Quantum Tomography of Neutral Mesons in Flavor Space

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2507.12513

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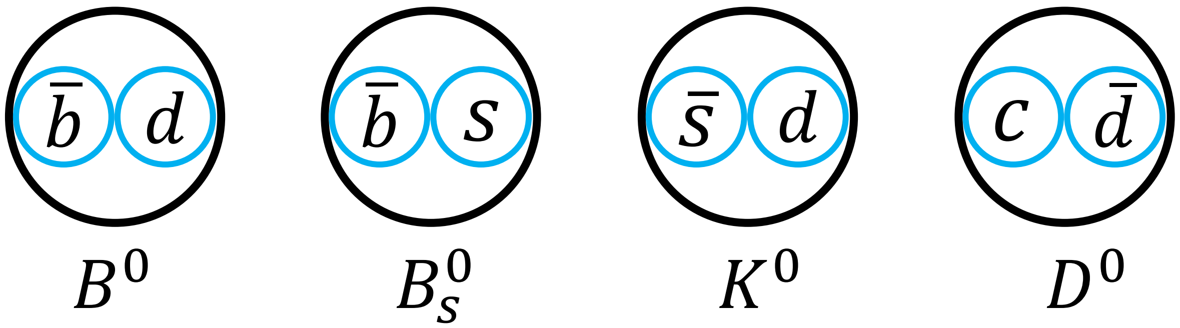
Introduction

Open-flavor neutral mesons:



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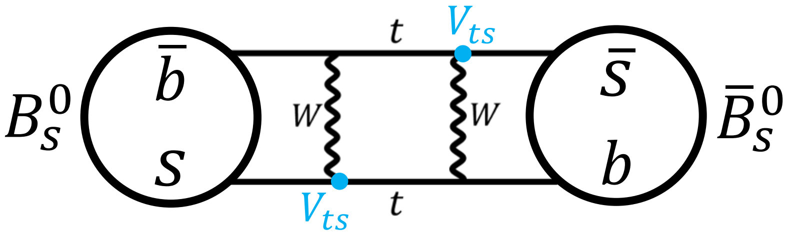
Flavor Mixing



$= \boldsymbol{m_{12}}$

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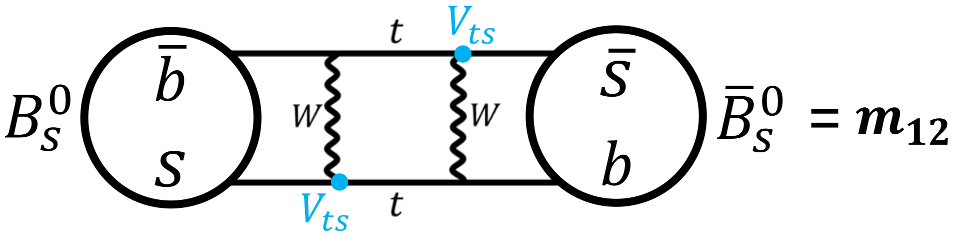

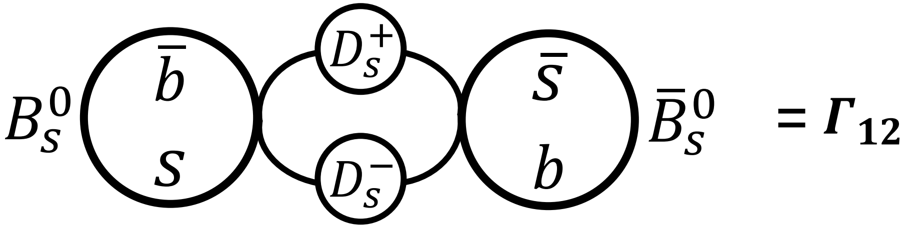

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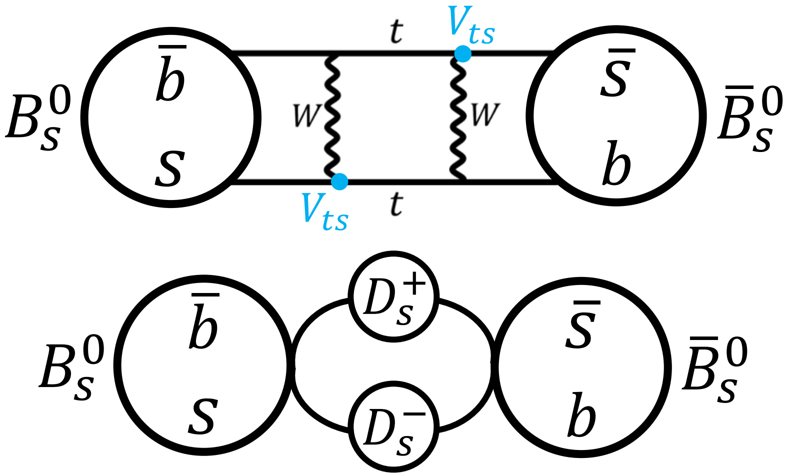

$= \boldsymbol{m_{12}}$

$= \boldsymbol{\Gamma_{12}}$

$\Rightarrow$

Non-diagonal Hamiltonian

$$H = \begin{pmatrix} m & m_{12} \\ m_{12}^* & m \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}$$

