Determination of the strong coupling constant from CT25 PDFs

Kirtimaan Mohan @PHENO-2026, University of Pittsburgh

Based on arXiv: 2512.23792

See also related talk by Tanishq Sharma

CTEQ-TEA
Asia: A. Ablat, S. Dulat, T.-J. Hou, I. Sitiwaldi

North America: A. Courtoy, Y. Fu, M. Guzzi, T.J. Hobbs, J. Huston, K. Mohan, H.-W. Lin, P. Nadolsky, M. Ponce-Chavez, K. Xie, C.-P. Yuan

MICHIGAN STATE UNIVERSITY

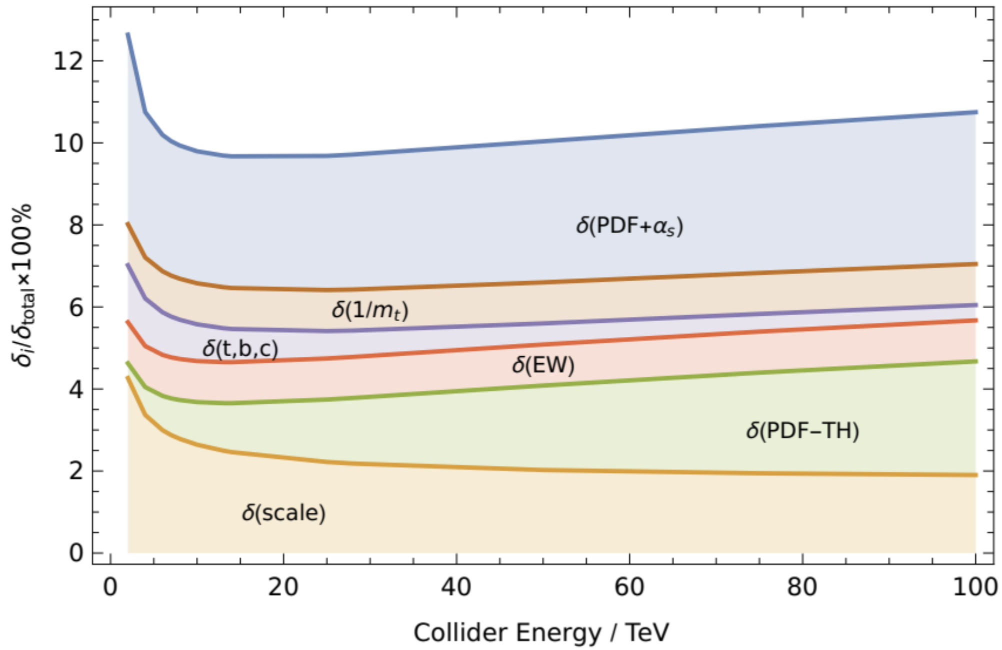
Strong coupling

Precise determination of the strong coupling is necessary for precision physics and has strong implications for BSM physics.

Least precisely known gauge coupling in the standard model.

Yet most important for determination of cross-sections at the LHC.

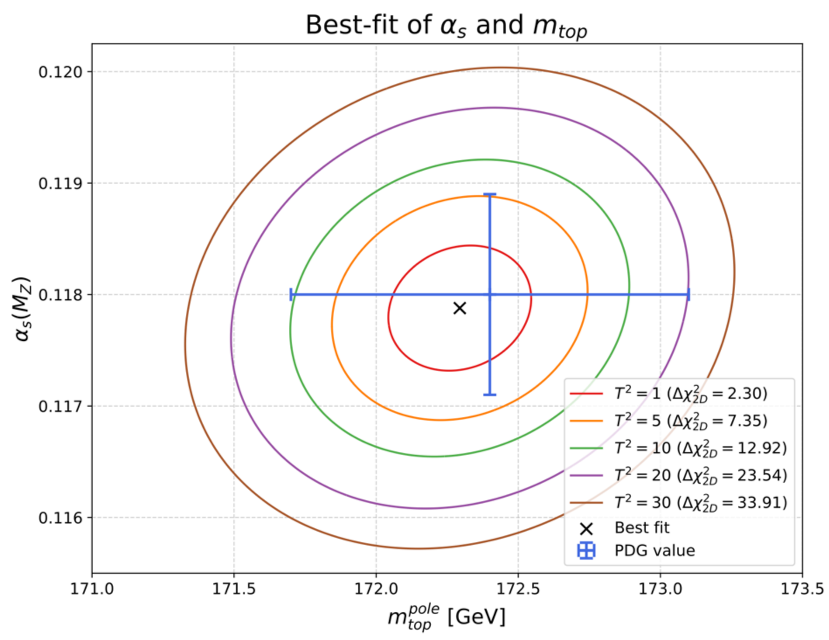
arxiv:1902.00134



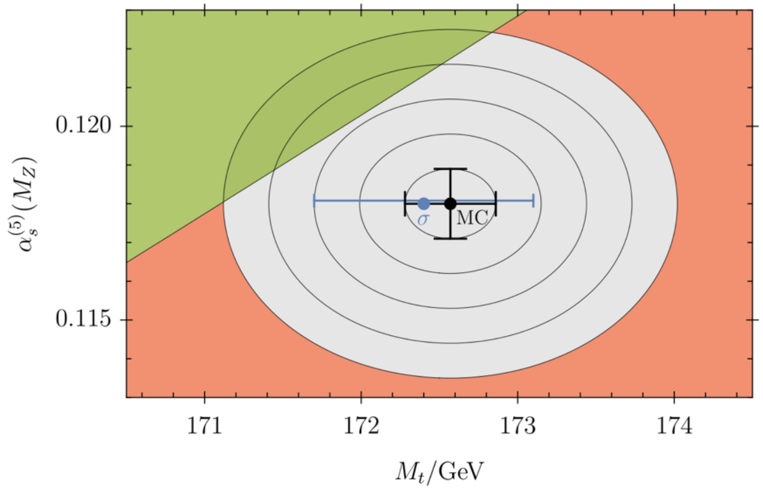
Implications for stability of electroweak vacuum

Results from CT25

See next talk by Tanishq Sharma



arxiv:2401.08811



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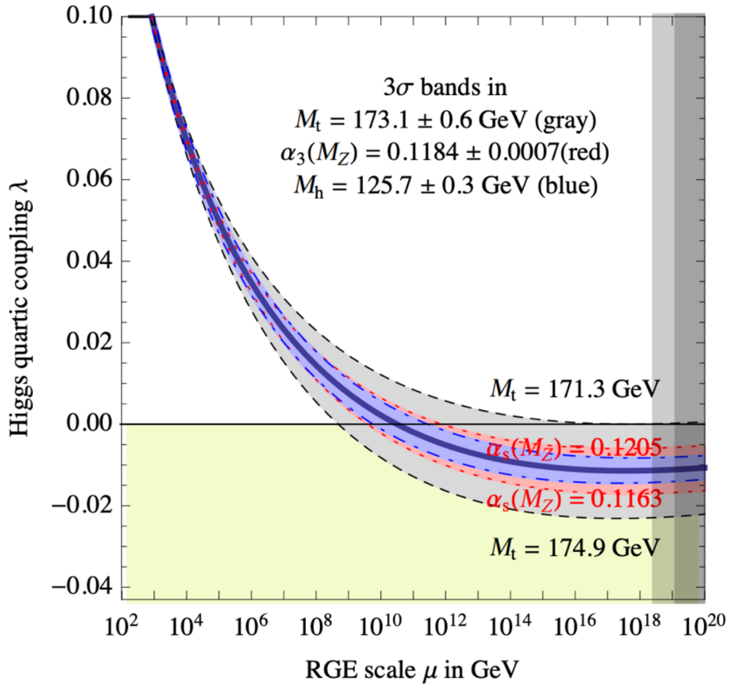


