Entanglement, Yang-Mills, and the Scattering-Matrix as an SU(N) - equivariant Kernel

Based on arXiv:2511.0962 (Kun-Feng Lyu, RM, Kuver Sinha)

Accepted in *JHEP*

Rahul Muraleedharan
University Of Oklahoma

S-Matrix

- **Encodes full information about the scattering process.**
- **Can be interpreted as an operator that maps initial states to finals states.**
- **Traditionally computed perturbatively using Feynman diagrams.**

S-matrix as a quantum logic gate

- **Incoming particles carry momentum, flavor, spin, color and other internal quantum numbers**

- **Together they define the full Hilbert space used to describe a particle is**

$$\mathscr{H}_{\textbf{total}} = \mathscr{H}_{\textbf{momentum}} \otimes \mathscr{H}_{\textbf{internal}}$$

- **Since interactions can in general produce correlations(<u>entanglement</u>) between them,**

S-matrix can be viewed as a quantum logic gate.

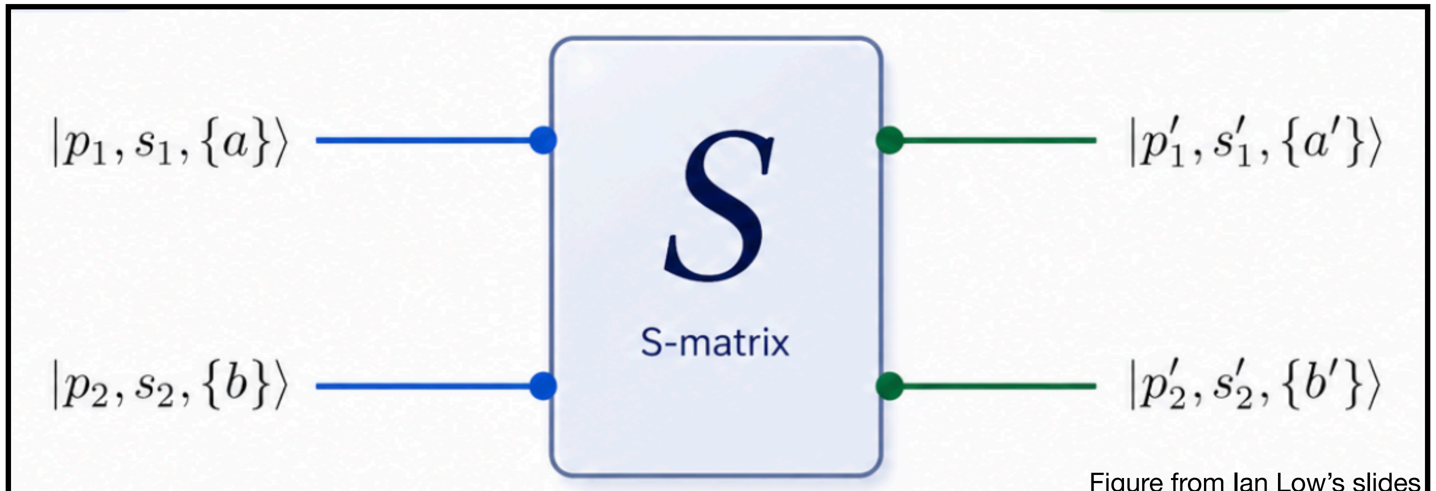
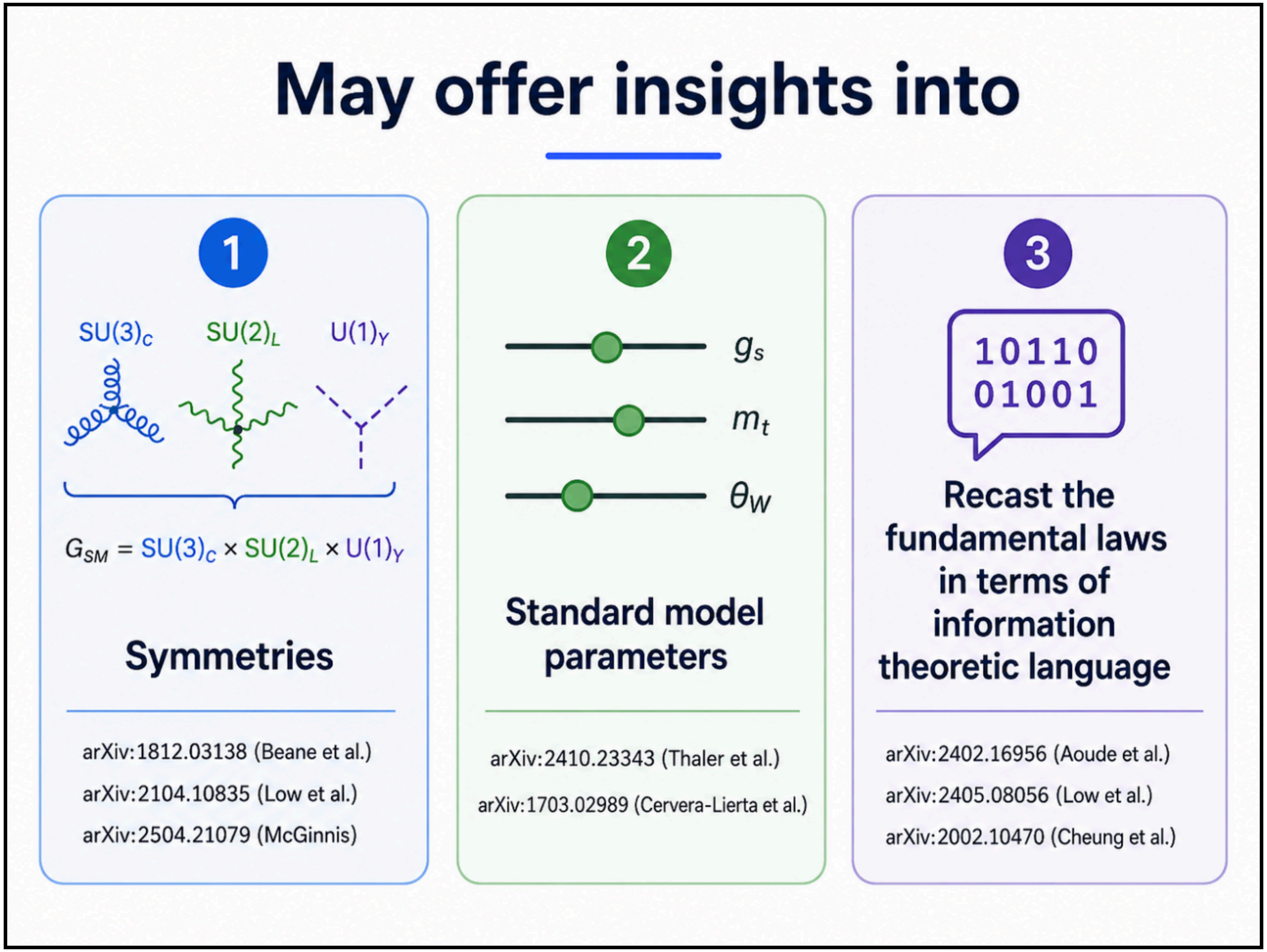


Figure from Ian Low's slides

Scattering is not just dynamics its a redistribution of information as well!!!

Buzz around “Entanglement”



"It from bit"

INFORMATION, PHYSICS, QUANTUM: THE SEARCH FOR LINKS

John Archibald Wheeler * †

Abstract

This report reviews what quantum physics and information theory have to tell us about the age-old question, How come existence? No escape is evident from four conclusions: (1) The world cannot be a giant machine, ruled by any preestablished continuum physical law. (2) There is no such thing at the microscopic level as space or time or spacetime continuum. (3) The familiar probability function or functional, and wave equation or functional wave equation, of standard quantum theory provide mere continuum idealizations and by reason of this circumstance conceal the information-theoretic source from which they derive. (4) No element in the description of physics shows itself as closer to primordial than the elementary quantum phenomenon, that is, the elementary device-intermediated act of posing a yes-no physical question and eliciting an answer or, in brief, the elementary act of observer-participancy. Otherwise stated, every physical quantity, every it, derives its ultimate significance from bits, binary yes-or-no indications, a conclusion which we epitomize in the phrase, *it from bit*.

