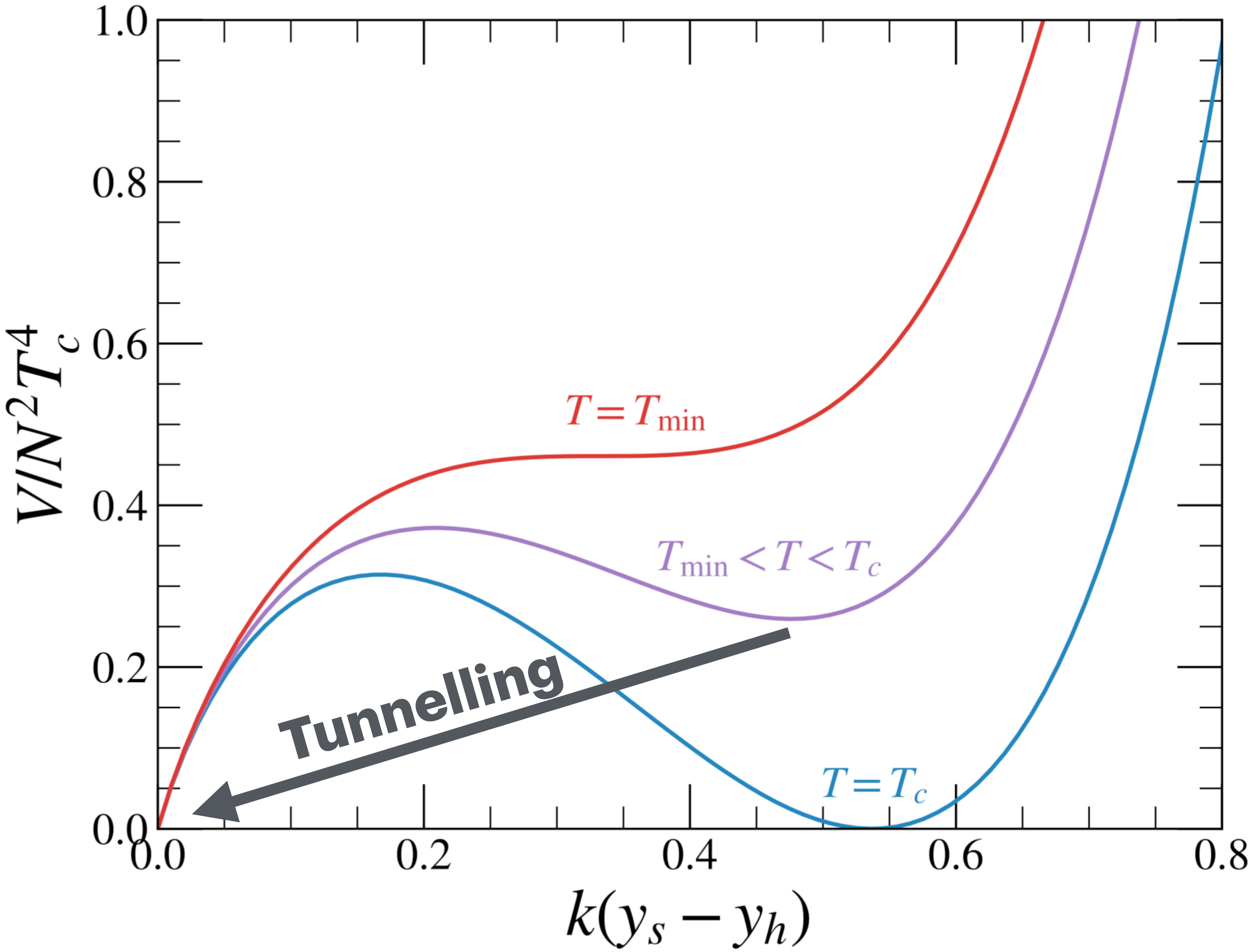
Parametrizing the bounce

We construct a 4D **effective action** for the horizon location

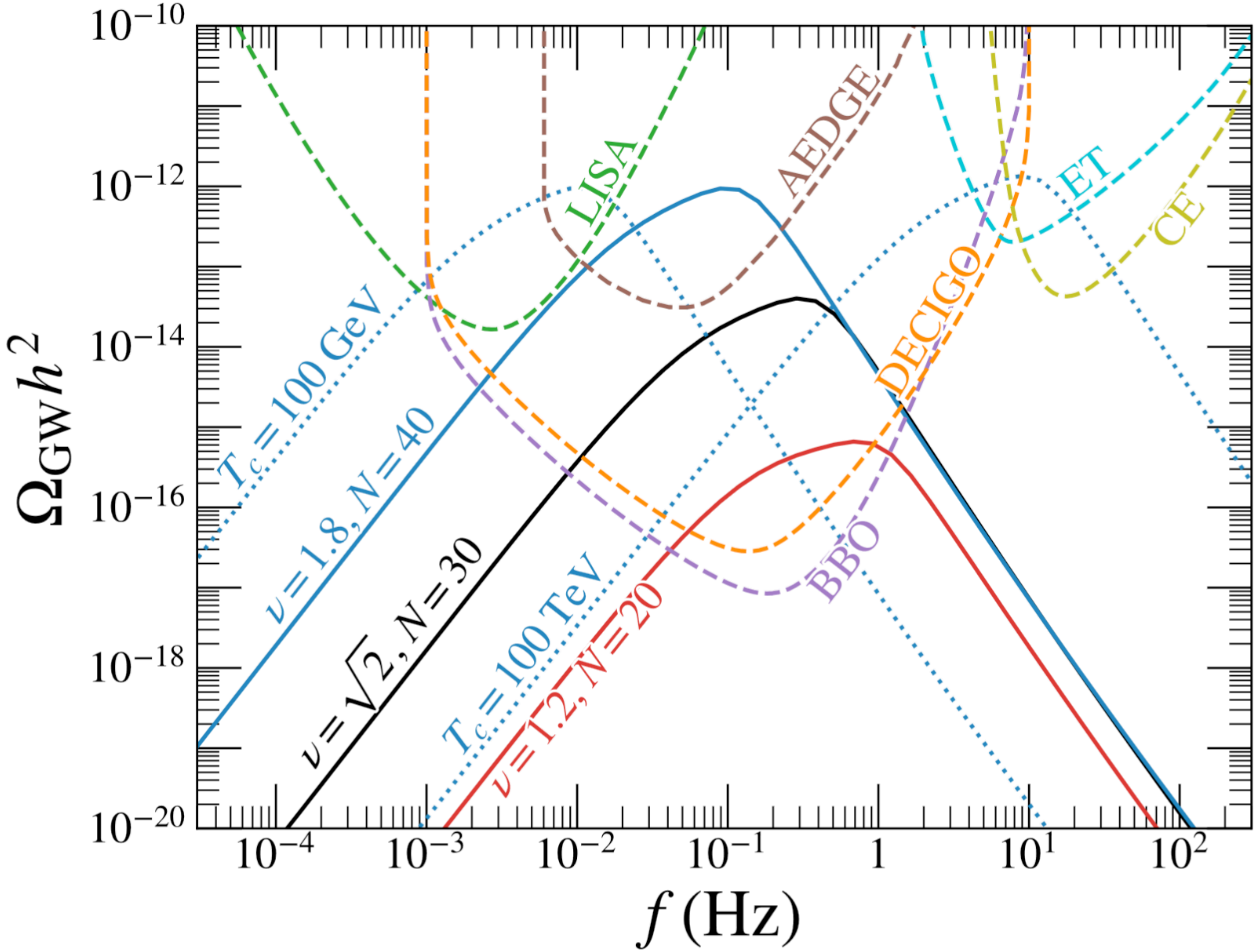
Phase diagram & effective potential



characteristic "shark-fin" structure



Gravitational wave signals

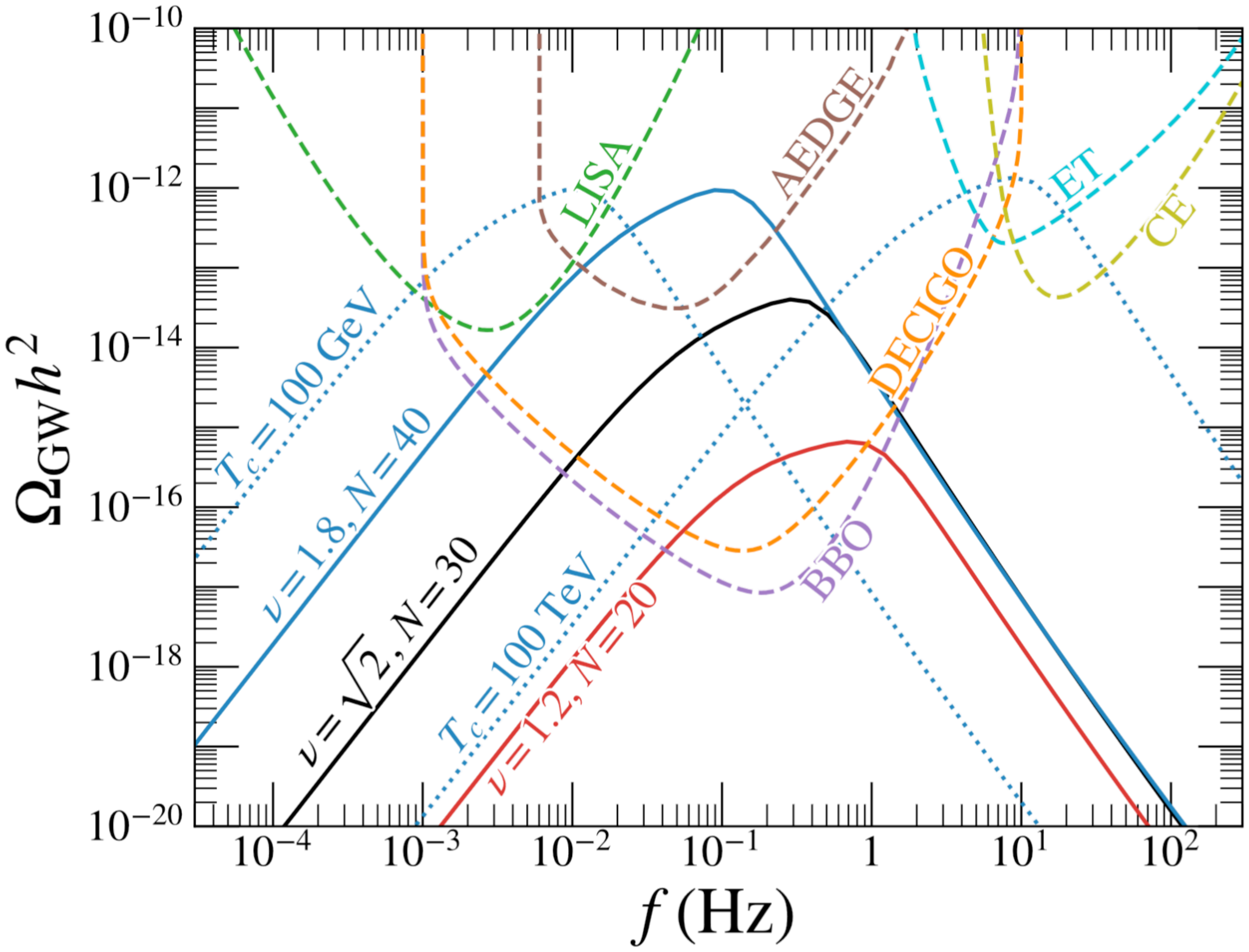


$\nu \in (1{,}2)$
parametrizes the "hardness" of the soft wall

N is # of colours in dual CFT

Summary

- Warped extra dimensions are a well-motivated scenario that lead to a first-order PT
- The PT is different in soft walls vs. vanilla RS; understanding the former is important if you care about top-down warped constructions
- GW signals visible to future space-based detectors
- Future goals: full 5D computation?



Thank you!

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Uncool soft-wall transitions

