

“GRAVITY’S GIFT: BARYONS FROM THE BIG BANG”



Andrew Long
Rice University
@ Pheno 2026
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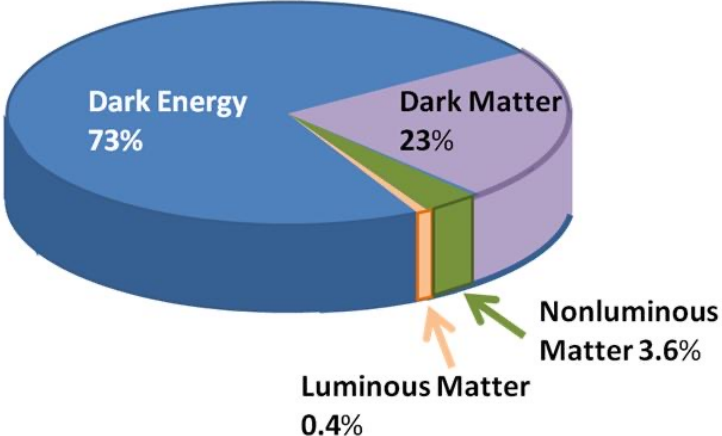
*based on: 2605.05304 with Tammi Chowdhury,
Leah Jenks, Rocky Kolb, and Evan McDonough*



What's the matter with matter?

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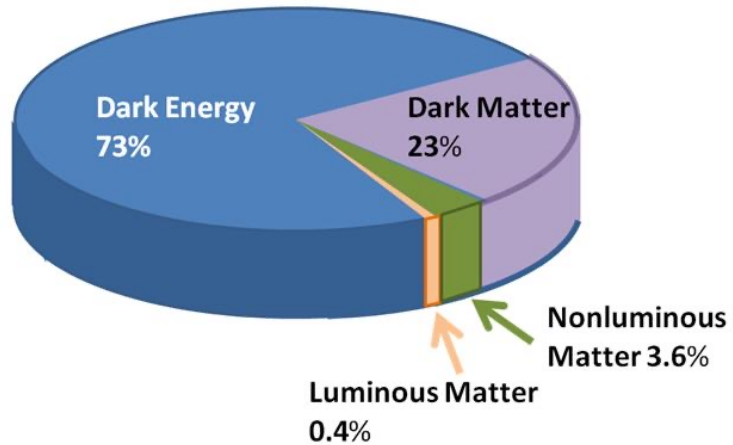
many cosmology talks start here:



“we don’t understand 95% of the Universe!”

What's the matter with matter?

many cosmology talks start here:



“we don't understand ~~95%~~ of the Universe!”

100%

Although, we've measured the amount of ordinary (baryonic) matter ...

CMB anisotropies tells us the cosmological energy fraction in baryons

$$\Omega_b h^2 = 0.0224 \pm 0.0001$$

[Planck (2018)]

for baryogenesis studies, we conventionally use the baryon-to-entropy ratio

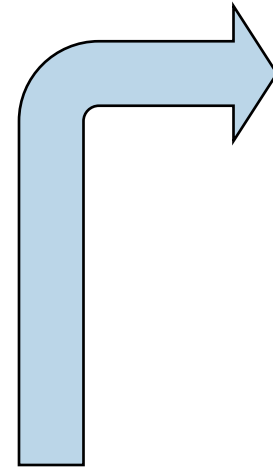
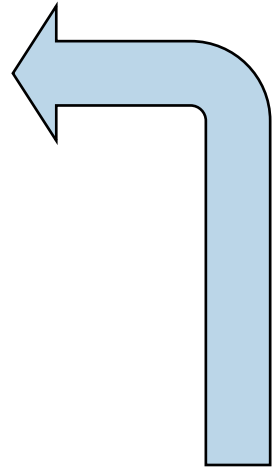
$$Y_B^{\text{obs}} = (0.879 \pm 0.004) \times 10^{-10}$$

... we still don't know *why* there's any matter at all.

baryogenesis = an event that we think occurred in the early universe and gave rise to the excess of matter over antimatter

A simple & testable
baryogenesis scenario

gravitational particle
production sets up
nonthermal leptogenesis



inflationary grav waves

A simple & testable
baryogenesis scenario

Baryogenesis scenario: *LG from CGPP*

(1) Type-I seesaw

heavy Majorana neutrinos: N_1, N_2, N_3

interaction with SM: $\mathcal{L}_{\text{int}} = -y_{ij}\epsilon_{ab}\bar{L}_{a,i}N_j\Phi_b^* + \text{h.c.}$

appealing features: light ν masses, GUT embedding

possible generalizations: Type-II or Type-III seesaw

(4) Stiff phase

(2) Cosmological gravitational particle production

(5) Reheating

(3) Nonthermal leptogenesis

Kobayashi, Yamaguchi, & Yokoyama (2010)

Hashiba & Yokoyama (2019)

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different implementations
of *LG from CGPP*

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CGPP = particles are created by the expansion of the universe during inflation & at the end of inflation

preferred parameters: $M_1 \approx H_e < H_{\text{inf}} \ll M_2 < M_3$

typically: 10^{-3} particles per Hubble volume [see next slides]

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e.g., SUSY flat direction

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inflaton's energy is converted into radiation

e.g., irruption

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What do we predict for the BAU?

