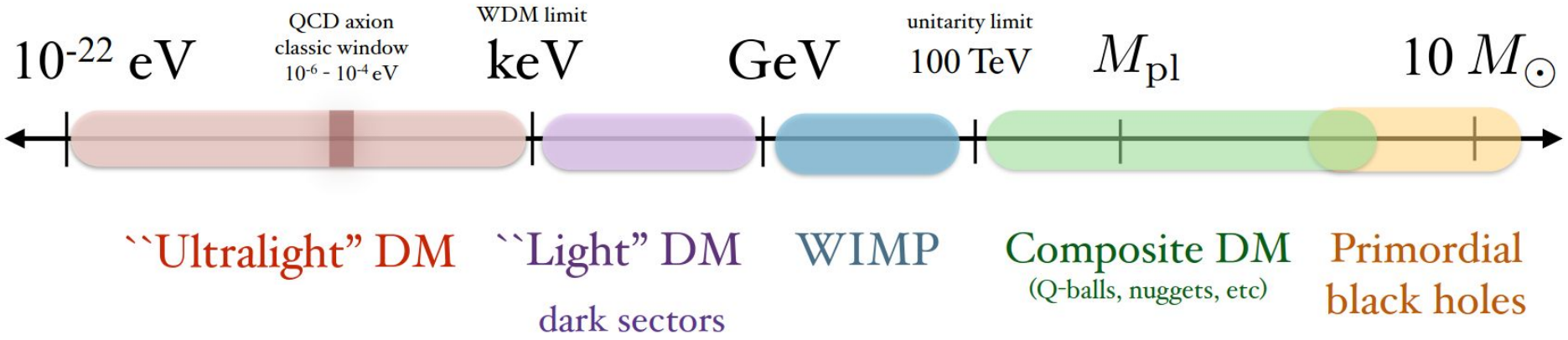
Dark matter-electron scattering with RPA dielectric screening

(and its implementation in QCDark2)

Megan Hott
Stony Brook University

Based on recent work:
Cyrus Dreyer, Rouven Essig, Marivi Fernandez-Serra, Megan Hott, Aman Singal,
All-electron dark matter-electron scattering with random-phase approximation dielectric screening and local field effects [2603.12326]

Our Focus



Tongyan Lin: [1904.07915]

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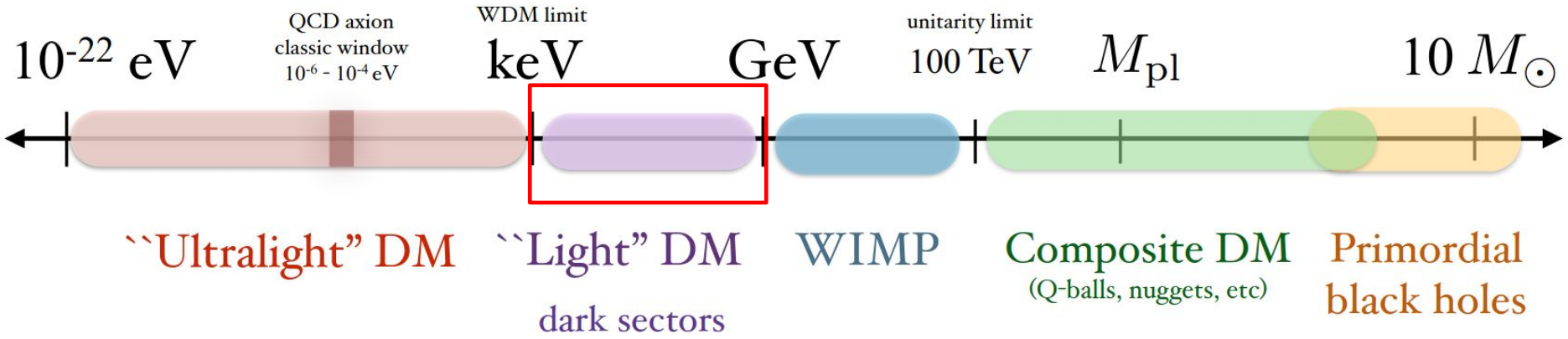

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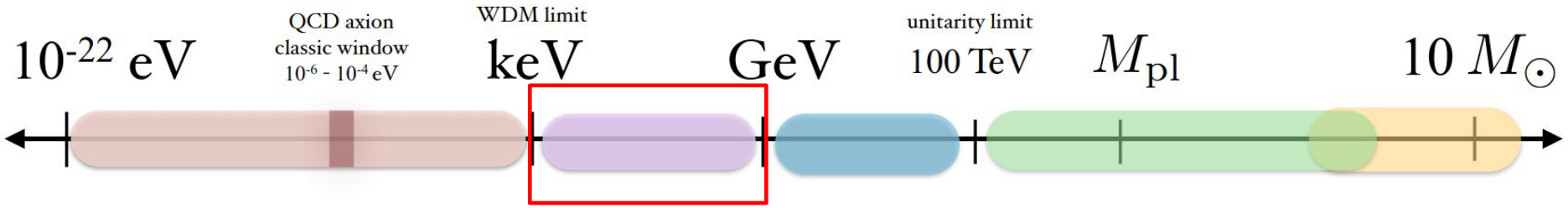
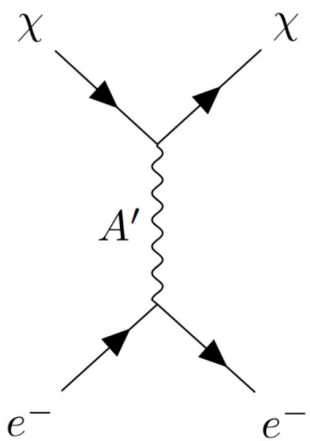


``Ultralight" DM ``Light" DM WIMP Composite DM (Q-balls, nuggets, etc) Primordial black holes

dark sectors



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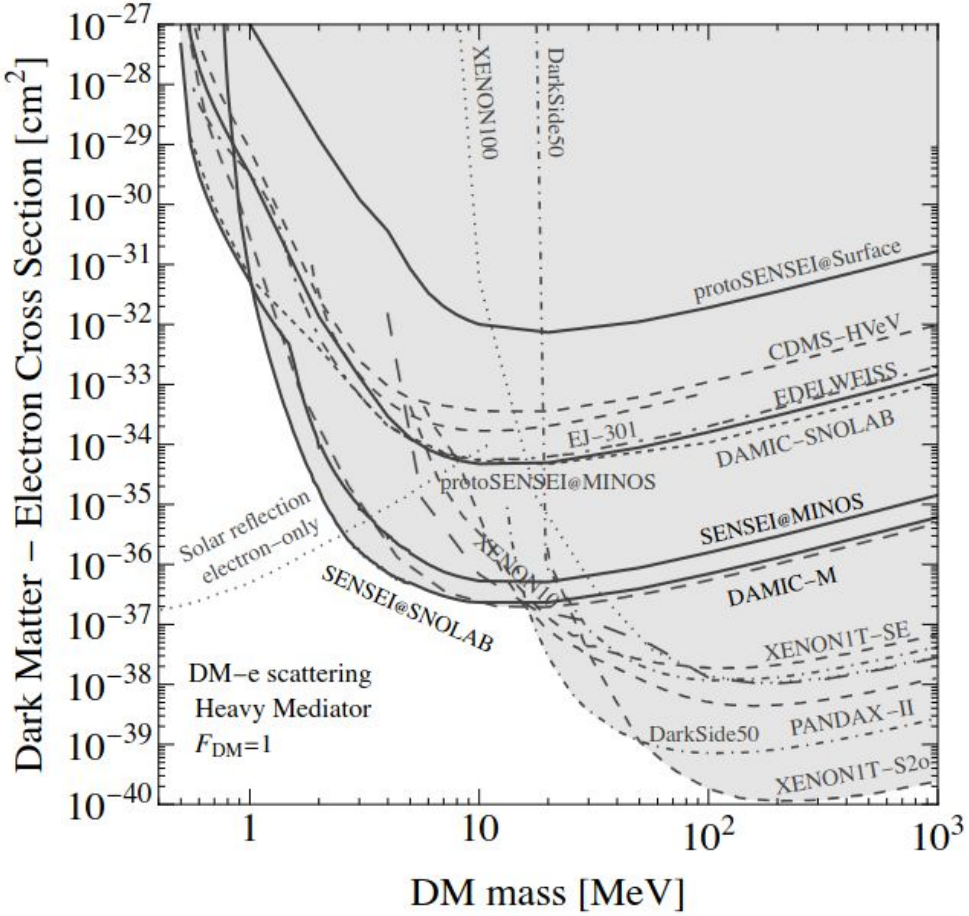
Dark Matter-Electron Direct Detection



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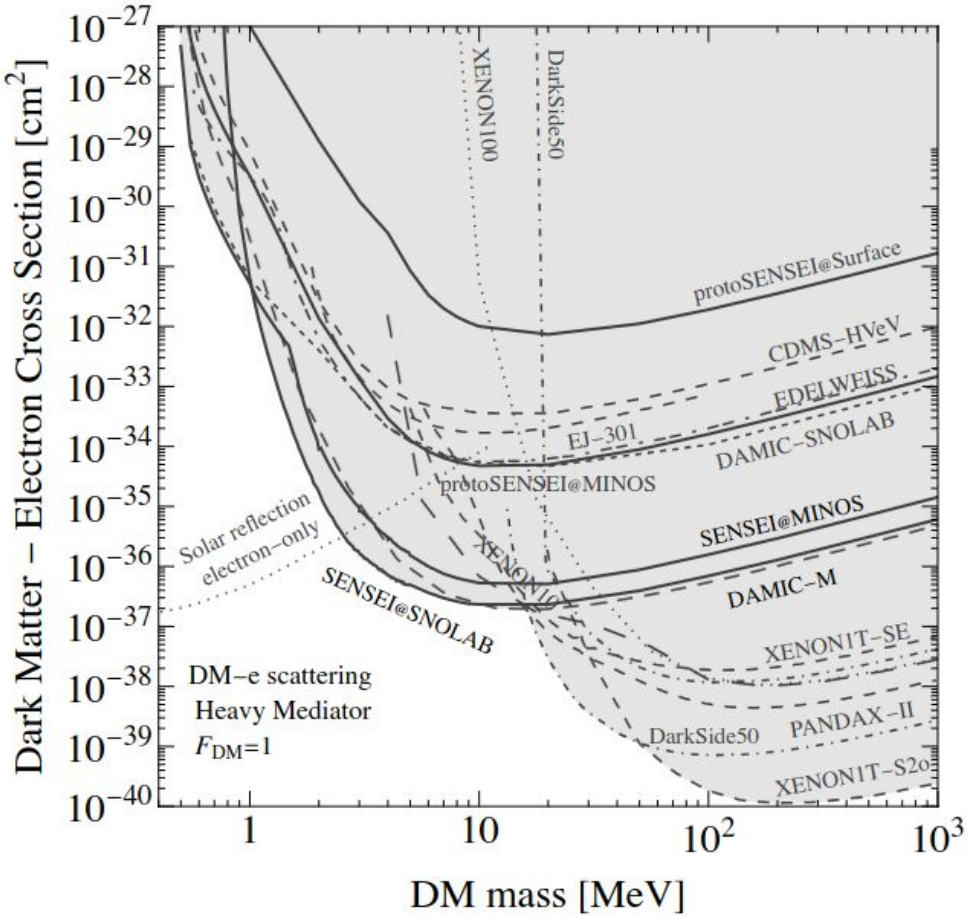