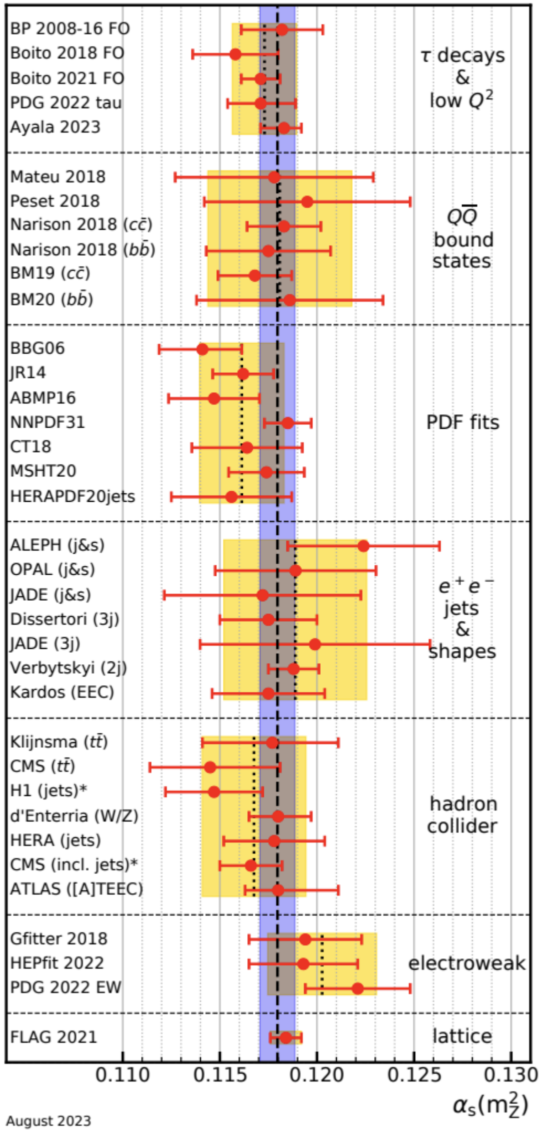
Strong coupling

Various methods to determine the strong coupling.

In PDF fits α_s shows up in the cross-section as well as the evolution of couplings masses and PDFs

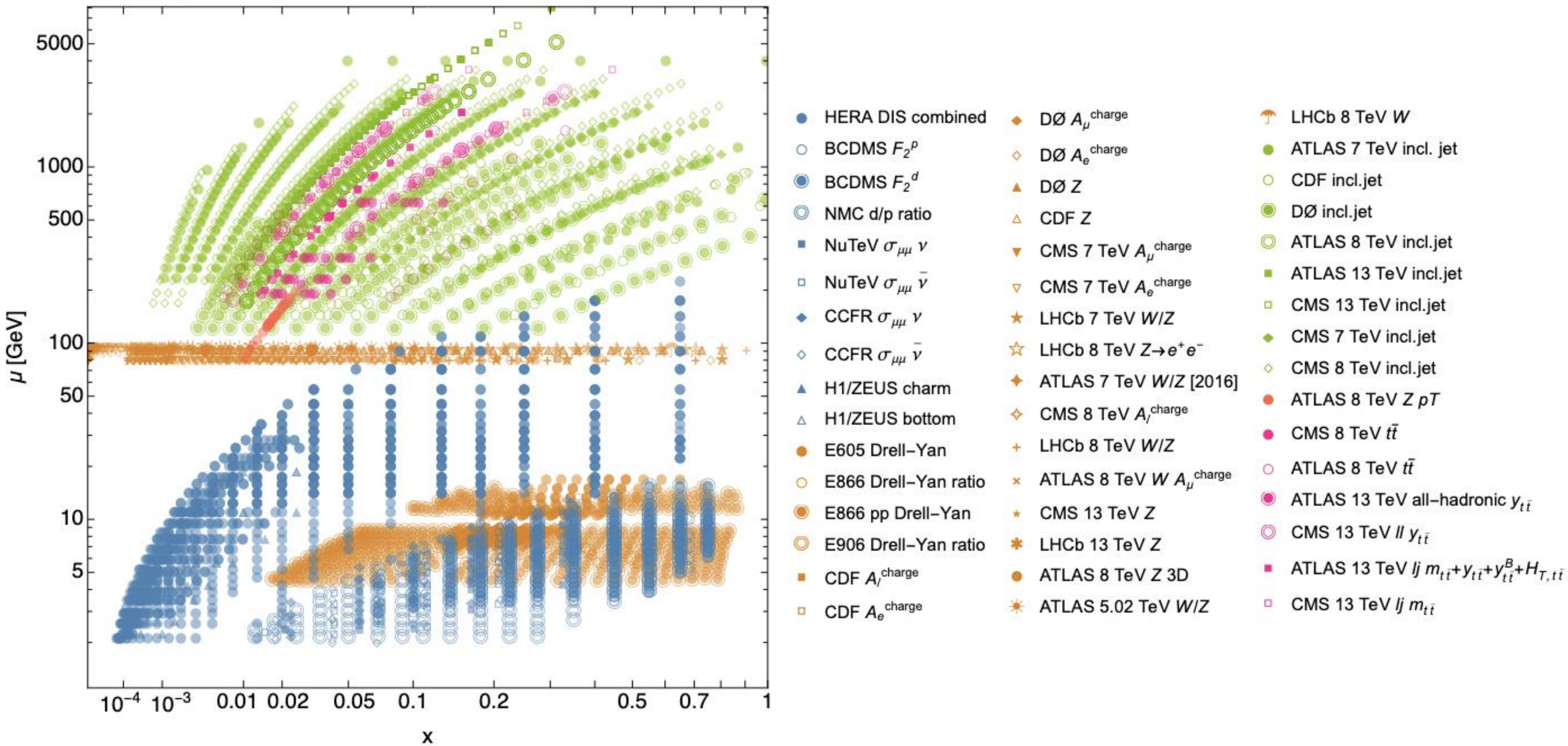
$$\sigma_{pp\to X} = \sum_s \int dx_1\, dx_2\, \hat{\sigma}^{(s)} \left(\sum_{i,j} C_{ij}^{(s)} f_i(x_1, Q^2) f_j(x_2, Q^2) \right)$$

