Quantum Tomography of Fermion Pairs in e^-e^+ Collisions: Longitudinal Beam Polarization Effects

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A Quick Review of Qubit

Bloch-Fano Decomposition: $\rho = \frac{1}{4}\left(\mathbb{I}_4 + \sum_{i=1}^{3} B_i^{\mathcal{A}} \left(\sigma_i \otimes \mathbb{I}_2\right) + \sum_{i=1}^{3} B_i^{\mathcal{B}} \left(\mathbb{I}_2 \otimes \sigma_i\right) + \sum_{i,j=1}^{3} C_{ij} \left(\sigma_i \otimes \sigma_j\right) \right)$

Fano coefficients:

- net polarization
  - $B_i^{\mathcal{A}} = \langle \sigma_i \otimes \mathbb{I}_2 \rangle$
  - $B_i^{\mathcal{B}} = \langle \mathbb{I}_2 \otimes \sigma_i \rangle$
- spin correlation matrix $C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$

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Quantum Information Observables

1. Concurrence $\mathcal{C}(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$

$\lambda_i (i = 1,2,3,4)$ are the eigenvalues of $R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ in descending order

$\mathcal{C}(\rho) = 0 \Leftrightarrow$ separable (unentangled) states $\rho = \sum p_i \rho_A^i \otimes \rho_B^i$

$0 < \mathcal{C}(\rho) \leq 1 \Leftrightarrow$ entangled states

2. Bell's Inequality $|P(\vec{a}, \vec{c}) - P(\vec{b}, \vec{c})| \leq 1 + P(\vec{a}, \vec{b})$

↓

CHSH Inequality $\mathcal{I}_2 \equiv \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \leq 2$

↓

Bell Nonlocality $\mathcal{B} \equiv \max(\mathcal{I}_2) = 2\sqrt{m_1 + m_2}$

m_1 and m_2 are the largest and the second largest eigenvalues of $C^T C$.

Quantum Information Observables: Magic

3. Stabilizer States: States that can be efficiently classically simulated. $\mathcal{M}_2 = 0$

e.g. $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)$ ...

Use Pauli string operators to create stabilizer states:

$|\psi\rangle = P|\uparrow\uparrow\rangle, P \in \mathcal{P}_n = \{\mathbb{1} \otimes \mathbb{1}, \mathbb{1} \otimes \sigma_i, \sigma_i \otimes \mathbb{1}, \sigma_i \otimes \sigma_j\}$

"Magic" States: States where quantum computing has advantages. $\mathcal{M}_2 \neq 0$

The Second Stabilizer Renyi Entropy ("Magic"):

$$\mathcal{M}_2 = -\log_2 \frac{\sum_{P\in\mathcal{P}_n} \mathrm{Tr}(\rho P)^4}{\sum_{P\in\mathcal{P}_n} \mathrm{Tr}(\rho P)^2}$$

$$= -\log_2 \frac{1 + \sum_i (B_i^+)^4 + \sum_i (B_i^-)^4 + \sum_{ij} (C_{ij})^4}{1 + \sum_i (B_i^+)^2 + \sum_i (B_i^-)^2 + \sum_{ij} (C_{ij})^2}$$

Quantum Information Observables: Magic

3. Stabilizer States: States that can be efficiently classically simulated. $\mathcal{M}_2 = 0$

e.g. $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \frac{1}{2}(|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + |\downarrow\downarrow\rangle\langle\downarrow\downarrow|)$ ...

Use Pauli string operators to create stabilizer states:

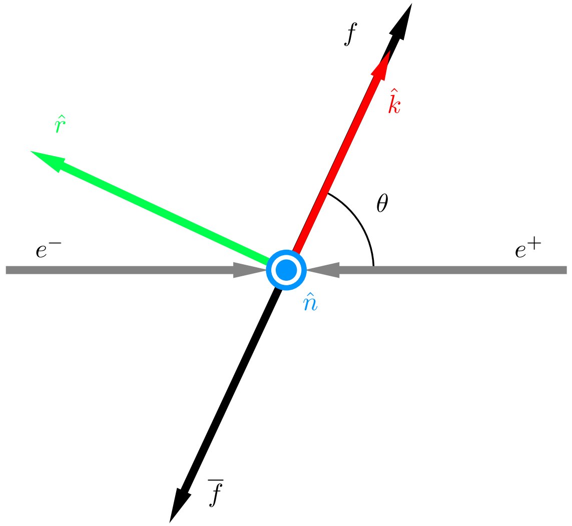
$|\psi\rangle = P|\uparrow\uparrow\rangle, P \in \mathcal{P}_n = \{\mathbb{1} \otimes \mathbb{1}, \mathbb{1} \otimes \sigma_i, \sigma_i \otimes \mathbb{1}, \sigma_i \otimes \sigma_j\}$

"Magic" States: States where quantum computing has advantages. $\mathcal{M}_2 \neq 0$

The Second Stabilizer Renyi Entropy ("Magic"):

It depends on the quantization basis we choose!

We use helicity basis here:



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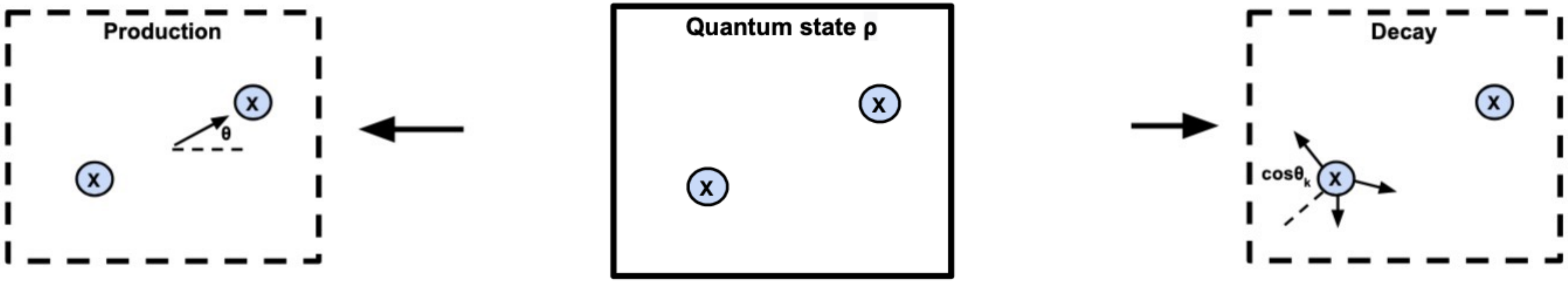
Quantum Tomography at Colliders

Two methods of quantum tomography:

Kinematic Method

Decay Method



Assuming SM at Production
Use kinematic variables

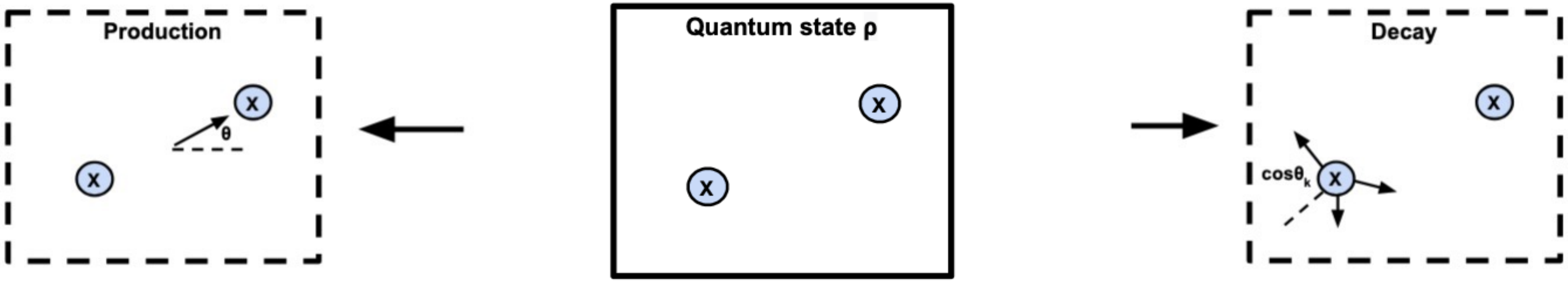
Assuming SM at Decay
Use decay particles

Both can reconstruct the quantum states

Quantum Tomography at Colliders

Two methods of quantum tomography:

Kinematic Method | **Decay Method**



$\mu^-\mu^+, e^-e^+$

No Decay at Colliders

$t\bar{t}$

Decay Channel: $t\bar{t} \to \ell q$

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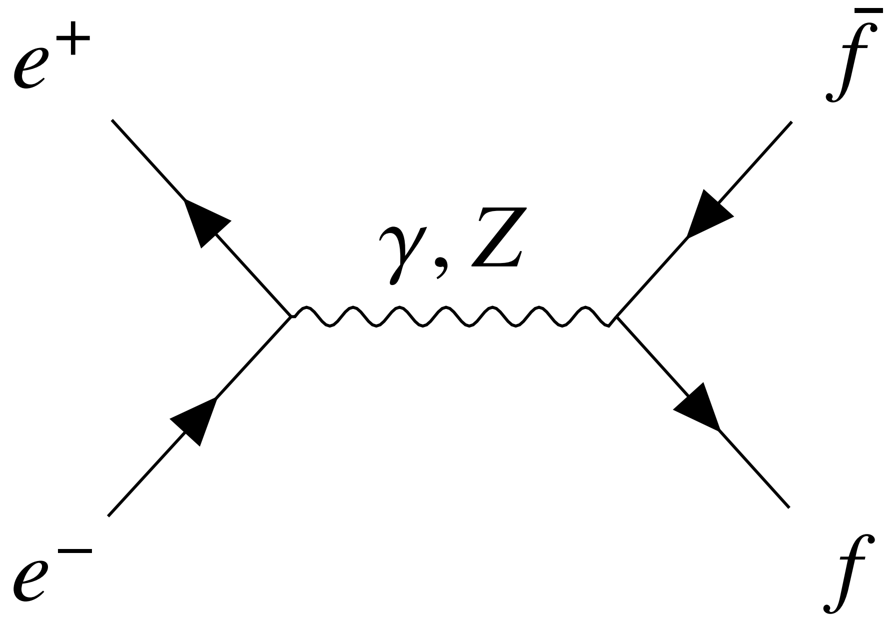
$e^-e^+ \to f\bar{f}$: Only s-channel Process



$f\bar{f} = t\bar{t}, \tau^-\tau^+, \mu^-\mu^+ \ldots$

Spin Density Matrix: $\rho \propto R = \mathcal{M}_{ss'}\mathcal{M}^*_{rr'}|ss'\rangle\langle rr'|$