- There are now many direct detection experiments placing limits on DM-electron scattering

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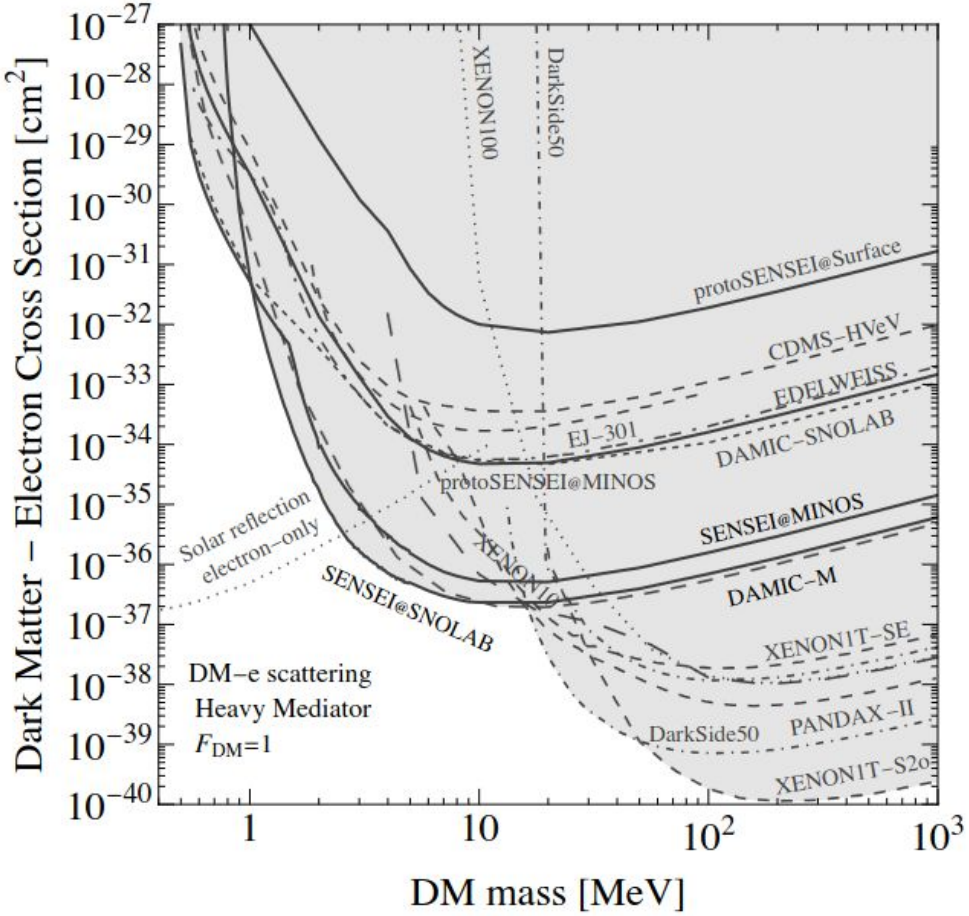

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- Accurate calculations of DM-electron scattering rates are needed to interpret data and optimize searches

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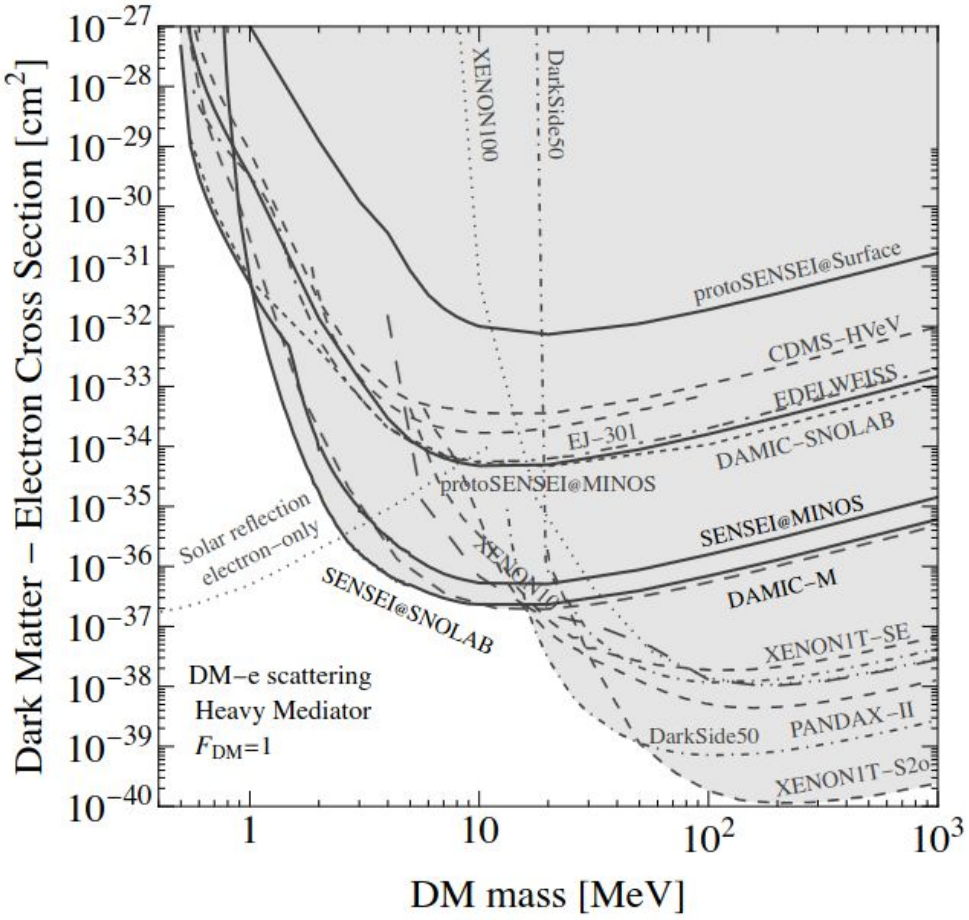
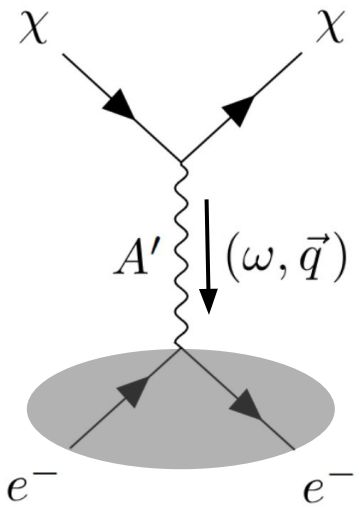


- There are now many direct detection experiments placing limits on DM-electron scattering
- Accurate calculations of DM-electron scattering rates are needed to interpret data and optimize searches
- QCDark2 computes the dielectric response of semiconductors which determines the scattering rate

Scattering Rate

$$\frac{dR}{d\omega} = \frac{1}{4\pi} \int \frac{dv}{v^2} \frac{d\Phi}{dv} \int_{q_{\min}}^{q_{\max}} dq \frac{\bar{\sigma}_e}{\mu_{\chi e}^2} q |F_{\mathrm{DM}}(q)|^2 \quad \frac{m_T}{\rho_T} S(\omega, q)$$



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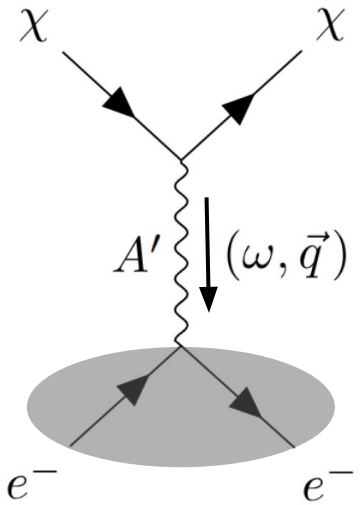
DM flux | DM model | Detector response



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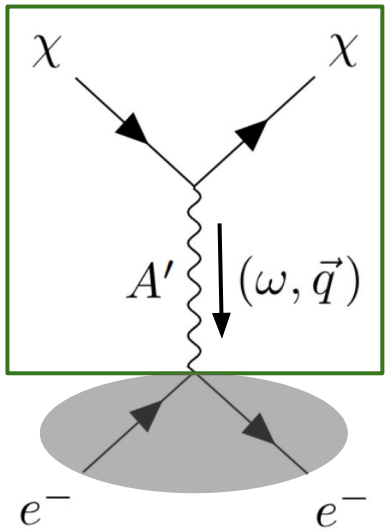

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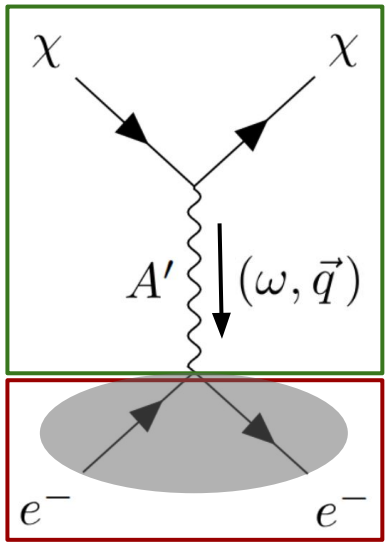

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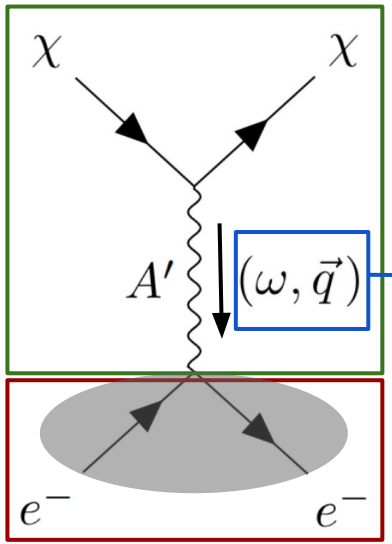

DM model : Simple

Flux: What is the relevant detector response?