$$f_i(x, Q_0) = a_0 x^{a_1 - 1} (1-x)^{a_2} P_i(y; a_3, a_4, \ldots).$$

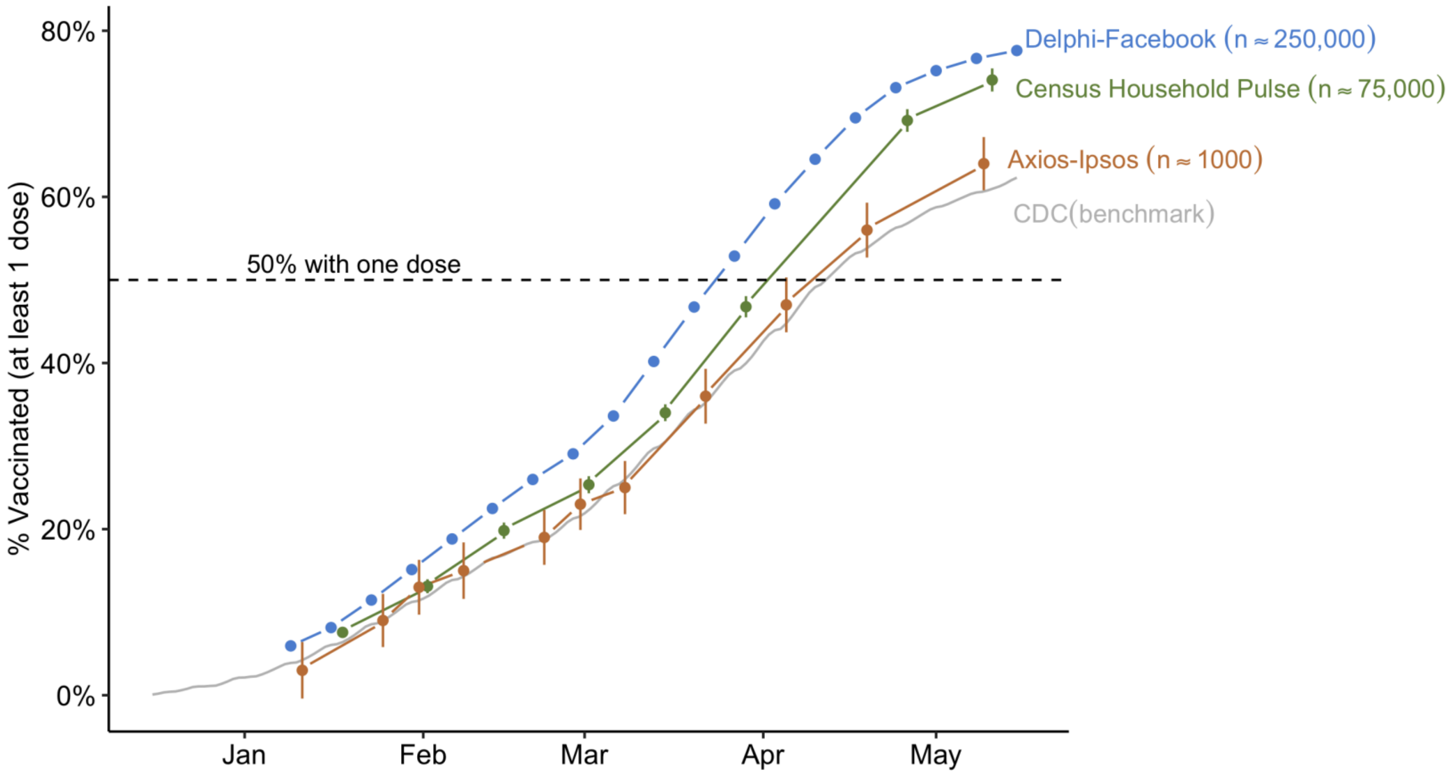


Fit to Data

$$\chi_E^2(\boldsymbol{a}, \boldsymbol{\lambda}, \alpha_s, \{m_q\}) = \sum_{k=1}^{N_{\rm pt}^{(E)}} \frac{1}{s_k^2} \left(D_k - T_k\left(\boldsymbol{a}, \alpha_s, \{m_q\}\right) - \sum_{\alpha=1}^{N_\lambda^{(E)}} \beta_{k\alpha}\left(\boldsymbol{a}, \alpha_s, m_q\right) \lambda_\alpha \right)^2 + \sum_{\alpha=1}^{N_\lambda^{(E)}} \lambda_\alpha^2 \,.$$



Big Data Paradox: *The bigger the data, the surer we fool ourselves*, if we do not pay attention to data quality



[Slide from Xiao Li Meng, PHYSTAT 2025]

Uncertainty Quantification

- **Aleatoric** (“dicey”) Uncertainty:
  - Statistical uncertainty in data that is reduced by improving data quantity and quality
- **Epistemic** (“knowledge”) Uncertainty:
  - Due to lack of knowledge, which can introduce bias. Improved through better (or at least) representative modelling. Aka Systematic uncertainty.

Hullermeier & Waegeman (2021)

This Talk

Present challenges of quantifying uncertainty in CTEQ-TEA's simultaneous determination of the strong coupling and PDFs using updated data sets.

NNLO fits with new data (nD) from LHC at 8 and 13 TeV

χ^2/N_{pt} for CT25 (*vs.* CT18A) NNLO fits; 68% CL

New and updated LHC data sets in the CT25 NNLO analysis

ID	Experiment	$N_{\rm pt}$	$\chi^2/N_{\rm pt}$ for $\alpha_s(M_Z)=0.118$, CT25 (CT18A)
	Lepton pair production		
211	ATLAS 8 TeV W [40]	22	$2.04^{+0.65}_{-0.52}$ ($4.35^{+2.65}_{-2.37}$)
212	CMS 13 TeV Z [41]	12	$2.02^{+1.03}_{-1.23}$ ($2.12^{+3.77}_{-0.17}$)
214	ATLAS 8 TeV Z 3D [42]	188	$1.16^{+0.14}_{-0.06}$ ($1.22^{+0.34}_{-0.14}$)
215	ATLAS 5.02 TeV W, Z [43]	27	$0.62^{+0.10}_{-0.09}$ ($0.77^{+0.34}_{-0.09}$)
217	LHCb 8 TeV W [44]	14	$1.39^{+0.34}_{-0.23}$ ($1.52^{+0.49}_{-0.36}$)
218	LHCb 13 TeV Z [45]	16	$1.09^{+0.49}_{-0.39}$ ($1.24^{+1.03}_{-0.38}$)
	Inclusive jet production		
553	ATLAS 8 TeV jets [46]	171	$1.60^{+0.10}_{-0.06}$ ($1.57^{+0.12}_{-0.07}$)
554	ATLAS 13 TeV jets [47]	177	$1.32^{+0.06}_{-0.05}$ ($1.26^{+0.09}_{-0.03}$)
555	CMS 13 TeV jets [6, 48]	78	$1.13^{+0.11}_{-0.04}$ ($1.29^{+0.23}_{-0.17}$)
556	CMS 7 TeV jets [6, 49–51]	118	$0.74^{+0.02}_{-0.03}$ ($0.7^{+0.12}_{-0.03}$)
557	CMS 8 TeV jets [6, 52]	164	$1.17^{+0.12}_{-0.06}$ ($1.16^{+0.18}_{-0.06}$)
	$t\bar{t}$ production at 13 TeV		
521	ATLAS all-hadronic $y_{t\bar{t}}$ [53]	12	$1.11^{+0.07}_{-0.07}$ ($1.06^{+0.10}_{-0.07}$)
528	CMS dilepton $y_{t\bar{t}}$ [54]	10	$1.28^{+0.43}_{-0.44}$ ($1.04^{+0.78}_{-0.44}$)
581	CMS lepton+jet $m_{t\bar{t}}$ [55]	15	$1.13^{+0.33}_{-0.31}$ ($1.36^{+0.89}_{-0.48}$)
587	ATLAS lepton+jet $m_{t\bar{t}}+y_{t\bar{t}}+y^B_{t\bar{t}}+H^{t\bar{t}}_T$ [56]	34	$1.07^{+0.19}_{-0.13}$ ($0.94^{+0.24}_{-0.10}$)