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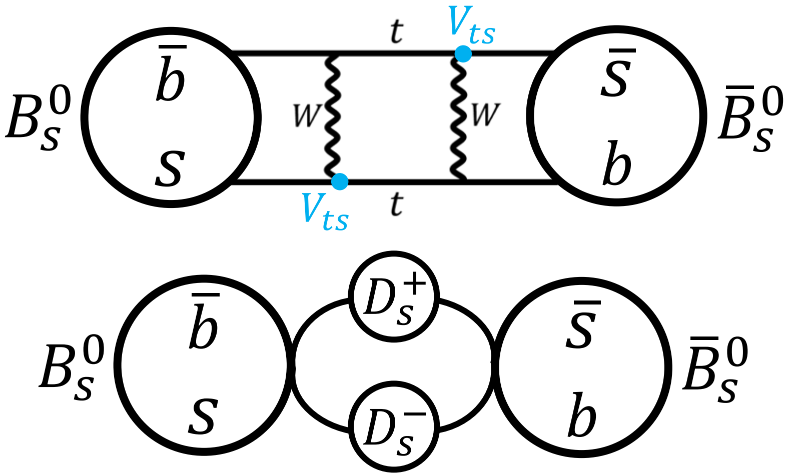

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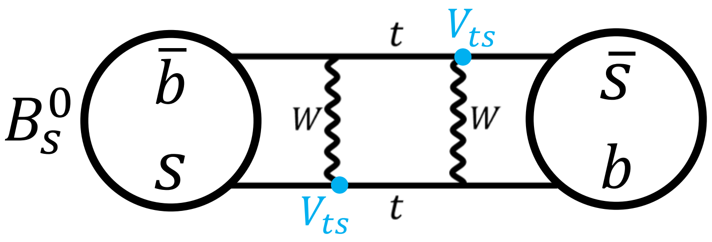

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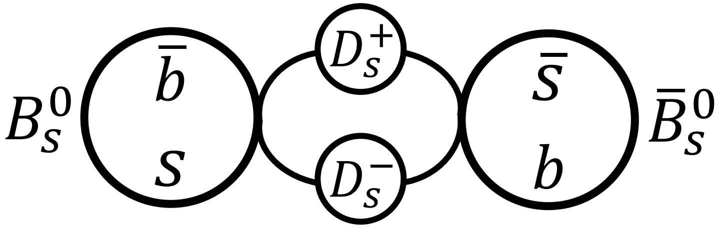

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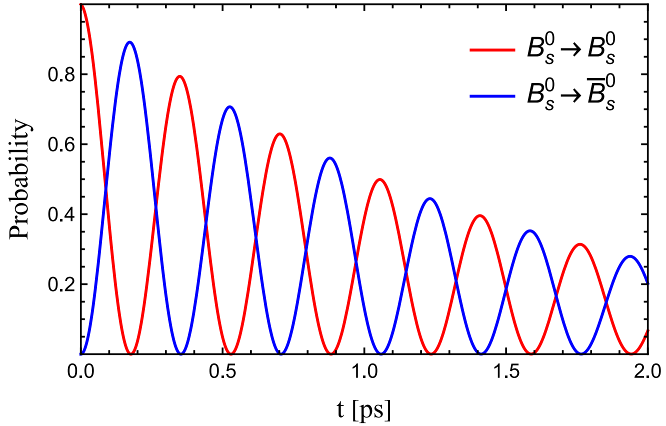
$\Downarrow$

$B_s^0 - \bar{B}_s^0$ oscillation



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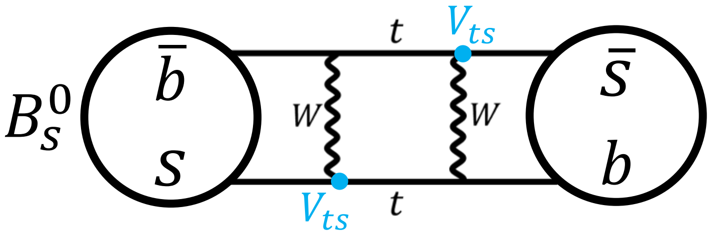

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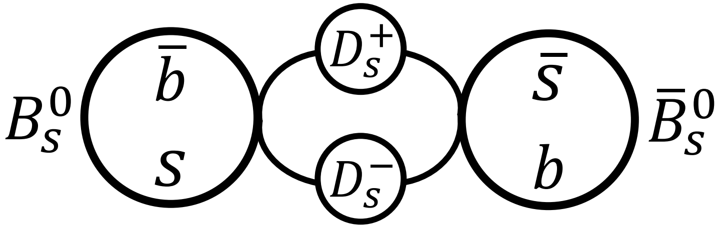

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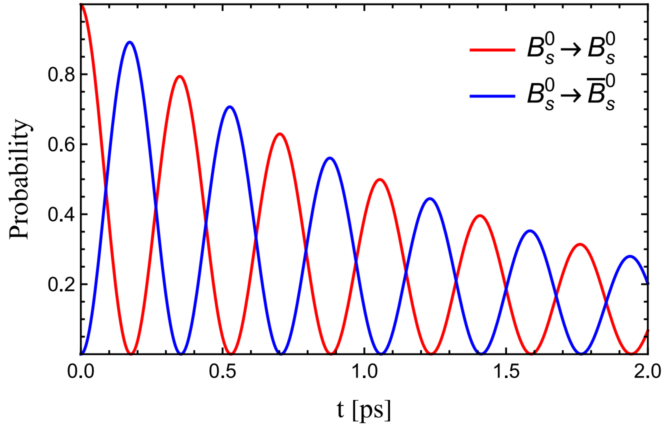