Fully polarized: Pure State $\xrightarrow{\hat{T}}$ Pure State

Unpolarized: Mixed State $\xrightarrow{\hat{T}}$ Usually Mixed State

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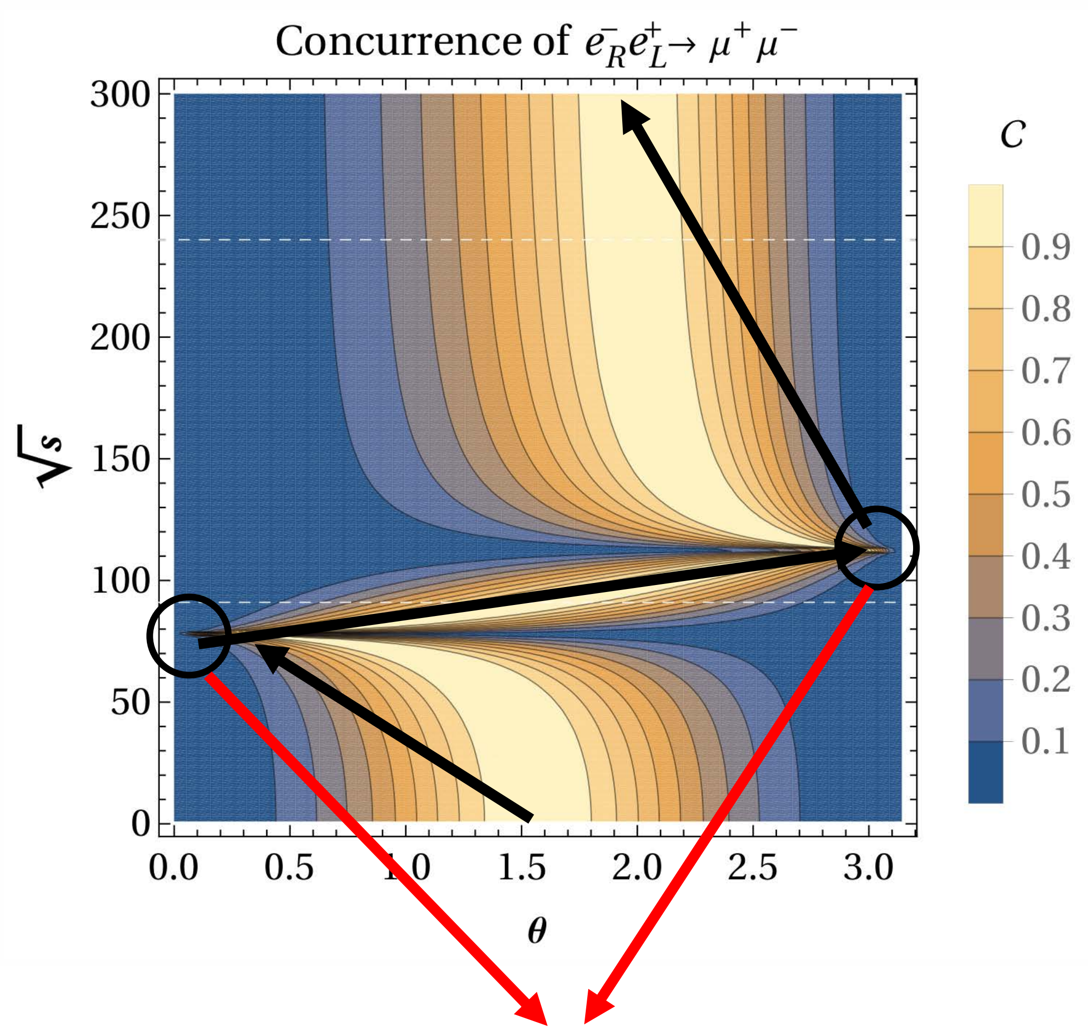
$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Massless and near Z pole

Fully polarized initial state: $e^-_R e^+_L$



$f_{L/R} = 0$, Max Parity Violation

$$\mathcal{M} = \frac{e^2 g_{\mu\nu}}{s} J^{\mu}_{in,\pm1}(f_V J^{\nu}_{out,V} + f_A J^{\nu}_{out,A})$$

Since γ/Z is spin 1 and in massless limit,

$|\psi\rangle \propto a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle$

Initial States: $|\uparrow\uparrow\rangle$ in z direction

↓ Project onto

Final States: $|\uparrow\uparrow/\downarrow\downarrow\rangle$ in k direction

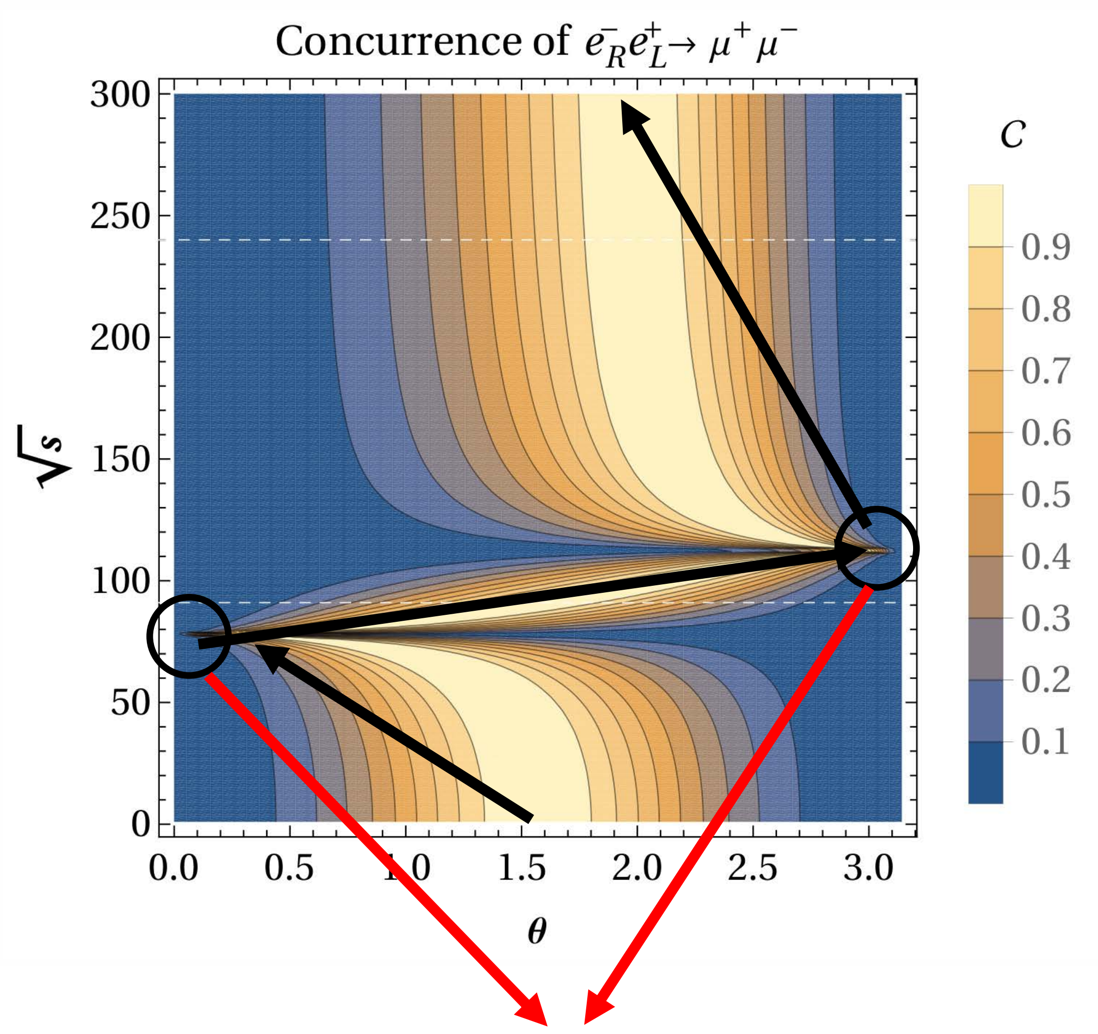
We can use the Wigner d-matrix:

$$|\psi\rangle \propto f_L d^1_{1,1}(\theta)|\uparrow\uparrow\rangle + f_R d^1_{-1,1}(\theta)|\downarrow\downarrow\rangle$$

$$f_{L,R} = (f_V \pm f_A)/2$$

$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Massless and near Z pole

Fully polarized initial state: $e^-_R e^+_L$



Max Entanglement at $\cos\theta_c \sim f_A/f_V$

$$f_V^R = Q_e Q_f + \frac{g_R^e(g_R^f + g_L^f)}{2s_W^2 c_W^2}\frac{s}{s - m_Z^2 + is\Gamma_Z/m_Z}$$

$$f_A^R = \frac{g_R^e(g_R^f - g_L^f)}{2s_W^2 c_W^2}\frac{s}{s - m_Z^2 + is\Gamma_Z/m_Z}$$

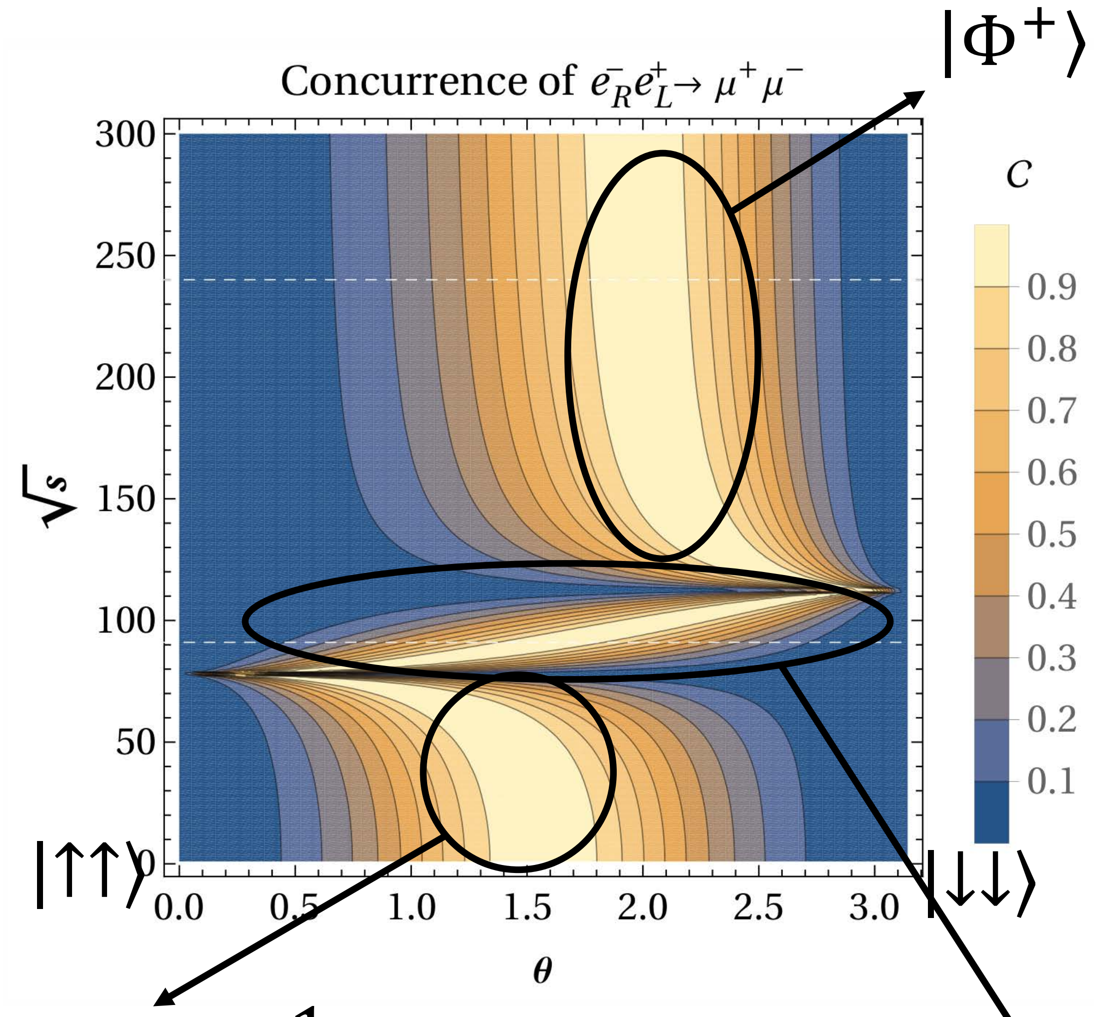
As $\sqrt{s}$ increases, f_V, f_A changes