(fits with 1 new process, 'nProcces')

nDY

nIncJet

$p\,p \to \bar{t}t$

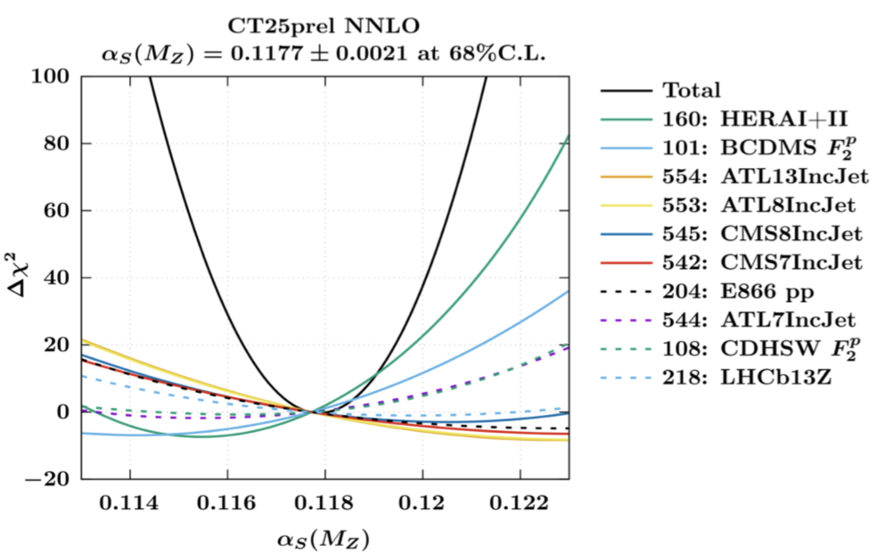
nD

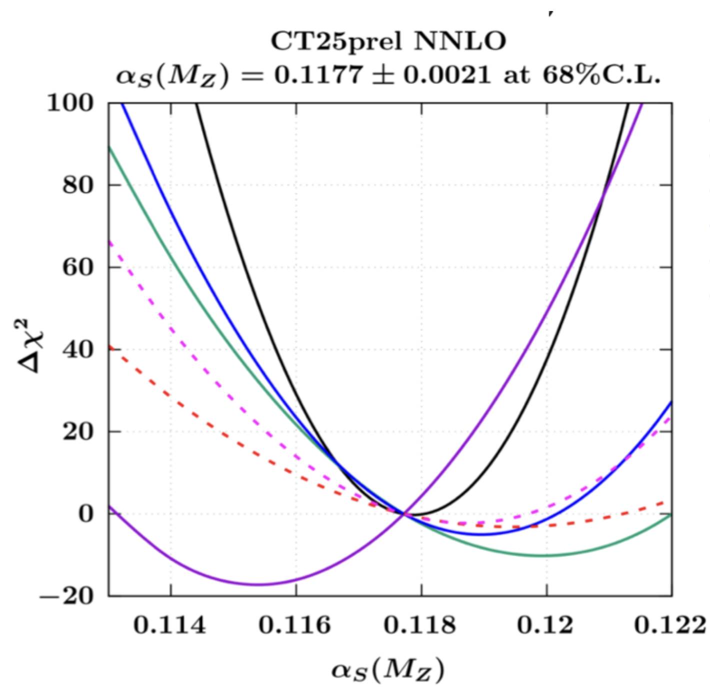
(combined fit)

Impact of new data on fits

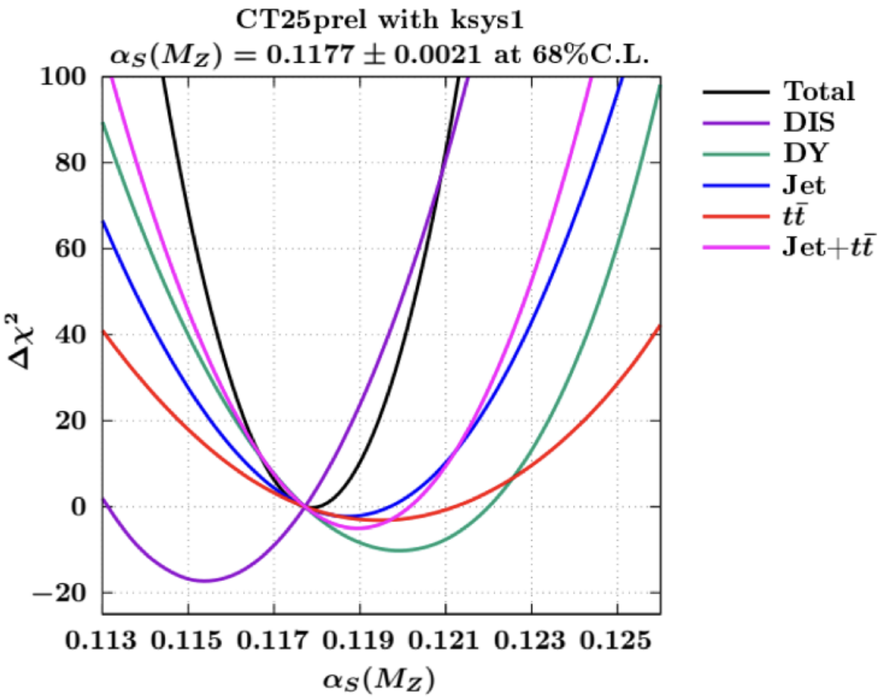
Updates from the upcoming CT25 NNLO fits

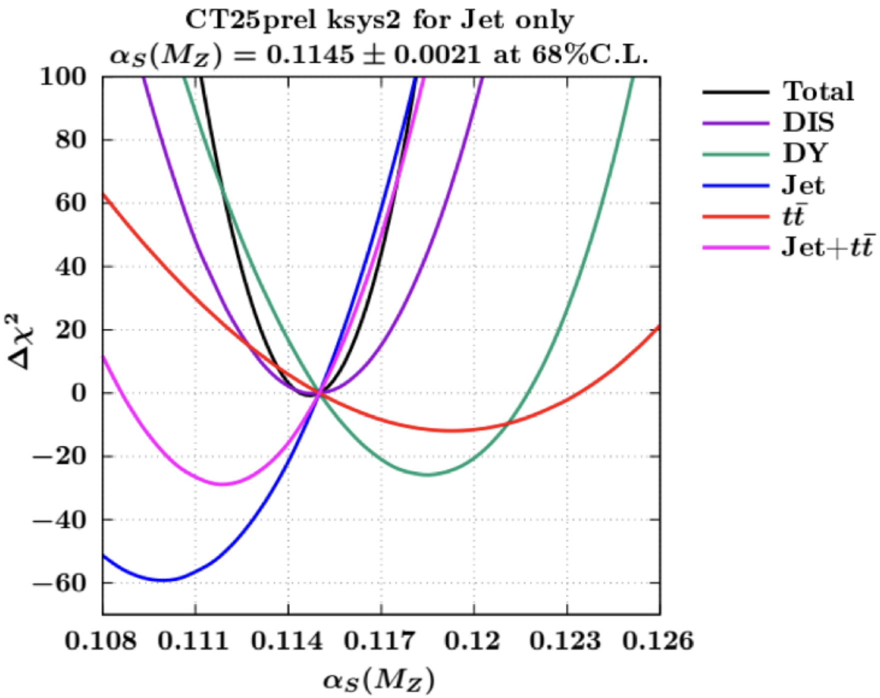
- Significant pulls on α_s from ATLAS Incl. Jets [553, 554] and 13 TeV LHCb Z data [218] and $t\bar{t}$ production data
- Large tension between DIS, DY and Jet+ $t\bar{t}$