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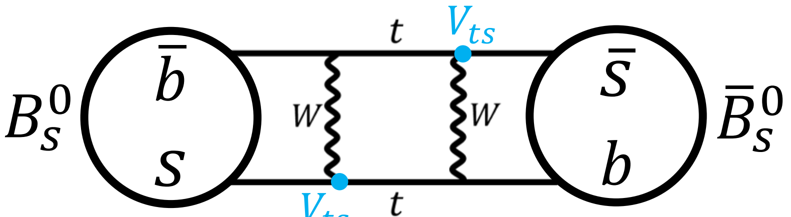

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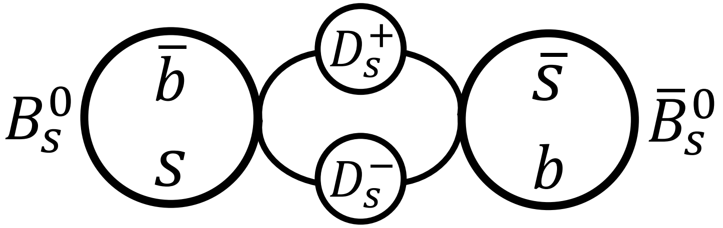

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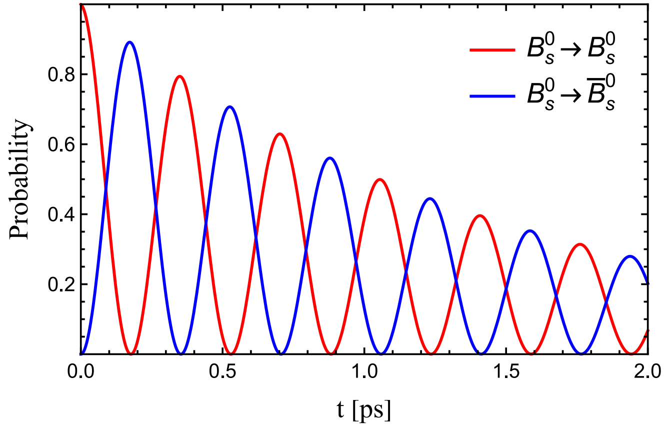

Inherently quantum mechanical!

Meson is a flavor qubit

$$|\psi\rangle = \alpha\,|B^0\rangle + \beta\,|\bar{B}^0\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

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Question:

Given a general meson flavor state, can we fully reconstruct the quantum state?

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Quantum tomography

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Quantum tomography

Given a general meson flavor state, can we fully reconstruct the quantum state?

Solution:

Flavor oscillation + Flavor tagging decay = Flavor polarizer in any direction !

Density matrix

The most general single meson state, 2×2 matrix:

$$\rho = \frac{1}{2}\left(\mathbb{1} + b_i\,\sigma_i\right)$$

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$$\begin{cases} b_z = 1 \\ b_z = -1 \end{cases} \Longleftrightarrow \begin{matrix} |M^0\rangle \\ |\bar{M}^0\rangle \end{matrix} \Rightarrow \text{Flavor eigenstate}$$

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$$\begin{cases} b_x = 1 \\ b_x = -1 \end{cases} \Leftrightarrow \begin{matrix} \frac{1}{\sqrt{2}}\left(|M^0\rangle + |\bar{M}^0\rangle\right) \\ \frac{1}{\sqrt{2}}\left(|M^0\rangle - |\bar{M}^0\rangle\right) \end{matrix}$$

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→ Mass eigenstate

(We ignore the small CPV)

Time evolution

Heisenberg picture: $\mathcal{O}(t) = U^\dagger(t)\,\mathcal{O}(0)\,U(t)$

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$$U = \begin{pmatrix} \frac{1}{2}(e^{-\Gamma_1 t/2 - i m_1 t} + e^{-\Gamma_2 t/2 - i m_2 t}) & \frac{1}{2}(e^{-\Gamma_1 t/2 - i m_1 t} - e^{-\Gamma_2 t/2 - i m_2 t}) \\ \frac{1}{2}(e^{-\Gamma_1 t/2 - i m_1 t} - e^{-\Gamma_2 t/2 - i m_2 t}) & \frac{1}{2}(e^{-\Gamma_1 t/2 - i m_1 t} + e^{-\Gamma_2 t/2 - i m_2 t}) \end{pmatrix}$$

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Heisenberg picture: $\mathcal{O}(t) = U^{\dagger}(t)\,\mathcal{O}(0)\,U(t)$

Observables:

① Total number of mesons

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Operator at time t:

$$\mathbb{1} \to U^\dagger(t)U(t)$$

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Operator at time t:

$$\mathbb{1} \to U^\dagger(t)U(t) = e^{-\Gamma t}\left(\cosh(\tfrac{\Delta\Gamma t}{2})\,\mathbb{1} - \sinh(\tfrac{\Delta\Gamma t}{2})\,\sigma_x\right)$$

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Polarizer in $\hat{x}$!