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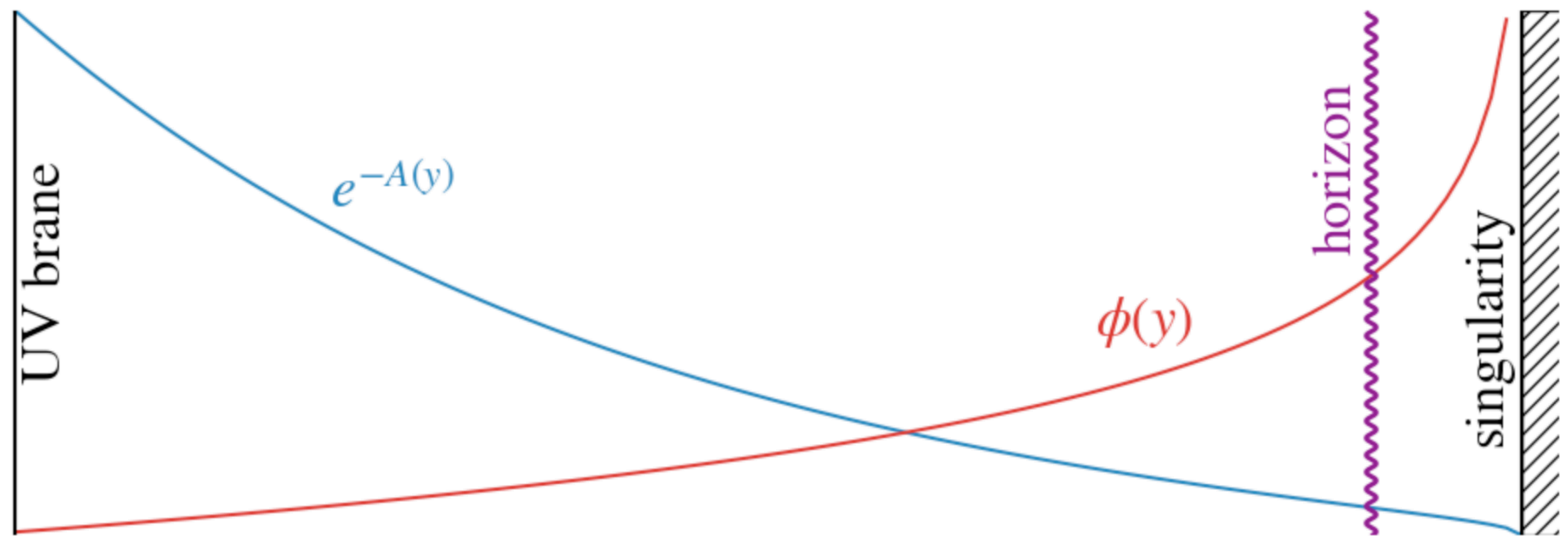

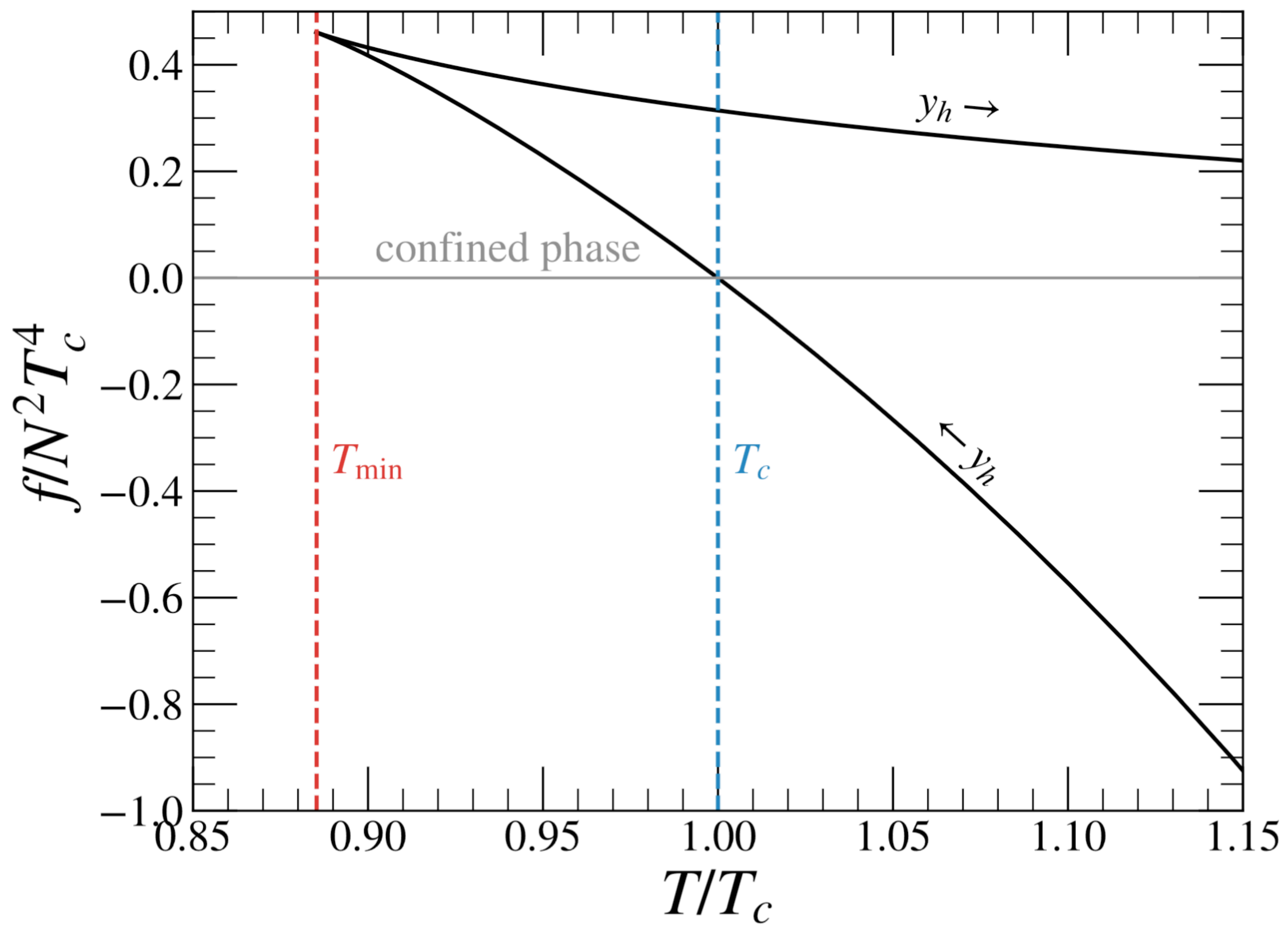

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Extra slides

Technical aside: superpotential trick

- Here are the Einstein equations (which also imply the scalar EOM):

$$\kappa^2 \phi'^2 = 3A''$$
$$\kappa^2 V(\phi(y)) = -6A'^2 + \frac{3}{2}A''.$$

- There's a clever trick to construct solutions to them.