⟶ $\cos\theta_c$ changes

$f_{L/R} = 0$, Max Parity Violation

$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Massless and near Z pole

Fully polarized initial state: $e^-_R e^+_L$



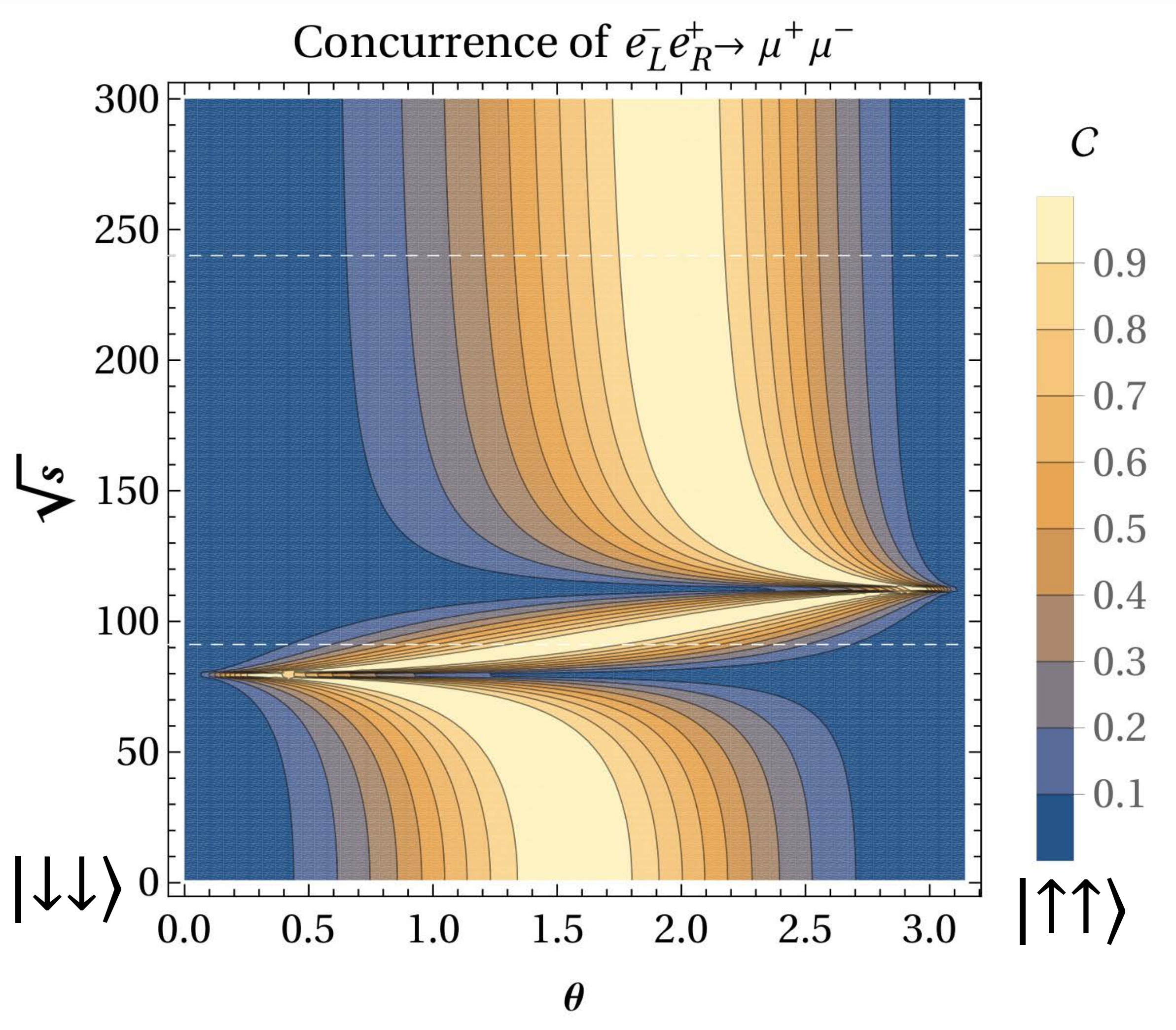
Different Bell state
at different energy scales

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

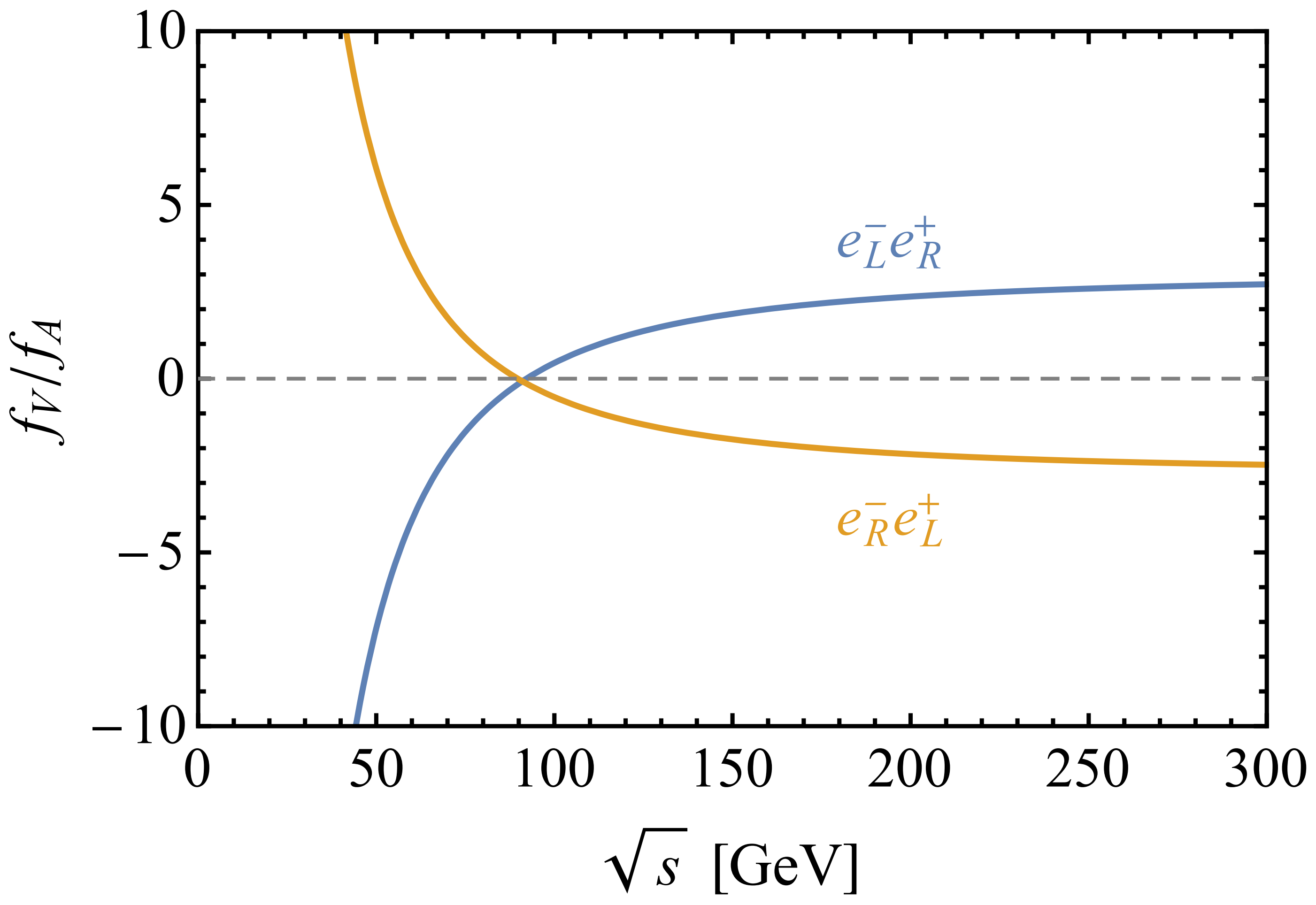
$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Massless and near Z pole

Fully polarized initial state: $e_L^- e_R^+$



$e_L^- e_R^+$ looks quite similar with $e_R^- e_L^+$!