x component controls rate of decay

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Expectation value:

$$N \propto \mathrm{Tr}(\rho \cdot \quad) \longrightarrow b_x$$

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Quantum tomography

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Flavors are tagged by semi-leptonic decay

$$M^0 \to \ell^+ \nu_\ell X, \qquad \bar{M}^0 \to \ell^- \bar{\nu}_\ell X$$

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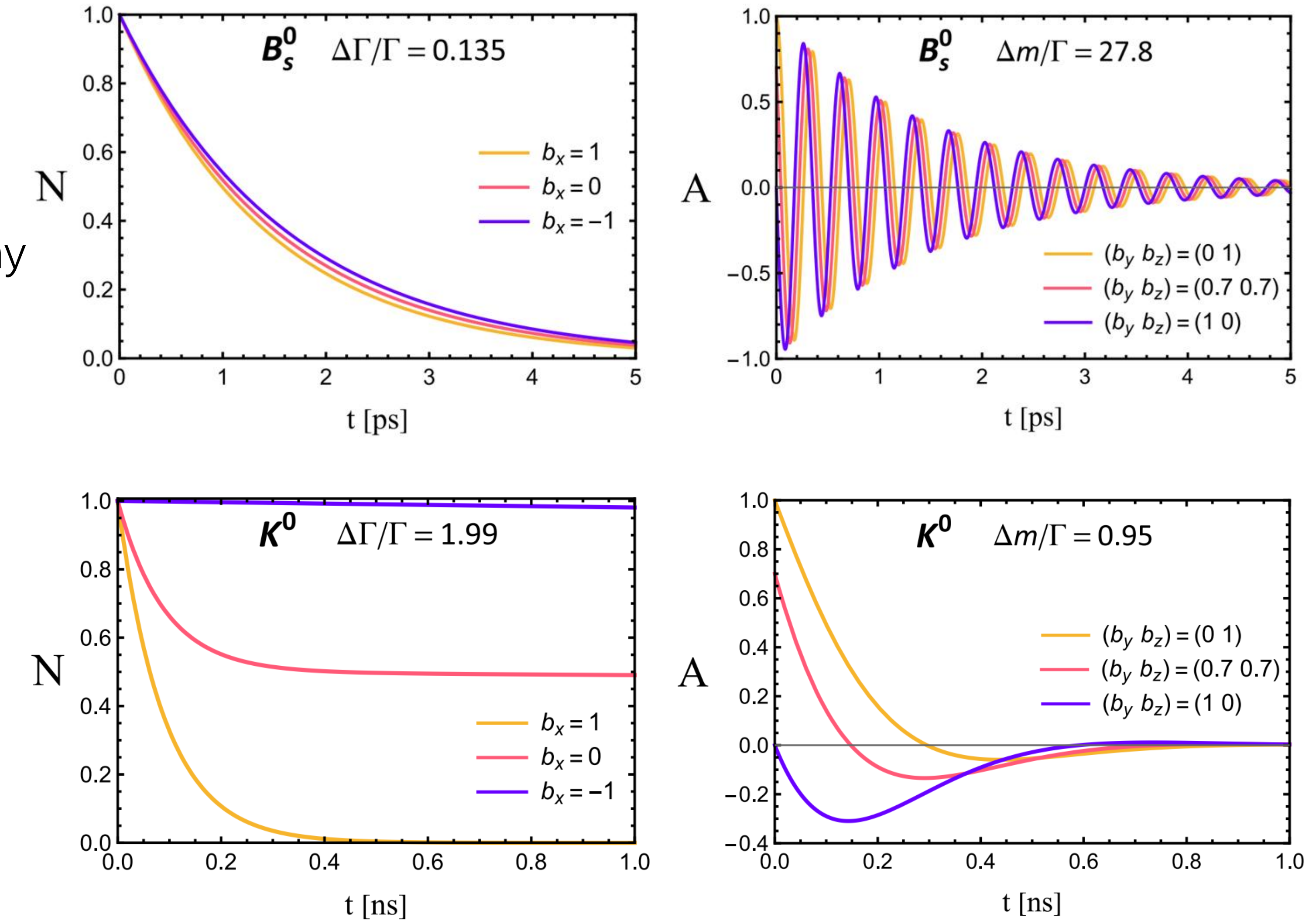
Fitting to the distributions gives $\vec{b}(0)$



Meson pair system

4×4 flavor density matrix:

$$\rho = \frac{a}{4}\left(\mathbb{1}_4 + b_i^{\mathcal{A}}\,\sigma_i \otimes \mathbb{1}_2 + b_i^{\mathcal{B}}\,\mathbb{1}_2 \otimes \sigma_i + C_{ij}\,\sigma_i \otimes \sigma_j\right)$$

Flavor correlation matrix

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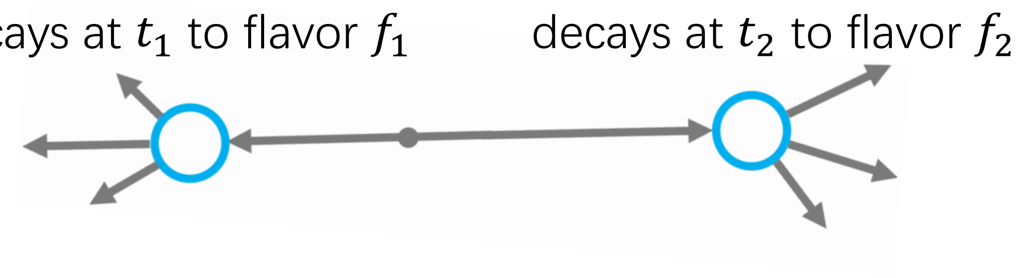