Technical aside: superpotential trick

- Given a “superpotential” $W[\phi]$, can construct solution to Einstein equations
  - $A'(y) = \frac{\kappa^2}{6} W[\phi]$
  - $\phi'(y) = \frac{1}{2} \frac{dW[\phi]}{d\phi}$
  - $V(\phi) = \frac{1}{8} \left(\frac{dW[\phi]}{d\phi} \right)^2 - \frac{\kappa^2}{6} W[\phi]^2$
- Often easier to make statements regarding confinement, etc. in terms of superpotential

Confining soft walls, generally

see 0707.1324, 0707.1349, 0907.5361

- Asymptotic superpotential $W[\phi] \sim \phi^n \exp \kappa\nu\phi/\sqrt{3}$ leads to confinement for $n \geq 0, 1 \leq \nu < 2^*$
- Near-wall geometry described by $A \sim -1/\nu^2 \log(1 - y/y_s) + \mathcal{O}[\log\log(1 - y/y_s)]^{**}$ and $\phi \sim -1/\nu \log \nu^2 k(y_s - y)$
- Can think of ν as parametrizing the hardness of the wall

* $\nu \geq 2$ is pathological. See Gubser, hep-th/0002160

**The subleading term matters for $\nu = 1$; I'll talk about this at the end, time permitting.

Temperature, entropy, free energy

- Standard trick for T - require absence of conical defect
- (at thermal equilibrium, $T_{\text{bath}} = T_h$)
- Bekenstein-Hawking for entropy:
- Note: $s \propto (M_5/k)^3 \sim N^{2*}$
- Free energy from $s = -\partial f/\partial T$

$$T_h = \beta^{-1} = \frac{1}{4\pi} e^{-A(y_h)} |b'(y_h)|.$$

$$s = \frac{2\pi}{\kappa^2} e^{-3A(y_h)}.$$

*remember N, it'll show up again later

Analytical solutions in the hot phase

(there aren't many)

- Constant (super)potential: AdS-Schwarzschild
  - $A(y) = ky$, $\phi = \text{const}\,.$
  - $b(y) = 1 - \exp 4k(y - y_h)$ (approaches 1 in deep UV, $y \to -\infty$, and vanishes at y_h)
- Exponential (super)potential: describes behaviour near a soft wall
  - $A(y) = -1/\nu^2 \log(1 - y/y_s)$, $\phi = -1/\nu \log \nu^2 k(y_s - y)$
  - $b(y) = 1 - [(y_s - y)/(y_s - y_h)]^{1-4/\nu^2}$

A simple ansatz

- UV limit ($y \to -\infty$): **constant** potential
- IR limit ($y \to y_s$): **exponential** potential
- Stitch together: $$A' = \begin{cases} k & y < y_i \equiv y_s - 1/k\nu^2 \\ \frac{1}{\nu^2 (y_s - y)} & y > y_i. \end{cases}$$

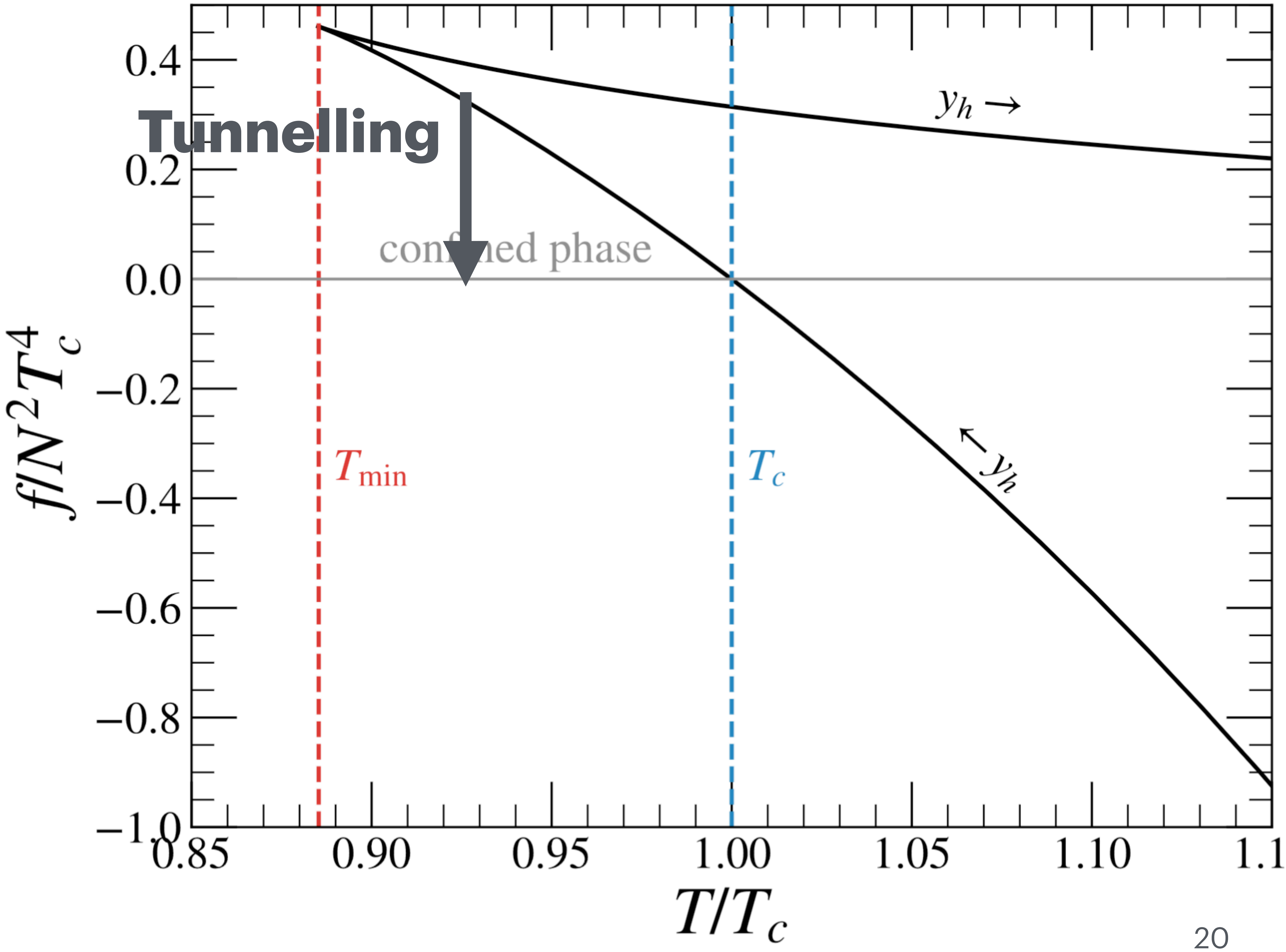
I'm not explicitly showing the profiles for the bulk scalar $\phi(y)$ or the blackening factor $b(y)$.

This parametrizes a family of confining soft walls for $1 < \nu < 2$.*

*Again, $\nu = 1$ needs a separate treatment, which I'll get to later.