initial N_1 abundance [comes from CGPP]

$$[a^3 n_N]_{\text{init}}$$

CP-violating decays to leptons

$$\varepsilon \equiv \frac{\sum_{ab} \sum_i [\text{Br}(N_1 \rightarrow L_{a,i} \Phi_b) - \text{Br}(N_1 \rightarrow \bar{L}_{a,i} \bar{\Phi}_b)]}{\sum_{ab} \sum_i [\text{Br}(N_1 \rightarrow L_{a,i} \Phi_b) + \text{Br}(N_1 \rightarrow \bar{L}_{a,i} \bar{\Phi}_b)]}$$

predicted B-number

$$a^3(t) n_B(t) = -\frac{28}{79} \varepsilon [a^3 n_N]_{\text{init}}$$

predicted B-to-entropy

$$Y_B = -\frac{28}{79} \varepsilon \frac{T_{\text{RH}} H_e}{4M_{\text{Pl}}^2} e^{3w N_{\text{RH}}} \left(\frac{[a^3 n_N]_{\text{init}}}{a_e^3 H_e^3} \right)$$

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upper bound (hierarchical spectrum)

$$|\varepsilon| < \varepsilon_{\text{DI}} \equiv \frac{3}{8\pi} \frac{M_N m_\nu}{v^2} \approx 9.86 \times 10^{-4} \left(\frac{M_N}{10^{13} \text{ GeV}} \right)$$

stiff phase after inflation

$$w = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \approx 1 \quad \Rightarrow \quad \dot{\phi}^2 \gg V(\phi)$$

BAU prediction

$$Y_B \simeq (1 \times 10^{-10}) \left(\frac{-\varepsilon}{10^{-1} \varepsilon_{\text{DI}}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \\ \times \left(\frac{H_e}{10^{13} \text{ GeV}} \right)^2 \left(\frac{T_{\text{RH}}}{10^9 \text{ GeV}} \right)^{-1} \left(\frac{[a^3 n_N]_{\text{init}} / a_e^3 H_e^3}{10^{-3}} \right)$$

Cosmological gravitational particle production

original work: Parker (1969, 70)
 fermions: Chung et. al. (2011)
 review: Kolb & AL (2023)

Dirac equation in an expanding Universe

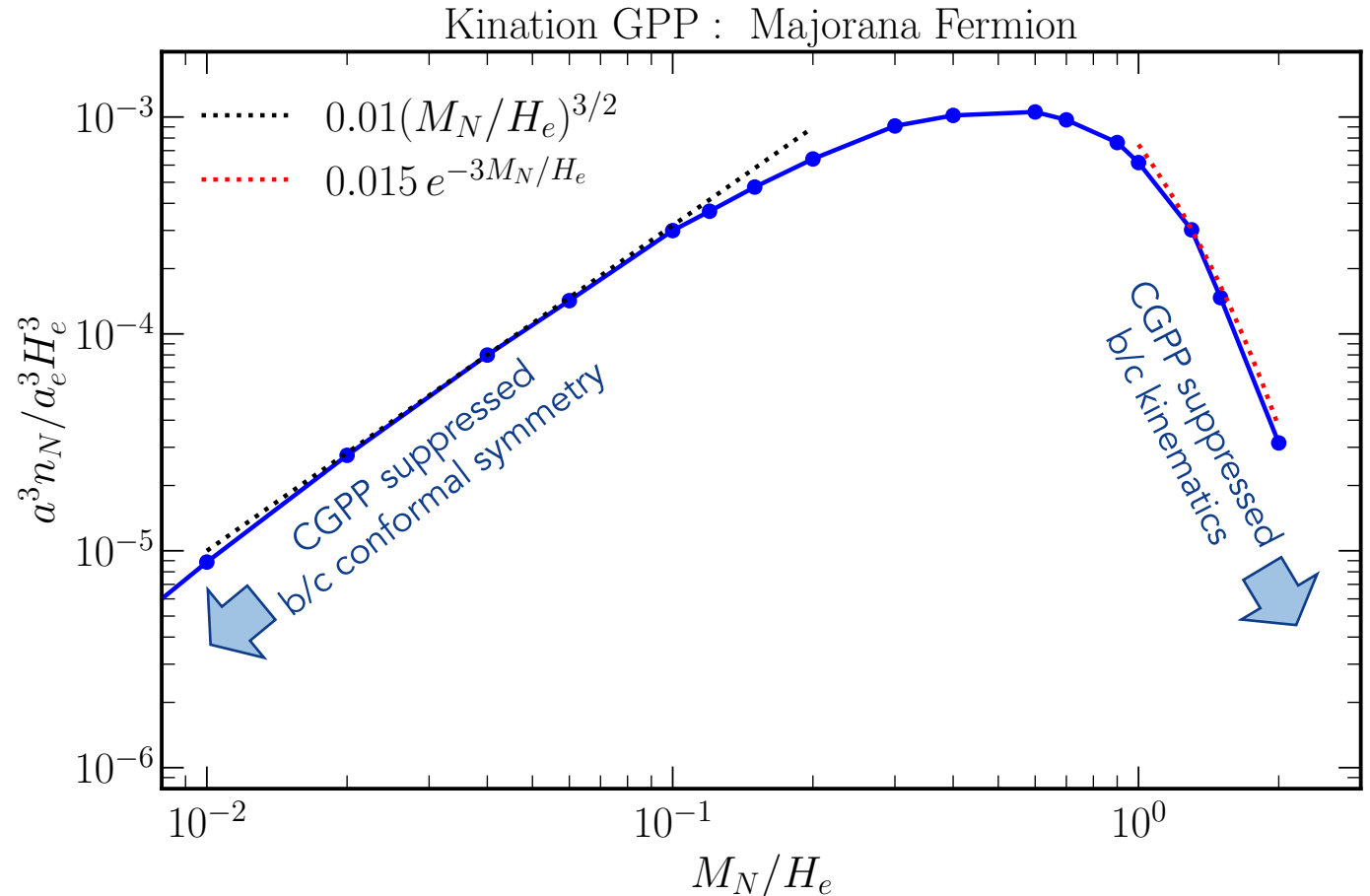
$$i \frac{d}{d\eta} \begin{pmatrix} u_{A,k} \\ u_{B,k} \end{pmatrix} = \begin{pmatrix} aM_N & k \\ k & -aM_N \end{pmatrix} \begin{pmatrix} u_{A,k} \\ u_{B,k} \end{pmatrix}$$

