# Dive deeper with SUBMARINE: SUB Mev dArk matter diRect detectIon using bilayer grapheNE


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Rinchen Sherpa [1,*] Anuvab Sarkar [1,2,†] Tarak Nath Maity [3,‡] Paramita Dutta [4,§] Ranjan Laha [1,¶] and Anirban Das [5,6,**]

[1] *Centre for High Energy Physics, Indian Institute of Science, C. V. Raman Avenue, Bengaluru-560012, India*
[2] *Department of Physics, Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA*
[3] *School of Physics, The University of Sydney and ARC Centre of Excellence for Dark Matter Particle Physics, NSW 2006, Australia*
[4] *Theoretical Physics Division, Physical Research Laboratory, Navrangpura, Ahmedabad-380009, India*
[5] *Theory Division, Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Kolkata-700064, India*
[6] *Homi Bhabha National Institute, Training School Complex, Anushaktinagar, Mumbai-400094, India*

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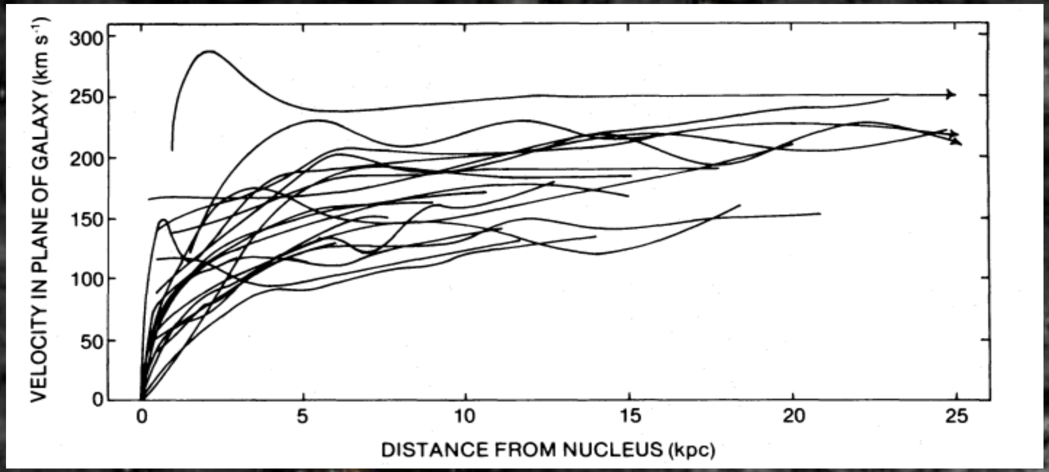
Novel target materials with anisotropic response will play a key role in detecting low-mass dark matter in upcoming experiments. Bilayer graphene is one such material that has been proposed for the detection of sub-MeV mass dark matter particles via electronic excitations. In this work, we calculate scattering rate via a massive mediator in bilayer graphene. With an exposure as small as $\sim 0.5$ mg-year, bilayer graphene can probe new regions of the parameter space. The anisotropic response function of bilayer graphene leads to a sidereal-day modulation in the scattering rate, depending on its orientation with respect to the Galactic dark matter wind. We find significant modulation in the scattering rate for sub-MeV mass dark matter, demonstrating bilayer graphene's promise for a future experiment. We hope that our work will motivate the community to investigate bilayer graphene as a novel target material, and that it may lead us to discover the particle nature of dark matter.

arXiv: 2604.21969

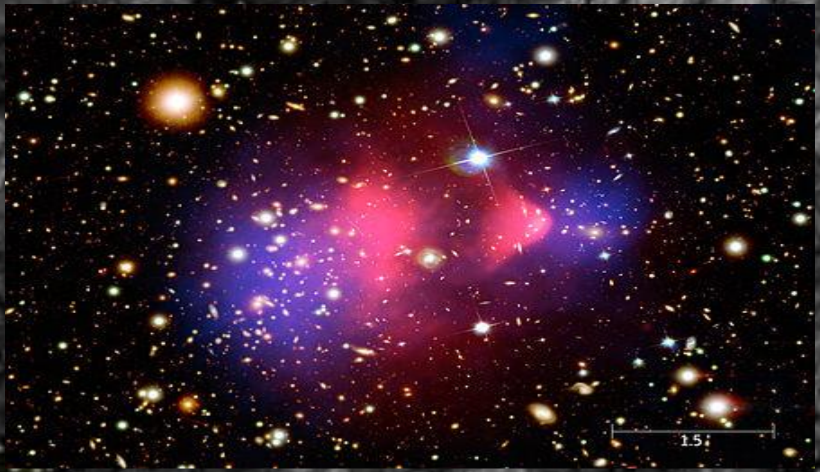
**Anuvab Sarkar**
Department of Physics,
Ohio State University

Credit: science.nasa.gov

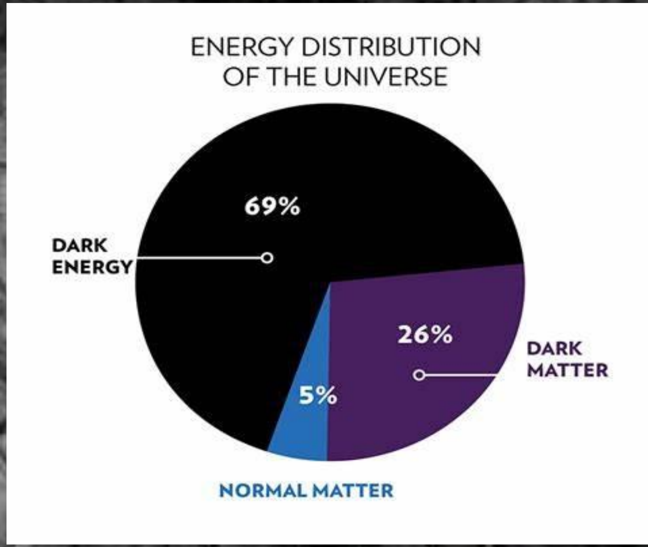
# Evidence for Dark Matter (DM)