Sensitivity to treatment of systematics





Left: **multiplicative errors** for all data sets

$\alpha_s(M_Z) \approx 0.118$

… but possible theory bias in syst. effects

Right: **additive errors** for jet data sets

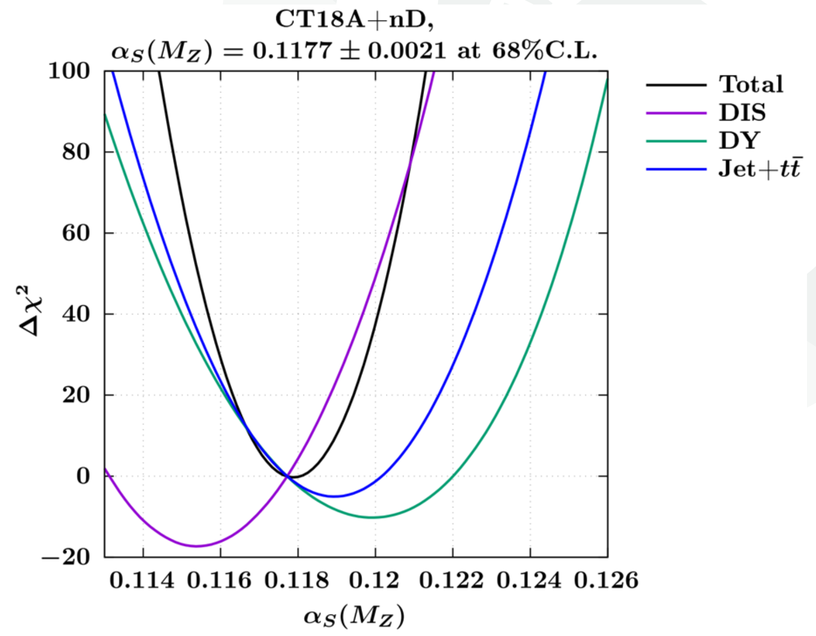
$\alpha_s(M_Z) \approx 0.115$

… likely reflects D'Agostini's bias – cf. also a similar shift in the 2025 NNPDF4.0 α_s study

The truth is within a range between the extremes

Tolerance for Tolerating Tension

- It is easy to see that there is a large tension between DIS and other data sets.
- Treat each of the sets (DIS, DY, Jet $+t\bar{t}$) as independent and identically distributed measurements of α_s
- $\chi^2_{tot} = \sum_i \chi^2_i$
- Mean and Error given by minimizing
- $\tilde{\chi}^2 = \sum_i \frac{(\alpha_{s_i} - \bar{\alpha}_s)}{\sigma_i^2}$
- $\bar{\alpha}_s = \sigma^2_{tot} \sum_i \frac{\alpha_{s_i}}{\sigma_i^2}$, $\frac{1}{\sigma^2_{tot}} = \sum_i \frac{1}{\sigma_i^2}$
- Large Tension: $\frac{\tilde{\chi}^2}{dof} \simeq 17$
- Yet small uncertainty:
  - $\alpha_s = 0.1184 \pm 0.000360$
  - (Profile Likelihood)



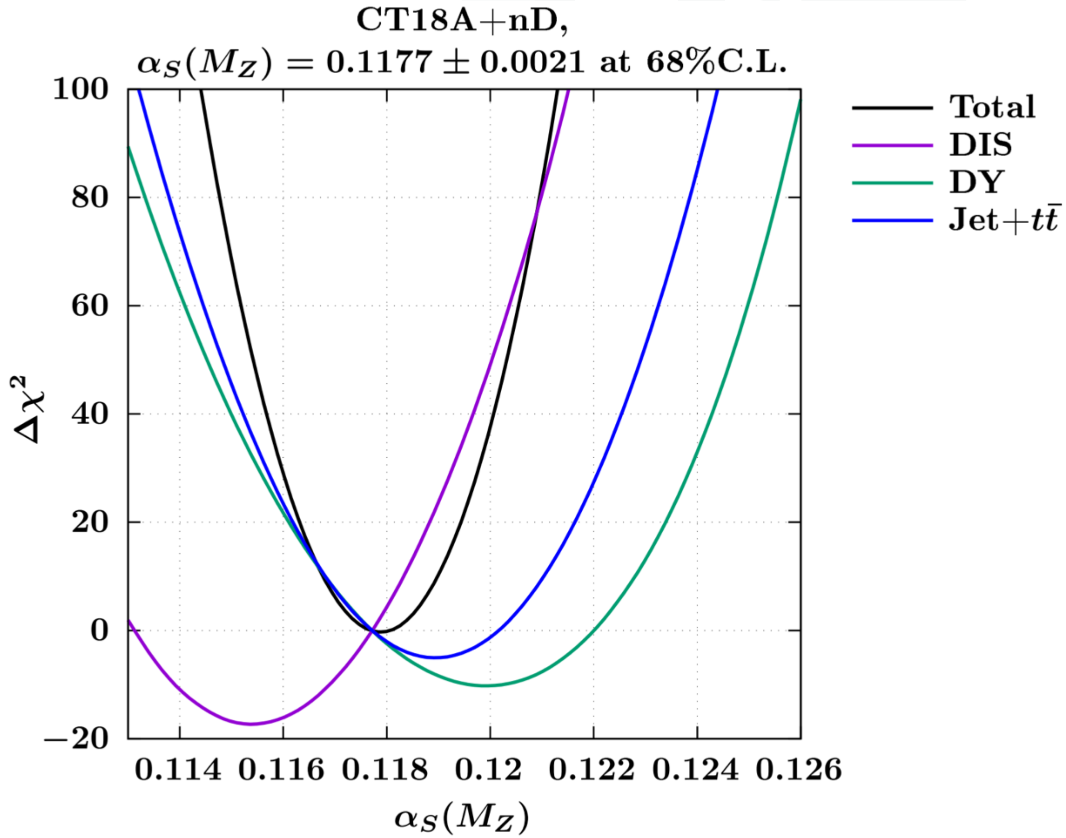
	DIS	DY	Jet $+t\bar{t}$	χ^2_{tot}
$\alpha_s \times 10^3$ $at\ \min of\ \chi^2_i$	115.14	119.91	118.94	**118.37**
Error $(\sigma_i \times 10^3)$ $(\Delta\chi^2_i = 1)$	0.629	0.743	0.543	**0.360**

How to proceed?

What should we do when we don’t know how to proceed?

Form committees!

Committee #1: Global Tolerance



Use global Tolerance:

$$\Delta\chi^2 = T^2$$

$$\alpha_s(M_Z) = 0.1177 \pm 0.0021 \; at \; 68\% \; CL$$

How should we justify the choice of Tolerance here?

Committee #2: Global Tolerance through Effective Gaussian variable

$$S_n = \sqrt{2\chi^2(N_n)} - \sqrt{N_n - 1}$$

Criteria determined by values of $S_n \approx 0.468$
for 68% CL for the total χ^2
Note: Related to Quantile of χ^2

$$\alpha_s(M_Z) = 0.1179 + 0.0024 - 0.0025$$

Committee #3: Dynamical Tolerance

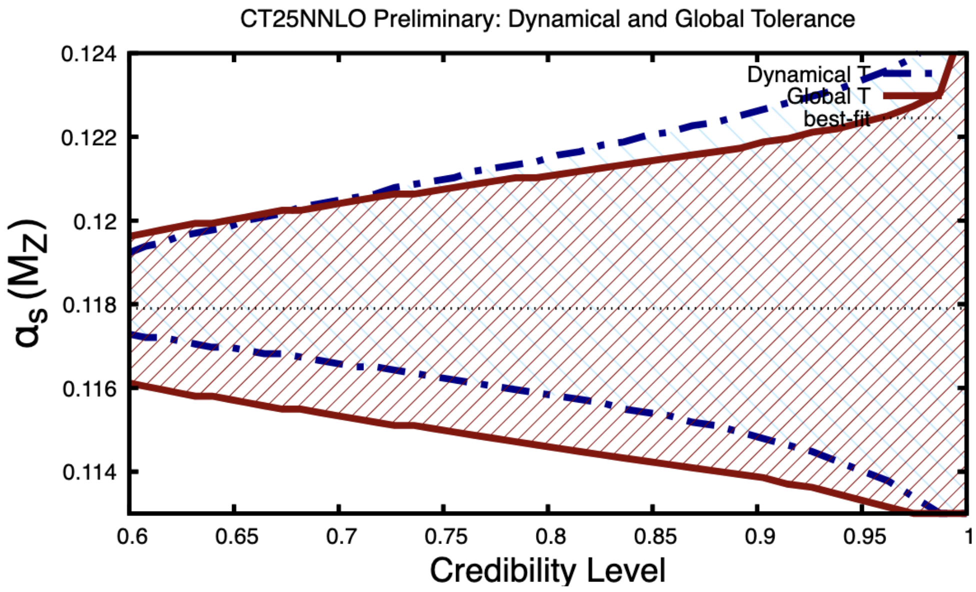
$$\chi^2_E(a,\lambda) = \sum_{k=1}^{N_{pt}} \frac{1}{s_k^2}\left(D_k - T_k(a) - \sum_{\alpha=1}^{N_\lambda} \beta_{k\alpha}\lambda_\alpha\right)^2 - \sum_{\alpha=1}^{N_\lambda} \lambda_\alpha^2 ,$$