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  - Computationally difficult
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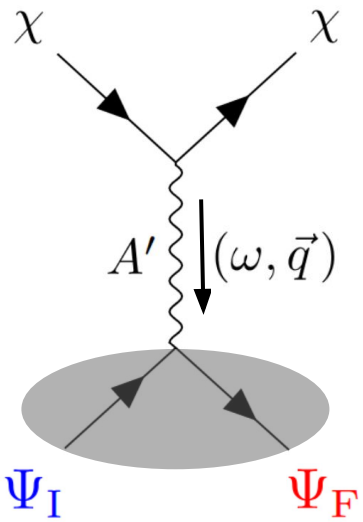

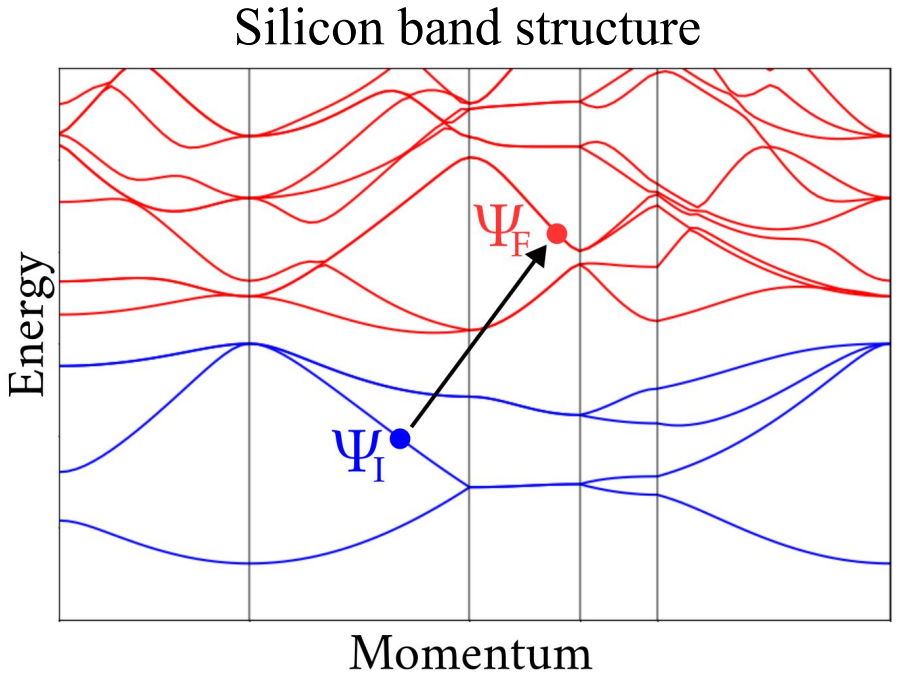

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We use the electronic states to calculate the dielectric function:

$$\epsilon^{\mathrm{RPA}}(\omega, \vec{q}) \sim \sum_{\mathrm{I,F}} \frac{\left| \langle \mathrm{F} | e^{i\vec{q}\cdot\vec{r}} | \mathrm{I} \rangle \right|^2}{\omega - (\omega_{\mathrm{F}} - \omega_{\mathrm{I}})}$$

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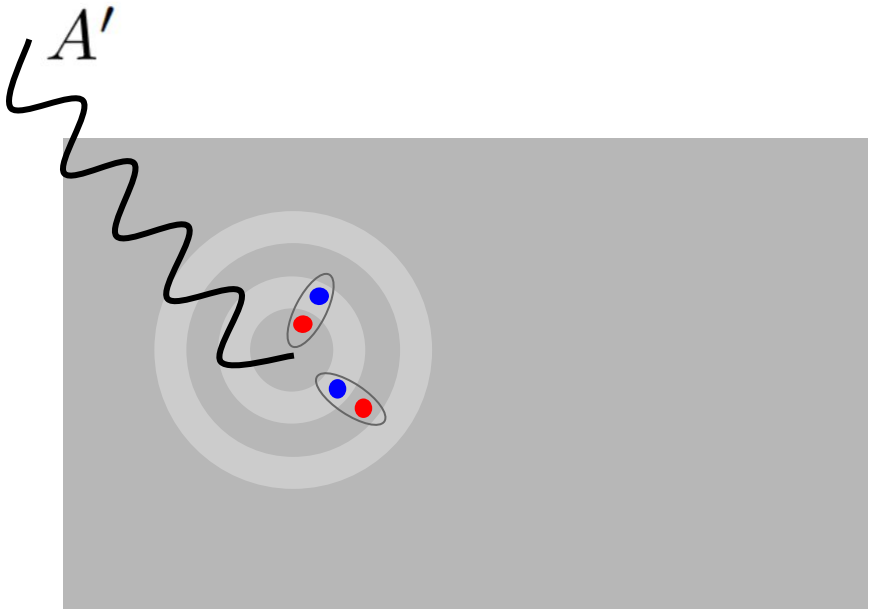

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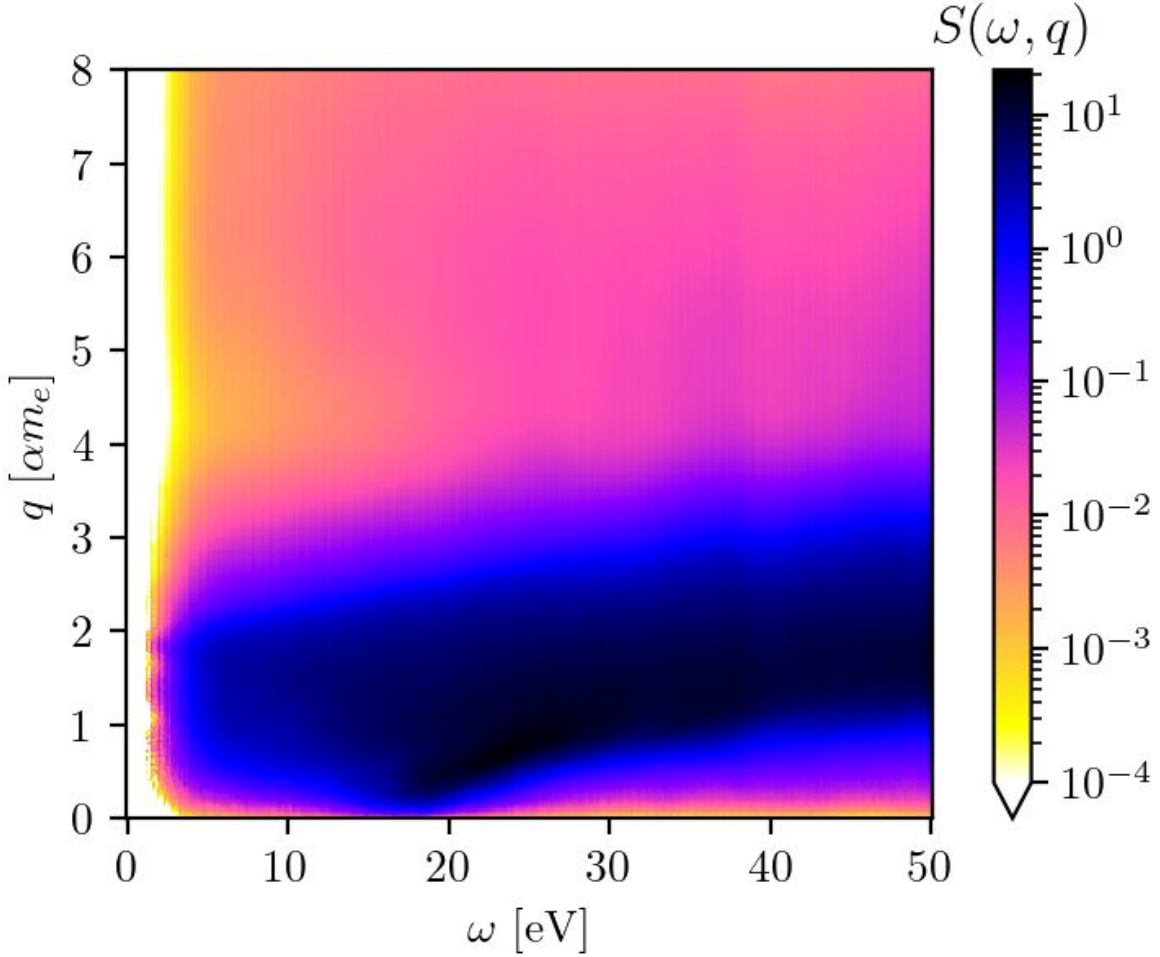

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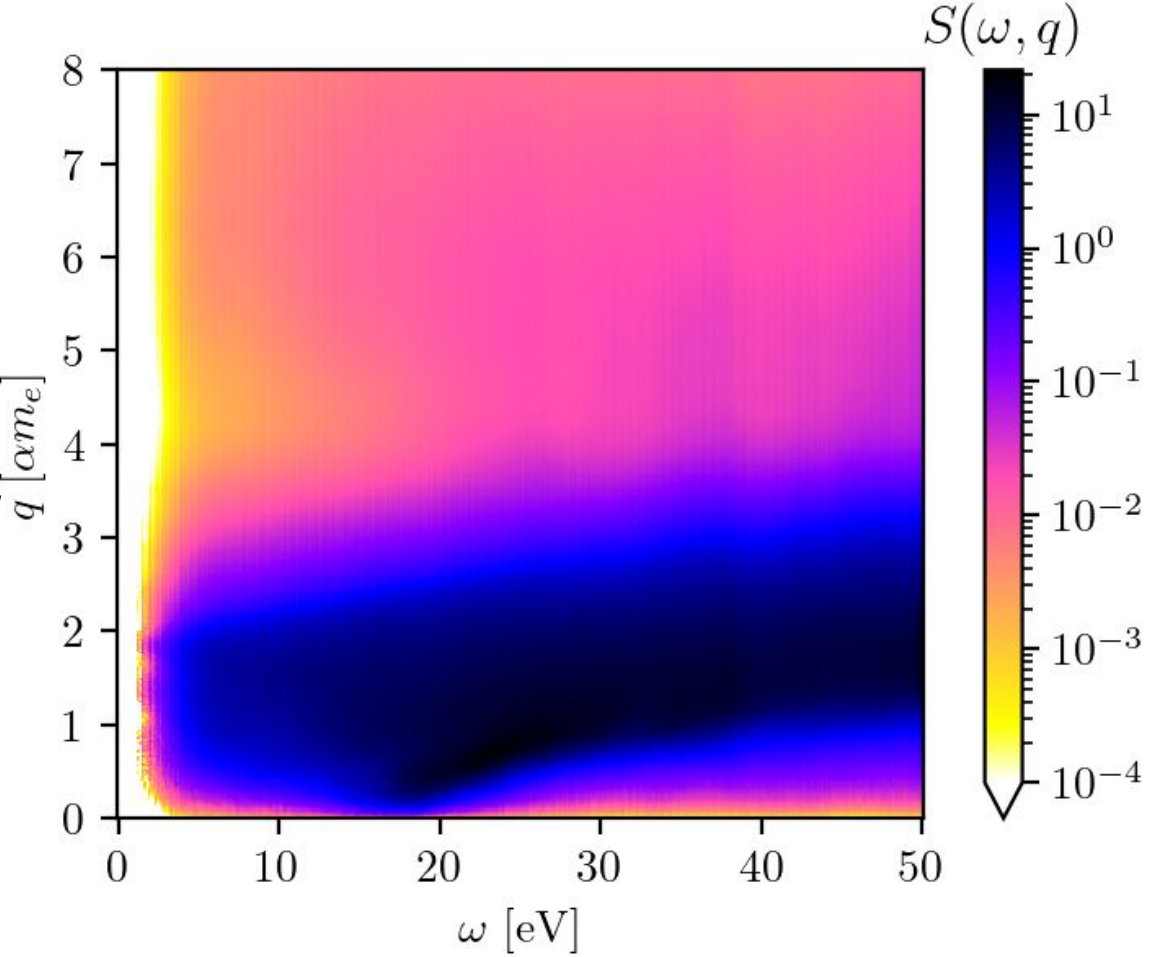
Momentum units

