Does NANOGrav favor massive gravity?

PTA angular correlations as a test of massive-gravity ORFs

Chris Choi

Based on
Suppression of extra polarizations in massive-gravity overlap reduction functions near the massless limit
(arXiv:2605.XXXX [gr-qc])

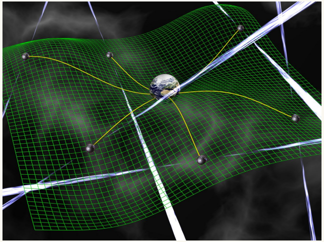
Does NANOGrav favor massive gravity?

(arXiv:2507.02059v2 [astro-ph.CO])

PTAs and massive gravity

Pulsar timing arrays

galactic millisecond pulsars – stable clocks



enables us to find a GW background in the cross-correlations

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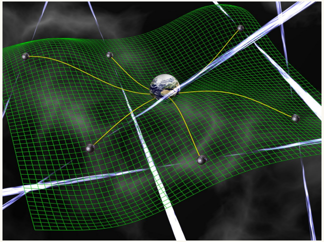

enables us to find a GW background in the cross-correlations

Why massive gravity

Massive gravity: graviton mass $m_g \neq 0$

Dark energy
self-acceleration without adding Λ.

Dark matter
extra spin-2 sectors can behave as ULDM.

PTAs turn this into a low-frequency test of gravity.

Sources: Agazie et al. 2023; Blas et al. 2024; D'Amico et al. 2011; Dvali 2013.

What massive gravity changes

Yukawa supp.

$$\Phi \propto e^{-m_g R}/R$$

finite range

Dispersion

$$\omega^2 = |\boldsymbol{k}|^2 + m_g^2$$

group vel.

$$A = \sqrt{1 - m_g^2/\omega^2}$$

Polarizations

$$\{+2, -2, +1, -1, 0\}$$

tensor, vector, scalar

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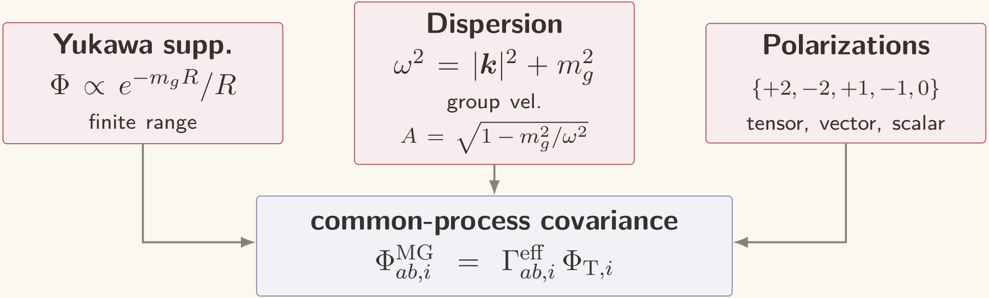

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tensor, vector, scalar

common-process covariance

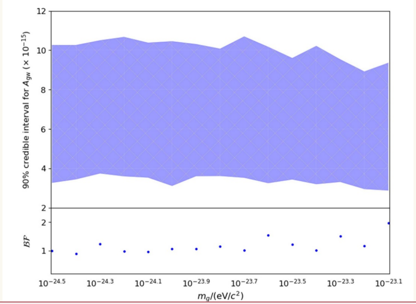
$$\Phi_{ab,i}^{\mathrm{MG}} = \Gamma_{ab,i}^{\mathrm{eff}} \, \Phi_{\mathrm{T},i}$$

effective ORF

$$\Gamma_{ab,i}^{\mathrm{eff}} = \beta_i \left[\Gamma_{ab,i}^{T} + \Gamma_{ab,i}^{V} \frac{\Omega_{V,i}}{\Omega_{T,i}} + \Gamma_{ab,i}^{S} \frac{\Omega_{S,i}}{\Omega_{T,i}} \right]$$

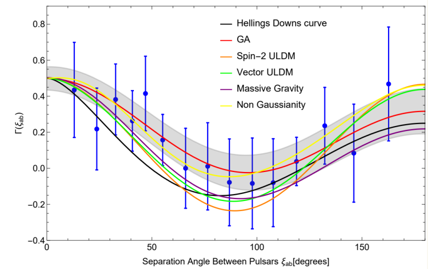
Effective ORF follows Liang and Trodden 2021.