The phase transition

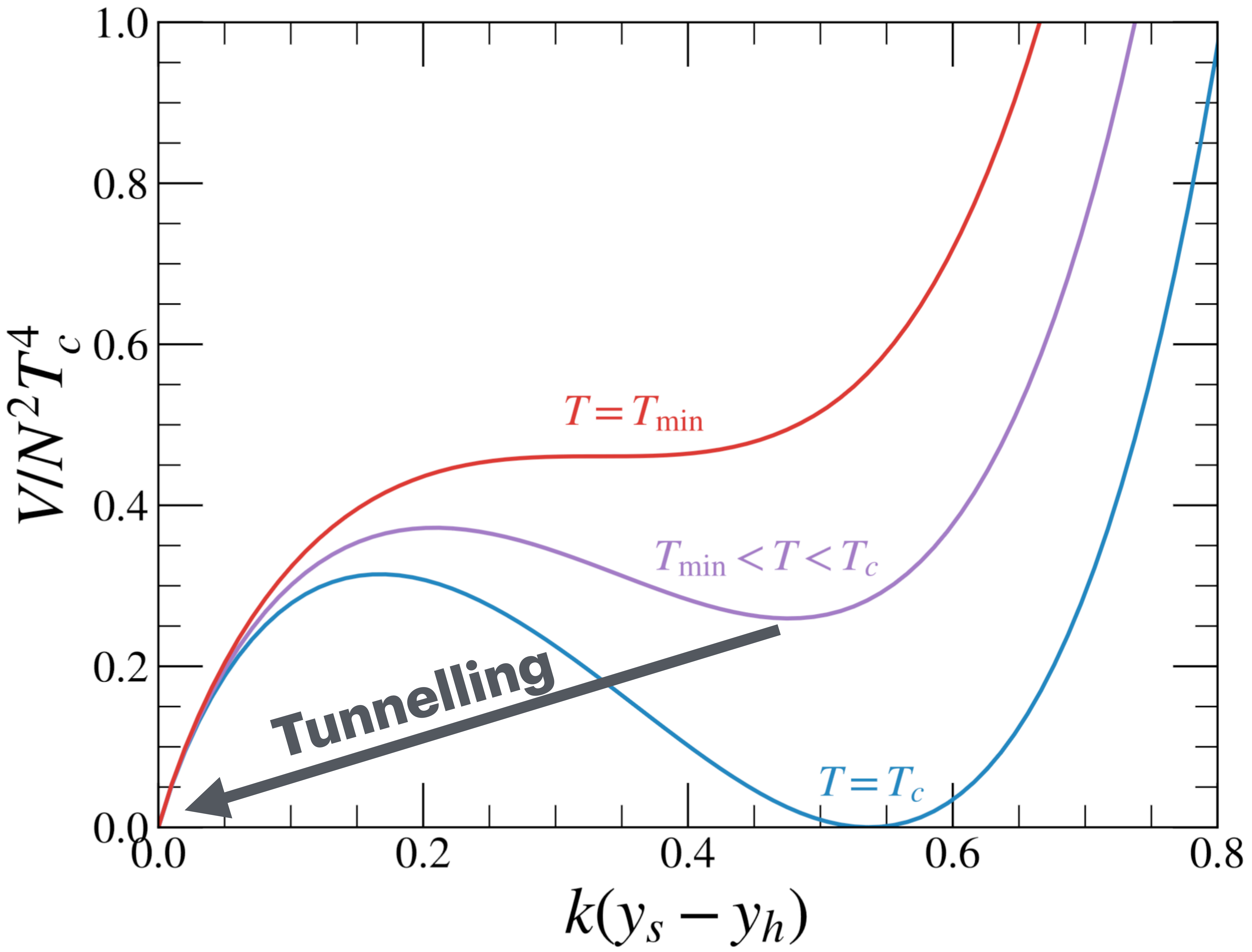
- How do we study bubbles?
- Ideally, we'd find a solution to 5D Euclidean-time EFE's, compute a bounce action, etc.
- This is really hard! Instead, take the equilibrium solution, but with y_h a function of bubble radial coordinate ρ
- Reduces to a standard 4D problem (compute eff. action for $y_h(\rho)$ and study tunnelling)