$\Rightarrow$ Measure the distribution of $N_{f_1 f_2}(t_1, t_2)$

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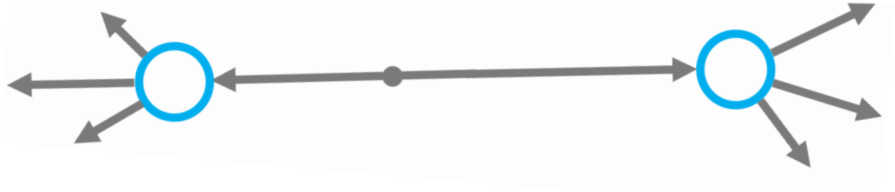
Flavor correlation matrix

Observables:

decays at t_1 to flavor f_1 decays at t_2 to flavor f_2



$\Rightarrow$ Measure the distribution of $N_{f_1 f_2}(t_1, t_2)$

$$N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}} = \text{Tr}\left[\rho \cdot (\mathbb{1}(t_1) \otimes \mathbb{1}(t_2))\right]$$

$$A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}} = \text{Tr}\left[\rho \cdot (\sigma_z(t_1) \otimes \sigma_z(t_2))\right]$$

$$A_f^{\mathcal{A}} = N_{ff} - N_{\bar{f}f} + N_{f\bar{f}} - N_{\bar{f}\bar{f}} = \text{Tr}\left[\rho \cdot (\sigma_z(t_1) \otimes \mathbb{1}(t_2))\right]$$

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Flavor correlation matrix

Observables:

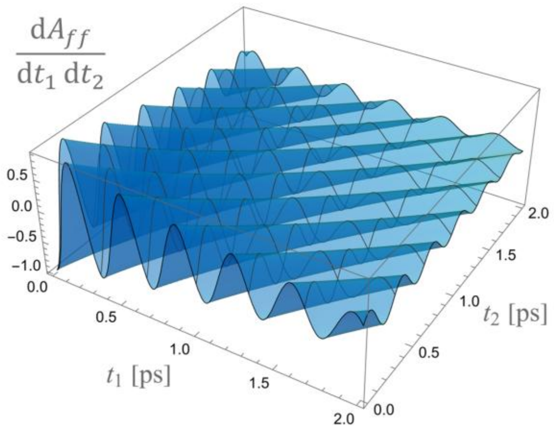
$$\frac{\mathrm{d}^2 N_{\mathrm{tot}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}\mathrm{ch}_{t_2} - \mathrm{ch}_{t_1}\mathrm{sh}_{t_2}\, b_x^{\mathcal{A}} - \mathrm{sh}_{t_1}\mathrm{ch}_{t_2}\, b_x^{\mathcal{B}} + \mathrm{sh}_{t_1}\mathrm{sh}_{t_2}\, C_{xx}\right)$$

$$\frac{\mathrm{d}^2 A_{ff}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{s}_{t_1}\mathrm{s}_{t_2}\, C_{yy} - \mathrm{c}_{t_1}\mathrm{s}_{t_2}\, C_{zy} - \mathrm{s}_{t_1}\mathrm{c}_{t_2}\, C_{yz} + \mathrm{c}_{t_1}\mathrm{c}_{t_2}\, C_{zz}\right)$$

$$\frac{\mathrm{d}^2 A_f^{\mathcal{A}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_2}(\mathrm{c}_{t_1}\, b_z^{\mathcal{A}} - \mathrm{s}_{t_1}\, b_y^{\mathcal{A}}) - \mathrm{sh}_{t_2}(\mathrm{c}_{t_1}\, C_{zx} - \mathrm{s}_{t_1}\, C_{yx})\right)$$

$$\frac{\mathrm{d}^2 A_f^{\mathcal{B}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}(\mathrm{c}_{t_2}\, b_z^{\mathcal{B}} - \mathrm{s}_{t_2}\, b_y^{\mathcal{B}}) - \mathrm{sh}_{t_1}(\mathrm{c}_{t_2}\, C_{xz} - \mathrm{s}_{t_2}\, C_{xy})\right)$$

$$\mathrm{c/s}_t = \cos / \sin(\Delta m t)$$
$$\mathrm{ch/sh}_t = \cosh / \sinh(\Delta\Gamma t/2)$$



Meson pair system

4×4 flavor density matrix:

$$\rho = \frac{a}{4}\left(\mathbb{1}_4 + b_i^{\mathcal{A}}\,\sigma_i \otimes \mathbb{1}_2 + b_i^{\mathcal{B}}\,\mathbb{1}_2 \otimes \sigma_i + \boxed{C_{ij}}\,\sigma_i \otimes \sigma_j\right)$$

Flavor correlation matrix

Observables:

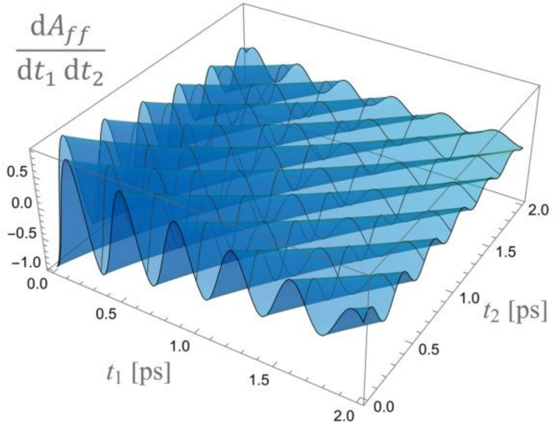
$$\frac{\mathrm{d}^2 N_{\mathrm{tot}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}\mathrm{ch}_{t_2} - \mathrm{ch}_{t_1}\mathrm{sh}_{t_2}\, b_x^{\mathcal{A}} - \mathrm{sh}_{t_1}\mathrm{ch}_{t_2}\, b_x^{\mathcal{B}} + \mathrm{sh}_{t_1}\mathrm{sh}_{t_2}\,\boxed{C_{xx}}\right)$$

$$\frac{\mathrm{d}^2 A_{ff}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{s}_{t_1}\mathrm{s}_{t_2}\boxed{C_{yy}} - \mathrm{c}_{t_1}\mathrm{s}_{t_2}\boxed{C_{zy}} - \mathrm{s}_{t_1}\mathrm{c}_{t_2}\boxed{C_{yz}} + \mathrm{c}_{t_1}\mathrm{c}_{t_2}\boxed{C_{zz}}\right)$$

$$\frac{\mathrm{d}^2 A_f^{\mathcal{A}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_2}(\mathrm{c}_{t_1}\, b_z^{\mathcal{A}} - \mathrm{s}_{t_1}\, b_y^{\mathcal{A}}) - \mathrm{sh}_{t_2}(\mathrm{c}_{t_1}\boxed{C_{zx}} - \mathrm{s}_{t_1}\boxed{C_{yx}})\right)$$

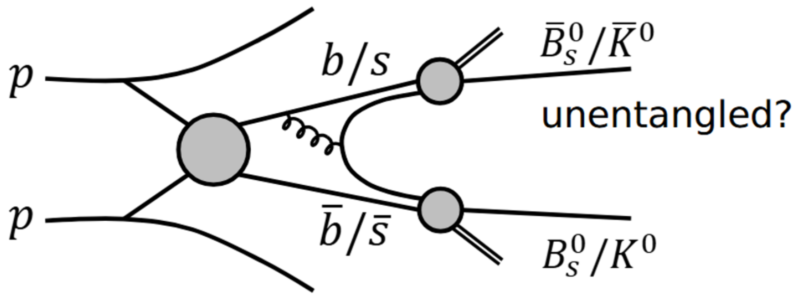
$$\frac{\mathrm{d}^2 A_f^{\mathcal{B}}}{\mathrm{d}t_1 \mathrm{d}t_2} = N_0\, e^{-\Gamma(t_1+t_2)}\left(\mathrm{ch}_{t_1}(\mathrm{c}_{t_2}\, b_z^{\mathcal{B}} - \mathrm{s}_{t_2}\, b_y^{\mathcal{B}}) - \mathrm{sh}_{t_1}(\mathrm{c}_{t_2}\boxed{C_{xz}} - \mathrm{s}_{t_2}\boxed{C_{xy}})\right)$$

$$\mathrm{c}/\mathrm{s}_t = \cos / \sin(\Delta m t)$$
$$\mathrm{ch}/\mathrm{sh}_t = \cosh / \sinh(\Delta\Gamma t/2)$$

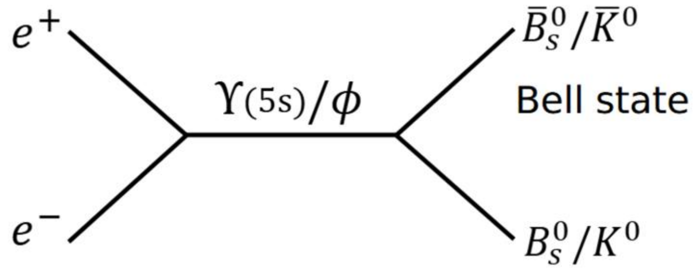


All the components of density matrix!

Sensitivity

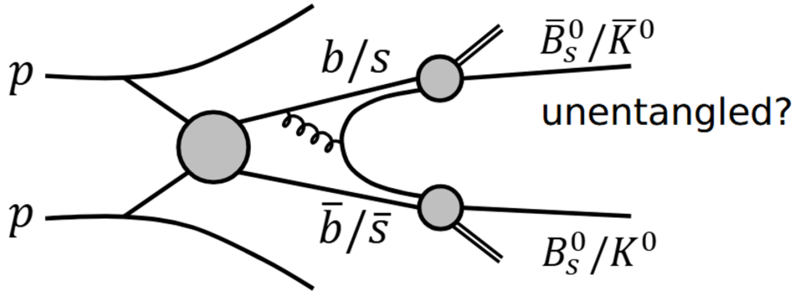


LHC



BELLE/KLOE

Sensitivity



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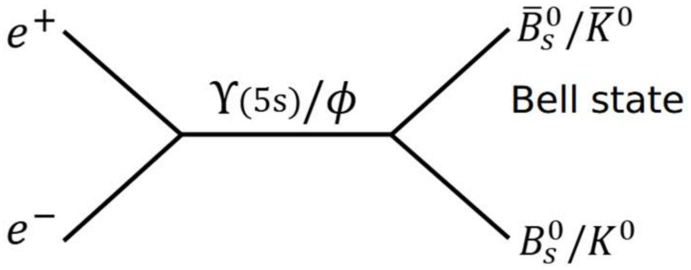