Motivation



Wu et al. 2024

Didn't include vector/scalar polarizations



Arjona et al. 2024

Keeps $\Omega_{V/S}/\Omega_T$ f-indep, doesn't properly suppress in $m_g \to 1$ limit

We contribute: Proper suppression of vector/scalar modes so GR limit is recovered

Arjona et al. 2024; Wu et al. 2024

Finite distance

$$\Gamma_{ab,I}(f_i) = \beta_I \, \Re \int d^2\hat{\Omega} \, \mathcal{N}_I(\hat{\Omega}; A_i) \, \frac{e^{i2\pi f_i L_a x_a} - 1}{x_a} \, \frac{e^{-i2\pi f_i L_b x_b} - 1}{x_b}$$

The exponentials are the **pulsar terms;** the "-1" pieces are the Earth terms.

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Why not take $L \to \infty$?
ORFs must be normalized
w/ finite pulsar distances

closest pulsar	J1630+3734
distance	$L \simeq 290 \pm 78$ ly
lowest bin	$f_{\min} \simeq 2$ nHz
smallest fL	$\simeq 18.1$

Finite-distance phases can survive normalization and reshape the small-angle ORF.

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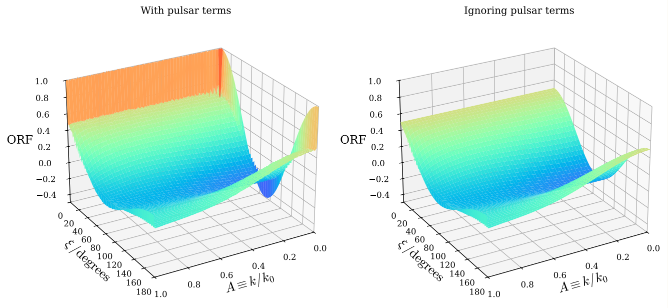

Recover GR w/ suppression

Near the massless limit

$$\Gamma_T \sim \mathcal{O}(1),$$

$$\Gamma_V \sim \left(\frac{m_g}{\omega}\right)^{-2},$$

$$\Gamma_S \sim \left(\frac{m_g}{\omega}\right)^{-4}.$$

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We need additional suppression!

$$\frac{\Omega_V}{\Omega_T} = \left(\frac{m_g}{\omega}\right)^{2+n_V},$$
$$\frac{\Omega_S}{\Omega_T} = \frac{3}{4}\left(\frac{m_g}{\omega}\right)^{4+n_S}.$$

We fix $n_V = 1$ and scan $n_S \in \{2, 4, 6, 8\}$.

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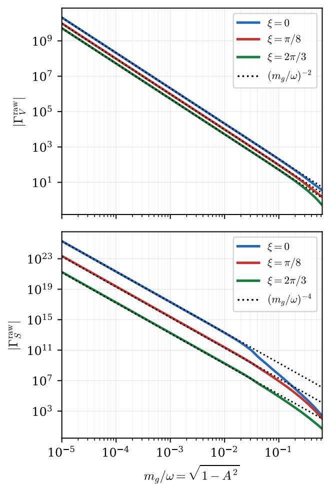

Bayesian methodology

Bayesian model selection

NANOGrav 15-year data
residuals + noise model

↓

Product-space hypermodel
sample between $\mathcal{H}_0$ and $\mathcal{H}_1(m_g, n_S)$

↓

MCMC chain
visits GR and MG branches

↓

Bayes factor from occupancy
$\mathcal{BF} = P(\mathcal{H}_1|D)/P(\mathcal{H}_0|D)$