Bounce preliminaries

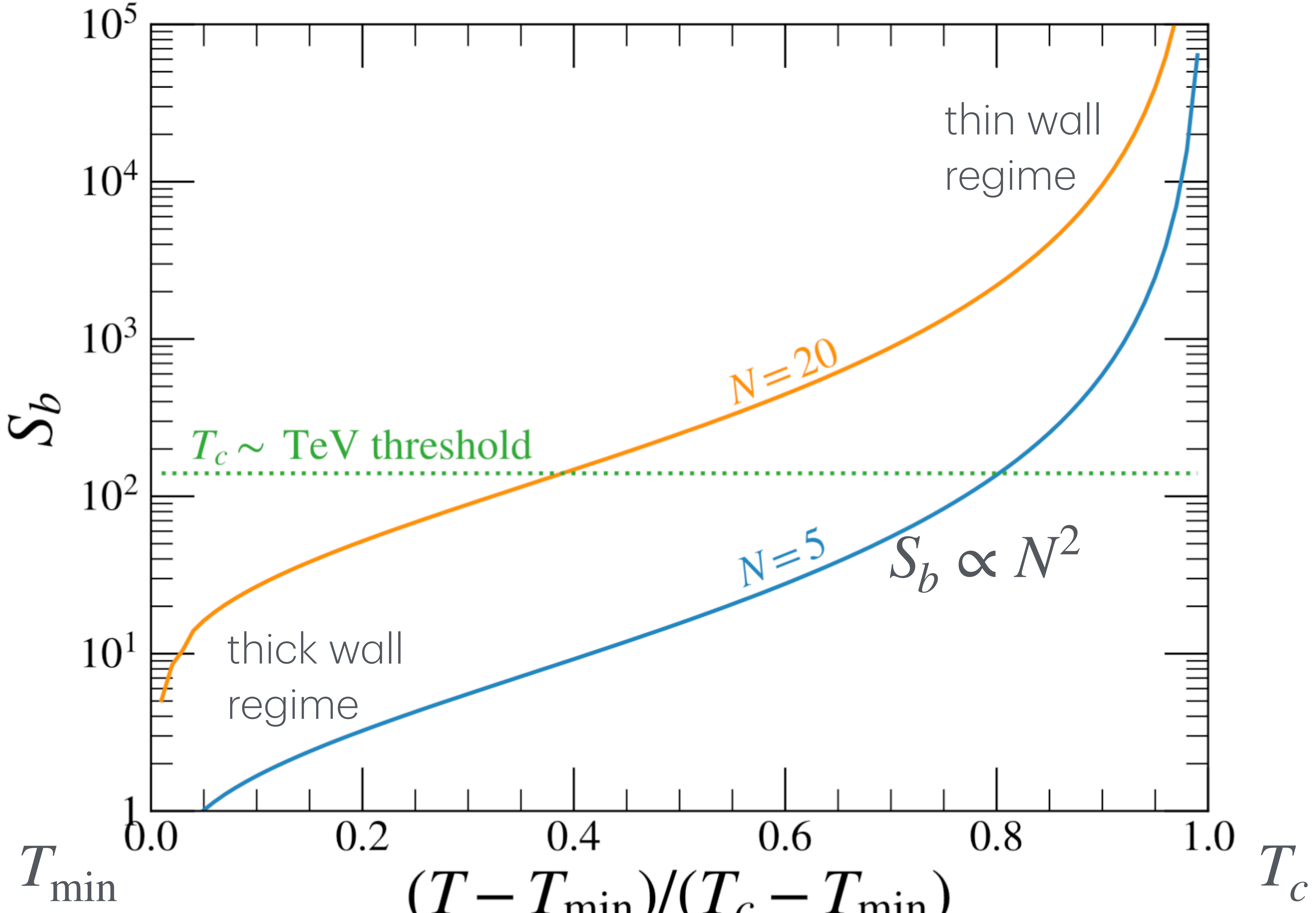
c.f. Coleman '77, Linde '83

- Tunnelling rate $\Gamma \sim T^4 e^{-S_b}$
- Bounce* interpolates b/w vacua
- This is minimized by the solution to the Euclidean-time EOM



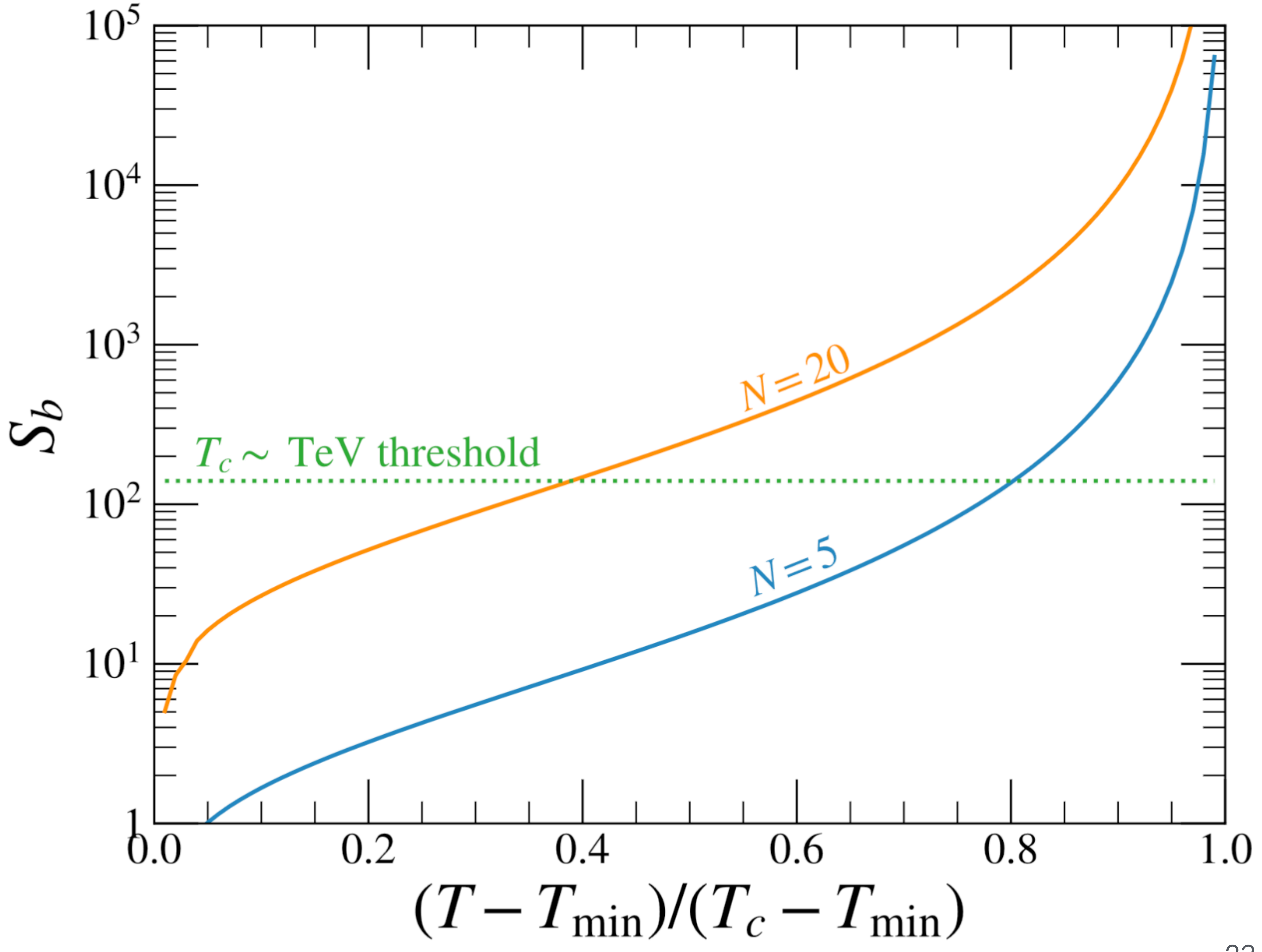
*for experts: the $O(3)$-symmetric bounce dominates over the $O(4)$ one throughout this work

Bounce action ($\nu = \sqrt{2}$)