Rotation curves




Bullet cluster

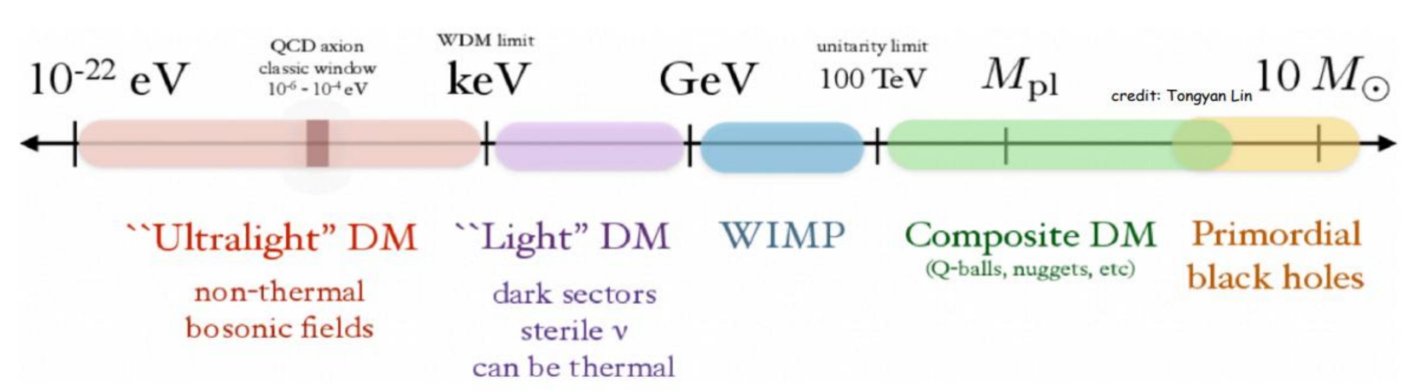



Energy density of the Universe

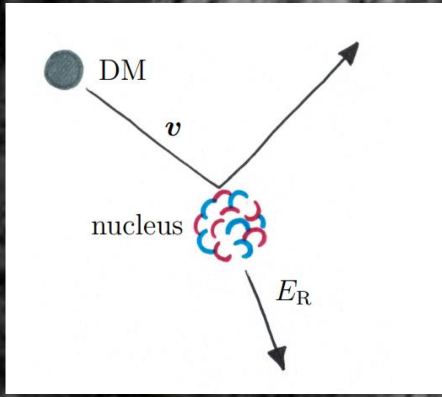
Galaxy Cluster

Image credit: Rubin et al 1980, Wikepedia, science.nasa.gov

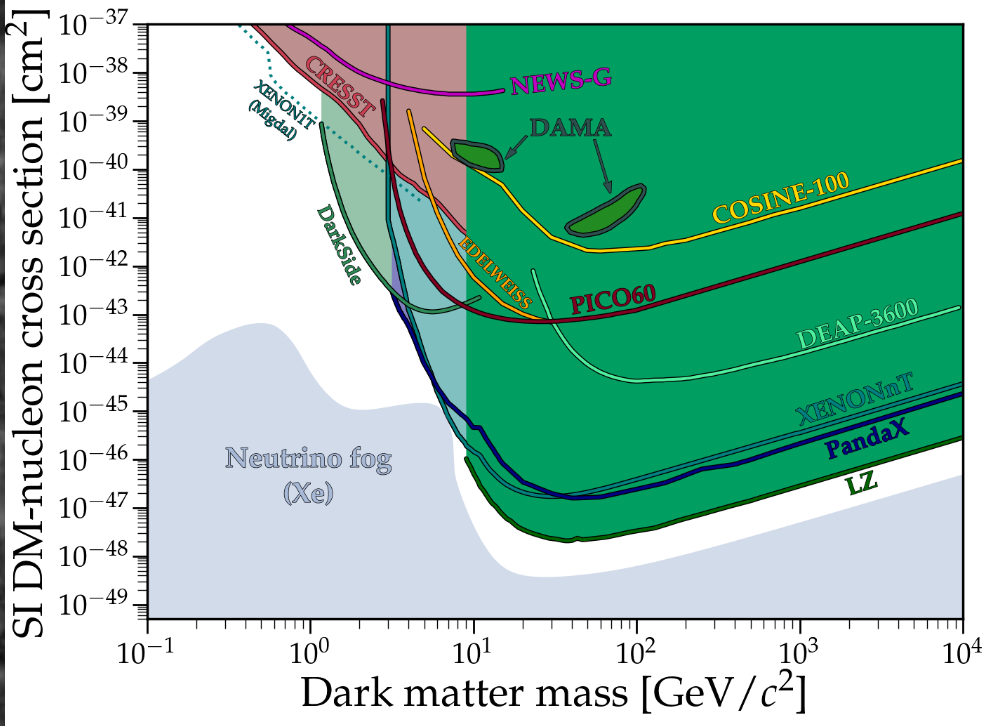
# Dark Matter Direct Detection




Wide range of candidate DM masses, about 80 orders of magnitude



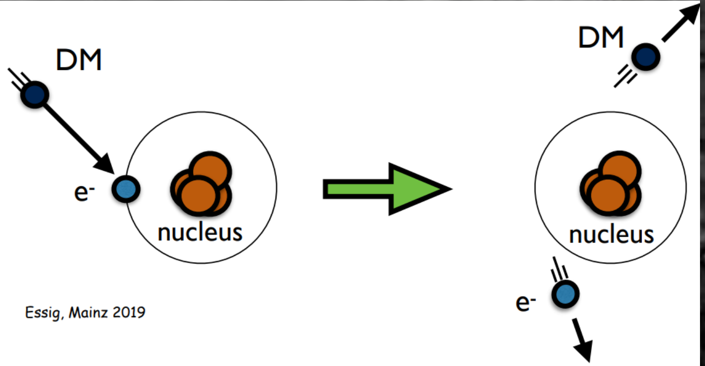

Traditional DM Direct Detection experiments



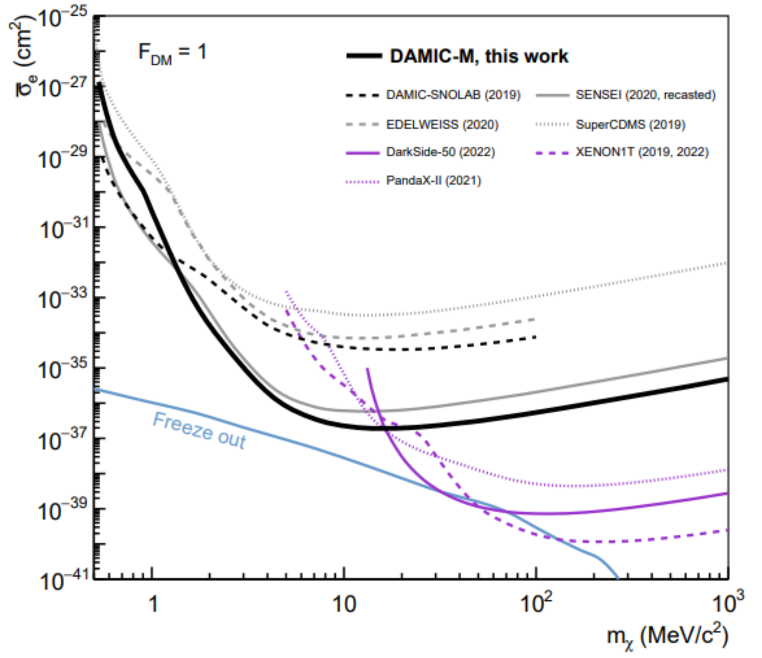

- Experiments lose sensitivity for lower masses as the halo DM particles do not have enough kinetic energy to produce a detectable nuclear recoil
- Condensed matter systems can overcome this challenge as they can detect much lower energy depositions by excitation of collective modes
- $m_\chi = 100.\ \mathrm{keV} \rightarrow E_\chi \sim 50\ \mathrm{meV}$ which is the energy of an optical phonon

Image credit: 2104.12785, cajohare

# Light Dark Matter (sub-GeV)



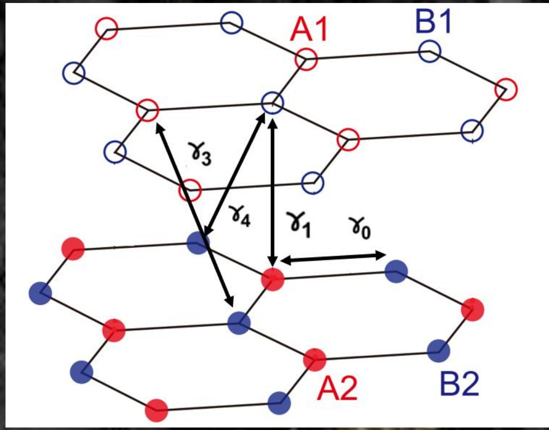

Representation of DM-electron scattering




Source: 2302.02372

- Here we assume some form of DM-electron interaction
- Generally sensitive to lower DM masses
- Bounds on DM-electron scattering cross section for massive mediators explored in the DAMIC-M experiment. The regions above the curve is ruled out
- These experiments (eg. DAMIC-SNOLAB, Darkside-50, PandaX-II) cannot probe masses less than 1 MeV
- Hence a new method is required which produce a signal for a small energy deposition
- As energy deposition is small, condensed matter systems are ideal to explore such detection phenomena

# Bilayer Graphene (BLG)




A schematic of Bilayer Graphene with the electrons described using the tight binding model

- We propose Bilayer Graphene (BLG) as a detector material for light DM
- Graphene in an allotrope of carbon where the atoms are arranged in a 2D honeycomb lattice
- BLG is two layers of graphene stacked over each other. Here we consider the more stable AB stacking

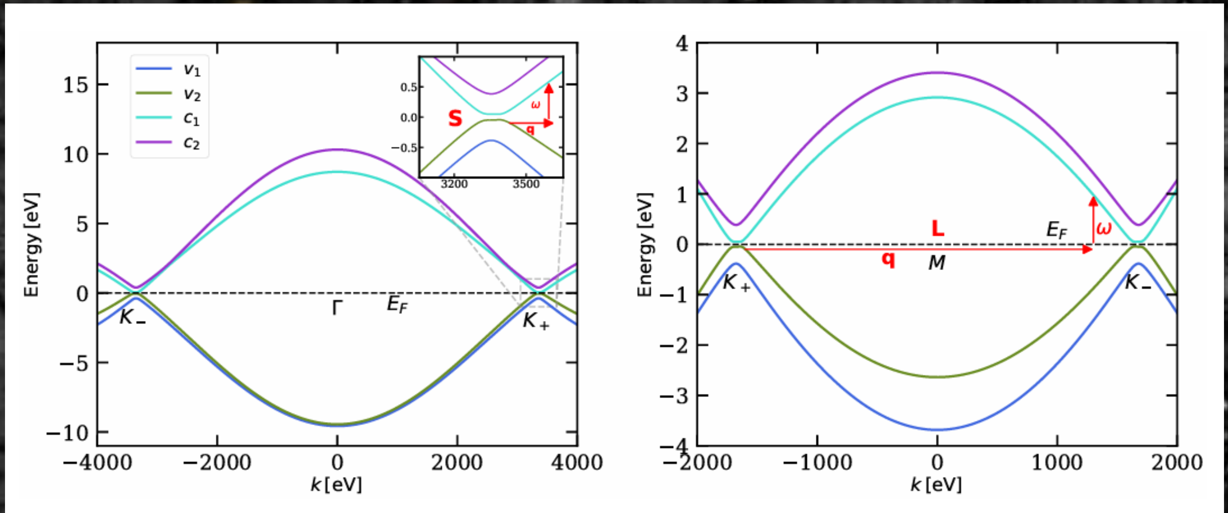
$$H = \begin{pmatrix} \varepsilon_{A1} & -\gamma_0 f(k) & \gamma_4 f(k) & -\gamma_3 f^*(k) \\ -\gamma_0 f^*(k) & \varepsilon_{B1} & \gamma_1 & \gamma_4 f(k) \\ \gamma_4 f^*(k) & \gamma_1 & \varepsilon_{A2} & -\gamma_0 f(k) \\ -\gamma_3 f(k) & \gamma_4 f^*(k) & -\gamma_0 f^*(k) & \varepsilon_{B2} \end{pmatrix}$$