comoving number density of N_1 's that are produced

$$a^3 n_N = \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \left(|\beta_{\mathbf{k},+}|^2 + |\beta_{\mathbf{k},-}|^2 \right)$$

model for background spacetime: inflation + kination

$$V(\phi) = \begin{cases} \frac{1}{2} m_\phi^2 \phi^2 & \text{for } \phi > 0 \\ = 0 & \text{for } \phi < 0 \end{cases}$$



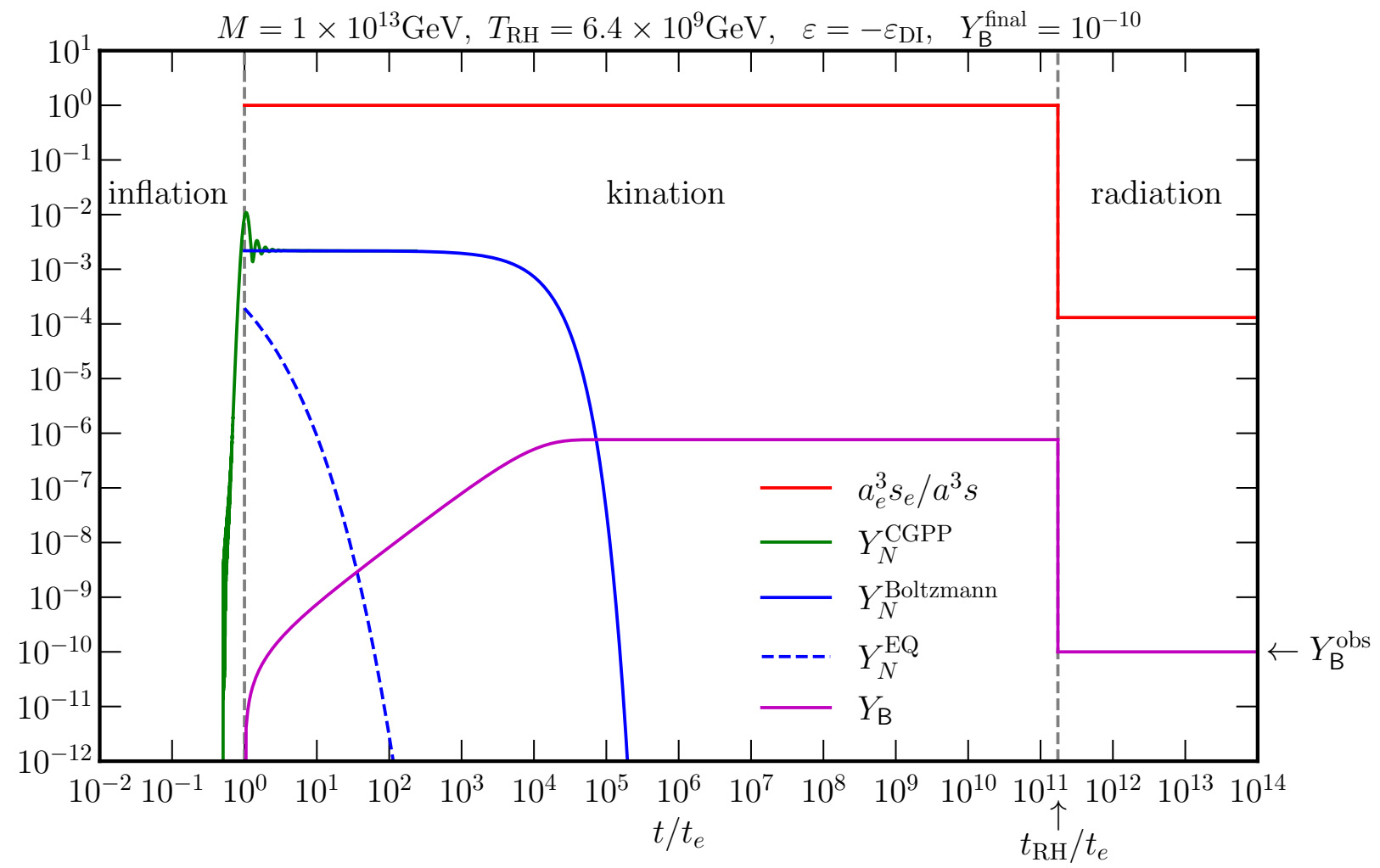
Evolution of densities via Boltzmann eqns