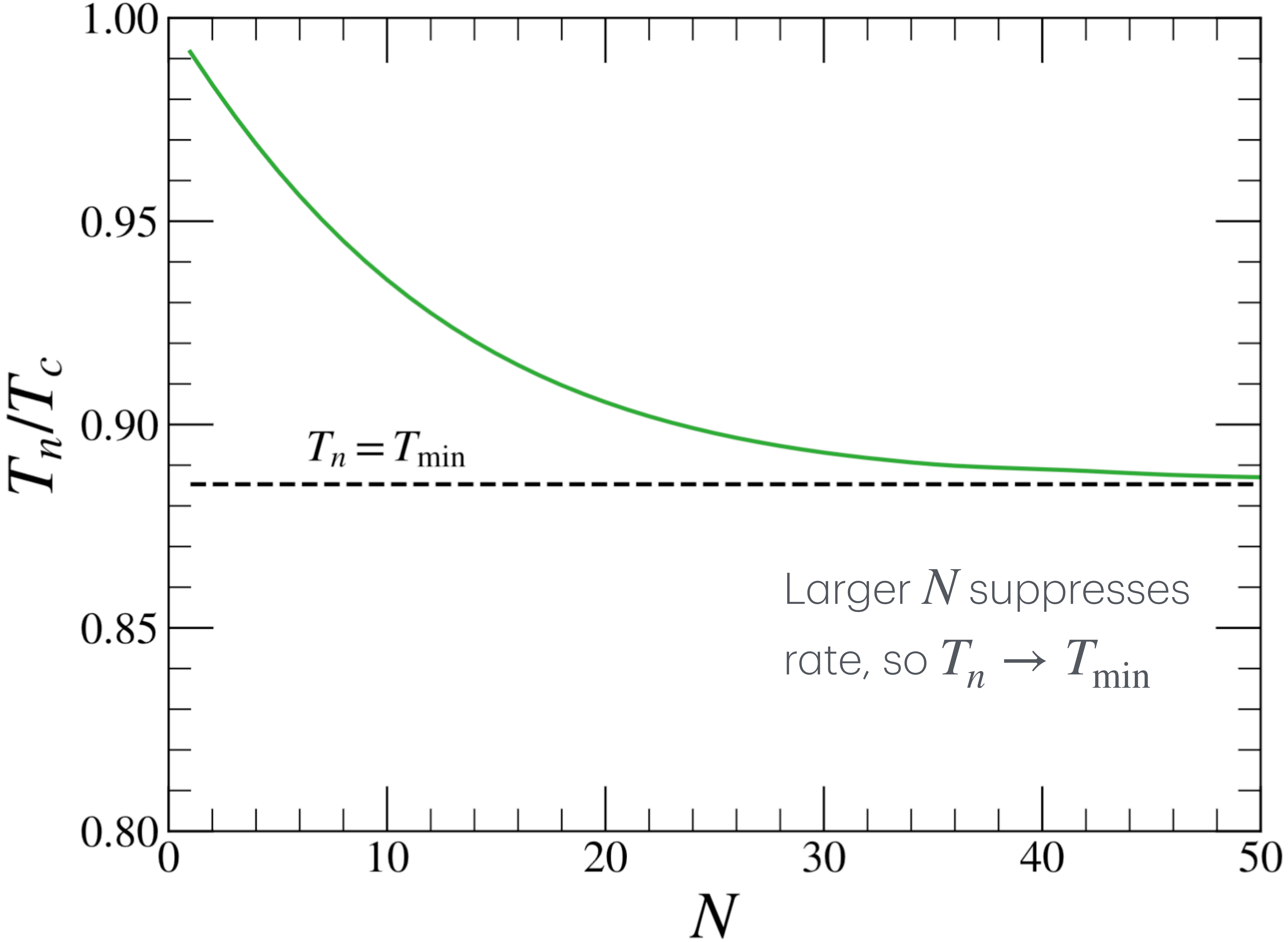


Finding the nucleation temperature

- PT completes when
  $\Gamma \sim H^4 \to T^4 e^{-S_b} \sim T^8/M_{\rm Pl}^4$
- So $S_b \sim 4 \log M_{\rm Pl}/T \approx 140$ for $T_c \sim$ TeV



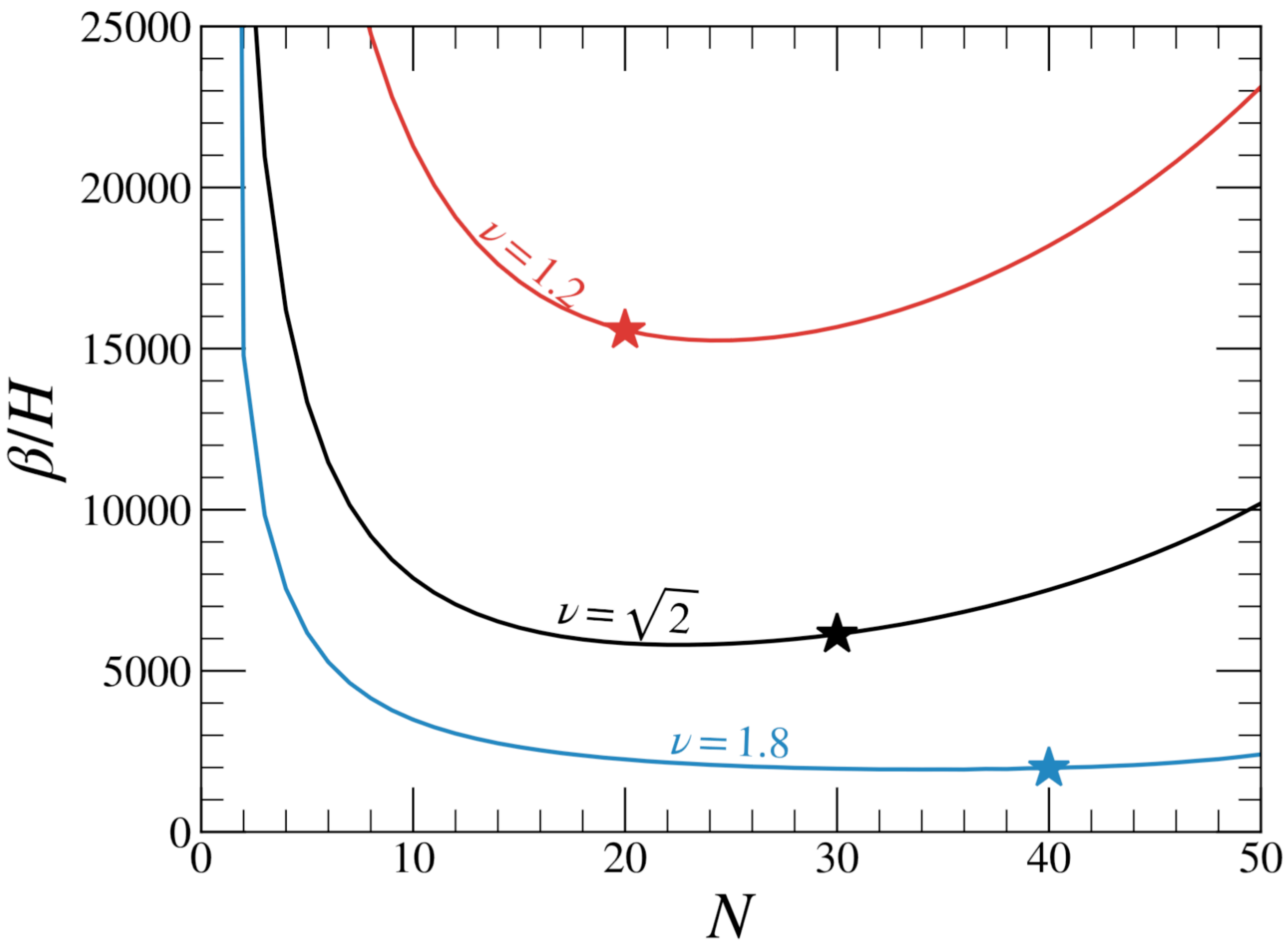
Nucleation temperature ($\nu = \sqrt{2}$)