The tight binding Hamiltonian for BLG

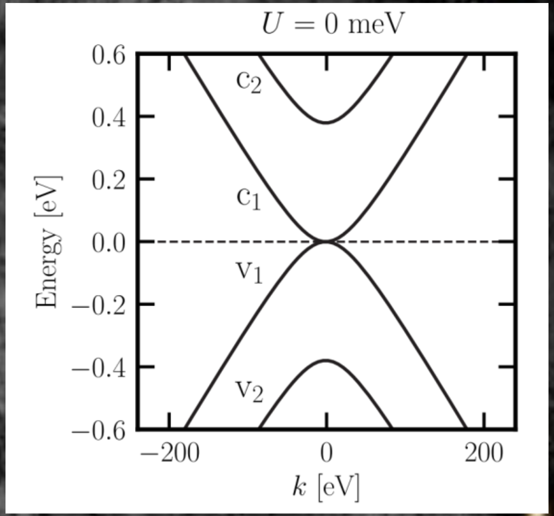
- We model the electronic excitations using the tight binding model
- We write the tight binding Hamiltonian. The diagonal terms are the on-site energies and the off diagonal terms, parameterised by the hopping parameters $\gamma$, are the amplitudes of an electron to jump from one atom to another and $f(k)$ is the form factor
- We obtain the band diagrams by solving for the eigenvalues of the Hamiltonian

Image credit: 2312.00866

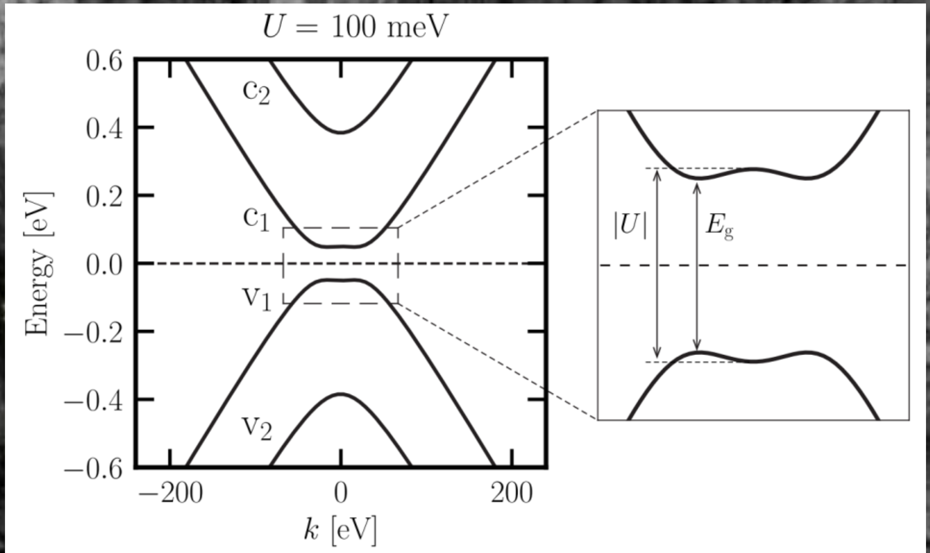
# BLG Band Diagram




Band diagram of BLG along two different paths. Note that the conduction and valence bands almost touch (touch exactly when no gate voltage is applied) at the K points. Also note the two different types of electronic excitations: small $\omega$ small $q$ and small $\omega$ large $q$, where $\omega$ is the energy deposited and $q$ is the momentum transfer




We can set a tunable band gap by applying a gate voltage




On applying a gate voltage, a small tuneable band gap opens which can be used as a threshold for light DM masses

Image credit: 2312.00866

# Scattering Rate

We consider spin-independent interactions, where the mediator only couples to electron density, at leading order in the non-relativistic limit. Treating the dark matter-electron interaction as a perturbation, the scattering rate is given by the Fermi's golden rule.

Differential Scattering Rate · DM density · DM-electron cross section · Dynamic Structure Factor · Velocity distribution

$$\frac{dR}{d\omega} = \frac{\rho_\chi}{\rho_T m_\chi} \int \frac{d^3q}{(2\pi)^3} \, \frac{\pi}{\mu_{\chi e}^2} \, \sigma_{\chi e} \, F_{\mathrm{DM}}^2(q) \, S(\mathbf{q}, \omega) \int d^3v \, f(\mathbf{v}) \, \delta(\omega - \omega_{\mathbf{q}})$$

Target Density · DM mass · DM-electron reduced mass · DM form factor · Energy transfer

Reference momentum

$$F_{\mathrm{DM}}(q) = \frac{q_0^2 + m_\phi^2}{q^2 + m_\phi^2}$$

Momentum transfer · Mediator Mass

# RESULTS

# Dynamic Structure Factor

Energy Loss Function

$$S_{2D}(\mathbf{q}_{||},\omega) = \frac{2\Theta(\omega)}{v_c(\mathbf{q}_{||})}\,\mathrm{Im}\left[-\frac{1}{\varepsilon(\mathbf{q}_{||},\omega)}\right]$$

2D Coulomb Potential

Dielectric Function

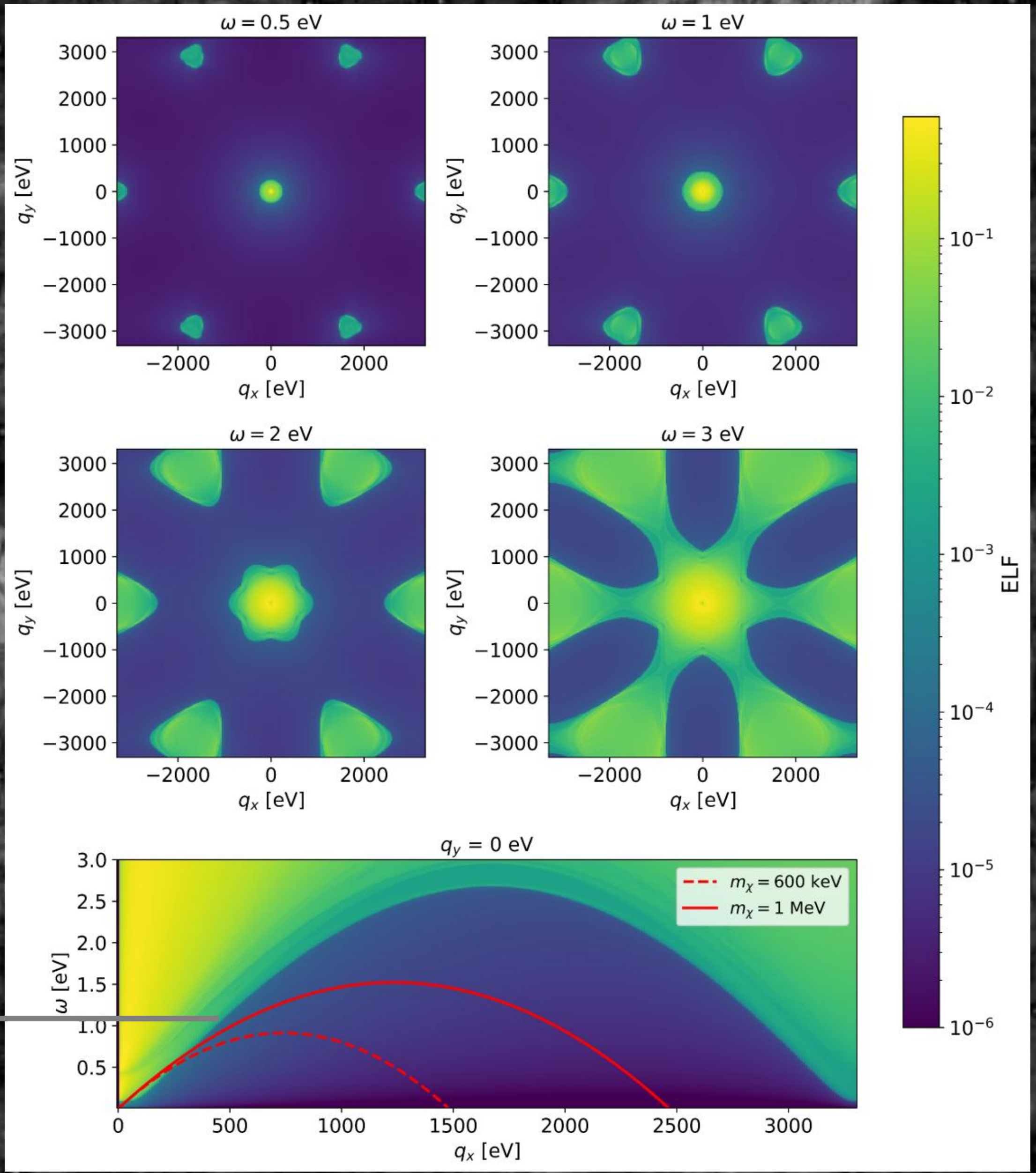
$$S(\mathbf{q},\omega) = \frac{1}{d}\,S_{2D}(\mathbf{q}_{||},\omega)\left(\frac{\sin(q_z d/2)}{q_z d/2}\right)^2$$