Does everything have an information-theoretic origin??

Information $\stackrel{?}{=}$ *Fundamental*

Motivation

- **Previous works looked at entanglement associated with helicity, momentum, flavor degrees of freedom in scattering processes.**

- **Color-space entanglement remain unexplored.**

- **We use Yang-Mills theory to study color space entanglement.**

Entanglement formalism

- **We study bipartite entanglement between outgoing states in the relevant internal space.**

- **For a 2-2 scattering process,**

$$g_a(p_1) + g_b(p_2) \to g_c(p_3) + g_d(p_4)$$

- **Scattering operator within the internal space(we may refer it "Kernel") is**

$$K(\Theta) : |a, b\rangle \xrightarrow{K_{cd,ab}(\Theta)} |c, d\rangle$$

a, b, c, d are internal labels.

- **For product input state** $|u\rangle \otimes |v\rangle$ **,out state is**

$$N_{cd} = K_{cd,ab} u_a v_b$$

- **Then normalized reduced matrix is**

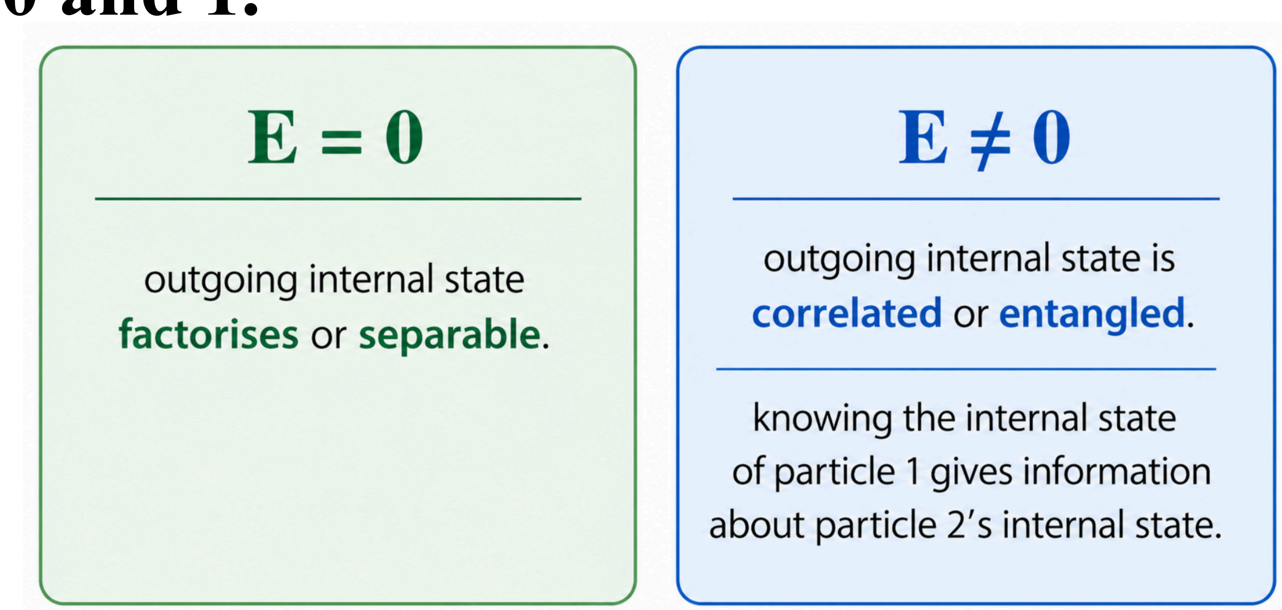
$$\rho_R = \frac{\mathcal{N}\mathcal{N}^\dagger}{\mathrm{Tr}[\mathcal{N}\mathcal{N}^\dagger]}$$