$\alpha m_e \sim 4$ keV



Local Field Effects

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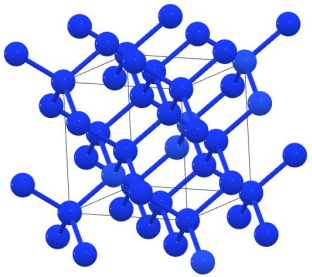
Macroscopic

Microscopic



Local Field Effects

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Large spatial scale

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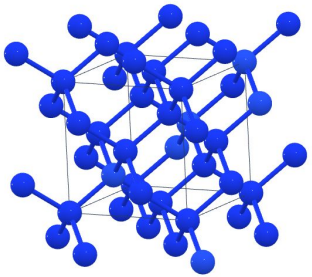

Small spatial scale

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Continuous
translational symmetry

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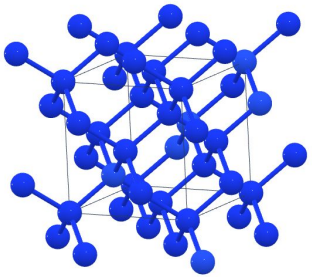

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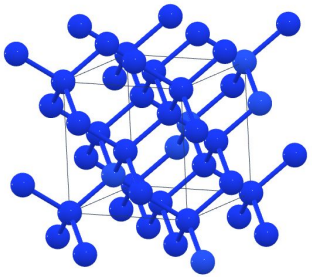

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	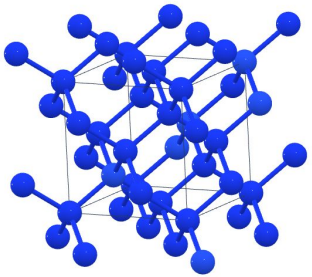
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	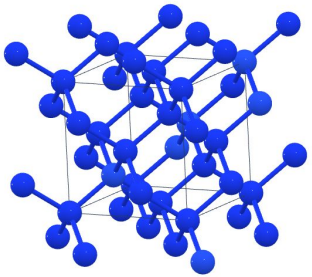
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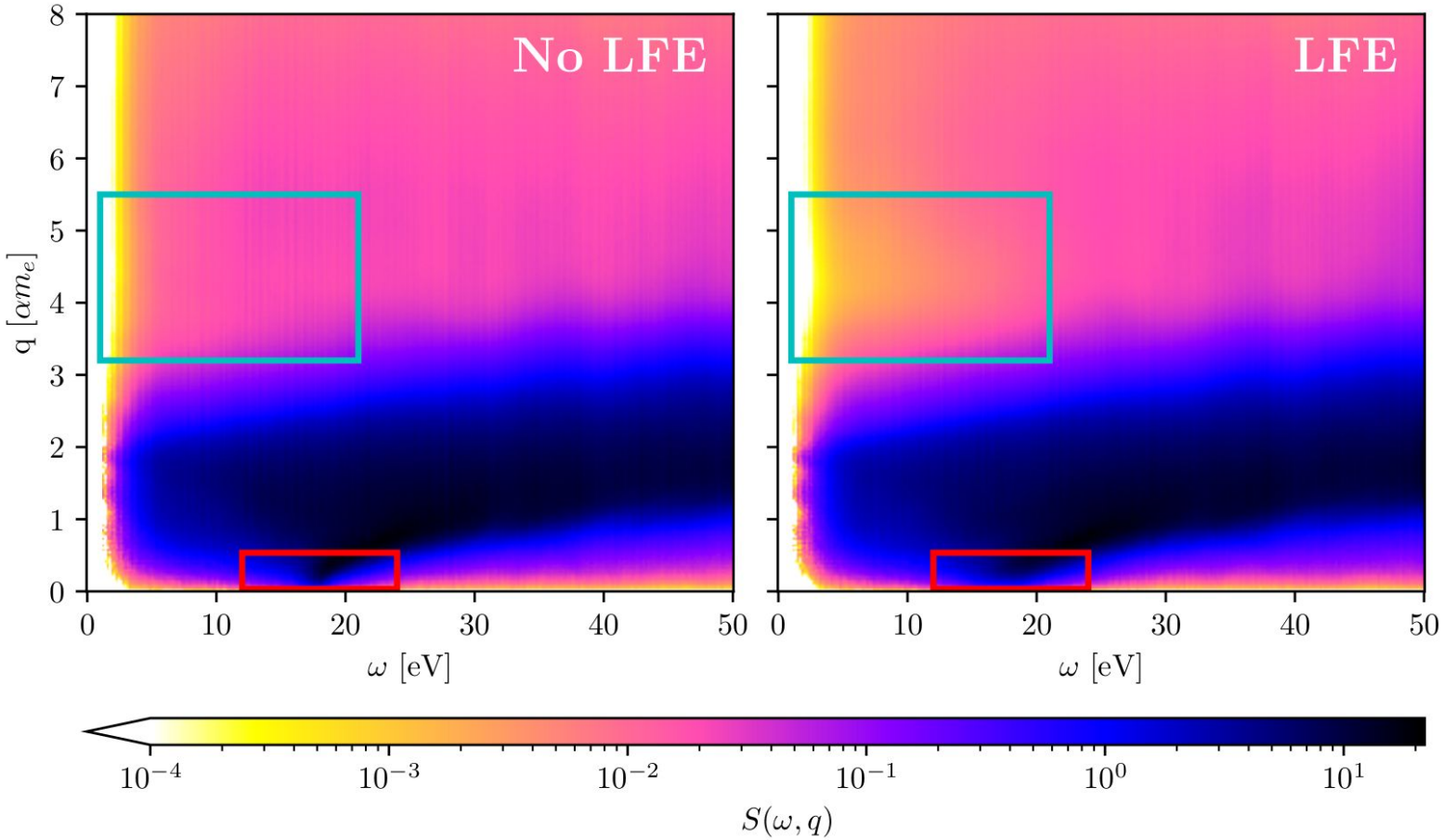
Taking the discrete translational symmetry into account changes the form of the dielectric function.

Local Field Effects



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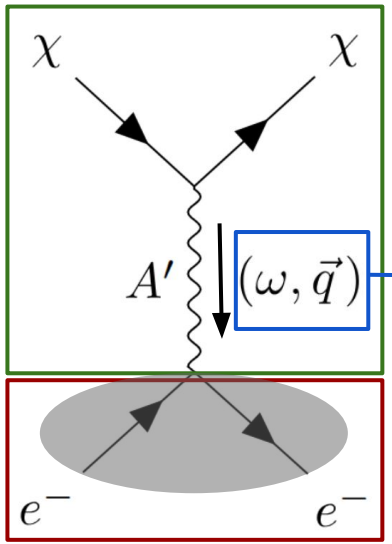

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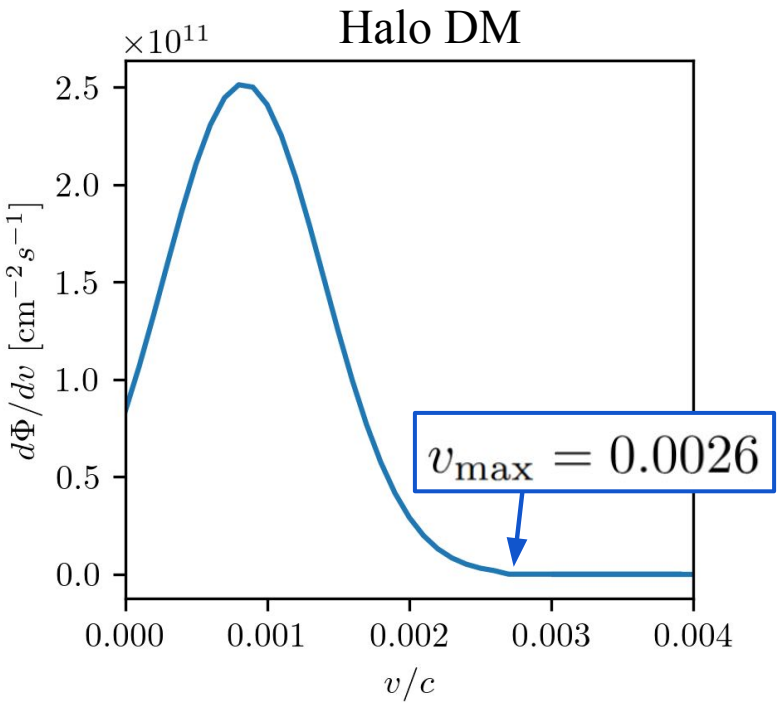

Dark Matter Flux

SRDM flux: T. Emken, R. Essig, H. Xu: [2404.10066]
SRDM theory: Emken et al. [1709.06573], An et al. [1708.03642], Emken [2102.12483], An et al. [2108.10332]