$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Polarization Effect



$$\frac{f_V^{RL}}{f_A^{RL}} \sim -\frac{f_V^{LR}}{f_A^{LR}}$$

+

The incoming current $J^{\mu}_{in,\pm 1}$

↓

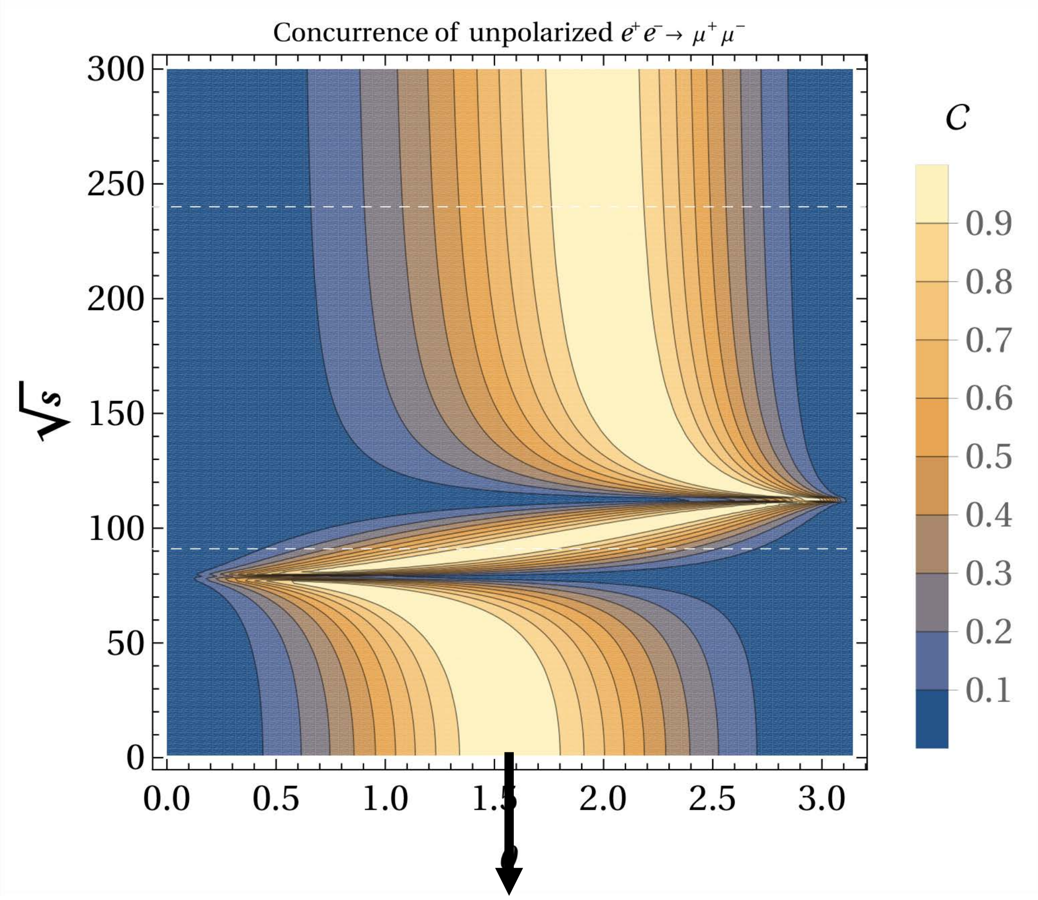
$\mathcal{C}^{RL}(s,\theta) \sim \mathcal{C}^{LR}(s,\theta)$