Dynamic Tolerance determine by using Lewis formula for 52 individual experiments

$$\alpha_s(M_Z) = 0.1179 + 0.0024 - 0.0012$$

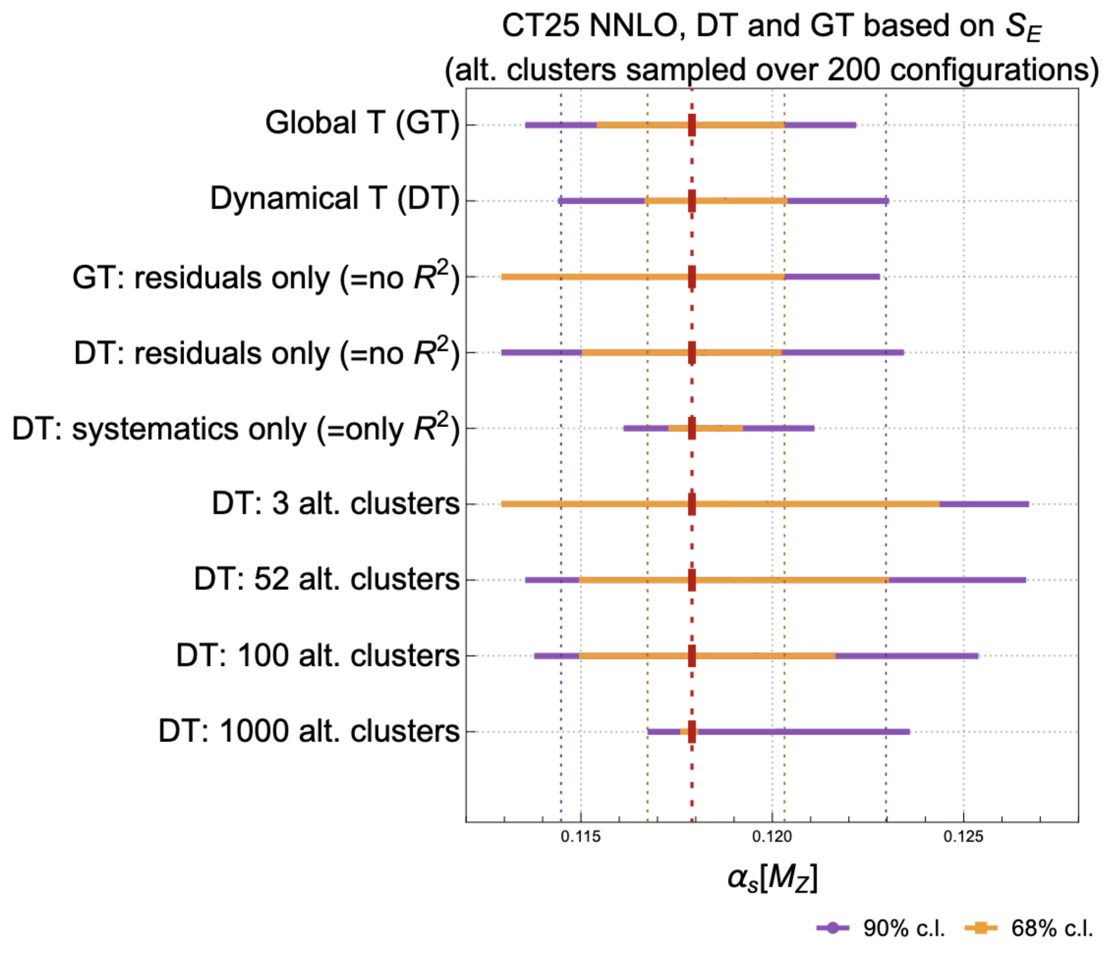
Comparison of Credibility level for Global Tolerance and Dynamic Tolerance

Dynamic Tolerance shows dependence one how data is partitioned



Sensitivity to partitioning of data sets

In an ideal scenario, in which all residuals are normally distributed according to $N(0,1)$, alternative clusters would produce approximately the same estimate for the uncertainty.



Committee #4: PDG Prescription: Scaling Errors

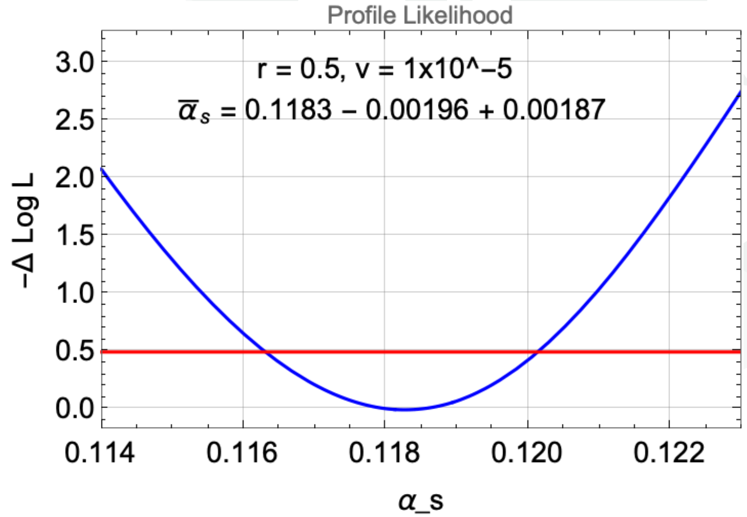
- **PDG proposal:** scale errors by a factor e_s to make fits more consistent, i.e. each $\sigma_i \rightarrow e_s\, \sigma_i$
- $e_{s_{PDG}} = \sqrt{\frac{\tilde{\chi}^2}{dof}} \simeq 4.7$ so that each $\sigma_i \rightarrow 4.7 \times \sigma_i$ and $\frac{\tilde{\chi}^2}{dof} \rightarrow 1$
- **Caveat:** For very large $\sqrt{\frac{\tilde{\chi}^2}{dof}}$, PDG recommends making an educated guess of the uncertainty rather than scaling the errors.
- $\bar{\alpha}_s = 0.1184 \pm 0.0017$