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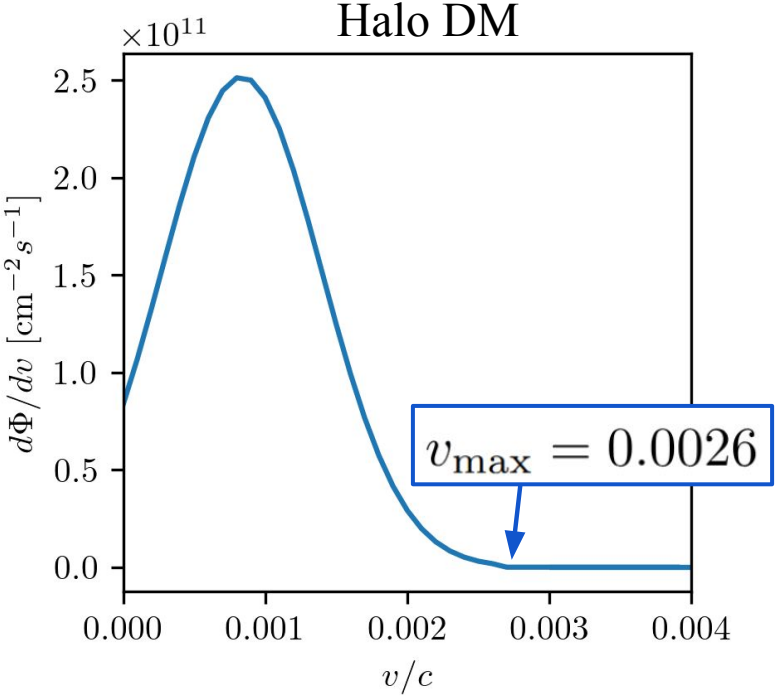
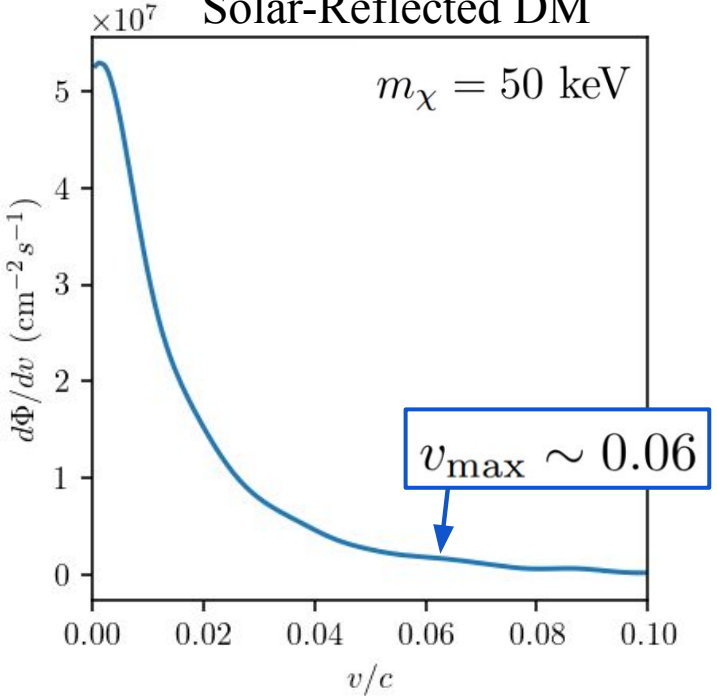
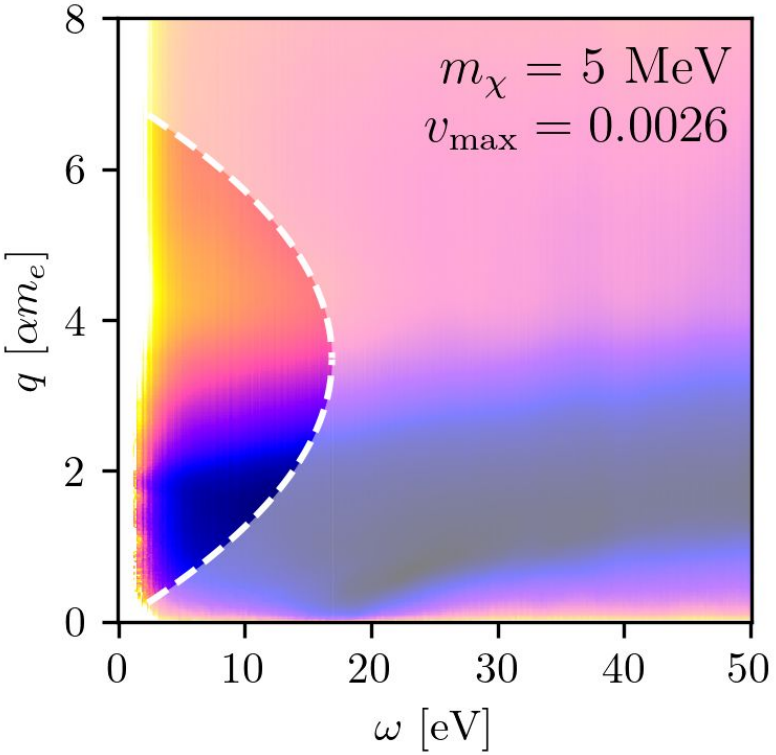
Halo DM



Solar-Reflected DM



Dark Matter Flux: Halo



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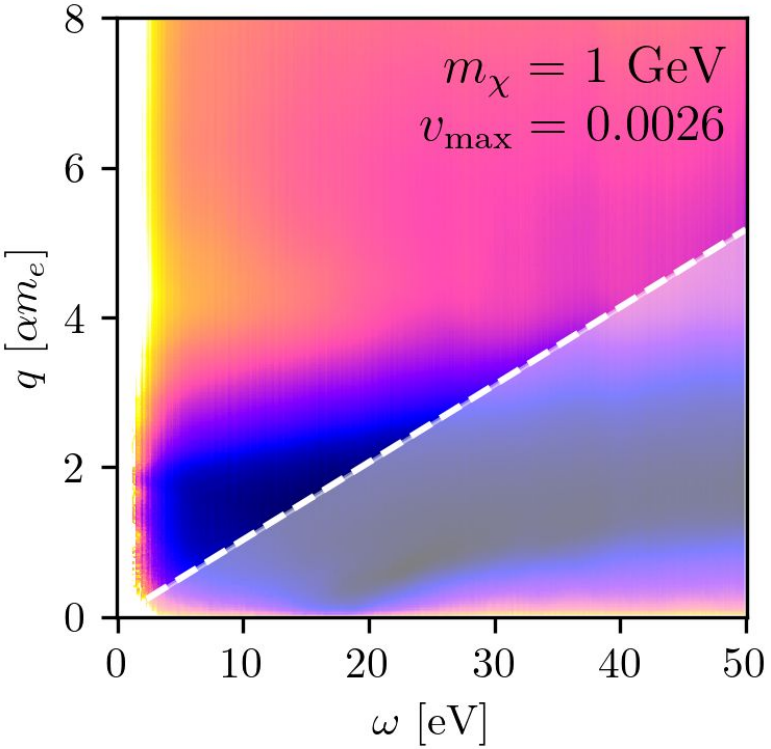

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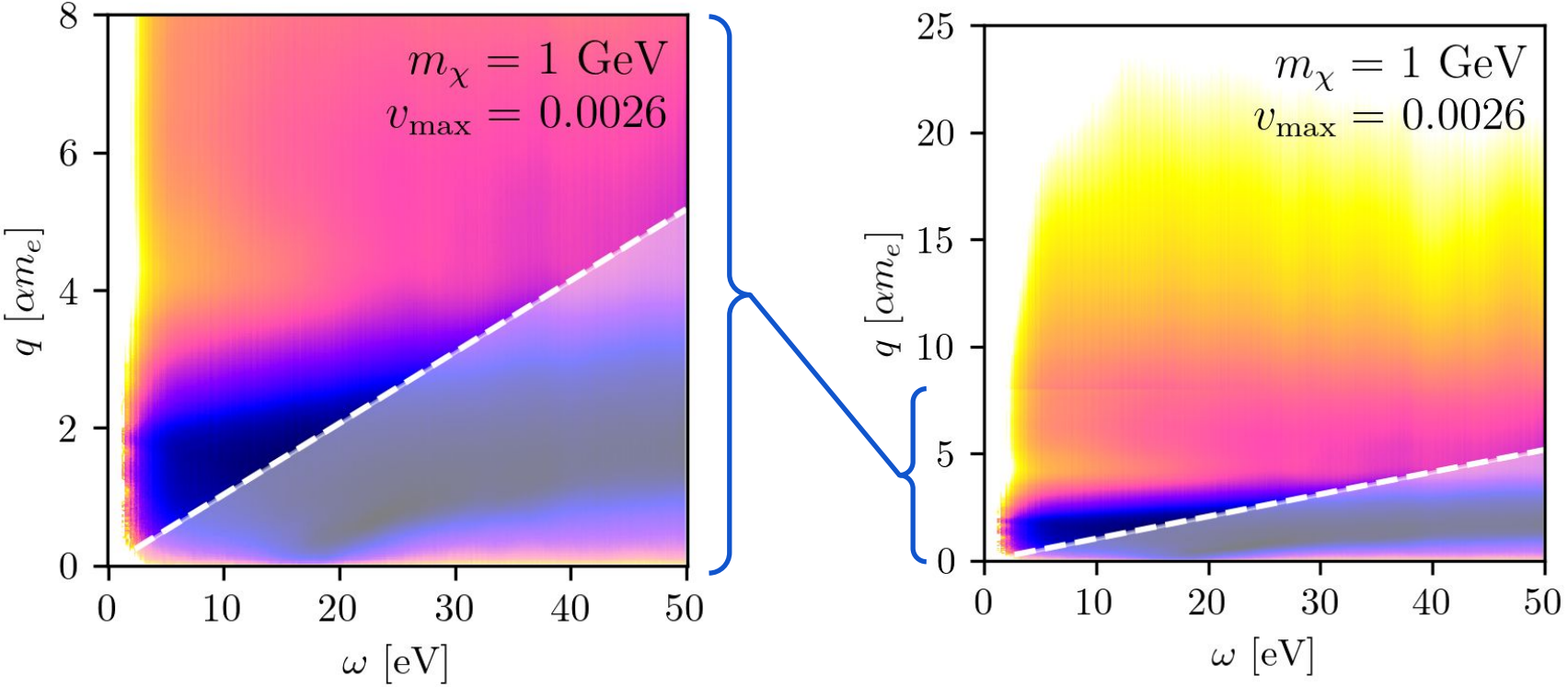

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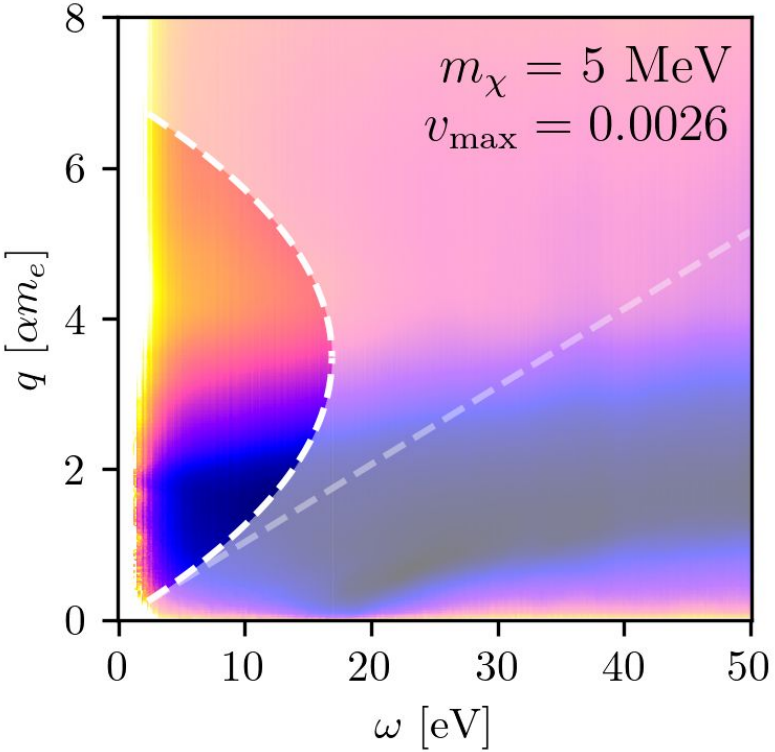