Boltzmann equations

$$\frac{d}{dt}n_N + 3Hn_N = -(n_N - n_N^{\text{eq}})[\Gamma + 2n_\gamma^{\text{eq}}\langle\sigma v\rangle_1]$$

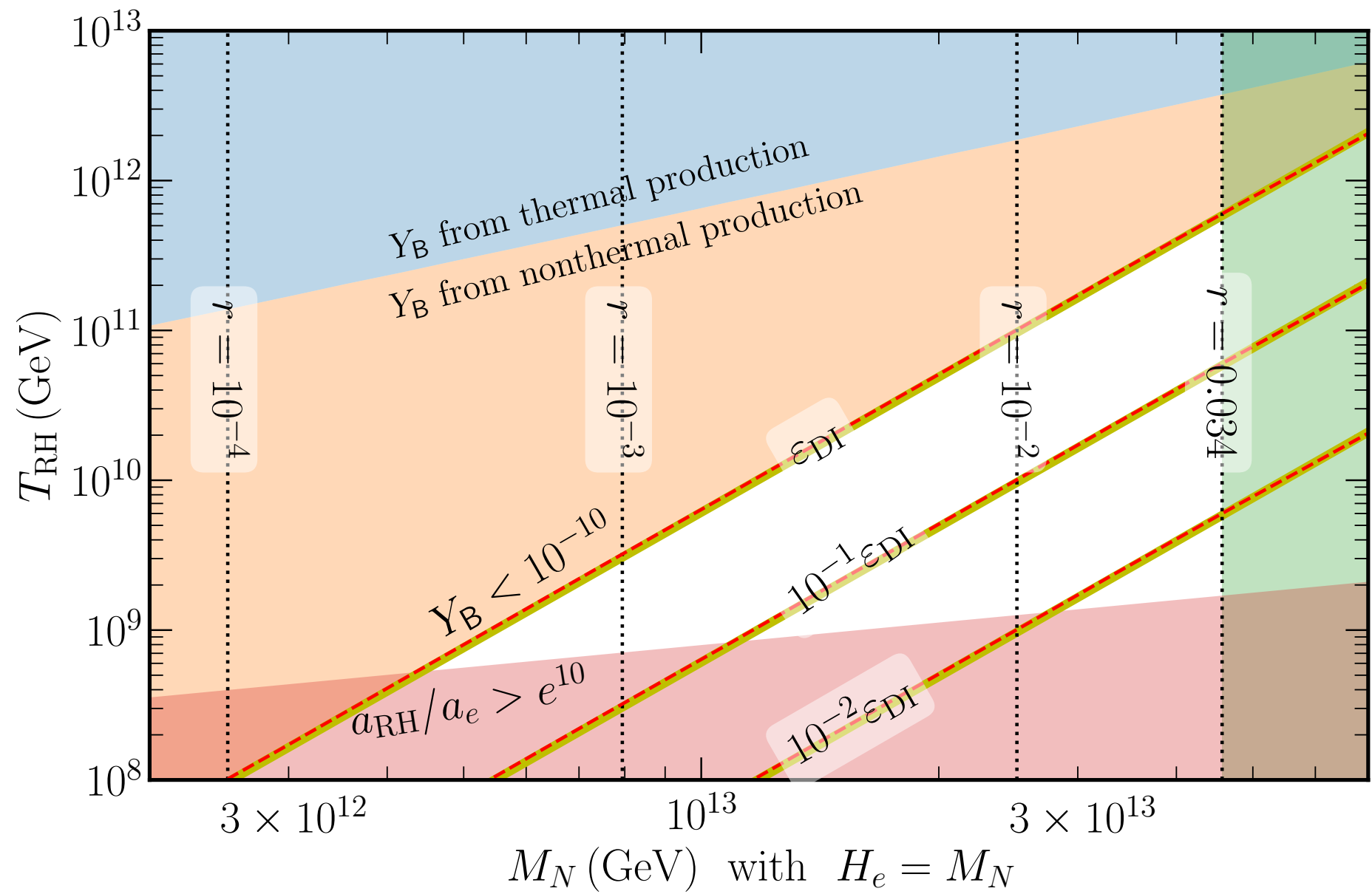
$$\frac{d}{dt}n_L + 3Hn_L = \varepsilon\Gamma(n_N - n_N^{\text{eq}}) - n_L\frac{n_L^{\text{eq}}}{n_\ell}\Gamma - 2n_Ln_N\langle\sigma v\rangle_1 - 4n_Ln_\Phi^{\text{eq}}\langle\sigma v\rangle_2$$

across the viable parameter space, inverse decays & washout are negligible



Main result

$$Y_B = -\frac{28}{79} \varepsilon \frac{T_{RH} H_e}{4M_{Pl}^2} e^{3wN_{RH}} \left(\frac{[a^3 n_N]_{init}}{a_e^3 H_e^3} \right) \simeq (1 \times 10^{-10}) \left(\frac{-\varepsilon}{10^{-1} \varepsilon_{DI}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \left(\frac{H_e}{10^{13} \text{ GeV}} \right)^2 \left(\frac{T_{RH}}{10^9 \text{ GeV}} \right)^{-1} \left(\frac{[a^3 n_N]_{init}/a_e^3 H_e^3}{10^{-3}} \right)$$



Plasma too hot at reheating.
Inverse decay creates N_1 's.
Nonthermal LG not viable.