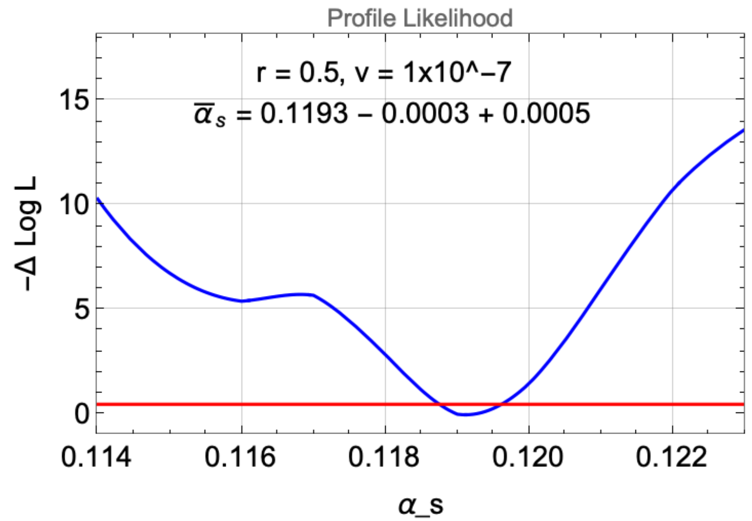
Committee #5: The "Error on the error"

$$\ln L(\boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}_{\mathbf{u}}^2) = \ln P(\mathbf{y}|\boldsymbol{\mu}, \boldsymbol{\theta})$$

$$-\frac{1}{2}\sum_{i=1}^{N}\left[\frac{(u_i-\theta_i)^2}{\sigma_{u_i}^2} + \left(1+\frac{1}{2r_i^2}\right)\ln\sigma_{u_i}^2 + \frac{v_i}{2r_i^2\sigma_{u_i}^2}\right]$$

$$P(\mathbf{y}|\boldsymbol{\mu}, \boldsymbol{\theta}) = \prod_{i=1}^{N}\frac{1}{\sqrt{2\pi}\sigma_{y_i}} e^{-(y_i-\varphi(x_i;\boldsymbol{\mu})-\theta_i)^2/2\sigma_{y_i}^2}$$





Needs as input an estimate of the uncertainty on the systematic uncertainty characterized by r.

Sensible values of r need to be chosen by analyst, making and educated guess.

G. Cowan arXiv:1809.05778, see also
M. Reader arxiv.org:2408.12922

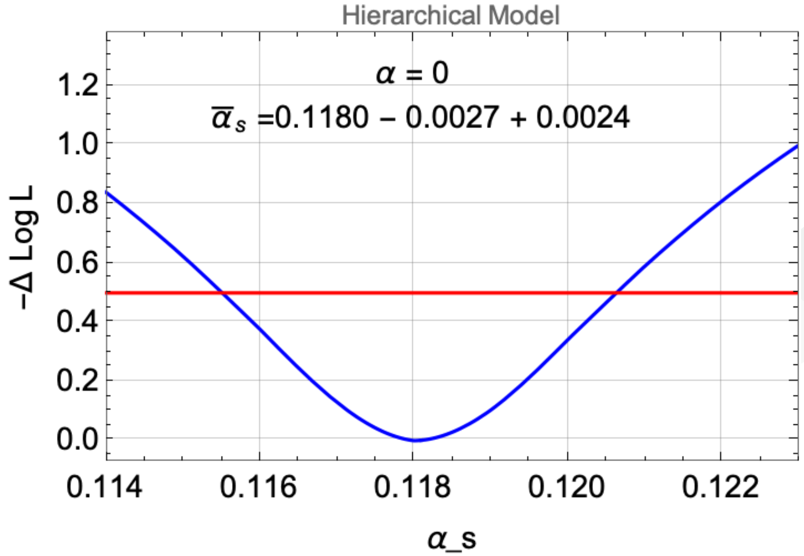
Committee #6: Bayesian Hierarchical Model

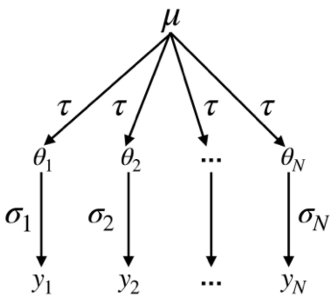
$$p(\mu|y_i) \propto \int_0^\infty \prod_{i=1}^{N} \left(\sigma_i^2 + \tau^2\right)^{-\frac{1}{2}\left(1+\frac{\alpha}{N}\right)} e^{-\frac{(\mu-y_i)^2}{2(\sigma_i^2+\tau^2)}} d\tau^2.$$

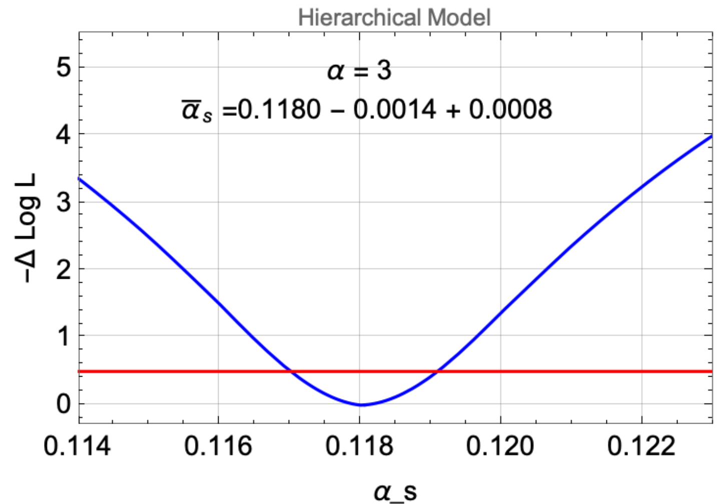
Depends on choice of α:

$0 \leq \ \alpha \leq 3$ for large unknown systematics.

Use large values of α if quoted uncertainties are trusted.

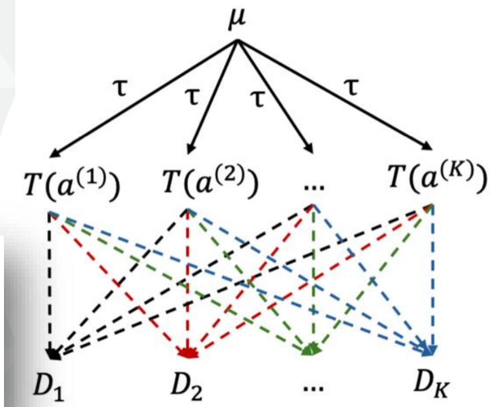






[J. Erler & R. Ferro-Hernandez, arXiv:2004.01219]

Gaussian Mixture Model



1. Modified Posterior

$$\prod_{i=1}^{N_D}\left(\sum_{k=1}^{K} P(T(a^{(k)})|D_i)\right) \propto \prod_{i=1}^{N_D}\left(\sum_{k=1}^{K} w_k \mathcal{N}(D_i|T(a^{(k)}),\sigma_i)\right)$$

2. Implementation via MLE of Mixture of Gaussians

$$\pi(Y|\vec{\theta}) = \prod_{j=1}^{N_{\rm pt}} \pi(y_j,\Delta y_j|\vec{\theta}) = \prod_{j=1}^{N_{\rm pt}}\sum_{i=1}^{K} \omega_i \mathcal{N}(y_j,\Delta y_j|\theta_i),$$

$$0 \le \omega_k \le 1 \quad \text{and} \quad \sum_k \omega_k = 1,$$

3. Calculate Mean

$$\mathbb{E}[\theta] = \sum_{i=1}^{K} \omega_i \hat{\theta}_i.$$

4. Estimate uncertainty via observed Fisher Information Matrix

$$\begin{aligned}\text{cov}_{\rm GMM} &= \sum_{i=1}^{K}\omega_i\ \text{cov}_{{\rm GMM},i} + \sum_{i=1}^{K}\omega_i(\mathbb{E}[\theta]-\hat{\theta}_i)^2 \\ &= \sum_{i=1}^{K}\omega_i\left(\sum_{j=1}^{N_{\rm pt}}\frac{1}{\Delta y_j^2}\left(\frac{\partial y_j(\theta_i)}{\partial\theta_i}\right)^2\frac{\mathcal{N}(y_j,\Delta y_j|\theta_i)}{\pi(y_j,\Delta y_j|\vec{\theta})}\right)^{-1} + \sum_{i=1}^{K}\omega_i(\mathbb{E}[\theta]-\hat{\theta}_i)^2.\end{aligned}$$