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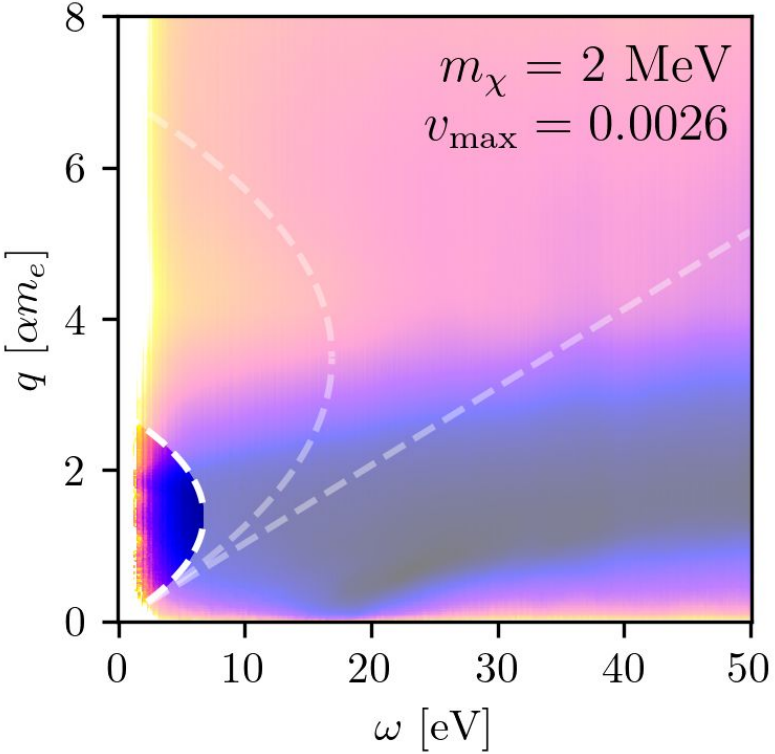

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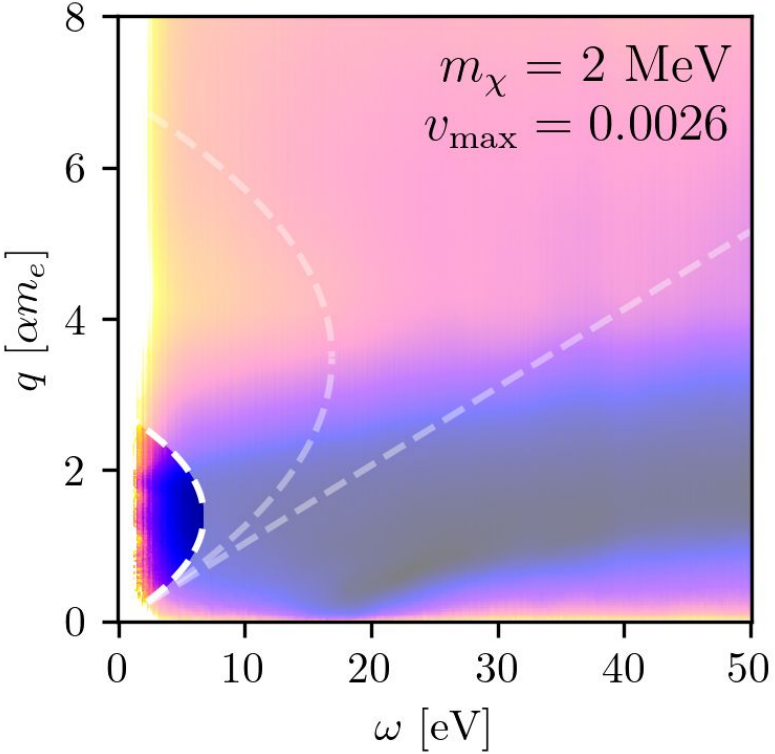
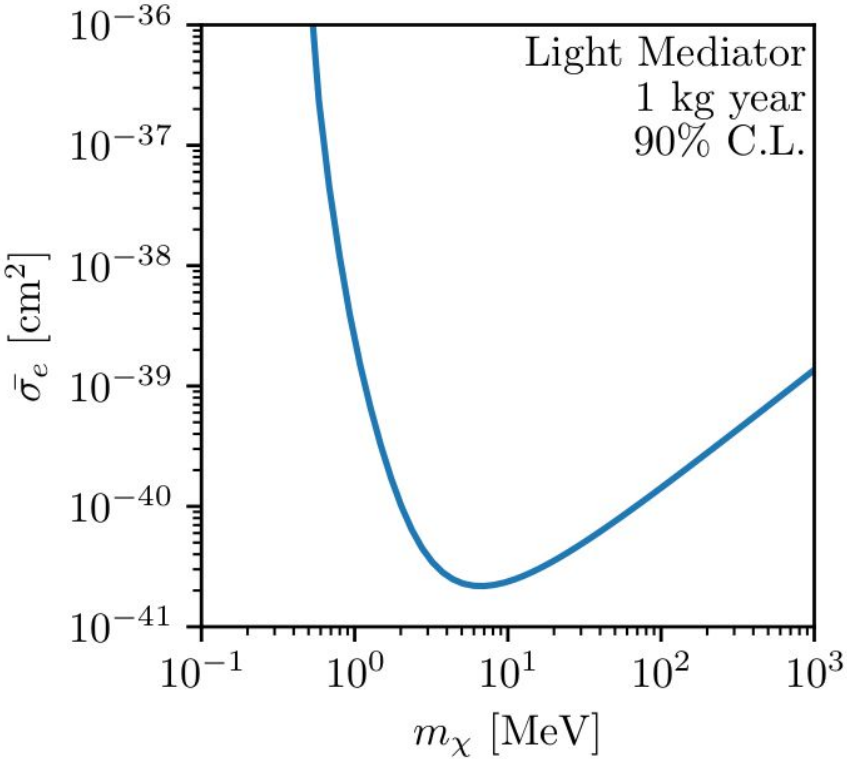
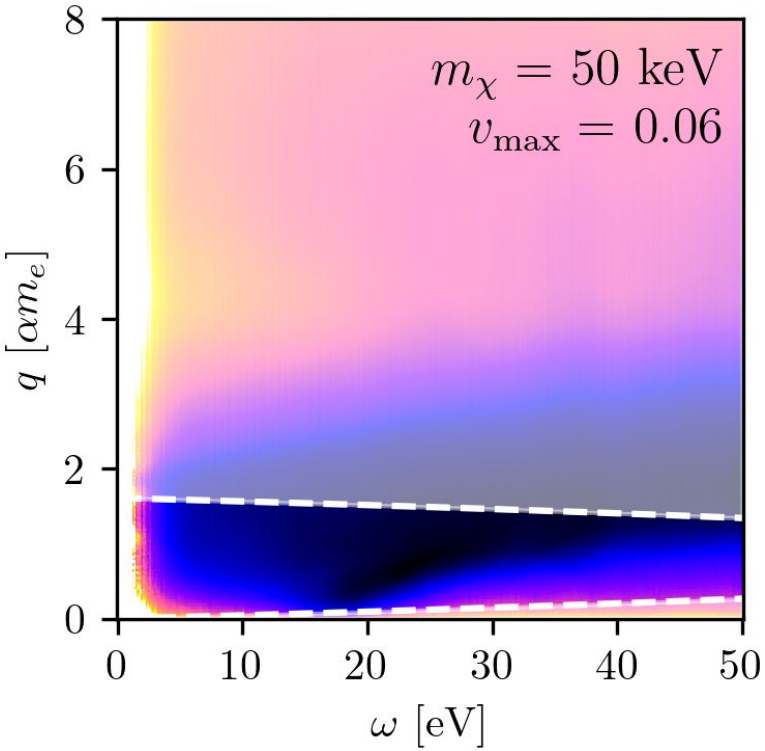
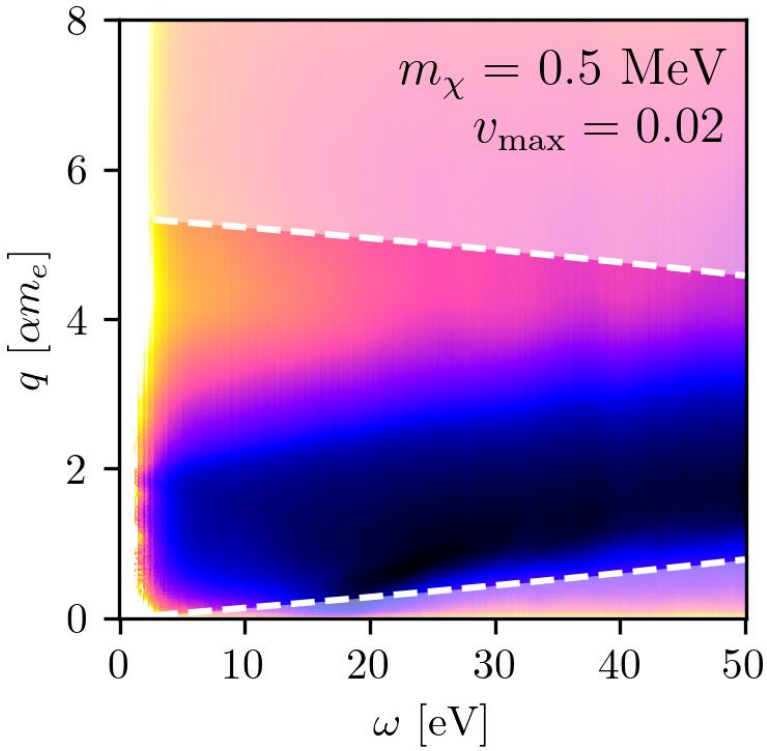