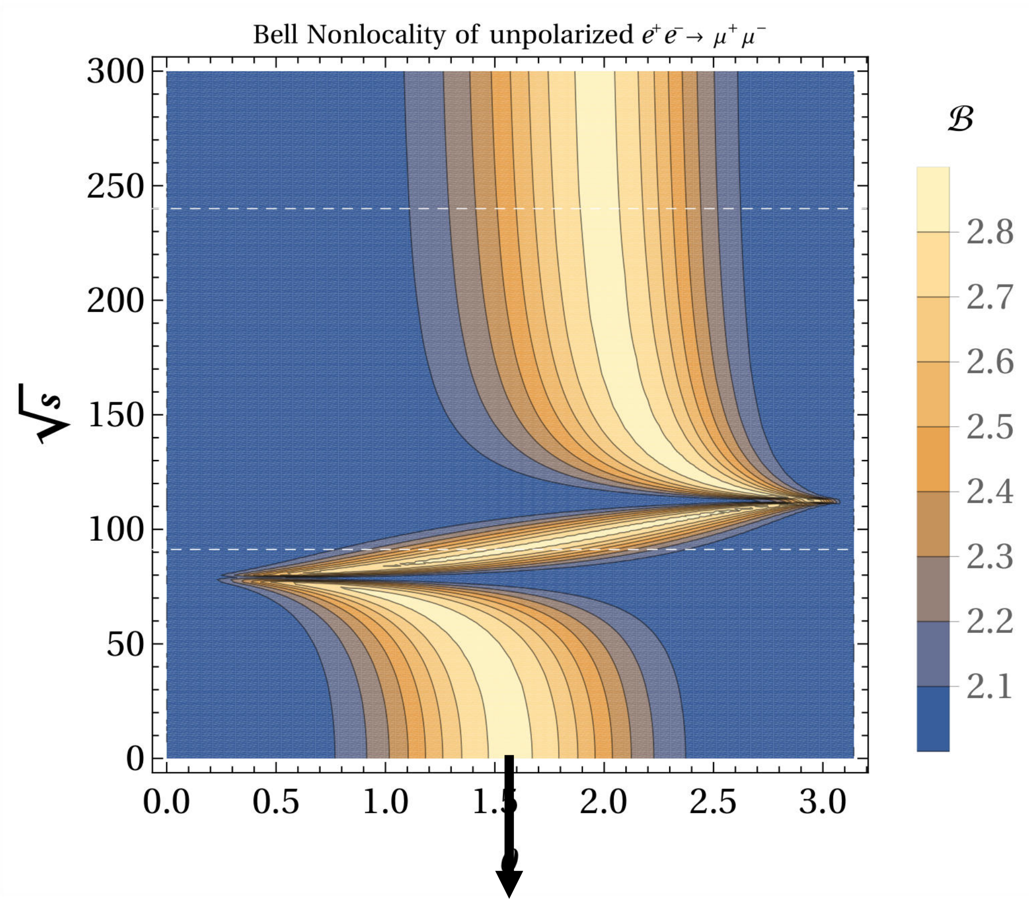
Similar Behavior

For Different Polarization

$e^- e^+ \to \mu^- \mu^+ / \tau^- \tau^+$: Unpolarized Beam

Unpolarized Beam: Classical mixing of fully polarized case



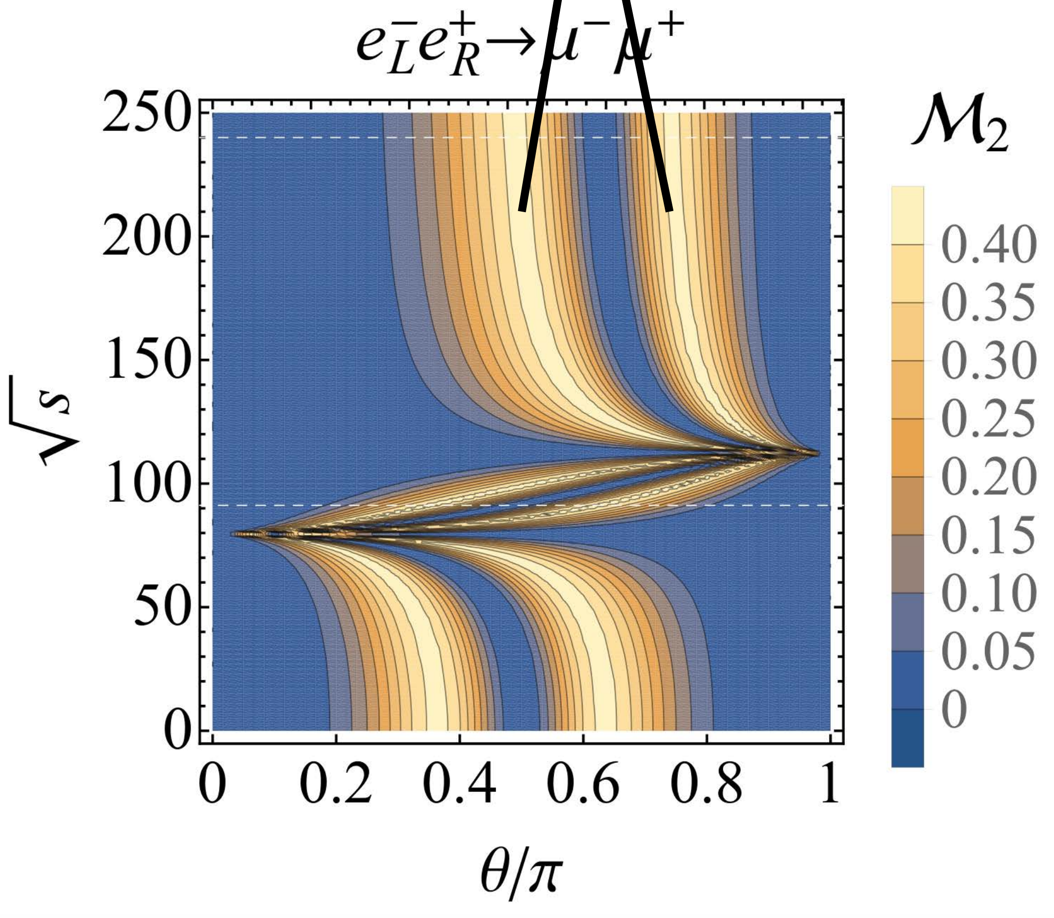


Max Entanglement
Close to pure state

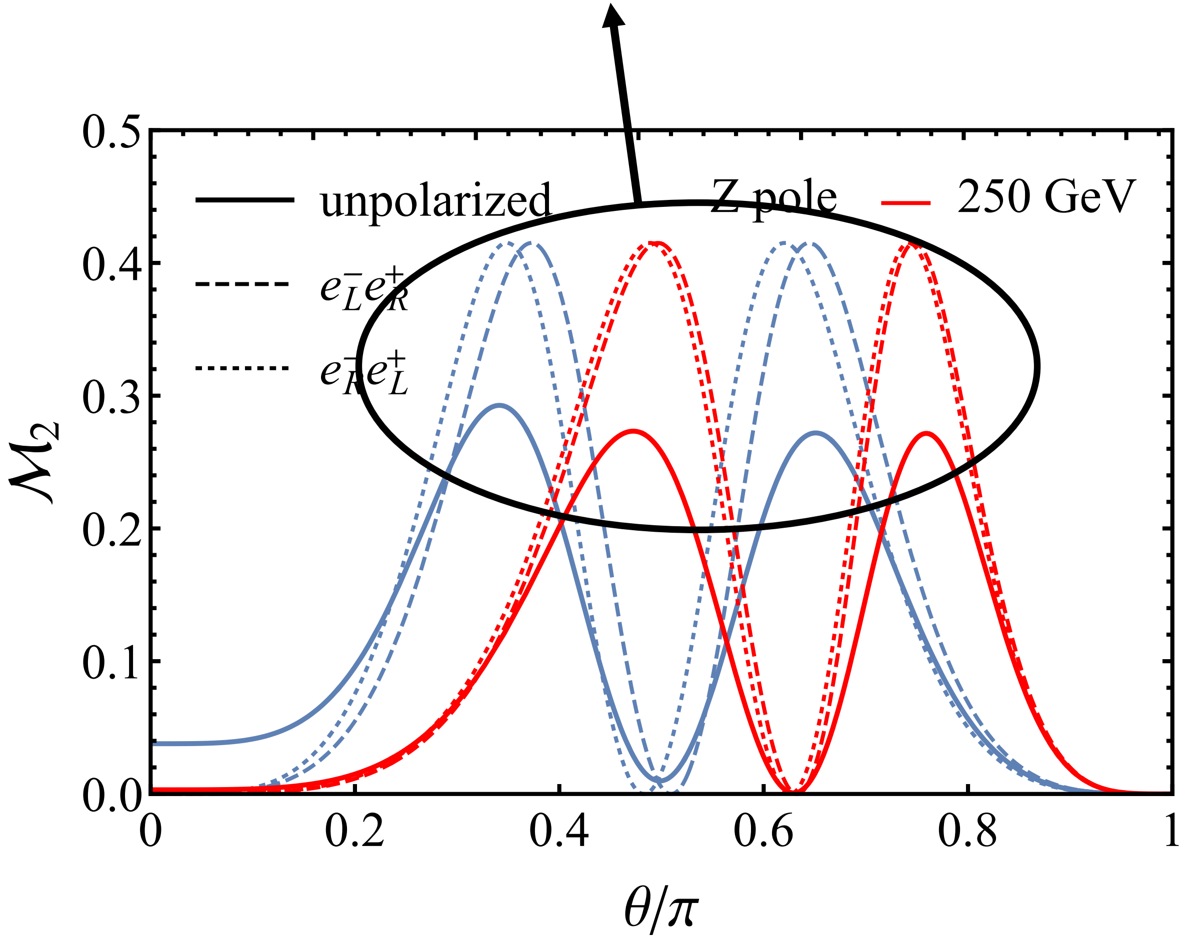
Bell State
$\mathcal{B} = 2\sqrt{2}$

$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Magic

Two Peaks of Magic

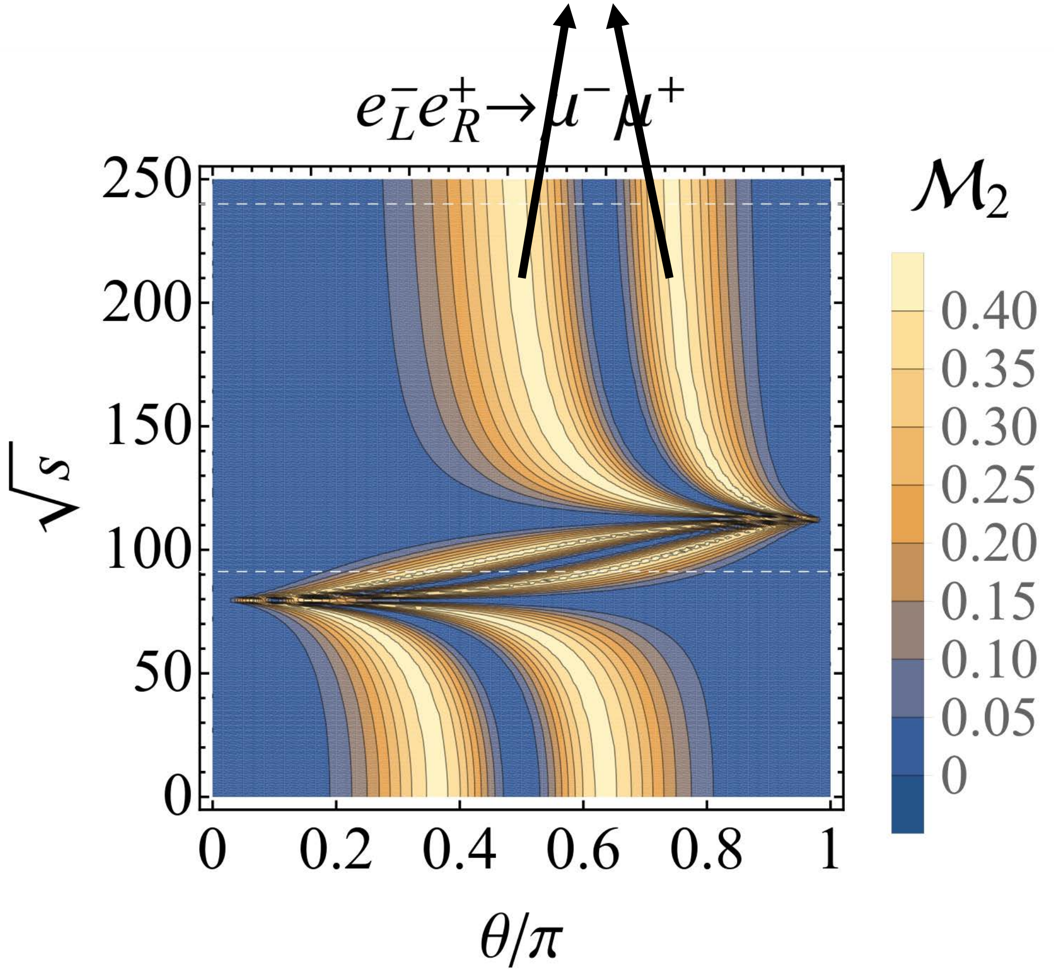


Magic is more sensitive to polarization

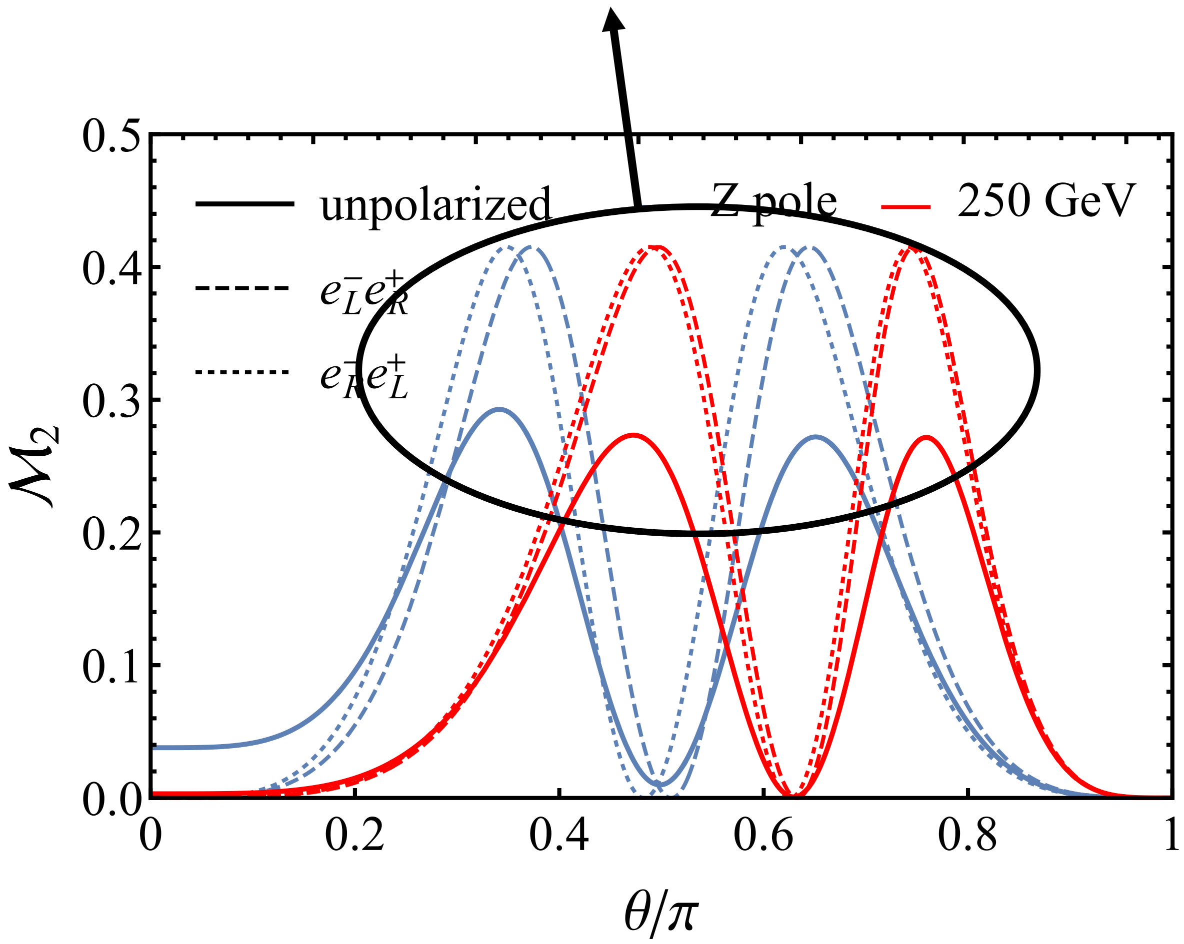


$e^-e^+ \to \mu^-\mu^+/\tau^-\tau^+$: Magic

Two Peaks of Magic



Magic is more sensitive to polarization



Magic is different quantum resource from entanglement!