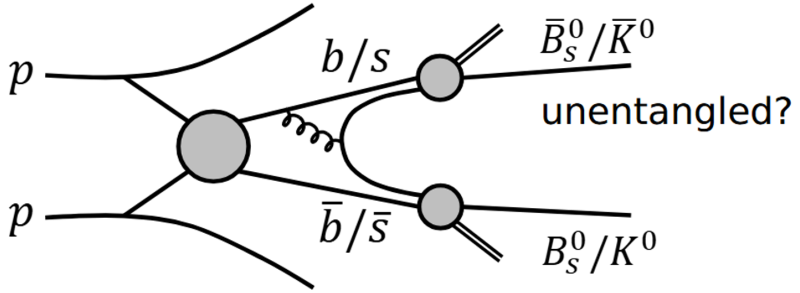
BELLE/KLOE

LHC: $\sim\mathbf{10^9}$ semileptonically decayed $\boldsymbol{B_s^0\bar{B}_s^0}$

BELLE II: $\sim\mathbf{10^7}$ semileptonically decayed $\boldsymbol{B_s^0\bar{B}_s^0}$

KLOE: $\sim\mathbf{10^5}$ semileptonically decayed $\boldsymbol{K^0\bar{K}^0}$

Sensitivity



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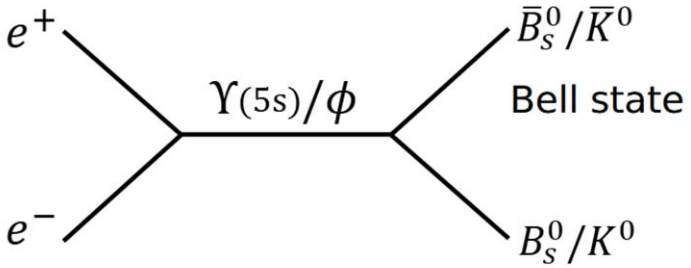


BELLE/KLOE

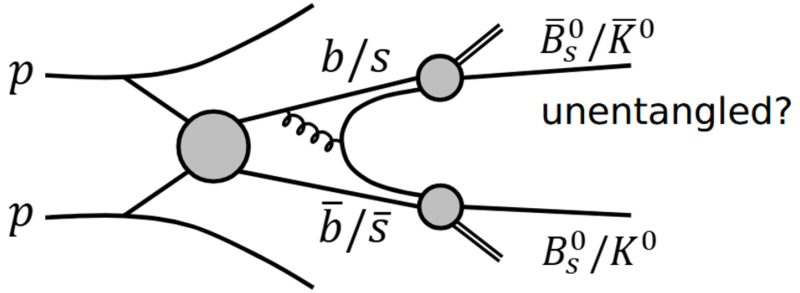
Benchmark:

#events: 10^7

Truth level quantum state:

$$\rho = 0.8\,\rho_{\text{Bell}} + 0.2\,\frac{\mathbb{1}_4}{4} \Leftrightarrow C = \begin{pmatrix} -0.8 & 0 & 0 \\ 0 & -0.8 & 0 \\ 0 & 0 & -0.8 \end{pmatrix}$$

Sensitivity



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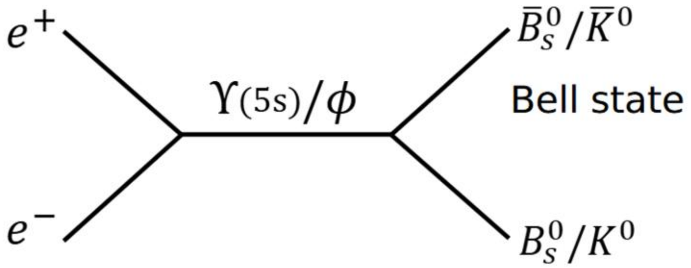

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Reconstructed (parton-level):

	$B_s^0\bar{B}_s^0$	$K^0\bar{K}^0$
$b_x^{\mathcal{A},\mathcal{B}}$	0 ± 0.017	0 ± 0.0020
$b_y^{\mathcal{A},\mathcal{B}}$	0 ± 0.0009	0 ± 0.0023
$b_z^{\mathcal{A},\mathcal{B}}$	0 ± 0.0009	0 ± 0.0015
C_{xx}	-0.8 ± 0.31	-0.8 ± 0.0036
$C_{xy,yx}$	0 ± 0.016	0 ± 0.0027
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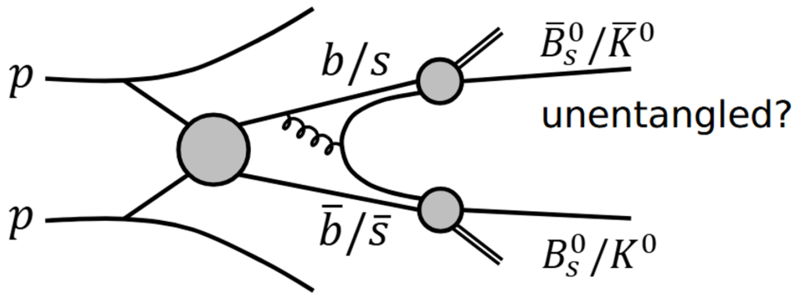

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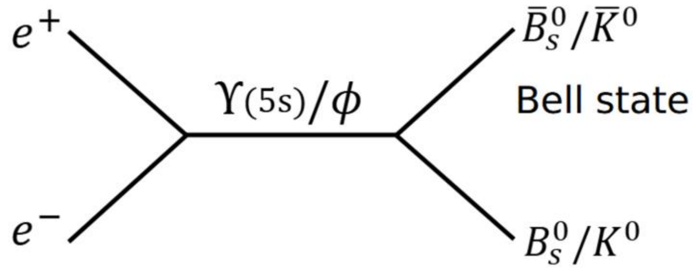

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Any QI quantities!