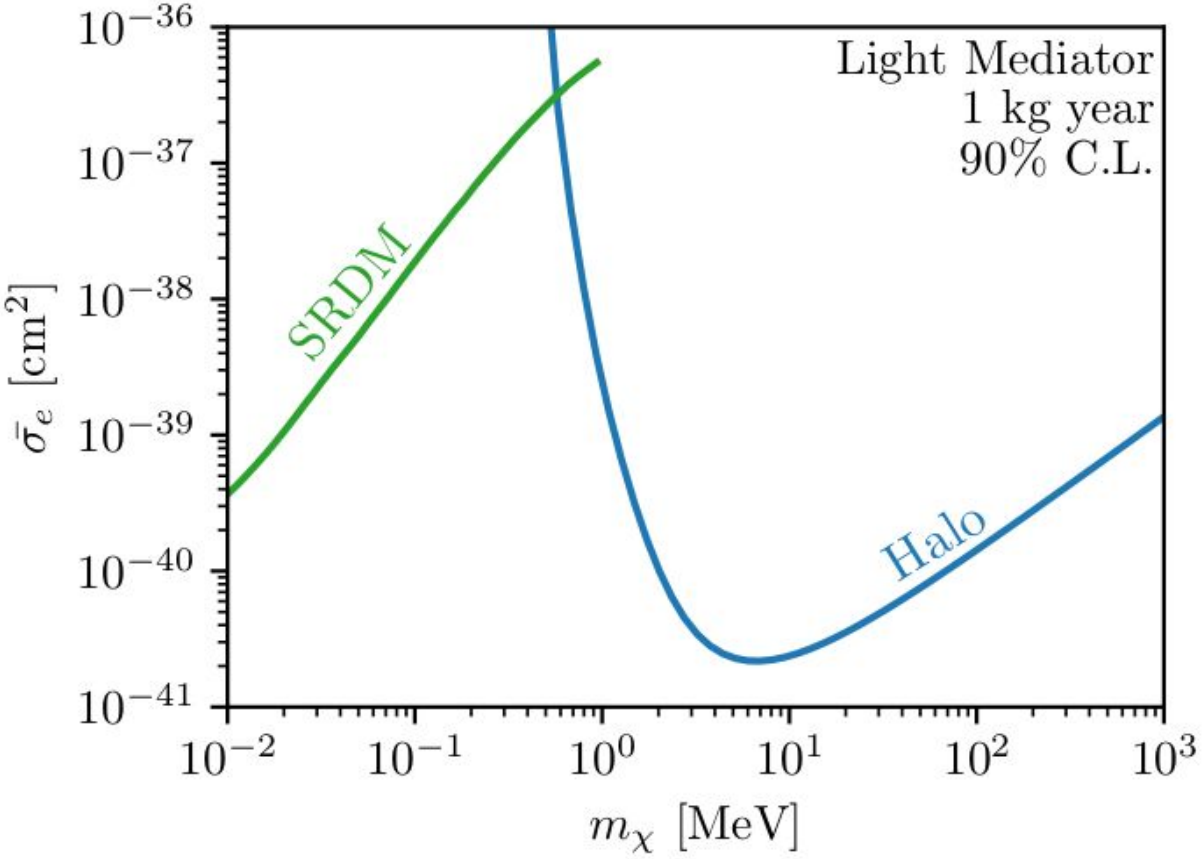


Dark Matter Flux: Solar-Reflected

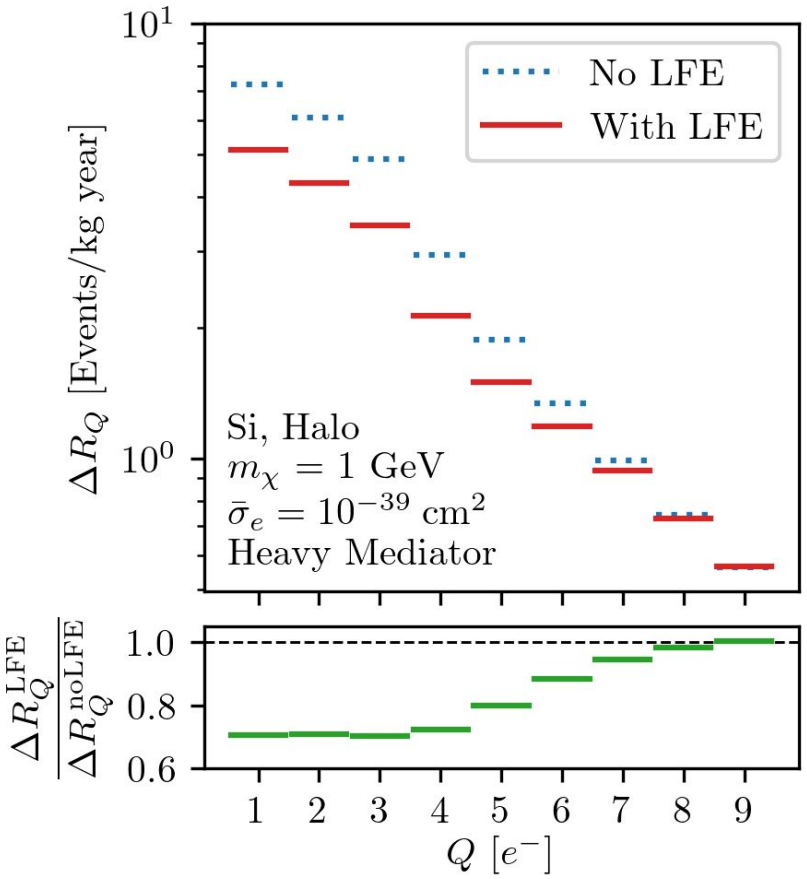


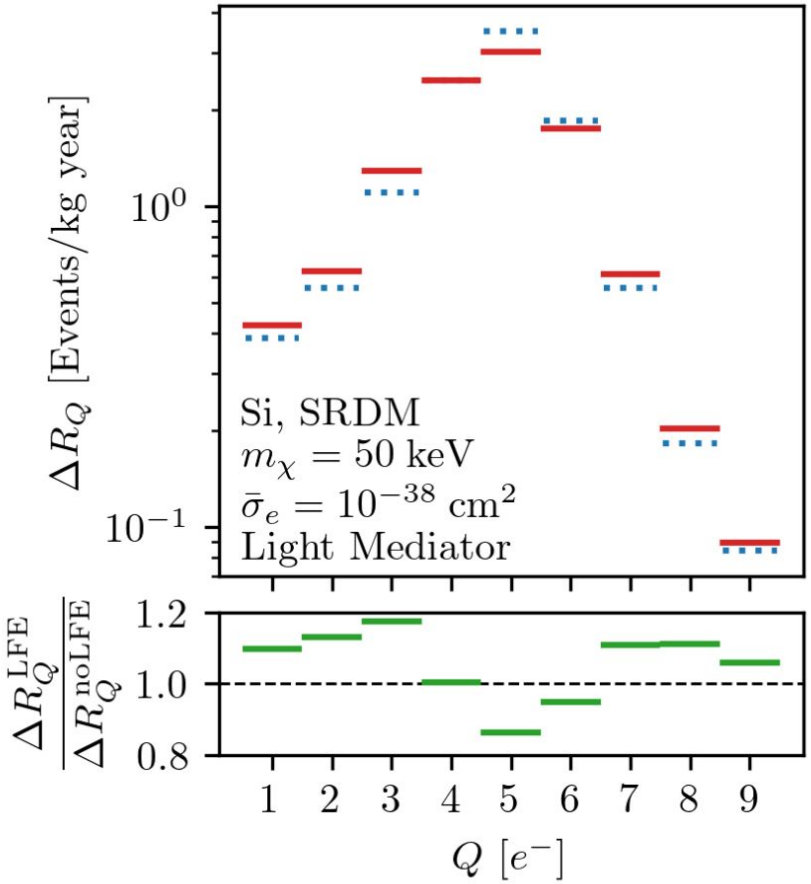


Dark Matter Flux: Solar-Reflected

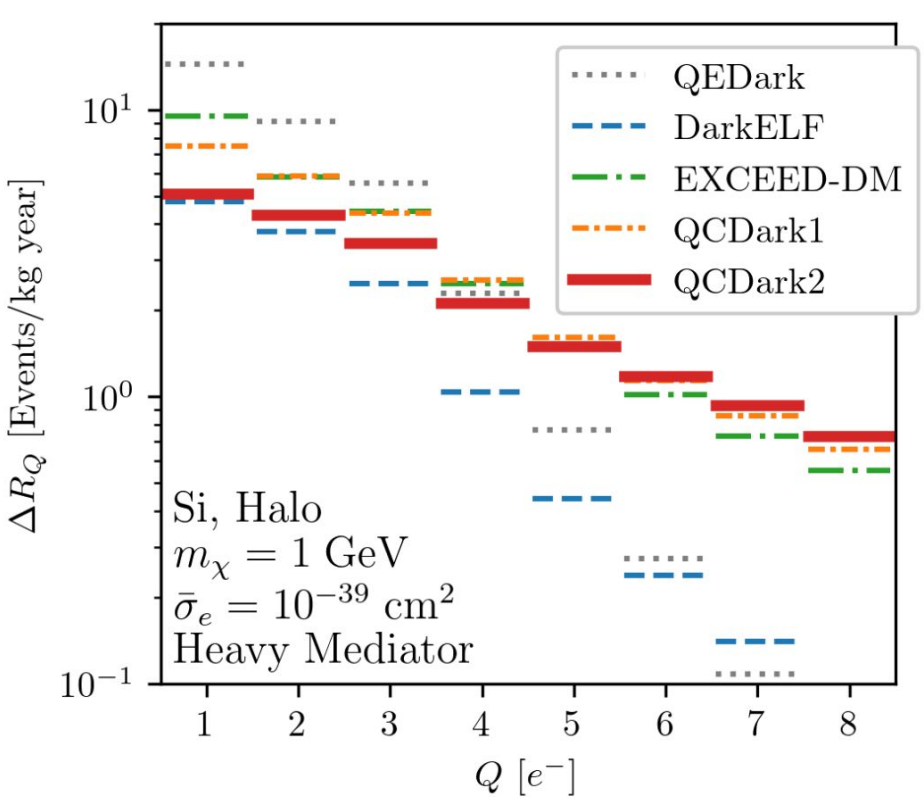


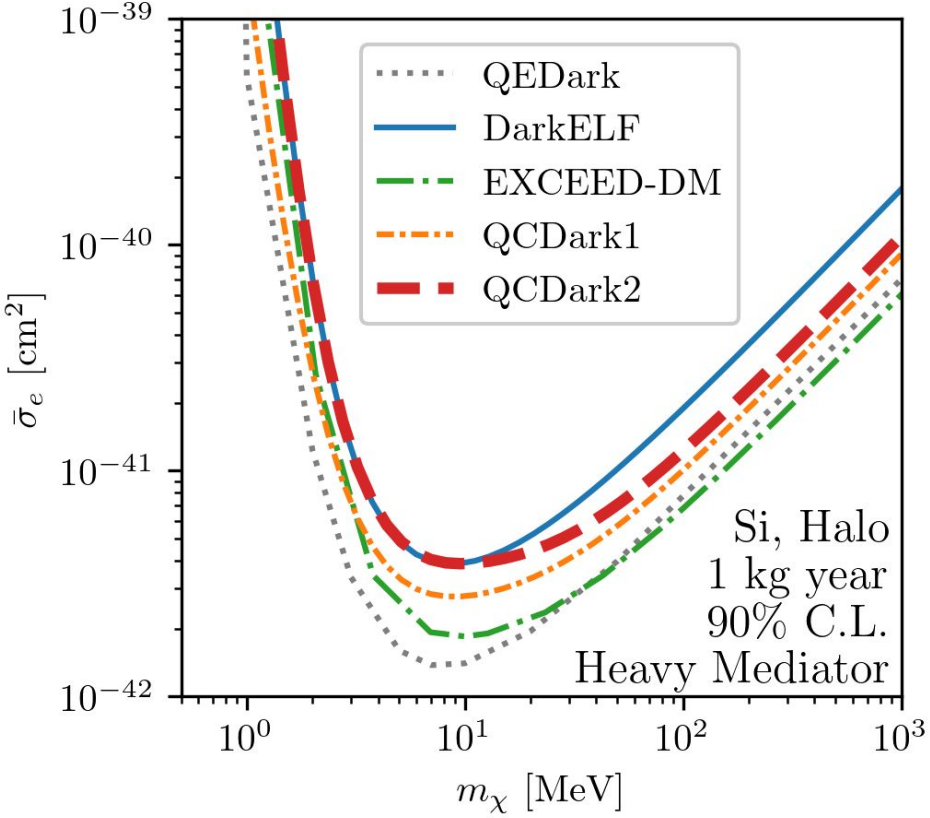
Electron Recoil Spectra





Comparing Codes for Halo DM





Conclusion

- Accurate calculations of DM-electron scattering rates are needed to interpret direct-detection data and place limits

Github: https://github.com/meganhott/QCDark2

Cyrus Dreyer, Rouven Essig, Marivi Fernandez-Serra, Megan Hott, Aman Singal, *All-electron dark matter-electron scattering with random-phase approximation dielectric screening and local field effects* [2603.12326]

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Thank you!

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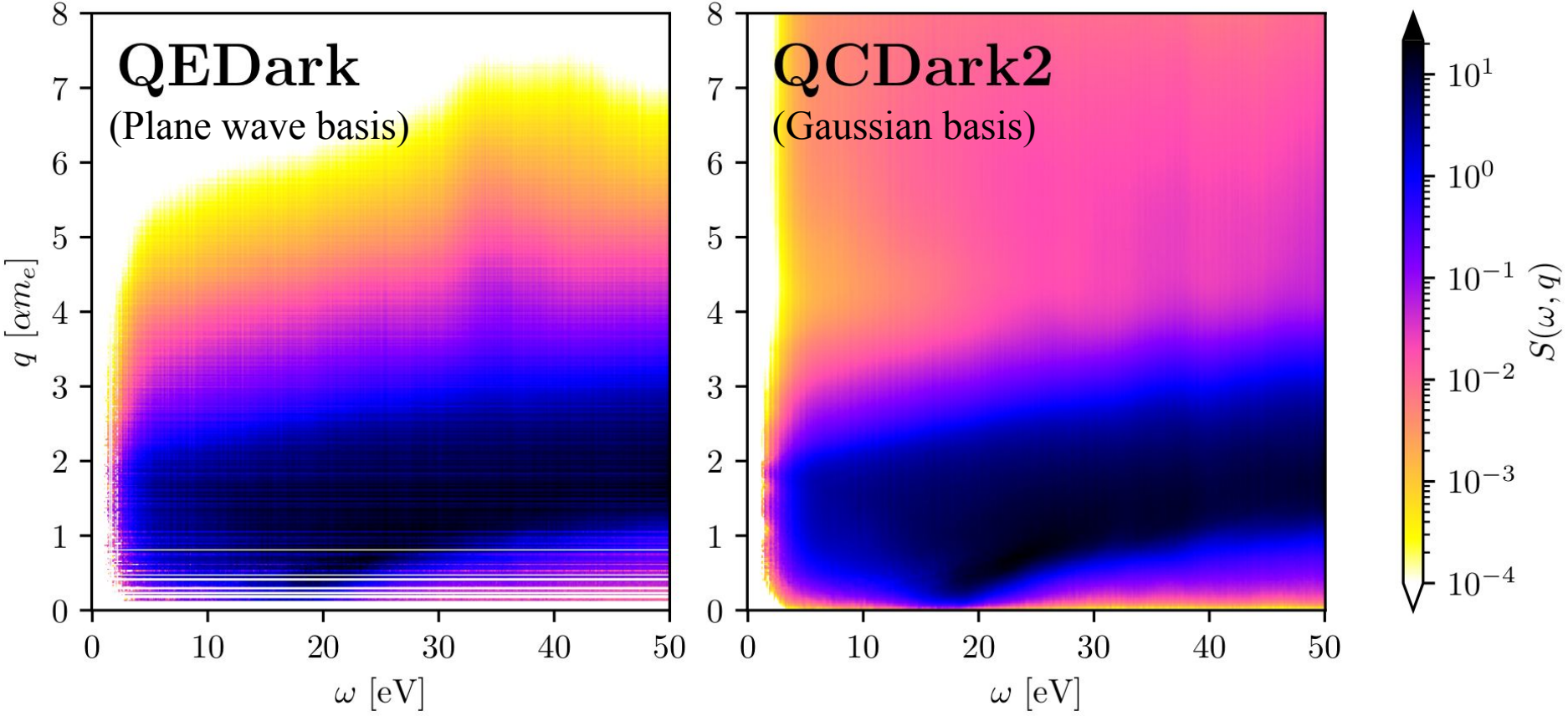
Extras

High momentum modes: Silicon



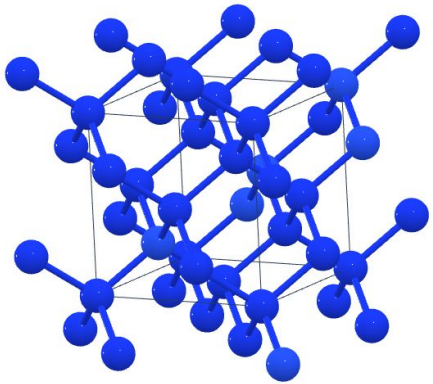
Dielectric Function

$$\epsilon^{\mathrm{RPA}}(\vec{q},\omega) = 1 - \frac{4\pi\alpha}{Vq^2}\sum_{i\vec{k}}\sum_{j\vec{k'}}\lim_{\eta\to 0}\frac{\left\langle i\vec{k}\middle|e^{-i\vec{q}\cdot\vec{r}}\middle|j\vec{k'}\right\rangle\left\langle j\vec{k'}\middle|e^{i\vec{q}\cdot\vec{r}}\middle|i\vec{k}\right\rangle}{\omega-(\omega_{j\vec{k'}}-\omega_{i\vec{k}})+i\eta}$$

$$\epsilon^{\mathrm{RPA}}_{\vec{G}\vec{G'}}(\vec{q},\omega) = \delta_{\vec{G}\vec{G'}} - \frac{4\pi\alpha}{V|\vec{q}+\vec{G}||\vec{q}+\vec{G'}|} \times \sum_{i\vec{k}}\sum_{j\vec{k'}}\lim_{\eta\to 0}\frac{\left\langle i\vec{k}\middle|e^{-i(\vec{q}+\vec{G})\cdot\vec{r}}\middle|j\vec{k'}\right\rangle\left\langle j\vec{k'}\middle|e^{i(\vec{q}+\vec{G'})\cdot\vec{r}}\middle|i\vec{k}\right\rangle}{\omega-(\omega_{j\vec{k'}}-\omega_{i\vec{k}})+i\eta}$$

The Microscopic Response & Local Field Effects