- **Then linear entropy is**

$$E = \frac{d_R}{d_R - 1}\left(1 - \mathrm{Tr}\,\rho_R^2\right) = \frac{d_R}{d_R - 1}\left(1 - \frac{\mathrm{Tr}[(\mathcal{N}\mathcal{N}^\dagger)^2]}{(\mathrm{Tr}[\mathcal{N}\mathcal{N}^\dagger])^2}\right)$$

where d_R = Hilbert space dimension of the subsystem

- **E tells us how the internal states of two out going particles are correlated.**
- **E vary between 0 and 1.**



Kernel as SU(N)-equivariant map

- **The kernel constructed from Lagrangian has to respect the symmetries it has. Thus,**

$$[K, U \otimes U] = 0, \quad U \in \mathrm{SU}(N)$$

- **Then Schurr's lemma suggests**

$$K = \sum_i c_i P_i$$

Where P_i are projectors and c_i are coefficients that depend on dynamics.

- **Representations of initial states in the underlying SU(N) symmetry thus reveal the entanglement structures.**

Kernel on SU(2) & SU(3): $Adj \otimes Adj$

- **Yang-Mills scattering amplitude at tree level is:**

$$K(\Theta) \;\propto\; A_s(\Theta)\, c_s \;+\; A_t(\Theta)\, c_t \;+\; A_u(\Theta)\, c_u$$

with $A_i(\Theta)$ **kinematic weights,** Θ **represent scattering angle, energy etc**

and,

$$c_s = f_{abe} f_{cde}, \qquad c_t = -f_{ace} f_{bde}, \qquad c_u = f_{ade} f_{bce}, \qquad c_s + c_t + c_u = 0$$

- **This is the scattering amplitude in color basis.**

- **But entanglement structures is transparent in invariant tensor basis.**

Projectors in terms of invariant tensors:

$\mathbf{3} \otimes \mathbf{3}$

1. $I_{cd,ab} = \delta_{ac}\delta_{bd}$
2. $S_{cd,ab} = \delta_{ad}\delta_{bc}$
3. $P_{singlet} = \frac{1}{3}\delta_{ab}\delta_{cd}$

$\mathbf{8} \otimes \mathbf{8}$

1. $I_{cd,ab} = \delta_{ac}\delta_{bd}$
2. $S_{cd,ab} = \delta_{ad}\delta_{bc}$
3. $P_{singlet} = \frac{1}{3}\delta_{ab}\delta_{cd}$
4. $(D_s)_{cd,ab} = d_{abe}\, d_{cde}$
5. $(D_t)_{cd,ab} = d_{ace}\, d_{bde}$
6. $(D_u)_{cd,ab} = d_{ade}\, d_{bce}$

YM Kernel in invariant tensor basis

- **Kernel for SU(2) is:**

$$K(\Theta) = (A_s - A_u)I + (-A_s + A_t)S + N_A(-A_t + A_u)P_{singlet} \quad ; \quad N_A = N^2 - 1$$

- **Kernel for SU(3) is:**

$$K(\Theta) = \frac{2}{3}(A_s - A_u)I + \frac{2}{3}(-A_s + A_t)S + \frac{2}{3}N_A(-A_t + A_u)P_{singlet} + (A_s - A_u)(D_t - D_u) + (A_t - A_u)(D_u - D_s)$$

- **Further simplification can be done using Color-Kinematic Duality and use it to compute entanglement entropy.**

- Similar decomposition of Kernel can be done for :