Larger N suppresses rate, so $T_n \to T_{\min}$

Gravitational waves

- Controlled by inverse duration β/H
- $\beta/H = TdS_b/dT\Big|_{T_n}$
- $\Omega_{\mathrm{GW}} \propto H^2/\beta^2$, $f_{\mathrm{peak}} \propto \beta/H$
- For comparison: for RS with Goldberger-Wise stabilization, $\beta/H \sim 10$ is typical



Note: α_{PT} is order-one, and I take $v_w \approx 1$.

The edge case

- Recall $W[\phi] \sim \phi^{p/2} \exp \kappa\nu\phi/\sqrt{3}$ leads to confinement for $1 \leq \nu < 2$ and $p \geq 0$
- For $\nu = 1$, asymptotic geometry near the soft wall:

$$A \sim -\log k(y_s - y) - p \log(-\log k(y_s - y))$$

(this is the subleading term that we ignored for $\nu > 1$)

- Some special cases:
  - $p = 0$: linear dilaton / gapped continuum spectrum
  - $p = 1/2$: linear KK spectrum $m_n^2 \propto n$

Metric ansatz redux

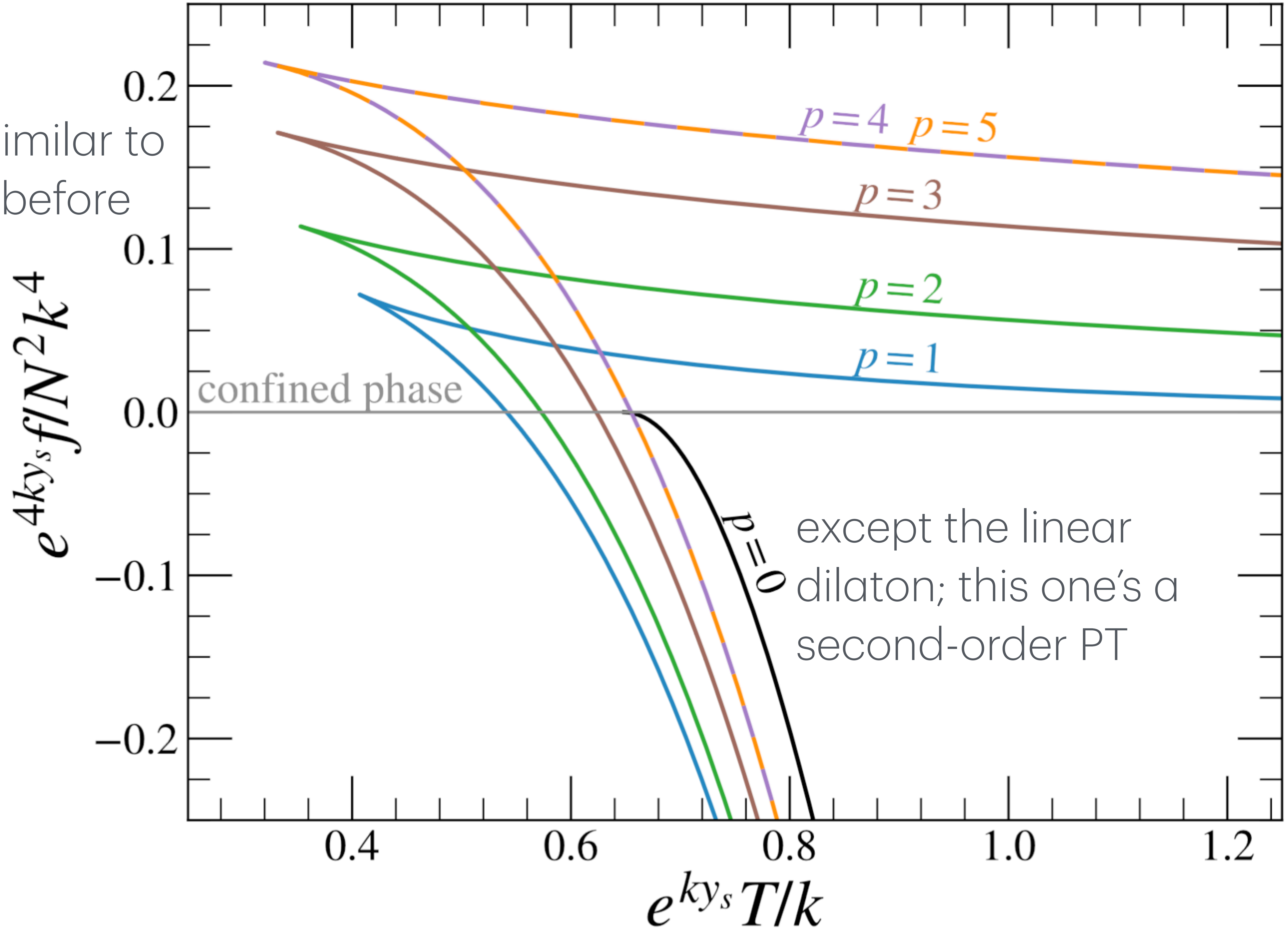
- Stitch together UV and IR behaviour:

$$A'(y) = \begin{cases} k & y < y_i \\ \frac{1}{y_s - y}\left(1 + \frac{p}{\log k(y_s - y)}\right) & y > y_i . \end{cases}$$

- (match at y_i such that $A'(y)$ is continuous)
- Numerically compute phase diagram, tunnelling, etc.

Edge case: phase diagram

qualitatively similar to what we saw before



except the linear dilaton; this one's a second-order PT

Edge case: GW signals