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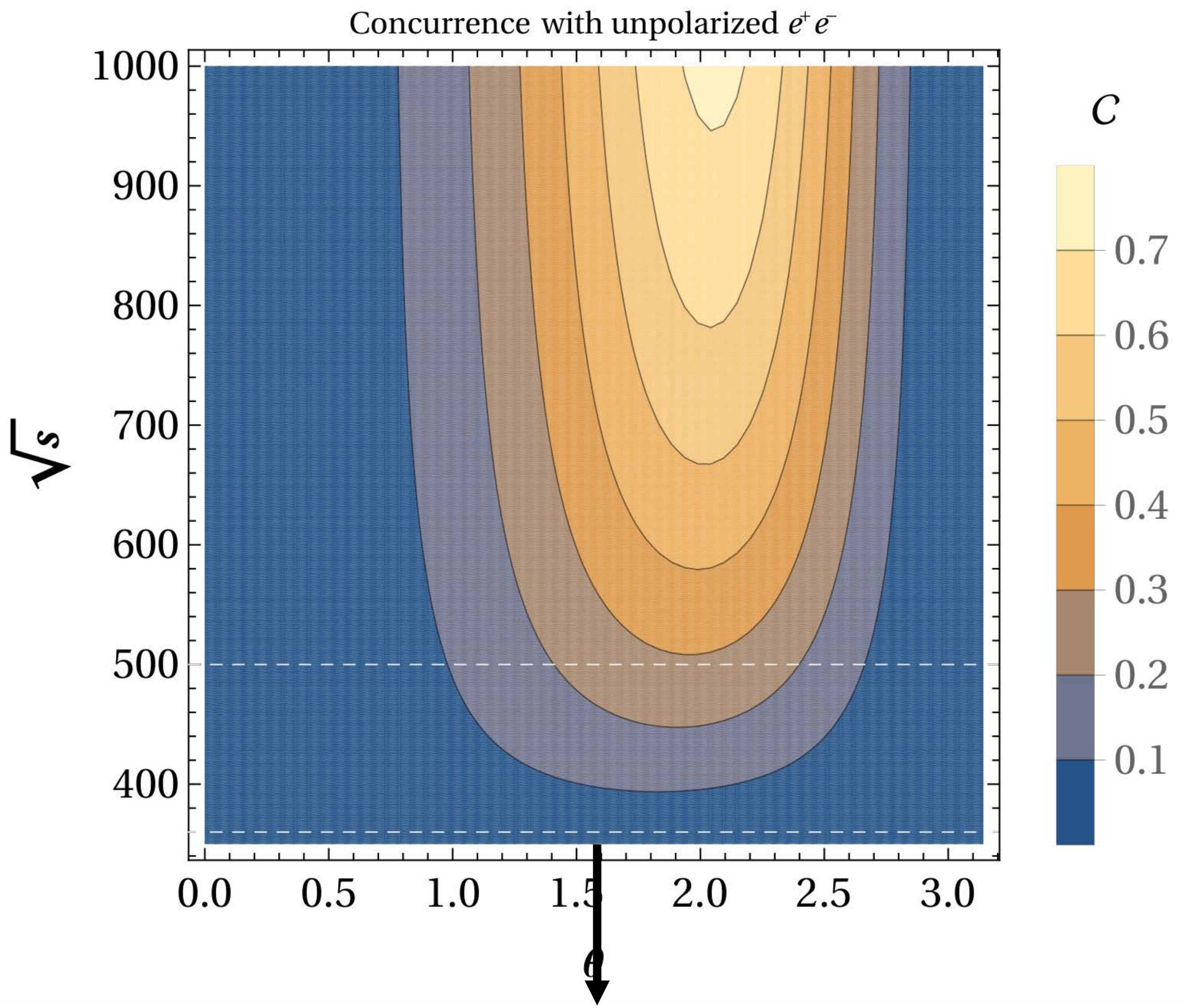
$e^- e^+ \to t\bar{t}$: Massive Particles



No entanglement at threshold!

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\downarrow\downarrow\rangle + c(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$

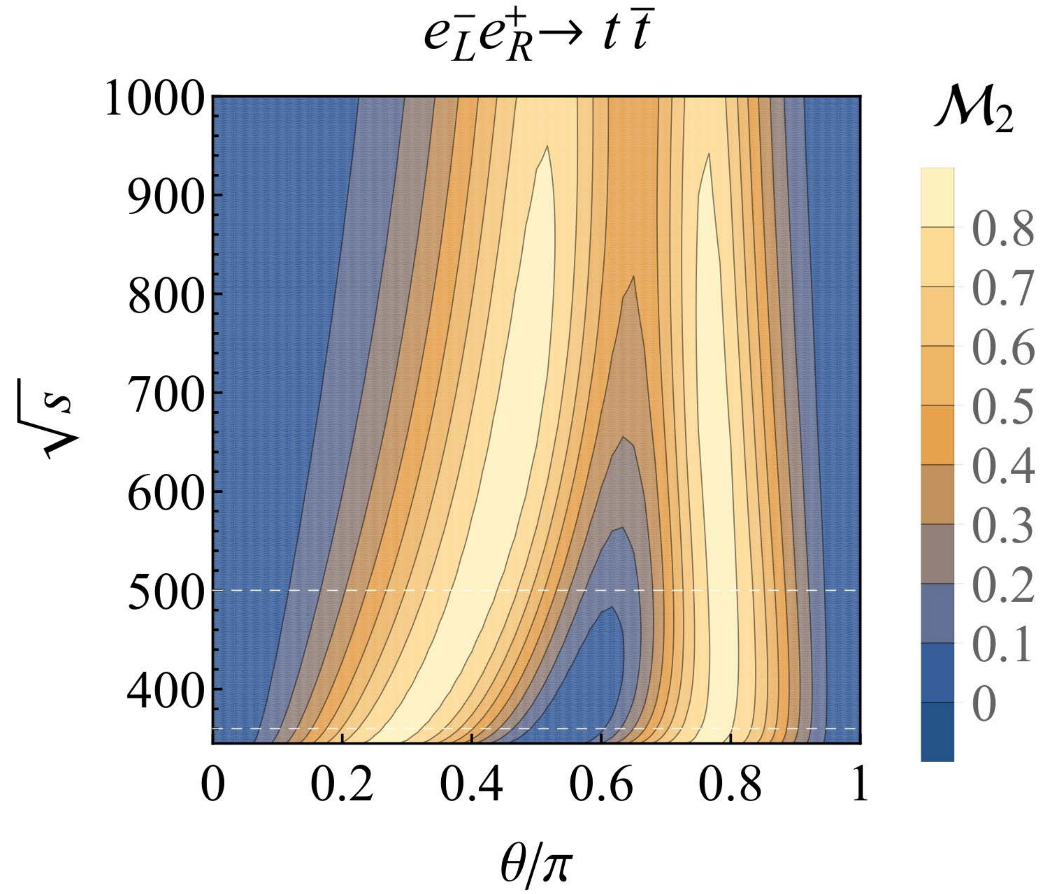
$c \propto m_t$

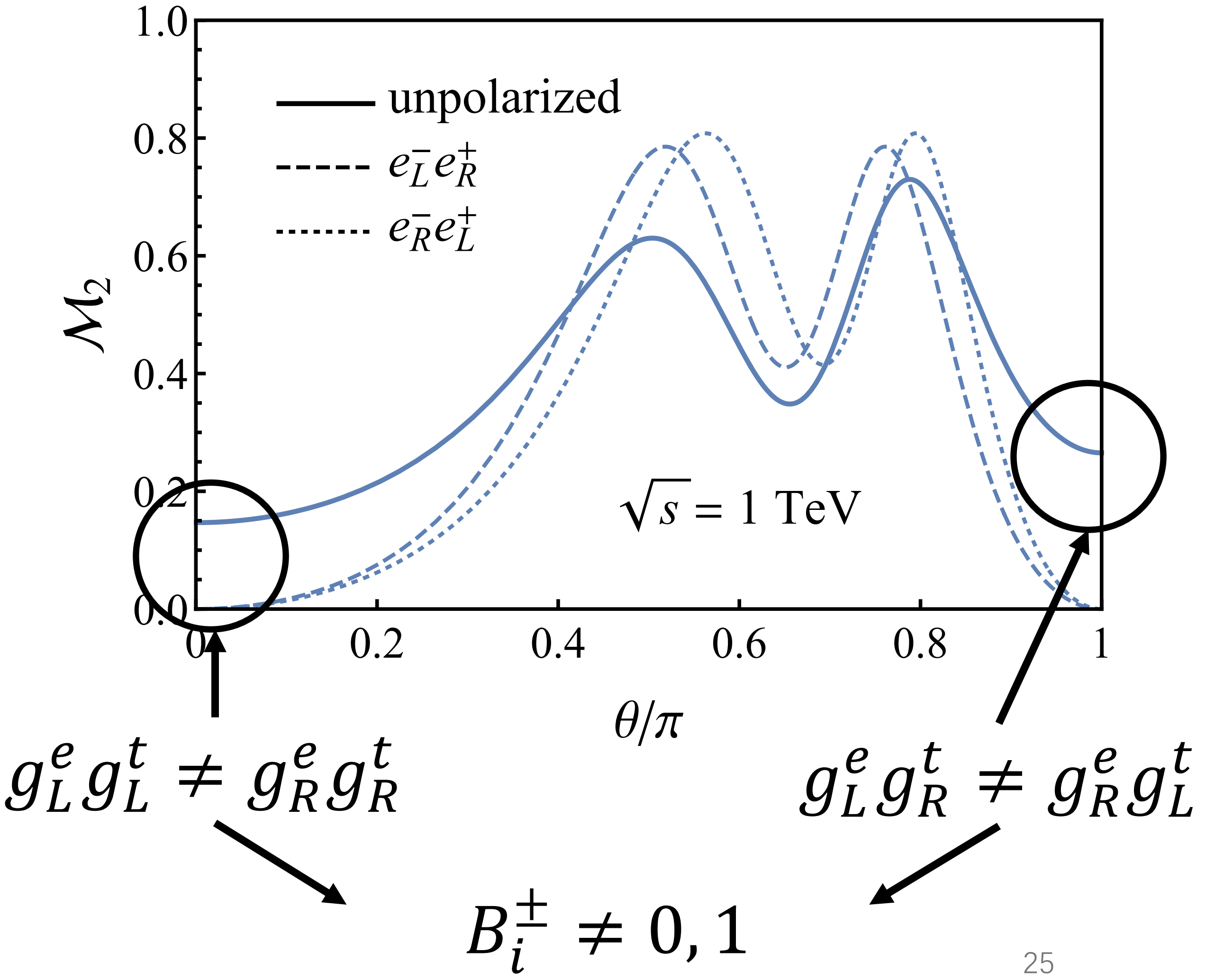
At threshold ($\beta \to 0$)

$|\psi\rangle = |LR\rangle$ or $|RL\rangle$
is separable state

$|LR\rangle$ or $|RL\rangle$ is the state of electron beam.
Quantization basis is z direction.

$e^-e^+ \to t\bar{t}$: Magic





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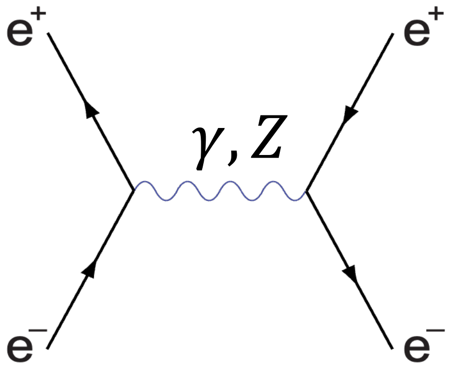
Bhabha Scattering: How t-channel influences quantum information?