1. $fund \otimes fund$

$$K_{f,f} = a(\Theta)I + b(\Theta)S$$

2. $fund \otimes antifund$

$$K_{f,\bar{f}} = a'(\Theta)I + b'(\Theta)P_{singlet}$$

Maximal entanglement in SU(N) *fund* $\otimes$ *fund*

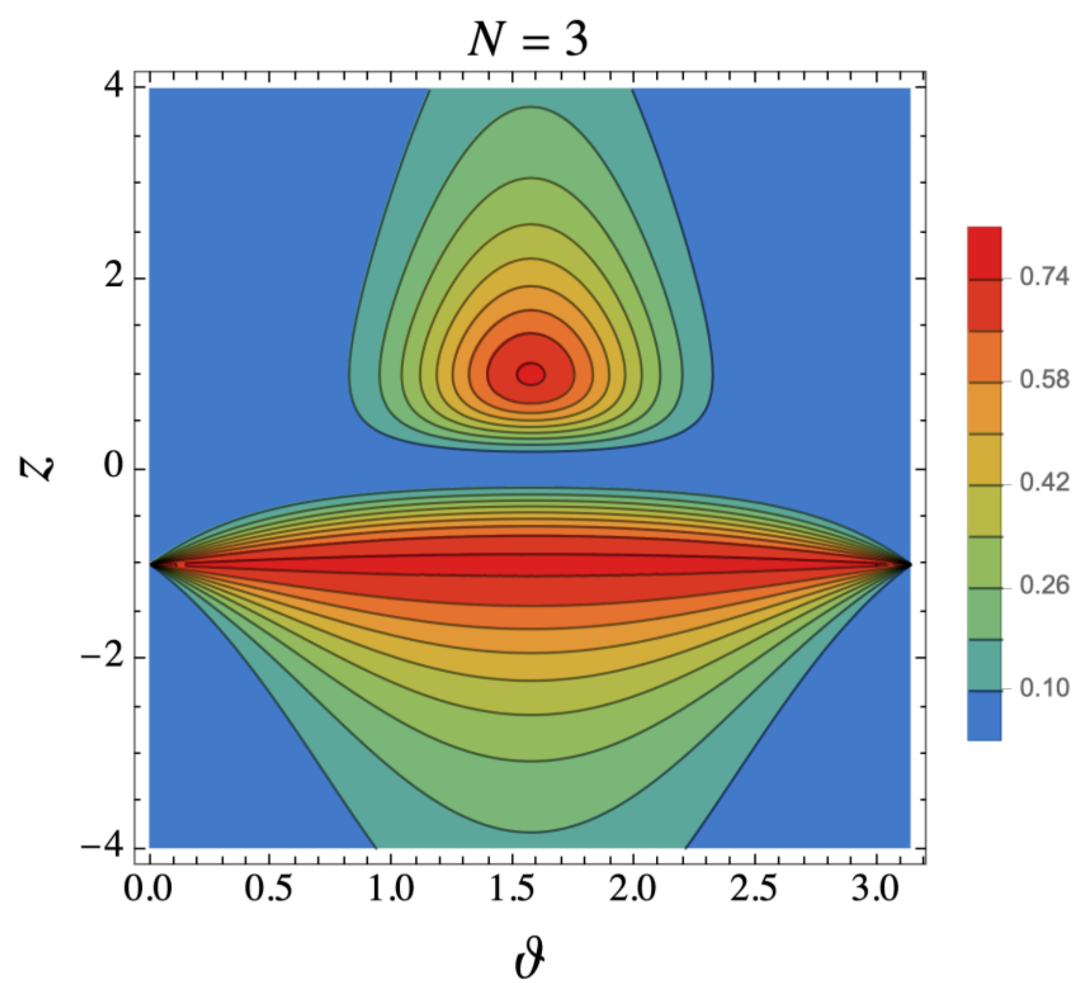
- **For** $K = I + zS$ **where** $z = \dfrac{b(\theta)}{a(\theta)}$ **and for input state** $|u\rangle \otimes |v\rangle$ **with** $cos\vartheta = |u^{\dagger}v|$**, we get**

$$E(|out\rangle) = \frac{2N z^2 \sin^4 \vartheta}{(N-1)(1+z^2+2zcos^2\vartheta)^2}$$

$$E_{max}^{SU(2)} = 1$$

$$E_{max}^{SU(3)} = \frac{3}{4}$$

Entanglement entropy as a function of z and ϑ for the case of Fundamental-Fundamental scattering. Here we take N = 3 as the benchmark.



$$K = I + zS$$

$$K(z) = (1 + z)P_S + (1 - z)P_A$$

Color space entanglement in YM

- **We have the SU(N) YM kernel in invariant tensor basis,**

$$K(\Theta) \propto a(\Theta)\, I + b(\Theta)\, S + c(\Theta)\, P_{singlet} + d_1(\Theta)(D_t - D_u) + d_2(\Theta)(D_u - D_s)$$