Anisotropic Structure Factor

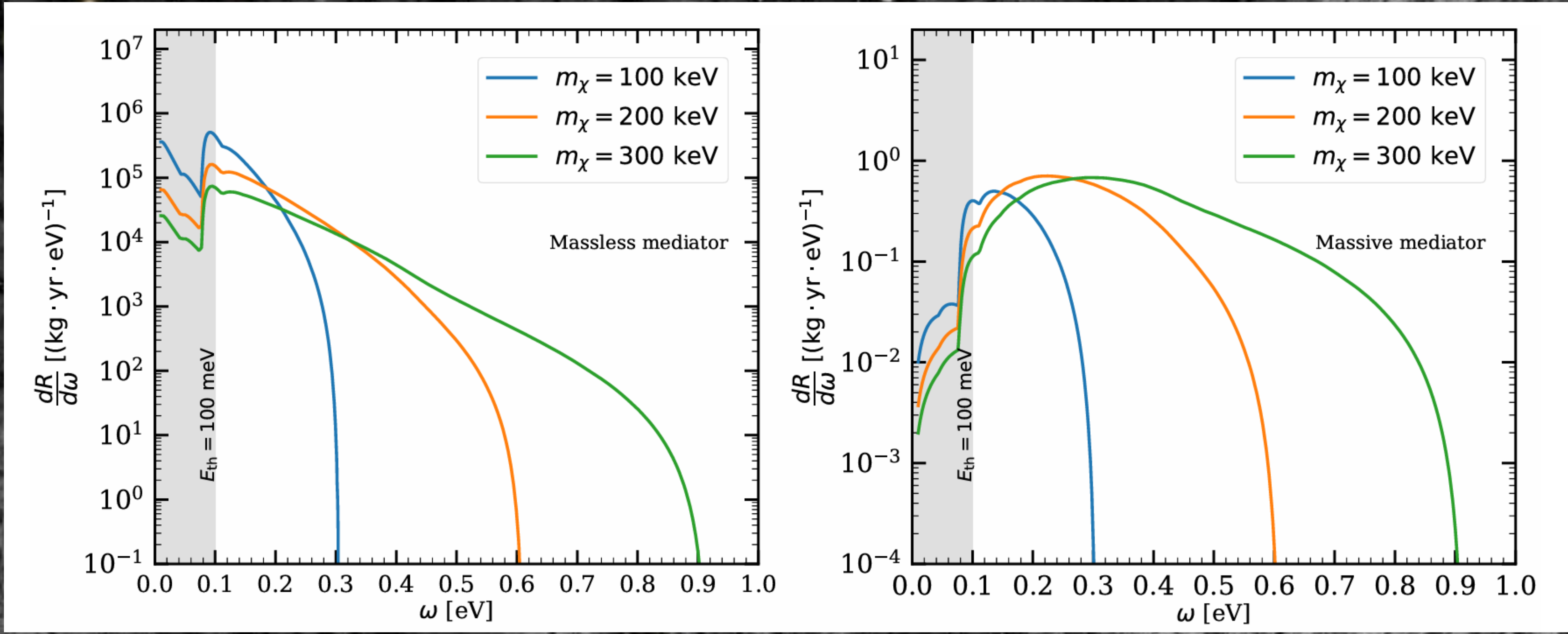
qz dependent part

$$\omega = \mathbf{q}.\mathbf{v} - \frac{q^2}{2m_\chi}$$

Energy and momentum conservation

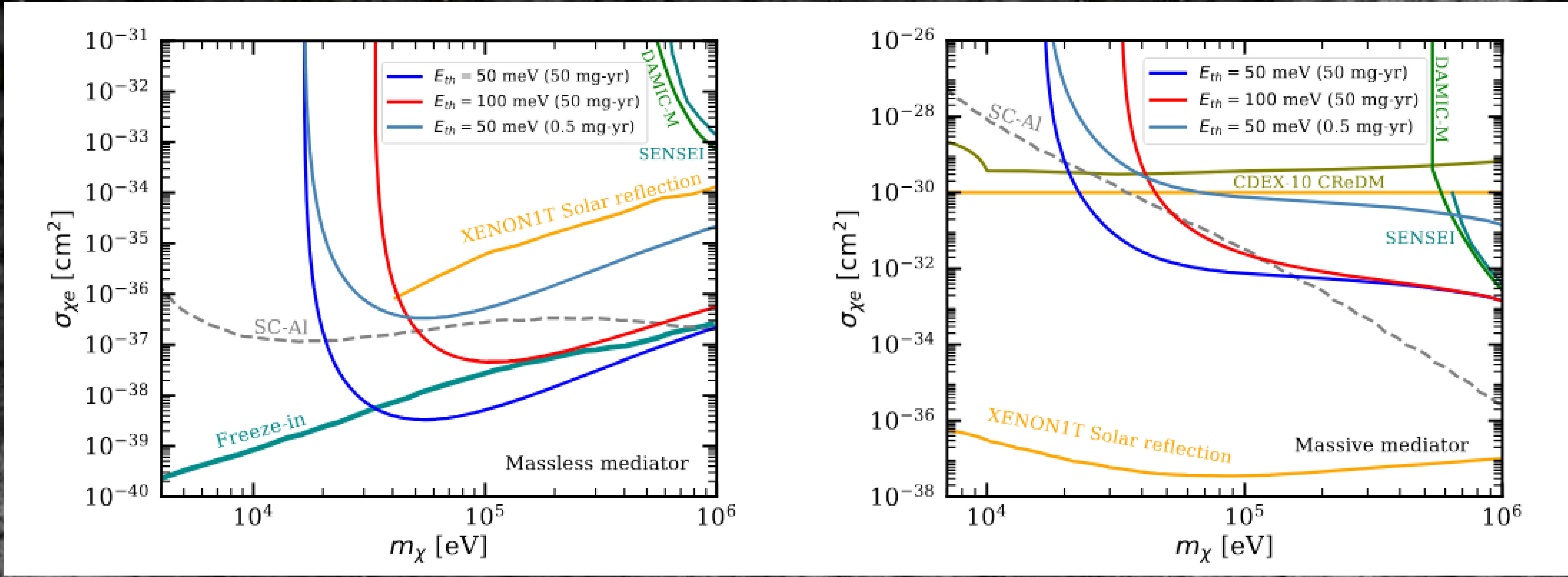


# Differential Scattering Rate




Differential DM-electron scattering rate via massless (left) and massive (right) mediator in BLG at t = 1 hr with band gap U = 100 meV and $\sigma_{\chi e} = 10^{-38}$ cm$^2$ for DM masses 100 keV (blue), 200 keV (orange) and 300 keV (green)

# Projected Sensitivity




Projected sensitivity on the upper limit of DM-electron scattering $\sigma_{\chi e}$ via a massless mediator (left) and massive mediator (right) for $E_{th} = 100$ meV (red) and $E_{th} = 50$ meV (blue) assuming 3 events in 50 mg-yr exposure and 0.5 mg-yr exposure (steel blue) with zero background. For comparison, the projection of the superconducting aluminium (SC-Al) for 50 mg-yr exposure is shown in as a dotted grey line. The limits for XENON1T on Solar reflected DM is shown in orange and CDEX-10 on cosmic ray electron boosted light DM shown in olive. The green and teal lines are the limits from DAMIC-M and SENSEI experiments and the freeze-in DM benchmark is shown in dark cyan.