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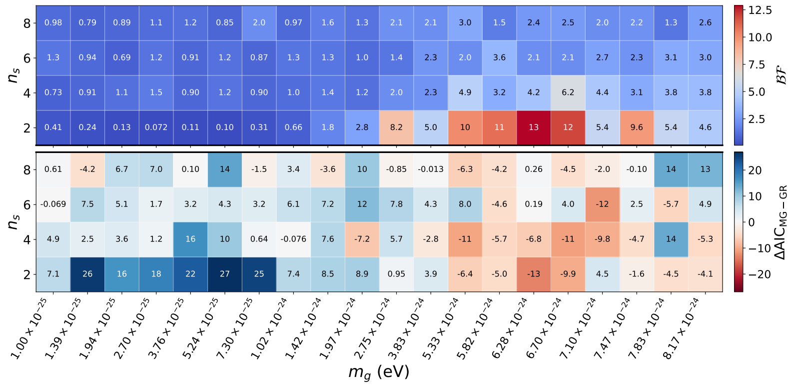
parameter	prior	desc.
E_k, Q_k, J_k	fixed	white noise
$A_{\mathrm{RN},a}$	log-U$[-20, -11]$	pulsar red noise
$\gamma_{\mathrm{RN},a}$	U$[0, 7]$	pulsar red noise
A_{gw}	log-U$[-18, -11]$	common-process amplitude
γ_{gw}	U$[0, 7]$	common-process spectrum
m_g	discrete 20-point grid	graviton mass
n_V	fixed, $n_V = 1$	vector suppression
n_S	discrete $\{2, 4, 6, 8\}$	scalar suppression

Mass scan



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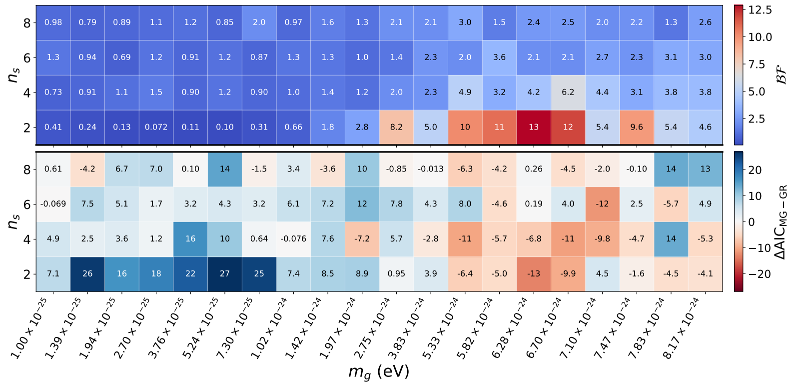
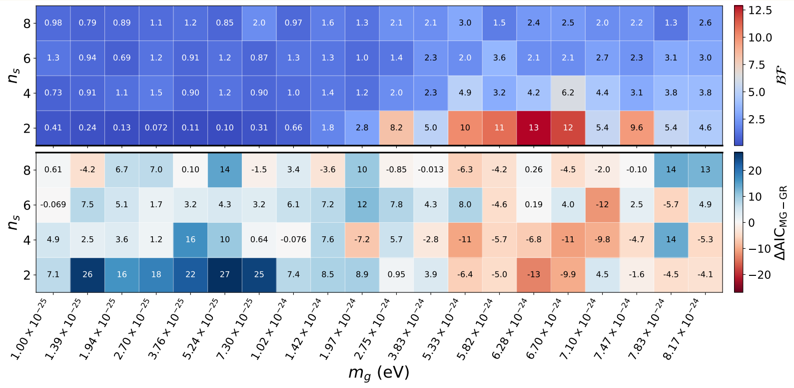


Strongest point: $m_g = 6.28 \times 10^{-24}\,\mathrm{eV}, \quad n_S = 2, \quad \mathcal{BF} \simeq 13$

Mass scan

Bayes-factor scale from Kass and Raftery 1995.



Bayes factor

$\mathcal{BF}$	Evidence against H_0
1–3.2	Not worth more than a bare mention
3.2–10	Substantial
10–100	Strong
> 100	Decisive

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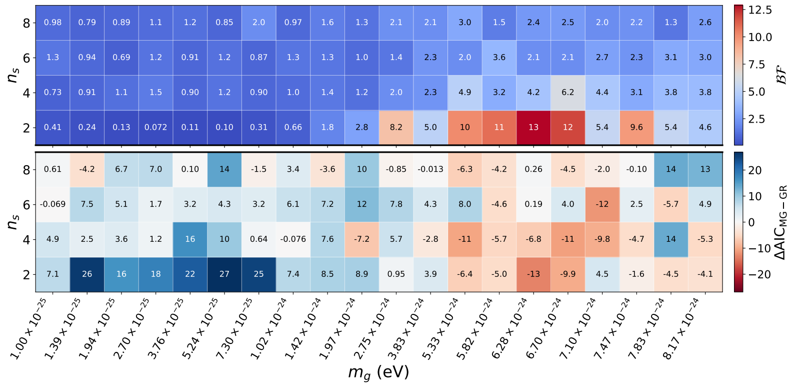