Kinaton phase is too short.
Reaching $Y_B = Y_B^{obs} = 10^{-10}$
would require $\varepsilon > \varepsilon_{DI}$.

Kinaton phase is too long.
Inflaton field fluctuations
would dominate & terminate
kination after ~ 10 e-foldings.

Eroncel et. al. (2025)

Inflationary GW too strong.

white triangle = just right!

Summary & Discussion

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summary

- A simple & testable scenario for baryogenesis: LG from CGPP
- Some abundance of heavy Majorana neutrinos are invariably produced at the end of inflation due to gravity alone
- Their CPV+OOE decays produce lepton number that leads to a baryon asymmetry
- If inflation is followed by a stiff phase (e.g., kination), the predicted baryon asymmetry can match the observed BAU

discussion

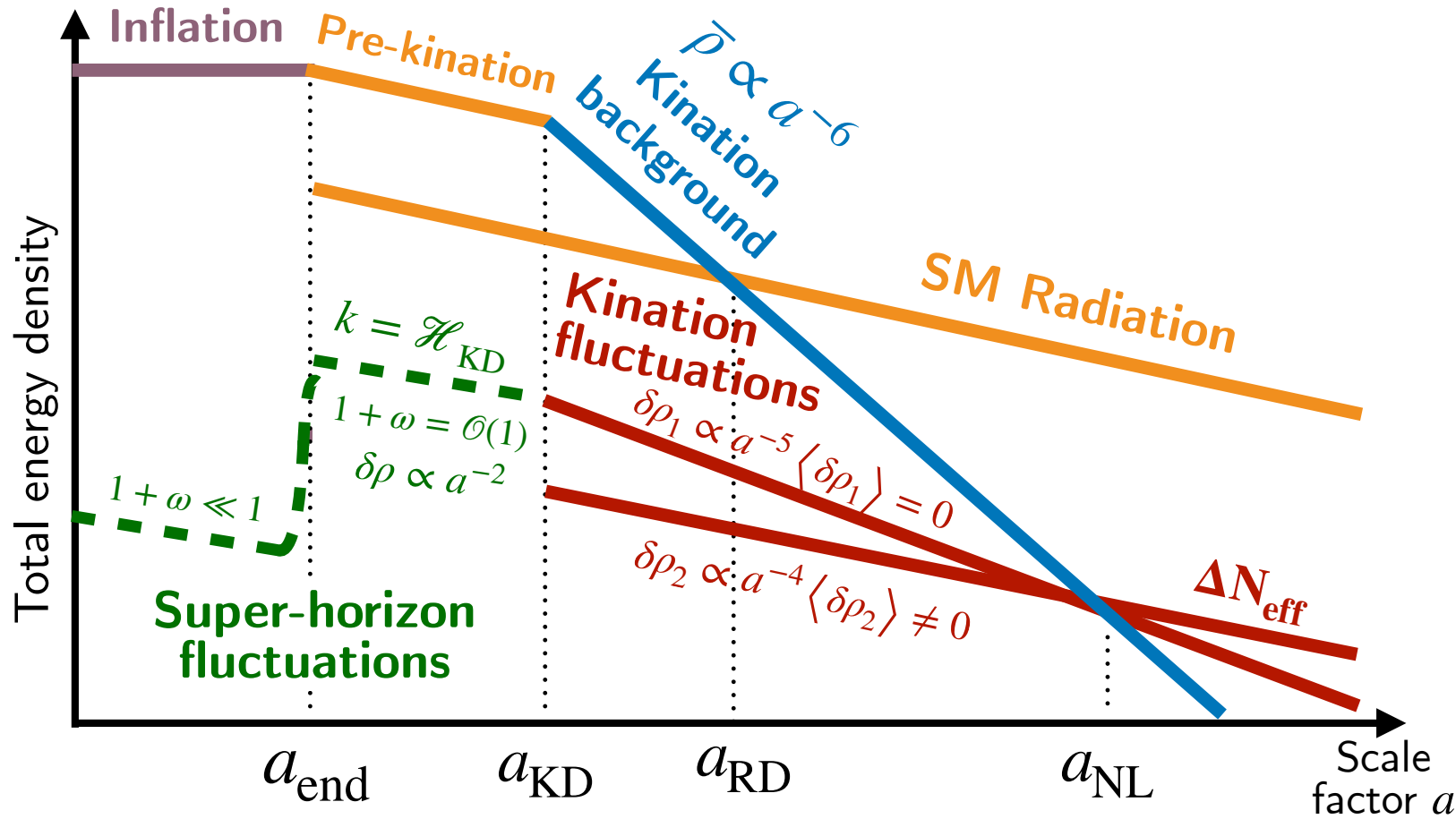
- Since N 's arise gravitationally via CGPP, high-scale inflation is required to achieve a sufficient abundance
- High-scale inflation predicts strong primordial **gravitational wave radiation**
- This baryogenesis scenario brings a welcome element of **falsifiability**
- If future observations push the upper bound on r below 10^{-3} , it would severely constrain our scenario

backup slides



“A universal bound on the duration of a kination era”