# Daily Modulation

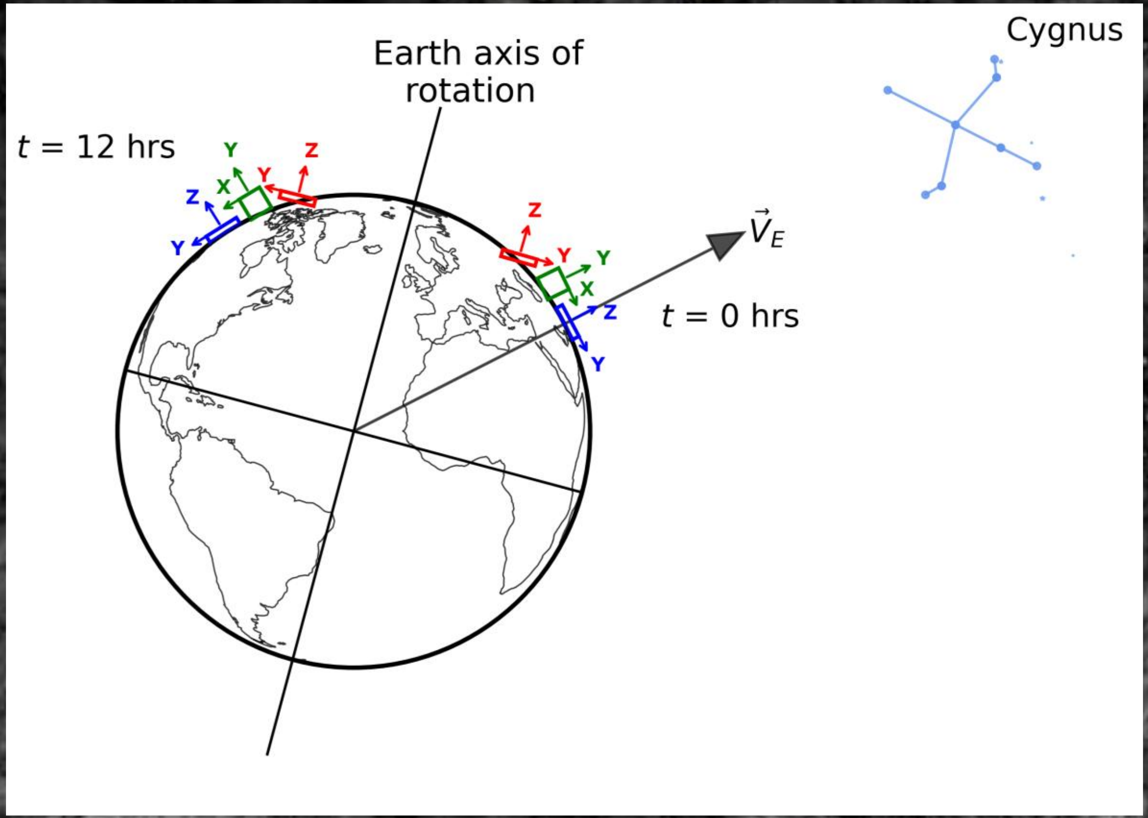
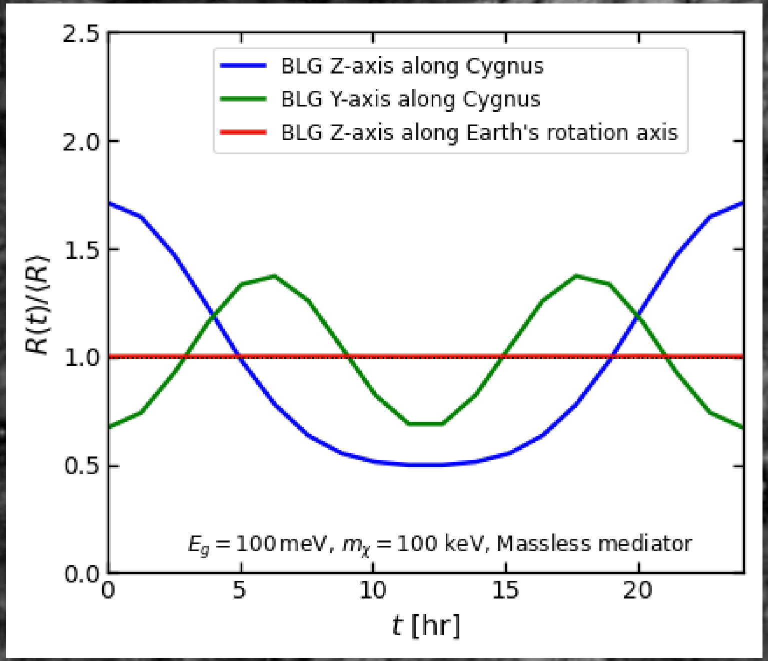



Illustration of rotation of Earth and the change of orientation of BLG with respect to the DM wind




Effect of BLG orientation on daily modulation signal for massless mediator

- Due to rotation and anisotropic response of BLG we have daily modulation of the scattering rate
- Orientation of BLG changes the daily modulation signal. This can be used to separate signal from background

# Summary

- Bilayer Graphene with a tuneable band gap, has the potential to be used as a dark matter detector target for sub-MeV masses
- With an exposure as small as ~ 0.5 mg-year, bilayer graphene has the potential to probe new regions of the parameter space
- Anisotropy in the bilayer graphene structure factor can lead to daily modulation, which is important for distinguishing the dark matter signal from various backgrounds
- Challenges include scaling the scaling the detector and achieving the required exposure

**THANK YOU!**
**QUESTION ARE WELCOME!**

# DARK MATTER-ELECTRON SCATTERING KINEMATICS

- **Energy conservation** of dark matter-electron system implies

$$\omega_{\boldsymbol{q}} = \frac{1}{2} m_\chi v^2 - \frac{(m_\chi \boldsymbol{v} - \boldsymbol{q})^2}{2 m_\chi} = \boldsymbol{q} \cdot \boldsymbol{v} - \frac{q^2}{2 m_\chi}$$

where $\omega_q$ is the energy deposited in the detector, $m_x$ is the DM mass, v is the incoming velocity of the DM particle and q is the momentum transfer to the electron.

- Taking sub-MeV mass DM and using 230km/s as the local DM velocity, we see we get kinetic energy in the meV range.
- We obtain the rate using Fermi Golden Rule. The matrix element can be evaluated using the Born rule

$$\Gamma(\boldsymbol{v}) = V \int \frac{d^3 q}{(2\pi)^3} \sum_f \left| \langle \boldsymbol{p}', f | \, \delta \hat{H} \, | \boldsymbol{p}, i \rangle \right|^2 2\pi \delta(E_f - E_i - \omega_{\boldsymbol{q}}) \qquad \langle \boldsymbol{p}' | \, \delta \hat{H} \, | \boldsymbol{p} \rangle = \frac{1}{V} \int d^3 x \, e^{i \boldsymbol{q} \cdot \boldsymbol{x}} \, \mathcal{V}(\boldsymbol{x}) = \frac{1}{V} \widetilde{\mathcal{V}}(-\boldsymbol{q})$$

- The Fourier Transform of the effective scattering potential can be written as

$$\widetilde{\mathcal{V}}(-\boldsymbol{q}) = \mathcal{M}(q) \, \mathcal{F}_T(\boldsymbol{q})$$

where M is the scattering matrix element and F is the target form factor which is related to the dynamic structure factor. M can also be wr

$$\mathcal{M}(q) = \mathcal{M}(q_0) \, \mathcal{F}_{\text{med}}(q) \,,$$

$$\mathcal{F}_{\text{med}}(q) = \begin{cases} 1 & \text{(heavy mediator)}, \\ (q_0/q)^2 & \text{(light mediator)}. \end{cases}$$

Source: Bloch et al., 2006.14521 JHEP

# WHY CONDENSED MATTER SYSTEMS?

- Experiments like XENON and LZ search for heavy dark matter particles with a smaller cross-section

$$p_\chi \simeq m_\chi v_\chi \simeq 100 \text{ MeV} \left(\frac{m_\chi}{100 \text{ GeV}}\right), \qquad E_\chi \simeq \frac{1}{2} m_\chi v_\chi^2 \simeq 50 \text{ keV} \left(\frac{m_\chi}{100 \text{ GeV}}\right),$$