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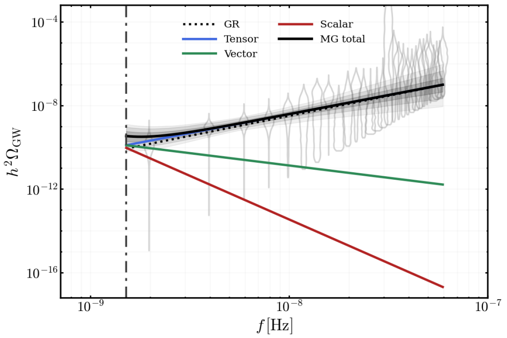
Akaike Information Criterion

$$\mathrm{AIC} = -2\ln\mathcal{L} + 2k$$

Negative values of $\Delta\mathrm{AIC}_{\mathrm{MG-GR}}$ favor massive gravity after penalizing extra parameters.

How should this age with more data?



The preferred mass implies

$$f_g = \frac{m_g}{2\pi\hbar} \simeq 1.5 \text{ nHz} \qquad (T_g \simeq 21 \text{ yr}).$$

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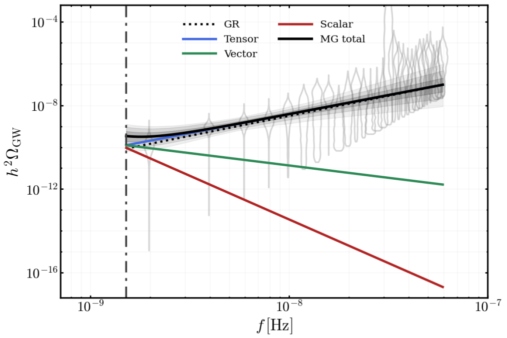

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- observed ‘red-tilt’

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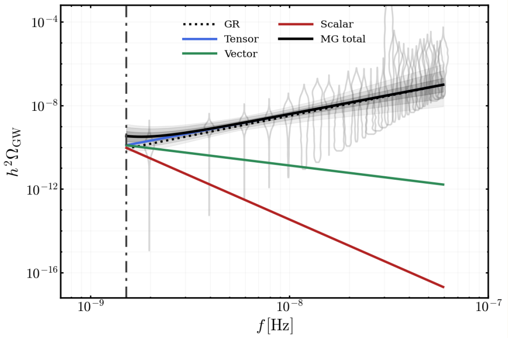

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Strengthens if

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- observed ‘red-tilt’

Weakens if

- SGWB power persists below f_g
- the preferred mass drifts with added data
- no observed ‘red-tilt’

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* **Caveats**

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Source-specific energy densities and longer PTA baselines are needed to test the interpretation.

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Convergence concerns

Acknowledgements

- Advisor: Prof. Tina Kahniashvili
- Collaborators and discussions: Barnabas Poczos, Marcus Bosca, Murman Gurgenidze, Qiuyue Liang, Alberto Roper Pol, Hippolyte Quelquejay Leclere
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Thank you!

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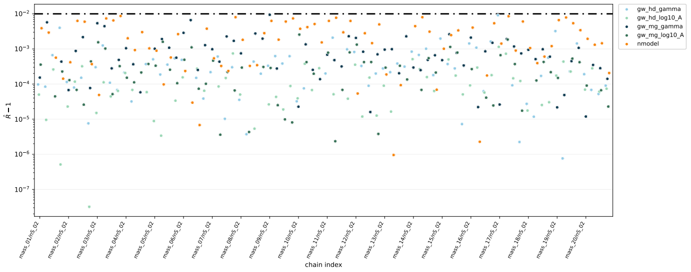
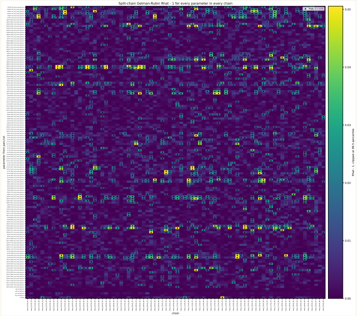
Backup slide: Convergence caveat





Interpretation

The $\mathcal{BF}$ is converged with respect to the model-comparison parameters:

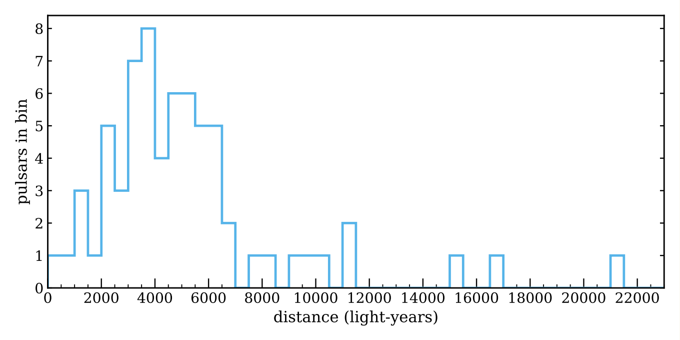
$$A_{\rm gw}^{\rm GR},\ \gamma_{\rm gw}^{\rm GR},\ A_{\rm gw}^{\rm MG},\ \gamma_{\rm gw}^{\rm MG},\ \texttt{nmodel}.$$

Not a fully converged posterior summary for all 139 nuisance parameters.

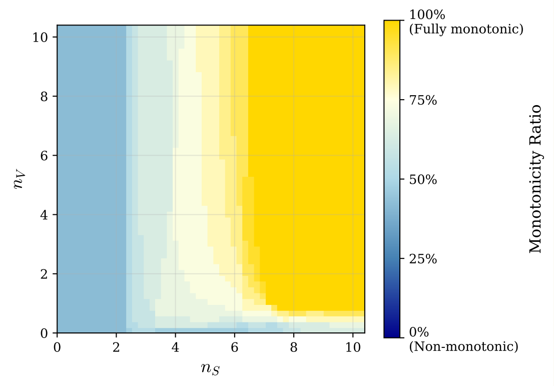
Backup: pulsar distances

NANOGrav 15-year distances

$$\text{median} \simeq 4500 \text{ ly},$$
$$\text{mean} \simeq 5190 \pm 140 \text{ ly}.$$



Backup: why scan n_S but fix n_V?

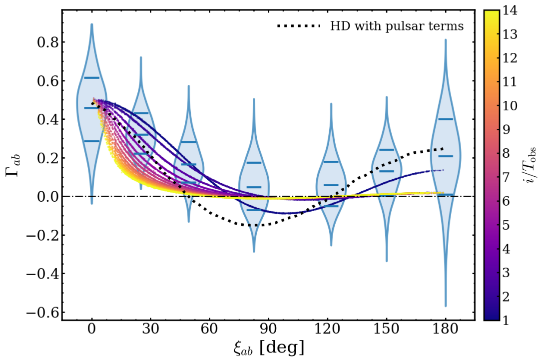


Phenomenological criterion

As A moves away from 1, the RMSE from Hellings–Downs should grow smoothly.

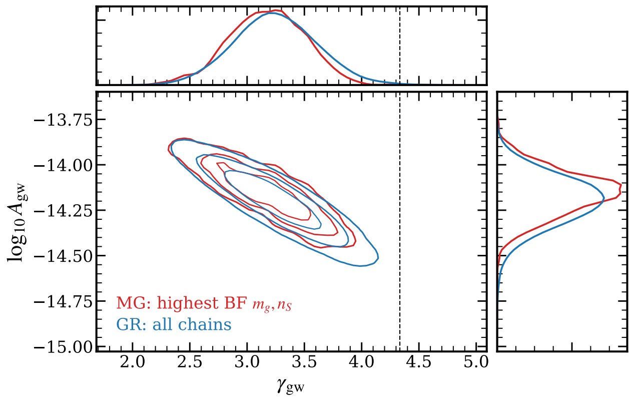
- $n_V = 1$ already captures the distinguishable vector behavior.
- Larger n_V becomes partially degenerate over the PTA band.
- $n_S = 2, 4, 6, 8$ remains visibly different.

Backup: all 14 Fourier-bin ORFs



The preferred MG point remains compatible with the spline reconstruction while carrying frequency dependence absent from a single HD curve.