- **Color Kinematic Duality - there exist a gauge choice where kinematic numerators in tree level YM amplitude satisfy same Jacobi identity as color factors (arXiv:0805.3993)**

BCJ relation

- $$K \equiv M_4^{\textbf{full,tree}} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

Or, $$K = A_s\, c_s + A_t\, c_t + A_u\, c_u$$

- **Color-Jacobi identity is ,** $$c_s + c_t + c_u = 0$$
- **CKD says ,** $$n_s + n_t + n_u = 0$$

Or, $$s\, A_s + t\, A_t + u\, A_u = 0$$

- **This expression is called BCJ relation.**
- **SU(N) YM Kernel after applying CKD**

$$K(\Theta) \propto a(\Theta) \left[\mathbb{I} + \left(\frac{u}{t}\right) \mathbb{S} + N_A \left(\frac{s}{t}\right) P_{\text{singlet}} + \frac{N}{2} (\mathbb{D}_t - \mathbb{D}_u) - \frac{N}{2} \left(\frac{s}{t}\right) (\mathbb{D}_u - \mathbb{D}_s) \right]$$

Color and helicity separation

- **Out state is,**

$$|out\rangle = C\,|C\rangle \otimes a(\Theta)\,|H\rangle$$

- **There is no entanglement between color-helicity sectors!!!!**

- **CKD** **is not just a tool for computing amplitudes, its also a statement that** **color and helicity degrees of freedom scatter independently in YM !!!**

- **In Info-theoretic language,**

CKD means color and helicity sectors doesn't entangle in YM!!

Finally, Color entanglement in YM

- **SU(N) YM Kernel after applying CKD**

$$K(\Theta) \propto a(\Theta) \left[\mathbb{I} + \left(\frac{u}{t}\right) \mathbb{S} + N_A \left(\frac{s}{t}\right) P_{\text{singlet}} + \frac{N}{2} (\mathbb{D}_t - \mathbb{D}_u) - \frac{N}{2} \left(\frac{s}{t}\right) (\mathbb{D}_u - \mathbb{D}_s) \right]$$

- **At right angle scattering**($\theta = \dfrac{\pi}{2}$), $t = u = -\dfrac{s}{2}$

$$K(\theta = \pi/2) \propto \begin{cases} \mathbb{I} + S - 6P_{\text{singlet}}, & N = 2 \\ \alpha_1 \mathbb{I} + \alpha_2 S + \alpha_3 P_{\text{singlet}} + \alpha_4 (D_t - D_u) + \alpha_5 (D_u - D_s), & N \geq 3 \end{cases}$$

Group invariant peak entanglement

- $E_*^{(N)} := \max_{|u\rangle\otimes|v\rangle} E\Big(\textbf{normalized out-state of } K(\pi/2)\,|\,u\rangle \otimes |\,v\rangle\Big)$

- **This value depends only on the underlying SU(N) gauge group of YM.**

- **We can compute this for SU(2) and SU(3).**

$$E_*^{(2)} = \frac{3}{4} \qquad \textbf{and} \qquad E_*^{(3)} \approx 0.9067$$

- **In the large N limit($N \to \infty$), D-tensors will dominate the kernel. We expect to get maximal entanglement i.e** $E_*^{(N\to\infty)} \to 1$

Probing EFT operators

- **Can we use the group invariant peak entanglement to probe higher dimensional operators?**
- $$\mathscr{L} = \mathscr{L}_{\text{YM}} + \frac{c_6}{\Lambda^2}\mathcal{O}_{F^3} + \frac{1}{\Lambda^4}\sum_j c_{8,j}\mathcal{O}_j^{(8)} + \cdots,$$
- **arXiv:1208.0876 (Broedal & Dixon) found that CKD holds at tree level for dim-6 as well.**
- **These operators hence won't modify color structures and won't modify $E_*^{(N)}$.**
- **Color-space entanglement is blind to dim-6 operators.**
- **But the amplitudes for dim-8 operators can interfere with pure YM amplitudes and dim-6 operators, modifying entanglement.**
- **So deviation in peak entanglement can signal new physics!**