	$B_s^0\bar{B}_s^0$	$K^0\bar{K}^0$
Quantum Discord	0.58 ± 0.10	0.617 ± 0.003
Concurrence	0.69 ± 0.15	0.701 ± 0.003
Steerability	0.48 ± 0.07	0.4818 ± 0.0017
Bell Variable	0.1856 ± 0.0015	0.187 ± 0.004
Conditional Entropy	-0.16 ± 0.31	-0.154 ± 0.007
SSRE (magic)	0.38 ± 0.15	0.389 ± 0.003

Conclusion and outlook

- We can do quantum tomography in flavor space
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contributes to b_i, C_{ij}

⟹ Hadronization mechanism

No first-principle calculation of the entanglement after hadronization

Thank you!

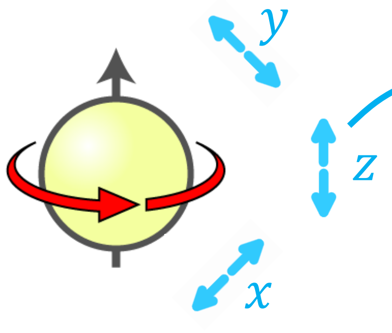
Introduction

Question:

Quantum tomography

Given a general meson flavor state, can we fully reconstruct the quantum state?

Compared to photon polarization:



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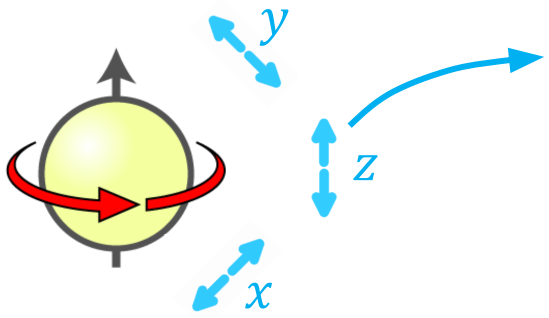

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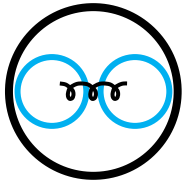

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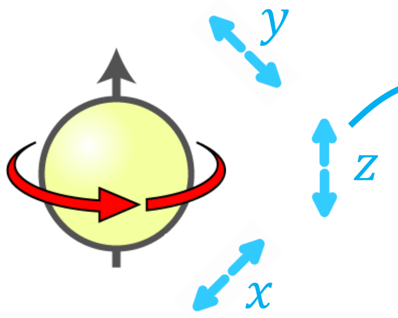

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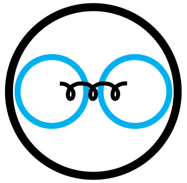

Flavor polarizer?

Solution:

Flavor oscillation + Flavor tagging decay = Flavor polarizer in any direction !

Time evolution

e^{-iHt}

Schrödinger picture: $\rho(t) = U(t)\,\rho(0)\,U^{\dagger}(t)$

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① Total number of mesons

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① Total number of mesons → $\begin{cases} b_x(0) = 1 & \Rightarrow |M_1\rangle \text{ initial state} \Rightarrow \text{decay as } \Gamma_1 \\ b_x(0) = -1 & \Rightarrow |M_2\rangle \text{ initial state} \Rightarrow \text{decay as } \Gamma_2 \end{cases}$

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$$= N_0\, e^{-\Gamma t}\big(b_z(0)\cos(\Delta m t) - b_y(0)\sin(\Delta m t)\big)$$

Time evolution

$$N = N_0\, e^{-\Gamma t}\left(\cosh\left(\tfrac{\Delta\Gamma t}{2}\right) - \boxed{b_x(0)}\sinh\left(\tfrac{\Delta\Gamma t}{2}\right)\right)$$

$$A = N_0\, e^{-\Gamma t}\left(\boxed{b_z(0)}\cos(\Delta m t) - \boxed{b_y(0)}\sin(\Delta m t)\right)$$

Flavors are tagged by semi-leptonic decay

$$M^0 \to \ell^+ \nu_\ell X, \qquad \bar{M}^0 \to \ell^- \bar{\nu}_\ell X$$

Fitting to the distributions gives $\vec{b}(0)$

$\Delta\Gamma$ provides sensitivity to $b_x(0)$

Δm provides sensitivity to $b_y(0)$, $b_z(0)$

	B_s^0	B^0	D^0	K^0
$\Delta m/\Gamma$	26.8	0.769	4.6×10^{-3}	0.95
$\Delta\Gamma/\Gamma$	0.135	4.0×10^{-3}	0.012	1.99