Backup: posterior overlap

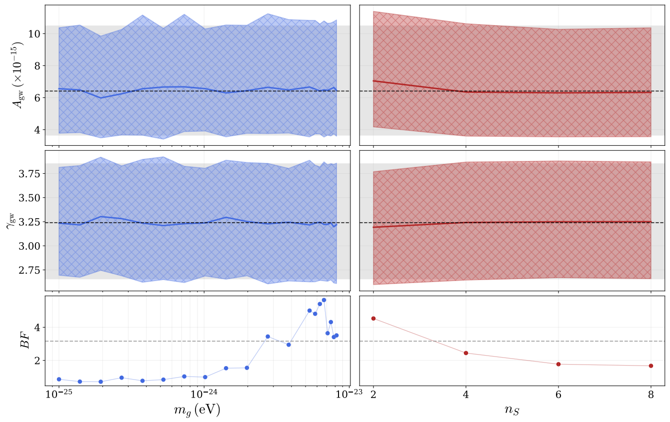


Interpretation

The Bayes-factor preference is not obviously caused by a different common-process amplitude or spectral index.

The dotted vertical line marks $\gamma_{\rm gw} = 13/3$, the SMBHB expectation.

Backup: $A_{\rm gw}$ and $\gamma_{\rm gw}$?



Posterior summaries

$$A_{\rm gw} \sim (3.5 - 11.3) \times 10^{-15}$$

$$\gamma_{\rm gw} \sim 2.6 - 3.9$$

The GR and MG common-process spectra overlap.

$$\Phi_{{\rm T},i} = \frac{A_{\rm gw}^2}{12\pi^2} \frac{1}{T} \left(\frac{f_i}{f_{\rm ref}} \right)^{-\gamma_{\rm gw}} f_{\rm ref}^{-3}, \qquad f_{\rm ref} = 1\,{\rm yr}^{-1}$$

Backup: Constraints on the graviton mass

m_g (eV)	λ_g (km)	Eq.	
			Yukawa
7.2×10^{-23}	**2.8×10^{12}**	(49)	A 2σ bound from the precession of Mercury (Talmadge *et al.*, 1988; Will, 1998).
6×10^{-32}	3×10^{21}	(53)	A 1σ bound from weak lensing of a cluster at $z = 1.2$ (Choudhury *et al.*, 2004). Sensitive to the dark matter distribution and cosmological model.
10^{-29}	10^{19}	(52)	From observations of gravitationally bound clusters of 0.5 Mpc (Goldhaber and Nieto, 1974; Hare, 1973). Sensitive to the dark matter distribution.
			Dispersion Relation
m_g (eV)	λ_g (km)	Eq.	
1.2×10^{-22}	**1.7×10^{12}**	(58)	A 90% confidence bound two 30 $M_\odot$ bh-bh merger (GW150916) (Abbott *et al.*, 2016d; Will, 1998).
7.6×10^{-20}	2.6×10^{9}	(65)	From pulsar timing of PSR B1913+16 and PSR B1534+12 (Finn and Sutton, 2002).
10^{-30}	10^{20}	(63)	Observations of power in B–mode polarization in CMB at low ℓ (Dubovsky *et al.*, 2010; Gumrukcuoglu *et al.*, 2012; Raveri *et al.*, 2015).
10^{-26}	10^{16}	(59)	A 10^4 to 10^7 $M_\odot$ merger by eLISA type experiment (Will, 1998).
10^{-24}	10^{14}	(60)	A dual messenger observation of IBWD by eLISA type experiment (Cooray and Seto, 2004; Cutler *et al.*, 2003; Larson and Hiscock, 2000).
10^{-23}	10^{13}	(66)	Pulsar timing array of 100ns accuracy with 10 year observation (Lee *et al.*, 2010).
10^{-20}	10^{10}	(61)	Dual messenger observation of SNe gamma ray burst and gravitational waves (Nishizawa and Nakamura, 2014).
			Fifth Force
m_g (eV)	λ_g (km)	Eq.	
10^{-32}	10^{22}	(77)	From earth-moon precession for cubic Galileon theories (Dvali *et al.*, 2003).
10^{-32}	10^{22}	(84)	From precession in full 5D DGP in the Solar System (Gruzinov, 2005; Lue and Starkman, 2003).
10^{-30}	10^{20}	(81)	From earth-moon precession for quartic Galileon theories (dRGT-like) (de Rham, 2014).
10^{-27}	10^{17}	(86)	From PSR B1913+16 pulsar in cubic Galileon theories (DGP) (de Rham *et al.*, 2013d).
10^{-33}	10^{23}	(89)	A prospective 4σ bound from weak lensing on next-gen surveys (Park and Wyman, 2015; Wyman, 2011). Sensitive to alternative DM halo profiles.
10^{-32}	10^{22}	(90)	Observations of altered structure formation from fifth force (Khoury and Wyman, 2009; Park and Wyman, 2015; Wyman, 2011; Zu *et al.*, 2014). Sensitive to the particular theory of massive gravity.