$$\epsilon^{\mathrm{RPA}}_{\vec{G}\vec{G}'}(\vec{q},\omega) = \delta_{\vec{G}\vec{G}'} - \frac{4\pi\alpha}{V|\vec{q}+\vec{G}||\vec{q}+\vec{G}'|} \times \sum_{i\vec{k}}\sum_{j\vec{k}'} \lim_{\eta\to 0} \frac{\left\langle i\vec{k}\left| e^{-i(\vec{q}+\vec{G})\cdot\vec{r}} \right| j\vec{k}' \right\rangle \left\langle j\vec{k}'\left| e^{i(\vec{q}+\vec{G}')\cdot\vec{r}} \right| i\vec{k} \right\rangle}{\omega - (\omega_{j\vec{k}'} - \omega_{i\vec{k}}) + i\eta}$$



Microscopic scale

$$\epsilon^{\mathrm{RPA}}_{\vec{G}\vec{G}'}(\vec{q},\omega) = \underbrace{\begin{pmatrix} \epsilon_{\vec{G}_1\vec{G}_1} & \epsilon_{\vec{G}_1\vec{G}_2} & \epsilon_{\vec{G}_1\vec{G}_3} \\ \epsilon_{\vec{G}_2\vec{G}_1} & \epsilon_{\vec{G}_2\vec{G}_2} & \epsilon_{\vec{G}_2\vec{G}_3} \\ \epsilon_{\vec{G}_3\vec{G}_1} & \epsilon_{\vec{G}_3\vec{G}_2} & \epsilon_{\vec{G}_3\vec{G}_3} \end{pmatrix}}$$

Number of $\vec{G}$ scales like $\sim \frac{4}{3}\pi q^3_{\mathrm{max}}$

DM-Electron Scattering Codes

QEDark: 2015 [1509.01598]
R. Essig, M. Fernandez-Serra, J. Mardon, A. Soto, T. Volansky, T. Yu
- DFT with plane wave basis

QCDark1: 2023 [2306.14944]
C. Dreyer, R. Essig, M. Fernandez-Serra, A. Singal, C. Zhen
- DFT with localized Gaussian basis
- Uses approximation for screening

★ **QCDark2: 2026** [2603.12326]
C. Dreyer, R. Essig, M. Fernandez-Serra, M. Hott, A. Singal
- Full RPA dielectric function to use for screening
- Better resolution and binning, q → 0 limit
- Faster and optimized
- Dielectric functions available for Si, Ge, GaAs, SiC, diamond
- Able to include crystal local field effects (LFEs)

DarkELF: 2021 [2101.08275]
S. Knapen, J. Kozaczuk, T. Lin
- Dielectric function formalism
  (See also Hochberg, Khan, Kurinsky, Lehmann, Yu, Berggren: [2101.08263])
- Importance of screening!
- GPAW dielectric functions (LFEs)

EXCEED-DM: 2022 [2105.05253]
S. Griffin, K. Inzani, T. Trickle, Z. Zhang, K. Zurek
- High-momentum response
- Localized & PW basis
- ab initio screening