- Many body effects don't come into play at these energy scales
- But Dark Matter might be light, with a larger cross section
- Because of low kinetic energy, sub –GeV DM may be invisible to ton-scale nuclear detectors

$$\mathcal{O}(\mathbf{q}) = \frac{\varepsilon e g_D}{q^2 + m_{A'}^2} \left( -\sum_k e^{i\mathbf{q}\cdot\mathbf{r}_k} + \sum_I Z_I e^{i\mathbf{q}\cdot\mathbf{r}_I} \right).$$

- Operator of interest in dark photon model
- For large momentum transfer, the cross terms will have large phases and average out to zero, incoherent scattering
- Effective two-body scattering, many body effects don't come into play

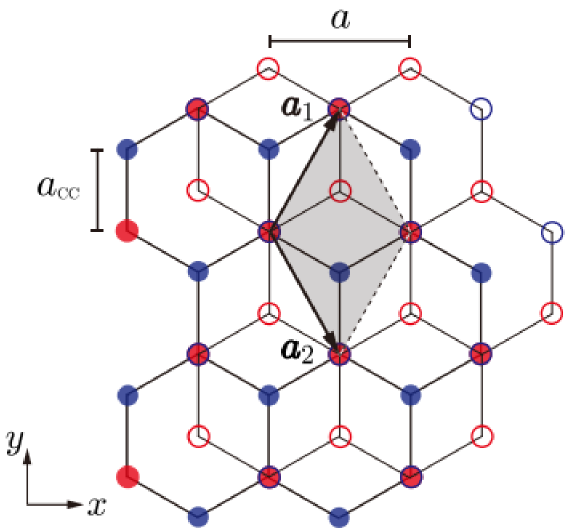
$$q_{\text{coh}} \lesssim \frac{2\pi\hbar}{a} \simeq 5 \text{ keV}/c,$$

a: lattice constant of crystal

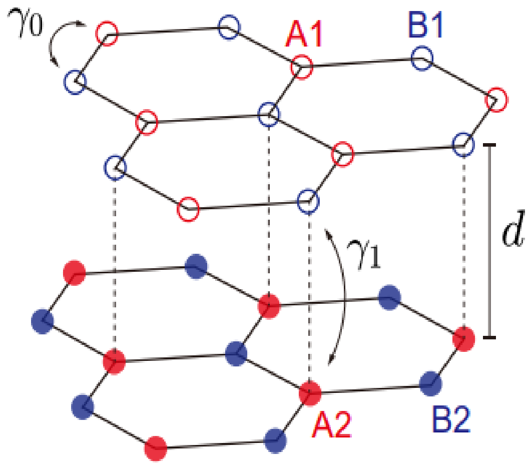
- For sub-MeV DM, maximum momentum ≈ 1 keV so $q < 1 \text{ keV} < q_{coh}$
- For light mediator, $m_A \rightarrow 0$, prefactor → $1/q^4$ , rate integral is weighted towards small kinetically allowed momentum transfer

Source: 2108.03239

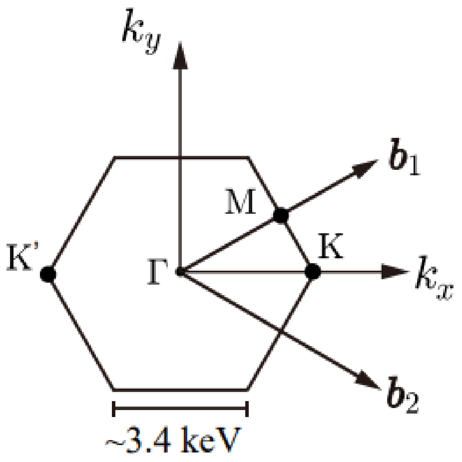
# BILAYER GRAPHENE




Bilayer graphene crystal structure showing the **unit cell** and the **lattice vectors**




Stacking sequence of BLG showing **hopping parameters $\gamma_0$** and **$\gamma_1$**




Reciprocal lattice showing **reciprocal lattice vectors** and **high symmetry points**

$$\boldsymbol{a}_1 = a\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \quad \boldsymbol{a}_2 = a\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$$

Lattice vectors

$$\boldsymbol{b}_1 = \frac{2\pi}{a}\left(1, \frac{1}{\sqrt{3}}\right), \quad \boldsymbol{b}_2 = \frac{2\pi}{a}\left(1, -\frac{1}{\sqrt{3}}\right)$$

Reciprocal lattice vectors

$\gamma_0$ = 3.16 eV
$\gamma_1$ = 0.381 eV

Source: 2312.00866

# BLG HAMILTONIAN AND BAND DIAGRAM

$$H_{\mathrm{b}} = \begin{pmatrix} \epsilon_{A1} & -\gamma_0 f(\mathbf{k}) & \gamma_4 f(\mathbf{k}) & -\gamma_3 f^*(\mathbf{k}) \\ -\gamma_0 f^*(\mathbf{k}) & \epsilon_{B1} & \gamma_1 & \gamma_4 f(\mathbf{k}) \\ \gamma_4 f^*(\mathbf{k}) & \gamma_1 & \epsilon_{A2} & -\gamma_0 f(\mathbf{k}) \\ -\gamma_3 f(\mathbf{k}) & \gamma_4 f^*(\mathbf{k}) & -\gamma_0 f^*(\mathbf{k}) & \epsilon_{B2} \end{pmatrix}$$

**Tight binding Hamiltonian** of bilayer graphene with all hopping parameters

⬇

$$f(\mathbf{k}) = e^{ik_y a/\sqrt{3}} + 2e^{-ik_y a/2\sqrt{3}} \cos(k_x a/2)$$

$$H_{\mathrm{b}} = \begin{pmatrix} \epsilon_{A1} & v\pi^\dagger & -v_4\pi^\dagger & v_3\pi \\ v\pi & \epsilon_{B1} & \gamma_1 & -v_4\pi^\dagger \\ -v_4\pi & \gamma_1 & \epsilon_{A2} & v\pi^\dagger \\ v_3\pi^\dagger & -v_4\pi & v\pi & \epsilon_{B2} \end{pmatrix}$$

**Continuum Hamiltonian** approximation, Taylor expansion about the K point for small k values

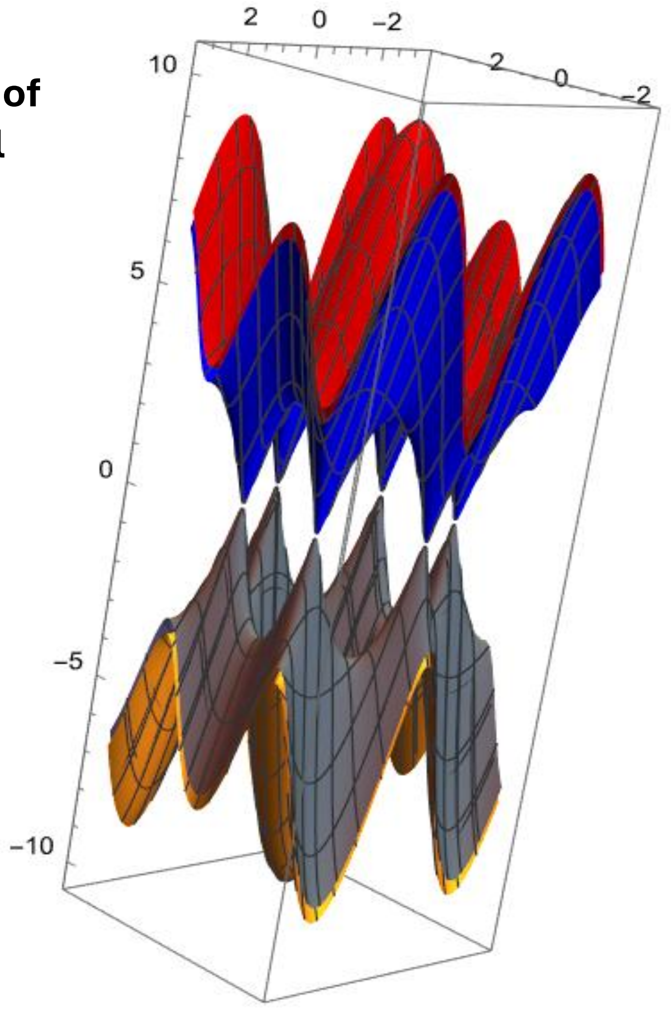
**(Derivations in thesis)**

⬇

$$\pi = \xi k_x + ik_y$$

$$H = \begin{pmatrix} \varepsilon_{A1} & v\pi^\dagger & 0 & 0 \\ v\pi & \varepsilon_{B1} & \gamma_1 & 0 \\ 0 & \gamma_1 & \varepsilon_{A2} & v\pi^\dagger \\ 0 & 0 & v\pi & \varepsilon_{B2} \end{pmatrix}$$