5. Use Information Criteria (AIC/BIC) to determine the number of Gaussians

$$\begin{aligned}\text{AIC} &= N_{\rm parm}\ \log N_{\rm pt} - 2\log L\big|_{\theta=\hat{\theta}}, \\ \text{BIC} &= 2N_{\rm parm} - 2\log L\big|_{\theta=\hat{\theta}}.\end{aligned}$$

$$N_{\rm parm} = 2K + (K-1).$$

M.Yan, T.-J. Hou, Z. Li, KM, C.-P. Yuan, arxiv: 2406.01664

Committee #7: α_s Uncertainty with GMM

GMM (K=2) (Yellow shaded)

$$\bar{\alpha}_s = 0.1183 \pm 0.0023$$

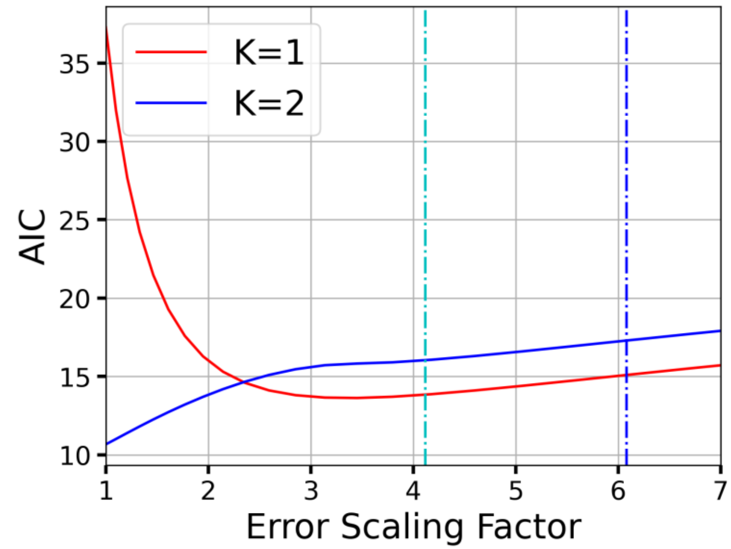
$e_{s_{PDG}} \sim 4.1$ *(Green shaded, Cyan line)*

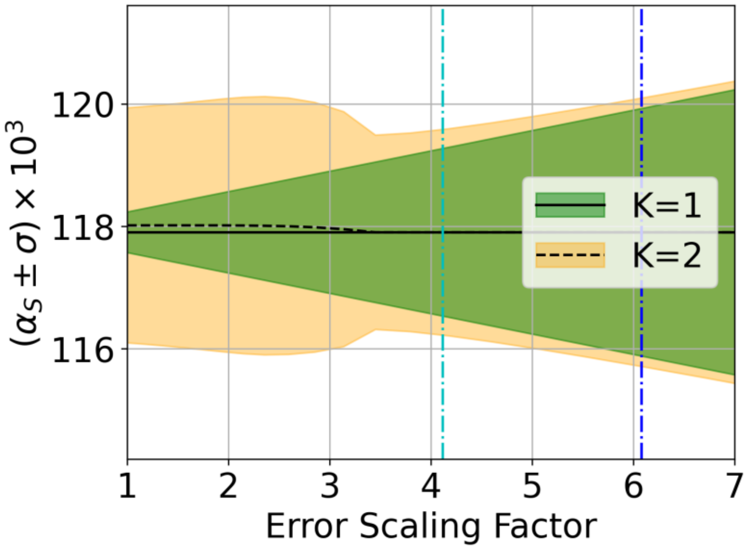
$$\bar{\alpha}_s = 0.1183 \pm 0.00135$$

$e_s \sim 6.1$ (Green shaded, Blue line)

$$\bar{\alpha}_s = 0.11795 \pm 0.0020$$

Caveat: How we partition the data sets does have an impact on uncertainty determination. More complete study is underway

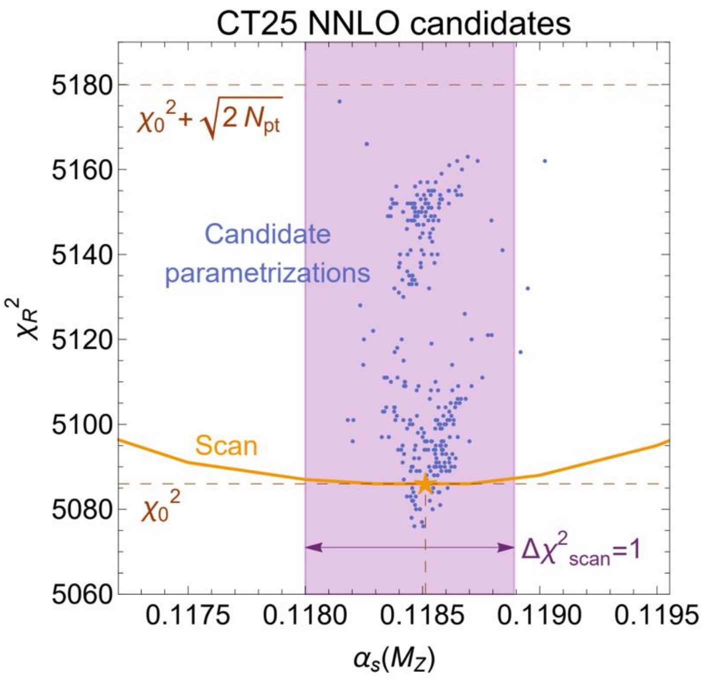




Committee #8: PDF Parametrization Uncertainty for α_s

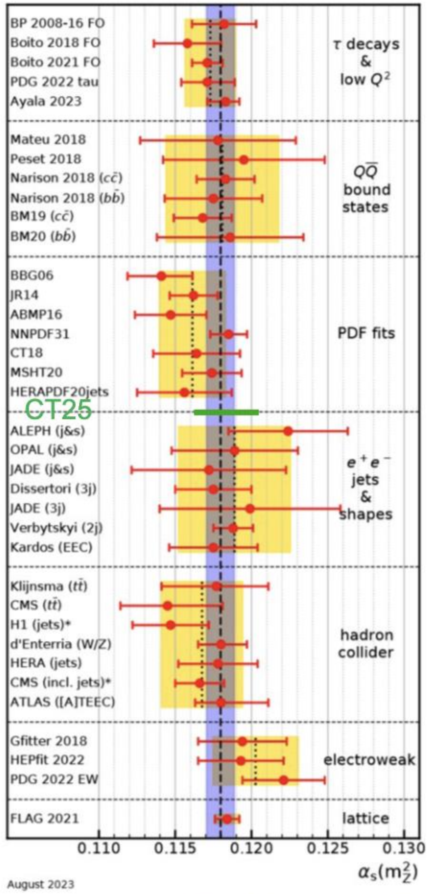
α_s has low correlation with PDF parametrizations.

Epistemic uncertainty from PDF parametrization is small



Central Committee

Statistical Method	Eq.	$\delta\alpha_s(M_Z)$
Global Tolerance	6.1	$0.1184^{+0.0026}_{-0.0028}$
Dynamical Tolerance	6.2	$0.1184^{+0.0024}_{-0.0012}$
BHM	5.3	$0.1181^{+0.0022}_{-0.0023}$
GMM	5.5	$0.1183^{+0.0023}_{-0.0023}$
Average		$0.1183^{+0.0023}_{-0.0020}$



$$\alpha_s(m_Z^2) = 0.1180 \pm 0.0009 \qquad \text{(PDG 2023 average)}\,.$$

Outlook

- Presented (Preliminary) updated CTEQ-TEA fits to α_s
- Significant upward pulls from Jet and ttbar data from LHC
- Statistical Modeling is challenging!
- Set up committees to determine the uncertainty on the strong coupling.
- Each committee has its own estimate on the uncertainty – represents uncertainty on the uncertainty
- Central committee to build consensus between committees.