Eroncel, Gouttenoire, Sato, Servant, & Simakachorn (2501.17226)



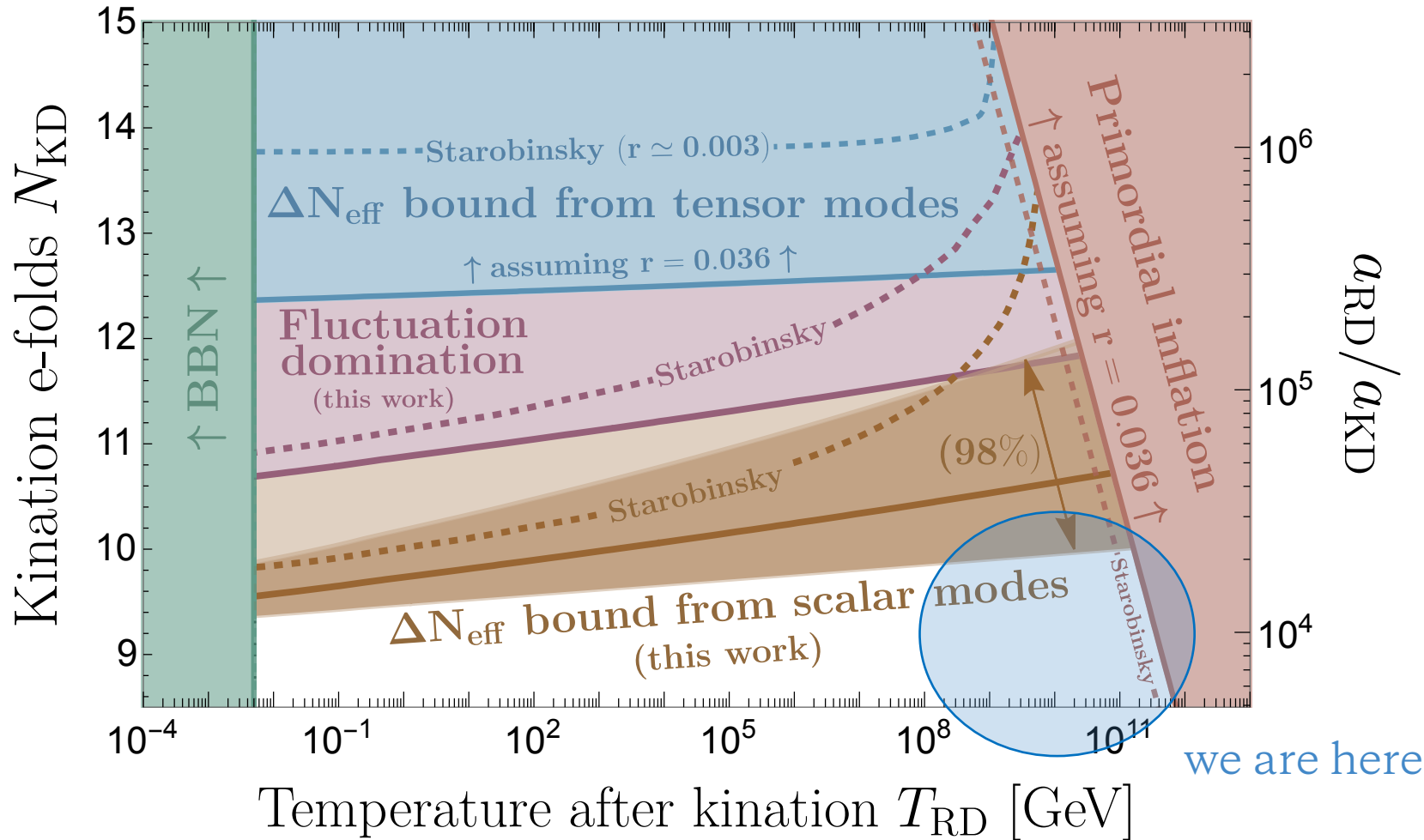
homogeneous inflaton $\sim a^6$
 inflaton fluctuations $\sim a^4$

Fluctuations dominate within ~ 10 e-foldings, and kination gives way to radiation domination.

$$N_{\text{KD}} \lesssim 9.9 + 0.5 \ln \left(\frac{2.099 \times 10^{-9}}{\mathcal{P}_{\mathcal{R}}(k_*)} \right) + 0.5 \ln n_s(k_{\text{KD}}) + 0.5(1 - n_s(k_{\text{KD}})) \ln \left(\frac{k_{\text{KD}}}{k_*} \right)$$

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Davidson-Ibarra bound

Yukawa interaction

$$\mathcal{L}_{\text{int}} = -y_{ij}\epsilon_{ab}\bar{L}_{a,i}N_j\Phi_b^* + \text{h.c.}$$