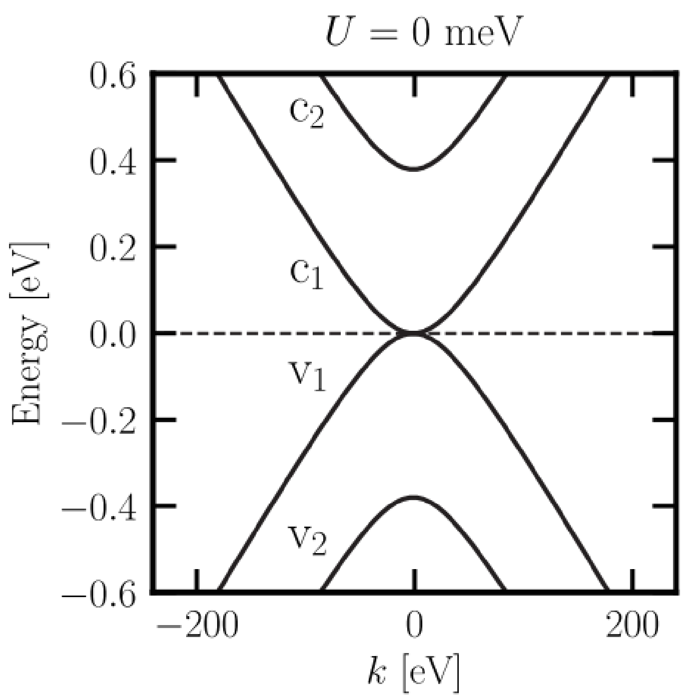
Here we use a simpler Hamiltonian, only using **nearest neighbour** and **interlayer nearest neighbouring** hopping



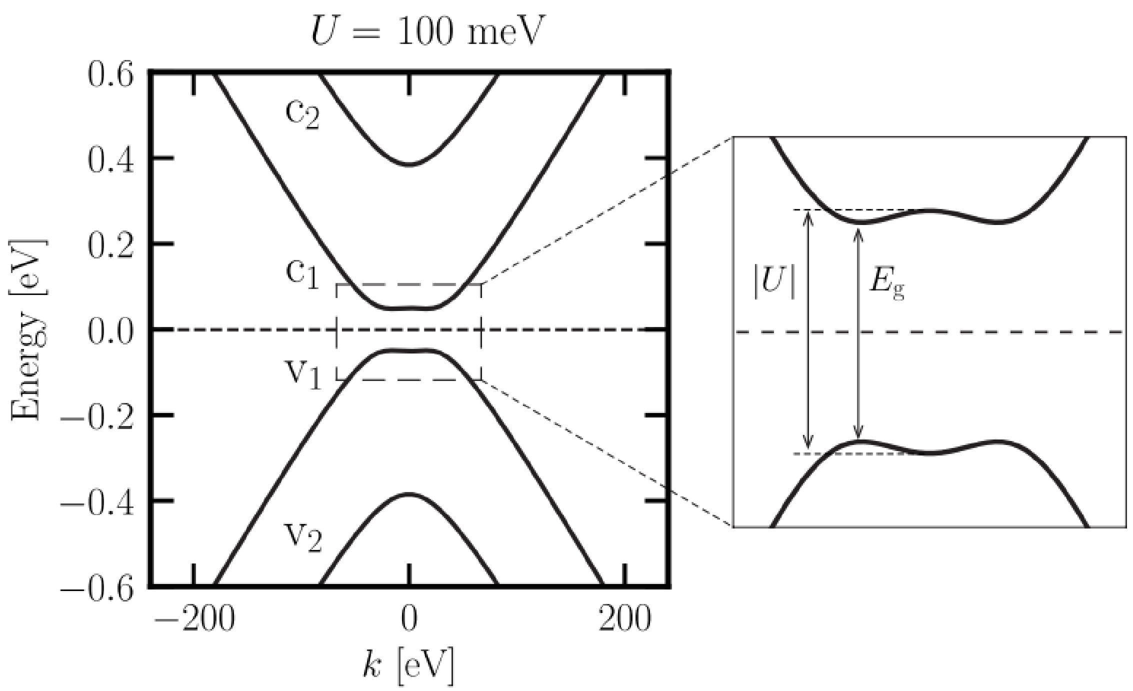

Plot of the **Eigenenergies** of BLG, also known as band diagram

Source: arXiv: 1205.6953

# APPLYING A VOLTAGE ACROSS BLG




When there is no voltage, U = 0 meV, the conduction and the valence bands touch and there is no band gap. Note that the nature at the K point is quadratic instead of linear which is the case in monolayer graphene.




When we apply a voltage, here U = 100 meV, a band gap opens up. This band gap depends on the applied voltage and is thus tunable. We will show that the lower mass threshold for DM detection depends on the applied voltage. Due to this band gap, we can cancel out a lot of the background noise.

$$E_g = |U|\gamma_1/\sqrt{\gamma_1^2 + U^2}$$

Source: 2312.00866

# RATE

- **DM scattering rate in any detector is given by**

$$R(t) = \frac{1}{\rho_T}\frac{\rho_\chi}{m_\chi}\int d\omega \int \frac{d^3q}{(2\pi)^3}\frac{\pi\sigma(q)}{\mu_{\chi e}^2} g(\boldsymbol{q},\omega,t) S(\boldsymbol{q},\omega)$$

- **We will go through each term in the next few slides**
- **The scattering cross section can be split into a constant factor and a momentum dependent mediator form factor**

$$\sigma(q) = \sigma_{\chi e} F_{\text{med}}(q)^2 \qquad \sigma_{\chi e} \equiv \frac{\mu_{\chi e}^2}{\pi}\left(\frac{g_e g_\chi}{q_0^2 + m_\phi^2}\right)^2, \qquad F_{\text{med}}(q) = \frac{q_0^2 + m_\phi^2}{q^2 + m_\phi^2}.$$

- **The velocity distribution of dark matter is not known and determining it is an open problem. For the purposes of this analysis, we take a Maxwellian velocity distribution with a mean velocity of 230 km/s and a cutoff at the local galactic escape velocity.**

$$g(\boldsymbol{q},\omega,t) = \frac{2\pi^2 v_0^2}{qN_0}\left[\exp\left(-\frac{v_-(\boldsymbol{q},t)^2}{v_0^2}\right) - \exp\left(-\frac{v_{\text{esc}}^2}{v_0^2}\right)\right], \quad v_-(\boldsymbol{q},t) = \min\{v_{\text{esc}}, \frac{\omega}{q} + \frac{q}{2m_\chi} + \hat{q}\cdot\boldsymbol{v}_{\text{lab}}(t)\}$$

  - **Time dependent velocity of the laboratory on earth in the galactic frame**

$$\boldsymbol{v}_{\text{lab}}(t) = |\boldsymbol{v}_E|\begin{pmatrix}\sin\theta_e \sin\phi \\ \sin\theta_e\cos\theta_e(\cos\phi - 1) \\ \cos^2\theta_e + \sin^2\theta_e\cos\phi\end{pmatrix}$$

# STANDARD HALO MODEL

- The Standard Halo Model (SHM) is a non-rotating, isotropic DM distribution. The corresponding local velocity distribution is an isotropic, isothermal Maxwell-Boltzmann velocity distribution

$$f_{\mathrm{SHM}}(\boldsymbol{w}) \propto e^{-w^2/v_0^2}$$

- Both density and velocity distributions formally extend to infinite values, which make it necessary to cutoff speeds at the escape velocity $v_{esc}$ of the Milky Way as faster particles are not gravitationally bound to the finite halo

$$f_\chi^{\mathrm{MB}}(\boldsymbol{v}) = \frac{1}{N_0} e^{-(\boldsymbol{v}+\boldsymbol{v}_{\mathrm{e}})^2/v_0^2}\, \Theta\left(v_{\mathrm{esc}} - |\boldsymbol{v}+\boldsymbol{v}_{\mathrm{e}}|\right) \qquad N_0 = \pi^{3/2} v_0^2 \left[ v_0 \,\mathrm{erf}\left(v_{\mathrm{esc}}/v_0\right) - \frac{2\, v_{\mathrm{esc}}}{\sqrt{\pi}} \exp\left(-v_{\mathrm{esc}}^2/v_0^2\right) \right]$$

- The quantity entering the rate equation is related to the velocity distribution as

$$g(\boldsymbol{q}, \omega) \equiv \int d^3v\, f_\chi(\boldsymbol{v})\, 2\pi \delta(\omega - \omega_{\boldsymbol{q}})$$