Summary

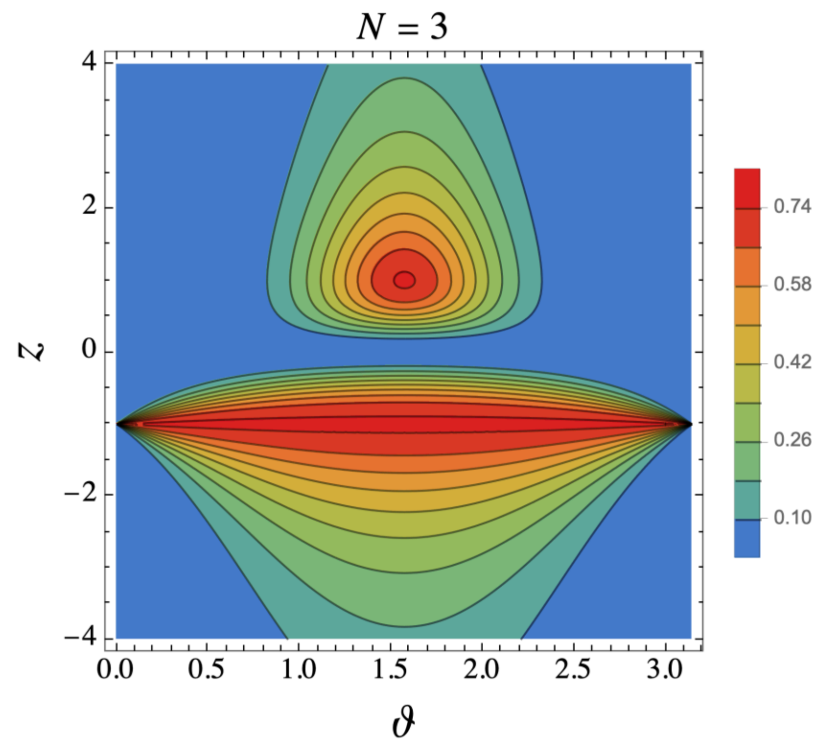
- **For theory with SU(N) symmetry, the representation of the initial states decide the entanglement structure in internal space.**

- **Color and helicity separates for YM.**

- **There exist group invariant peak color entanglement in YM that can probe new physics.**

Thank You!

BACKUP



$$K = I + zS$$

$$K(z) = (1+z)P_S + (1-z)P_A$$

- $v = cos\vartheta u + sin\vartheta w$
- The symmetric projected state is $|u \otimes v\rangle + |v \otimes u\rangle$
- This is equal to $2cos\vartheta\,|u \otimes u\rangle + sin\vartheta(\,|u \otimes w\rangle + |w \otimes u\rangle)$.
- For $\vartheta = pi/2$ this is maximally entangled bell state.

Action of $P_{singlet}$ on product state

- We have $(P_{\mathbf{sing}})_{cd,ab} = \dfrac{1}{N}\delta_{ab}\delta_{cd}$
- Input state is $|u\rangle \otimes |v\rangle = u_a v_b |a,b\rangle \equiv T_{ab}|a,b\rangle$,
- Then, $(P_{\mathbf{sing}}T)_{cd} = \dfrac{1}{N}\langle u|v\rangle\,\delta_{cd}\,.$
- Finally, $P_{sing}|u,v\rangle = \dfrac{\langle u|v\rangle}{N}\sum_{c=1}^{N}|c,c\rangle\,.$
- This is entangled unless u orthogonal to v.

Action of $d - tensor$ on product state

- **Similarly,** $D_s\,|u,v\rangle \;=\; \sum_e \left(d_{abe}\,u_a v_b\right)|\psi_e\rangle\,;\quad |\psi_e\rangle \;\equiv\; d_{cde}\,|c,d\rangle$

- **The output is entangled whenever** $d_{abe}\,u_a v_b \;\neq\; 0,$ **i.e whenever the input state** $|u\rangle \otimes |v\rangle$ **have nonzero projection onto the symmetric subspace.**

- **The d-tensors are constrained :**

$$D_s + D_t + D_u \;=\; \frac{1}{3}(N_A P_{\text{singlet}} + I + S)$$

where $N_A = N^2 - 1$