$e^-_{R/L}e^+_{L/R} \to e^-_{R/L}e^+_{L/R}$:



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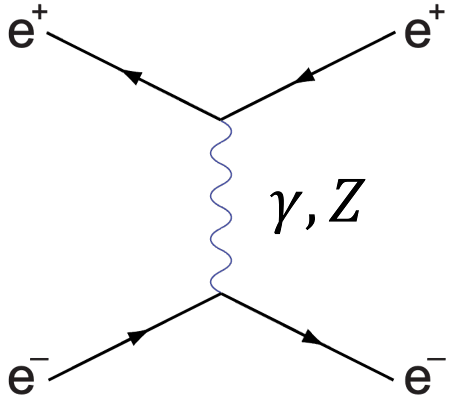
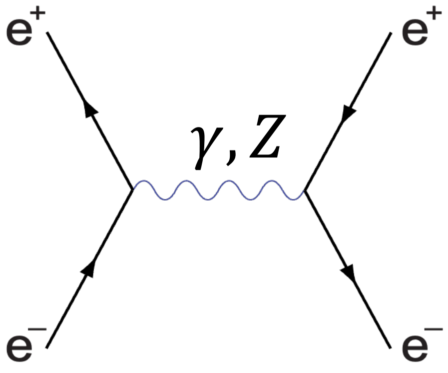


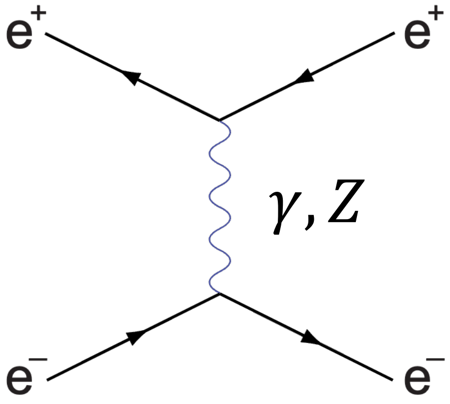
$e^-_{R/L}e^+_{L/R} \to e^-_{L/R}e^+_{R/L}$:



$|\psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\downarrow\downarrow\rangle$

Entangled State

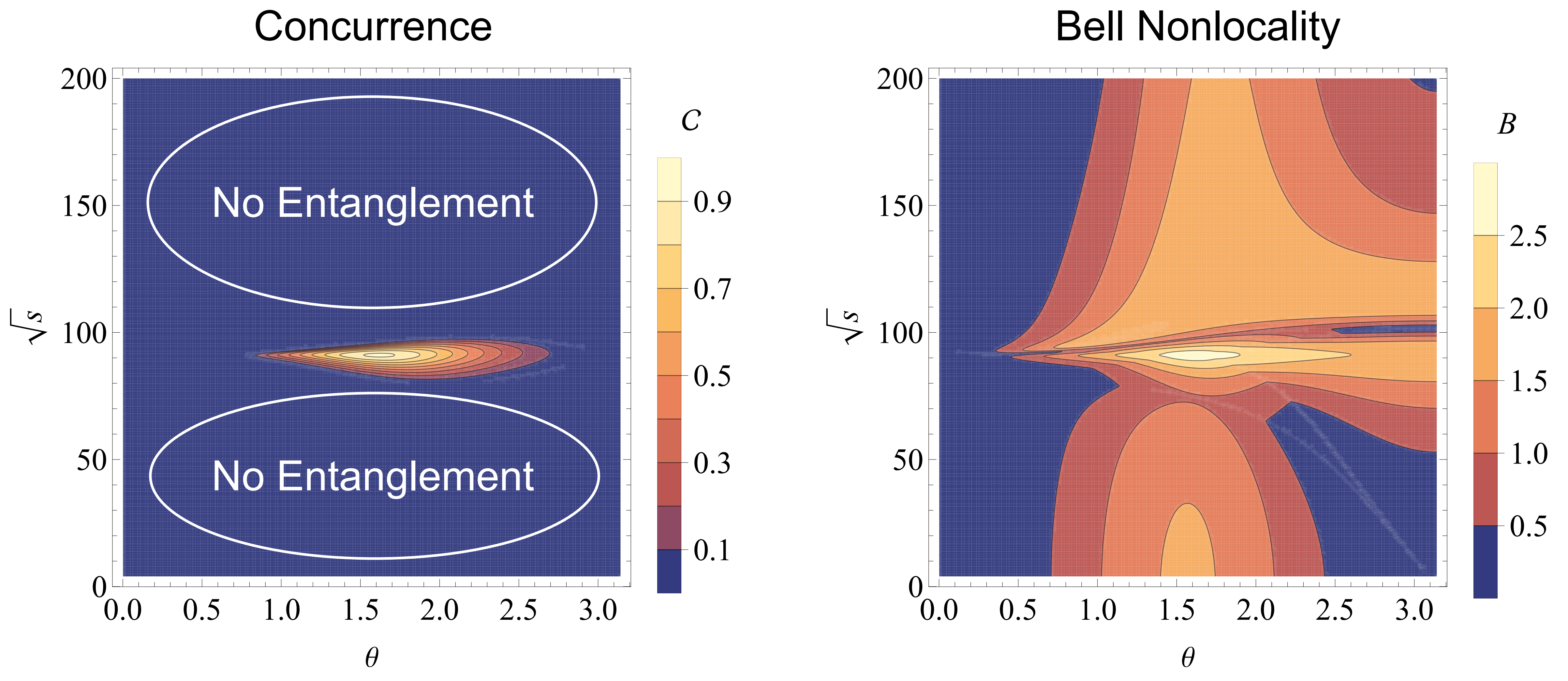
$e^-_{R/L}e^+_{R/L} \to e^-_{R/L}e^+_{R/L}$:



$|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$

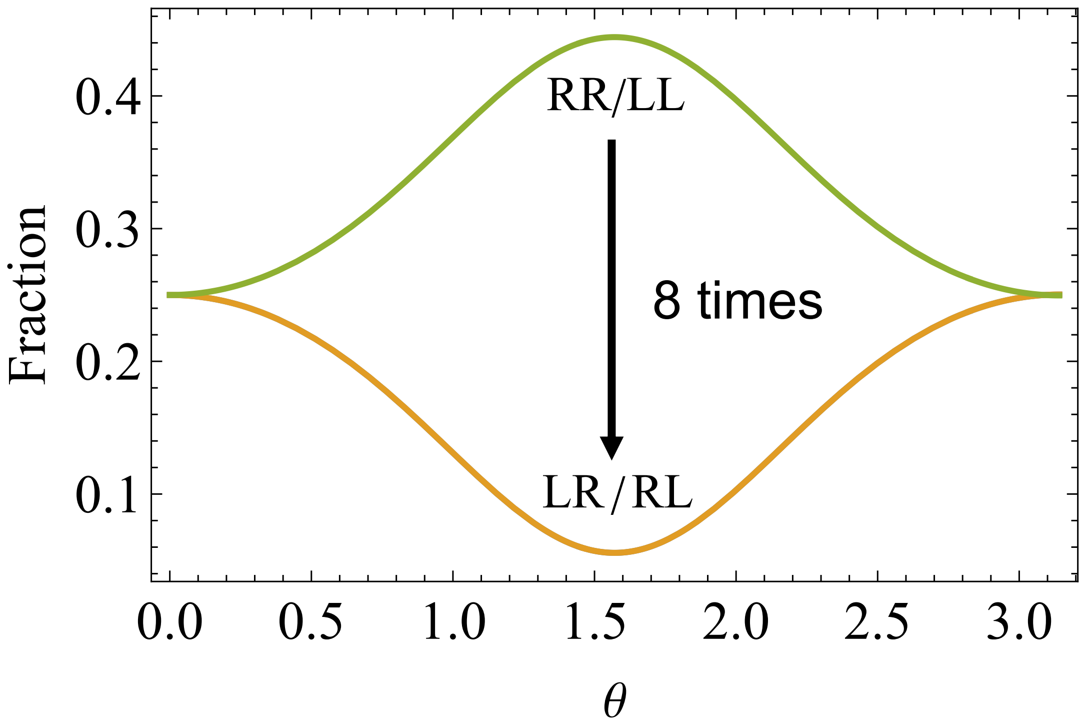
Separable State

Concurrence and Bell Nonlocality for Unpolarized Case



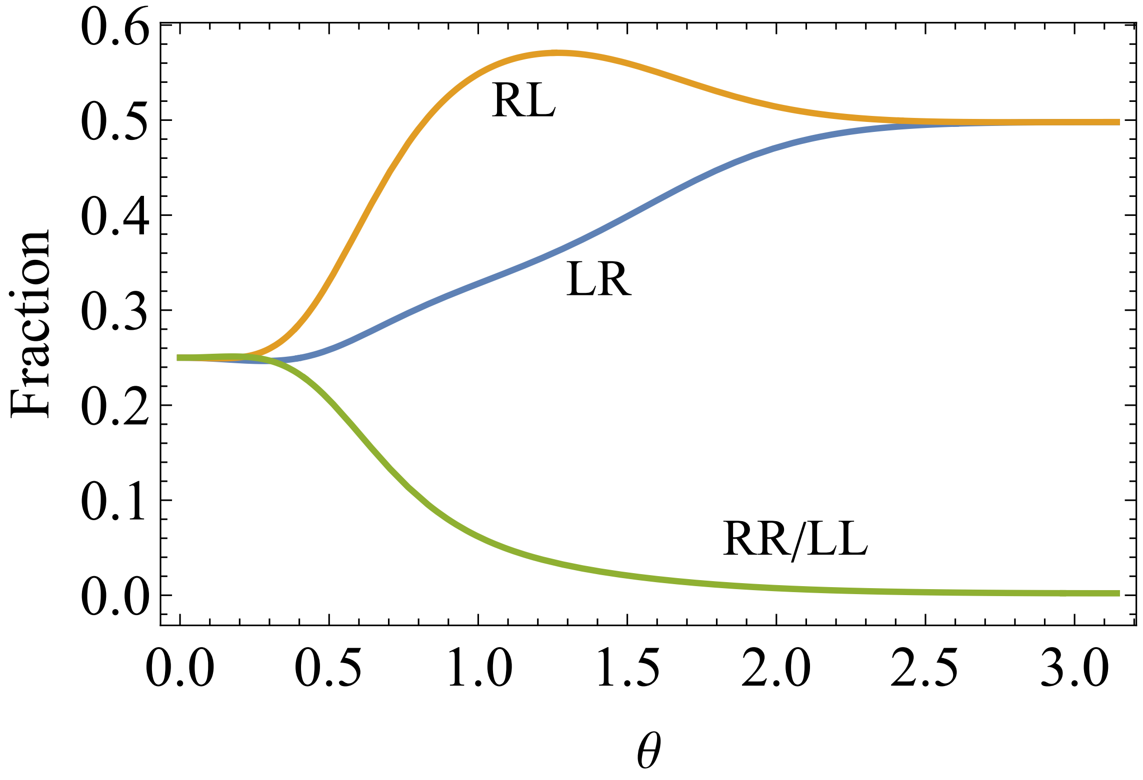
Reasons of the unpolarized case

QED Level



t-channel dominates
especially unentangled process

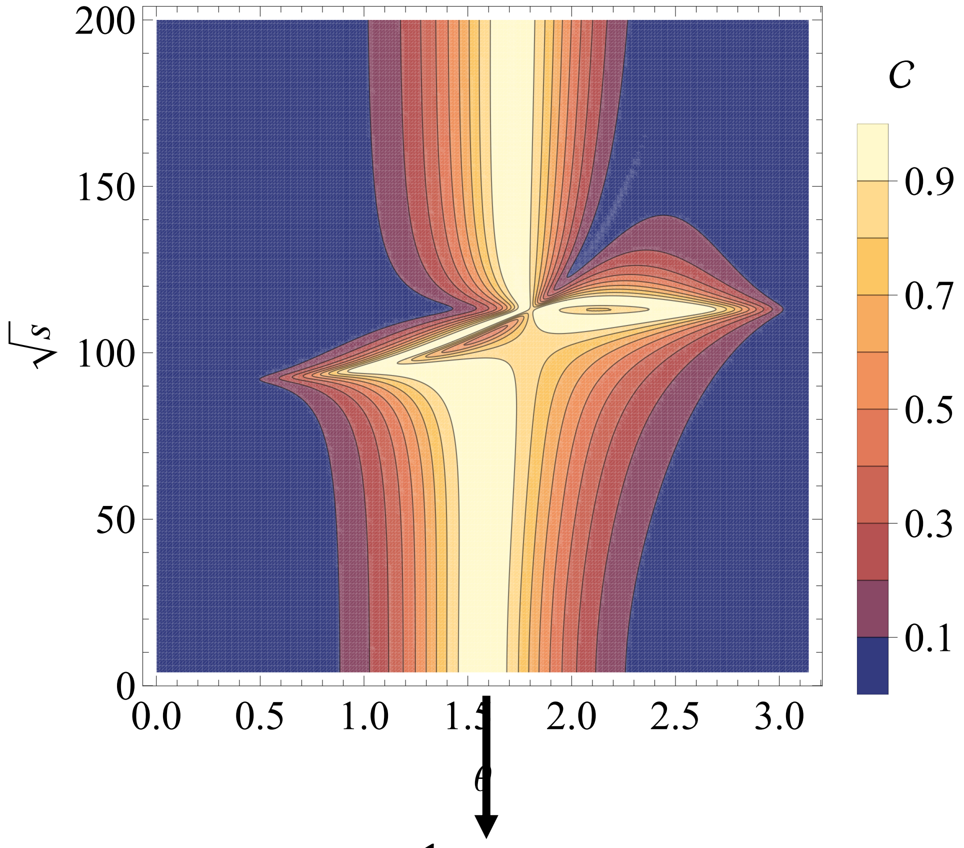
Around Z pole



s-channel dominates
similar with only s-channel process

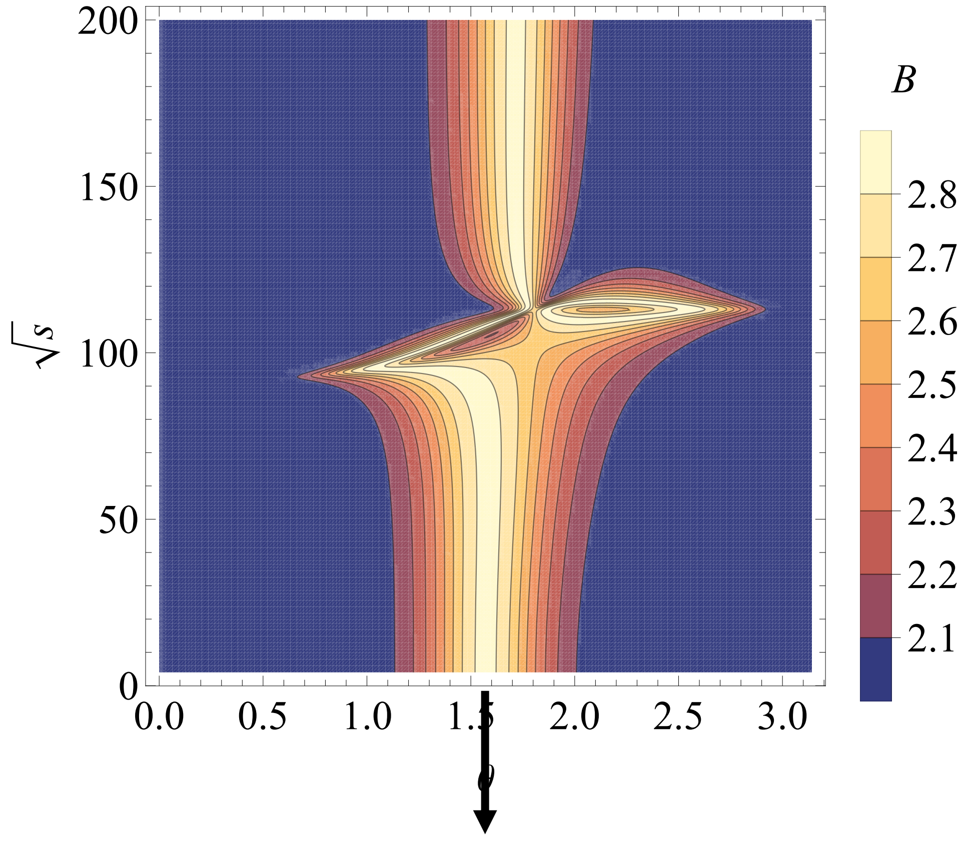
Polarized Case: Remove the unentangled pairs

$e_R^- e_L^+$



$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)$$

High Entanglement

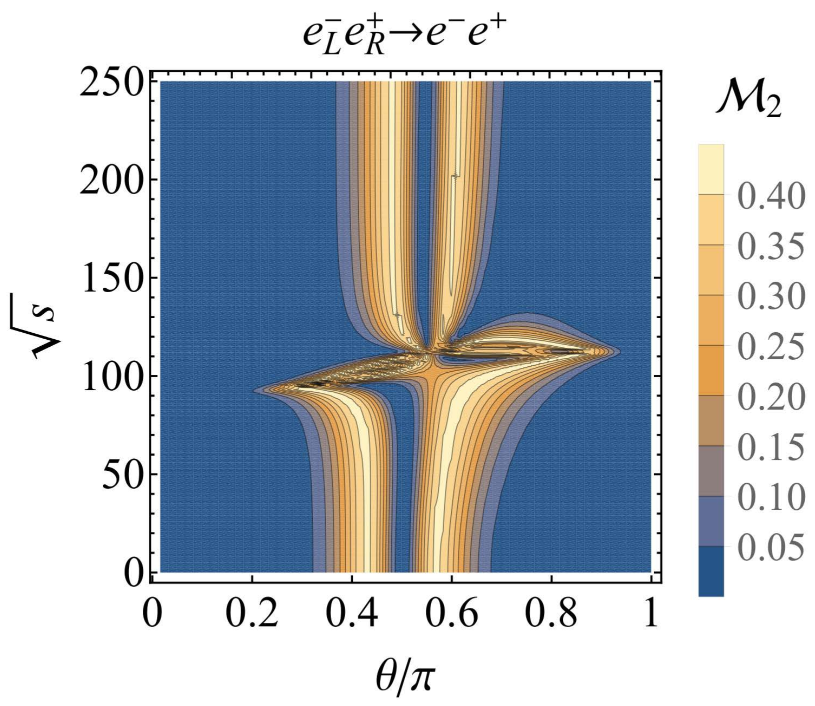


$$\mathcal{B} = 2\sqrt{2}$$

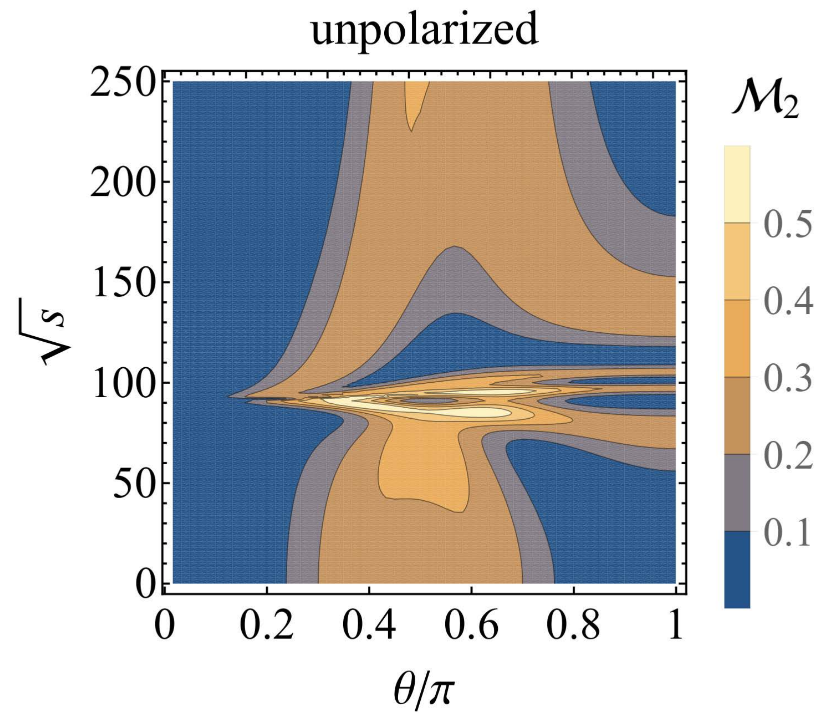
Magic of Bhabha Scattering

Magic still exhibits two-peak structure.

Unlike entanglement, magic remains nonzero largely.



Full polarized
two peaks similar with only s-channel



Unpolarized
changed by t-channel
with RR/LL initial states

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Polarization Effect on Tomography ($\mathcal{C}$ and $\mathcal{B}$ of $t\bar{t}$ as an example)

$\sqrt{s} = 1$ **TeV**		Entanglement Measure ($\mathcal{C}$)					Bell Nonlocality ($\mathcal{B} - 2$)				
(P_{e^-}, P_{e^+})	Δ_{sys}	$\mathcal{C}$	$\Delta_{\text{tot}}(\mathcal{C})$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$	$\mathcal{B} - 2$	$\Delta_{\text{tot}}(\mathcal{B})$	$\mathcal{S},\ \mathcal{S}^{-1}$	θ_c	$\Delta\theta$
(0, 0)	1%	0.49	0.022	$> 5\sigma$, 4.5%	119°	92°	0.37	0.090	4.2σ	119°	37°
	2%	0.55	0.035	$> 5\sigma$, 6.4%	119°	75°	0.41	0.133	3.1σ	119°	29°
(+0.8, −0.6)	1%	0.41	0.025	$> 5\sigma$, 6.1%	124°	110°	0.34	0.10	3.3σ	124°	38°
	1%	0.41	0.032	$> 5\sigma$, 7.9%	124°	110°	0.37	0.14	2.6σ	124°	31°
(−0.8, +0.6)	1%	0.49	0.020	$> 5\sigma$, 4.2%	116°	101°	0.41	0.074	$> 5\sigma$, 18%	116°	34°
	2%	0.49	0.032	$> 5\sigma$, 6.5%	116°	101°	0.45	0.12	3.7σ	116°	25°

Using $L = 5\,\text{ab}^{-1}$, the polarized cross section is larger.

→ Significance is larger.

Thank you!