- The rate is given by

$$R = \frac{1}{\rho_T} \frac{\rho_\chi}{m_\chi} \int d^3v\, f_\chi(\boldsymbol{v})\, \Gamma(\boldsymbol{v}) \qquad \frac{d\Gamma}{d\omega} = \frac{\pi \bar{\sigma}}{\mu^2} \int \frac{d^3q}{(2\pi)^3} \mathcal{F}_{\mathrm{med}}^2(q)\, S(\boldsymbol{q}, \omega_{\boldsymbol{q}})\, \delta(\omega - \omega_{\boldsymbol{q}})$$

# DYNAMIC STRUCTURE FACTOR

- The dynamic structure factor encodes information about the correlation of the density fluctuations in the system. It can be expressed using the electron density operator as the Fourier transform of the density-density correlation function.

$$S(\boldsymbol{q},\omega) = \frac{1}{V}\int dt e^{i\omega t}\langle \hat{n}(\boldsymbol{q},t)\hat{n}(-\boldsymbol{q},0)\rangle$$

- Here we make the assumption that the electron density remains constant between the two layers of BLG.

$$\hat{n}(\boldsymbol{r},t) = \hat{n}_{2\mathrm{D}}(\boldsymbol{r}_{\parallel},t)\left[\frac{1}{d}\Theta(d/2-|r_z|)\right]$$

- Doing the calculation, we see the 2-dimensional dynamic structure factor factors out.

$$\begin{aligned} S(\boldsymbol{q},\omega) &= \frac{1}{V}\int dt e^{i\omega t}\langle \hat{n}(\boldsymbol{q},t)\hat{n}(-\boldsymbol{q},0)\rangle \\ &= \frac{1}{d}\left(\frac{1}{A}\int dt e^{i\omega t}\langle n_{2\mathrm{D}}(\boldsymbol{q}_{\parallel},t)n_{2\mathrm{D}}(-\boldsymbol{q}_{\parallel},0)\rangle\right)\left(\frac{\sin q_z d/2}{q_z d/2}\right)^2 \\ &= \frac{1}{d}S_{2\mathrm{D}}(\boldsymbol{q}_{\parallel},\omega)\left(\frac{\sin q_z d/2}{q_z d/2}\right)^2, \end{aligned}$$

- This can be written as the imaginary part of the 2D density-density response function derived from the fluctuation dissipation theorem

$$S_{2\mathrm{D}}(\boldsymbol{q}_{\parallel},\omega) = -\frac{2}{1-e^{-\beta\omega}}\mathrm{Im}\left[\chi_{2\mathrm{D}}(\boldsymbol{q}_{\parallel},\omega)\right]$$

# POLARISATION FUNCTION

- We use the 2D version of the dielectric function. It is defined as the ratio of the screened potential to the bare potential of the material. The term $v_c$ is the 2D coulomb interaction.

$$\frac{1}{\varepsilon(\boldsymbol{q}_{\|}, \omega)} = 1 + v_{\mathrm{C}}(\boldsymbol{q}_{\|})\chi_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega)$$

- The dynamic structure factor is related to the imaginary part of the inverse dielectric function, known as the energy loss function. The ELF represents electronic energy loss in materials and is thus related to an experimentally measurable quantity.

$$S_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega) = -2\Theta(\omega)\mathrm{Im}\chi_{2\mathrm{D}}(\omega) = \frac{2\Theta(\omega)}{v_{\mathrm{C}}(\boldsymbol{q}_{\|})}\mathcal{W}(\boldsymbol{q}_{\|}, \omega), \quad \mathcal{W}(\boldsymbol{q}_{\|}, \omega) = \mathrm{Im}\left[-\frac{1}{\varepsilon(\boldsymbol{q}_{\|}, \omega)}\right]$$

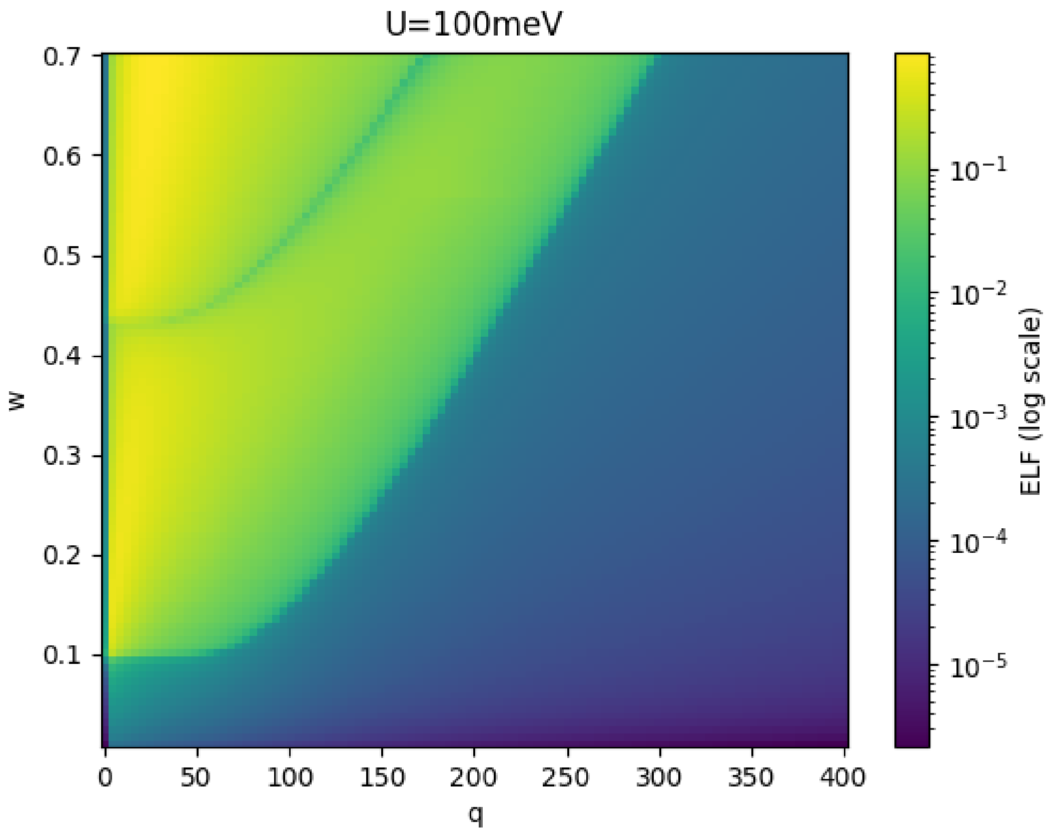
- We use random phase approximation to compute ELF. RPA corresponds to an infinite series of non-interacting polarisation functions. The response function and the dielectric function can be expressed in terms of this polarisation function.

$$\chi_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega) = \frac{\chi^{(0)}_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega)}{1 - v_{\mathrm{C}}(\boldsymbol{q}_{\|})\chi^{(0)}_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega)}, \quad \varepsilon(\boldsymbol{q}_{\|}, \omega) = 1 - v_{\mathrm{C}}(\boldsymbol{q}_{\|})\chi^{(0)}_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega)$$

- The non-interacting polarisation function is given by

$$\chi^{(0)}_{2\mathrm{D}}(\boldsymbol{q}_{\|}, \omega) = g\sum_{n,m}\int \frac{d^2k}{(2\pi)^2}\frac{f_{\boldsymbol{k},n} - f_{\boldsymbol{k}+\boldsymbol{q}_{\|},m}}{\omega + \varepsilon_{\boldsymbol{k},n} - \varepsilon_{\boldsymbol{k}+\boldsymbol{q}_{\|},m} + i\eta}|\langle \boldsymbol{k}, n|\boldsymbol{k} + \boldsymbol{q}_{\|}, m\rangle|^2$$

# NEW RESULTS




**Colour plot** of energy loss function (**ELF**) of Bilayer Graphene (**BLG**) plotted for **small momentum transfer** values, using the continuum model Hamiltonian, in the **q (in eV)** and **ω (in eV)** plane

- Wrote a **python code** for the ELF, obtained results **did not match** the results from 2312.00866
- **Contacted** the **original authors** and they shared their **code**
- Checked the code line-by-line and found two **errors**
- Informed the original authors, they **confirmed** the errors

## INTERPRETATION

- Origin of ELF is the **interband transitions of electrons**
- Sharp colour change across the **electron-hole boundary**, represents the case where momentum transfer by q and energy transfer by ω are possible from an occupied band to an empty band
- In the region $\omega < E_g$ the ELF is close to **zero**
- Since the Fermi energy lies within the gap, **intraband transitions do not contribute** to the ELF
- Red curves in Fig. 3 of 2312.00866 denote the kinematic phase space of DM, places where they overlap with the yellow-green regions is where DM-electron scattering occurs
- Applied **U** determines **minimum detectable mass** threshold