From Rham, Deskins, et al. 2017

Backup: Full table of theoretical models of modified gravity

theoretical model	+	×	b	l	x	y	parity viol.	Lorentz viol.	References
General Relativity (4D)	✓	✓	–	–	–	–	×	×	–
GR in noncompactified 5D Minkowski	✓	✓	✓*	✓*	✓	✓	×	×	Nishizawa et al. 2010
GR in noncompactified 6D Minkowski	✓	✓	✓	✓	✓	✓	×	×	Nishizawa et al. 2010
5D Kaluza–Klein theory	✓	✓	✓	–	✓	✓	×	×	Alesci and Montani 2005
Randall–Sundrum braneworld	✓	✓	–	–	–	–	×	×	Frolov and Kofman 2003
DGP braneworld (normal branch)	✓	✓	–	–	–	–	×	×	Charmousis et al. 2006
DGP braneworld (self-accelerating)	✓	✓	–	–	–	–	×	×	Charmousis et al. 2006
Brans–Dicke theory	✓	✓	✓	✓	–	–	×	×	Capozziello et al. 2006
Will–Nordtvedt theory	✓	✓	–	–	✓	✓	×	×	Eardley et al. 1973
$f(R)$ gravity	✓	✓	✓	✓	–	–	×	×	Alves et al. 2009
Horndeski theory	✓	✓	–	–	✓	✓	×	×	Alves et al. 2009
Bimetric theory	✓	✓	✓	✓	✓	✓	×	×	Chatziioannou et al. 2012
dRGT massive gravity	✓	✓	✓*	✓*	✓	✓	×	×	Rham and Gabadadze 2010
Minimal Theory of Massive Gravity	✓	✓	–	–	–	–	×	✓	De Felice and Mukohyama 2016
Chern–Simons gravity	✓	✓	–	–	–	–	✓	×	Jackiw and Pi 2003
Einstein–Aether theory	✓	✓	✓*	✓*	✓	✓	×	✓	Hou et al. 2019
Generalized TeVeS	✓	✓	✓	✓	✓	✓	×	×	Hou et al. 2019

Table: Asterisk * in the b and l columns indicates that the two scalar modes are correlated, and thus reduce the overall degrees of freedom from 6 to 5. Adapted from my grant proposal.

Backup: ORF with free mode weights

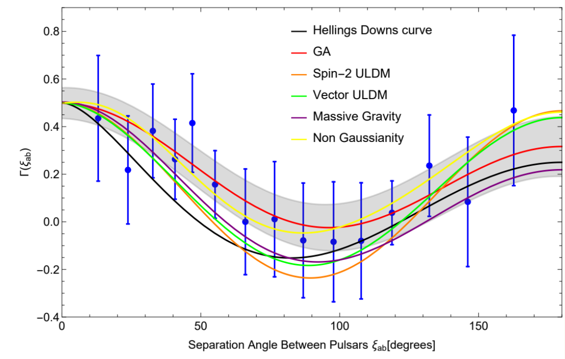
Rather than fixing equipartition, introduce ratios

$$r_V(f) \equiv \frac{\Omega_V(f)}{\Omega_T(f)}, \qquad r_S(f) \equiv \frac{\Omega_S(f)}{\Omega_T(f)}.$$

- A more general analysis fits (r_V, r_S)

Models	Best fit parameters
Ultralight vector DM	$\mu = 1 \times 10^{-24}$
Spin-2 ultralight DM	$m = 4.4 \times 10^{-24}, \alpha = 5.5 \times 10^{-6}$
Massive gravity	$\mathbf{A = 0.73},\ \mathbf{\Omega = 0.46}$
Non-Gaussianity	$\alpha = -0.99$

- They used two free parameters: $A \equiv |\boldsymbol{k}|/k_0$ and $\Omega \equiv r_V = r_S$.
- Did not include finite-distance effects.



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