Decay channels

$$N_1 \rightarrow \begin{cases} L_{a,i}\Phi_b & \Rightarrow \Delta L = +1 \\ \bar{L}_{a,i}\bar{\Phi}_b & \Rightarrow \Delta L = -1 \end{cases}$$

Total decay rate

$$\Gamma = (yy^\dagger)_{11}M_N/8\pi$$

CP-violation parameter

$$\varepsilon \equiv \frac{\sum_{ab}\sum_i[\text{Br}(N_1 \rightarrow L_{a,i}\Phi_b) - \text{Br}(N_1 \rightarrow \bar{L}_{a,i}\bar{\Phi}_b)]}{\sum_{ab}\sum_i[\text{Br}(N_1 \rightarrow L_{a,i}\Phi_b) + \text{Br}(N_1 \rightarrow \bar{L}_{a,i}\bar{\Phi}_b)]}$$

If the spectrum is hierarchical

$$\varepsilon = -\frac{3}{16\pi} \frac{1}{(yy^\dagger)_{11}} \sum_{i=2,3} \text{Im}[(yy^\dagger)_{1i}^2] \frac{M_1}{M_i}$$

Meanwhile, seesaw mass relation

$$m_\nu \sim y^2 v^2 / M_N$$

In combination:

$$|\varepsilon| < \varepsilon_{\text{DI}} \equiv \frac{3}{8\pi} \frac{M_N m_\nu}{v^2} \approx 9.86 \times 10^{-4} \left(\frac{M_N}{10^{13} \text{ GeV}} \right)$$

Hamaguchi, Murayama, & Yanagida (2002)

Davidson & Ibarra (2002)

Buchmuller, Di Bari, & Plumacher (2003)

Hambye, Lin, Notari, Papacci, & Strumia (2003)

CGPP details

a homogeneous & isotropic spacetime

$$(ds)^2 = (dt)^2 - a(t)^2 |d\mathbf{x}|^2$$

expansion is driven by inflaton field

$$\frac{\dot{a}}{a} = \sqrt{\frac{\frac{1}{2}\dot{\phi}^2 + V(\phi)}{3M_{\text{Pl}}^2}}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

inflaton rolls down a potential

$$V(\phi) = \begin{cases} \frac{1}{2}m_\phi^2\phi^2 & \text{for } \phi > 0 \\ 0 & \text{for } \phi < 0 \end{cases}$$

$$\phi_i = 10M_{\text{Pl}}$$

$$m_\phi \approx 2 \times 10^{13} \text{ GeV}$$

$$H_e \approx 1 \times 10^{13} \text{ GeV}$$

QFT in curved spacetime: Majorana spinor

$$S[\Psi(x), e_\mu^a(x)] = \int d^4x e \left[\frac{i}{4} \bar{\Psi} \gamma^\mu \nabla_\mu \Psi - \frac{i}{4} \bar{\Psi} \overleftarrow{\nabla}_\mu \gamma^\mu \Psi - \frac{1}{2} M_N \bar{\Psi} \Psi \right]$$

mode function EOM

$$i \frac{d}{d\eta} \begin{pmatrix} u_{A,k} \\ u_{B,k} \end{pmatrix} = \begin{pmatrix} aM_N & k \\ k & -aM_N \end{pmatrix} \begin{pmatrix} u_{A,k} \\ u_{B,k} \end{pmatrix}$$

$$i \frac{d}{d\eta} \begin{pmatrix} \tilde{\alpha}_k \\ \tilde{\beta}_k \end{pmatrix} = \begin{pmatrix} 0 & -i\mu e^{2i\Phi} \\ i\mu^* e^{-2i\Phi} & 0 \end{pmatrix} \begin{pmatrix} \tilde{\alpha}_k \\ \tilde{\beta}_k \end{pmatrix}$$

$$\mu = \frac{a^2 k H M_N}{2(k^2 + a^2 M_N^2)} \quad \Phi = \int_{-\infty}^{\eta} d\eta' \sqrt{k^2 + a(\eta')^2 M_N^2}$$

average occupation number

$$a^3 n_N = 2 \lim_{\eta \rightarrow \infty} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} |\tilde{\beta}_p(\eta)|^2$$

Boltzmann equations

reactions involving N_1 and leptons

$$\text{decay: } \begin{cases} N_1 \rightarrow L_{a,i} + \Phi_b \\ N_1 \rightarrow \bar{L}_{a,i} + \bar{\Phi}_b \end{cases}$$

$$\text{inverse decay: } \begin{cases} L_{a,i} + \Phi_b \rightarrow N_1 \\ \bar{L}_{a,i} + \bar{\Phi}_b \rightarrow N_1 \end{cases}$$

$$\Delta L = \pm 2 \text{ scattering: } \begin{cases} L_{a,i} + \Phi_b \leftrightarrow \bar{L}_{c,j} + \bar{\Phi}_d \\ L_{a,i} + \bar{L}_{b,j} \leftrightarrow \bar{\Phi}_c + \bar{\Phi}_d \\ \bar{L}_{a,i} + \bar{L}_{b,j} \leftrightarrow \Phi_c + \Phi_d \end{cases}$$

$$\Delta L = \pm 1 \text{ scattering: } \begin{cases} N_1 + L_{a,i} \rightarrow \bar{\Phi}_b^* \rightarrow \text{SM} + \text{SM} \\ N_1 + \bar{L}_{a,i} \rightarrow \Phi_b^* \rightarrow \text{SM} + \text{SM} \end{cases}$$

transport coefficients

we adopted the rates & cross sections from Buchmuller, Di Bari, & Plumacher's Leptogenesis for Pedestrians (hep-ph/0401240)

thermal environment

we suppose that CGPP creates a plasma of SM particles with

$$T_e \approx H_e / 2\pi g_*^{1/4}$$

at the end of inflation

number density of lepton number

$$n_L \supset \sum_{a,i} (n_{L_{a,i}} - n_{\bar{L}_{a,i}})$$

Boltzmann equations

$$\frac{d}{dt} n_N + 3H n_N = -(n_N - n_N^{\text{eq}}) [\Gamma + 2 n_\gamma^{\text{eq}} \langle \sigma v \rangle_1]$$

$$\begin{aligned} \frac{d}{dt} n_L + 3H n_L = & \varepsilon \Gamma (n_N - n_N^{\text{eq}}) - n_L \frac{n_L^{\text{eq}}}{n_\ell} \Gamma \\ & - 2n_L n_N \langle \sigma v \rangle_1 - 4n_L n_\Phi^{\text{eq}} \langle \sigma v \rangle_2 \end{aligned}$$

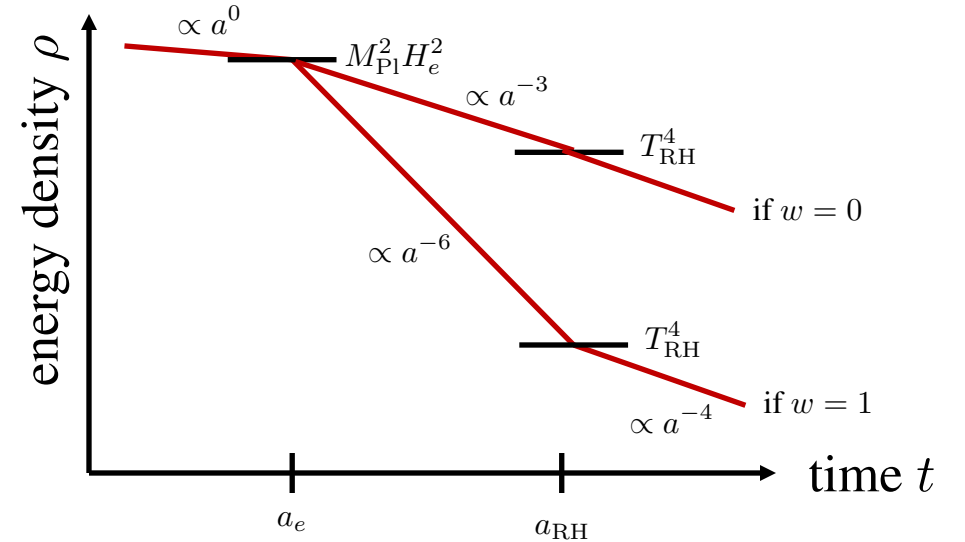
take away

scattering & inverse decay are **negligible** across the viable parameter space (i.e., the white triangle)

Why is the stiff phase needed?

B-to-entropy ratio prediction

$$Y_B = -\frac{28}{79} \epsilon \frac{T_{RH} H_e}{4M_{Pl}^2} e^{3wN_{RH}} \left(\frac{[a^3 n_N]_{init}}{a_e^3 H_e^3} \right)$$



Varying the equation of state between inflation & reheating

$$Y_B \approx \begin{cases} (1.5 \times 10^{-23}) \left(\frac{-\epsilon}{10^{-1} \epsilon_{DI}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \left(\frac{H_e}{10^{13} \text{ GeV}} \right) \left(\frac{T_{RH}}{10^9 \text{ GeV}} \right) \left(\frac{[a^3 n_N]_{init}/a_e^3 H_e^3}{10^{-3}} \right) & , \quad w = 0 \\ (3.9 \times 10^{-17}) \left(\frac{-\epsilon}{10^{-1} \epsilon_{DI}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \left(\frac{H_e}{10^{13} \text{ GeV}} \right)^{3/2} \left(\frac{[a^3 n_N]_{init}/a_e^3 H_e^3}{10^{-3}} \right) & , \quad w = 1/3 \\ (1.8 \times 10^{-13}) \left(\frac{-\epsilon}{10^{-1} \epsilon_{DI}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \left(\frac{H_e}{10^{13} \text{ GeV}} \right)^{9/5} \left(\frac{T_{RH}}{10^9 \text{ GeV}} \right)^{-3/5} \left(\frac{[a^3 n_N]_{init}/a_e^3 H_e^3}{10^{-3}} \right) & , \quad w = 2/3 \\ (1.0 \times 10^{-10}) \left(\frac{-\epsilon}{10^{-1} \epsilon_{DI}} \right) \left(\frac{M_N}{10^{13} \text{ GeV}} \right) \left(\frac{H_e}{10^{13} \text{ GeV}} \right)^2 \left(\frac{T_{RH}}{10^9 \text{ GeV}} \right)^{-1} \left(\frac{[a^3 n_N]_{init}/a_e^3 H_e^3}{10^{-3}} \right) & , \quad w = 1 \end{cases}$$